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From the Cosmological Model to the generation of the Hubble flow

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Abstract. This paper reviews various approaches to the question (pioneered by Sakharov) of how the observed Hubble flow forms. By extrapolating the Cosmological Standard Model to the past, the geometrical properties of and conditions in the early Universe are determined. A new cosmogenesis paradigm based on geodesically complete black/white hole geometries with an integrable singularity is discussed.

1. Introduction

In his papers (see, for example, Refs [1, 2]), Andrei Dmitrievich Sakharov repeatedly expressed the idea that cosmological flows may build up from superdense singular states of matter as a result of quantum transitions accompanied by changes in the world constants, signature, time arrow, and other geometrical characteristics of space-time and matter. The way gravitating systems or their parts get into such special states and how they leave them have spawned discussions over a long period of time, which continue even now.

Large curvatures and densities arise in a natural way in separated space-time domains during the collapse of compact astrophysical objects, leading to the formation of black holes. Yet the question of how the collapse passes into expansion still awaits a solution. The concept of the multisheeted universe proposed in Sakharov's works, as well as the paradigm of multiple universes (Multiverse) (see, for example, book [3]), widely accepted today, requires the existence of an explicit and simple physical mechanism generating multiple flows of expanding matter. However, the mechanism of *cosmogenesis* — the formation of cosmological flows of matter — remains vague thus far. To explore it, we need to know the initial structure of our material flow, which we call the early Universe, with sufficient accuracy.

The task of determining the geometrical properties of the early Universe was successfully solved at the turn of this century with the appearance of the Cosmological Standard Model (CSM) which describes all the totality of experimental and observational data in the energy range $10^{-3} - 10^{12}$ eV. With the creation of the CSM, it has become possible to recover the initial state of the Universe through the direct extrapolation in the past, having made only the assumption that the general relativity (GR) is valid for energies up to that of Grand Unification ($\sim 10^{25}$ eV). The subsequent direct extrapolation (toward even larger energies) faces difficulties because of the inflationary stage of the Big Bang (BB) during which the Hubble radius goes beyond the light horizon of the past, where the prevailing part of the information on the pre-inflationary flow geometry is confined (Fig. 1) [4]. The deviations from the quasi-Friedmann model increase at the inflationary stage (for the extrapolation in the past); therefore, the structure of cosmological flow at the beginning of inflation could be substantially different from the Friedmann one and have another symmetry and topology.

Owing to the appearance of the CSM, the problem of generation of the initial state of expanding flow (the cosmogenesis problem) has acquired a rigorous scientific formulation in the framework of GR, since energies do not exceed the Planck value. Additionally, because the gravitating system passes through its ultrahigh energy and curvature phases very fast, in considering models with the transition of collapse into expansion it is sufficient to use only local conservation laws which can be represented in the general geometric form of the Bianchi identities. For doing so, it is convenient to move all kinds of gravity modifications and quantum-gravity corrections to the mean Einstein tensor to the right-hand side of gravity equations, associating all these terms with the *effective* energy–momentum tensor which now

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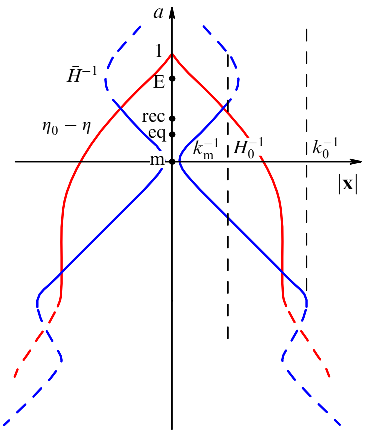

Figure 1. The ordinate is the scale factor a, and the abscissa is the comoving coordinate $|\mathbf{x}|$ (observers's world line is $\mathbf{x} = 0$). The line $\bar{H}^{-1}$ corresponds to the Hubble radius ($\bar{H} = aH$), $\eta_0 - \eta$ is the light cone of the past. The $k_{\rm m}^{-1}$, H_0^{-1}, and k_0^{-1} scales correspond to some fractions of a millimeter, the value of 4.3 Gpc, and the size of the Friedmann world. The time moments labelled as m, eq, E, and rec mark the end of the inflationary stage of the BB and the origins of DM, DE, and recombination epochs.

