#### METHODOLOGICAL NOTES

### Angular beam width of a slit-diffracted wave with noncollinear group and phase velocities

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DOI: 10.3367/UFNe.0182.201212e.1327

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<u>Abstract.</u> Taking magnetostatic surface wave diffraction as an example, this paper theoretically investigates the 2D diffraction pattern arising in the far-field region of a ferrite slab in the case of a plane wave with noncollinear group and phase velocities incident on a wide, arbitrarily oriented slit in an opaque screen. A universal analytical formula for the angular width of a diffracted beam is derived, which is valid for magnetostatic and other types of waves in anisotropic media and structures (including metamaterials) in 2D geometries. It is shown that the angular width of a diffracted beam in an anisotropic medium can not only take values greater or less than  $\lambda_0/D$  (where  $\lambda_0$  is the incident wavelength, and *D* is the slit width), but can also be zero under certain conditions.

#### 1. Introduction

Waves of different physical natures propagating through various media and structures, as is well known, obey common physical laws (see, for instance, books [1–4]). In particular, the application of momentum and energy conservation laws in the first-approximation description of the propagation, reflection, and refraction of waves in homogeneous isotropic and anisotropic media permitted revealing

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Received 4 April 2011, revised 5 September 2011 Uspekhi Fizicheskikh Nauk **182** (12) 1327–1343 (2012) DOI: 10.3367/UFNr.0182.201212e.1327 Translated by E N Ragozin; edited by A Radzig several common laws: the laws of geometrical optics for isotropic media (see, for instance, review [5]) and, for anisotropic media, the laws determined by the mathematical properties of the wave isofrequency dependence,<sup>1</sup> e.g., the inherence of mathematical attributes like symmetry axes, asymptotes, inflection points, and extreme points in the dependence (for more details, see Ref. [5]).

In the description of spatially limited wave processes and diffraction phenomena in homogeneous isotropic media, it has also been possible to reveal several laws common for waves of different physical natures. Among the best known results undoubtedly is the formula describing the diffracted beam angular width in the incidence of a plane wave on a slit in an opaque screen as the ratio of the initial wavelength  $\lambda_0$  to the slit width *D*. As is commonly known, the  $\lambda_0/D$  ratio defines the Rayleigh resolution criterion which plays an important role in estimation calculations and interpretation of physical effects in isotropic media (see, for instance, monograph [1]). Naturally, this brings up the question: Is it possible to derive a similar universal formula for describing the diffracted beam angular width, at least for two-dimensional geometries<sup>2</sup> of anisotropic media?

Diffraction phenomena in anisotropic media have been studied primarily by the examples of electromagnetic waves in plasmas (see monograph [2, Chapters 7 and 8] and references cited therein), light in optical crystals [2, 6, 7], acoustic waves

<sup>&</sup>lt;sup>1</sup> Notice that several papers also introduce the terms 'isofrequency', 'section of the isoenergy surface', 'equifrequency line', 'section of the wave vector surface', etc. to denote the wave isofrequency dependence (for more details, see Ref. [5, Section 4]).

 $<sup>^2</sup>$  The term 'two-dimensional geometries' will be used in reference to different anisotropic plane-parallel structures and those special geometry cases realized in three-dimensional anisotropic media wherein the plane of incidence coincides with one of the symmetry planes of the medium isofrequency surface (for more details, see Ref. [5]).

[8–10], and dipole spin waves (see Refs [11–13] and references cited therein), which are usually referred to as magnetostatic waves (MSWs) [14]. To date, however, an analytical dependence for the far-field diffracted beam angular width has not been obtained for any of these waves.

In this paper, an attempt is made to fill in this gap. In Sections 2–9, in particular, by the example of an MSW propagating in a ferrite slab, a theoretical investigation was made of the two-dimensional far-field diffraction pattern emerging in the incidence of a plane MSW on a wide slit in an opaque screen in the most general case, when the group and phase velocities of the initial wave are noncollinear and the orientation of the screen is arbitrary.

MSWs, which are efficiently excited and propagate in different ferrite structures, are a rather convenient subject both for theoretical and for experimental investigations of diffraction phenomena in anisotropic media. Owing to the low phase velocity of MSWs, their wavenumber k at microwave frequencies is on the order of  $10-10^4$  cm<sup>-1</sup> (wavelength  $\sim 10-10^4 \,\mu$ m), which is many times greater than the wavenumber  $k_0$  of an electromagnetic microwave wave in a vacuum  $(k \gg k_0 \equiv \omega/c \sim 1 \text{ cm}^{-1})$ . This circumstance permits describing the MSW characteristics in the magnetostatic approximation [14] (neglecting the terms  $\sim \partial/\partial t$  in the Maxwell equations and applying the equations of magnetostatics). This makes it possible to analytically investigate the properties and dispersion of these waves in different structures for an arbitrary propagation direction of a wave with noncollinear group and phase velocities.

Historically, the investigation of diffraction phenomena in the propagation of MSWs sprang from several experimental and theoretical papers concerned with MSW propagation through periodic inhomogeneities produced by different methods in a ferrite slab (see, for instance, Refs [12, 13, 15– 21]).<sup>3</sup> More recently, papers have appeared which formulate diffraction problems for MSWs [25, 26], which are concerned with the methods for the solution of the parabolic equation [27–30], and which investigate the diffraction of an MSW beam limited in width and excited by a finite aperture radiator [11, 12, 31–36]. In the last-named studies, investigations were primarily made of the case in which an exciting linear transducer of finite length was oriented parallel to one of the symmetry axes of the MSW isofrequency dependence.<sup>4</sup>

An analysis of the literature, therefore, shows that the farfield diffracted beam parameters in the diffraction from a wide slit in an opaque screen have so far remained unexplored for a plane MSW with noncollinear group and phase velocities.

## 2. Basic relations describing the propagation of a magnetostatic wave through a ferrite slab

Consider an infinite plane-parallel ferrite slab (or film) 2 (Fig. 1) of thickness *s*, which is magnetized to saturation by a tangent uniform magnetic field  $H_0$  and surrounded by vacuum half-spaces *l* and *3*. The fields in media *l*–*3* or their parameters will be marked with the corresponding subscripts

**Example 1.** Arrangement of a ferrite slab in a Cartesian system of coordinates  $\Sigma_D$  associated with the field  $\mathbf{H}_0$ : *I* and *3* are vacuum half-

j = 1, 2, 3. We introduce a Cartesian system of coordinates  $\Sigma_{\rm D} = \{x, y, z\}$  with the *z*-axis parallel to vector  $\mathbf{H}_0$ , and with the *x*-axis perpendicular to the slab surface. In the description of the wave processes which depend on the time as  $\sim \exp(i\omega t)$ , the ferrite slab in the selected coordinate system is characterized, as is well known, by permeability tensor  $\mu_2$  with diagonal and off-diagonal components  $\mu$  and  $\nu$ , whose dependences on the electromagnetic oscillation frequency  $\omega = 2\pi f$ , the magnitude of  $H_0$ , and the ferrite slab saturation magnetization  $4\pi M_0$  can be found in Refs [12, 14, 37]. The half-spaces I and 3 will be assumed to have permeabilities  $\mu_{1,3} = 1$ .

Since the problem of the MSW propagation in a ferrite slab has been repeatedly solved [12, 14], below we only outline briefly the information and relations required for the subsequent discussion.

In the solution to this problem, we shall apply the Maxwell equations in the magnetostatic approximation: rot  $\mathbf{h} = 0$ , and div  $\mathbf{b} = 0$ . By introducing the magnetic potential  $\Psi_j$  for each of the media (j = 1, 2, 3) in accordance with relationship  $\mathbf{h}_j = \text{grad } \Psi_j$  and proceeding from the continuity conditions for the magnetic potential and the normal component of magnetic induction  $\mathbf{b}$  at the media interfaces, it is possible to derive the differential equations for the potential inside and outside the ferrite slab, find the coordinate dependences of the magnetic potential:

$$\begin{aligned} \Psi_1 &= C \exp(-k_{1x}x - ik_yy - ik_zz), \\ \Psi_2 &= \left[G \exp(k_{2x}x) + B \exp(-k_{2x}x)\right] \exp(-ik_yy - ik_zz), \quad (1) \\ \Psi_3 &= F \exp(k_{3x}x - ik_yy - ik_zz), \end{aligned}$$

and obtain the dispersion relation<sup>5</sup> describing the propagation of a magnetostatic surface wave (MSSW) in the ferrite slab plane:

$$\mu^{2}k_{2x}^{2} - \nu^{2}k_{y}^{2} + k_{1x}^{2} + 2\mu k_{1x}k_{2x} \coth(k_{2x}s) = 0, \qquad (2)$$

where  $k_{1x}$ ,  $k_{2x}$ ,  $k_{3x}$ ,  $k_y$ , and  $k_z$  are the wave vector components along the coordinate axes ( $k_{1x}$ ,  $k_{2x}$ , and  $k_{3x}$  are positive numbers), which are given by the relations

$$k_{1x} = \sqrt{k_y^2 + k_z^2},$$
  

$$k_{2x} = \sqrt{k_y^2 + \frac{k_z^2}{\mu}},$$
  

$$k_{3x} = k_{1x}.$$
(3)

<sup>5</sup> A more detailed derivation of the dispersion relation for an MSSW can be found in Refs [12, 14, 37], the notation employed in Ref. [37] coinciding with the notation of our work. A table with dispersion relations for different structures can also be found in paper [37].

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spaces, 2 is the ferrite slab.



<sup>&</sup>lt;sup>3</sup> Recently, investigations of MSWs in spatially periodic structures termed magnonic or photonic crystals have received a new impetus in connection with the emergence of interest in the properties of metamaterials [13, 22–24].

<sup>&</sup>lt;sup>4</sup> Exceptions are provided by Refs [31, 33], in which measurements were made of the near-field profile of an MSW beam excited by an arbitrarily oriented (relative to the external magnetic field) limited-length radiator.

In the ferrite slab plane, we also introduce a polar coordinate system  $\Sigma_p = \{x, r, \varphi\}$  corresponding to the Cartesian system  $\Sigma_D$ , so that angles  $\varphi$  are measured from the *y*-axis, which is the axis of collinear propagation for an MSSW. In this case, it is assumed that positive angles are counted counterclockwise. The coordinates of the  $\Sigma_p$  and  $\Sigma_D$  systems are related through the equalities  $y = r \cos \varphi$ , and  $z = r \sin \varphi$ . The wavenumbers  $k_{1x}$ ,  $k_{2x}$ ,  $k_y$ , and  $k_z$  used in the  $\Sigma_D$  system are related to the modulus of the wave vector **k** used in the  $\Sigma_p$  system as follows:

$$k_y = k \cos \varphi \,, \tag{4}$$

$$k_z = k \sin \varphi \,,$$

$$k_{2x} = \alpha k , \qquad (5)$$

$$k_{1x} = k \,,$$

where

$$\alpha = \sqrt{\cos^2 \varphi + \frac{\sin^2 \varphi}{\mu}} = \sqrt{\frac{(\mu - 1)\cos^2 \varphi + 1}{\mu}}.$$
 (6)

Substituting expressions (4), (5) into coordinate dependences (1), it is possible to write, with the inclusion of the time dependence  $\sim \exp(i\omega t)$ , the expressions for the magnetic potential  $\Psi_j$  inside and outside the film in the coordinate system  $\Sigma_p$  in the following form

$$\Psi_j = \Psi_{jx} \exp\left(\mathrm{i}\omega t - \mathrm{i}kr\right),\tag{7}$$

where  $\Psi_{jx}$  are the *x*-dependent amplitudes of the potential in the *j*th medium.

In view of expressions (4)–(6), the dispersion relation (2) in the polar coordinate system takes on the form (see paper [37])

$$\frac{1}{\mu} + \mu_{\perp} \cos^2 \varphi + \sin^2 \varphi + 2\alpha \coth(\alpha ks) = 0, \qquad (8)$$

where  $\mu_{\perp} = (\mu^2 - v^2)/\mu$ .

In addition to the polar angle  $\varphi$  describing the orientation of the MSSW wave vector **k**, we introduce another polar angle in the ferrite slab plane — an angle  $\psi$ , which describes the orientation of the MSSW group (ray) velocity vector **V**. As is well known, the vector **V** is perpendicular to the isofrequency curve at the point corresponding to the wave vector **k** (see, for instance, Refs [5, 12]). Finding the angle  $\psi$  from the known dispersion relation represents a cumbersome procedure,<sup>6</sup> but not a sophisticated task. We do not reproduce it here and consider the dependence  $\psi(\varphi)$  to be known.

#### 3. Formulation of diffraction problem

We now address the investigation of diffraction phenomena.

