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PHYSICS OF OUR DAYS

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Unsolved problems in particle physics

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Contents

Abstract. **Among the problems yet to be solved by particle theory, we consider those that are most important in our opinion, some of which require physics beyond the Standard Model (neutrino oscillations, probably inconsistent with the usual three-generation scheme; astrophysical evidence for new physics; electroweak symmetry breaking, and the hierarchy of parameters) and some of which do not (description of strong interactions at low and intermediate energies).**

1. Introduction: status and parameters of the Standard Model

The current state of quantum field theory and its applications to particle physics may be compared with the situation 20 – 30 years ago, with the curious result that all principal statements in this field of physics are practically unchanged, in contrast to the rapid progress in condensed matter physics. Indeed, most of the experiments performed over the last two decades have supported the correctness of predictions that had been made earlier, derived from the models developed earlier.

This success of particle theory has resulted in considerable stagnation in its development. However, we may expect that particle physics will again become an intensively developing area in the next few years. First, there is a certain amount of collected experimental results (primarily related to cosmology and astrophysics, but also obtained in laboratories) suggesting that the Standard Model (SM) is incomplete. Second, the theory was being developed under the guidance of the principle of naturalness, that is, the requirement that any hierarchy in model parameters be explained quantitatively (for the SM, this is possible only within a larger fundamental theory yet to be constructed). Finally, one of the most important arguments for the coming excitement in particle physics is the expectation of new results from the Large Hadron Collider. As we explain shortly, this accelerator will be able to study the *full* range of energies where the physics responsible for electroweak symmetry breaking should appear, and we therefore expect interesting discoveries in the next few years in any case: it will be the Higgs boson, or some other new particles, or (in the most interesting case) no new particle will be found, which would suggest a serious reconsideration of the SM.

The Large Hadron Collider (LHC; see, e.g., [1, 2]) is an accelerator that allows colliding protons with the center-of-mass energy up to 14 TeV (currently working at 7 TeV) and heavy nuclei. In a tunnel 30 km long, on the border between Switzerland and France, there are four main experimental installations (general-purpose detectors ATLAS and CMS; LHCb, which is oriented to the study of B mesons; and ALICE, specialized in heavy-ion physics) as well as a few smaller experiments. The first results of the work of the collider have brought a large amount of new data on particle interactions, which we mention whenever necessary in what follows.

The purpose of the present review is to briefly discuss the current state of particle physics and possible prospects for its development. For such a wide subject, the selection of topics is necessarily subjective and the estimates of the importance of particular problems and of the potential of particular

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approaches reflect our personal preferences, while the bibliography cannot be exhaustive.

The contemporary situation in particle physics can be described as follows. Most of the modern experimental data are well described by the Standard Model of particle physics, which was created in the 1970s. At the same time, there are a considerable number of indications that the SM is not complete and is not more than a good approximation to the correct description of particles and interactions. We are not speaking now about minor deviations of certain measured observables from theoretically calculated ones — these deviations may be related to the insufficient precision of either the measurement or the calculations, to unaccounted systematic errors, or to insufficient sets of experimental data (statistical fluctuations); it so happens that these deviations disappear after a few years of more detailed study. On the contrary, we emphasize more serious qualitative problems of the SM, which is considered an instrument of the quantitative description of elementary particles. These problems include the following:

(1) experimental indications of the incompleteness of the SM, namely, the well-established experimental observations of neutrino oscillations (which are impossible in the SM; see Section 2.4) and the incapability of the SM to describe the results of astrophysical observations, in particular, those related to the structure and evolution of the Universe;

(2) values of the SM parameters that are not fully natural and not calculable in the theory, notably, the fermion mass hierarchy, the hierarchy of symmetry-breaking scales, and the absence of a light Higgs boson (with mass $\lesssim 100$ GeV);

(3) purely theoretical difficulties in describing hadrons by means of the available methods of quantum field theory.

We discuss these unsolved problems of the SM and related prospects for the development of the particle theory.

For future reference, it is useful to briefly recall the structure of the SM (see, e.g., [3, 4] and the appendix in [5]). The model includes a certain set of particles and their interactions.

Of the four known interactions (Fig. 1), three are described by the SM — electromagnetic, weak, and strong. The first two have a common *electroweak* gauge interaction behind them. The symmetry of this interaction, $SU(2)_L \times U(1)_Y$, manifests itself at energies higher than ~ 200 GeV. At lower energies, this symmetry is broken down to $U(1)_{EM} \neq U(1)_Y$ (electroweak symmetry breaking); in the SM, this breaking is associated with the vacuum expectation value of a scalar field, the Higgs boson. The parameters of the electroweak breaking are known with high precision; experimental data are in perfect agreement with the theory. The Higgs boson has not been observed yet; its mass, being a free parameter of the theory, is bound by direct experimental searches (see Table 1 and more details in Section 4.1).

