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On the existence conditions for a fast surface wave

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Contents

Abstract. **Conditions are obtained for the existence of a fast-moving surface electromagnetic wave (with a speed close to the speed of light in the vacuum) on a flat interface between the vacuum and an isotropic dissipative medium with a permittivity $\varepsilon = \varepsilon' + i\varepsilon''$. The interfaces considered include vacuum – seawater, vacuum–metal, vacuum–plasma, and vacuum–dielectric. Conditions for the existence of negligibly damped surface waves are considered for extremely high (vacuum–seawater, vacuum–metal) and very low (vacuum–plasma, vacuum–dielectric) ε'' values. It is shown that at least in these two limit cases, the phase wave velocity V_p and the group wave velocity V_g pass synchronously through the speed of light c in the vacuum, which can be considered the reason why surface waves exist at the interface between the vacuum and a collisionless plasma (with $\varepsilon' < -1$ and $V_{p,g} < c$) and do not exist at the interface between the vacuum and a weakly absorbing dielectric (with $\varepsilon' > 1$ and $V_{p,g} > c$). In the first limit case, it is shown that both the phase and group velocities pass c at $\varepsilon' = -3/4$, implying that a surface wave exists at the vacuum–metal interface (with $\varepsilon' < -3/4$), but that a surface wave (Zenneck's wave) cannot exist at the vacuum–seawater interface (with $\varepsilon' > -3/4$).**

1. Introduction

Immediately after Marconi [1] accomplished a radio signal transmission at the frequency of 30 kHz across the Atlantic Ocean in 1901, two competing hypotheses emerged on the mechanism of electromagnetic wave propagation beyond the horizon. The Kennedy–Heaviside hypothesis [2, 3] explained this effect by the alleged presence in Earth's upper atmosphere of ionospheric layers reflecting the signal, which were actually discovered later. The second, no less plausible, hypothesis by Sommerfeld and Zenneck [4, 5] referred to the effect of 'sticking' to the spherical surface of Earth of a surface wave (SW) propagating almost without damping; this wave was later named the Zenneck wave.

Currently, no one doubts that global radio communication is realized due to the existence of the ionosphere. These effects have been well studied both theoretically and experimentally; this is reflected in the extensive literature that can be found, for example, in [6]. According to numerous experiments and the corresponding mathematical models, global radio communication is realized in the spectrum of higher-order modes of a spherical shielded waveguide, with one of the 'walls' being the well-conducting earth (sea) surface and the second wall being the ionosphere. The constructed theoretical models, where Earth's surface was assumed for simplicity to be perfectly conducting, demonstrate [7] that the bulk, 'cable', mode of the spherical waveguide is not involved in the effects of radio transmission beyond the horizon. The reason is that the field of this mode is increasingly driven, with increasing frequency, to the concave surface of the ionosphere, and not to the convex surface of Earth. As a result, it is considered [7] that the efficiency of cable mode excitation by artificial terrestrial antennas is very low. Only lightning can excite low-frequency Schumann resonances with the field structure of the cable mode.

These are, in general, the fundamentals of the current theory that explains the global effects of the excitation of natural and artificial electromagnetic fields above Earth's spherical surface in the frequency range of 0 – 10 MHz, with the ionospheric influence being decisive. Probably the only vulnerable point of mathematical models constructed on the basis of this theory is that Earth's surface is not a perfect conductor in reality. This approximation is valid only for very low frequencies, because the sea water conductivity σ is about seven orders of magnitude lower than that of copper. As a result, we cannot exclude the possibility that the cable mode field can actually be driven to Earth's surface as well, already for frequencies of the order of several kHz and higher. In any case, this important issue requires special consideration, as we

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mention here, because it may be closely related to the main topic of this paper focusing primarily on the consideration of the second hypothesis — the Sommerfeld–Zenneck hypothesis.

The history of this issue is very instructive for many reasons. The main result obtained in this paper is that the hypothetical Zenneck surface wave over a *plane* sea or Earth surface cannot exist because its *group* velocity, and not only the phase velocity, is higher than the speed of light.

We give a brief historical background of the Zenneck SW.

The idea of the existence of electromagnetic waves on the surface of a conducting medium occurred to Sommerfeld in 1896 [8] when he studied diffraction of a homogeneous plane electromagnetic wave by a perfectly conducting surface. This idea was further developed by Sommerfeld's student Zenneck, who solved a model problem of SW propagation over an *infinite* imperfectly conducting plane [4]. In hindsight, this is a very modest result, of course, but at the time, when the theory of shielded waveguides, and then open waveguides, was still very far away, this result inspired many experimental physicists to what turned out to be a generally unsuccessful search for the wave; despite everything, this pursuit is still going on [9].

A characteristic feature of the Zenneck SW is that for media with a positive real part ε' of the dielectric permittivity $\varepsilon = \varepsilon' + \mathrm{i}\varepsilon''$ of the underlying half-space (sea water, earth soil), the wave phase velocity exceeds the speed of light in the vacuum, $V_{\mathrm{p}} > c$. This wave differs, for example, from conventional SWs, which were found later in the wave spectrum of dielectric waveguides and which constitute the physical basis for the transmission of electromagnetic signals in optical communication lines. This also makes it different from the Sommerfeld SW with $V_{\mathrm{p}} < c$, which can propagate over a cylindrical conductor, such as in wire antennas. The condition $V_{\mathrm{p}} < c$ is a characteristic feature of all actually observed SWs, regardless of the cross-sectional shape of an open waveguide or the physical state of its internal material medium, of whether it is an insulator or a conductor or a collisionless plasma discharge.

At present, theoretical and experimental studies related to SWs propagating over good conductors with a flat cross-sectional configuration and thickness much larger than the skin layer depth are performed in the terahertz frequency range [10]. It is important that $\varepsilon' < 0$ in this case (in contrast to the case over the sea surface or Earth soil). For SWs propagating along a plane vacuum–metal interface (in other words, for propagating surface plasmons [10] or plasmon polaritons [11]), the phase velocity is $V_{\mathrm{p}} < c$ [10]. As we already mentioned, this is so for all conventional SWs. Therefore, although these two cases resemble each other, there is no reason to name a normal SW, with $V_{\mathrm{p}} < c$, a Zenneck wave, for which $V_{\mathrm{p}} > c$ in general. However, the authors of Ref. [10], as well as many others in the list of references in [10] do exactly so, in contrast to the authors of Ref. [11], for example. On the other hand, the authors of Ref. [11] (and, of course, not only they) usually name a common SW ($V_{\mathrm{p}} < c$) over a plane metal or plasma surface a Fano wave. Over a cylindrical metal surface, an SW is sometimes referred to as a Sommerfeld wave. Are there not too many names invented for the same physical phenomenon? But this is not so bad, if a variety of names cover the same *physical* phenomenon. An utter historical curiosity is that, for the mathematically correct solution of a singular boundary value Sturm–Liouville problem corresponding to the Zenneck wave in the original sense of this term ($V_{\mathrm{p}} > c$), there is no physical phenomenon. The proof of this statement is the main subject of our work here. To make this case a lesson for the future, we need to understand how it could happen that the result of mathematical calculations tricked the expectations of physicists, and this delusion has lasted a whole century. For this, we need to continue considering the historical background.