We shall highlight the diffraction pattern when a plane MSW is incident on a slit of width D in an infinitely thin screen, which is opaque for the MSW. Let the incident wave have a frequency  $\omega_0 = 2\pi f_0$ , a wave vector  $\mathbf{k}_0$ , and a group velocity vector  $\mathbf{V}_0$ ,  $\mathbf{k}_0$  and  $\mathbf{V}_0$  being noncollinear vectors (hereinafter such a wave will be termed noncollinear) arbitrarily oriented relative to the slit line (Fig. 2).

A typical isofrequency dependence for an MSSW with frequency  $f_0$  in the plane of wave numbers is given in Fig. 3 (curves *I* and *2*), which also shows the point S corresponding



**Figure 2.** General geometry of the incidence of a plane MSSW on a slit in an opaque screen. The dashed lines depict the wave fronts of the initial wave. Indicated for an arbitrary elementary slit portion dl with coordinate l are the phase incursion  $k(\varphi')l\sin\varphi'$ , which arises due to the difference between the distances  $r_l$  and  $r_1$ , and the phase incursion  $k_0l\sin\varphi'_0$ , which is due to the fact that the initial wave does not simultaneously reach the elementary exciters with coordinates 0 and *l*. The corresponding geometry in the wavenumber plane is illustrated in Fig. 3. As one can see from this figure, the direction  $\varphi' = 12^\circ (\varphi = 32^\circ)$  to the point P<sub>k</sub> corresponds to a direction  $\psi' = -72^\circ (\psi = -52^\circ)$  to a point P<sub>V</sub> (if the corresponding vectors **k** and **V** are plotted in the isofrequency dependence).

to the initial noncollinear MSSW. Also shown are the vectors  $\mathbf{k}_0$ ,  $\mathbf{V}_0$  and the corresponding angles  $\varphi_0$  and  $\psi_0$ , which describe the orientation of these vectors in the coordinate system  $\Sigma_D$ . The vector  $\mathbf{k}_0$  is inclined to the vector  $\mathbf{V}_0$  by an angle  $\chi_0$  defined by the formula

$$\chi_0 = \varphi_0 - \psi_0 \,. \tag{9}$$

Below, we shall also use the Cartesian  $\Sigma'_D = \{x, y', z'\}$ and polar  $\Sigma'_p = \{x, r, \varphi'\}$  coordinate systems, which are rotated about the x-axis by the angle  $\theta$  (see Figs 2 and 3) relative to the systems  $\Sigma_D$  and  $\Sigma_p$ , so that the y'-axis is perpendicular to the slit line. The angles  $\varphi'$  and  $\psi'$ , which define the orientation of an arbitrary wave vector **k** and a group velocity vector **V** in the system  $\Sigma'_p$ , are related to the similar angles  $\varphi$  and  $\psi$  in the system  $\Sigma'_p$ , are related to the similar angles  $\varphi$  and  $\psi' = \psi - \theta$ . In accordance with the Huygens principle, it will be assumed that the incidence of a noncollinear plane MSW on the slit gives rise to secondary point-like MSW sources along the slit line, and that estimating their action at a distant point requires integrating (i.e. calculating the superposition of) the contributions from all infinitely small elements (secondary MSW sources) located on the slit.

Since the magnetic potential  $\Psi$  of an MSW is a *scalar* quantity and all curves of the MSW isofrequency dependence describe waves of the *same polarization*, to calculate the resultant field of the secondary sources at a distant point of observation one simply needs to sum up the magnetic

<sup>&</sup>lt;sup>6</sup> This procedure is described at length, for instance, in Refs [12, 38].



Figure 3. General geometry in the wavenumber plane for the incidence of a noncollinear MSSW on a screen.

potentials of the perturbations at this point caused by all secondary MSW sources located along the slit line.

It should be emphasized that we are always dealing with two directions,  $\varphi$  and  $\psi$  (or  $\varphi'$  and  $\psi'$  in the coordinate systems  $\Sigma'_D$  and  $\Sigma'_p$ ), which define the respective orientations of the wave vector  $\mathbf{k}$  and the group velocity vector  $\mathbf{V}$  of the wave. This distinguishes the problem under consideration from the similar diffraction problem for isotropic media. In this case, the conditions that define the constructive interference of secondary MSW sources will be written for their wave vectors, i.e., for the direction  $\varphi$  (or  $\varphi'$ ), but the wave energy transfer in the event of this constructive interference will be effected not in the direction  $\varphi$  (or  $\varphi'$ ), but in the direction  $\psi$  (or  $\psi'$ ) of the corresponding group velocity vector. Therefore, to describe the diffraction problem under investigation, we introduce two points sufficiently distant from the slit:  $P_k$  and  $P_V$ , such that the direction to the point  $P_k$  will coincide with the orientation  $\varphi$  of the wave vectors of the secondary MSW sources, while the direction to the point  $P_V$  will be aligned with the orientation  $\psi$  of the corresponding group velocities of the secondary MSW sources.

To facilitate our consideration, the dependences  $k(\varphi)$  and  $\psi(\varphi)$  are assumed to be *unique* (i.e., a single ordinate value corresponds to every value of the argument). Our treatment will be performed by the example of an MSSW propagating in a free ferrite slab. Such a wave is always characterized by unique dependences  $k(\varphi)$  and  $\psi(\varphi)$ , with the  $k(\varphi)$  dependence for this wave being explicitly expressed from dispersion relation (8) written in the polar coordinate system. From the single-valued character of the  $\psi(\varphi)$  dependence, it follows that a *single* point P<sub>V</sub> will correspond to every point P<sub>k</sub>. For the sake of simplicity we also assume that the inverse relationship  $\varphi(\psi)$  is also single-valued, i.e., a single point P<sub>k</sub> we note that in what follows it

will be possible to abandon the assumption of the singlevalued behavior of the dependences  $k(\varphi)$  and  $\psi(\varphi)$ , as well as of the dependence  $\varphi(\psi)$ , and to consider the consequences of the ambiguity of these dependences (see Section 7).

As is well known, to precisely calculate the direction of electromagnetic wave energy transfer requires finding the Poynting vector. However, in those cases when calculating the Poynting vector is a very arduous task (as with a noncollinear MSW), this calculation can be performed with sufficiently high accuracy by taking advantage of the group velocity vector. The notion of group velocity is fully applicable to an MSW propagating in a free ferrite slab, because the conditions of small damping and sufficiently weak dispersion are fulfilled for these waves (for more details on this issue, see Lektsii po Nekotorym Voprosam Teorii Kolebanii (Lectures on Some Issues of the Theory of Oscillations) by Mandelstam in book [39]). Furthermore, in several papers it was confirmed that the calculated direction of the group velocity corresponded to the measured direction of MSW energy propagation [31, 40-47]. It is noteworthy that a similar correspondence was also established in the investigation of noncollinear acoustic wave propagation (see, for instance, Refs [48, 49]). Therefore, it is safe to say that the direction of the group velocity vector adequately describes the direction of propagation of the wave power flux.

# 4. Expression for the total magnetic potential arising when a plane magnetostatic wave is incident on a slit

For simplicity of reasoning, we temporarily fix the xcoordinate when considering the two-dimensional problem in the (y', z') plane on that surface of the ferrite slab near which the initial MSSW is localized.

We introduce an auxiliary coordinate *l* along the slit line, so that l = 0 at the left slit edge, and l = D at the right one, where *D* is the slit length. We divide the slit line into a set of *equal elementary* portions of length *dl*, each of which may be treated as a secondary wave source producing an elementary magnetic potential  $d\Psi_j$ . All elementary sources radiate with different phases, since the initial MSSW is inherently noncollinear and its phase fronts, which are shown with dashed lines in Fig. 2, are not parallel to the slit line. Furthermore, since the medium is anisotropic, every direction  $\varphi'$  corresponds to its own wavenumber *k*, because *k* is a function of the angle  $\varphi'$ :  $k = k(\varphi')$ .

So then, let us find the total magnetic potential  $\Psi_j$  at an arbitrary<sup>7</sup> distant point  $P_V$ , which corresponds to a polar angle  $\psi'$ . This potential is the result of the interference of all secondary waves whose wave vectors are oriented in the  $\varphi'$  direction to a distant point  $P_k$ (which corresponds to the point  $P_V$  and the angle  $\psi'$ ) and is the sum of the secondary potentials  $d\Psi_j$ , with every slit element d/ exciting a potential  $d\Psi_j$  with an amplitude  $\Psi(r, \varphi')/D = \Psi(r)\Psi(\varphi')/D = \Psi_r\Psi_{\varphi}/D$ . To calculate the  $\Psi_j(\psi')$  dependence, one must first find the dependence  $\Psi_j(\varphi')$  and then put the value of  $\psi'$  in correspondence to

<sup>&</sup>lt;sup>7</sup> We emphasize that the arbitrary manner of selecting the point  $P_V$  or directions  $\psi$  and  $\psi'$  in the case under consideration (when the isofrequency dependence of the wave has cutoff angles) is limited: the  $\psi$  (or  $\psi'$ ) direction must be such that it can be aligned with the group velocity vector of at least one point S lying in the isofrequency curves, because the energy transfer in other directions  $\psi$  is evidently impossible, and for them we must put  $\Psi_j \equiv 0$ .

$$\Psi_{j} = \int d\Psi_{j} = \int_{0}^{D} \frac{1}{D} \Psi_{jx} \Psi_{r} \Psi_{\varphi}$$

$$\times \exp\left(i\omega_{0}t + i\beta_{1} + ik_{0}l\sin\varphi_{0}' - ik(\varphi')r_{1} - ik(\varphi')l\sin\varphi'\right) dl.$$
(10)

Here, all terms (except  $i\omega_0 t$ ) in the exponent describe the phase incursions that emerge as the MSSW propagates (see Fig. 2):  $\beta_1$  characterizes the phase with which the initial wave arrives at the first elementary source of secondary waves on a slit with coordinate l = 0; the quantity  $k_0 l \sin \varphi'_0$ describes the phase difference between the elementary secondary source with an arbitrary coordinate l and the elementary secondary source with coordinate l = 0 (this phase difference emerges due to the fact that the initial MSSW does not simultaneously reach all elements of the slit); the quantity  $-k(\varphi')r_1 - k(\varphi')l\sin\varphi'$  stands for the phase incursion occurring over the distance  $r_l$  between the point  $P_k$  and the arbitrary elementary secondary source dl on a slit with coordinate l, with the first term,  $-k(\varphi')r_1$ , describing the phase incursion between the point  $P_k$  and the elementary secondary source with coordinate l = 0, and the second term describing the phase incursion due to the difference between the distances  $r_l$  and  $r_1$ . The phase incursions listed above should be calculated with the inclusion of the signs of angles  $\varphi'_0$  and  $\varphi'$ .

Introducing the designation

$$\xi = k_0 \sin \varphi_0' - k(\varphi') \sin \varphi', \qquad (11)$$

expression (10) can be represented in the form

$$\Psi_{j} = \frac{1}{D} \Psi_{jx} \Psi_{r} \Psi_{\varphi} \exp\left(\mathrm{i}\omega_{0}t + \mathrm{i}\beta_{1} - \mathrm{i}k(\varphi')r_{1}\right) \int_{0}^{D} \exp\left(\mathrm{i}\xi l\right) \mathrm{d}l.$$
(12)

We calculate the integral on the right-hand side of expression (12):

$$\int_{0}^{D} \exp(i\xi l) dl = \frac{1}{i\xi} \exp(i\xi l) \Big|_{0}^{D} = -\frac{i}{\xi} \left[ \exp(i\xi D) - 1 \right]$$
$$= -\frac{i}{\xi} \exp\left(\frac{i\xi D}{2}\right) 2i \sin\left(\frac{\xi D}{2}\right) = \exp\left(\frac{i\xi D}{2}\right) \frac{\sin(\xi D/2)}{\xi/2}$$
$$= D \exp\left(\frac{i\xi D}{2}\right) \frac{\sin \Phi}{\Phi}, \qquad (13)$$

where  $\Phi$  denotes (in view of the relation  $k_0 = 2\pi/\lambda_0$ ) the following quantity:

$$\Phi = \frac{1}{2} D\xi = \frac{1}{2} D(k_0 \sin \varphi_0' - k(\varphi') \sin \varphi')$$
$$= \pi \frac{D}{\lambda_0} \left[ \sin \varphi_0' - \frac{k(\varphi')}{k_0} \sin \varphi' \right]$$
$$= \pi \frac{D}{\lambda_0} \left[ \sin (\varphi_0 - \theta) - \frac{k(\varphi)}{k_0} \sin (\varphi - \theta) \right].$$
(14)

Substituting expression (13) into formula (12), in view of designation (11), we obtain the following expression for  $\Psi_i$ :

$$\Psi_{j} = \Psi_{jx}\Psi_{r}\Psi_{\varphi}\frac{\sin\Phi}{\Phi}\exp\left(\mathrm{i}\omega_{0}t + \mathrm{i}\beta_{1} - \mathrm{i}k(\varphi')r_{1}\right)\exp\left(\mathrm{i}\xi\frac{D}{2}\right)$$
$$= \Psi_{\sigma}\Psi_{\sigma}\Psi_{\sigma}\Psi_{\sigma}\Phi\exp\left(\mathrm{i}\omega_{0}t + \mathrm{i}\beta_{0} - \mathrm{i}k(\varphi)r\right) \tag{15}$$

 $= \Psi_{jx}\Psi_r\Psi_{\varphi}A\exp\left(\mathrm{i}\omega_0t + \mathrm{i}\beta - \mathrm{i}k(\varphi)r\right), \qquad (15)$ 

where A is the modulated amplitude of the total magnetic potential  $\Psi_j$ :

$$A = \frac{\sin \Phi}{\Phi} \,, \tag{16}$$

r is the distance of point  $P_k$  from the slit center:

$$r = r_1 + \frac{1}{2} D \sin \varphi',$$
 (17)

and the quantity

$$\beta = \beta_1 + \frac{1}{2} Dk_0 \sin \varphi_0' \tag{18}$$

is the wave phase of the center of a slit with length *D*, or the phase of the central elementary slit exciter (see Fig. 2).

