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A D SAKHAROV'S 90TH BIRTHDAY COMMEMORATION

Cosmological Sakharov oscillations and quantum mechanics of the early Universe

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Contents

Abstract. **This is a brief summary of a talk delivered at the Special Session of the Physical Sciences Division of the Russian Academy of Sciences, Moscow, 25 May 2011. The meeting was devoted to the 90th anniversary of the birth of A D Sakharov. The focus of this contribution is on the standing-wave pattern of quantum-mechanically generated metric (gravitational field) perturbations as the origin of subsequent Sakharov oscillations in the matter power spectrum. Other related phenomena, particularly in the area of gravitational waves, and their observational significance are also discussed.**

1. Sakharov's first cosmological paper

The ideas and results of Andrei Sakharov's remarkable paper [1] have influenced the course of cosmological research and are still at the center of theoretical and observational studies. The title of his paper was "The initial stage of an expanding universe and the appearance of a nonuniform distribution of matter." The paper was submitted to JETP on 2 March 1965, that is, in the days when not only the existence of the cosmic microwave background radiation (CMB) was not yet established, but even the nonstationarity of the Universe was still debated. The second sentence of the abstract says: "It is assumed that the initial inhomogeneities arise as a result of quantum fluctuations of cold baryon–lepton matter at densities of the order of 10^{98} baryons/cm^3. It is suggested that at such densities gravitational effects are of decisive importance in the equation of state...."

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In what follows, we discuss recent attempts to explain the appearance of cosmological perturbations (density inhomogeneities, gravitational waves, and possibly rotational perturbations) as a result of quantum processes. In our approach, the perturbations arise as a consequence of superadiabatic (parametric) amplification of quantum mechanical fluctuations of the appropriate degrees of freedom of the gravitational field itself. For us, therefore, gravity is of decisive importance not so much because of its contribution to the equation of state of primeval matter but because the gravitational field (metric) perturbations are the primary object of quantization. Nevertheless, it must be stressed that the mind-boggling idea suggesting that something microscopic and quantum mechanical can be responsible for the emergence of fields and observed structures at astronomical scales was first formulated and partially explored in Sakharov's paper.

A considerable part of paper [1] is devoted to the evolution of small density perturbations, rather than to their origin. The spatial Fourier component of the relative density perturbation is denoted as $z_\kappa(t)$, where κ is the wavenumber. The function $z_\kappa(t)$ satisfies a second-order differential equation, numbered in the paper as Eqn (15), which follows from the perturbed Einstein equations. The calculation leading to the phenomenon that was later named Sakharov oscillations is introduced by the following words [1, p. 350]:

Yu.M. Shustov and V. A. Tarasov have at our request solved Eqn (15), *with the aid of an electronic computer, for different values of* κ. *The calculations were made for the simplest equation of state, satisfying* $\varepsilon = nM$ *with* $n^{1/3} \ll M$ *and* $\varepsilon = An^{4/3}$ *with* $n^{1/3} \gg M$ *(A is a constant* ~ 1)

$$\varepsilon = n(M^2 + A^2 n^{2/3})^{1/2} \qquad (16).$$

In [1], the quantity, $n = 1/a^3(t)$ is the particle number density, ε is the energy density in the rest frame of matter, and $p = n\,\mathrm{d}\varepsilon/\mathrm{d}n - \varepsilon$ is the pressure. Obviously, interpolating formula (16) describes the transition from the relativistic equation of state $p = \varepsilon/3$, applicable at early times of evolution and relatively large n, to the nonrelativistic equation of state $p = 0$, valid at small n and late times. During the transition, the speed of sound decreases from $c_s = c/\sqrt{3}$ to $c_s = 0$.

It is important to realize that the physical nature of the discussed transition from $p = \varepsilon/3$ to $p = 0$ can be quite general. Being guided by the physical assumptions of his time, Sakharov speaks about cold baryon–lepton matter, the degenerate Fermi gas of relativistic noninteracting particles, and so on. But it is important to remember that the perturbed Einstein equations, such as Eqn (15), do not require knowledge of the microscopic causes of elasticity and the associated speed of sound. Gravitational equations operate with the energy–momentum tensor of matter and its bulk mechanical properties, such as the energy density, pressure, and the link between them, the equation of state. These are postulated by Eqn (16), and we can now think of the results of the performed calculation as a qualitative model of what can occur in other transitions, for example, in the transition from a fluid dominated by a photon gas with the equation of state $p = \varepsilon/3$ to a fluid dominated by cold dark matter (CDM) with the equation of state $p = 0$.