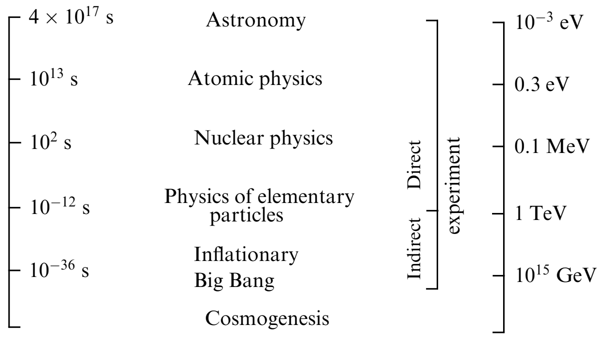

Figure 2. Experimental basis of the early Universe. The left scale corresponds to the Universe curvature radius expressed in seconds, and the right one shows characteristic energies.

includes not only material but also space-time degrees of freedom. Such an approach enables us to keep the notion of mean metric space-time (independently of values of density and curvature components) and stay in the class of geometries with *integrable* singularities [5], which facilitates the construction of full maps of black (white) holes on radial geodesics and the understanding of physics behind the gravitational transformation of the internal T-region in a black hole to the anticollapse of created matter in the white hole (cosmological expansion).

In what follows, we consider the lessons from the extrapolation, determine the initial conditions in the early Universe, discuss the physical nature of the multisheet universe, and present models for the formation of cosmological flows of matter in the framework of the cosmogenesis concept proposed by us.

2. Lessons of extrapolation

The Cosmological Standard Model rests on extensive observational and experimental bases in the energy range spanning 15 orders of magnitude — from the current cosmological density ($\sim 10^{-3}$ eV) to energies of electroweak transition ($\sim 10^{12}$ eV) explored with the Large Hadron Collider and corresponding to the age of the Universe of several picoseconds (Fig. 2). No extrapolation is involved here — this is the available scientific knowledge, well studied and tested on Earth. Extrapolation begins further, in advancing over the next 13 orders of magnitude in energy, up to that of Grand Unification.

The most important outcome of this knowledge is our ideas about the geometry of the early Universe, which is identified in GR with the structure of the metric tensor and the energy–momentum tensor. The empirically derived model contains a small parameter — the amplitude of cosmological inhomogeneities of the metrics ($\sim 10^{-5}$) — which allows the perturbation theory to be applied to its description. In the zeroth order, we deal with the spatially flat Friedmann model described by a single function of time — the scale factor $a(t)$ depending on the physical composition of matter. In the first order, the structure of tensors proves to be more complex — it can be reduced to three irreducible forms [6]: scalar (density perturbations), tensor (gravitational waves), and vector (for example, magnetic fields), each represented by its power spectrum: $S(k)$, $T(k)$, and $V(k)$, where k is the wave number (inverse perturbation scale). For a random spatial phase of perturbation, the second and third orders do not introduce new free functions.

We conclude that the initial cosmological flow of matter is fully deterministic and possesses a laminar, quasi-Hubble character (a weakly inhomogeneous, or quasi-Friedmann universe). Having specified the initial conditions and matter composition, we obtain as the result of evolution the whole palette of physical processes and events in the current world. Of the four functions mentioned above, we know only the first two in the domains of definition accessible for observational cosmology. The ongoing Planck experiment will unveil the spectrum of cosmological gravitational waves, assuming it is successful. The detection of the vector mode lies beyond the present-day experimental possibilities.

The explanation of the initial properties of cosmological flow is the main task of cosmogenesis. The statement of physical problems follows from the lessons of extrapolation [4], of which seven are considered below.