In 1907, Zenneck [4] obtained a solution of the corresponding boundary value problem for an individual 'mode' with the above properties. To prove the theoretical possibility of the existence of a single mode, it is necessary, of course, to solve the problem of its excitation by a real antenna.

First, Sommerfeld, having examined the problem of wave excitation over a conducting plane by a vertical Hertz dipole, proved in 1909 [5] that the Zenneck wave field is present in the dipole field at large distances from it. However, later Weyl [12] and then Fock [13], van der Pol [14], and others [15] found an error in Sommerfeld's calculations. It turned out that because $V_{\mathrm{p}} > c$ for the Zenneck wave, the wave cannot be isolated as a separate mode from the plane wave integral describing the dipole field. In 1926, Sommerfeld corrected his error and showed the following [16]. The pole of the spectral function corresponding to the Zenneck wave, although not directly captured (due to $V_{\mathrm{p}} > c$) by the integration contour when deforming it into a saddle contour, is in close proximity to that contour; therefore, a kind of imitation of the Zenneck wave field takes place in the vicinity of an antenna, later called the 'Sommerfeld numerical distance'. But at large distances from the antenna, outside the numerical distance, the Zenneck wave field is absent in the dipole field. We note that the latest measurement results obtained in the field experiment conducted in 2009 on a two-layer ice–salt-water 1.2 km path at frequencies of 10 and 15 MHz [9] are fully consistent with Sommerfeld's result and confirm that with increasing the distance outside the Sommerfeld numerical distance, the field simulating the Zenneck SW within it is exponentially damped.

Hence, the *mathematical* fact was established by collective efforts of prominent scientists in theoretical studies [12–14, 16] that the boundary condition at infinity, which is well satisfied by the Zenneck wave, does no play any role for the actual existence of the wave. Indeed, the actual existence of so-called 'leaky waves' not satisfying the boundary condition at infinity in the transverse direction was subsequently proved. This was the second example that indicated a pattern.

The result obtained in Refs [12–14, 16] was the first warning sign for theoreticians as well as experimentalists, but they did not notice it because that was a purely mathematical result, and it was possible, as is often the case, to confront it against a result obtained by another mathematical tool, etc., until 'natural selection' occurred. Because this has not happened, discussions of the physical meaning of the Zenneck SW ($V_{\mathrm{p}} > c$) continue to appear in the literature [17, 18], although a century has already passed since its 'discovery'.

In the considered historical context, a good mathematical tool formulated in terms of an idealized model of an infinite regular open waveguide did not take long to appear in the theory of regular open waveguides [19]. The discrete spectrum of normal waveguide modes includes all SWs without distinguishing the cases $V_{\mathrm{p}} > c$ and $V_{\mathrm{p}} < c$ because from the *mathematical* standpoint, they do not differ from each other. In both cases, the SW field of an *infinite* waveguide can of course exist, according to mathematical calculations, over a regular guiding path, because this field is a solution of the eigenfunction problem in a singular boundary value Sturm–Liouville problem on an infinite interval. The question is, however, in the physical correctness of the formulation itself for the Sturm–Liouville problem on an *infinite* interval under the condition when there is radiation in the system. The radiation characteristics should correspond well to the physics of electromagnetic energy channeling along the path. This requirement is, in theory [19], well satisfied for leaky waves, which, however, are more related to the description of the effects of energy leaking into outer space than the effects of directed energy channeling along the path. As a key element of the theory, this requirement should also be applied to surface modes in the spatial domain, where they are just starting to form or, vice versa, finishing their propagation along the path. It is clear that the idealized model of an infinite waveguide and the related formulation of the Sturm–Liouville problem on an *infinite* interval are not suitable for that. This is the essence of the problem. To solve it, we need a different mathematical tool, with the basics stated in Refs [20–22], and a complete description given in thesis [23].

Briefly, the essence of the approach used in Refs [20–23] to solve the problem can be reduced to the replacement of an infinite interval on which the singular Sturm–Liouville problem is formulated by a semi-infinite interval. As a result, modal (or rather, quasimodal) functions of a semi-infinite (i.e., irregular) open waveguide acquire a form factor — a complex Fresnel integral with a parametric dependence on the wave number of the mode (no matter which one: surface, leaky, or any other mode from the continuum of waves of the continuous spectrum). This completely changes the situation. The mathematical techniques described in Refs [20–23] are very different from the tools described in [19]. Although there is a certain similarity between them, which at some important points is reflected in the formulation of the mathematical apparatus in [20–23], the calculation results differ significantly for these two theories. The results of mathematical calculations now agree with the results in Refs [12–16]. In particular, the presence of Fresnel integrals naturally makes the nature of the radiation fields consistent 'by default' with the nature of the guided mode fields. It turns out that SWs for which $V_p > c$ cannot be matched to the radiation that accompanies the beginning of the formation of this mode, and therefore the mode cannot form. This is manifested in the fact that these modes simply cannot appear in the semi-infinite open waveguide mode spectrum due to the properties of the Fresnel integral, where, in fact, the characteristics of those radiation fields that accompany the process of the beginning of the mode formation are encoded.

Thus, the story could make full circle if it was proven that the SW with $V_p > c$ do not satisfy some other requirements that must necessarily be satisfied by a physical wave. That has not yet been done.

That is why in Europe there is currently, as there was before, continuing financial support for projects [24] based on the idea that the Zenneck wave ($V_p > c$) can be excited over a locally flat sea surface by real coastal antennas. Moreover, in Russia, there is already an operating radio-monitoring system [25] in the decameter range of wavelengths with the operation range of 300 km, which is positioned by its developers as the coastal over-the-horizon surface wave radar (BZGR Podsolnukh-E). If the claim (of over-the-horizon surface wave propagation) in Ref. [25] is correct, and not related to more probable diffraction of 'ground rays', the question arises: what kind of a surface wave can be discussed if the Zenneck wave, as we demonstrate below, has all the features of a *mathematical ghost wave*, because its group velocity exceeds the speed of light in the vacuum?

2. Analysis of the dispersion equation

We consider a plane interface between two media at $z = 0$. In the region $z \leqslant 0$, the medium is described by a complex dielectric permittivity $\varepsilon(\omega)$, and in the region $z \geqslant 0$ we have the vacuum with the dielectric constant $\varepsilon_0 = 1$. Along the interface, parallel to the x axis, a surface electromagnetic wave can propagate,[1] decaying on both sides of the boundary with an increase in the transverse coordinate. The surface wave is the E-type wave (TM wave) with nonzero field components E_z, E_x, and B_y, and E_z satisfies the equation [26]

$$\Delta E_z + \frac{\omega^2}{c^2}\,\varepsilon(\omega) E_z = 0\,. \tag{1}$$

In obtaining this equation from the system of Maxwell equations, we assumed the electromagnetic field to be monochromatic, depending on time as $\exp(-\mathrm{i}\omega t)$, and set $\varepsilon(\omega) \equiv 1$ in the vacuum. Other field components are expressed in terms of the longitudinal component by simple relations (see [26], Ch. 9).