For simple models of matter, such as $\varepsilon = n^\gamma$ and $p = (\gamma - 1)\varepsilon$, Eqn (15) can be solved in elementary functions. Sakharov writes:

When $\gamma =$ const, the solution of this equation is expressed in terms of Bessel functions; for example, when $\gamma = 4/3$ we have increasing and decreasing solutions of the form ($\theta \sim t^{1/2}\kappa$)

$$z \propto \begin{cases} \cos\theta - \theta^{-1}\sin\theta\,, \\ \sin\theta + \theta^{-1}\cos\theta\,. \end{cases}$$

Indeed, these are the well-known solutions for $z_\kappa(t)$ in the $p = \varepsilon/3$ medium. The general solution of Eqn (15) is a linear combination of these two branches with arbitrary (in general, complex) coefficients. The first solution can be called increasing and the second decreasing because they respectively behave as θ^2 and θ^{-1} at very small θ. At later times, not long before the transition to the $p = 0$ stage, the functions $z_\kappa(t)$ represent ordinary acoustic waves with the oscillatory time dependence $\cos\theta$ and $\sin\theta$. We do not learn anything new from matching the increasing/decreasing part of the solution to the oscillatory part of the same solution; the general solution is already given by the formula above. At the $p = 0$ stage, the solutions for $z_\kappa(t)$ do not oscillate as functions of time; they are power-law functions of t.

The crucial observation in Sakharov's paper is contained in the following quotation:

The function $a(t)$ can be obtained in the case of Eqn (16) *analytically* (*Shustov*). *Shustov and Tarasov find, by integrating* (15), *the limiting value as $t \to \infty$ of the auxiliary variable*

$$\zeta = z\left(1 + \frac{a^2M^2}{A^2}\right)^{-1/2},$$

putting $\mathrm{d}\zeta/\mathrm{d}t = \mathrm{d}z/\mathrm{d}t \sim z_0$ *as* $t \to 0$. *It is obvious that* $\zeta(\infty) \propto z_0 B$.

In accordance with the results of the sections that follow, we put $z_0 \sim \kappa$. $\zeta(\infty)$ is a function of the parameter $A^{1/2}\kappa$. This function is oscillating and sign-alternating, but attenuates rapidly with increasing κ.

The last sentence of this quotation is a surprising statement of incredible importance. It says that well after the transition to the $p = 0$ stage ($t \to \infty$), the density fluctuation $z_\kappa(t)$ becomes an oscillating and sign-alternating function of the wavenumber κ. The square of this function is what can be called the power spectrum. Sakharov uses $z_\kappa(t) = z_{0\kappa}/\dot{a}^2$ at the very early times and takes $z_{0\kappa}$ as $z_0 \sim \kappa$ from his quantum mechanical considerations. It is therefore stated that the initial smooth power spectrum $z_{0\kappa}^2$ transforms into an oscillatory final power spectrum that has a series of zeros and maxima at some specific wavenumbers κ. If we imagine that in the era before the transition to the $p = 0$ stage, the field of sound waves was represented by a set of harmonic oscillators with different frequencies, then the claim is that well after the transition, some oscillators find themselves 'lucky', in the sense that they occur at the maxima of the resulting power spectrum, while others are 'unlucky', because they are at the zeros of the resulting power spectrum.

Certainly, such a striking conclusion cannot be unconditionally true. After all, a computer can be asked to make a similar calculation, but backwards in time. In this calculation, we can postulate a smooth power spectrum at the late $p = 0$ stage and evolve the spectrum back in time to derive the functions $z_\kappa(t)$ at the early $p = \varepsilon/3$ stage. The derived functions do not coincide with what was taken as the initial conditions in the original calculation [1], but such new initial conditions are possible in principle. By construction, these new initial conditions would not lead to the final power-spectrum oscillations. On the other hand, if the oscillations do arise from physically justified initial conditions, then this is an extremely important phenomenon. It dictates the appearance of a periodic structure in the Fourier space (a 'standard ruler' with characteristic spatial scales), which can be recognized in observations and can be used as a tool for other measurements.

The point of this remark is to stress that, as is argued below in more detail, the initial conditions leading to the Sakharov oscillations are inevitable if the primordial cosmological perturbations were indeed generated quantum mechanically.

The oscillatory transfer function $B(\kappa)$ participates in further calculations in [1], but it plays quite a modest role there. Sakharov himself did not elaborate on the discovered phenomenon in later publications. However, it seems to me that he was perfectly well aware of the importance of his observation, and he attentively followed subsequent developments. Some evidence for this is given in Section 4.

It was Ya B Zeldovich who assigned significant value to the discovered oscillations and named them the Sakharov oscillations. In conversations, at seminars, in papers with R A Sunyaev and A G Doroshkevich, and in a book with I D Novikov, Zeldovich discussed the physics of the phenomenon and its possible observational applications. Zeldovich and coauthors deserve credit for seeing the relevance of Sakharov's work for their own studies and for mentioning his paper. For example, one of the first papers on the subject in the context of a 'hot' model of the Universe [2] remarks: "at a later stage of expansion the amplitude of density perturbations turns out to be a periodic function of a wavelength (mass). Such a picture was previously obtained by Sakharov (1965) for a cold model of the Universe." And further [2]: "The picture presented above is only a rough approximation since the phase relations between density and velocity perturbations in standing waves in an ionized plasma were not considered. As mentioned in the introduction, Sakharov (1965) showed that the amplitude of perturbations of matter at a later stage when pressure does not play a role (in our case after recombination) turns out to be a periodic function of wavelength." Zeldovich and Novikov [3] discuss the phenomenon at some length and note that "The distribution of astronomical objects with respect to mass will thus reflect the Sakharov oscillations in a very smoothed-out form only. It is possible that they may not be noticed in a study of the mass spectrum." Fortunately, as we see below, there was significant observational progress in revealing Sakharov oscillations.