So then, the final expression (15) describes the dependence  $\Psi_i(\varphi)$  [or  $\Psi_i(\varphi')$ ], and now, to find the dependence  $\Psi_i(\psi)$  [or  $\Psi_i(\psi')$ ], every value of  $\varphi$  (or  $\varphi'$ ) must be put in correspondence to the value of  $\psi$  (or  $\psi'$ ), thus representing the angle  $\varphi$ in formulas (14) and (15) as a  $\varphi(\psi)$  dependence. Furthermore, in going from the conventional observation point  $P_k$  to the true observation point  $P_V$ , we must replace in the final formula (15) the distance r (of the point  $P_k$  from the slit center) by the corresponding distance R (of the corresponding point P<sub>V</sub> from the slit center) using the formula<sup>8</sup>  $r = R \cos \chi$ , so that the factor  $\exp(-ik(\varphi)R\cos\chi)$  will describe the phase of the wave beam excited by the slit at the point  $P_V$ . In expressions (14)–(16), the function  $k(\varphi)$  is defined by the dispersion relation (8);  $\Psi_{jx} = \Psi_{jx}(x)$  describes, according to relations (1), the dependence of magnetic potential  $\Psi_i$  on the *x*-coordinate inside and outside the ferrite slab (j = 1, 2, 3);  $\Psi_r = \Psi_r(r)$  stands for the dependence of the potential  $\Psi_i$  on the coordinate r; and the factor  $\Psi_{\varphi}$  and the modulated amplitude A define the dependence of the potential  $\Psi_i$  on the polar angles  $\varphi$  and  $\psi$  (or  $\varphi'$  and  $\psi'$ ).

Our further concern is only with the dependence of the magnetic potential on the polar angles, and therefore a comment about the factor  $\Psi_{\varphi}$  in expression (15) is in order. This quantity is physically similar to the Kirchhoff factor for isotropic media (see, for instance, Ref. [6, § 38]) and describes how the amplitude induced by every auxiliary source depends on the polar angles  $\varphi$  and  $\psi$ . For an MSSW, this dependence will evidently be more complicated<sup>9</sup> than the Kirchhoff

<sup>&</sup>lt;sup>8</sup> The quantities *r* and *R* in Fig. 2 and in Fig. 4, which appears in Section 5, do not satisfy the relationship  $r = R \cos \chi$  owing to the limitation of the drawing size.

<sup>&</sup>lt;sup>9</sup> Calculation of the Kirchhoff factor for a specific anisotropic medium is beyond the scope of the present paper. Evidently, for anisotropic media the polar angle dependence of the amplitudes of auxiliary sources will be more complicated than the function ~  $(1 + \cos \varphi')/(2\lambda)$  describing the Kirchhoff factor for isotropic media (see, for instance, Ref. [6, § 38]). However, since the MSSW wave vector **k** cannot be oriented at angles  $\varphi$ that lie in the intervals  $\varphi_{cut1} < \varphi < \varphi_{cut2}$  and  $\varphi_{cut3} < \varphi < \varphi_{cut4}$  (where  $\varphi_{cut1}, \varphi_{cut2}, \varphi_{cut3}$ , and  $\varphi_{cut4}$  are the wave vector cutoff angles aligned with the asymptotes of isofrequency curves *I* and *2* in Fig. 3), factors  $\Psi_{\varphi}$  and, therefore,  $\Psi_j$  will evidently be equal to zero for these intervals of angle  $\varphi$ (and the corresponding intervals of angle  $\psi$ ).

factor. But similar to the case of isotropic media, the factor  $\Psi_{\varphi}$  may be treated (in comparison with the rapidly oscillating modulated amplitude  $A = \sin \Phi/\Phi$ ) as being practically constant in the small interval of the values of angle  $\varphi$  between two zeroes of the function  $\sin \Phi/\Phi$  nearest the principal maximum [this interval is very narrow when  $D/\lambda_0 \ge 1$ , for which expressions (14)–(16) were obtained]. That is why in the calculation of the diffracted beam width one may assume that  $\Psi_{\varphi} = \text{const}$  and analyze only the dependences  $A(\varphi(\psi))$  or  $A(\varphi'(\psi'))$ .

It should be emphasized that everything set forth in this section and Section 3 is *true* not only for an MSSW propagating in a free ferrite slab, but also *for MSWs of other types in different structures*, because the wave characteristic enters into the final expressions (14)–(16) only in the most general form — in the form of the dependences  $k(\varphi)$  and  $\psi(\varphi)$  [or  $\varphi(\psi)$ ].

#### 5. Diffraction patterns arising in a ferrite slab

We consider now typical diffraction patterns emerging when an MSSW is incident on a wide slit in an opaque screen.

For the geometry wherein the wave vector  $\mathbf{k}_0$  of the initial MSSW is normal to the slit line,<sup>10</sup> expression (14) for the phase function  $\Phi$  becomes simpler. In this case, the angle  $\varphi'_0 = \varphi_0 - \theta = 0$  and, therefore,  $\theta = \varphi_0$ , while expression (14) assumes the form<sup>11</sup>

$$\Phi = \frac{1}{2} Dk(\varphi') \sin \varphi' = \frac{1}{2} Dk(\varphi) \sin (\varphi - \theta), \qquad (19)$$

where  $\varphi'$  and  $\varphi$  are the dependences  $\varphi'(\psi')$  and  $\varphi(\psi)$ .

The geometry wherein the wave vector  $\mathbf{k}_0$  and the group velocity vector  $\mathbf{V}_0$  of the initial wave are collinear and perpendicular to the slit line (Fig. 4) is the most simple, because, in this case,  $\varphi_0 = \psi_0 = \chi_0 = 0$ , the coordinate systems  $\Sigma_p$  and  $\Sigma'_p$  coincide (i.e.,  $\varphi' = \varphi$  and  $\theta = 0$ ), and  $\Phi$  is expressed as

$$\Phi = \frac{1}{2} Dk(\varphi') \sin \varphi' = \frac{1}{2} Dk(\varphi) \sin \varphi .$$
<sup>(20)</sup>

As is easily seen, formula (20) for  $k(\varphi) \equiv \text{const} = k_0 = 2\pi/\lambda_0$  coincides with the well-known similar formula for isotropic media <sup>12</sup> (see, for instance, Refs [1, § 9.6; 6, § 39]).

The results of numerical calculations of the modulated amplitude A for the geometry depicted in Fig. 4 are given in Fig. 5 in the polar coordinate system (curve I).<sup>13</sup> In the execution of these calculations, we assumed a magnitude  $H_0 = 300$  Oe of the external uniform magnetic field; the ferrite slab possessed the parameters used most frequently: a

<sup>10</sup> This geometry is easy to imagine if Fig. 3 and the vectors  $\mathbf{k}_0$  and  $\mathbf{V}_0$  in Fig. 2 are rotated counterclockwise in the plane of the drawing so that the vector  $\mathbf{k}_0$  turns out to be vertically oriented and perpendicular to the line of the screen.

<sup>11</sup> The minus sign in front of expressions (19) and (20) was omitted, because the ratio  $\sin \Phi/\Phi$  remains unchangeable under the replacement  $\Phi \to -\Phi$ . Notice that the minus sign in front of  $k(\phi')$  in expression (14) is due to describing the traveling MSW in the form  $\exp(i\omega t - ikr)$  [see expression (7)], while the traveling waves, for instance, in Ref. [1] are described in the form  $\exp(ikr - i\omega t)$ . As is well known, both these descriptions are equivalent.



**Figure 4.** Geometry of MSSW incidence on a screen in the case of  $\theta = \varphi_0 = \psi_0 = 0$ , i.e., the wave vector  $\mathbf{k}_0$  and the group velocity vector  $\mathbf{V}_0$  of the initial MSSW are normal to the screen ( $\mathbf{k}_0$  and  $\mathbf{V}_0$  are collinear in that event). The dashed lines exhibit the wave fronts of the initial wave. From the isofrequency curve of the initial MSSW with f = 2900 MHz shown in the inset, it is possible to determine, for instance, that the direction  $\varphi = 12^\circ$  to a point P<sub>k</sub> corresponds to the direction  $\psi = -30^\circ$  to a point P<sub>V</sub>.



**Figure 5.** Far-field diffraction pattern occurring when an MSSW with collinear vectors  $\mathbf{k}_0$  and  $\mathbf{V}_0$  is normally incident on a slit (curve *I*). Initial MSSW and geometry parameters are as follows:  $f_0 = 2900$  MHz,  $\lambda_0 = 138.9 \ \mu\text{m}$  ( $k_0 = 452.5 \ \text{cm}^{-1}$ ),  $\theta = \psi_0 = \varphi_0 = 0$ , and  $\lambda_0/D = 0.1$  (only the diffraction pattern behind the screen is displayed, i.e., the pattern for angles  $|\psi'| \leq 90^\circ$ ). Shown for comparison is the diffraction pattern occurring in isotropic media under normal wave incidence on the slit (curve 2).

thickness  $s = 10 \ \mu\text{m}$  and a saturation magnetization  $4\pi M_0 = 1750 \text{ G}$ ; an initial MSSW frequency  $f_0 = 2900 \text{ MHz}$ , and  $\lambda_0/D = 0.1$ .

Shown for comparison in Fig. 5 is the similar dependence of the modulated amplitude for an isotropic medium and the

<sup>&</sup>lt;sup>12</sup> Notice that  $2\pi$  enters in lieu of  $\pi$  in the formula for  $\Phi$  in Ref. [1], since the latter makes use of the ratio  $\sin (\Phi/2)/(\Phi/2)$  rather than  $\sin \Phi/\Phi$ .

<sup>&</sup>lt;sup>13</sup> Shown in Fig. 5 is only that part of the behind-screen diffraction pattern, which corresponds to the angular interval  $-56.5^{\circ} < \psi < 56.5^{\circ}$ .

same ratio  $\lambda_0/D = 0.1$  (curve 2). As is clear from the comparison of curves 1 and 2 in Fig. 5, the width of the principal maximum for curve 1, which describes MSSW diffraction, is approximately three times broader<sup>14</sup> than the corresponding peak of curve 2.

Next, let us calculate the modulated amplitude A by formulas (16) and (14) for the most general geometry shown in Figs 2 and 3, when the vectors  $\mathbf{k}_0$  and  $\mathbf{V}_0$  of the initial wave are noncollinear and arbitrarily oriented relative to the screen. The  $A(\psi')$  dependence in the case of such a geometry is plotted in Fig. 6a, the coordinate systems in Figs 2, 3, and 6a being equally oriented. Figure 6b depicts the  $A(\psi')$  dependence for the geometry in which the initial noncollinear wave with the same parameters is normally incident on the screen (the vector  $\mathbf{V}_0$  is perpendicular to the screen).<sup>15</sup> The upper half-planes in Fig. 6, on which  $|\psi'| < 90^\circ$ , correspond to the diffraction pattern emerging behind the screen, and the lower half-planes,  $|\psi'| > 90^\circ$ , correspond to the diffraction pattern emerging in front of the screen (which corresponds to reflection from the slit).

Below, we note several differences and common features characteristic of the dependences  $A(\psi')$  and  $A(\psi)$  in Figs 5 and 6.