The solution of Eqn (1) for the surface wave is written as

$$E_z(x, z) = \exp(\mathrm{i}kx)\begin{cases} \exp(-\kappa_0 z)\,, & z > 0\,,\\ \exp(\kappa_\varepsilon z)\,, & z < 0\,, \end{cases} \tag{2}$$

where

$$\kappa_0 = \sqrt{k^2 - \frac{\omega^2}{c^2}}\,, \qquad \kappa_\varepsilon = \sqrt{k^2 - \frac{\omega^2 \varepsilon(\omega)}{c^2}}\,. \tag{3}$$

It follows from the requirement for the SW field to decrease on both sides of the media interface away from it that the conditions

$$\mathrm{Re}\,\kappa_0 > 0\,, \qquad \mathrm{Re}\,\kappa_\varepsilon > 0 \tag{4}$$

must be satisfied.

Substituting solution (2) in the electromagnetic boundary conditions (continuity conditions for E_z and B_y on the plane $z = 0$; see [26], Ch. 9), we obtain the dispersion relation for a surface electromagnetic wave:

$$\varepsilon\kappa_0 + \kappa_\varepsilon = 0\,. \tag{5}$$

To simplify the analysis of Eqn (5), we introduce the normalization $K = k/k_0$ ($k_0 = \omega/c$), reducing the equation to

$$\varepsilon\tilde{\kappa}_0 + \tilde{\kappa}_\varepsilon = 0\,. \tag{6}$$

Here, $\tilde{\kappa}_0 = \sqrt{K^2 - 1}$ and $\tilde{\kappa}_\varepsilon = \sqrt{K^2 - \varepsilon}$, and it is generally assumed that ε and K are complex: $\varepsilon = \varepsilon' + \mathrm{i}\varepsilon''$ and $K = K_1 + \mathrm{i}K_2$. Equation (6) in the complex plane has a

[1] In the *Great Soviet Encyclopedia*, the following definition of the Zenneck wave is given: "J Zenneck theoretically showed in 1907 that along plane surface of the Earth's ground (or sea), a surface wave, similar to the wave propagating along a wire, can propagate."

simple solution [24]:

$$K^2 = \frac{\varepsilon}{\varepsilon + 1} \,. \tag{7}$$

Surprisingly, a detailed and consistent physical analysis of such a simple equation as Eqn (6) that would cover the whole range of possible values of ε, $\varepsilon' \in [-\infty, +\infty]$ and $\varepsilon'' \geqslant 0$, has not been carried out, apparently. And when such analysis was conducted anyway, it was not completed by conclusions important for physics. Indeed, otherwise, as we show, it would have become clear long ago that a wave such as the Zenneck wave, with the phase velocity greater than the speed of light in the vacuum, cannot exist, not only for this reason (stated in Ref. [18] as a consequence of the mathematical calculations in Refs [20–23]), but also, and, more importantly for physics, because its group velocity is greater than the speed of light, $V_g > c$.

This previously unknown result that puts everything in place follows from a consistent physical analysis of Eqn (6). Such an analysis is nothing special, and it is unlikely that to date no one has done it, but we did not find it in the literature available to us. Therefore, given the importance of the conclusions arising from this analysis, we provide it here in full.

We begin with a general remark regarding the form of writing field (2) in two adjacent media, which, in fact, leads to an equation like (6). The question of the form of this equation just seems to be trivial; in the literature, it is usually not given proper attention, and that can lead to misunderstandings. The case is as follows.

In [24], the cases in expression (2) involve the imaginary exponentials with the *same positive* sign in the argument before the imaginary unit. This leads to a change from the positive to the negative sign in the second term in Eqn (6); in addition, in both square roots in Eqn (6), their arguments are permuted; we symbolically display that by the corresponding transformation: $\tilde{\kappa}_{0,\varepsilon} \to \bar{\kappa}_{0,\varepsilon}$. It is easy to see that if $\varepsilon > 0$, then expression (7) is a real root of a 'dispersion relation' derived from the fields written in Ref. [24]:

$$\varepsilon\bar{\kappa}_0 - \bar{\kappa}_\varepsilon = 0 \,. \tag{8}$$

This is very convenient, but only for real, positive ε. However, the problem has no relation in this case to the boundary value Sturm–Liouville problem, i.e., to the eigenfunction and eigenvalue problem. It is not difficult to conjecture to which boundary value problem the real solution (7) of Eqn (8) corresponds in this case.

Writing fields in the form of imaginary exponentials in both half-spaces means having two homogeneous plane waves propagating at different angles in the two adjacent media. It is essential that there is energy transfer in the direction of the increasing coordinate z, i.e., (i) there is energy supply to the interface from below ($z < 0$), this energy is not reflected from the boundary ($z = 0$), and it is fully emitted into the vacuum ($z > 0$). Property (i) is a characteristic, invariant feature of the considered problem that we need to remember. It is necessary for the correct interpretation of an analytic continuation of real solution (7) to the complex domain, where (7) is a solution of the dispersion equation written in form (8).

We conclude that there is no energy emission downwards, i.e., the interface does not reflect a plane wave incident at some angle. Consequently, the wave comes to the interface from below at the Brewster angle, because the wave has the polarization necessary for that. Equation (8) in the real domain is an equation for this angle, and not a dispersion equation. Equation (8) is the vanishing condition for the Fresnel expression for the amplitude of a plane wave reflected down (from the interface of two media). This solution of the boundary value problem is known from optics. The physical meaning of the square root in Eqn (8) is obvious:

$$K \equiv \cos\theta_0 = \sqrt{\frac{\varepsilon}{\varepsilon + 1}} \,, \tag{9}$$

where θ_0 is the slip angle of the wave transmitted to the vacuum, counted from the interface plane. It can be easily shown that the Brewster angle θ_ε in the lower medium, measured from the normal to the boundary, can be calculated in a similar way: $\sin\theta_\varepsilon = \sqrt{\varepsilon/(\varepsilon + 1)}$.

In this regard, an analytic continuation of real solution (9) to the complex domain would have the same meaning. Nevertheless, complex values of the angles θ_0 and θ_ε (complex Brewster angle) can be given the meaning of wave numbers of 'modes' directed by the interface, and Eqn (8) can now be treated as a *dispersion equation*. But these 'modes' are not surface modes because they inevitably inherit the invariant property (i) of the problem considered. According to property (i), these 'modes' must be closely related not to the effect of directed energy channeling along the boundary without radiation, to which the structure of the SW field is adjusted, but to the effects of radiation of the channeled energy into the external medium (vacuum). As is known, these 'modes' are leaky waves.