In the more detailed paper by Peebles and Yu [4], which paralleled [2], a modulated spectrum, with maxima and zeros, is explicitly presented in Fig. 5 and the relevance of the "first big peak in Fig. 5" for future experimental searches for irregularities in the microwave background radiation is noted. The spectral modulation was derived as a result of numerical calculations. Later private correspondence on the physical interpretation of oscillations inevitably ended up with lucky and unlucky oscillators [5]: "The Sakharov oscillations you mention also were considered by Jer Yu and me (a few years after Sakharov)..... Here there truly are modes that are unlucky, in the sense that they carry negligible energy."

To better understand the Sakharov oscillations, as well as other closely related phenomena, we have to make some formalization of the problem. We do this in the next section. Before that, it is interesting to note as a side remark that in his quantum mechanical considerations, Sakharov discusses the "initial stage of the expansion of the universe," and in particular with the scale factor $a = \exp(\lambda t)$ as $t \to -\infty$. He found this evolution in two cases, c and d, out of the four considered. A scale factor of this type is now advertized as inflation. However, Sakharov himself was sceptical about cases c and d. He finds arguments against them and concludes: "For these reasons we turn to curves a and b." (Criticism of contemporary inflationary claims can be found in [6, 7].)

2. Wave fields of different natures in time-dependent environments

The main physical reason behind Sakharov oscillations, and indeed behind many other similar phenomena, is the time dependence of the parameters characterizing the environment in which a wave field is given. This can be a changing speed of sound, or a changing background gravitational field, or all such factors together. In cosmology, the central object is the gravitational field (metric) perturbations. Other quantities, such as fluctuations of the density and velocity of matter (if they are present; we recall that they are absent in the case of gravitational waves), are calculable from the metric perturbations via the perturbed Einstein equations. Only in special conditions and for relatively short-scale variations can the gravitational field perturbations be neglected.

The gravitational field perturbation h_{ij} is defined by

$$\begin{aligned} \mathrm{d}s^2 &= -c^2\,\mathrm{d}t^2 + a^2(t)(\delta_{ij} + h_{ij})\,\mathrm{d}x^i\,\mathrm{d}x^j \\ &= a^2(\eta)\left[-\,\mathrm{d}\eta^2 + (\delta_{ij} + h_{ij})\,\mathrm{d}x^i\,\mathrm{d}x^j\right]. \end{aligned} \tag{1}$$

For each of the three types of cosmological perturbations (density perturbations, gravitational waves, and rotational perturbations), the field h_{ij} can be expanded in spatial Fourier modes with wave vectors $\mathbf{n}$:

$$\begin{aligned} h_{ij}(\eta,\mathbf{x}) &= \frac{\mathcal{C}}{(2\pi)^{3/2}} \int_{-\infty}^{\infty} \mathrm{d}^3\mathbf{n} \sum_{s=1,2} p_{ij}^s(\mathbf{n})\,\frac{1}{\sqrt{2n}} \\ &\times \left[h_n^s(\eta)\exp(\mathrm{i}\mathbf{n}\mathbf{x})\,c_{\mathbf{n}}^s + h_n^{s*}(\eta)\exp(-\mathrm{i}\mathbf{n}\mathbf{x})\,c_{\mathbf{n}}^{s\dagger}\right]. \end{aligned} \tag{2}$$

The power spectrum (variance) of a given field is a quadratic combination of the field averaged over space, or over the known classical probability density function, or over the known quantum mechanical state. In all cases, we arrive at an expression of the structure

$$\langle 0|\,h_{ij}(\eta,\mathbf{x})\,h^{ij}(\eta,\mathbf{x})|0\rangle = \frac{\mathcal{C}^2}{2\pi^2}\int_0^{\infty} n^2 \sum_{s=1,2} \left|\,h_n^s(\eta)\right|^2 \frac{\mathrm{d}n}{n}. \tag{3}$$

The quantity

$$h^2(n,\eta) = \frac{\mathcal{C}^2}{2\pi^2}\,n^2 \sum_{s=1,2} \left|h_n^s(\eta)\right|^2 \tag{4}$$

is called the metric power spectrum. At each instant of time, the metric power spectrum is determined by the absolute value of the gravitational mode functions $h_n^s(\eta)$ (in general, complex). (We often suppress the index $s = 1, 2$, which marks two polarization states present in metric perturbations of each type of cosmological perturbations.) To calculate power spectra of other quantities participating in the problem, we have to expand these quantities as in Eqn (2) and then use their mode functions in expressions for their power spectra, similar to Eqn (4).

The gravitational mode functions $h_n^s(\eta)$, as well as mode functions of other quantities participating in our problem, satisfy one version or another of the second-order differential 'master equation' [8]

$$f'' + f\left[n^2\,\frac{c_s^2}{c^2} - W(\eta)\right] = 0\,, \tag{5}$$

where the 'speed of sound' c_s and the 'potential' $W(\eta)$ are functions of time in general. In particular, the Sakharov mode functions $z_\kappa(t)$ for density perturbations obey a specific equation of this kind (written in the t time). And the above-quoted Sakharov solution, for $\gamma = 4/3$, expressed in terms of Bessel functions with the argument θ, is a particular case where $c_s = c/\sqrt{3}$, whereas $W(\eta)$ is a simple function of the scale factor $a(\eta)$. Gravitational wave equations are also equations of this form with $c_s = c$.

