on radiointerferometry, which made it possible to determine the phase structure and velocity of propagation of waves along the Earth's surface.

During WWII, he developed radio navigation and radar systems for the air force.

In 1951, V V Migulin was appointed Director of the Sukhumi Physical-Technical Institute, where work on the Soviet Atomic project was conducted, including work on rocket design. The research staff of the Institute involved many German scientists brought to the USSR from defeated Germany.

I will make a short detour here. It is likely that many of you knew the German scientist Klaus Thiessen, or heard about him. In 1953–1954, he was a student at Moscow State University. At the time, I was a student at the Moscow Institute of Physics and Technology (FizTekh), and we were acquainted. Well, his father Peter Adolf Thiessen was the person who developed the fuel for the German V-2 rocket. As you know, once WWII ended, the Americans got hold of rocket designer von Braun, and we Soviets got hold of P A Thiessen, Hitler's former science advisor, and this very P A Thiessen, essentially the technical director of the Sukhumi Institute, then worked for Stalin.

Between 1957 and 1959, V V Migulin was the acting deputy to the IAEA Director General in Vienna.

In 1962–1969, V V Migulin headed the Department of Parametric and Electronic Devices at the RAS Institute of Radioengineering (IRE). He appears to be the first person to bring attention in 1968 to applying a physical novelty, SQUIDs, as potential quantum interference sensors of magnetic fields. Our first SQUIDs were designed under his guidance at this Department. He was able to assemble a team of experts known to many of you: V P Koshelets, G A Ovsyannikov, and some others. That is all his scientific school, and today it is to a great extent thanks to V V Migulin and the school he created that the IRE RAS has stayed at the forefront of SQUID development and applications. My subsequent talk on biomagnetic measurements will partly be devoted to applications of SQUIDs possessing extremely high sensitivity.

In 1969, V V Migulin became Director of the RAS Institute of Terrestrial Magnetism, Ionosphere and Radiowave Propagation (IZMIRAN); he headed this Institute for 20 years. V V Migulin made a great contribution to progress in a new field of research — solar-terrestrial physics. He was also a scientific leader of the Interkosmos-19 program; the pioneering results of space exploration which made us all so proud were actually obtained under his guidance.

The spectrum of V V Migulin's interests in science was exceptionally wide. I even remember, although I cannot expand on this topic here, that he once described how in some obscure way he and Zel'dovich together had even discovered somewhere quarks. In addition, Vladimir Vasil'evich never stopped doing work of great social significance. He was Deputy Secretary-Academician of the Physical Sciences Division of the RAS for nearly 30 years. It is fair to say that in reality he shared the management of the Division with the Secretary-Academician.

V V Migulin also took part in the work of international organizations. One such organization was URSI (the French abbreviation of Union Radio-Scientifique Internationale). Even though a large number of international organizations exist in the fields of radiophysics, radio electronics, etc., most of them are applications-oriented. For example, in the long run the IEEE (Institute of Electrical and Electronics Engineers) is precisely the institute for engineers in the fields of radio electronics, electrical engineering, etc., and thus mostly concerns itself with the practical applicability of the results of research. URSI is actually the only organization focusing primarily on fundamental problems. Well, for five years Migulin was the Vice-President of the entire URSI, and for 20 years he headed the URSI Russian Committee. At the moment, I am in fact his legal successor, as I now head the same URSI committee. Consequently, I am well informed about his activities in the international arena as well.

Vladimir Vasil'evich Migulin had an exceptionally wonderful personality. Very important among his characteristic features was the absolute reliability of his word; he was a model workaholic and was always very upset if he was late arriving for anything — his time was planned to the minute, and not only regarding his job but in dealing with people as well. When Vladimir Vasil'evich talked to you, he listened attentively, looking straight at you, and always tried hard to do something and help. Even after resigning from the IRE directorship in 1988, when I became IRE Director, he would be a regular visitor, enquiring about progress in the Department of Superconductor Electronics, which was very much his brainchild; he was always very close to it.