(1) All  $A(\psi)$  dependences in Figs 5 and 6, like the isofrequency MSSW dependence in Fig. 3, are undefined in the same polar angle intervals  $\psi_{cut2} < \psi < \psi_{cut1}$  and  $\psi_{cut4} < \psi < \psi_{cut3}$  of the coordinate system  $\Sigma_p$ . This situation takes place owing to the fact that the isofrequency MSSW dependence resembles a hyperbola, and, therefore, the angle  $\varphi$  describing the direction of the wave vector **k** may not belong to the intervals  $\varphi_{cut1} < \varphi < \varphi_{cut2}$  and  $\varphi_{cut3} < \varphi < \varphi_{cut4}$ , and the group velocity vector **V** may not possess a direction corresponding to the above-indicated  $\psi$  angular intervals [for the isofrequency dependence of the MSSW with a frequency f = 2900 MHz (see Fig. 3), the **k** vector cutoff angles, which are defined by the asymptote directions, are as follows:  $\varphi_{cut1} = -33.5^{\circ}$ ,  $\varphi_{cut2} = 146.5^{\circ}$ ,  $\varphi_{cut3} = -146.5^{\circ}$ , and  $\varphi_{cut4} = -33.5^{\circ}$ , while the corresponding **V** vector cutoff angles are  $\psi_{cut1} = -56.5^{\circ}$ ,  $\psi_{cut2} = -123.5^{\circ}$ ,  $\psi_{cut3} = 123.5^{\circ}$ , and  $\psi_{cut4} = 56.5^{\circ}$ ].

(2) As is clear from the changes in  $A(\psi)$  dependences shown in Fig. 6, two beams always emerge in this example of plane MSSW diffraction from a slit.<sup>16</sup> The amplitude of the *transmitted* beam emerging behind the screen always peaks at  $\varphi = \varphi_0$  and  $\psi = \psi_0 = 43.7^\circ$ . In this case, since the incidence geometries in Figs 6a and 6b are different, the angle  $\psi'$  values corresponding to the highest beam amplitude are also different:  $\psi' = 23.7^\circ$  in Fig. 6a, and  $\psi' = 0$  in Fig. 6b. The *reflected* beam emerges in front of the screen; in this case, the peak in the  $A(\psi)$  dependence corresponding to the reflected beam may lie in curve 2 (Fig. 6a), as well as in curve 1 (Fig. 6b).



**Figure 6.** Far-field diffraction pattern occurring when  $\lambda_0/D = 0.1$  for an arbitrary (a) and normal (b) incidence on the slit for an MSSW with noncollinear vectors  $\mathbf{k}_0$  and  $\mathbf{V}_0$  for the following parameters:  $f_0 = 2900$  MHz,  $\psi_0 = 43.7^\circ$ ,  $\varphi_0 = -22.6^\circ$ ,  $\lambda_0 = 96.3 \,\mu\text{m}$ , and  $k_0 = 652.7 \,\text{cm}^{-1}$  (the wave parameters correspond to the point S in the isofrequency curve *I* in Fig. 3). The values of polar angle  $\psi'$ , which is measured from the normal to the screen, are indicated. The screen normal orientation is  $\theta = 20^\circ$  in Fig. 6a, and  $\theta = 43.7^\circ$  in Fig. 6b. The incidence geometry corresponding to Fig. 6a is illustrated in Fig. 2.

(3) The two maxima of the  $A(\psi)$  dependence in Fig. 6 possess the same amplitude due to the fact that our treatment disregarded the factor similar to the Kirchhoff factor (see Section 4). It was believed that this factor may be considered as having a constant value in a narrow interval of values of angle  $\psi'$  (or  $\psi$ ) between the two zeroes nearest some maximum of the dependence  $A(\psi) = \sin \Phi(\psi)/\Phi(\psi)$ . Therefore, the assumption made permits us to find the position and angular width of each of the maxima of the  $A(\psi)$  dependence, but does not enable comparing the amplitudes of different maxima of the  $A(\psi)$  dependence (which was not our intention in this paper).

(4) The diffraction patterns displayed by curves I and 2 in Fig. 6 correspond to excitation, by secondary sources, of waves localized, respectively, near the upper and lower (in Fig. 1) ferrite slab surfaces (curves I and 2 in Fig. 3 also correspond to the waves localized near the upper and lower ferrite slab surfaces). Since the initial MSSW itself is localized at the upper slab surface (a point S corresponds to the initial MSSW in curve I in Fig. 3), the excitation, by secondary sources, of waves localized at the opposite surface (the wave

<sup>&</sup>lt;sup>14</sup> A physical explanation of this fact is provided in the discussion of the corresponding geometry in Section 9.

<sup>&</sup>lt;sup>15</sup> This geometry is not given in the drawings, but it is easily imagined if, without changing the screen orientation, the entire Fig. 3 and the vectors  $\mathbf{k}_0$  and  $\mathbf{V}_0$  in Fig. 2 are rotated clockwise through an angle of 23.7° (to make  $\mathbf{V}_0$  oriented vertically, normally to the screen).

<sup>&</sup>lt;sup>16</sup> The  $A(\psi)$  dependence in Fig. 5, which corresponds to the geometry of Fig. 4, also possesses, strictly speaking, two maxima (the peak corresponding to the reflected beam emerges at  $\varphi = \psi = 180^{\circ}$ ). However, if it is assumed, as for isotropic media, that the secondary waves with wave vectors oriented in the direction  $\varphi = \varphi_0 + 180^{\circ}$  (in opposition to the vector  $\mathbf{k}_0$  of the initial MSSW) are not excited, this maximum may be disregarded.

vectors of such waves terminate in curve 2 in Fig. 3) will evidently be quite inefficient. Therefore, a portion of the  $A(\psi)$ dependence described by curves 2 in Fig. 6 must have amplitudes which will be one-two orders of magnitude lower than the amplitudes of the other portion of the  $A(\psi)$ dependence described by curves *I* in Fig. 6. For the purpose of clarity, however, we plotted curves 2 in Fig. 6 neglecting this factor (i.e., on the scale presented in Fig. 6, curve 2, unlike

curve 1, in reality must virtually coincide with the circumfer-

#### 6. Physical meaning

#### of phase function $\Phi$ from the standpoint of the momentum conservation law

ence complying with the zero amplitude).

An analysis of the  $A(\psi)$  dependences displayed in Figs 5 and 6 brings up the natural question of what determines the highestamplitude diffracted beam directions and how many diffracted beams may arise from the diffraction of a certain wave by a slit.

To answer these questions, we turn to the analysis of expressions (14) and (16), which describe the quantities  $\Phi$  and A. As is evident from expression (14),  $\Phi$  is, as regards the dimensionality and substance, a phase function, and the values of angle  $\varphi$ , whereat  $\Phi = 0$  and  $A = \sin \Phi/\Phi \rightarrow 1$ , have the following physical meaning: these are the directions of secondary MSW wave vector orientations whereby constructive interference of the waves of all secondary sources occurs. For other wave vector directions  $\varphi$ , secondary MSW sources also interfere, but the interference in these directions is not constructive, and the inequality A < 1 is always fulfilled.

By putting  $\Phi = 0$  in expression (14), we can obtain the following equations

$$k(\varphi')\sin\varphi' = k_0\sin\varphi'_0, \text{ or } k(\varphi)\sin(\varphi-\theta) = k_0\sin(\varphi_0-\theta).$$
(21)

Evidently,  $\varphi' = \varphi'_0$  (or  $\varphi = \varphi_0$ ) are always the solutions of these equations. Since  $k(\varphi'_0) = k(\varphi_0) \equiv k_0$ , equations (21) at  $\varphi' = \varphi'_0$  (or  $\varphi = \varphi_0$ ) turn into an identity. The beam which emerges for the  $\varphi = \varphi_0$  orientation of the wave vectors of secondary sources and possesses a maximum amplitude in the  $\psi = \psi_0$  direction (in the direction of vector  $\mathbf{V}_0$  of the initial wave) will be referred to as *principal*, because it always emerges (when vector  $\mathbf{V}_0$  is directed at the screen, of course) and is characterized by the values of  $\varphi$  and  $\psi$ , corresponding to the initial wave parameters.

How many other directions  $\varphi'$  or  $\varphi$  satisfying Eqn (21) emerge for some given geometry?

As is clear from Fig. 3, which complies with the general geometry of MSSW incidence on a slit, the product  $k_0 \sin \varphi'_0$  on the right-hand side of Eqn (21) represents the projection  $k_{0pr}$  of the initial MSSW wave vector  $\mathbf{k}_0$  onto the slit line or on the screen (on the *z'*-axis). That is why the left-hand side of equation (21) will be satisfied by those values of  $\varphi'$  (or  $\varphi$ ) whereto correspond wave vectors  $\mathbf{k}$  also having projection  $k_{0pr}$  (with due regard to the sign) onto the slit line.

Evidently, when finding all vectors  $\mathbf{k}$  possessing projection  $k_{0\text{pr}}$  onto the slit line, one must draw a normal to the z'-axis through the tip of vector  $\mathbf{k}_0$  and find all points of intersection of this normal with isofrequency curves. On completing this construction, we will find, apart from point  $S_0$ , another intersection point  $S_1$  lying in isofrequency curve 2. This point corresponds to wave vector  $\mathbf{k}_1$  directed at an angle

 $\varphi_1 = -147^\circ$ , and to a group velocity vector  $\mathbf{V}_1$  directed at an angle  $\psi_1 = 125^\circ$  (see Figs 3 and 6a). Thus, two diffracted beams emerge in the geometry considered in Fig. 3—the principal transmitted beam, and the reflected beam possessing a maximum amplitude in the  $\psi_1$  direction.

It is evident that the parameters of the reflected beam depend heavily on the geometry of incidence of the initial MSSW. For instance, if a similar construction is made for the geometry wherein the initial MSSW with the same parameters is normally incident on the screen (vector  $V_0$  is oriented normally to the screen), then point  $S_1$ , which corresponds to the reflected beam, will lie on the left end of isofrequency curve *I* and will have the parameters  $\varphi = 32^\circ$  and  $\psi = -56.5^\circ$  (which is evident from Fig. 6b, which shows the  $A(\psi)$  dependence for this geometry).

Therefore, in searching for the answer to the question about the directions of diffracted beams, we arrive at the following conclusion.

Equation  $\Phi = 0$  or equations (21) are, in essence, one of the forms of writing down the momentum conservation law, which establishes the equality between the projection  $k_0 \sin \varphi'_0$  of the wave vector of the initial wave and the projections of the wave vectors of secondary sources onto the slit line in the emergence of constructive interference (for instance, this equality has the form  $k_1 \sin \varphi'_1 = k_0 \sin \varphi'_0$  for the geometry of Fig. 3). Therefore, the orientation  $\varphi$  of the wave vectors of the secondary sources, whereat constructive interference occurs, and the direction  $\psi$ , in which the amplitude of the diffracted beam goes through a maximum, can be calculated on the basis of the momentum conservation law in the framework of geometrical optics.

#### 7. Peculiarities of diffraction from a slit for a magnetostatic wave with an isofrequency dependence of arbitrary form

Relying upon the conclusions drawn in Section 6, it is possible to provide an answer to the question about the number of emerging diffracted beams.

Since the condition  $\Phi = 0$  for the emergence of diffracted beams is equivalent to the momentum conservation law, to find all diffraction beams in the general case—for an arbitrary isofrequency dependence  $k(\varphi)$  and an arbitrary beam incidence geometry—it is evidently convenient to apply the well-known methods and rules of geometrical optics for two-dimensional anisotropic geometries, which are described at length in review [5].

By taking advantage of these rules, it is easy to ascertain that two diffracted beams always emerge in the example of MSSW diffraction from a slit under consideration. An exception is provided by the geometries in which the normal to the screen is aligned with one of the asymptotes of the isofrequency dependence: in these geometries, only the principal diffracted beam emerges, while the reflected diffracted beam is nonexistent. Evidently, the geometries wherein the reflected diffracted beam does not emerge may also be realized in the MSW diffraction in other anisotropic structures whose isofrequency dependences satisfy the requirements stated in Ref. [5, Section 8.5].

Let us consider the consequences of abandoning the single-valued character of the condition for the  $\varphi(\psi)$  dependence, which was introduced in Section 3. Let the  $\psi(\varphi)$  dependence corresponding to the initial MSSW be single-valued, and the inverse dependence  $\varphi(\psi)$  be ambig-



**Figure 7.** Dependences  $\psi(\varphi)$  (a) and  $d\psi(\varphi)/d\varphi$  (b) calculated at  $H_0 = 300$  Oe,  $4\pi M_0 = 1750$  G, and  $s = 10 \mu \text{m}$ . Curves *1*-6 correspond to an MSSW with a frequency *f* taking the following values: 2200, 2330, 2500, 2700, 2900, and 3100 MHz.

uous. Calculations suggest that this situation is typical of MSSWs with relatively low frequencies lying near the initial frequency in the spectrum, for instance, of an MSSW with a frequency f = 2200 MHz, for which the  $\psi(\varphi)$  dependence is described by curve *I* in Fig. 7a. With reference to this drawing, for this curve there is an interval of angles  $\psi_{a1} < \psi < \psi_{a2}$  wherein, to every value of  $\psi$ , there corresponds one  $\varphi'$  value lying in the interval  $\varphi_{a1} < \varphi' < \varphi_{a2}$  and one more  $\varphi''$  value lying in the interval  $\varphi_{a2} < \varphi'' < \varphi_{a3}$ .