Two linearly independent high-frequency solutions (i.e., solutions of master equation (5) without $W(\eta)$ and with $c_s = \mathrm{const}$) are usually taken as $f_n(\eta) = \exp\left[\pm\,\mathrm{i}n(c_s/c)\,\eta\right]$. If these mode functions $f_n(\eta)$ represent sound waves not long before the transition to the $p = 0$ stage, then using them for calculating the power spectrum would lead to $|f_n|^2 = 1$ and hence to the absence of oscillations in the power spectrum of density perturbations. Therefore, we do not expect any segregation into lucky and unlucky oscillators in the post-transition era. The general decomposition (2) should be considered more thoroughly.

The general high-frequency solution of Eqn (5) (for simplicity, we temporarily set $c_s/c = 1$) is given by $f_n(\eta) = A_n\exp(-\mathrm{i}n\eta) + B_n\exp(\mathrm{i}n\eta)$, where complex coefficients A_n and B_n are in general arbitrary functions of n. The $\mathbf{n}$-mode of the field

$$h_{\mathbf{n}}(\eta,\mathbf{x}) = f_n(\eta)\exp(\mathrm{i}\mathbf{n}\mathbf{x}) + f_n^*(\eta)\exp(-\mathrm{i}\mathbf{n}\mathbf{x})$$

is a sum of two waves traveling in opposite directions with arbitrary amplitudes and arbitrary phases. One particular traveling wave is chosen by setting $|A_n| = 0$ or $|B_n| = 0$. By contrast, the choice $|A_n| = |B_n|$ makes the field a standing wave, that is, the product of a function of η and a function of $\mathbf{nx}$:

$$h_{\mathbf{n}}(\eta, \mathbf{x}) = 4\rho_A \cos\left(n\eta + \frac{\phi_B - \phi_A}{2}\right)\cos\left(\mathbf{nx} + \frac{\phi_B + \phi_A}{2}\right),$$

where we use $A_n = \rho_{A_n}\exp(\mathrm{i}\phi_{A_n})$, $B_n = \rho_{B_n}\exp(\mathrm{i}\phi_{B_n})$ without the label n.

The power spectrum of the general solution is

$$|f_n|^2 = \rho_A^2 + \rho_B^2 + 2\rho_A\rho_B\cos(2n\eta + \phi_B - \phi_A)\,.$$

Clearly, for a given time instant η, the spectrum is a modulated function of n. For the modulation to take the form of strictly periodic oscillations, the phase $\phi_B - \phi_A$ must be a linear function of n. The oscillations vanish for traveling waves and have the maximal depth, up to the appearance of zeros, for standing waves. In principle, ρ_A and ρ_B could themselves be complicated functions of n, but for the moment we do not consider this possibility.

In Fig. 1, for illustration, we show a model spectrum $h^2(n,\eta) = \sin^2[n(\eta - \eta_e)]$ ($\eta_e = \mathrm{const}$) plotted for a discrete set of wavenumbers n. The zeros in the spectrum, marked by blue stars, move and proliferate in the course of time, in the sense that they gradually arise at new frequencies, and the distance between them decreases. The moving zeros and moving maxima are inherited and fixed (possibly with a phase shift) in the spectrum at the $p = 0$ stage after the transition.

Indeed, the general solution of Eqn (5) after the transition is $f_n(\eta) = C_n + D_n\eta$. The coefficients C_n and D_n then become oscillating functions of n. The moving features become fixed features at some particular wavenumbers, thus defining the lucky and unlucky oscillators. If the transition can be approximated as a sharp event occurring at some η_{eq}, then by matching the general solutions for the function $f_n(\eta)$ and its first time derivative $f_n'(\eta)$ at $\eta = \eta_{\mathrm{eq}}$, we find the coefficient in the increasing solution as

$$|D_n|^2 = n^2\left(\frac{c_s}{c}\right)^2 \times\left[\rho_A^2 + \rho_B^2 - 2\rho_A\rho_B\cos\left(2n\left(\frac{c_s}{c}\right)\eta_{\mathrm{eq}} + \phi_B - \phi_A\right)\right].$$

Obviously, there are no final spectrum modulations if the incoming field consists of traveling waves ($\rho_A = 0$ or $\rho_B = 0$), and the modulations have maximal depth if the waves are standing ($\rho_A = \rho_B$). The relevant set of maxima is determined by the set of n where the function $\sin^2[(c_s/c)\,n\eta_{\mathrm{eq}} + (\phi_B - \phi_A)/2]$ has a maximum, starting from $(c_s/c)\,n\eta_{\mathrm{eq}} + (\phi_B - \phi_A)/2 = \pi/2$. The smallest n and hence the largest spatial scale $\lambda = 2\pi a(\eta)/n$ is expected to be the most pronounced observationally. For such long wavelengths, metric perturbations cannot be neglected in general. We note that if the $p = 0$ post-transition medium is CDM, then there must be oscillations in the CDM power spectrum.