The mathematical mechanism to implement the idea to inherit invariance properties (i) consists in the analytic continuation of real solution (7) to the complex domain mentioned above; in software environments of modern applied (computer) mathematics, it reduces to the following.

Built-in square root functions of a complex number in standard software environments of computer mathematics (e.g., MatLab) return the principal value of the double-valued square root function as the value with a positive real part. For Eqn (6) in the complex domain, this means that conditions (4) for surface waves are automatically satisfied when substituted into an equation for the analytic continuation of solution (9). For Eqn (8), the same actions also give a solution of this equation, with the only difference that for the above reasons, it has the property $\mathrm{Re}\,\bar{\kappa}_{0,\varepsilon} > 0$, but not property (4) as required for an SW. Therefore, and also because $K_{1,2} > 0$ (we consider passive media and waves traveling in the direction of the increasing longitudinal coordinate x), the wave equation in the vacuum for complex solutions of Eqn (8) is satisfied $(1 = K^2 + \bar{\kappa}_0^2)$ only for $\mathrm{Im}\,\bar{\kappa}_0 < 0$. Consequently, for transverse displacement of the observation point from the interface into the vacuum, the amplitude of the wave field always increases. If the additional condition $K_1 < 1$ is also satisfied, such a field in the external medium can be attributed to the physically realizable case of a leaky wave. But Eqn (8) never gives solutions that would correspond to an SW field in the external medium, $\mathrm{Im}\,\bar{\kappa}_0 > 0$.

Therefore, the correct form of the dispersion equation for an SW remains indeed Eqn (6) rather than Eqn (8), as is actually assumed in Ref. [24]. Hence, the form of writing the fields in both media is important. Namely, by choosing to analyze Eqn (6) rather than Eqn (8), we exclude leaky waves from consideration in the complex domain and keep only surface waves, while in the real domain the 'optical branch' solutions are excluded as not being relevant to the Sturm–Liouville problem. The advantage of form (6) of the dispersion equation is also related to the fact that in the real domain, but only under condition $\varepsilon < -1$, it is suitable, in contrast to Eqn (8), for solving the boundary value Sturm–Liouville problem applied for $V_p < c$ to a regular SW that exists on the vacuum–plasma interface. This property is inherited by the analytic continuation of the corresponding real root. As above, the eigenvalue corresponding to this root is calculated by Eqn (7), where the real number $K = K_1 > 1$ stands for the SW deceleration rate relative to the speed of light in the vacuum.

The following is key to our analysis.

The analytic continuation of solutions to the complex domain are always correct from the mathematical standpoint [Eqn (6) with Eqn (7) substituted is automatically satisfied for any values ε from the half-plane $\varepsilon' \in [-\infty, +\infty]$, $\varepsilon'' > 0$], as well as from the physical standpoint in the sense that the eigenfunction *automatically* satisfy requirements (4). But Eqn (6), as well as condition (4), is not satisfied in this case for *positive* real ε: $\varepsilon' \in [0, +\infty]$, $\varepsilon'' = 0$. Moreover, the interval $\varepsilon' \in [-1, 0]$, $\varepsilon'' = 0$ of the real axis is also excluded because conditions (4) are not satisfied in this case, although Eqn (6) is satisfied as before. The solution in this case occurs exactly on the cut $\mathrm{Re}\,\tilde{\kappa}_{0,\varepsilon} = 0$, and therefore does not satisfy conditions (4). However, this case (normal incidence on the interface of an *inhomogeneous* plane wave having no reflection from the interface) is also not relevant to the problem of determining the spectrum of eigenwaves, and refers to solutions of 'complex optics'.

Thus, we have shown that a systematic analysis of the nature of the solution of dispersion equation (6) in the whole range of possible values of complex ε with due exceptions should be reduced, according to formula (9), to a simple operation of taking the square root of the corresponding complex number. The crucial feature of the wave is its phase velocity (and, of course, group velocity); therefore, in the complex half-plane, we first of all need to definitely establish the boundary where the phase velocity passes through the speed of light in the vacuum. Without that, the analysis of solutions is not complete and physically consistent. In the half-plane ε, this boundary is realized as a semi-infinite curve, which, being mapped into the complex quarter-plane K, $K_1, K_2 \in [0, \infty]$, should be a saddle-like integration path (where the equality $K_1 = 1$ is realized) in the theory of normal waves of an open waveguide [19]. Therefore, when extracting the square root of the right-hand and left-hand sides of relation (7) and determining the above boundary of transiting the speed of light, we follow the procedure given below.

After separating real and imaginary parts in the right-hand side and left hand side of relation (7), we obtain a set of two algebraic equations for K_1 and K_2:

$$K_1^2 - K_2^2 = \frac{\varepsilon'\chi + \varepsilon''^2}{a}\,, \tag{10}$$

$$K_2 = \frac{\varepsilon''}{2K_1 a}\,, \tag{11}$$

where we introduce the notation

$$\chi = 1 + \varepsilon'\,, \qquad a = \chi^2 + \varepsilon''^2\,. \tag{12}$$

After substituting Eqn (11) in Eqn (10), we obtain a biquadratic equation for K_1, with the solution

$$K_1 = \frac{1}{\sqrt{2}}\sqrt{\sqrt{\left(1 - \frac{\chi}{a}\right)^2 + \left(\frac{\varepsilon''}{a}\right)^2} + 1 - \frac{\chi}{a}}\,. \tag{13}$$

An explicit expression for K_2 is obtained by substituting (13) in Eqn (11); after simple transformations, we then have

$$K_2 = \frac{1}{\sqrt{2}}\sqrt{\sqrt{\left(1 - \frac{\chi}{a}\right)^2 + \left(\frac{\varepsilon''}{a}\right)^2} - \left(1 - \frac{\chi}{a}\right)}\,. \tag{14}$$

An equation for the line in the plane $(\varepsilon', \varepsilon'')$ of transiting the speed of light can be obtained after substituting unity instead of K_1 in the left-hand side of Eqn (13). The required equation is

$$\varepsilon''^2 = \frac{\chi^3}{1/4 - \chi}$$

or, finally,

$$\varepsilon'' = (1 + \varepsilon')\sqrt{\frac{1 + \varepsilon'}{-(\varepsilon' + 3/4)}}\,. \tag{15}$$

We see that the real branch of Eqn (15) is realized in the strip $-1 < \varepsilon' < -3/4$ of the plane $(\varepsilon', \varepsilon'')$. Hence, there are three conventional regions for analysis in the plane $(\varepsilon', \varepsilon'')$: this strip and two distinct areas on the right side and the left side of it. But, in fact, there are only two fundamentally distinct regions — the regions on the right side and the left side of the curve on which $V_p = c$.

These regions, as well as the boundary between them described by Eqn (15), where $V_p = c$, are shown in Fig. 1.