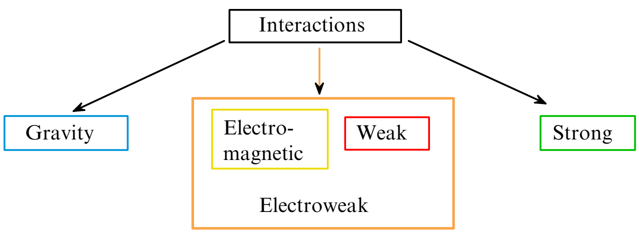

Figure 1. Particle interactions.

Table 1. Parameters of the Standard Model. For parameters with significant energy dependence, the energy scales to which the numerical values correspond are given in parentheses.

Parameter	Value
$\alpha_s(M_Z)$	0.114 ± 0.0007
$1/\alpha(M_Z)$	127.916 ± 0.015
$\sin^2\theta_W(M_Z)$	0.23108 ± 0.00005
Θ	$\lesssim 10^{-10}$
m_u (2 Gev)	$2.5^{+0.8}_{-1.0}$ MeV
m_d (2 GeV)	$5.0^{+1.0}_{-1.5}$ MeV
m_s (2 GeV)	105^{+25}_{-35} MeV
$m_c(m_c)$	$1.266^{+0.031}_{-0.036}$ GeV
$m_b(m_b)$	4.198 ± 0.023 GeV
$m_t(m_t)$	173.10 ± 1.35 GeV
m_e	510.998910 ± 0.000013 keV
m_μ	105.658367 ± 0.000004 MeV
m_τ	1.77682 ± 0.00016 GeV
θ_{12}	$13.02^\circ \pm 0.05^\circ$
θ_{23}	$2.35^\circ \pm 0.06^\circ$
θ_{13}	$0.199^\circ \pm 0.011^\circ$
δ	1.20 ± 0.08
$v(m_\mu)$	246.221 ± 0.002 GeV
M_H	115.5 – 127.0 GeV (95% confidence level)

The *strong* interaction is described in the SM by quantum chromodynamics (QCD), a theory with the gauge group $SU(3)_c$. The effective coupling constant of this theory increases as the energy decreases. As a result, particles involved in this interaction cannot exist as free states and appear only in the form of bound states called hadrons. Most of the modern methods of quantum field theory work for small values of coupling constants, that is, for QCD, at high energies.

The fourth known interaction, the *gravitational* one, is not described by the SM, but its effect on microscopic physics is negligible.

The particle content of the SM is summarized in Fig. 2. Quarks and leptons, so-called SM matter fields, are described by fermionic fields. Quarks take part in strong interactions and compose observable bound states, hadrons. Both quarks and leptons participate in the electroweak interaction. The matter fields constitute three generations;

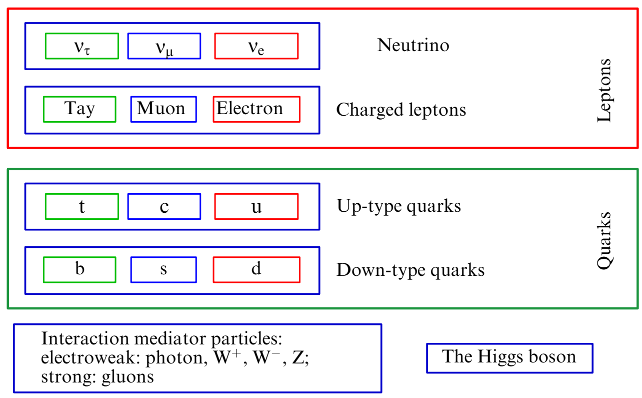

Figure 2. Particles described by the Standard Model.

particles from different generations interact identically but have different masses. The full electroweak symmetry forbids fermion masses, and therefore nonzero masses of quarks and leptons are directly related to the electroweak breaking; in the SM, they appear due to the Yukawa coupling to the Higgs field and are proportional to its vacuum expectation value. For neutrinos, these Yukawa interactions are forbidden as well, and hence neutrinos are strictly massless in the SM. Gauge bosons, which are carriers of interactions, are massless for the unbroken gauge groups $U(1)_{EM}$ (electromagnetism: photons) and $SU(3)_c$ (QCD: gluons); the masses of $W^{\pm}$ and Z bosons are determined by the electroweak symmetry breaking mechanism. All SM particles except the Higgs boson have been found experimentally.

From the standpoint of quantum field theory, quarks and leptons can be described as states with a definite mass. At the same time, gauge bosons interact with superpositions of these states; in another formulation, when the basis is chosen to consist of the states interacting with the gauge bosons, the SM symmetries allow not only the mass terms $m_{ii}\bar{\psi}_i\psi_i$ for each ith fermion ψ_i but also a nondiagonal *mass matrix* $m_{ij}\bar{\psi}_i\psi_j$. Up to unphysical parameters in the SM, this matrix is trivial in the leptonic sector and is related to the Cabibbo–Kobayashi–Maskawa (CKM) matrix in the quark sector. The CKM matrix can be expressed through three independent real parameters (quark mixing angles) and one complex phase (for more details, see [4, 6]).