It follows that only a very high degree of organization of the field before the transition — standing waves with phases proportional to n — can lead to the emergence of periodic Sakharov oscillations in the post-transition pressureless matter and in the associated metric perturbations.

The power spectra of cosmological fields in the recombination era determined the angular power spectrum of the cosmic microwave background anisotropies observed today. The CMB spectra differ greatly depending on whether the perturbations are realized as traveling or standing waves. This is best illustrated with the help of gravitational waves, in which case only gravity is involved, and hence we should not worry about the 'acoustic physics' and the role of various matter components. The decoupling of photons from baryons at the last-scattering surface $\eta = \eta_{\mathrm{dec}}$ has no effect on gravitational waves themselves, but for the photons it is very important in which gravitational field they start their journey and propagate.

In Fig. 2, we show two power spectra of gravitational waves given at $\eta = \eta_{\mathrm{dec}}$ and two corresponding CMB temperature spectra caused by them (more details are given in [8]). The red (wavy) line describes the physical spectrum

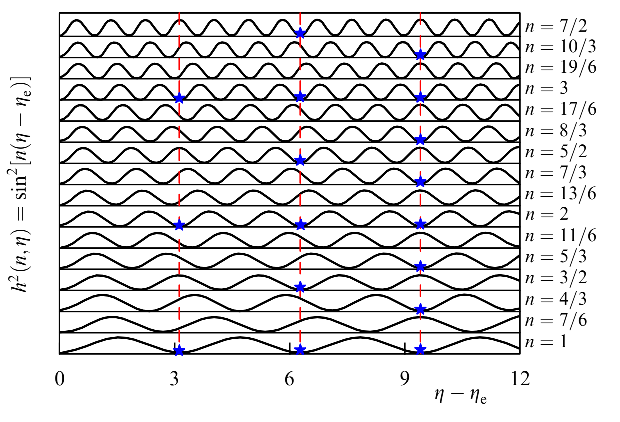

Figure 1. A model spectrum of the pre-transition wave field with moving (proliferating) zeros.

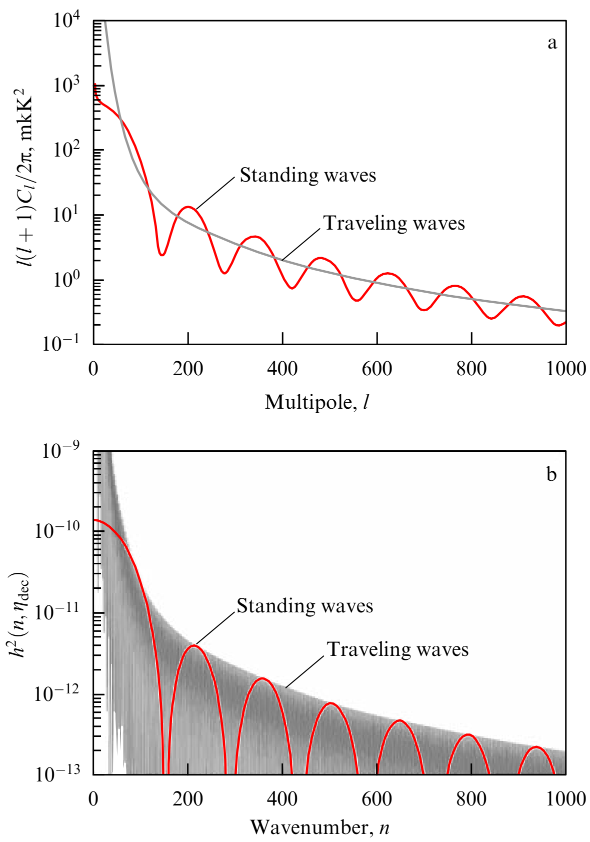

Figure 2. Angular power spectra of CMB temperature anisotropies (upper graph) generated by power spectra of standing or traveling gravitational waves (lower graph).

formed by (quantum-mechanically generated) standing waves, whereas the grey (smooth) line shows the alternative background formed by traveling waves. The power spectrum of the alternative background was chosen to be an envelope of the physical one, and therefore the broad-band powers in the two spectra are approximately equal, except at very small n. The CMB spectra are placed right above the underlying gravitational wave spectra in order to demonstrate the almost one-to-one correspondence between their features in n-space and l-space. A similar correspondence holds for the power spectrum of the first time derivative of the h_{ij} field and the CMB polarization spectra for which it is responsible [9]. It is important to note that the planned new sensitive measurements of CMB polarization and temperature (see, e.g., [10]) may be capable of identifying the first cycle of oscillations in the physical gravitational wave background.

3. Current observations of oscillations in the power spectra of matter and CMB

It should be clear from the discussion above that the Sakharov oscillations are not trivial acoustic waves in relativistic plasma. Waves such as variability in space and time always exist, in the sense that they are the general solution of the density fluctuation equation. The Sakharov oscillations are something much more subtle. They are the variability in the post-transition power spectrum, that is, oscillations in Fourier space. At late times, the oscillatory shape of the matter power spectrum remains fixed. The oscillations define the preferred wavenumbers and spatial scales, in agreement with the standing-wave pattern of the pre-transition field.