1. *The Universe is large*. This fact can be explained by the presence of a short inflationary stage of the BB preceding the radiation-dominated period of expansion.

The current Hubble radius (4-curvature radius) equals $H_0^{-1} \simeq 4.3$ Gpc which, on the time scale, lies 60 orders of magnitude away from the Planck value. According to the CSM, the scale factor could have changed by only 30 orders of magnitude over this period, as follows from the Friedmann equations describing the events in the principal order of the perturbation theory:

$$H \equiv \frac{\dot{a}}{a} = H_0 \sqrt{\frac{10^{-4}}{a^4} + \frac{0.3}{a^3} + 0.7} \;\; \rightarrow \;\; \frac{H_0}{100a^2}\,, \tag{1}$$

$$\gamma \equiv -\frac{\dot{H}}{H^2} = \frac{2 \times 10^{-4} + 0.4a}{10^{-4} + 0.3a + 0.7a^4} \in (2,\, 0.4)$$

(the three terms under the root sign correspond to the radiation, nonrelativistic matter, and dark energy; the scale factor is normalized to unity at the present time). By performing extrapolation in the past, we recover the dominance of radiation at the early times and a value of several millimeters for the initial size of the Universe, which is a very large magnitude exceeding the Planck value by 30 orders of magnitude. To explain such a size, a preceding inflationary stage is needed with a value of $\gamma < 1$ and the number of Hubble epochs not less than 70 ($\sim$ 30 ln 10).

2. *The causality principle*, witnessing independently the existence of the inflationary phase of the BB. According to formulas (1), the galactic scales turn out to be outside the causal zone in the radiation-dominated period of expansion (see Fig. 1). They could have got into this zone from the causally connected domain provided that a short inflationary stage of the BB exists.

3. *The smallness of tensor mode,* also pointing to the inflationary stage of the BB, and the Gaussian character of density perturbations.

While the zeroth order of the perturbation theory is described by the Friedmann equations, the first order corresponds to oscillators (see the Appendix). The S- and T-modes evolve as massless scalar fields $q = (q_S, q_T)$ experiencing external gravitational action of the nonstationary Hubble flow, which leads to the parametric amplification of fields q in the course of cosmological expansion [4, 7, 8]. Under sufficiently general assumptions about the expansion rate, the equations for elementary oscillators admit a general solution; however, their excitation amplitudes depend on the initial data. For oscillators residing initially in their ground (nonexcited) state, the power spectra of the generated perturbations have the form

$$T(k) \simeq \frac{H^2}{M_{\mathrm{P}}^2} < 10^{13}\ \mathrm{GeV}, \qquad \frac{T}{S} \simeq 4\gamma < 0.1\,, \tag{2}$$

where $\langle q_{S,T}^2 \rangle = \int (S, T)\, \mathrm{d}k/k$, $\langle \ldots \rangle$ implies averaging over the state, and $M_{\mathrm{P}} \equiv G^{-1/2} \simeq 10^{19}$ GeV is the Planck mass. As we can see, the tensor mode is on equal footing with the scalar one in the theory, whereas their ratio depends on the magnitude of γ during the parametric generation epoch. The inequalities (2) reflect the present-day observational limitations on cosmological gravitational waves. From the second inequality, it follows that the quantity γ was below unity in the early Universe, which indirectly points to the inflationary character of the early Hubble flow. A rigorous proof of the primary inflation will become possible in the case of direct detection of the tensor mode by exploring the relic radiation and confirming the theoretically predicted relationship between the slope of the exponent of the T-spectrum and the ratio between perturbation mode amplitudes ($n_T \equiv \mathrm{d}\ln T/\mathrm{d}\ln k \simeq -2\gamma \simeq -0.5T/S$).

