collinear states marked in the figure. Using experimental data, we find that the 'magnetically dependent' parts of transition resistances $R_{1,2}$ (see formula (13)) differ by 20% or less. There are two points to note from the results of this experiment. First, the two tunneling junctions in series that make up the structure under study share a sufficiently high degree of identity. Second, anisotropy plays a fundamental role in the formation of noncollinear states.

6. Conclusion

To summarize, this paper

— predicts new transport, optical, and neutron-optical effects for ferromagnetic systems with a noncoplanar magnetization distribution;

— develops nanolithography and probe microscopy techniques that create vortex, antivortex, and spiral magnetization distributions in ferromagnetic nanostructures;

— establishes that, in multilayered ferromagnetic particles of anisotropic shape, stable collinear states of different resistance exist, making these systems promising for application in information storage and processing devices.

While there has been some success in the study of inhomogeneously magnetized ferromagnetic structures, thus far none of the predicted 'exchange' effects have been observed. Noting also that many relevant topics are left unaddressed in this paper [including those related to the magnetoelectric effect in inhomogeneous magnets [33, 34], to phenomena in nonstationary and inhomogeneous magnetic structures (see, for example, Ref. [35]), etc.], it is safely concluded that the topic is far from exhausted and further research remains to be done.

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Nonlinear wave processes in a deformable solid as in a multiscale hierarchically organized system

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1. Introduction

In this report, we theoretically and experimentally substantiate the conception of a multiscale description of a deformable solid as a nonlinear hierarchically organized system. The surface layers and all internal interfaces are considered as an independent planar functional subsystem with a short-range order. The channelled plastic flow in the planar subsystem is primary. It is responsible for the formation and emission of all types of strain-induced defects into the crystalline subsystem. This process is developed through the mechanism of nonlinear waves which determine the law of self-consistency of

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plastic flow in hierarchically organized systems. The fracture is related to a wave-like structural phase decomposition of the material.

The science of plasticity and strength of solids is going through a stage of replacing a paradigm. For a long time, the description of plastic deformation and fracture of solids was developed in terms of linear approximations of the mechanics of continuum media (macroscale level) and of the physics of strain-induced defects in a loaded solid (microscale level). However, it has become obvious in recent decades that a deformable solid represents a multiscale hierarchically organized system which should be described in terms of nonlinear mechanics and nonequilibrium thermodynamics [1].

At present, mechanisms of deformation on the nano-, micro-, meso-, and macroscale levels are being widely discussed in the literature. Unfortunately, the classification of scales reduces in most cases to only the size factor, retaining within the single-level approach. The problems of a multiscale self-organization and allowance for the nonlinearity of a hierarchically organized system have so far remained undeveloped.

A fundamentally new proposition in the multiscale approach is the conception of classification of surface layers and all internal interfaces as functional nonlinear planar subsystems in which translational invariance is absent [3–5], rather than as planar defects in crystals (according to the approach accepted, e.g., in monograph [2]).

The primary plastic shears emerge not in a translationally invariant crystal but rather in planar strongly excited subsystems in the form of nonlinear waves of channelled local structural transformations. Upon such wave fluxes propagating in a planar subsystem, strain-induced defects of various types are generated. A periodic emission of defects into the crystalline subsystem is developed as a nonlinear wave process. The thermodynamic stability of the crystalline subsystem in the course of plastic deformation decreases continuously, causing a nonlinearity of the behavior of the deformable solid.

This paper is devoted to a theoretical and experimental substantiation of the fundamental importance of the role of nonlinear waves in the plastic deformation and fracture of solids.

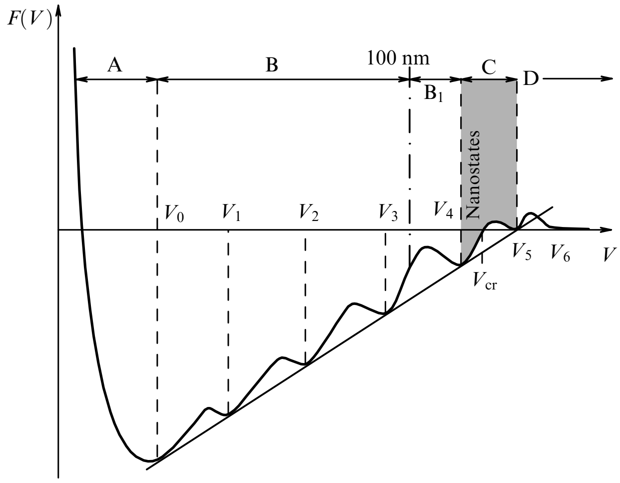

Figure 1. Dependence of the Gibbs thermodynamic potential $F(V)$ on the molar volume V with allowance for local zones of hydrostatic tension of various scales, in which defect structures arise. The regions of different states are as follows: A, hydrostatic compression; B, mesosubstructures of various structural scale levels; B_1, nano-sized structures, C, nanostructured states, and D, the emergence of porosity and fracture.