Consider a geometry example in which the initial MSSW incident on the slit possesses parameters  $\varphi_0$  and  $\psi_0$  such that the value of angle  $\varphi_0$ , which defines the orientation of vector  $\mathbf{k}_0$ , lies in the interval  $\varphi_{a1} < \varphi_0 < \varphi_{a2}$  and, therefore, the value of angle  $\psi_0$ , which specifies the orientation of vector  $\mathbf{V}_0$ , lies in the interval  $\psi_{a1} < \psi_0 < \psi_{a2}$ . As is easily seen, the energy transfer in this case, for instance, *in the direction*  $\psi_0$ , will be effected not only by *constructively interfering* secondary waves with wave vectors  $\mathbf{k}_0$  oriented at an angle  $\varphi_0$ , but also by *other* secondary waves with wave vectors  $\mathbf{k}''$  oriented at some angle  $\varphi'''$  (belonging to the interval  $\varphi_{a2} < \varphi'' < \varphi_{a3}$ ), which the value  $\psi_0$  also corresponds to. If the result of interference of the secondary waves is nonzero, *two* waves of the same frequency but different amplitudes and wavelengths will propagate in the  $\psi_0$  direction. Evidently, both of these

waves will interfere throughout their propagation path, somewhere adding up in phase and somewhere out of phase, with the like interference pattern of the two waves observed not only in the direction  $\psi = \psi_0$ , but also in other directions  $\psi$ from the interval  $\psi_{a1} < \psi < \psi_{a2}$ . Therefore, a *spatially nonuniform diffraction pattern* will be observable in the angular interval  $\psi_{a1} < \psi < \psi_{a2}$ : in every direction  $\psi$  from this interval, the resultant amplitude  $A(\psi)$  will not simply decrease with distance *R* of the observation point P<sub>V</sub> from the slit, but depend on *R* in a complicated manner (becoming alternatively higher and lower).

Now let us consider briefly several features of MSW diffraction occurring for arbitrary  $k(\varphi)$  and  $\psi(\varphi)$  dependences. In this case, not only one or two, but also several transmitted or reflected diffracted beams may emerge for a certain geometry of initial wave incidence on the slit (see Ref. [5, Section 8.6]). For instance, when the isofrequency dependence has inflection points, as in a ferrite-insulatormetal structure (see, for instance, curve 2 in Fig. 10 of Ref. [5]), one reflected and two transmitted beams emerge for a certain slit orientation and a specific selection of initial MSW parameters in accordance with the momentum conservation law.<sup>17</sup> Similarly, several transmitted or reflected diffracted beams will emerge if we abandon the single-valuedness condition for the  $k(\varphi)$  and  $\psi(\varphi)$  dependences, which was introduced in Section 3 for ease of treatment. Evidently, the  $k(\varphi)$  and  $\psi(\varphi)$  dependences may then consist, for instance, of several  $k_1(\varphi), k_2(\varphi), \ldots, k_m(\varphi) \ldots$  curves.<sup>18</sup> In this case, for a certain geometry, each mth mode, described by the curve  $k_m(\varphi)$ , will give a reflected beam and a transmitted one owing to diffraction, whose maxima will be oriented in directions specified by the momentum conservation law. Furthermore, the  $\psi$ -angle ranges corresponding to each isofrequency curve overlap, as a rule, for multivalued dependences  $k(\varphi)$  and  $\psi(\varphi)$ . Therefore, a spatially nonuniform diffraction pattern also emerges in the overlap intervals of the  $\psi$  values (see above), when a multitude of waves of the same frequency but different amplitudes and wave numbers will add up in every direction  $\psi$ .

## 8. Formula for the angular width of diffracted beams

As is clear from Figs 5 and 6, the width  $\Delta \psi$  of the maxima of the  $A(\psi)$  dependence (or the angular width of diffracted beams) may vary greatly depending on the geometry of wave incidence and parameters of the initial MSSW. Of interest to us is elucidating at what parameters of the medium and geometry the quantity  $\Delta \psi$  turns out to be greater than the corresponding angular beam width in isotropic media, and at what parameters it turns out to be smaller. For this purpose, we will obtain an expression describing the angular width  $\Delta \psi$ of the diffracted beam.

Let there be some structure in which an MSW is characterized by the isofrequency dependence  $k(\varphi)$ , and let a plane initial MSW with parameters  $\mathbf{k}_0, \varphi_0, \mathbf{V}_0, \psi_0$ , and  $f_0$  be

<sup>&</sup>lt;sup>17</sup> In particular, for a horizontal orientation of the slit line in Fig. 10 of Ref. [5] and a selection of the initial MSW with parameters such that its wave vector projection  $k_b$  onto the abscissa axis lies between the  $k_{b1}$  and  $k_{b2}$  values.

<sup>&</sup>lt;sup>18</sup> As is the case, for instance, with a magnetostatic backward volume wave (MSBVW), which comprises a set of an infinite number of modes [14]. The isofrequency curves and  $\psi(\varphi)$  dependences for BBMSW modes may be found, for example, in Refs [5; 12, § 5.3].

incident on an arbitrarily oriented slit (see Fig. 2). Let us next assume that diffraction gave rise to a certain number of reflected and transmitted diffracted beams, none of them lying in the angular sector where a nonuniform diffraction pattern may exist (see Section 7). Each *n*th diffracted beam emerges for an orientation of the wave vectors of secondary sources in the direction  $\varphi = \varphi_n$ , and has a maximum amplitude in the direction  $\psi = \psi_n$  (n = 0, 1, ..., N), with the directions  $\varphi = \varphi_0$  and  $\psi = \psi_0$  (n = 0) attributable to the *principal* transmitted diffracted beam.

Mathematically, as discussed in Section 6, the emergence of the *n*th diffracted beam signifies that, for  $\varphi = \varphi_n$ , the phase function is equal to zero,  $\Phi = 0$ , i.e.,  $\varphi_n$  satisfy equation (21) and, therefore, the following equality holds true:

$$k_0 \sin (\varphi_0 - \theta) = k(\varphi_n) \sin (\varphi_n - \theta).$$
(22)

As is well known from mathematics, for any function  $\psi(\varphi)$  differentiable in the neighborhood of some point  $\varphi = \varphi_n$ , the increment  $\Delta \varphi$  of argument and the increment  $\Delta \psi$  of the function itself are approximately related as

$$\Delta \psi = \frac{\mathrm{d}\psi}{\mathrm{d}\varphi} (\varphi = \varphi_n) \Delta \varphi \,. \tag{23}$$

We select the value of  $\Delta \varphi$  in such a way that the phase function  $\Phi$  is equal to  $\pi$  for the argument value of  $\varphi = \varphi_n + \Delta \varphi$ , i.e., the value  $\varphi = \varphi_n + \Delta \varphi$  corresponds to one of the two modulated amplitude zeroes nearest the principal maximum:  $A(\varphi_n + \Delta \varphi) = \sin \pi/\pi = 0$ . Therefore, the argument increment  $\Delta \varphi$  may be thought of as corresponding to about half of the angular separation of the two zeroes nearest the principal maximum or to the angular width  $\Delta \psi$  of the diffracted beam at a level of 0.5. Proceeding from this and from formula (14), we can write down the relationship

$$\pi = \pi \frac{D}{\lambda_0} \left[ \frac{k(\varphi_n + \Delta \varphi)}{k_0} \sin(\varphi_n + \Delta \varphi - \theta) - \sin(\varphi_0 - \theta) \right]$$
(24)

With reference to the last relation, the greater the ratio  $D/\lambda_0$ , the smaller is the increment  $\Delta \varphi$  required to change the value  $\Phi = 0$  of the phase function to  $\Phi = \pi$  (recall that the expression in brackets on the right-hand side of relationship (24) is equal to zero at  $\varphi = \varphi_n$ ). That is why for  $D/\lambda_0 \ge 1$  it is safe to say that  $\Delta \varphi$  is rather small. By expanding the sine of the sum of angles  $\varphi_n - \theta$  and  $\Delta \varphi$  and assuming that  $\cos \Delta \varphi \approx 1$  and  $\sin \Delta \varphi \approx \Delta \varphi$  owing to the smallness of  $\Delta \varphi$ , we can represent relationship (24) as

$$\frac{2\pi}{D} = k(\varphi_n + \Delta \varphi) \left[ \sin(\varphi_n - \theta) + \Delta \varphi \cos(\varphi_n - \theta) \right] - k_0 \sin(\varphi_0 - \theta).$$
(25)

We replace the last term in expression (25) in accordance with equality (22) and divide both parts of expression (25) by  $\Delta \varphi$  to obtain

$$\frac{2\pi}{\Delta\varphi D} = \frac{k(\varphi_n + \Delta\varphi) - k(\varphi_n)}{\Delta\varphi} \sin(\varphi_n - \theta) + k(\varphi_n + \Delta\varphi)\cos(\varphi_n - \theta).$$
(26)

Since  $\Delta \varphi$  is small when  $D/\lambda_0 \ge 1$ , the ratio  $[k(\varphi_n + \Delta \varphi) - k(\varphi_n)]/\Delta \varphi$  is the value of the derivative  $dk/d\varphi$  at the point  $\varphi = \varphi_n$ , and it can be assumed that

 $k(\varphi_n + \Delta \varphi) = k(\varphi_n)$  in the second term on the right-hand side of expression (26). As a result, the expression for  $\Delta \varphi$  takes on the form <sup>19</sup>

$$\Delta \varphi = \frac{2\pi}{D} \left[ \frac{\mathrm{d}k}{\mathrm{d}\varphi}(\varphi_n) \sin\left(\varphi_n - \theta\right) + k(\varphi_n) \cos\left(\varphi_n - \theta\right) \right]^{-1}.$$
 (27)

Substituting expression (27) into relation (23), we find that the angular width  $\Delta \psi$  of the *n*th (emerging at  $\varphi = \varphi_n$  and  $\psi = \psi_n$ ) diffracted beam at a level of 0.5 is expressed as<sup>20</sup>

$$\Delta \psi = \left| \frac{2\pi}{D} \frac{\mathrm{d}\psi}{\mathrm{d}\varphi}(\varphi_n) \left[ \frac{\mathrm{d}k}{\mathrm{d}\varphi}(\varphi_n) \sin(\varphi_n - \theta) + k(\varphi_n) \cos(\varphi_n - \theta) \right]^{-1} \right|$$
$$= \frac{\lambda_n}{D} \left| \frac{\mathrm{d}\psi}{\mathrm{d}\varphi}(\varphi_n) \left[ \frac{1}{k(\varphi_n)} \frac{\mathrm{d}k}{\mathrm{d}\varphi}(\varphi_n) \sin(\varphi_n - \theta) + \cos(\varphi_n - \theta) \right]^{-1} \right|,$$
(28)

where  $\lambda_n$  designates the wavelength for  $\varphi = \varphi_n$ , i.e.,  $\lambda_n = 2\pi/k(\varphi_n)$ .

For the *principal transmitted* diffracted beam (which corresponds to n = 0, and therefore the relations  $\varphi_n = \varphi_0$ ,  $k(\varphi_n) = k(\varphi_0) \equiv k_0$ , and  $\lambda_n = \lambda_0$  hold true), formulas (27) and (28) assume the form

$$\Delta \varphi = \frac{\lambda_0}{D} \left[ \frac{1}{k_0} \frac{dk}{d\varphi}(\varphi_0) \sin(\varphi_0 - \theta) + \cos(\varphi_0 - \theta) \right]^{-1}$$
$$= \frac{\lambda_0}{D} \left[ \frac{1}{k_0} \frac{dk}{d\varphi}(\varphi_0) \sin\varphi_0' + \cos\varphi_0' \right]^{-1}, \qquad (29)$$

$$\Delta \psi = \frac{\lambda_0}{D} \left| \frac{\mathrm{d}\psi}{\mathrm{d}\varphi}(\varphi_0) \left[ \frac{1}{k_0} \frac{\mathrm{d}k}{\mathrm{d}\varphi}(\varphi_0) \sin(\varphi_0 - \theta) + \cos(\varphi_0 - \theta) \right]^{-1} \right|$$
$$= \frac{\lambda_0}{D} \left| \frac{\mathrm{d}\psi}{\mathrm{d}\varphi}(\varphi_0) \left[ \frac{1}{k_0} \frac{\mathrm{d}k}{\mathrm{d}\varphi}(\varphi_0) \sin\varphi_0' + \cos\varphi_0' \right]^{-1} \right|, \tag{30}$$

where, it will be recalled, angle  $\varphi'_0 = \varphi_0 - \theta$  describes the orientation of the vector  $\mathbf{k}_0$  of the initial MSSW relative to the normal to the screen in the coordinate system  $\Sigma'_p$ .