We stress that this assertion rests on the hypothesis that the early Hubble flow is ideal, as expressed by the vacuum initial condition for the fields q. This assumption is supported by the observed *Gaussian* random spatial distribution of large-scale density perturbations (the property of quantum fluctuations linearly transferred to the field of inhomogeneities) and a pronounced temporal phase of acoustical oscillations that corresponds to a *growing* adiabatic evolution branch (the implication of the parametric amplification effect).

4. *The presence of dark matter*. Nonlinear halos 'inhabited' by galaxies inside them are composed of nonrelativistic particles of dark matter (DM) that do not interact with baryons and radiation. The nature of DM particles is currently unknown, but there are observational arguments favoring the assertion that the origin of DM is rooted in the baryon asymmetry of the Universe. Here are two of them: the cosmological mass densities of DM and baryons are close to each other (their ratio equals 5), and their large-scale distribution scales in space coincide (the cosmological horizon at the instant of equal densities of relativistic and nonrelativistic components of matter is identical to the acoustical horizon at the instant of hydrogen recombination). If we take into account that the ratio between densities for two nonrelativistic media stays constant with time, we are led to conclude that the reasons for the appearance of DM and baryon asymmetry are interconnected. One can suppose that both the particles of DM and excessive baryons have been formed in nonequilibrium processes of particle transformation in a hot radiative plasma of the Hubble flow. In this case, their origin is not related to the pre-inflatory history of the BB.

5. *Indications in favor of the existence of dark energy*. The matter forming the structure of the Universe is measured by gradients of gravitational potential, derived from dynamical observations of galaxies and gas, as well as with the help of gravitational lensing. Its share does not exceed 30% of the critical density. The remaining 70% constitute a uniformly distributed subsystem that does not interact with light or baryons. This is the so-called dark energy (DE) possessing a negative effective pressure comparable in absolute value to its energy density. By all probability, here we are dealing with a relic superweak field 'conserved' at the stage dominated by radiation and particles that came into motion (in the slow-roll state) under the action of self-gravity 3.5 billion years ago. If it is indeed so, we are witnessing the relaxation of a massive field, which suggests a different view of the Hubble flow history.

6. *The history of the Universe's evolution*. We see that evolution passed through the periods of accelerated ($\gamma < 1$) and decelerated ($\gamma > 1$) expansion. The first case includes the inflationary stages of the BB and DE, and the second one covers the stages dominated by radiation and matter. We know that small perturbations fade away for $\gamma < 1$ but grow for $\gamma > 1$. Hence, it follows that the history of the Universe has seen periods of both *build-up* (recovery) of the Hubble flow and phases of its *destruction* (and then we are talking about structure formation). This is a manifestation of the dual character of long-range gravity, capable of creating strongly ordered systems from rather general initial distributions and states of matter. It is the anticollapse, or inflation (the build-up of an ideal Hubble flow), and its inverse, the process of collapse (the formation of gravitationally bound halos and black holes). Thus, we can view the dynamical history of the flow as a process, covering 14 billion years, of relaxation of massive fields to the minimum energy state. Here, we come close to the seventh and last lesson from the extrapolation of the CSM toward a pre-inflationary universe. It is how to create the conditions necessary for the occurrence of an expanding material flow, taken over by inflation and transformed into the observed Hubble flow.

3. Conditions of cosmogenesis

Thus, solving the cosmogenesis problem means answering three questions.

- How do large densities form?
- Where does the expansion come from?
- What is the origin of cosmological symmetry?

Inflation leaves these questions unanswered. In its different variants (e.g., Refs [9, 10]), new physical fields are introduced, which from the very beginning are in a superdense state. The birth of the Universe from 'nothing' [11] again leads to the idea of 'false' vacuum with a high density, whereas in bouncing models, having been developed for already more than 40 years, the question about the initial state no longer makes sense (owing to modifications of the equation of state), and the Friedmann symmetry is introduced axiomatically.