The SM therefore has 19 independent parameters, the values of 18 of which are determined experimentally. They include three gauge coupling constants α_s, α_2, and α_1 for the respective gauge groups $SU(3)_c$, $SU(2)_W$, and $U(1)_Y$ (the last two are often expressed through the electromagnetic coupling constant α and the mixing angle θ_W), the QCD Θ-parameter, nine charged-fermion masses $m_{u,d,s,c,b,t,e,\mu,\tau}$, three quark mixing angles $\theta_{12,13,23}$, one complex phase δ of the CKM matrix, and two parameters of the Higgs sector, which are conveniently expressed through the known Higgs-boson vacuum expectation value v and its unknown mass M_H. The experimental values of these parameters, recalculated from the 2010 data [7] (bounds on the mass of the Higgs boson based on LEP, Tevatron and LHC data are given as of December, 2011), are given in Table 1.

It is worth remembering that the observable world is mostly made of atoms, and therefore, out of the full variety of elementary particles, only a few are encountered 'in everyday life'. These are the u and d quarks in the form of protons (udd) and neutrons (uud), electrons, and, as regards the interaction carriers, the photon. The reasons for this are different for different particles. In particular, the neutrino does not interact with the electromagnetic field and is therefore very difficult to detect; heavy particles are unstable and decay into lighter ones; strongly interacting quarks and gluons are confined in hadrons. The full variety of SM particles reveal themselves either in complicated dedicated experiments or indirectly by their effects seen in astrophysical observations.

Hence, before proceeding with the description of unsolved problems, we recall that all experimental results concerning the physics of charged leptons, photons, and W and Z bosons at all available energies and quarks and gluons at high energies are in excellent agreement with the SM for the given set of its parameters.

2. The observed deviation from the Standard Model: neutrino oscillations

We discuss the unique evidence, well-established in laboratory experiments, in favor of the incompleteness of the SM, the phenomenon of neutrino oscillations, that is, the mutual conversion of neutrinos of different generations into one another. A more detailed modern description of the problem can be found in [8], in the appendix to textbook [5], and in reviews [9–11].

2.1 Theoretical description

In analogy with the case of charged leptons, we consider three generations of neutrinos: the electron neutrino (ν_e), the muon neutrino (ν_μ), and the tau neutrino (ν_τ). The corresponding fermion fields are coupled to the gauge bosons W and Z via weak charged and neutral currents. These couplings are responsible for both the creation and the experimental detection of neutrinos.

Similarly to the quark case, we can suppose that neutrinos have a nonzero mass matrix (although it cannot be incorporated into the SM, the low-energy effective theory, electrodynamics, does not forbid it), which can be nondiagonal. It is convenient to describe this system in terms of linear combinations $\nu_{1,2,3}$ of the original fields $\nu_{e,\mu,\tau}$ with the diagonal mass matrix

$$\nu_i = \sum_{\alpha=e,\mu,\tau} U_{i\alpha}\nu_\alpha\,,$$

where $U_{i\alpha}$, $i = 1, 2, 3$ and $\alpha = e, \mu, \tau$, are elements of the leptonic mixing matrix.

To demonstrate the phenomenon of neutrino oscillations, we restrict ourselves to the case of two flavors, ν_e and ν_μ. Let their linear combinations

$$\begin{aligned} \nu_1 &= \cos\theta_{12}\,\nu_e + \sin\theta_{12}\,\nu_\mu\,, \\ \nu_2 &= -\sin\theta_{12}\,\nu_e + \cos\theta_{12}\,\nu_\mu \end{aligned} \tag{1}$$

be the eigenvectors of the mass matrix with the respective eigenvalues m_1^2 and m_2^2. The inverse transformation expresses (ν_e, ν_μ) through (ν_1, ν_2):

$$\begin{aligned} \nu_e &= \cos\theta_{12}\,\nu_1 - \sin\theta_{12}\,\nu_2\,, \\ \nu_\mu &= \sin\theta_{12}\,\nu_1 + \cos\theta_{12}\,\nu_2\,. \end{aligned}$$

We suppose that at the instant $t = 0$, in a certain weak-interaction event, an electron neutrino ν_e, that is, the superposition of ν_1 and ν_2 with known coefficients, was created:

$$\begin{aligned} \nu_1(0) &= \cos\theta_{12}\,\nu_e(0)\,, \\ \nu_2(0) &= -\sin\theta_{12}\,\nu_e(0)\,. \end{aligned}$$