Oscillations in the final power spectrum do not arise simply as a result of a 'snapshot' of oscillations in the baryon–photon fluid or as an 'impression' of acoustic waves in the hot plasma of the early universe onto the matter distribution. And they are neither the result of the propagation of spherical sound waves up to the 'sound horizon' before recombination, nor the result of the 'freezing out' of traveling sound waves at decoupling. The event when the plasma becomes transparent can make the Sakharov oscillations visible, but this is not the reason why they exist. Periodic structures in the final power spectrum arise only if sound waves in relativistic plasma (as well as the associated metric perturbations) are standing waves with special phases. The oscillations in the power spectrum do not arise at all if the sound waves are propagating. It is also clear from the discussion above that the phenomenon of oscillations is not specific to baryons. The oscillations are present, for example, in the power spectrum of metric perturbations accompanying matter fluctuations and in gravitational waves.

It appears that actual observations have revealed convincing traces of Sakharov oscillations in the distribution of galaxies. Existing and planned surveys concentrate on the distribution of luminous matter (baryons), and the spectral features are therefore called baryon acoustic oscillations (BAOs). The structures in the power spectrum are Fourier-related to the spikes in the two-point spatial correlation function. Both characteristics have been measured in galaxy surveys (see, e.g., [11 – 14]; the last citation contains many references to previous work).

Of course, the ideal picture of standing waves in the early plasma is blurred by the multicomponent nature of cosmic fluid and by the variety of astrophysical processes occurring on the way to the observed spatial distribution of nonrelativistic matter. This makes the oscillatory features much smoother and much more difficult to identify. Moreover, the measurement of our own particular realization of the inherently random field is only an estimate of the theoretical, statistically averaged, power spectrum, such as Eqn (4). Nevertheless, the impressive observations of recent years have given significant evidence of the existence of Sakharov oscillations.

A similar situation occurs in the study of the CMB temperature and polarization. The difference between smooth and oscillatory underlying spectra for the ensuing CMB anisotropies was illustrated by gravitational waves in Fig. 2. Density perturbations are more complicated because they include the individual power spectra of fluctuations in matter components, the velocity of the fluid that emits and scatters CMB photons (the velocity and the associated Doppler terms require careful definitions), and gravitational field perturbations. Surely, the observed peaks and dips in the CMB temperature angular spectrum C_l^{TT}, now measured up to high multipoles l [15], are a reflection of oscillations in the underlying power spectra at the time of decoupling $\eta_{\rm dec}$. (A link with the phenomenon of Sakharov oscillations, in some generalized sense, was mentioned in [16, 8].) It is very likely that the oscillations in C_l^{TT} at relatively high l are a direct reflection of the standing-wave pattern of density variations in the baryon–electron–photon plasma itself, and are therefore 'acoustic' signatures. By contrast, the structures at the lowest multipoles l probably have a considerable contribution from the pre-transition metric perturbations, which were inherited at the time of transition $\eta_{\rm eq}$, mostly by the gravitationally dominant cold dark matter, and hence the structures are more like 'gravitational' peaks and dips [8]. (The current cosmological literature emphasizing the 'acoustic' side of the problem incorrectly claims that there should not be oscillations in the power spectrum of CDM.)

It should be remembered, however, that the decomposition of the total CMB signal into different contributions is not unambiguous, and the interpretation may depend on the coordinate system (gauge) chosen for the description of fluctuations. The decomposition of the total signal in the so-called Newtonian gauge is presented in Fig. 3, taken from [17]. The dominating Sachs–Wolfe contribution is a combination of variations of the metric and photon density.

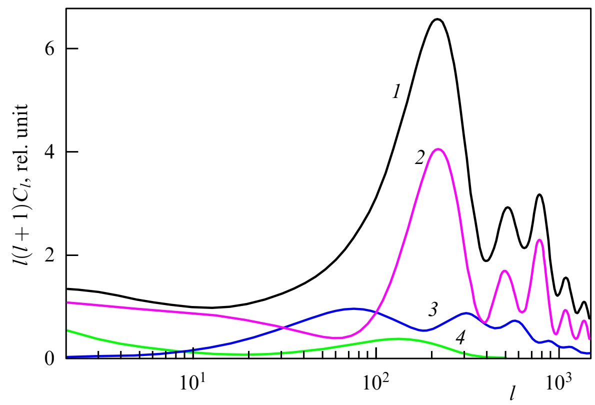

Figure 3. Various contributions to CMB temperature anisotropies [17]. Curve *2*, the contribution due to the Sachs – Wolfe effect; curve *3*, the Doppler contribution; curve *4*, the contribution of the integral Sachs–Wolfe effect.