The fundamental principle in the natural sciences, stating that all measurable quantities should remain finite in a solution that describes Nature, allows us to advance in solving the cosmogenesis problem. Indeed (see Ref. [5]), if we consider realistic models of black/white holes with *smoothed* metric singularities, it becomes possible to constrain tidal forces (despite the divergence of some curvature components) and recover a geodesically complete metric space-time based on dynamic solutions stemming from the energy–momentum conservation laws. In the vicinity of a singularity forming around a collapsed object, there is the effective matter which we model in a broad class of equations of state. Radial geodesics now do not end at the singular hypersurface, but continue in the T-domain of a white hole. Hence, we arrive at the conclusion that the T-domain of a black/white hole originated from the collapse of a compact astrophysical system may spawn a new (daughter, or astrogenic) universe that is in the absolute future with the respect to the maternal black hole. In that case, the answers to the posed questions are almost obvious:

- The ultrahigh curvatures and densities at the initial stages of cosmological evolution arise because of extremely strong rapidly varying gravitational fields existing inside the black/white hole and generating matter belonging to the daughter universe;
- The initial push to the cosmological flow of effective matter comes from the expanding T-domain of the white hole that was formed as a result of the collapse of a compact object in the maternal universe. Relatedly, the BB phenomenon is of a purely gravitational nature and is, in essence, the manifestation of gravitational (tidal) instability;
- The symmetry of the inner domain of the black/white hole outside the body of a collapsed system is that of a homogeneous cosmology, in which the material flow in the white hole can be isotropized through the known inflation mechanisms, and in this way the white hole will transform into the Friedmann world.

4. Black/white holes with integrable singularity

The application of the aforementioned principle to the general type spherically symmetric metrics implies the finiteness of real-valued functions N and Φ in $\mathbb{R}^2 \in (r, t)$:

$$ds^2 = N^2(1+2\Phi)\,dt^2 - \frac{dr^2}{1+2\Phi} - r^2\,d\Omega\,, \tag{3}$$

where r and t are the radial and temporal Eulerian coordinates in the R-domains of space-time ($\Phi > -1/2$) and, accordingly, the temporal and radial coordinates of the same solution in the T-domain ($\Phi < -1/2$ (see Ref. [12])), and $d\Omega$ is the interval squared on the surface of a 2-sphere.

According to the equations of GR, we have

$$\Phi = -\frac{Gm}{r}\,, \tag{4}$$

where the everywhere continuous function of mass

$$m = m(r,t) = 4\pi \int_0 T_t^t r^2\,dr \tag{5}$$

becomes zero in the inversion line $t = 0$ because of the requirement that Φ be finite, with T_t^t being the tt component of the energy–momentum tensor. The integrability of the function $T_t^t r^2$ at a zero point (for a finite black-hole mass) leads us to the notion of the *integrable* singularity $r = 0$ surrounded by the effective matter. [1] In the absence of spatial flows in the T-domain, the energy–momentum tensor takes the form $T_\mu^\nu = \mathrm{diag}\,(-p, \varepsilon, -p_\perp, -p_\perp)$. In Section 5, we shall give examples of models in which the energy density is generated by variations of transverse pressure $p_\perp$ changing in a triggered manner at certain time instants r. In these models, the tidal forces for radial geodesic lines are everywhere finite, and world lines of test particles continue from the T-domain of the black hole into the white one (see Ref. [5] for more details). Thus, the tidal gravitational interaction in the vicinity of integrable singularity attains the form of time oscillation connecting the inner regions of the black and white holes. We call this effect collapse *inversion*.