2. Nonlinear waves of channelled local structural transformations in a planar subsystem as the main mechanism of generating strain-induced defects. Law of plastic deformation flow self-consistency at various structural scale levels

Figure 1 illustrates the thermodynamic foundations of the evolution of the nucleation of strain-induced defects as local structural transformations by the example of the curve tracing the dependence of the Gibbs thermodynamic potential $F = F(V)$ on the molar volume V which is considered as a generalized thermodynamic parameter [3]. It follows from the expression $F = U - TS + PV - \sum_{i=1}^{n} \mu_i C_i$ that with increasing F in a deformable solid body, due to the presence of the terms U and PV, local minima can appear caused by the production of entropy and a redistribution of alloying elements (or impurities). In accordance with the nonequili-

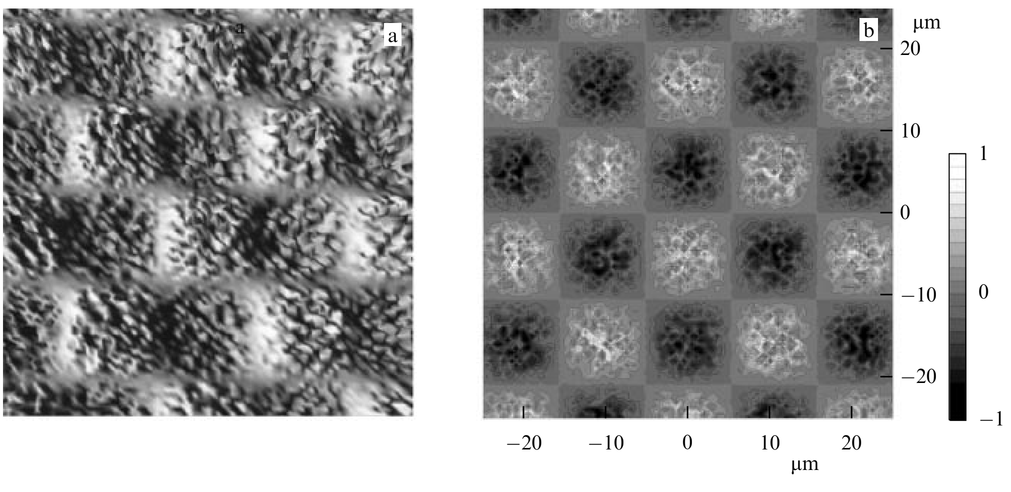

Figure 2. A 'chessboard' deformation profile on the surface of a deformable material; elasticity modulus of the surface layer is $E_s = 0.5E_b$ (where E_b is the appropriate elasticity modulus of the material's bulk). The thickness of the substrate was assumed to be infinitely large as compared to the thickness of the surface layer [7].

brium thermodynamics [6], zones of nonequilibrium states arise in a deformable crystal with increasing V because of the appearance in it of an inhomogeneous mechanical field. In these states, the entire set of strain-induced defects is formed, i.e., dislocations, disclinations, and mesobands and macrobands of localized plastic deformation. Finally, for $V > V_{\mathrm{cr}}$, when the potential $F(V)$ becomes positive, the crystal in local zones of strongly nonequilibrium states loses thermodynamic stability and undergoes a structural phase decomposition. Cracks (or pores) are formed in such zones because of the excess molar volume.

The treatment of a deformable solid as a nonlinear multiscale system made it possible to establish the mechanisms of the formation of local zones of strongly nonequilibrium states, in which strain-induced defects of various scale levels are nucleated [3]. The necessity of self-consistency of shears at different structural-scale levels and the 'chessboard' character of the distribution of tensile and compressive normal stresses at the interfaces of structural subsystems (Fig. 2) [7] is responsible for the propagation of nonlinear waves of channelled local structural transformations in a solid under deformation. The appearance of zones of nonequilibrium states is associated with such nonlinear waves, with relaxation of these states occurring as a result of the generation of strain-induced defects in the crystalline subsystem. The effect of channeling fluxes of local structural transformations on mesoscale levels is a necessary condition for the propagation of multiscale nonlinear waves with allowance for the dissipative process of dislocation motion at the microscale level.

The multiscale development of nonlinear waves of plastic flow was predicted in our theoretical work [8–10]. The results of experimental investigations of such processes were generalized in Refs [3, 11–19].

In Refs [12–14, 18], the nonlinear waves of channelled plastic flow have been studied in the course of uniaxial tension in nanostructured surface layers of planar metallic samples and in thin films deposited onto a substrate. In all cases, nonlinear waves in the form of double spirals have been revealed (Figs 3, 4). Their quantitative treatment made it possible to compare the experimental data with the results of the scaling theory of nonlinear waves [10], which is described in Section 3.