We can make the following intermediate conclusions. First, for the Sakharov oscillations to appear in the final matter power spectrum, they must the encoded from the very beginning in the power spectrum of primordial cosmological perturbations as a consequence of standing waves. Therefore, the Sakharov oscillations must have a truly primordial origin (quantum mechanical, as we argue below). Second, the very existence of periodic structures in power spectra of matter and CMB gives us no less information about the Universe than do those discoveries that will hopefully be made with the help of these 'standard rulers'. In particular, in the case of data from galaxy surveys, it is important to be sure that we are dealing with manifestations of Sakharov oscillations, and not with something else. If they are Sakharov oscillations, then the phases were remembered for 13 billion years. Third, at some elementary level, the Sakharov oscillations can be tested in laboratory conditions. This is a difference in the fates of traveling and standing waves in a medium in which the speed of sound changes from large values to zero. It would be useful to perform this experimental demonstration.

4. Quantum mechanics of the very early Universe

It is appropriate to start this section with one of the last photographs of Sakharov (see Fig. 4). It shows an intermission in the meeting chaired by Sakharov at which the present author (among other enthusiastic speakers) argued that if primordial cosmological perturbations were generated quantum mechanically, then the result would be not just something but very specific quantum states known as squeezed vacuum states, and why this should be important observationally. The notions of the vacuum, a squeezed vacuum, and a displaced vacuum (coherent states) sounded suspicious to the audience, but Sakharov remained silent. At some crucial point he astonished me by the question "which variable specifically is squeezed?" Such a question can be asked only by someone who is perfectly well familiar with the subject and deeply understands its implications.

Figure 4. One of the last photographs of Sakharov.

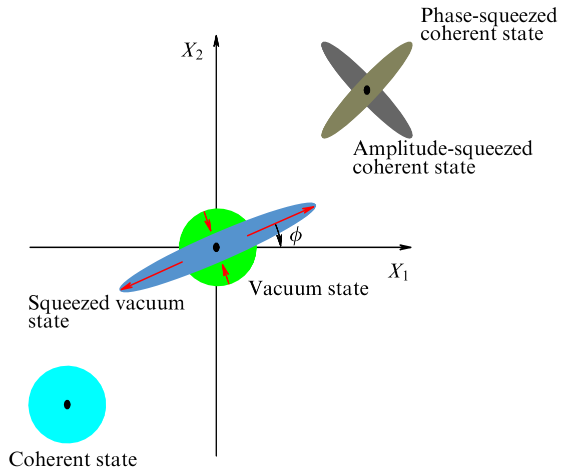

Figure 5. Some quantum states of a harmonic oscillator.

Indeed, from the sketch in Fig. 5, we can see that simple quantum states of a harmonic oscillator can greatly differ in the mean values and variances of conjugate variables. For example, squeezed coherent states can be squeezed, i.e., have very small uncertainties, either in the number of quanta or in the phase. This leads to different observational results. I was glad to answer Sakharov's question, because a squeezed vacuum state can be squeezed only in phase. The arising correlation of the $\mathbf{n}$ and $-\mathbf{n}$ modes is equivalent to the generation of a standing wave (a two-mode squeezed vacuum state; more details are given in [18] and [6]). The appearance of a standing-wave pattern is not surprising if we think of the generation process as the creation of pairs of particles with equal energies and oppositely directed momenta. Moreover, the phase, almost free of uncertainties in strongly squeezed vacuum states, smoothly depends on n, as the oscillators with different frequencies n start free evolution (rotation of a highly squeezed ellipse in the X_1, X_2 plane) after the completion of the generation process (squeezing of the vacuum circle into an ellipse). This provides the prerequisites for the future Sakharov oscillations.

The generation of excitations in physically different degrees of freedom — relic gravitational waves and primordial density perturbations — is described by essentially the same equations. The equation for gravitational wave mode functions is

$$h'' + 2\,\frac{a'}{a}\,h' + n^2 h = 0\,, \tag{6}$$

while the equation for metric perturbations describing the density perturbation degree of freedom is

$$\zeta'' + 2\,\frac{(a\sqrt{\gamma})'}{a\sqrt{\gamma}}\,\zeta' + n^2\zeta = 0\,, \tag{7}$$

where the variable $\zeta(\eta)$ is also known as the curvature perturbation. Surely, Eqns (6) and (7) can also be written in the form of the master equation, Eqn (5). The function $\gamma(\eta) \equiv 1 + (a/a')'$ in Eqn (7) is not the constant γ that Sakharov [1] uses in the equation of state, but the scale factor $a(\eta)$ is a power-law function for simple equations of state, and $\gamma(\eta)$ is then a constant. In this case, Eqns (6) and (7) are identical, and they have general solutions in terms of the Bessel functions.

The two-mode Hamiltonian

$$H = n c_{\mathbf{n}}^{\dagger} c_{\mathbf{n}} + n c_{-\mathbf{n}}^{\dagger} c_{-\mathbf{n}} + 2\sigma(\eta)\, c_{\mathbf{n}}^{\dagger} c_{-\mathbf{n}}^{\dagger} + 2\sigma^*(\eta)\, c_{\mathbf{n}} c_{-\mathbf{n}} \tag{8}$$

is common for these two degrees of freedom, with the coupling function $\sigma(\eta) = (\mathrm{i}/2)[a'/a]$ for gravitational waves and $\sigma(\eta) = (\mathrm{i}/2)[(a\sqrt{\gamma})'/(a\sqrt{\gamma})]$ for density perturbations. The coupling functions coincide if $\gamma(\eta) = \mathrm{const}$. As a result of the Schrödinger evolution, the initial vacuum state of cosmological perturbations (ground state of the corresponding time-dependent Hamiltonian) evolves into a two-mode squeezed vacuum (multiparticle) state. In other words, cosmological perturbations are quantum mechanically generated as standing waves [6, 18].