A brief note is in order. If we abandon the assumption made at the beginning of this section and consider the case when some diffracted beam lies in the angular sector where there is a spatially nonuniform diffraction pattern (if many diffracted beams emerge, the angular intervals in which there are other diffracted beams do not overlap with the angular interval containing the diffracted beam under investigation), it makes sense to speak only about the angular width  $\Delta \psi_{av}$  of this beam averaged over an extended portion of its trajectory. Evidently, the average angular width  $\Delta \psi_{av}$  can also be calculated using formula (28).

#### 9. Variation of the angular width of the principal diffracted beam for different incidence geometries

Let us consider how formulas (29) and (30), which describe the parameters of the principal diffracted beam, are trans-

<sup>&</sup>lt;sup>19</sup> If we put  $k(\varphi_n + \Delta \varphi) = k(\varphi_n)$  directly in relation (24), the expression for  $\Delta \varphi$  will produce significant errors in calculations.

<sup>&</sup>lt;sup>20</sup> The angular beam width  $\Delta \psi$  may be both positive and negative, because its determinant quantities  $d\psi/d\varphi$ ,  $dk/d\varphi$ ,  $\sin(\varphi_n - \theta)$ , and  $\cos(\varphi_n - \theta)$ may be of any sign. However,  $\Delta \psi$ , like distances, is conveniently described by positive numbers, and, therefore, we introduce the modulus sign in formulas for  $\Delta \psi$ .

formed for the most commonly encountered geometries of incidence of initial MSWs.

When the wave vector  $\mathbf{k}_0$  of the initial MSSW is oriented normally to the slit line (for an arbitrary orientation of  $\mathbf{V}_0$ ), i.e., at  $\varphi'_0 = \varphi_0 - \theta = 0$ , formulas (29) and (30) take on the form

$$\Delta \varphi = \frac{\lambda_0}{D} \,, \tag{31}$$

$$\Delta \psi = \frac{\lambda_0}{D} \left| \frac{\mathrm{d}\psi}{\mathrm{d}\varphi}(\varphi_0) \right|. \tag{32}$$

Mathematically and physically, this geometry is equivalent to the geometry of MSW excitation by means of a linear transducer <sup>21</sup> of length *D*, if two conditions are simultaneously fulfilled at the MSW excitation frequency: (i) the transducer is much longer than the MSW wavelength,  $D/\lambda_0 \ge 1$ , and (ii) the transducer is much shorter than the electromagnetic wavelength at the corresponding frequency,  $D/\lambda_{\rm EMW} \ll 1$ , because only in this case can the entire transducer aperture be treated as being in-phase. As is easily seen, in microwaves, where  $\lambda_{\rm EMW} \sim 3-30$  cm, and  $\lambda_0 \sim 0.05-1$  mm, it is almost always possible to satisfy both of these conditions <sup>22</sup> by selecting  $D \sim 2-10$  mm.

For the simplest geometry, wherein a wave with *collinearly oriented* vectors  $\mathbf{k}_0$  and  $\mathbf{V}_0$  is normally incident on the slit (see Fig. 4), the quantity  $\Delta \varphi$  is described by formula (31) as before, and the expression for  $\Delta \psi$  may be derived from formula (32) by putting  $\varphi_0$  equal to zero:<sup>23</sup>

$$\Delta \psi = \frac{\lambda_0}{D} \left| \frac{\mathrm{d}\psi}{\mathrm{d}\varphi} (\varphi = 0) \right|. \tag{33}$$

In isotropic media, where the isofrequency dependence of a wave is fitted by a circle and vector **k** and the corresponding group velocity vector **V** are always collinear, the  $\psi(\varphi)$  dependence has the form  $\psi = \varphi$ , and hence  $d\psi/d\varphi \equiv 1$  in *all* cases. For isotropic media, expression (33) transforms, therefore, to the well-known formula

$$\Delta \psi = \frac{\lambda_0}{D} \,. \tag{34}$$

Now, relying on formulas (33) and (34), it is possible to provide a physical explanation for the results of the numerical calculations given in Section 5. In particular, one can see from formula (33) that the *factor* defining, at the same  $\lambda_0/D$  ratio, the angular beam width  $\Delta \psi$  is the value of the derivative  $d\psi/d\varphi$  at  $\varphi = 0$ . Therefore, in the discussion of the diffraction pattern emerging when a collinear MSSW with a frequency  $f_0 = 2900$  MHz is normally incident on the slit (see Figs 4 and 5), it is easily seen that  $d\psi/d\varphi = -3$  for this geometry and  $\varphi = 0$  (see curve 5 in Fig. 7b), while for an isotropic medium  $d\psi/d\varphi \equiv 1$  in all cases. That is why the angular width of the principal maximum of MSSWs with a frequency  $f_0 = 2900$  MHz turns out to be three times greater than the angular width of the principal maximum for waves propagating in isotropic media at the same  $\lambda_0/D$  ratio.

For a difference  $\varphi_0 - \theta = \varphi'_0 = \pm \pi/2$ , i.e., for a geometry wherein the wave vector  $\mathbf{k}_0$  of the initial MSSW is parallel to the slit line (this diffraction regime is impossible to realize in isotropic media), formulas (29) and (30) are also simplified to assume the form

$$\Delta \varphi = \pm \frac{2\pi}{D} \left( \frac{\mathrm{d}k}{\mathrm{d}\varphi}(\varphi_0) \right)^{-1},\tag{35}$$

$$\Delta \psi = \frac{2\pi}{D} \left| \frac{\mathrm{d}\psi}{\mathrm{d}\varphi}(\varphi_0) \left( \frac{\mathrm{d}k}{\mathrm{d}\varphi}(\varphi_0) \right)^{-1} \right| = \frac{2\pi}{D} \left| \frac{\mathrm{d}\psi}{\mathrm{d}k}(\varphi_0) \right|. \tag{36}$$

As may be inferred from formulas (28), (30), (32), (33), and (36), the angular width  $\Delta \psi$  of the *n*th diffracted beam (n = 0, 1, ..., N) is defined in anisotropic media not only by the ratio  $\lambda_n/D$  (or  $\lambda_0/D$  for the principal beam), but also by the value of the derivative  $d\psi/d\varphi$  at  $\varphi = \varphi_n$  (which defines the orientation of the wave vectors of the secondary sources), whereat constructive interference occurs to give rise to the *n*th beam. And so we will consider briefly how the  $\psi(\varphi)$ dependence and the  $d\psi/d\varphi$  derivative vary in the case of an MSSW<sup>24</sup> in a ferrite slab located in a free space (see Fig. 7).

Since  $d\psi/d\varphi \equiv 1$  for isotropic media, the value of derivative  $d\psi/d\varphi$  is conveniently compared with unity. As may be seen from Fig. 7b, the inequality  $|d\psi/d\varphi| > 1$  is always fulfilled for MSSWs with frequencies lying in the upper part of the spectrum (curve 6). With a decrease in MSSW frequency (curves 3–5), a progressively broadening interval of  $\varphi$  values appears in which  $|d\psi/d\varphi| < 1$ , while for the frequencies lying in the initial part of the MSSW spectrum (curves 1 and 2), such  $\varphi$  values even appear at which  $d\psi/d\varphi = 0$ . As is clear from formulas (28) and (30), if  $\varphi_n$  and  $\varphi_0$  turn out to be equal to the  $\varphi$  value whereat  $d\psi/d\varphi = 0$ , the angular width  $\Delta\psi$  of the appropriate *n*th diffracted beam will also be equal to zero!

In anisotropic media, it is convenient to calculate not the quantity  $\Delta \psi$  itself in degrees, but the ratio  $\sigma$  between the angular width  $\Delta \psi$  (in radians) and  $\lambda_0/D$  (the angular width of the diffracted beam in an isotropic medium in radians):

$$\sigma = \frac{\Delta \psi}{\lambda_0 / D} \,. \tag{37}$$

From the physical standpoint,  $\sigma$  may be termed the *relative angular width of the principal diffracted beam*: when  $\sigma$  turns out to be smaller (greater) than unity, this signifies that  $\Delta \psi$  is smaller (greater) than in isotropic media.

We now turn to calculating the relative angular width  $\sigma$  of the principal diffracted beam. These calculations will be performed in two ways: directly with formula (37), and applying numerical methods, first determining the  $A(\psi)$ dependence from formulas (16) and (14) and then finding from this dependence the angular width  $\Delta \psi$  at a level of 0.5 and the corresponding  $\sigma$  value. Comparing the results of calculations done by the two methods makes it possible to elucidate how precisely formulas (30) and (37) (which were derived under certain assumptions) describe the quantities  $\Delta \psi$ and  $\sigma$ .

Figure 8 depicts the calculated dependences of  $\sigma$  on  $\chi_0$  (on the angle between  $\mathbf{k}_0$  and  $\mathbf{V}_0$ ) in two cases: (i) the wave vector

<sup>&</sup>lt;sup>21</sup> Neglecting excitation effects at the transducer ends.

<sup>&</sup>lt;sup>22</sup> To investigate the case when the transducer aperture may not be considered as being in-phase, it is possible to use the geometry wherein the vector  $\mathbf{k}_0$  of the initial MSSW is inclined to the slit line by some angle. <sup>23</sup> In the example of an MSSW in a ferrite slab, as in the description of waves in the majority of other anisotropic structures, the  $\varphi$  angles are so reckoned that the collinear wave corresponds to a value of  $\varphi = 0$ .

<sup>&</sup>lt;sup>24</sup> The  $\psi(\phi)$  dependences for an MSSW and other waves are discussed in greater detail, for instance, in Refs [5, 12, 38].

**k**<sub>0</sub> of the initial MSSW is oriented normally to the screen (Fig. 8a), and (ii) the group velocity vector **V**<sub>0</sub> of the initial MSSW is oriented normally to the screen (Fig. 8b). Varying the  $\chi_0$  angle of the initial MSSW from  $-90^{\circ}$  to  $90^{\circ}$  corresponds to a displacement of point S from one end of the isofrequency curve *1* to the other one, the screen orientation simultaneously changing in such a way that the normal to the screen (the *y*'-axis) remains aligned with the vector **k**<sub>0</sub> in the former case, and aligned with the vector **V**<sub>0</sub> in the latter case. The  $\sigma(\chi_0)$  dependences in Fig. 8 were calculated for two frequencies of the initial MSSW:  $f_0 = 2900$  MHz, and  $f_0 = 2330$  MHz; for the latter frequency,<sup>25</sup> as is clear from Fig. 7b, a situation is realized with the derivative  $d\psi/d\varphi = 0$  (!) at  $|\chi_0| = 73^{\circ}$  (at  $|\varphi_0| = 45^{\circ}$ ).

When discussing the  $\sigma(\chi_0)$  dependences given in Fig. 8, first of all we note that, even at  $\lambda_0/D = 0.01$ , curves 1 and 4 calculated by formula (37) agree closely with curves 2 and 5, respectively, calculated by numerical methods from the  $A(\psi)$  dependence. At the same time, only approximate agreement is observed between the respective curves (curves 1, 4 and curves 3, 6) at  $\lambda_0/D = 0.1$ .

Also, it is easily seen that the  $\sigma(\chi_0)$  dependence presented in Fig. 8a is, in fact, the inverted  $d\psi/d\phi$  dependence, which was depicted in Fig. 7b, for the appropriate frequencies: when the wave vector  $\mathbf{k}_0$  of the initial MSSW is oriented normally to the screen,  $\Delta\psi$  is described by formula (32), whence follows the relation

$$\sigma = \left| \frac{\mathrm{d}\psi}{\mathrm{d}\varphi}(\varphi_0) \right|. \tag{38}$$

When the group velocity vector  $\mathbf{V}_0$  of the initial MSSW is oriented normally to the screen (Fig. 8b) and the  $\sigma(\chi_0)$ dependence is defined by the general formula (37), an appreciable effect on the magnitude of  $\sigma$  is also exerted by the derivative  $dk/d\varphi$ : near the collinear *y*-axis (see Fig. 3), where *k* changes only slightly, the value of the derivative  $dk/d\varphi$  is small, but near the asymptotes<sup>26</sup> of the isofrequency curves (as  $|\chi_0| \rightarrow 90^\circ$ ) this derivative, which appears in the denominator of formula (37), becomes rather large, which explains why all the curves in Fig. 8b tend to zero when  $|\chi_0| \rightarrow 90^\circ$ .