The memory of him is very dear to us, very clear and very warm, and I think most people who knew him feel that way. He was a wonderful person.

PACS numbers: 04.80.Nn, **07.60.–j**, **42.60.–v**
DOI: 10.3367/UFNe.0182.201203g.0324

Parametric oscillatory instability in laser gravitational antennas

S P Vyatchanin

1. Introduction

I had the honor of knowing Vladimir Vasil'evich Migulin when I was an undergraduate student, a postgraduate student, and then a staff member in the chair headed by him. Unfortunately, I did not maintain close contact with him, but I was well aware of the high prestige he enjoyed among the chair staff members.

V V Migulin is known for his work on parametric processes [1–3]. The subject of my report is the undesirable parametric instability effect, which is also inherently parametric.

An obvious illustration of parametric oscillatory instability is the model of a two-circuit parametric amplifier (Fig. 1) which consists of two parallel oscillatory circuits connected with a variable coupling capacitor $C_0(t) = C_0 + \delta C \cos \omega_0 t$ [4]. As is well known, the operation of the parametric amplifier becomes unstable for sufficiently strong pumping (i.e., when the modulation part δC of the coupling capaci-

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S P Vyatchanin M V Lomonosov Moscow State University,
Moscow, Russian Federation
E-mail: svyatchanin@phys.msu.ru

Uspekhi Fizicheskikh Nauk **182** (3) 324 – 327 (2012)
DOI: 10.3367/UFNr.0182.201203g.0324
Translated by E N Ragozin; edited by A Radzig

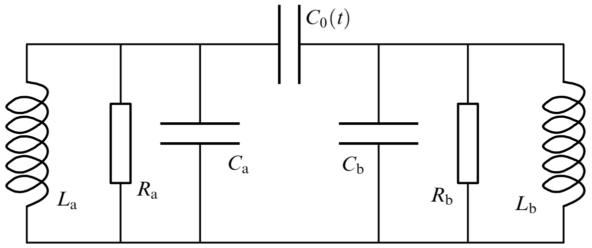

Figure 1. Two-circuit parametric amplifier model. The partial circuit frequencies ω_a, ω_b and the modulation frequency ω_0 of the coupling capacitance $C_0(t) = C_0 + \delta C \cos \omega_0 t$ obey the relationship $\omega_0 \simeq \omega_a + \omega_b$.

tance is large enough). The instability condition for a perfect synchronism (i.e., when $\omega_0 = \omega_a + \omega_b$) has the following form:

$$\frac{\delta C^2}{C_a C_b} > \frac{\gamma_a \gamma_b}{\omega_a \omega_b}\,, \qquad \gamma_a \equiv \frac{R_a}{2L_a}\,, \qquad \gamma_b \equiv \frac{R_b}{2L_b}\,. \tag{1}$$

It is also known that parametric pumping is responsible for the insertion of *antidamping*, and that condition (1) describes a situation where the inserted damping is higher than the intrinsic one [5].

2. Parametric oscillatory instability

To qualitatively consider the phenomenon of parametric oscillatory instability (POI), we address ourselves to the configuration of a Fabry–Perot resonator (Fig. 2) which is excited through resonance pumping at the frequency ω_0. One of the resonator mirrors is mobile and comprises a mechanical oscillator of frequency ω_m. Let there exist a Stokes optical-resonator mode with an eigenfrequency ω_1, so that the following condition is satisfied:

$$\omega_0 \simeq \omega_1 + \omega_m\,. \tag{2}$$

In this case, a parametric interaction between these modes is possible, which may give rise to parametric instability [6–10]. In the presence of small oscillations in the optical Stokes mode, a ponderomotive force emerges which acts on the mobile mirror at the difference frequency $\omega_0 - \omega_1 \simeq \omega_m$, which *resonantly* drives mechanical oscillations. On the other hand, small mechanical oscillations of the mirror owing to the Doppler effect give rise to the mirror-reflected waves with combination frequencies $\omega_0 \pm \omega_m$. One of these waves (with the frequency $\omega_0 - \omega_m \simeq \omega_1$) resonantly excites oscillations in the optical Stokes mode. With increasing pump power at the frequency ω_0, the stated mechanisms evidently result in progressively greater energy transfer. In accordance with the Manley–Rowe relations, the energy of the pump wave will be transferred to the optical Stokes mode and the mechanical mode. This effect may be considered as the insertion of *antidamping*, and therefore parametric instability will set in on attainment of some threshold value of the pump power.

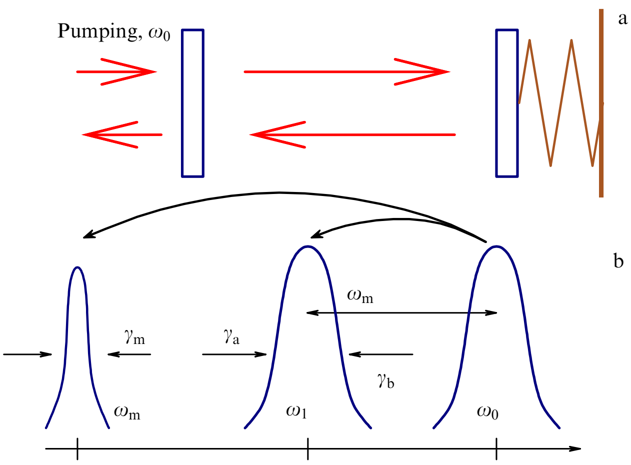

Figure 2. (a) Schematic of a Fabry–Perot resonator, one of its mirrors being mobile and a mechanical oscillator of frequency ω_m. (b) Mode diagram. The arrows indicate energy fluxes emerging in accordance with Manley–Rowe relations.

POI represents a threshold effect, its onset condition being conveniently described by factor $\mathcal{R}$:

$$\mathcal{R} = \frac{W\omega_1}{cLm\omega_m\gamma_1\gamma_m}\,\frac{\Lambda}{1 + (\Delta_1/\gamma_1)^2} > 1\,, \tag{3}$$

$$\Delta_1 = \omega_0 - \omega_1 - \omega_m\,, \tag{4}$$

$$\Lambda = \frac{V_m \left|\int \mathcal{A}_{0\mathrm{in}} \mathcal{A}^*_{1\mathrm{in}} u_\perp \, \mathrm{d}\mathbf{r}_\perp\right|^2}{\int \left|\mathcal{A}_{0\mathrm{in}}(\mathbf{r}_\perp)\right|^2 \mathrm{d}\mathbf{r}_\perp \int \left|\mathcal{A}_{1\mathrm{in}}(\mathbf{r}_\perp)\right|^2 \mathrm{d}\mathbf{r}_\perp \int \left|\mathbf{u}(\mathbf{r})\right|^2 \mathrm{d}\mathbf{r}}\,, \tag{5}$$

where W is the power circulating in the fundamental mode of the Fabry–Perot resonator, γ_1 and γ_m are the damping coefficients of the optical Stokes and elastic modes, L is the distance between Fabry–Perot resonator mirrors, c is the speed of light, m is the mirror mass, Δ_1 is the frequency mismatch, Λ is the overlap factor for the optical fundamental, optical Stokes, and elastic mode distributions, $\mathcal{A}_0$, $\mathcal{A}_1$ are the light field distribution functions over the beam section for the fundamental and Stokes modes, $u_\perp$ is the component of the elastic-mode displacement vector $\mathbf{u}$ perpendicular to the mirror surface, $\mathrm{d}\mathbf{r}_\perp$ corresponds to integration over the surface area of the mirror, and $\mathrm{d}\mathbf{r}$ corresponds to integration over the mirror volume V_m.