3. Gauge theory of nonlinear waves of channelled local structural transformations

The introduction of dislocations and disclinations into the mechanics of a deformable solid is performed on the basis of the gauge theory of defects [20–22]. In papers [8–10], we suggested considering, as a group of gauge transformations, a simple nine-parameter group of transformations of a real three-dimensional space, GL(3, R); we also introduced sources of Yang–Mills fields — quasielastic microdistortions. The wave equations obtained, when analyzed with allowance for the nonequilibrium thermodynamics of discrete subsystems, make it possible to substantiate, in terms of the multiscale approach, both the wave nature of the channelled plastic deformation and the dissipative process of motion of strain-induced defects on a common structural scale level.

One of the particular cases of wave equations derived in paper [10] concerns the equations for the dimensionless quantities of the flux $\mathbf{J}$ and the density α of linear defects

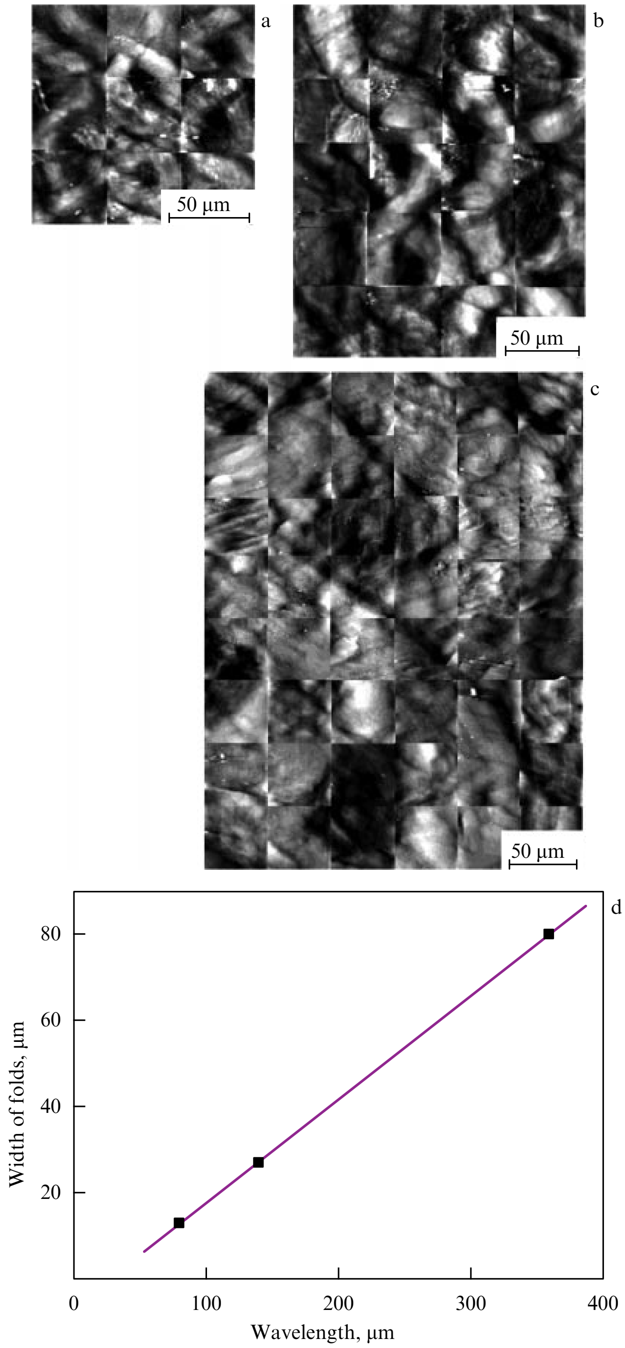

Figure 3. Double spirals of the extruded material in the mesobands of channelled deformation on the surface of planar samples of ferritic–martensitic steel with a nanostructured surface layer subjected to various tension ε: (a) $\varepsilon = 17\%$, the layer thickness is 100 μm; (b) $\varepsilon = 16\%$, the layer thickness is 200 μm, and (c) $\varepsilon = 10\%$, the layer thickness is 300 μm. The temperature reaches 293 K [14]. (d) Linear dependence of the width of the mesoband on the wavelength of double spirals.

(discontinuities of the displacement vector $\mathbf{u}$):

$$\frac{\partial}{\partial x_\alpha} J_\mu^\alpha = -\frac{\partial \ln u_\mu}{\partial t}\,, \tag{1}$$

$$\varepsilon_{\mu\chi\delta} \frac{\partial J_\delta^\alpha}{\partial x_\chi} = -\frac{\partial \alpha_\mu^\alpha}{\partial t}\,, \tag{2}$$