The evolution of mass eigenstates for a plane monochromatic wave moving in the z direction is described as

$$\nu_i(z,t) = \exp\left(-i\omega t + i\sqrt{\omega^2 - m_i^2}\,z\right)\nu_i(0)\,, \qquad i = 1, 2\,,$$

where ω is the energy and $\sqrt{\omega^2 - m_i^2}$ is the momentum. While propagating, the wave packets corresponding to ν_1 and ν_2 disperse in different ways, and hence the relation $(\cos\theta_{12}, -\sin\theta_{12})$ between their coefficients no longer holds, which means that an admixture of an orthogonal state, ν_μ, appears. In the (commonly considered) ultrarelativistic limit, $\omega \gg m_i$ and $\sqrt{\omega^2 - m_i^2} \simeq \omega - m_i^2/(2\omega)$. The probability of detecting ν_μ at a point (t, z) for each emitted ν_e is then

$$P(\nu_\mu; z, t) = |\nu_\mu(z, t)|^2 = \sin^2 2\theta_{12} \sin^2\left(\frac{m_2^2 - m_1^2}{4\omega} z\right). \quad (2)$$

It follows that this probability is an oscillating function of the distance z, which is the origin of the term 'neutrino oscillations'. As expected, no oscillations occur either in the case of equal (even nonzero) masses (similar dispersions of ν_1 and ν_2) or for a diagonal mass matrix ($\theta_{12} = 0$, $\nu_1 = \nu_e$, etc.).

A similar description of oscillations of three neutrino flavors determines, in analogy with Eqn (1), three mixing angles θ_{12}, θ_{13}, θ_{23}.

When individual neutrinos propagate over large distances, the oscillation formalism described above stops working because particles of different masses require different times to propagate from the source, which results in a loss of coherence; nevertheless, transformations of neutrinos are possible and their probability is calculable.

2.2 Experimental results: standard three-flavor oscillations

We now turn to the history (see, e.g., [8]) and the modern state (see, e.g., [9–12]) of the problem of neutrino oscillations. In 1957, Pontecorvo [13, 14] suggested the possibility of oscillations in a neutrino–antineutrino system, similar to K meson oscillations already known at that time. This first mentioning of the possibility of neutrino oscillations was aimed at explaining Davis's preliminary results on the observation of the reaction $\bar{\nu} + {}^{37}\mathrm{Cl} \to {}^{37}\mathrm{Ar} + \mathrm{e}^-$ with reactor neutrinos. On the one hand, this experimental result has not been confirmed; on the other hand, it has become clear that the Pontecorvo model is not able to describe it, even if the effect were true. The first mention of mutual transformations of ν_e and ν_μ is by Maki, Nakagawa, and Sakata [15], while the first successful description of oscillations in the system of two-flavor neutrinos was given by Pontecorvo [16] and by Gribov and Pontecorvo [17]. The theory of neutrino oscillations in its present form was developed in 1975–1976 by Bilenky and Pontecorvo [18, 19], Eliser and Swift [20], Fritch and Minkowski [21], Mikheyev, Smirnov [22, 23], and Wolfenstein [24].

The first experimental evidence in favor of neutrino oscillations was obtained more than a half century ago, although their interpretation remained an open question for a considerable period of time. We are speaking about the so-called 'solar neutrino problem': the observed flux of neutrinos form the Sun was considerably lower than the flux predicted by the model of solar nuclear reactions. This solar neutrino deficit was first found in the Homestake experiment (USA) already in 1968 [25] and subsequently confirmed by Kamiokande (Japan) [26], SAGE (Russia, Baksan neutrino observatory of INR, RAS) [27], GALLEX/GNO (Italy, Gran-Sasso Laboratory) [28], and Super-K (Japan) [29] experiments, which used various experimental techniques and were sensitive to neutrinos from different nuclear reactions. Because only electron neutrinos are produced in the Sun, and only these were detected in the experiments, the deficit might be explained by the transformation of a part of the electron neutrinos into other flavors.

The natural source of *muon* neutrinos is provided by cosmic rays, that is, charged particles (protons and nuclei) of extraterrestrial origin that interact with atoms in Earth's atmosphere and produce secondary particles. A significant part of the latter are charged π mesons. Neutrinos from the decays of these π mesons, as well as from decays of secondary muons, are called atmospheric neutrinos. The first indications of oscillations of atmospheric neutrinos were obtained in the late 1980s in the Kamiokande [30] and IMB [31] experiments, with subsequent confirmation in Soudan-2 [32], MACRO [33], and Super-K [34]. Their result is an anisotropy in the flux of muon neutrinos: a greater flux arrives from above, i.e., from the atmosphere, than from below (through the Earth). Without oscillations, the flux would be isotropic, because it is determined by an isotropic flux of primary cosmic rays, while the interaction of neutrinos with terrestrial matter is negligible. This anisotropy is not seen for electron neutrinos; hence, it is natural to suppose that ν_μ oscillates mainly to ν_τ (the latter were not detected in these experiments).

In the first decade of this century, significant experimental progress in the questions we are discussing has been achieved, and we now have a reliable experimental proof of neutrino transformations with measured parameters.