When parametric instability condition (3) is fulfilled, the energy $\mathcal{E}_0$ in the fundamental mode at the ω_0 frequency ceases to increase on a further rise in laser pump power W, while the energies of the Stokes and mechanical modes begin to grow [11] (Fig. 3). This growth may have the effect that the Stokes mode will serve as the pump for the excitation of the next appropriate pair of Stokes and mechanical modes. Thus, a cascade development of parametric instability becomes possible.

The POI phenomenon was observed in optical microcavities [12–14] for a modest optical pump power, on the order of 10^{-4} W; this is due to the high Q factors of optical modes (on the order of 10^9) and the small effective mass of mechanical oscillations (about 10^{-10} kg). It is relatively easy to obtain a cascade POI in these microcavities, which makes it possible to generate optical combs [15–18].

It should be emphasized that an appropriate optical *anti-Stokes* mode with a frequency $\omega_{1a} \simeq \omega_0 + \omega_m$ may also exist in the resonator. Then, in accordance with the Manley–Rowe relations, the pumping will insert *positive* damping into the mechanical mode, which may exceed the negative friction inserted by the Stokes mode and make parametric instability impossible [19]. In this case, a photon of the pump wave will be scattered by the elastic phonon to produce a photon of the anti-Stokes wave, while a part of the energy will be borrowed from the elastic wave. And so, on the one hand, the presence

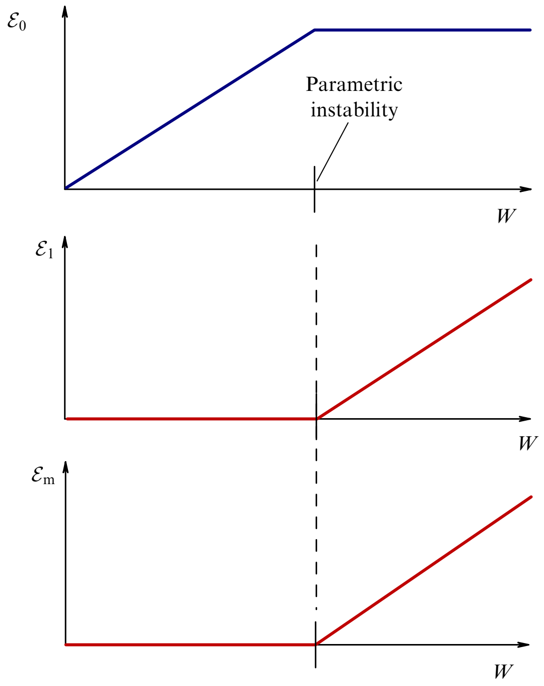

Figure 3. Respective energies $\mathcal{E}_0$, $\mathcal{E}_1$, and $\mathcal{E}_\mathrm{m}$ of the fundamental, Stokes, and mechanical modes as functions of pump power W below and above the parametric instability threshold.

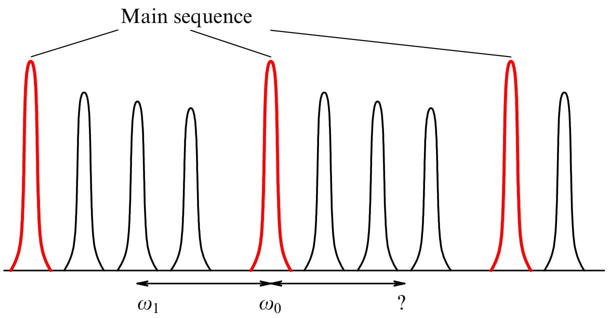

Figure 4. Structure of the optical modes of a Fabry–Perot resonator. The modes of the main sequence correspond to the high peaks. The optical Stokes mode has a frequency ω_1, and the possible anti-Stokes mode is indicated with a question mark.

of the optical Stokes mode is responsible for the insertion of antidamping into the mechanical mode (and therefore for the parametric instability effect), and, on the other hand, the presence of the anti-Stokes mode brings about the mechanical mode damping. However, the probability that the anti-Stokes mode suppresses the parametric instability completely is low enough, as illustrated by Fig. 4.