5. Astrogenic universes

The matter in T-domains of spherically symmetric vacuum geometries can be generated with the help of time variations of the function $p_\perp(r)$, for example, jumps of the first kind, because the equations of motion do not contain its derivatives (the energy is supplied by the gravitational field, and the metric is automatically adjusted in accordance with GR). The longitudinal pressure can conveniently be chosen 'vacuum-like' ($p = -\varepsilon$) for the sake of simplicity, then $N = 1$ everywhere in $\mathbb{R}^2$ and the reference frame (3) is comoving with matter, and the energy density follows from the Bianchi identity

$$\frac{d(\varepsilon r^2)}{r\,dr} = -2p_\perp\,. \tag{6}$$

Consider two simple variants of the behavior of function $p_\perp$ (Fig. 3 and 4):

A) Asymmetric step, $p_\perp^{(A)} = p_0\theta\big(r(r_0 - r)\big) - p_1\theta(-r)$;

B) Symmetric step, $p_\perp^{(B)} = p_0\theta(r_0^2 - r^2)$, where $r_0 \leqslant 2GM$ and p_1 are real-valued positive constants, and $M \equiv 8\pi r_0^3 p_0/3$ is the black-hole mass. Integration of Eqn (6) subject to the initial condition $\varepsilon(r \geqslant r_0) = 0$ gives the following continuous $\varepsilon(r)$ functions:

$$\varepsilon^{(A)} = -p_\perp^{(A)} + p_0\,\frac{r_0^2}{r^2}\,\theta(r_0 - r)\,, \quad \varepsilon^{(B)} = p_\perp^{(B)}\left(\frac{r_0^2}{r^2} - 1\right). \tag{7}$$

Accordingly, A is the model of the astrogenic universe ($\varepsilon^{(A)} \to p_1$ for $r \to -\infty$), while B offers an example of an

[1] We assume that this matter can be induced by an intense rapidly varying gravitational field (because of quantum-gravity processes of vacuum polarization and matter creation) existing outside the collapsed object in the T-domain of the black/white hole. In that case, the symmetry of the full solution preserves the global Killing t-vector contained in the original Schwarzschild metric in a vacuum, and all the physical variables considered here are functions of r only (we suppose that $r > 0$ in the maternal black hole, and that $r < 0$ in the metric continuation into the T-domain through the $r = 0$ line).

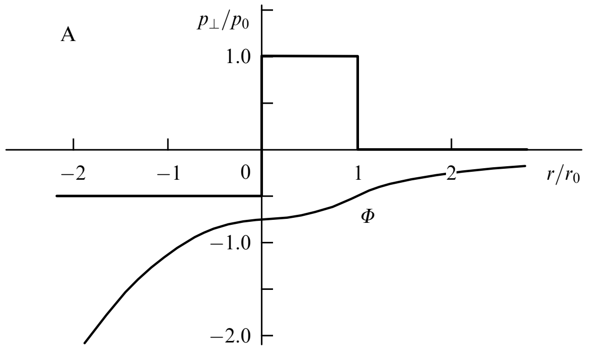

Figure 3. An asymmetric profile of transverse pressure (bold broken line), leading to the transformation of collapse into the cosmological expansion that asymptotically tends to the de Sitter solution. The curve depicts the evolution of gravitational potential in a model in which matter fills the entire T-domain of a black hole ($r_0 = 2GM$). Additionally, as an example, it is assumed that $p_1/p_0 = 0.5$.

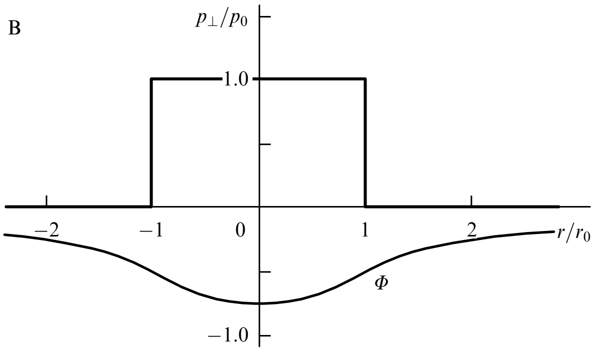

Figure 4. A symmetric profile of transverse pressure (bold broken line) leading to the transformation of a black hole into a white hole of the same mass. The curve plots the evolution of gravitational potential in a model in which matter fills the T-domains of black and white holes completely ($r_0 = 2GM$).