The simplest models of the initial stage of expansion of the Universe are described by power-law scale factors $a(\eta)$. (The four cases of the initial stage considered by Sakharov [1] also belong to this category.) Such gravitational pump fields $a(\eta) \propto |\eta|^{1+\beta}$ generate gravitational waves (t) and density perturbations (s) with approximately power-law primordial spectra:

$$P_{\rm t}(k) = A_{\rm t}\left(\frac{k}{k_0}\right)^{n_{\rm t}}, \qquad P_{\rm s}(k) = A_{\rm s}\left(\frac{k}{k_0}\right)^{n_{\rm s}-1}, \qquad (9)$$

where $n_{\rm s} - 1 = n_{\rm t} = 2(\beta + 2)$, and we use $k_0 = 0.002\ \mathrm{Mpc}^{-1}$. The amplitudes $(A_{\rm t})^{1/2}$ and $(A_{\rm s})^{1/2}$ are independent unknowns, but according to the theory based on Eqns (6)–(8), they should be of the same order of magnitude: $A_{\rm s}^{1/2} \sim A_{\rm t}^{1/2} \sim H/H_{\rm Pl}$, where H is the Hubble parameter at the initial stage of expansion. [The inflation theory also uses the same superadiabatic (parametric) amplification mechanism, which was originally worked out for gravitational waves [19, 6]. However, after blind wanderings between variables and gauges, inflationists arrived at what they call the 'standard', or even 'classic', result of the inflation theory. Namely, the prediction of arbitrarily large $A_{\rm s}$ in the limit of the Harrison–Zeldovich–Peebles spectrum $n_{\rm s} = 1$ and, moreover, for any strength of the generating gravitational field, i.e., for any value of the Hubble parameter H of the inflationary de Sitter expansion $\dot{H} = 0$.] It is common to characterize the contribution of gravitational waves to the CMB by the ratio $r \equiv A_{\rm t}(k_0)/A_{\rm s}(k_0)$.

Our analysis [20] of the 7-year Wilkinson Microwave Anisotropy Probe data (WMAP7) has resulted in $r = 0.285$ and $r = 0.2$ as the respective maximum likelihood values in 3-parameter and marginalized 1-parameter searches. The uncertainties are still large, and therefore these numbers can only be regarded as indications of a possible real signal. The relic gravitational waves are very difficult to register, but they are the cleanest probe of the very early Universe [19, 21, 6]. This is why they are in the center of several programs aimed at their identification. The Sakharov oscillations are an element of the whole picture of quantum-mechanically generated cosmological perturbations, and hence the detection of relic gravitational waves would be a huge support for the entire theoretical framework.

5. Expected results of the ongoing observations. Conclusions

The prospects of measuring relic gravitational waves with the help of data from the currently operating Planck mission appear to be good. In Fig. 6, taken from [20], we show the expected signal-to-noise ratio with which the signal will be observed assuming that the indications found in WMAP7 data are real. A big obstacle is the foreground contamination, which should be carefully dealt with. The ability, ranging from excellent to none, of removing contamination is parameterized by the parameter $\sigma^{\rm fg} = 10^{-2}$, 10^{-1}, 10^0. We also work with the pessimistic case, in which $\sigma^{\rm fg} = 1$ and the nominal instrumental noise in the BB polarization channel at each frequency is increased by a factor of 4. We see from the figure that the S/N ratio can be as large as S/N ≈ 6, and even in the pessimistic scenario, it remains at the interesting level S/N > 2.

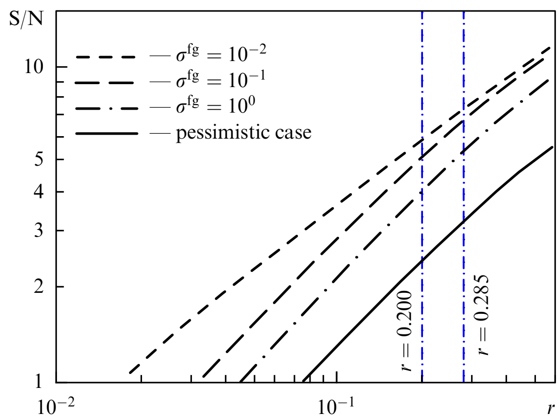

Figure 6. The expected S/N ratio in the detection of relic gravitational waves by the Planck mission.

As was already mentioned above, the planned dedicated observations [10] may even be able to outline the first cycle in the oscillatory power spectrum of the gravitational wave background.

In general, we can conclude that the originally proposed Sakharov oscillations, as well as related phenomena whose existence can be traced back to the earliest moments of our Universe, are right in the focus of current fundamental research.

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