2.2.1 $\nu_e - \nu_\mu$ oscillations. In addition to largely model-dependent results on the solar neutrino deficit (ν_e *disappearance*), the SNO experiment, in 2001 [35], detected the *appearance* of neutrinos of other flavors from the Sun, in full agreement with the flux expected in the oscillational picture. This has therefore closed the 'solar neutrino problem' and at the same time supported the standard solar model. The KamLAND experiment [36] registered the disappearance of electron antineutrinos born in reactors of atomic power plants (in contrast to the case of the Sun, the initial flux of the particles can be directly determined in this case).

The parameters of oscillations measured in these very different experiments are in excellent agreement (Fig. 3). The SNO results, together with the even more precise results of the BOREXINO experiment (Italy) [38], confirm the expected energy dependence of the number of disappeared solar neutrinos, in agreement with the predictions in [22, 23] and [24], where a theory of neutrino oscillations in plasma was

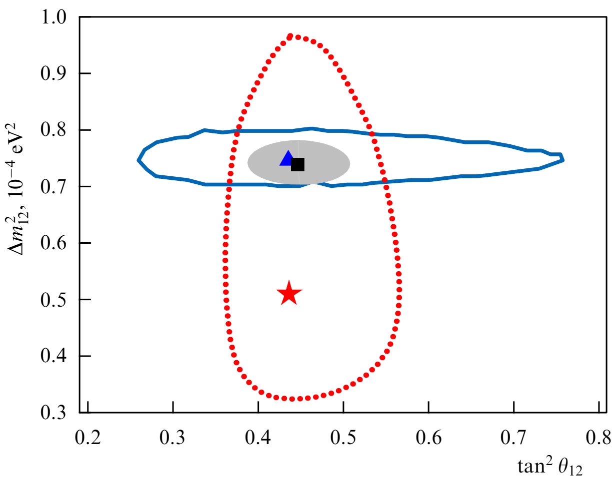

Figure 3. Bounds (95% confidence level) on the $\nu_e - \nu_\mu$ oscillation parameters from the analysis taking three neutrino flavors into account [37]. The dotted line corresponds to the combination of solar experiments, the solid line represents the KamLAND constraints, and the gray ellipse gives the constraints from the combination of all data. The star, the triangle, and the square correspond to the most probable oscillation parameters respectively obtained in these three analyses.

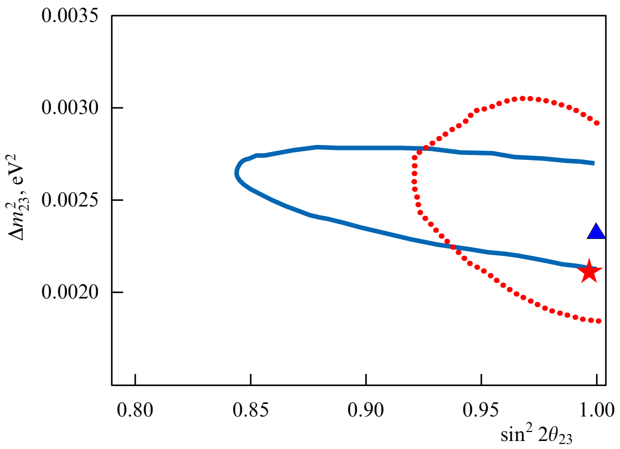

Figure 4. Bounds (90% confidence level) on the $\nu_\mu - \nu_\tau$ oscillation parameters. The dotted line represents the results of the SuperK analysis with three neutrino flavors [40]; the solid line represents constraints by MINOS [43]. The star and the triangle respectively denote the most probable oscillation parameters for these two analyses.

developed; because electrons are present in plasma, unlike muons and tau leptons, the coupling to a medium is different for different neutrino types. As a result, the oscillation formalism is modified and the resonance enhancement of oscillations becomes possible.

2.2.2 $\nu_\mu - \nu_\tau$ oscillations. In addition to the Super-K experiment, which measured deviations from isotropy in atmospheric ν_μ and $\bar{\nu}_\mu$ with a high accuracy [39, 40], the disappearance of ν_μ has been measured directly in neutrino beams created by particle accelerators (experiments K2K [41] and MINOS [42]; see Fig. 4). Finally, in 2010, the OPERA detector, which is located in the Gran Sasso Laboratory (Italy), detected the first (and currently unique) case of the *appearance* of ν_τ in the ν_μ beam from the SPS accelerator (CERN, Switzerland) [44].