As we already mentioned, in terms of the variables K_1 and K_2, the separating curve shown in Fig. 1 is the result of mapping the saddle integration path from the complex K plane to the complex plane $(\varepsilon', \varepsilon'')$. In Fig. 1, the image of an infinitely distant point of this contour is the point $(-1, 0)$,

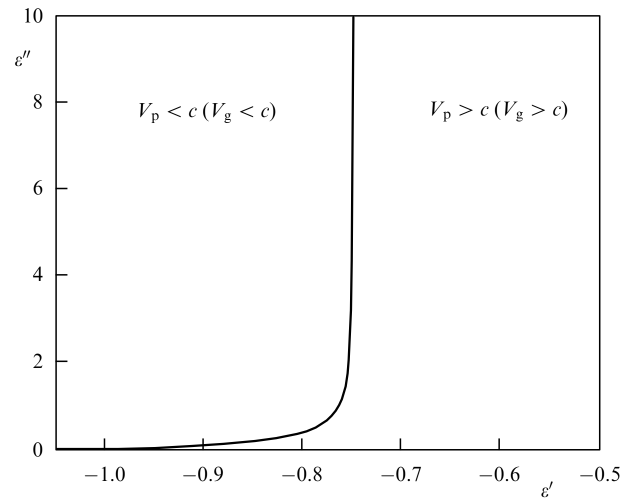

Figure 1. Two regions separated by the line $V_p = c$ of the roots of dispersion equation (6) in the plane $(\varepsilon', \varepsilon'')$, to which solutions characterized by the respective relations $V_{p,g} > c$ and $V_{p,g} < c$ correspond.

because $K_2 = \infty$ there, and the image of the saddle point of the integration contour in the plane $(\varepsilon', \varepsilon'')$ is the point $(-3/4, \infty)$ at infinity, where $K_2 = 0$. Everywhere to the right of the 'saddle line' in Fig. 1, the surface wave phase velocity is greater than the speed of light in the vacuum, and everywhere to the left, it is less than the speed of light.

From the standpoint of the mathematical techniques used by the authors of Refs [12–16], as well as those completely different ones given in Refs [20–23], the region to the right of the 'saddle path' in Fig. 1 is not related to actual SWs that can be maintained by an interface as waveguide modes capable of leaving the source. Solutions of the boundary value problem corresponding to that region are related to solutions of the 'complex optics' problems for the complex Brewster angle values of an inhomogeneous plane wave satisfying the boundary conditions at infinity in both media. Here, only those solutions of the dispersion equation for SWs that are to the left of the saddle path have the physical meaning of guided modes able to separate from the source.

From the standpoint of the mathematical techniques used in the formulation of the theory of open waveguides described in book [19], all solutions — to the right as well as to the left of the shown saddle path — have the physical meaning of wave numbers for discrete spectrum modes of an infinite regular open waveguide; this leads, as we see below, to the point of physical absurdity.

We now show that at least for those regions with very low damping where the wave group velocity is well defined, solutions to the right of the 'saddle path' in the plane $(\varepsilon', \varepsilon'')$ must be *unconditionally* disregarded, because we have not only $V_{\rm p} > c$ but also $V_{\rm g} > c$ for these SWs (weakly damped Zenneck waves).

Conversely, for all solutions from this region, but to the left of the saddle paths that correspond to conventional SWs with $V_{\rm g} < c$ (surface plasmon polaritons), we have $V_{\rm p} < c$ as well. In other words, these are undoubtedly observable SWs.

3. Vacuum–metal, vacuum–seawater, and vacuum–ground interfaces

We do not impose any restrictions on the sign of ε', but we assume that the condition $\varepsilon'' \gg |\varepsilon'|$ is satisfied for metals, sea water, and Earth soil within their respective frequency ranges. In this case, it is easy to obtain the appropriate approximations from expressions (13) and (14). Indeed, we then have

$$a \approx \varepsilon''^2, \qquad 1 - \frac{\chi}{a} \approx 1 - \frac{1+\varepsilon'}{\varepsilon''^2},$$

$$\left(1 - \frac{\chi}{a}\right)^2 + \left(\frac{\varepsilon''}{a}\right)^2 \approx 1 - \frac{1+2\varepsilon'}{\varepsilon''^2}.$$

Substituting these expressions in (13) and (14), we obtain the estimates

$$K_1 \approx 1 - \frac{3+4\varepsilon'}{8\varepsilon''^2}, \qquad K_2 \approx \frac{1}{2\varepsilon''}. \tag{16}$$

Taking into account that $\varepsilon'' \to 4\pi\sigma/\omega \gg 1$ for metals (as well as for sea water and Earth soil, but at frequencies lower than those for metals), we see that the wave is weakly absorbed in the medium; therefore, its phase and group velocities are given by

$$V_{\rm p} \cong c\left[1 - \frac{(\varepsilon'+3/4)\omega^2}{2(4\pi\sigma)^2}\right]^{-1}, \tag{17}$$

$$V_{\rm g} \cong c\left[1 - \frac{3(\varepsilon'+3/4)\omega^2}{2(4\pi\sigma)^2}\right]^{-1}. \tag{18}$$

As it should be, the boundary line, as can be seen from Fig. 1 and formulas (17) and (18), in the limit $\varepsilon'' \gg |\varepsilon'|$ corresponds to the value $\varepsilon' = -3/4$. To the left of that, $\varepsilon' < -3/4$, the phase velocity, as well as the group velocity, is less than the speed of light, and such a surface wave is a physical wave. For $\varepsilon' > -3/4$, the wave phase velocity is greater than the speed of light, but its group velocity, as can be clearly seen from expression (18), is also greater than the speed of light in the vacuum.

Of course, formula (18) was obtained as a result of a very simple operation: $V_{\rm g} \approx [{\rm d}k_1/{\rm d}\omega]^{-1}$, i.e., in the limit of negligibly small losses. Hence, a more correct formulation of the obtained result should be as follows.

In the region $\varepsilon' < -3/4$ for $\varepsilon'' \gg |\varepsilon'|$, the group velocity of an SW over a plane interface between two media in the limit of negligibly small losses approaches the speed of light from below, and in the region $\varepsilon' > -3/4$, it approaches the speed of light from above.

Transition through the speed of light for the phase velocity $\varepsilon' = -3/4$ occurs synchronously with the physically forbidden transition of the group velocity (although defined only in the limit of negligibly small losses). This result is also sufficient to permanently reject the idea that a weakly damped Zenneck SW over a locally plane Earth or sea surface can be excited by real antennas, because we always have $V_{\rm p} > c$, and hence $V_{\rm g} > c$ for the SW in this case.

This is just the indisputable *physical* argument so lacking for a valid solution to the issue of the physical meaning of SWs with $V_{\rm p} > c$, the issue that had to arise immediately for physicists, as soon as Weyl's article [12] was published and its key findings were confirmed in the work of other highly reputable scientists [13–16]. Arguably, even if this question has not yet been finally resolved (because the question of more accurately determining the energy transfer rate by weakly damped surface waves remains), at least its solution is within reach.