Parametric instability in laser interference antennas. To date, several laboratories in different countries have demonstrated the operation of first-generation gravitational wave detectors (the Laser Interferometer Gravitational Wave Observatory (LIGO) [20, 21], VIRGO [22], GEO-600 [23], and TAMA [24] projects), and efforts are underway by now to create second-generation detectors (Advanced LIGO (AdLIGO), Advanced VIRGO, GEO-HF, etc.), which will make it possible to detect gravitational waves in the near future. It is planned to substantially increase in the second-generation detectors the optical power circulating in the interferometer arms (up to 800 kW in AdLIGO). Therefore, the probability of developing POI in the second-generation laser gravitational detectors will be high enough.

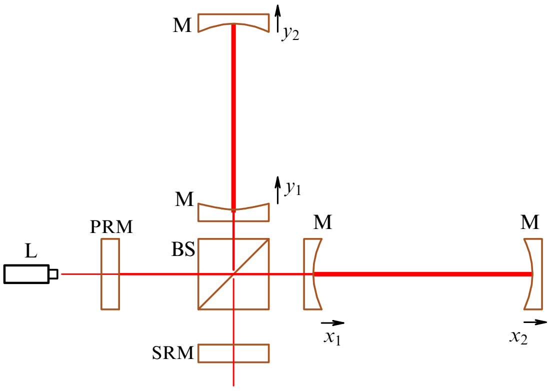

Figure 5. Simplified AdLIGO configuration: L — laser, BS — beam splitter, M — mirrors in the interferometer arms, PRM — power recycling mirror, SRM — signal recycling mirror. The light passing through the dark port (through the SRM) carries information about the difference in length between the interferometer arms $[(x_2 - x_1) - (y_2 - y_1)]$.

A simplified schematic representation of the AdLIGO interferometer is given in Fig. 5. Two cylindrically shaped mirrors suspended at a long distance (4 km) from each other make up the first Fabry–Perot resonator in one interferometer arm; two other mirrors, the same as the two first, make up another arm with the second Fabry–Perot resonator, which is perpendicular to the first one. The laser beam of a pump laser passes through a beam splitter located at the arms intersection point. The light is assumed to experience multiple reflections from the mirrors inside each of the arms prior to its return to the beam splitter. The interferometer arms are so aligned that all reflected light is directed back to the laser (the so-called bright port), but the light does not arrive at a detector port (the so-called dark port). Some difference in arm lengths appears under the action of a gravitational wave, and a part of the light travels to the dark port and is recorded by a photodetector. The AdLIGO is targeted to attain the parameter values collected in Table 1.

Table 1. Parameters of the AdLIGO interferometer.

Power	0.83×10^6 W
Arm length	4000 m
Mirror mass	40 kg
Mirror radius	0.17 m
Mirror height	0.2 m
Material	Fused silica

The influence exerted by a gravitational wave, which gives rise to the difference in arm lengths, is extremely small: the AdLIGO interferometer is supposed to measure a difference in displacements on the order of $10^{-17} - 10^{-16}$ cm in a time of $\sim 10^{-2}$ s for an arm length of 4 km. Detection of so small a displacement implies extremely small mirror displacements caused by other factors — thermal, seismic, and technical noise. Even minor mirror displacements alter the phases of the beams that arrive at the beam splitter, thereby simulating the detection of a gravitational wave.