2.2.3 The mixing angle θ_{13}. For a long time, solar ($\nu_e - \nu_\mu$) and atmospheric ($\nu_\mu - \nu_\tau$) oscillation data have been described independently (see discussions in [5], Appendix C), while the relatively low precision of experiments has allowed a zero value of the mixing angle θ_{13}. The situation changed recently and, analyzed together, the data of various experiments point to a nonzero θ_{13} value [45]. In summer 2011, two accelerator experiments, T2K (Japan) [46] and MINOS [47], which are both searching for the appearance of ν_e in ν_μ beams, published their results, which are incompatible with $\theta_{13} = 0$. A quantitative analysis of all data on solar and atmospheric neutrinos, jointly with accelerator and reactor experiments, which are studying the same region of the parameter space, points to a nonzero value of θ_{13} at a confidence level better than 99% [48]. The results of this analysis are quoted in Table 2.

2.3 Experimental results: nonstandard oscillations

The combination of all the experiments described above is in good quantitative agreement with the picture of oscillations of three types of neutrinos with certain parameters. However, results exist that do not fit this picture and may suggest that a fourth (or maybe even a fifth) neutrino exists. As we have seen above, one of the principal oscillation parameters is the mass-square difference $\Delta m^2_{ij} = m_j^2 - m_i^2$. The results concerning atmospheric and solar neutrinos, jointly with the accelerator and reactor experiments, are explained by two unequal Δm^2_{ij} (see Table 2):

Table 2. Parameters of oscillations of three neutrino flavors obtained with all relevant experimental data as of summer 2011 taken into account [48].

Parameter	Value
Δm^2_{12}	$(7.58^{+0.22}_{-0.26}) \times 10^{-5}$ eV^2
Δm^2_{23}	$(2.31^{+0.12}_{-0.09}) \times 10^{-3}$ eV^2
$\sin^2\theta_{12}$	$0.312^{+0.017}_{-0.016}$
$\sin^2\theta_{13}$	0.025 ± 0.007
$\sin^2\theta_{23}$	$0.42^{+0.08}_{-0.03}$

$$\Delta m^2_{12} \ll \Delta m^2_{23} \sim 2 \times 10^{-3}\ \text{eV}^2 .$$

In the case of three neutrinos, these two values compose the set of linearly independent Δm^2_{ij} and

$$|\Delta m^2_{13}| = |\Delta m^2_{12} - \Delta m^2_{23}| \sim \Delta m^2_{23} .$$

Therefore, the observation of any neutrino oscillations with $\Delta m^2_{ij} \gg \Delta m^2_{23}$ implies either the existence of a new neutrino flavor $(i, j > 3)$ or some other deviation from the standard picture. On the other hand, there is a very restrictive bound on the number of relatively light $(m_i < M_Z/2)$ particles with the neutrino quantum numbers. This bound comes from precise measurements of the Z-boson width and implies that there are only three such neutrinos. This means that the fourth neutrino, if it exists, is not coupled to the Z boson; in other words, it is 'sterile'.

We now turn to a certain experimental evidence in favor of $\Delta m^2_{ij} \gtrsim 0.1$ eV^2. We note that the oscillations related to this Δm^2_{ij} should reveal themselves at relatively short distances and may be detected in so-called short-baseline experiments.

2.3.1 $\bar{\nu}_\mu - \bar{\nu}_e$ oscillations. The LSND experiment [49] studied muon decay at rest, $\mu^+ \to e^+\nu_e\bar{\nu}_\mu$, and measured the $\bar{\nu}_e$ flux at a distance of about 30 m from the location where muons were held. The excess of this flux over the background rate has been detected and interpreted as the appearance of $\bar{\nu}_e$ as a result of $\bar{\nu}_\mu$ oscillations, for a range of possible parameters. A similar experiment, KARMEN [50], excluded a significant part of this parameter space; however, in 2010, the MiniBooNE experiment [51] also detected an anomaly that is compatible with the LSND results and, within statistical uncertainties, does not contradict the KARMEN data for a certain range of parameters (Fig. 5).

Another group of short-baseline experiments studying possible $\bar{\nu}_e - \bar{\nu}_\mu$ oscillations is searching for the disappearance of $\bar{\nu}_e$ in the antineutrino flux from nuclear reactors. These experiments continued for decades; recently, their results were reanalyzed jointly with a more precise theoretical calculation of the expected fluxes [53]. It has been shown that there is a statistically significant deficit of $\bar{\nu}_e$ in the detectors, which is compatible with $\Delta m^2 \gtrsim 0.1$ eV^2 — the so-called reactor neutrino anomaly. The corresponding bounds on the parameters are also shown in Fig. 5 for convenience. However, we should keep in mind that while LSND, KARMEN, and MiniBooNE detected $\bar{\nu}_e$ in the $\bar{\nu}_\mu$ flux, thus constraining $\bar{\nu}_e - \bar{\nu}_\mu$ oscillations, the reactor experiments only fixed the disappearance of $\bar{\nu}_e$. The lack of this disappearance excludes $\bar{\nu}_e - \bar{\nu}_\mu$ oscillations, but its presence may be explained as a transformation of $\bar{\nu}_e$ into antineutrinos of any other type.