The characteristic feature of the physics of wave processes in open guided structures should be noted in comparison with the physics of energy channeling in shielded waveguides.

As soon as the phase velocity of an SW propagating in open systems becomes greater than the speed of light, the wave ceases to exist, because its group velocity transits beyond the physically permitted limit at this point. This, in particular, makes a difference between cases occurring in the theory of open and shielded waveguides, where the group velocity of propagating modes is always less than the speed of light, although the wave phase velocity can be greater than the speed of light.

Our final remark about the Zenneck wave is related to actual projects of radio observations of sea areas, based on a wrong assumption about the possibility of wave excitation by onshore maritime antennas.

To assess the Zenneck wave characteristics, ε' is usually [24] assumed to be the static dielectric permittivity of water, $\varepsilon' \approx 80 > -3/4$. In this case, if the sea water conductivity is assumed to be $\sigma \approx 5$ (in practical units $1/\Omega$ m) [24], the condition $\varepsilon'' \gg |\varepsilon'|$ is satisfied in the frequency range $0-100$ MHz and, consequently, the wave group velocity exceeds the light speed. Such a wave cannot exist, of course, and projects to construct radio observation systems by using the 'surface wave radar', as is still the case [24, 25], in reality can no longer be based on this 'effect'. The physical basis for this effect (on which the operation of the decameter coastal over-the-horizon surface wave radar BZGR Podsolnukh-E [25] is based), if it exists, should be significantly reconsidered. For this, there are the following theoretical prerequisites.

First, the model of a plane interface between two media does not correspond to the real conditions because it does not take the finite curvature radius of the surface of Earth into account. It is impossible to underestimate this factor, as has been done to date, because the curvature of the wave guiding path of an open waveguide (when, due to the *arbitrary* assumption of the presence of the effect of the surface wave field driven to the *convex* surface of the path, the influence of the ionosphere is ignored) brings the physical correctness of the concept of a surface wave in its pure form into question. This is because at each point of a continuously curved path, electromagnetic energy is radiated into the space surrounding the path. However, the structure of the surface wave field in its purest form, which is perfectly appropriate for the physics of energy channeling in *regular* open waveguides, is not 'adjusted' to the existence of a *continuous* leaking of radiation energy in the case where $V_\text{p} < c$. The field structure of so-called leaky waves of a regular open waveguide is not 'tuned' to that, and it is driven away from the surface of the path rather than driven to this surface! This is the essence of the problem that occurs when locally approximating the spherical surface of Earth with a plane surface, to which theoreticians still resort for simplicity, in particular, when supporting projects to create a surface wave radar [24].

The paradox lies in the fact that the argument of a small value of the ratio of the wavelength to Earth's radius, usual in such cases, is not applicable here. It is merely sufficient to slightly bend the path of a regular open waveguide for the radiation accompanying the propagation of a guided wave to inevitably occur in the system. We emphasize again that the undeniable presence of this factor on the slightly curved surface of Earth is *absolutely* incompatible with the concept of a surface wave on the convex surface of Earth. Due to the reason mentioned above, there is no way for this wave to be considered a purely surface one. There is a question about the structure of the guided wave field, because the design and performance of terrestrial antennas depend exactly on that. What will be the nature of the transverse field distribution in such a wave, if, according to the most general physical reasons, it cannot be purely surface? This question, of course, is far beyond the scope of this paper.[2] But it is already clear that for this type of wave, whose characteristics must necessarily be linked to the presence of the accompanying radiation field, and the field cannot be strongly driven to the surface of Earth. In such circumstances, it is impossible to disregard the influence of the ionosphere. In theoretical terms, this means passing from the theory of open waveguides to the theory of shielded waveguides, because at frequencies up to 10 MHz, the ionospheric plasma behaves like a screen reflecting the electromagnetic field. We note that the coastal over-the-horizon surface wave radar BZGR Podsolnukh-E works in this frequency range. We have already said that the only candidate for the role of a surface wave, which, in accordance with the above considerations could replace the Zenneck wave, which is nonphysical in given circumstances, is the basic 'cable mode' of a closed spherical waveguide, which, due to the finite conductivity of the Earth or sea surface, can also be driven to this surface, and not only to the surface of the ionosphere. Other options for the theoretical support for projects to create a surface wave radar in the decameter range of wavelengths ($f < 10$ MHz), in our opinion, simply do not exist.

Thus, we see that the term 'Zenneck wave' historically originated in relation to a wave having no physical sense, as has been established almost surely. Indeed, initially this wave was clearly associated with a surface wave that should, by Zenneck's assumption, propagate over a locally flat sea or Earth surface, where the wave group velocity, as we have shown above, exceeds the speed of light. But the term was then unreasonably, in our opinion, applied to physical waves that can propagate over a metal surface, where $\varepsilon' < -3/4$. In this case, usual *slow* surface waves can exist in the sense that their phase velocity, and hence the group velocity, is less than the speed of light. Waves with exactly the same characteristics propagate in optical communication lines, regardless of the configuration of the fiber cross section, and no one calls them Zenneck waves. However, this name was assigned to exactly the same slow ($V_\text{p} < c$) surface waves that can actually propagate, as is the case in dielectric waveguides, over metal plates as well, but with a plane cross-sectional configuration [10]. After all, exactly the same slow and physical surface wave propagating over a *cylindrical* conductor were given the name of Zenneck's teacher, Sommerfeld.

The history of science does not know examples where the name of a phenomenon or an effect would depend on such irrelevant features of its development as, in this case, on the particular shape of a waveguide cross section. The true historical reasons for the term 'Zenneck wave' to appear in the scientific literature were based on a plausible assumption that Zenneck discovered a wave that should exist in reality over the corresponding flat underlying surface. That assumption was mistaken for a plausible one, and we now know why. If the corresponding effect is discovered in the future, its existence will be impossible to explain by the Zenneck wave. In fact, Zenneck had 'discovered' a 'good' mathematical solution that misled so many for such a long time, and which in reality does not correspond to anything. While this curious historical phenomenon credits a good researcher, the name 'Zenneck wave' may continue to exist in the literature, but only in this historical context, which is not at all offensive to the memory of Zenneck.