To predict unstable mode combinations requires complete information both about elastic modes and about optical Stokes modes. The frequencies and optical field distributions over the mirror surfaces are easy to calculate analytically for the Gaussian modes of a Fabry–Perot resonator [25], while the frequencies and displacement vector distributions for elastic modes can be determined only by numerical techniques. (The superposition technique is employed in application only to the axially symmetric elastic modes of a cylindrical mirror [26].) One can see from condition (3) that the uncertainties in calculating the eigenfrequencies of elastic modes must be smaller than the value of the damping coefficient of the optical Stokes mode. For typical mechanical frequencies ranging from 10 kHz to 100 kHz and an optical relaxation time corresponding to a 10–100 Hz frequency interval, this signifies that the relative uncertainty of numerical calculations of elastic mode frequencies should be within 10^{-4}. This requirement is not always fulfilled in the calculation of elastic modes using, for instance, the COMSOL numerical code package. It is also noteworthy that the nonuniformities in the distribution of the density and Young modulus of the mirror material may give rise to an elastic mode frequency shift at a level of 10^{-3}. For instance, with the use of the ANSYS package to numerically solve grid problems, the accuracy of elastic mode calculations was about 0.5% [8–10, 27]. Therefore, the existing precision of elastic mode calculations is insufficiently high.

To this we can add the uncertainty introduced by the material inhomogeneity of the mirrors; for fused silica, for instance, the relative density variations are at a level of $\delta\rho/\rho \approx 10^{-3}$.

3. Conclusions

So, let us list the possible ways of avoiding parametric instability in second-generation laser gravitational antennas.

First, there is good reason to search for precursors, i.e., discover weak oscillations at Stokes and mechanical frequencies. This will permit inserting noiseless damping into the acoustic modes or change the spectrum of optical modes.

Second, since the theoretical and numerical analyses are insufficient, it is necessary to observe parametric instability in a real antenna. This will enable working out different techniques of suppressing the instability.

The third way is to lower the optical power circulating in interferometer arms: second-generation gravitational antennas are intended to attain the accuracy of the standard quantum limit [5] at frequencies of about 100 Hz. This frequency is determined by the high quality of isolation from seismic noise. It is significant that the requisite optical power is proportional to the *cube* of the frequency. If attempts to go over from the 100 Hz frequency to the 30 Hz frequency meet with success, the circulating power will become much lower: it would suffice to have about 20 kW instead of 800 kW. Furthermore, the gravitational radiation intensity is higher in a lower frequency range, according to predictions.

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PACS numbers: 91.25.Mf, **94.30.−d**, **96.60.−j**
DOI: 10.3367/UFNe.0182.201203h.0327

Solar-terrestrial physics and its applications

V D Kuznetsov

1. Introduction

The N V Pushkov Institute of Terrestrial Magnetism, Ionosphere and Radio Wave Propagation of the RAS (IZMIRAN in *Russ. abbr.*), which Vladimir Vasil'evich Migulin led for nearly 20 years (from 1969 to 1988), is a multidisciplinary institute. It embraces different realms of physics: astronomy and astrophysics, geophysics, radiophysics and plasma physics, nuclear physics, etc. Solar–terrestrial physics, which studies processes in the Sun–Earth system, is one of IZMIRAN's main lines of inquiry. V V Migulin, as IZMIRAN's Director and Chair of the Scientific Council on the Problem of Solar–Terrestrial Relations of the RAS (the Sun–Earth Council), for many years was involved in solar–terrestrial physics and supervised a number of programs and projects.

Along with investigations into basic phenomena and processes in the Sun–Earth system, the solar–terrestrial physics of our days has many practical applications related

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V D Kuznetsov N V Pushkov Institute of Terrestrial Magnetism, Ionosphere and Radio Wave Propagation, Russian Academy of Sciences, Troitsk, Moscow region, Russian Federation. E-mail: kvd@izmiran.ru

Uspekhi Fizicheskikh Nauk **182** (3) 327 – 336 (2012)
DOI: 10.3367/UFNr.0182.201203h.0327
Translated by E N Ragozin; edited by A Radzig