$$\frac{\partial \alpha_\mu^\alpha}{\partial x_\alpha} = 0\,, \tag{3}$$

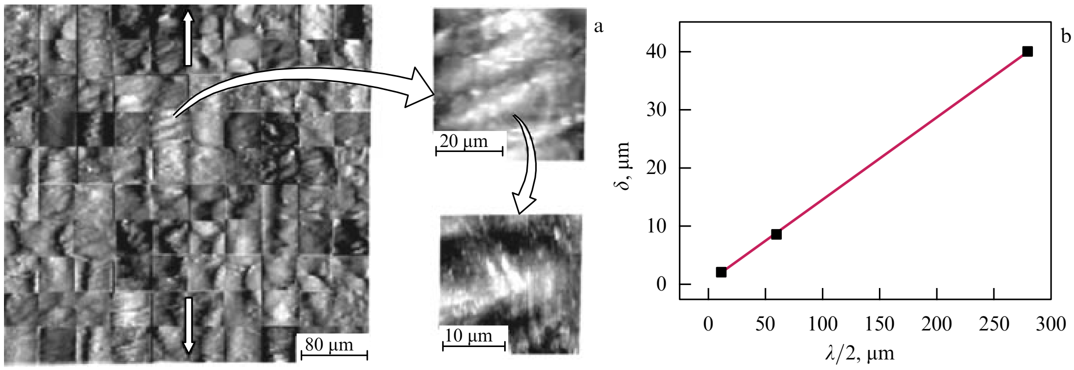

Figure 4. (a) Nonlinear waves in the form of double spirals on the surface of a sample of commercial titanium VT1-0 subjected to tension $\varepsilon = 16\%$ at $T = 293$ K after preliminary ultrasonic treatment and electrolytic hydrogenation for 1 h. The micrographs were obtained using scanning tunneling microscopy [18]. (b) Linear dependence of the thickness of extruded lamellae of various scales on their length.

$$\varepsilon_{\mu\chi\delta}\,\frac{\partial \alpha^{\alpha}_{\delta}}{\partial x_{\chi}} = \frac{1}{\tilde{c}^2}\,\frac{\partial J^{\alpha}_{\mu}}{\partial t} + \sigma^{\alpha}_{\mu} - P^{\beta}_{\nu}\,\frac{C^{\mu\nu}_{\alpha\beta}}{E}\,, \tag{4}$$

$$\frac{1}{c^2}\,\frac{\partial v_{\mu}}{\partial t} = \frac{\partial \sigma^{\alpha}_{\mu}}{\partial x_{\alpha}} - \frac{\partial P^{\beta}_{\nu} C^{\mu\nu}_{\alpha\beta}}{\partial x_{\alpha}\, E}\,, \tag{5}$$

where $v_{\mu} = \partial \ln u_{\mu}/\partial t$ is the rate of the elastic deformation of the medium with defects; $\sigma^{\alpha}_{\mu} = \partial \ln u_{\nu}/\partial x_{\beta}\,(C^{\mu\nu}_{\alpha\beta}/E)$ are the elastic stresses in such a medium; c and $\tilde{c}$ are the speed of sound and the speed of propagation of the plastic disturbance front, respectively; $P^{\beta}_{\nu}(x, t)$ is the plastic part of the distortion; $\varepsilon_{\mu\chi\delta}$ is the Levi–Civita symbol, and, finally, $C^{\mu\nu}_{\alpha\beta}$ are the elastic constants.

Equations (1)–(5) have the following meaning: (1) is the equation of the continuity of the medium with defects, from which it follows that the source of the plastic flow is the rate of the rearrangement of defects; (2) is the condition of the compatibility of plastic deformation [it is fundamentally important that the change in the density of the medium in time is determined in this case by the operation rot of the flux (i.e., by its spatial inhomogeneity), rather than by the operation div]; (3) is the condition of the continuity of defects, which reflects the absence of charges of the vortex component of a field of plastic deformation ($\alpha^{\beta}_{\chi} = \varepsilon_{\chi\mu\nu}\,\partial_{\mu} P^{\beta}_{\nu}$); (4) is the constitutive equation for a medium with plastic flow, and (5) is the equation of the quasielastic equilibrium, which in fact is an equation known in continuum mechanics, but here contains in its right-hand part, apart from the term responsible for elastic deformation, a term that describes plastic distortions, which in fact reflects the generation of strain-induced defects in local zones of hydrostatic extension produced by the stress concentrator.

Equation (4), which is inherent only in a medium with plastic flow, relates the temporary changes in the plastic flux to the anisotropic spatial changes in the densities of defects ($\varepsilon_{\mu\chi\delta}\,\partial \alpha^{\alpha}_{\delta}/\partial x_{\chi}$) and sources ($\sigma^{\alpha}_{\mu} - P^{\beta}_{\nu} C^{\mu\nu}_{\alpha\beta}/E$). The difference of equations (4) and (5) and the corresponding equations of the theory of elasticity lies in the fact that the change in the rate of plastic deformation with time is determined by the stresses themselves rather than by the derivatives $\partial \sigma^{\alpha}_{\mu}/\partial x$, as in the elastic case. In addition, the right-hand part of equation (4) contains, as a source, the plastic distortion $P^{\beta}_{\nu}(x, t)$ itself, which indicates the dual nature of the defects as field sources.