<sup>25</sup> To dispel doubt concerning the validity of the calculations at a frequency  $f_0 = 2330$  MHz, which is close to the origin of MSSW spectrum (the doubt arising from the circumstance that the magnetostatic approximation may be invalid), we present the additionally calculated parameters of the MSSW at this frequency:  $k_0 = \omega/c = 0.488 \text{ cm}^{-1}$ , the *minimal* value of the wave vector modulus  $k_{\min} = k(\varphi = 0^\circ) = 52.47 \text{ cm}^{-1}$ , and  $k \approx 140 \text{ cm}^{-1}$  for  $|\varphi| = 45^\circ$ . That is, for an MSSW with  $f_0 = 2330 \text{ MHz}$ , the inequality  $k \ge k_0$  is fulfilled for all orientations of the wave vector. Furthermore, it was earlier shown that the MSSW characteristics calculated proceeding from the complete system of Maxwell equations and in the magnetostatic approximation hardly differ even for  $k \ge 3 \text{ cm}^{-1}$  (compare curves *I* and *2* in Fig. 1 of Ref. [50]).

<sup>26</sup> Strictly speaking, the results of calculations presented here are not exact near the asymptotes of the isofrequency curves, because a more accurate calculation of the isofrequency dependence itself is required in this domain. Indeed, the isofrequency dependence is fitted by curves *I* and *2* in Fig. 3 only for MSWs (to state it in other words, for the waves with wave numbers  $k \approx 10-10^4$  cm<sup>-1</sup>). However, near the asymptotes of the MSW isofrequency dependence (where  $k \rightarrow \infty$  and the values of *k* are beyond the specified range), one has to take into account exchange interaction in the calculation of dispersion dependences even for  $k \sim 10^5$ , i.e., the isofrequency dependence will deviate from the asymptotes to transform to the isofrequency dependence of exchange spin waves.



**Figure 8.** Relative angular width  $\sigma$  of the principal diffracted beam as a function of  $\chi_0$  (the angle between vectors  $\mathbf{V}_0$  and  $\mathbf{k}_0$  of the initial MSSW): the normal to the screen (the *y*'-axis) coincides (a) with the vector  $\mathbf{k}_0$ , and (b) with the vector  $\mathbf{V}_0$ . Curves *I*–3 were obtained for  $f_0 = 2900$  MHz, and curves *4*–6 for  $f_0 = 2330$  MHz; curves *I*, 4 were calculated by formula (37), curves 2, 5 are the results of numerical calculations at  $\lambda_0/D = 0.01$ , and 3, 6 are the results of numerical calculations at  $\lambda_0/D = 0.1$ .

As suggested by our calculations, at  $\varphi_0 = 0$ ,  $\psi_0 = 0$ , and  $\chi_0 = 0$ , i.e., when an MSSW with collinearly oriented vectors  $\mathbf{k}_0$  and  $\mathbf{V}_0$  is normally incident on the slit, the angular beam width always turns out to be greater than in isotropic media (see Fig. 8). In this case, the quantity  $\sigma$  always remains greater than unity, although it decreases with decreasing frequency (at  $\varphi_0 = 0$  and  $\chi_0 = 0$ , the quantity  $\sigma$  is described by formula (38) and, as is clear from Fig. 7b, one has  $|d\psi/d\phi| > 1.3$  for all the curves at  $\varphi = 0$ , i.e., always greater than unity). It should be emphasized that it does not transpire from this fact that in the normal incidence of a collinear MSW on a slit the primary diffracted beam always possesses a greater angular width than a similar beam in isotropic media: this situation occurs for an MSSW in a free ferrite film, but for an MSW in other structures or for another type of MSW the situation may be different<sup>27</sup>—everything depends on the magnitude of

<sup>&</sup>lt;sup>27</sup> Evidently, for another type of MSW or in other structures the dependences of  $\psi$  and  $d\psi/d\varphi$  on  $\varphi$  will be different from those in Fig. 7, and the cases where  $d\psi/d\varphi < 1$  or even  $d\psi/d\varphi \ll 1$  will be possible for a collinear MSW. For instance, the MSW isofrequency dependences for a metal–ferrite–magnetic-wall structure in the neighborhood of  $\varphi_0 = 0$  are quite close to a straight line (see Fig. 6 in Ref. [37]), i.e.,  $d\psi/d\varphi \ll 1$  for a collinear MSW in this structure, and  $\sigma \ll 1$  in the case of *normal incidence of a collinear MSW on a slit*.



**Figure 9.** Relative angular width  $\sigma$  of the principal diffracted beam as a function of angle  $\chi_0$  between vectors  $\mathbf{V}_0$  and  $\mathbf{k}_0$  in the initial MSSW in the case where the wave vector  $\mathbf{k}_0$  of the initial MSSW is parallel to the slit line (to the screen). Curves I-3 were obtained for  $f_0 = 2900$  MHz, and curves 4-6 for  $f_0 = 2330$  MHz. Curves 1, 4 were calculated by formula (37), curves 2, 5 resulted from numerical calculations for  $\lambda_0/D = 0.01$ , and curves 3, 6 came from numerical calculations for  $\lambda_0/D = 0.1$ .

 $|d\psi/d\phi|$  for the value of  $\phi$  corresponding to a wave of collinear nature.

As is clear from Fig. 8, the angular beam width gradually decreases as the absolute values of  $\chi_0$  increase, with the angular beam width decreasing to zero for those frequencies lying in the initial region of the spectrum. Specifically, for instance, at  $f_0 = 2330$  MHz the quantity  $\sigma = 0$  for  $|\phi_0| = 45^\circ$  and  $|\chi_0| = 73^\circ$ , because  $d\psi/d\phi = 0$  at  $|\phi_0| = 45^\circ$  (see curve 2 in Fig. 7). Physically, this signifies that the beam transmitted through the slit does not expand and retains its absolute width for the given parameters of the initial MSSW.<sup>28</sup>

Figure 9 presents the results of  $\sigma$  calculations in the case where the initial MSSW wave vector  $\mathbf{k}_0$  is oriented parallel to the slit line and to the screen. For definiteness, we assumed in the calculations that the vector  $\mathbf{k}_0$  is directed from the left slit edge to the right one. Evidently, the group velocity vector  $\mathbf{V}_0$ of the initial MSSW will then be directed towards the slit only for those waves in which the angle  $\chi_0$  between the vectors  $\mathbf{V}_0$ and  $\mathbf{k}_0$  adopts negative values from the interval  $-90^\circ < \chi_0 < 0$  (the vector  $\mathbf{V}_0$  will not be directed to the slit for waves with positive angles  $\chi_0$ ). The  $\sigma(\chi_0)$  dependences in Fig. 9 were calculated for the same two frequencies of the initial MSSW as in Fig. 8.

With reference to Fig. 9, curves *1* and *4*, which were calculated by formula (37), nearly perfectly coincide with the corresponding curves 2 and 5, which were calculated using numerical methods from the  $A(\psi)$  dependence for  $\lambda_0/D = 0.01$ . On the other hand, curves *1* and *4* show a less exact coincidence with curves *3* and *6* calculated by numerical methods for  $\lambda_0/D = 0.01$ . For angles  $\chi_0$  close to  $-90^\circ$ , the behavior of  $\sigma(\chi_0)$  dependences in Figs 9 and 8b is identical, because the geometry of wave incidence in both cases is such



**Figure 10.** Relative angular width  $\sigma$  of the principal diffracted beam for fixed parameters of the initial MSSW with  $f_0 = 2900$  MHz and different slit orientations  $\theta$  relative to the orientation  $\psi_0$  of the vector  $\mathbf{V}_0$  of the initial MSSW (the angular difference  $\theta - \psi_0$  on the abscissa axis varies in the interval  $-90^\circ < \theta - \psi_0 < 90^\circ$ ). Curves *I*, 2 were obtained at  $\varphi_0 = 0$ ( $\psi_0 = 0, \chi_0 = 0$ ); 3, 4 at  $\varphi_0 = -10^\circ$  ( $\psi_0 = 26.3^\circ$ ,  $\chi_0 = -36.3^\circ$ ); 5, 6 at  $\varphi_0 = -22.6^\circ$  ( $\psi_0 = 43.7^\circ, \chi_0 = -66.3^\circ$ ), and 7, 8 for  $\varphi_0 = -33^\circ$ ( $\psi_0 = 54.9^\circ, \chi_0 = -87.9^\circ$ ). Curves *I*, 3, 5, and 7 were calculated with formula (37), and curves 2, 4, 6, and 8 were calculated by numerical methods for  $\lambda_0/D = 0.01$ .

that the vector  $\mathbf{k}_0$  of the initial MSSW is parallel or almost parallel to the slit line, and the vector  $\mathbf{V}_0$  is normal or almost normal to the slit line. However, the value of  $\sigma$  in Fig. 9 rises sharply with a decrease in  $\chi_0$ , and for  $\chi_0 = 0$  the  $\sigma(\chi_0)$ dependence looses its meaning, because the vectors  $\mathbf{V}_0$  and  $\mathbf{k}_0$  of the initial MSSW become collinear in this case ( $\chi_0 = 0$ ,  $\varphi_0 = 0$ , and  $\psi_0 = 0$ ), and the wave is no longer incident on the slit, propagating parallel to the screen. The growth of  $\sigma$  for  $\chi_0$ close to zero is easily explained by formula (36) which describes the angular beam width for the  $\mathbf{k}_0$  vector orientation along the line slit: the closer  $\chi_0$  and  $\varphi_0$  to zero, the smaller is the quantity  $dk/d\varphi$  appearing in the denominator of formula (36). We also note that the values of  $\sigma$  for a frequency  $f_0 = 2330$  MHz in Fig. 9, same as in Fig. 8, approach zero at  $|\chi_0| = 73^\circ$  (see curves 4–6 in Fig. 9).

Figure 10 demonstrates the calculated values of  $\sigma$  when parameters  $\varphi_0, \psi_0$ , and  $\chi_0$  of the initial MSSW are fixed, while the screen normal orientation  $\theta$  changes relative to the vector  $V_0$  of the initial MSSW in such a way that the value of  $\theta$  differs from  $\psi_0$  by no more than 90° (i.e., the angular difference  $\theta - \psi_0$  plotted on the abscissa axis varies from  $-90^\circ$  to  $90^\circ$ ). Curves 1 and 2 in Fig. 10 serve to illustrate the  $\sigma(\theta - \psi_0)$ dependence for the initial MSSW with collinear vectors  $V_0$ and  $\mathbf{k}_0$  ( $\chi_0 = 0$ ,  $\varphi_0 = 0$ , and  $\psi_0 = 0$ ), and, therefore, the value of  $\sigma$  coincides at  $\theta - \psi_0 = \theta = 0$  with the corresponding  $\sigma$ values for similar geometries of incidence in Fig. 8 (at  $\chi_0 = 0$ ). With an increase in the absolute values of  $|\theta - \psi_0| = |\theta|$ , the  $\sigma$ values rise steeply in about the same way as in the right part of Fig. 9 (because the limiting cases  $\chi_0 \rightarrow 0$  in Fig. 9, and  $|\theta| \rightarrow 90^{\circ}$  for curves 1 and 2 in Fig. 10 correspond to the same geometries of incidence). As is clear from Fig. 10, the greater the absolute value  $|\chi_0|$  of the angle between the vectors  $\mathbf{k}_0$  and  $\mathbf{V}_0$  of the initial MSSW, the lower lie the corresponding dependences  $\sigma(\theta - \psi_0)$ : in particular, for curves 7 and 8, which correspond to  $\chi_0 = -87.9^\circ$ ,  $\sigma < 1$  for almost any

<sup>&</sup>lt;sup>28</sup> It seems likely that in experiment the MSSW beam angular width may turn out to be not exactly equal to zero at  $d\psi/d\varphi = 0$ , because in an actual ferrite slab the angular beam width may be affected by different factors (for instance, an insufficiently small  $\lambda_0/D$  ratio, or some nonuniformity of an external magnetic field  $\mathbf{H}_0$  or of ferrite magnetization  $4\pi M_0$  in the region of beam propagation). It is nevertheless evident that, in a real medium,  $\sigma$  and  $\Delta \psi$  will assume the smallest possible values at  $d\psi/d\varphi = 0$ .