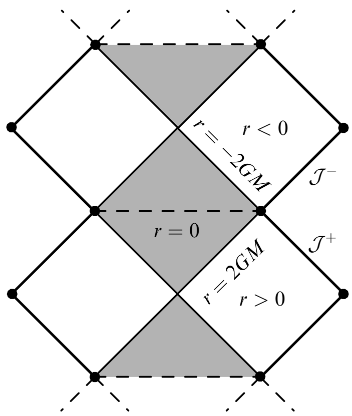

Figure 5. The Penrose diagram of a pulsating flow with a symmetric function $p_\perp(r)$ (see Fig. 4). The shaded domain is occupied with matter. $\mathcal{J}^+$ is the light infinity of the future for observers in the domain $r > 0$; $\mathcal{J}^-$ is the light infinity of the past for observers in the domain $r < 0$, and I^0 are spatial infinities of the R-domains.

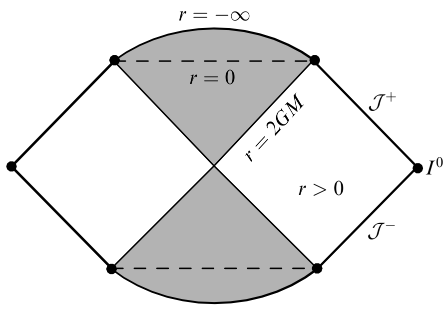

Figure 6. The Penrose diagram of the astrogenic universe with an asymmetric function $p_\perp(r)$ (see Fig. 3).

oscillating (eternal) black/white hole. The potential $\Phi(r)$ belongs to the class of C^1 functions [see Eqns (4) and (5), and Figs 3 and 4].

Consider the limiting cases of option B. As $r_0 \to 0$, we have an eternal black/white hole maximally continued into a vacuum, with a δ-shaped source localized in the region $r = 0$ [5]:

$$\varepsilon = 2p_\perp = M\,\frac{\delta(r)}{2\pi r^2}\,. \tag{8}$$

For the limiting extension, $r_0 = 2GM$, we obtain a stationary hole with an oscillating flow of matter in the T-domain, where $r = -2GM\sin(H\tau)$, $H^{-1} \equiv 2\sqrt{2}GM$, τ is the oscillation frequency and the proper time of the flow:

$$\varepsilon = \frac{3H^2}{8\pi G}\cot^2(H\tau)\,, \tag{9}$$

$$\mathrm{d}s^2 = \mathrm{d}\tau^2 - \frac{1}{2}\left(\cos^2(H\tau)\,\mathrm{d}t^2 + \frac{\sin^2(H\tau)}{H^2}\,\mathrm{d}\Omega\right).$$

Here, we are dealing with a spatially homogeneous, anisotropic and pulsating flow of matter, the full Penrose diagram of which is plotted in Fig. 5. Phase transitions in matter at the stage of its volume expansion can lead to inflation and isotropization of the flow, establishing in this manner the Friedmann symmetry in an arbitrarily large volume.

A simple example of such a scenario is illustrated by case A. Indeed, for $\tau \geqslant 0$ from Eqn (7) we get a solution that asymptotically tends to the de Sitter one (Fig. 6):

$$r = -\frac{\sinh(H_1\tau)}{\sqrt{2}H_1}\,, \qquad \varepsilon = \frac{3H_1^2}{8\pi G}\coth^2(H_1\tau)\,, \tag{10}$$

$$\mathrm{d}s^2 = \mathrm{d}\tau^2 - \frac{1}{2}\left(\cosh^2(H_1\tau)\,\mathrm{d}t^2 + \frac{\sinh^2(H_1\tau)}{H_1^2}\,\mathrm{d}\Omega\right),$$

where the constant $H_1 = (8\pi G p_1/3)^{1/2}$ can take any values, independent of the value of external mass of the maternal black hole. This elementary example of the astrogenic universe allows easy generalization to more complex models, with inclusion of massive scalar fields, radiation, and other elements of the modern ‘kitchen’of the CSM.