From the set of equations (1)–(5), we can derive wave equations for the dimensionless quantities of the flux density $\mathbf{J}$ and the defect density α:

$$\frac{1}{\tilde{c}^2}\,\frac{\partial^2 J^{\mu}_{\alpha}}{\partial t^2} - \frac{\partial^2 J^{\mu}_{\alpha}}{\partial x^2_{\nu}} = \frac{\partial}{\partial t}\left(\frac{\partial \ln u_{\alpha}(x, t)}{\partial x_{\mu}} - \frac{1}{E}\,\frac{\partial \ln u_{\beta}}{\partial x_{\nu}}\,C^{\mu\nu}_{\alpha\beta} - \frac{1}{E}\,P^{\beta}_{\nu} C^{\mu\nu}_{\alpha\beta}\right), \tag{6}$$

$$\frac{1}{\tilde{c}^2}\,\frac{\partial^2 \alpha^{\mu}_{\alpha}}{\partial t^2} - \frac{\partial^2 \alpha^{\mu}_{\alpha}}{\partial x^2_{\nu}} = \varepsilon_{\mu\chi\sigma}\left(\frac{\partial^2 \ln u_{\beta}(x, t)}{\partial x_{\chi}\partial x_{\nu}}\,C^{\sigma\nu}_{\alpha\beta} - \frac{\partial P^{\beta}_{\nu}}{\partial x_{\chi}}\,C^{\sigma\nu}_{\alpha\beta}\right)\frac{1}{E}\,, \tag{7}$$

under the condition of the compatibility of the sources:

$$\frac{\partial N_{\mu}}{\partial t} + \varepsilon_{\mu lm}\,\frac{\partial M_m}{\partial x_l} = 0\,, \tag{8}$$

where M stands for the right-hand side of Eqn (6), N is the right-hand side of Eqn (7), and $u(x, t)$ are the inelastic displacements in the inelastic localized strain wave.

The right-hand side of equation (6) characterizes the sources of the defect flux, which are determined by the rate of the quasielastic deformation, $\partial/\partial t\,(E^{\alpha}_{\mu} E - E^{\beta}_{\nu} C^{\mu\nu}_{\alpha\beta})\,(1/E)$, where E^{α}_{μ} and E^{β}_{ν} are the spherical and deviator components of the strain tensor, respectively, and $E^{\alpha}_{\mu} E - E^{\beta}_{\nu} C^{\mu\nu}_{\alpha\beta}$ is the difference in the internal stresses of compression–tension and shear, which are related to the distribution of stresses in the zone of the stress concentrator. The relaxation processes of the defect rearrangement (of the type of clusters of various atomic configurations or their conglomerates) are represented in equation (6) by the term $P^{\beta}_{\nu} C^{\mu\nu}_{\alpha\beta}/E$.

The right-hand side of equation (7) characterizes the source of the strain-induced defect density, which is represented by the vorticity $\varepsilon_{\mu\chi\delta}\,\partial/\partial x\,(E^{\beta}_{\nu} - P^{\beta}_{\nu})\,(C^{\mu\nu}_{\alpha\beta}/E)$ of the shear deformation caused by the relaxation of shear stresses upon the generation of strain-induced defects in local zones of hydrostatic tension.

The character of the wave fluxes of strain-induced defects is determined by the right-hand side of equations (6) and (7). The plastic distortion $P^{\beta}_{\nu}(x, t)$ plays a fundamentally important role here.

Prior to the interpretation of equations (1)–(7), note that the wave equations of the plastic flow in solids have also been obtained in papers [20–22], but they have not been interpreted there as plastic waves. The conclusion on the wave character of the propagation of a disturbance in a medium is always connected with the problem of the disturbance group velocity. In the absence of a dispersion of the group velocity, a wave is well defined. The inhomogeneity of a single-scale medium leads to a dispersion and the splitting of a wave packet. Therefore, in terms of the single-scale approach, no plastic waves can arise at all.

However, the conclusion on the existence of nonlinear plastic and fracture waves obtains convincing substantiation when a deformable solid is considered as a multiscale system, with allowance for the presence of planar subsystems in the form of surface layers and internal interfaces. Moreover, it is impossible to ensure beyond the scheme of nonlinear waves the reproduction of stress concentrators upon the propagation of plastic shears as local structural transformations.

Let us consider a localized flux of defects in a planar structure where the deformation along the direction L is developed as channeling between two layers of a crystalline material. Let us choose the common coordinate system so that the z-axis is oriented along L, while the x and y coordinates are varied within the thickness of the deformable layer. According to paper [10], the distribution of the plastic flux in the local ($r < L$) region has the form

$$\mathbf{J} = \frac{b_1 - b_2}{4\pi}\,\chi(s,t)\,\mathbf{b}(s,\mathbf{t}_\mathrm{n})\left(\ln\frac{2L}{r} - 1\right) - \nabla f, \tag{9}$$

where $\mathbf{b}$ is the vector of the binormal in the local coordinate system (perpendicular to the normal to a turn of the wave spiral and to its tangent); χ is the change in the curvature of the region (the change in the curvature of the axis of the region) caused by the external load; $\mathbf{t}_\mathrm{n}$ is the tangent; s is the current value of the region length; b_1, b_2 are the moduli of the so-called Burgers vector of the bulk translational and subsurface or rotational incompatibility, respectively, and ∇f is the gradient part of the flux caused by foreign sources.