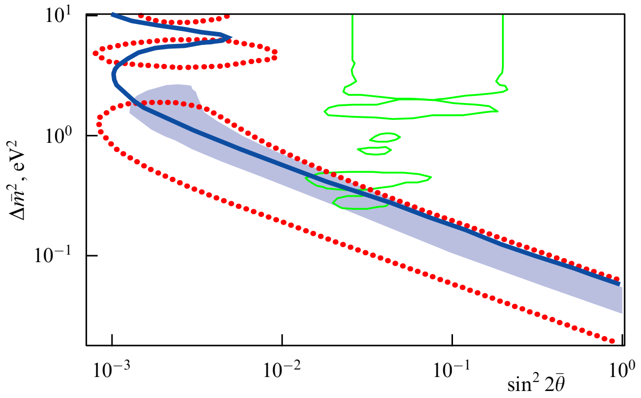

Figure 5. Bounds (90% confidence level) on the parameters of $\bar{\nu}_\mu - \bar{\nu}_e$ oscillations. The shaded region is compatible with the LSND signal [49]; the region inside the dotted curve is compatible with the MiniBooNE signal [52]. The thin solid lines bound the parameter regions compatible with a joint reanalysis of reactor data [53]. The KARMEN2 experiment excludes the region above and to the right from the thick solid line [50].

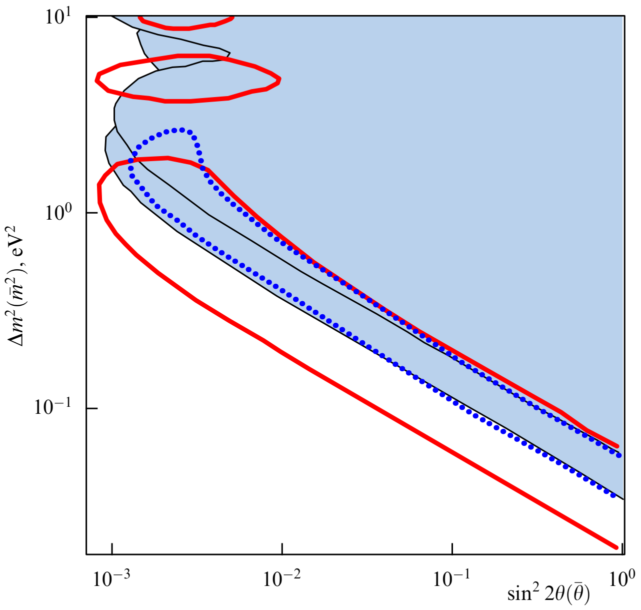

Figure 6. Bounds (90% confidence level) on the $\nu_\mu - \nu_e$ and $\bar{\nu}_\mu - \bar{\nu}_e$ oscillation parameters. The shaded area corresponds to the parameter space region that is excluded from neutrino oscillations by MiniBooNE [54] and KARMEN [50], while the thick contours bound the region that corresponds to the signal in antineutrino oscillations (solid lines, MiniBooNE [52]; dotted line, LSND [49]).

We see that there are several independent indications in favor of $\Delta m^2 \gtrsim 0.1$ eV^2, which, as we discussed above, require either the introduction of more than three neutrino flavors or some other new physics (see below).

2.3.2 Other anomalies. Recent intense exploration of neutrino oscillations has also revealed a range of other anomalies, which are currently being thoroughly discussed and rechecked.

Possible difference between neutrino and antineutrino oscillations. The MiniBooNE experiment studied neutrino and antineutrino beams separately. The appearance of $\bar{\nu}_e$ was detected [51, 52], while that of ν_e was not [54] (Fig. 6). If equal oscillation parameters for ν and $\bar{\nu}$ are assumed, the MiniBooNE result contradicts LSND, but without this assumption, on the contrary, the LSND claim is supported. It is worth noting that the MINOS experiment also performed separate measurements with neutrino and antineutrino beams (studying a range of much smaller Δm^2). First, the results for the two cases were incompatible at the 98% confidence level; however, subsequent analysis of a larger amount of data did not confirm this difference [55]. This last result agrees with the Super-K data: although this experiment cannot distinguish neutrino from antineutrino in each particular case, it may limit [56] antineutrino oscillation parameters statistically based on the known contribution of $\bar{\nu}_\mu$ to the atmospheric neutrino flux.

Calibration of gallium detectors. The GALLEX [57, 58] and SAGE [59, 60] experiments, constructed to detect solar neutrinos with the help of gallium detectors, calibrated their instruments with the help of artificial radioactivity sources. They detected a deficit of electron neutrinos compatible with oscillations with $\Delta m^2 \gtrsim 0.1$ eV^2 (see also [61]). This mass-square difference, which by itself does not agree with the standard three-neutrino oscillation picture, agrees with the antineutrino results of LSND, MiniBooNE, and reactor experiments; however, the corresponding mixing angle differs from these predictions [62].