6. Conclusions

Extrapolation of the CSM in the past implies the expanding initial Hubble flow of matter with ultralarge curvatures and densities. In models of black/white holes with integrable singularities, cosmological flows may arise in the expanding T-domains of these geometries (white holes) lying in the absolute future with respect to the maternal black hole. In the framework of the proposed concept, we arrive at the notion of *astrogenic cosmology* — the cosmology obtained through the inversion of the collapse of some astrophysical compact system into the expanding flow of effective matter outside the maternal body of the collapsed object proper. Speaking figuratively, black holes in such models play a role of matches igniting other worlds.

It is conceivable that the multisheet universes with complex topology, anticipated and discussed by Sakharov, owe their existence to collapsed systems that completed their evolution in the maternal Universe. Scientific theories have predictive skills and have to be tested against experiments and observations. These questions inspire us to new studies of riddles and problems of cosmogenesis.

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7. Appendix

It should be kept in mind [4] that $q_S = \delta a/a + Hv$ and v are the perturbations of a comoving scale factor and the matter velocity potential, respectively, and $q_T = (q_\lambda)$ are the amplitudes of gravitational waves with the polarization $\lambda = \oplus, \otimes$. The conformal fields $\tilde{q} = \tilde{\alpha} q/\sqrt{8\pi G}$ behave like classical harmonic oscillators with variable frequencies in the Minkowski space:

$$\tilde{q}'' + (\omega^2 - U)\,\tilde{q} = 0\,, \qquad (11)$$

where the prime denotes a derivative with respect to the Minkowski time $\eta = \int \mathrm{d}t/a$, and

$$U \equiv \frac{\tilde{\alpha}''}{\tilde{\alpha}}\,, \quad U_T = (2-\gamma)\,a^2 H^2\,, \quad \tilde{\alpha}_S = \frac{a\sqrt{2\gamma}}{\beta}\,, \quad \tilde{\alpha}_T = a\,,$$

where $\omega = \beta k$, β_S is the speed of sound in the light speed units, and $\beta_T = 1$. For two or more media, a term describing the action of isometric perturbations has to be added to the right-hand side of equations for S-oscillators.

The dependence of the effective frequency $(\omega^2 - U)$ on time leads to the parametric excitation of elementary oscillators in the course of the Universe's evolution. Assuming the initial vacuum state of fields in the wave zone $(\omega^2 > |U|)$, which transforms with time into the parametric one $(|U| > \omega^2)$, we get the required solution (11) (for more details, see Ref. [4]):

$$\frac{\exp\left(-\mathrm{i}\int \omega\,\mathrm{d}\eta\right)}{\tilde{\alpha}\sqrt{2\omega}} \to \frac{\boldsymbol{c} - \mathrm{i}}{C\sqrt{2k}} \to \frac{M_{\mathrm{P}}\sqrt{\pi}}{2k^{3/2}}\,q_k\,, \qquad (12)$$

where C is the matching constant in the region $|U| \simeq \omega$, and the function $\boldsymbol{c} = -kC^2 \int \tilde{\alpha}^{-2}\,\mathrm{d}\eta \to \mathrm{const}$ converges on the upper limit for $\gamma < 3$. The 'frozen' fields q_k correspond to the growing branch of the general solution, their phase is random, and the module defines spectral amplitudes $S = |q_{kS}|^2$ and $T = |q_{k\oplus}|^2 + |q_{k\otimes}|^2$. For $\beta = 1$ and $\gamma \simeq \mathrm{const}$, equations (11) become identical for all modes and $T/S = 2\tilde{\alpha}_S^2/\tilde{\alpha}_T^2 = 4\gamma$; for $\gamma < 1$, we get the T-spectrum (2) up to a factor of order unity.

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