**Figure 11.** Relative angular width  $\sigma$  of the principal diffracted beam as a function of  $\chi_0$  (the angle between vectors  $\mathbf{V}_0$  and  $\mathbf{k}_0$  in the initial MSSW) for different parameters  $\varphi_0$ ,  $\psi_0$ , and  $\chi_0$  of the initial MSSW with f = 2900 MHz and a fixed screen orientation  $\theta$ . Curves 1, 2 were obtained at  $\theta = 0$ ; curves 3, 4 at  $\theta = 20^\circ$ ; 5, 6 at  $\theta = 32^\circ$ ; 7, 8 at  $\theta = 40^\circ$ ; 9, 10 at  $\theta = 45^\circ$ ; 11, 12 at  $\theta = 70^\circ$ , and 13, 14 at  $\theta = 120^\circ$ . Curves 1, 3, 5, 7, 9, 11, and 13 were calculated by formula (37), and curves 2, 4, 6, 8, 10, 12, and 14 were obtained by numerical methods for  $\lambda_0/D = 0.01$ .

screen orientation (i.e., the angular beam width is smaller than in isotropic media). We note, however, that the position of the  $\sigma(\theta - \psi_0)$  curves may also vary nonmonotonically with parameter  $\chi_0$ , because  $\sigma$  is determined largely by the values of  $d\psi/d\phi$  and  $dk/d\phi$  at  $\phi = \phi_0$ . For instance, the dependences  $\sigma(\theta - \psi_0)$  for  $f_0 = 2330$  MHz (not plotted in the drawing) will vary as follows: initially, with an increase in  $|\chi_0|$ , the dependences  $\sigma(\theta - \psi_0)$ , as in Fig. 10, shift progressively closer to the abscissa axis. However, even at  $\phi_0 = -45^\circ$ ,  $\psi_0 = 28^\circ$ , and  $\chi_0 = -73^\circ$ , the  $\sigma(\theta - \psi_0)$  dependence is hardly different from the abscissa axis [since  $d\psi/d\phi = 0$  for  $\phi = \phi_0 = -45^\circ$  (see curve 2 in Fig. 7b)], and next, on a further increase in  $|\chi_0|$ , the  $\sigma(\theta - \psi_0)$  dependences will shift upwards and then downwards again.

Finally, Fig. 11 exhibits the calculated values of  $\sigma$  in the case where the screen orientation is fixed, and the parameters  $\varphi_0$ ,  $\psi_0$ , and  $\chi_0$  of the initial MSSW (with a frequency  $f_0 = 2900$  MHz) are varied (the position of point S, which defines the parameters  $\varphi_0$ ,  $\psi_0$ , and  $\chi_0$  of the initial MSSW, varies from one end of the isofrequency curve 1 in Fig. 3 to the other). Under this variation of the position of the point S, the direction  $\psi_0$  of the group velocity of the initial wave will vary within the interval  $-56.53^{\circ} < \psi_0 < 56.53^{\circ}$ , and, therefore, for  $|\theta| > 90^{\circ} - 56.53^{\circ} = 33.47^{\circ}$  (i.e., when the screen angle of inclination  $\theta$  to the y-axis exceeds 33.47°) the vector  $\mathbf{V}_0$  of the initial wave for some values of  $\psi_0$  may turn out to be directed away from the screen rather than towards it. And so the  $\sigma(\chi_0)$  dependences corresponding to the values of  $|\theta| < 33.47^{\circ}$  (curves 1–6) are located in the entire interval of possible values of  $\chi_0$  in Fig. 11, while the  $\sigma(\chi_0)$  dependences, which correspond to  $|\theta| > 33.47^\circ$  (curves 7-14), occupy only a part of the interval of possible values of  $\chi_0$ . In this case, the greater  $|\theta|$ , the broader is the interval of  $\chi_0$  values in which diffraction does not occur (simply because the vector  $\mathbf{V}_0$  of the initial wave is not directed towards the screen). Similar dependences for the initial MSSW with a frequency  $f_0 = 2330$  MHz (not shown in the drawing) vary overall like the  $\sigma(\chi_0)$  dependences in Fig. 11. The only difference is that, for  $f_0 = 2330$  MHz, the value of  $\sigma$ 

becomes equal to zero for  $|\varphi_0| = 45^\circ$ ,  $|\psi_0| = 28^\circ$ , and  $|\chi_0| = 73^\circ$  (i.e., when  $d\psi/d\varphi = 0$ ).

#### 10. Discussion of results and summary

Evidently, by analogy with the calculations of the angular width of the MSSW principal diffracted beam cited above for different geometries, it is also possible to calculate the angular width of other MSW diffracted beams for various ferrite structures (containing, for instance, ferrite, dielectric, metal, and magnetic wall layers).

As with other relationships common for waves in general, the results of our treatment, which was carried out using the example of an MSSW propagating in a ferrite slab, will hopefully permit performing calculations of the angular diffracted beam width not only for the slit diffraction of other MSWs in different structures, but also for the slit diffraction of waves of other natures that propagate through different anisotropic media and structures (for two-dimensional geometries). In particular, since the results obtained in this paper are based on (i) the momentum conservation law, (ii) the Huygens principle, and (iii) the proposition that the direction of the group velocity vector adequately describes the direction of propagation of the wave power flux, these results may also be employed for determining the angular width of the diffracted beams of other waves (waves of another nature) that obey the listed physical regularities.

Consider, for instance, the two-dimensional case of plane light wave diffraction from a slit in a medium characterized by an ellipse type isofrequency dependence. As in the case with an MSW, the momentum conservation law will determine the direction of the amplitude maximum of the principal diffracted beam, i.e., this direction will coincide with the orientation of the group velocity vector of the initial wave. Unlike the MSW field, the light wave field cannot be described by a scalar function, and the electric field vectors of the light field will have different orientations in different directions owing to the anisotropy of the medium. However, when the absolute angular width of the diffracted beam is small enough (this is always possible to achieve by taking the value of  $\lambda_0/D$  to be sufficiently small), the electric vectors of the light field will be almost collinear in the very narrow angular interval contained between the two zeroes closest to the diffraction maximum, and, therefore, their magnitudes may be summed. In this way, we can find the light amplitude distribution (the diffraction pattern) in some narrow angular interval, which is quite sufficient for the subsequent determination of the angular width of the diffracted beam by the method identical to that employed for isotropic media and MSWs.

To apply the resultant formulas in the investigation of diffraction of another type of wave (possessing another nature), one actually has to evaluate, proceeding from the dispersion relation describing the propagation of this wave, the dependences  $k(\varphi)$  and  $\psi(\varphi)$  and calculate their derivatives  $dk/d\varphi$  and  $d\psi/d\varphi$  at  $\varphi = \varphi_n$ —the value of  $\varphi$  corresponding to the *n*th diffracted beam (or, at  $\varphi = \varphi_0$ , when an investigation is made of the parameters of the principal diffracted beam).

For instance, by applying the results of our investigation to the analysis of the possible angular beam width in twodimensional geometries of uniaxial optical crystals (in which the isofrequency dependence for the extraordinary wave is an ellipse), it is possible to conclude from the form of the  $\psi(\varphi)$  dependence for an ellipse (see curve 2 in Fig. 3a in Ref. [5]) that the derivative  $d\psi/d\phi$  may not be equal to zero and, furthermore, it is also easy to establish that the derivative  $dk/d\phi$  may never tend to infinity in the case of an ellipse. Consequently, the angular beam width in the two-dimensional geometries of uniaxial optical crystals may never be equal to zero. At the same time, when analyzing the isofrequency dependences of acoustic waves, for instance, in paratellurite and rutile (the general form of these 'four-lobe' dependences can be found in book [51]), it is easily seen that there are points in these dependences at which  $d\psi/d\phi = 0$ . Therefore, for acoustic waves propagating through such crystals a situation is possible whereat the angular beam width is equal to zero.

The formula obtained for the angular width of the diffracted beam may supposedly be employed in the investigation of metamaterials, because the latter (which comprise various periodic structures) may be characterized by isofrequency dependences, too (see Refs [52–54]).

Experimental research on the diffraction of MSWs and other types of waves in different anisotropic media and structures (for two-dimensional geometries) takes on added importance for verifying the theoretical results obtained above.

#### 11. Conclusions

Employing the example of MSSWs, a theoretical investigation was made of the far-field diffraction pattern in a ferrite slab, emerging when a plane wave is incident on a wide slit in an opaque screen; the investigation was carried out for the most general case wherein the group and phase velocities of the initial MSSW are not collinear and the screen orientation is arbitrary (see Fig. 2). The problem of MSW diffraction was analytically solved in the magnetostatic approximation with the use of a method similar to those employed for isotropic media. In this case, by the farfield of secondary sources was meant the total magnetic potential of the set of MSW elementary secondary sources located along the slit line.

It was established that the far-field action of the set of MSW secondary sources is described, as for isotropic media, by the function of the form  $\sin \Phi/\Phi$ . However, the phase function  $\Phi$ , unlike that in isotropic media, is defined by a more complex expression (14), into which enter not only the parameters of the initial MSW and the slit, but also the isofrequency dependence  $k(\varphi)$  of the wave in the polar coordinate system.

It was found that the slit diffraction of a plane MSSW in a ferrite slab located in a free space gives rise to one (transmitted) or two (transmitted and reflected) diffracted beams. In the general case, the number of reflected and transmitted beams was shown to depend on the geometry of incidence, the initial wave parameters, and the mathematical properties of the isofrequency dependence  $k(\varphi)$  (multivaluedness, the existence of inflection points, etc.).

Mathematically, the emergence of each *n*th diffracted beam signifies that the phase function is equal to zero,  $\Phi(\varphi = \varphi_n) = 0$ , when the wave vectors of the secondary sources are oriented in the direction  $\varphi = \varphi_n$  (n = 0, 1, ..., N), and that the modulated amplitude passes through a maximum in this case:  $A = \sin \Phi/\Phi \rightarrow 1$ . The equation  $\Phi = 0$  is, in essence, one of the forms of writing down the momentum conservation law, which establishes the equality between the projection of the initial wave vector and the projections of the wave vectors of the secondary sources onto the slit line on emergence of the constructive interference. Therefore,  $\varphi_n$  can be calculated proceeding from the momentum conservation law in the framework of geometrical optics. Owing to the anisotropy of the medium, however, the amplitude of each *n*th beam is a maximum not in the direction  $\varphi = \varphi_n$ , but in the direction  $\psi = \psi_n$  defined by the orientation of the group velocity, and the  $\psi(\varphi)$  dependence may be calculated from the known isofrequency dependence  $k(\varphi)$  of the wave. The principal transmitted beam (which corresponds to n = 0) is characterized by the values of  $\varphi = \varphi_0$  and  $\psi = \psi_0$ , which correspond to the initial wave parameters.

A universal analytical formula was derived for evaluating the angular width  $\Delta \psi$  of each MSW diffracted beam. It was established that the angular width  $\Delta \psi$  of the *n*th diffracted beam described by formula (28) is defined by the width D and orientation  $\theta$  of the slit, as well as by the parameters of the isofrequency dependence of the wave  $(\lambda_n, d\psi/d\varphi, and dk/d\varphi)$ at the point corresponding to the value of  $\varphi = \varphi_n$ . The angular widths  $\Delta \psi$  calculated by the resultant formula are in perfect agreement with those obtained by numerical simulations of  $\Delta \psi$  from diffraction patterns for  $\lambda_0/D = 0.01$ . It was found that the angular width  $\Delta \psi$  of the MSW *n*th diffracted beam not only may be greater or smaller than the corresponding beam width in isotropic media, but may also turn out to equal zero under certain conditions. Physically, this signifies that the emergent beam retains constant absolute width in the course of its propagation. This situation takes place when a given *n*th beam corresponds to a value of  $\varphi = \varphi_n$  such that  $d\psi/d\varphi = 0$ . For the geometry wherein the wave vector  $\mathbf{k}_0$  of the initial MSW is oriented normally to the slit line, the formula for the angular width  $\Delta \psi$  of the principal diffracted beam assumes the simplest form and consists of two multipliers:  $\lambda_0/D$ , and the value of  $d\psi/d\varphi$  at  $\varphi = \varphi_0$ . Therefore, how many times the value of  $d\psi/d\varphi$  is greater or smaller than unity, so many times the width  $\Delta \psi$  of the principal beam is greater or smaller than the similar beam width in isotropic media. The resultant formulas may be employed in the calculation of the angular width of the diffracted beams for not only different types of MSWs, but also waves of other natures in various anisotropic media and structures (including metamaterials).

When the  $\psi(\varphi)$  dependence corresponding to the initial wave is single-valued and the inverse dependence  $\varphi(\psi)$  is ambiguous, an interval of polar angles was shown to emerge in the medium (in the structure), wherein two or several waves of the same frequency but with different amplitudes and wave numbers propagate in every direction  $\psi$ . That is, a spatially nonuniform diffraction pattern is observed in this interval, in which the amplitude angular distribution  $A(\psi)$  depends on the distance of the observation point from the slit.

#### Acknowledgments

The author expresses his appreciation to A V Vashkovsky, V G Veselago, S V Gerus, and A Yu Annenkov for fruitful discussions of the results presented in this paper.

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