4. Vacuum–plasma and vacuum–dielectric interfaces

To conclude our extended systematic physical analysis of dispersion equation (6) for SWs, we consider another case,

[2] While this article was in press, papers [27, 28], directly related to the clarification of this issue, were published. In Ref. [27], it was shown that the Sommerfeld wave over a cylindrical conductor with the conductivity of sea water and a radius equal to Earth's radius becomes the Zenneck wave at a frequency of approximately 1 MHz. For frequencies below the critical frequency, the phase velocity of an SW over the cylinder is less than c, and for frequencies above 1 MHz, it is more than c. This justifies the correctness of the local approximation (in the given physical context) of a cylindrical guiding surface by a planar surface. In [28], where 'pseudosurface modes' of a spherical open waveguide with the parameters of Earth's actual surface were considered, the concrete calculations proved that the local approximation of Earth's spherical surface by a plane surface is incorrect. In particular, it was shown that the field of these waves is strongly driven away from the spherical surface of Earth. Thus, the views expressed above on the absolute incompatibility of the SW concept with the physical situation that occurs over a curved guiding path of Earth's spherical surface have been fully confirmed in Ref. [28].

interesting in theory as well as in practice: that of very small losses in a discharge plasma and in a dielectric. For this, we rewrite expressions (13) and (14) as

$$K_{1,2} = \frac{1}{\sqrt{2}} \sqrt{\frac{a-\chi}{a}\left[\sqrt{1+\left(\frac{\varepsilon''}{a-\chi}\right)^2} \pm 1\right]} \,. \tag{19}$$

Next, we introduce the conditions

$$\varepsilon'(1+\varepsilon') > 0\,, \tag{20}$$

$$\varepsilon''^2 \ll \varepsilon'(1+\varepsilon')\,, \tag{21}$$

$$\varepsilon''^2 \ll (1+\varepsilon')^2\,. \tag{22}$$

Condition (20) is important for the plasma and is satisfied in what follows by assuming the restriction $\varepsilon' < -1$. Thus, the analysis excludes the region $0 > \varepsilon' > -1$ (see Fig. 1) fully containing the zone of transition of the SW phase velocity through the speed of light. Consequently, in the negative region, where $\varepsilon' < -1$, we always have $V_p < c$, i.e., the case corresponding to observable waves. On the contrary, in the positive region, $\varepsilon' > 0$, $V_p > c$ always holds. From this area, we exclude the strip $1 \geqslant \varepsilon' > 0$, and to the right of it we consider the case of a dielectric, $\varepsilon' > 1$. Therefore, we consider the case $|\varepsilon'| > 1$ below.

Under the assumed constraints, we have

$$\frac{a-\chi}{a} = \frac{\varepsilon'(1+\varepsilon') + \varepsilon''^2}{(1+\varepsilon')^2 + \varepsilon''^2} \approx \frac{\varepsilon'}{1+\varepsilon'}\,,$$

$$\sqrt{1+\left(\frac{\varepsilon''}{a-\chi}\right)^2} \approx 1 + \frac{\varepsilon''^2}{2\big(\varepsilon'(1+\varepsilon')\big)^2}\,.$$

As a result, from formula (19), we obtain the following approximate expression for the normalized values of the real and imaginary parts of the surface mode longitudinal wave number:

$$K_1 \approx \sqrt{\frac{\varepsilon'}{1+\varepsilon'}}\left[1 + \frac{\varepsilon''^2}{8\big(\varepsilon'(1+\varepsilon')\big)^2}\right], \tag{23}$$

$$K_2 \approx \frac{1}{2}\sqrt{\frac{\varepsilon'}{1+\varepsilon'}}\,\frac{\varepsilon''}{\varepsilon'(1+\varepsilon')}\,. \tag{24}$$

With regard to the vacuum–plasma interface, $\varepsilon' < -1$, a usual SW with $V_p < c$ corresponds to approximations (23) and (24). As $\varepsilon'' \to 0$, expressions (23) and (24) give a value for the real root in Eqn (6) corresponding to the usual SW propagating at the vacuum–plasma interface without damping: $K_1 = \sqrt{\varepsilon'/(1+\varepsilon')} > 1$, $K_2 = 0$. There is no doubt about the physical meaning of this wave.

Quite a different situation occurs for roots in Eqn (6) in the region $\varepsilon' > 1$ and $\varepsilon'' \neq 0$, i.e., those roots that approach the positive ε' half-axis infinitely closely, but do not merge with it. In this case, as we mentioned in Section 2, Eqn (6) is not satisfied. But no matter how small the value of ε'' is, Eqn (6) is satisfied, and this solution, as is clear from (23) and (24), corresponds to a weakly damped SW field ($K_2 \to 0$), for which $V_p > c$ because $K_1 < 1$.

This is complete physical nonsense, because it is well known that no SW can exist at the boundary of the vacuum and a weakly absorbing dielectric. If it were possible, such effects would have been seen for a long time in optics. The physical reason for the absence of SWs at the vacuum–weakly absorbing dielectric interface is the same as in the case of the vacuum–sea-water and vacuum–ground interfaces. From expression (23), we have $K_1 \approx \sqrt{\varepsilon'/(1+\varepsilon')}$ in the limit of vanishingly small losses. It follows that for $\varepsilon' > 1$, the group velocity is $V_g > c$. We obtain exactly the same result for metals in the X-ray frequency range.

Resolving these contradictions can be very easy; we just need to recognize that the entire region (with no exceptions) to the right of the separating curve on which $V_p = c$ and which is shown in Fig. 1 has no relation to physical SWs on the interface between two respective media, those waves that can freely propagate along the boundary, breaking away from the source.

This general conclusion covers both cases in which small losses are realized: large and small values of ε''. In the first case, we are talking about SWs over the vacuum–sea-water and vacuum–ground interfaces, and in the second case, about SWs over the vacuum–weakly absorbing dielectric interface. The physical reasoning in both cases is the same: $V_g > c$. However, while no questions arise in the latter case, they do in the first case, and it is advisable to correct this situation.

5. Kinetic model of a plasma-like medium

In this section, we consider the simplest realistic model of a medium describing models of electron–ion plasma and metal in the opposite limits. We also show that for such a model medium, the real part of the dielectric permittivity is always negative. It is in plasmas and metals that surface waves are observed in the radio, VHF, and microwave bands. We consider the so-called Lorentz model of a plasma-like medium taking only the elastic scattering of electrons by ions (or by the ion lattice) into account. It is quantitatively justified in the case of multiply charged ions, when electron–electron scattering can be neglected; and we write the kinetic equation for the electron distribution function as (see [26], Ch. 3)

$$\frac{\partial f_e}{\partial t} + \mathbf{v}\,\frac{\partial f_e}{\partial \mathbf{r}} + e\left\{\mathbf{E} + \frac{1}{c}[\mathbf{v}\times\mathbf{B}]\right\}\frac{\partial f_e}{\partial \mathbf{p}} = \frac{v_i}{2}\,\frac{\partial}{\partial v_i}\,(v^2\delta_{ij} - v_i v_j)\,\frac{\partial f_e}{\partial v_j}\,, \tag{25}$$

where $v_i \equiv 4\pi e^2 e_i^2 N_i L/m^2 v^3$. From this equation, we find the dielectric permittivity for a stationary and homogeneous plasma-like medium in the absence of external fields and with the equilibrium electron distribution $f_0(p)$. We neglect the spatial dispersion because we are interested in fast waves with velocities much higher than the electron thermal velocity. For a small monochromatic perturbation of the distribution function

$$f_e = f_0 + \delta f_e \exp(-i\omega t)\,, \tag{26}$$

Eqn (25) yields

$$-i\omega\delta f_e + \frac{e}{m}\,\mathbf{E}\,\frac{\partial f_0}{\partial \mathbf{v}} = \frac{v_i}{2}\,\frac{\partial}{\partial v_i}\,(v^2\delta_{ij} - v_i v_j)\,\frac{\partial \delta f_e}{\partial v_j}\,. \tag{27}$$