Let us determine the change in the shape of the region of the localized deformation flux of length L with the initial dimensions δ. The space–time changes in the shape of $\mathbf{E}(s,t)$ in the process of deformation can be found from the equation

$$\mathbf{J} = \frac{\partial \mathbf{E}(s,t)}{\partial t}\,. \tag{10}$$

Using the expressions for $\mathbf{J}$ and making a replacement $t' \to t(b_1 - b_2)/(4\pi)[\ln(2L/r) - 1]$, we arrive at

$$\frac{\partial \mathbf{E}(s,t)}{\partial t} = \chi\mathbf{b} - \frac{4\pi}{(b_1 - b_2)[\ln(2L/r) - 1]}\,\nabla f. \tag{11}$$

The first term on the right-hand side of equation (11) describes the curvature of the defect flux (its vorticity). By solving equation (11) simultaneously with the equation $\partial\mathbf{E}/\partial s = \mathbf{t}$ and with Frenet equations [23], it can be shown that the change in the shape of the region under consideration is described as follows:

$$E_x(s,t) = -\frac{2}{\beta(v^2+1)}\left\{\operatorname{sech}\left[2\beta(s+4vt)\right]\sin\left[2\beta(s+4vt)\right] - \operatorname{sech}(8\beta vt)\,\sin(8\beta v vt)\right\}, \tag{12}$$

$$E_y(s,t) = -\frac{2}{\beta(v^2+1)}\left\{\operatorname{sech}\left[2\beta(s+4vt)\right]\cos\left[2\beta(s+4vt)\right] - \operatorname{sech}(8\beta vt)\,\cos(8\beta v vt)\right\}, \tag{13}$$

$$E_z(s,t) = s - \frac{1}{\beta(v^2+1)}\left\{\tanh\left[2\beta(s+4vt)\right] - \tanh(8\beta vt)\right\}. \tag{14}$$

Equations (12)–(14) govern the change in the shape of the region whose axis is a spiral curve with a constant torsion $\tau = 2v$. In these equations, $v = -v/\beta$, where v characterizes the velocity of the displacement of local structural transformations in the region of the spiral curve along the direction L, and the parameter β is related to the curvature χ as follows:

$$\chi(s,t) = 4\beta\operatorname{sech}\left[2\beta(s+4vt)\right]. \tag{15}$$

The curvature χ of the spiral is an important parameter of the channelled wave propagation of the localized plastic flow. The influence of this parameter on the shape of the spiral and on the local velocity v of the transverse change in the shape of the deformable region is illustrated in Fig. 5. As is seen from Fig. 5a, the velocity v of the transverse change in the shape is small at a slight curvature χ, and the spiral suffers a weakly pronounced torsion with a large transverse wavelength. Such a picture is observed when the plastic shear is developed under strongly nonequilibrium conditions, e.g., in nanostructured surface layers. With increasing curvature χ, the transverse wavelength decreases sharply, and the rates of the transverse shape changes increase (Fig. 5b). This is a very important effect, since in the zones of strong curvature a sharp increase is observed in the local effective potential $U(V,\alpha)$ in the expressions for the nonequilibrium Gibbs thermodynamic potential $F(V,\alpha) = F(V) - U(V,\alpha)$, where α characterizes the field of disturbances caused by the local violation of the translational invariance of the crystal [6]. This leads to a decrease in $|F(V,\alpha)|$, an increase in the molar volume V in the zone of curvature, and an increase in the rates of all types of atomic redistributions. After the condition $V > V_\mathrm{cr}$ is

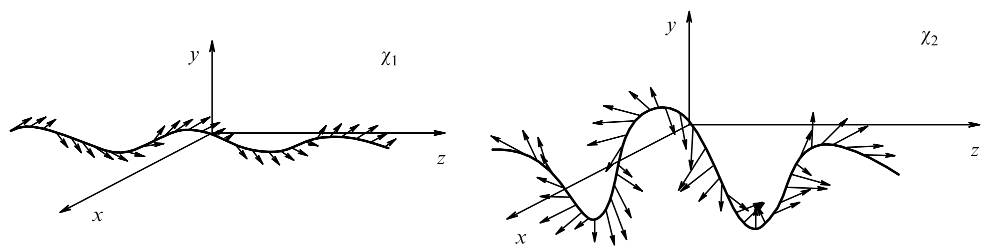

Figure 5. Dependences of the shape and rate of localized plastic deformation on the curvature χ of the region under deformation; $\chi_1 < \chi_2$.

reached, a structural–phase decomposition of the crystalline state occurs in the zone of strong curvature, and the material undergoes fracture. The manifestation of this effect is widely known in engineering practice.

4. Experimental verification of predictions of the gauge theory of nonlinear waves of channelled plastic deformation

Figure 3 displays nonlinear plastic waves that were channelled in nanostructured surface layers of planar samples of a ferritic–martensitic steel under conditions of uniaxial tension [14]. The observation of these waves makes it possible to perform an experimental verification of the predictions of the gauge theory of wave deformation in planar subsystems. A change in the thickness of the nanostructured surface layer leads to a change in both the nonlinear wavelength λ and the width δ of the spiral channelled flux. From formula (9), an expression that relates δ to λ can easily be obtained. According to paper [10], one has

$$\delta = L \exp\left[-\frac{4\pi(\nabla f \mathbf{b})}{\chi(b_1 - b_2)}\right]. \tag{16}$$

At a given counter field ∇f from the side of the crystalline substrate, we obtain a scalar product $\nabla f \mathbf{b} = 0$ if $\nabla f \perp \mathbf{b}$. This condition corresponds to the double spirals of the extruded material shown in Fig. 3. This statement is especially well illustrated by the structure of nonlinear waves in titanium samples with a surface layer enriched in hydrogen [18]. The mechanism of extrusion of the surface layer material in a nonlinear wave has been investigated in Ref. [18] in the course of a uniaxial tension of planar samples of polycrystalline titanium whose surface layers were nanostructured and enriched in hydrogen. Titanium has a very low shear stability (a stacking-fault energy of only 10 mJ m^{-2}). The nanostructuring and hydrogenating of a surface layer additionally reduce this shear stability.

The use of scanning tunneling microscopy permitted us [18] to reveal the mechanism of the extrusion of the material in a channelled nonlinear wave. It is seen from Fig. 4 that the wave extrusion of the material occurs as a result of the mutual displacements of individual lamellae, with each lamella being extruded via mutual displacements of even finer transverse lamellae. The binormal to each lamella is perpendicular to the sample's plane and, consequently, to the direction of the counter field ∇f from the side of the crystalline substrate. These data indicate a hierarchically organized structure of the nonlinear wave of the extrudible material. Correspondingly, a linear dependence should exist between the quantities δ and λ at each scale level. This is indeed confirmed experimentally.

Figure 3d displays the $\delta = f(\lambda)$ dependence calculated from the data of Ref. [14] for a ferritic–martensitic steel. This dependence corresponds to a straight line $\delta = k\lambda$ with a coefficient $k = 0.75$. The three scales of nonlinear waves shown in Fig. 4 are also described by a linear dependence $\delta = k_1\lambda$ (Fig. 4b) if the length of the lamella is assumed to be $1/2\lambda$, and its thickness to be δ. The coefficient $k_1 = 0.17$ is a factor of 4.4 less than the coefficient k for steel. This means that in titanium with its low shear stability a mesoband of a given power at a distance equal to its wavelength λ is capable of traveling a distance that is a factor of 4.4 greater than an analogous mesoband at a distance equal to its wavelength in high-strength steel. This regularity is also confirmed by the characteristics of nonlinear waves in nanostructured surface layers of other materials.

A good agreement between the predictions of the gauge theory of nonlinear waves of channelled structural transformations in planar subsystems and the experimental data indicates the validity of the conception of the authors of this paper focused on the necessity of representation of surface layers and internal interfaces as the leading functional subsystem in a deformable solid. The theoretical and experimental investigations of nonlinear waves of channelled plastic flow in planar subsystems with allowance for the well-developed theory of strain-induced defects in crystals opens the way for constructing a general theory of a deformable solid as a nonlinear hierarchically organized system [4].

5. Nonlinear wave processes of fracture

In general, the fracture of a solid constitutes a dissipative process. A crack represents a rotation deformation mode on a macroscale level. According to the law of conservation of angular momentum, this mode should be equal to the sum of rotation modes at lower scale levels. In a continuum medium, this is a dissipative process.

However, if we create conditions for a channelled propagation of a crack and maximally localize the dissipation effects, the crack will propagate via a nonlinear wave mechanism. This follows from the nonequilibrium thermodynamics of its propagation. The opening of a crack is caused by the structural phase decomposition of the material ahead of its tip, and this is a threshold process. The propagation of a crack is related to the relaxation of the stress concentrator in its tip. The growth of a crack should periodically cease for the restoration of the stress concentrator and the establishment of the state of structural phase decomposition in a new zone of the material. This is a typical nonlinear wave process which can be described by equation (6) if we neglect the term $P_\nu^\beta C_{\alpha\beta}^{\mu\nu}/E$ in it. An increase of the term $\partial \ln u_\alpha(x,t)/\partial x_\mu$ related to normal stresses will be periodically compensated for by quasielastic forces that are represented by the second term, $(\partial \ln u_\beta/\partial x_\nu)\, C_{\alpha\beta}^{\mu\nu}/E$. This will cause a nonlinear wave character of the propagation of the crack with periodic stops. Naturally, the rate of change of the right-hand part of Eqn (6) should be small for revealing such wave processes, i.e., the nonlinear fracture waves should be slow.