A solution of this equation can be found easily and is written as

$$\delta f_e = -\frac{ie\mathbf{E}\,\partial f_0/\partial \mathbf{p}}{\omega + i v_i}\,. \tag{28}$$

Substituting this expression in the formula for the induced current perturbation

$$J_i = e \int \mathrm{d}\mathbf{p}\, v_i \delta f_e = \sigma \delta_{ij} E_j \,, \tag{29}$$

we find the conductivity and then the dielectric permittivity of the medium:

$$\varepsilon(\omega) = 1 + \frac{4\pi i \sigma}{\omega} = 1 + \frac{4\pi e^2}{3\omega} \int \mathrm{d}\mathbf{p}\, \frac{v^2\, \partial f_0/\partial\tilde{\varepsilon}}{\omega + i v_i} \,, \tag{30}$$

where $\tilde{\varepsilon} = p^2/2m$ is the electron energy.

Integral (30) is easily calculated in the case of the degenerate Fermi distribution, $f_0 = f_F$, where $\partial f_F/\partial\tilde{\varepsilon} = -\delta(\tilde{\varepsilon} - \varepsilon_F)$. Here, $\varepsilon_F = p_F^2/2m$ is the Fermi energy and p_F is the Fermi momentum of electrons. As a result of calculations, we obtain the well-known formula [29]

$$\varepsilon(\omega) = 1 - \frac{\omega_{Le}^2}{\omega(\omega + i v_{iF})} \,, \tag{31}$$

where $\omega_{Le} = \sqrt{4\pi e^2 N_e/m}$ is the Langmuir frequency of electrons and $v_{iF} = 4\pi e^2 e_i^2 N_i L/m^2 v_F^3$ is the electron collision rate.

In the case of the Maxwell distribution, $f_0 = f_M$, the integral can be reduced to a special function, which is known in asymptotic limits. Therefore, we directly write these limits [29]:[3]

$$\varepsilon(\omega) = 1 - \frac{\omega_{Le}^2}{\omega^2}\left(1 - i\,\frac{v_{ei}}{\omega}\right), \qquad \omega \gg v_{ei} \,, \tag{32}$$

$$\varepsilon(\omega) = 1 + i\,\frac{\omega_{Le}^2}{\omega v_{ei}}\,\frac{32}{3\pi}\left(1 + i\,\frac{105\omega}{16\pi v_{ei}}\right), \qquad \omega \ll v_{ei} \,, \tag{33}$$

where

$$v_{ei} = \frac{4}{3}\sqrt{\frac{2\pi}{m}}\,\frac{e^2 e_i^2 N_i L}{T_e^{3/2}} \,. \tag{34}$$

We see from expressions (32) and (33) that in the plasma frequency region where $\omega \gg v_{ei}$, the real part of the dielectric constant is a large negative value for $\omega^2 \ll \omega_{Le}^2$. And it is exactly in this frequency range where weakly absorbed surface waves can exist on the plasma boundary [26]. In the opposite limit of the high collision rate, the dielectric permittivity has a large imaginary part under the condition $\omega_{Le}^2 \gg \omega v_{ei}$, and the possibility of the existence of a surface wave, as we have seen already, depends on the sign of the real part of the dielectric permittivity determined by the respective expressions

$$\varepsilon' = 1 - \frac{\omega_{Le}^2}{v_{iF}^2} \,, \qquad \varepsilon' = 1 - \frac{105\omega_{Le}^2}{3\pi^2 v_{ei}^2} \tag{35}$$

for degenerate and nondegenerate electrons. These values, according to the applicability conditions for the gas approximation, are negative and have large absolute values [26, 29].[4]

We note, however, that we have considered plasma-like media described by the classical Vlasov theory with the Landau collision integral in the Lorentz model. In the case of bad conductors (such as sea water or semiconductors and semimetals with a small number of carriers), when the contribution to the real part of the dielectric permittivity is for some reason positive in a certain frequency range at a sufficiently high conductivity, a fast surface wave cannot exist.

To conclude, we note that an objection is possible that introducing the group velocity is unjustified for times exceeding the decay time of the wave amplitude. The question of the speed of the wave energy propagation with a complex frequency, as may be the case in Eqn (6), is solved in the wave theory by studying the wave packet spreading in the vacuum (see [31], Ch. 4 and 5). In this case, it is shown that the value $\mathrm{Re}\,(\partial\omega/\partial k)$ characterizes the energy propagation rate of the packet, i.e., is the wave group velocity and cannot exceed the speed of light in the vacuum. The value $\mathrm{Re}\,(\partial\omega/\partial k)$ describes the energy flux along the interface. If there is energy dissipation in the underlying medium, then outside there is a normal component of the energy flux directed to the lower medium, and the magnitude of this component gives a measure of the SW field absorption by the medium. This flux, as well as the rate of field absorption by the medium, is negligible [32].

6. Brief conclusions

It follows from the foregoing that at least in the two limit cases — very large (vacuum–sea water, vacuum–metal) and very small (vacuum–plasma, vacuum–dielectric) imaginary part values for the dielectric permittivity of the underlying half-space — when a weakly damped surface wave can propagate along the boundary of two media, we have a synchronous transition of the wave phase and group velocities through the speed of light in the vacuum.

As soon as the surface wave phase velocity becomes greater than the speed of light, its group velocity also begins to exceed the speed of light. Hence, in the region where the SW phase velocity is greater than the speed of light, the wave cannot exist.

The region of a surface wave with the phase velocity less than the speed of light in the vacuum is characterized by negative values of the real part of the dielectric permittivity of the underlying medium, and therefore surface waves can exist on vacuum–plasma and vacuum–metal interfaces. And vice versa, they cannot exist on vacuum–weakly absorbing dielectric and vacuum–sea-water or vacuum–ground interfaces, because the real part of the dielectric permittivity is larger than zero in these media.

It follows that the Zenneck wave as a wave that according to its mathematical properties could propagate over land or sea surfaces, is a ghost wave from the physical standpoint.

This wave in the history of natural science should take the same place as phlogiston, electric fluids, and similar concepts that have gradually, due to scientific progress, been ousted from real physics, but have taken their rightful place in the history of science.

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[3] We note that (31) does not take a degeneracy of electron states in scattering processes into account. Allowing for this degeneracy of electrons results in the replacement $v_{eF} \to v_{eF} T_e/\varepsilon_F$, where $\varepsilon_F = m v_F^2/2$ is the Fermi energy, in (31).

[4] Strictly speaking, the value ε_0 due to the contribution of field scattering on bound electrons [30] instead of unity should be present in (35). However, $\varepsilon_0 \neq 1$ is only the case in the optical frequency range, and we have $\varepsilon_0 > 5-7$.

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