Such wave processes of a channelled fracture were described in paper [17] for the case of the propagation of fatigue cracks in two-layer composites. At first, zigzag mesobands of localized plastic deformation developed at the interface of unlike media in the course of their cyclic alternate bending. Then, a channelled fatigue crack of shears–rotations propagated as a nonlinear wave process in one of the mesobands.

In recent work [24], the channeling of fractures and the minimization of dissipation processes were realized in the case of tension of planar samples of submicrocrystalline titanium with a chevron notch (Fig. 6a). Under the conditions of uniaxial tension, an opening mode crack was nucleated in the tip C of a thin planar layer in the form of a chevron notch, which propagated in a channeling mode along the longitudinal section of the sample. The high degree of departure from thermodynamic equilibrium of the submicrocrystalline state caused rapid structural phase decomposition of the material in the region ahead of the crack tip. Further in this region, a transverse fracture wave propagated, which shifted

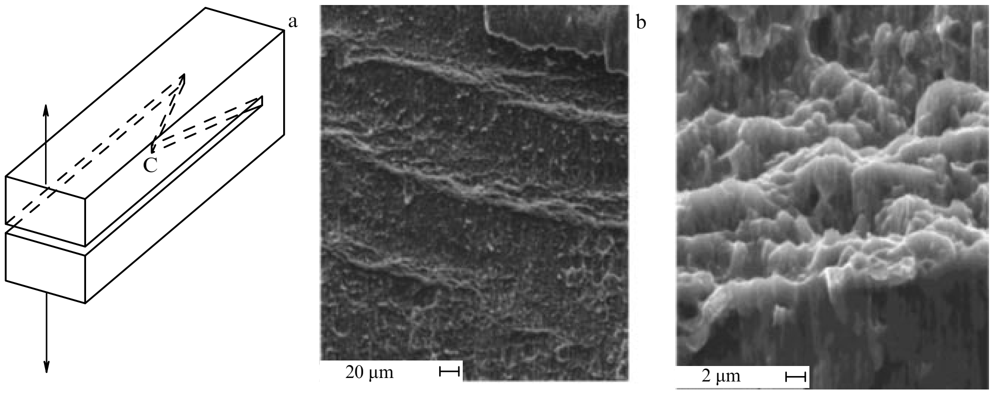

Figure 6. Nonlinear waves on the fracture surface after tensile test of a sample with a chevron notch [24]: (a) shape of the sample; (b) periodic white transverse bands of the material which underwent structural phase decomposition, and (c) porous nanostructure of a transverse band of the material. The micrographs were obtained using scanning electron microscopy.

the decomposition products of the crystalline material toward the periphery of this region (Fig. 6b). The porous nanostructure of the material that underwent a structural phase decomposition and was rejected by the fracture wave to the periphery of the transverse band is demonstrated in Fig. 6c. This material exhibits secondary-electron emission and can thus easily be detected. These results are of important value for the explanation of the fracture mechanism in solids.

Such nonlinear fracture wave processes are characteristic of many nanostructured objects (multilayer nanostructured coatings, thin-film structures in microelectronics, nanostructured surface layers of a functional designation in materials science, etc.). These wave processes can be controlled on the basis of the theory developed in paper [10].

6. Conclusions

It is suggested that a deformable solid be treated as a nonlinear hierarchically organized system consisting of two self-consistent subsystems. The deformation of a three-dimensional translationally invariant crystalline subsystem is described on the basis of the theory of strain-induced defects. In this case, local structural transformations in the cores of the strain-induced defects and an increase in the magnitude of departure from thermodynamic equilibrium of the deformable crystal should be taken into account. The surface layers and all internal interfaces should be considered as an independent planar nonlinear subsystem with a disrupted translational invariance rather than as planar defects in a three-dimensional crystal. The primary plastic shears in a loaded solid are related to the nonlinear waves of channelled structural transformations in a planar subsystem rather than to dislocations. The propagation of nonlinear channelled waves is accompanied by a periodic generation of strain-induced defects in the zones of strong curvature, whose emission into the crystalline subsystem provides a plastic change of its shape.

A theoretical and experimental substantiation of the conception developed is given. It is shown that the theory of nonlinear waves of channelled plastic deformation developed in paper [10] satisfactorily describes the regularities of the development of nonlinear wave processes which determine the law of self-consistency of plastic flow in multiscale hierarchically organized systems. The violation of such self-consistency leads to the fracture of a loaded solid. The nonequilibrium thermodynamics of fracture is related to the structural phase decomposition of the condensed state of a solid in regions where the Gibbs thermodynamic potential proves to be positive. The channelled propagation of cracks in multiscale systems is also developed as a nonlinear wave process.

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