Other puzzles. When speaking about unexplained results of neutrino experiments, we also mention the unexpected excess of events with energies $\lesssim 400$ MeV detected by MiniBooNE for neutrinos [63] and antineutrinos [52]; possible seasonal variations of the neutrino flux in the Troitsk-ν mass [64] and MiniBooNE [65] experiments; and the result of the OPERA experiment [66], which measured the speed of muon neutrinos, which happened to be higher than the speed of light. All these very interesting anomalies currently await confirmation in independent experiments.

Possible theoretical explanations. The experimental results listed above are rather difficult to explain. On the one hand, a series of experiments suggest neutrino transformations compatible with $\Delta m^2 \gtrsim 0.1$ eV^2, which cannot be described in the framework of the standard three-generation scheme. On the other hand, the addition of the fourth neutrino cannot help explain the difference between the neutrino and antineutrino oscillations [67, 68]. Alternatively, we can consider (a) two generations of sterile neutrinos (see, e.g., [69] and the references therein), (b) breaking of the CPT invariance [70],[1] or (c) a nonstandard interaction of neutrinos with matter, which may distinguish particles and antiparticles [75, 76]. A critical analysis of some of these suggestions can be found, e.g., in [69, 77, 78]. These scenarios experience considerable difficulties with simultaneous explanations of the full set of the experimental data, although they cannot be totally excluded; it might happen that a certain combination of these possibilities is realized in Nature.

A confirmation of the result on superluminal neutrino motion would require a serious reconsideration of the basic ideas of particle physics. A successful theory that explains the OPERA result quantitatively should also agree with very restrictive bounds on the Lorentz invariance violation in the sector of charged particles, with the absence of dispersion of

[1] The invariance under simultaneous charge conjugation (C) and reflection of both space (P) and time (T) coordinates (see, e.g., [71]) is a fundamental symmetry that inevitably exists in any $(3+1)$-dimensional Lorentz invariant local quantum field theory. However, phenomenologically acceptable models exist with CPT violation (with a higher number of spatial dimensions or with Lorentz invariance violation, or with a nonlocal interaction). In the context of neutrino oscillations, they are discussed, e.g., in [70, 72–74].

the neutrino signal from the supernova 1987A and with the absence of intense neutrino decays that are characteristic of many models with deviations from the relativistic invariance.

2.4 The neutrino mass

Conversions of neutrinos of one type to another have been proved experimentally, and the set of numerous independent and very different experiments are in a good agreement with the oscillation picture. The oscillatory behavior of neutrino conversions is proved by a comparison of the results obtained for different energies [cf. the argument of the sine squared in Eqn (2)]. The last step is to measure the neutrino flux at different distances along a single path [the distance dependence in Eqn (2)], which is planned for the nearest future. Up to this last detail, neutrino oscillations are experimentally confirmed.

Because the oscillations are possible only for different masses of neutrinos of different types, they also prove that (at least some of) the neutrino masses are nonzero. At the same time, direct experimental searches for neutrino masses have not been successful; the most restrictive bounds, determined by the Troitsk-ν mass (INR RAS) and Mainz experiments with tritium beta decay are $m_{\nu_e} \lesssim 2$ eV [79, 80]. For other neutrino types, experimental bounds on the mass are much weaker. An indirect bound on the sum of the neutrino masses can be obtained from studies of anisotropies of the cosmic microwave background and of the hierarchy of structures in the Universe [81]; it is given by $\sum_i m_{\nu_i} \lesssim 0.35$ eV.

At the same time, the lepton numbers are conserved separately for each generation in the SM, that is, changes of the neutrino flavor are forbidden. Using the SM fields, it is impossible to construct a gauge invariant renormalizable interaction resulting in a neutrino mass, even after the electroweak symmetry breaking. Therefore, neutrino oscillations represent an experimental proof of the incompleteness of SM.

How can the SM be modified so as to incorporate massive neutrinos? We first note that at energies below the electroweak breaking scale, the neutrino field is gauge invariant (uncharged and colorless). For such fermion fields, two kinds of mass terms exist, the Dirac term $m_D \bar{\nu}_R \nu_L$ (all charged SM fermions have similar masses) and the Majorana term $m_M \nu_L C \nu_L$, where C is the charge conjugation matrix and ν_L and ν_R denote the left-handed and right-handed neutrino spinors. In the SM, only left-handed neutrinos are present; therefore, to have Dirac masses, new fields $\nu_{R,i}$ must be introduced. At first sight, the Majorana mass does not require new fields; however, like the Dirac one, it cannot be obtained from a renormalizable interaction. Going beyond the renormalizability means that the SM is a low-energy limit of a more complete theory (similarly to how the nonrenormalizable Fermi theory is a low-energy limit of the SM), and hence the introduction of new fields is again inevitable. In any case, neutrinos are several orders of magnitude lighter than charged fermions, and a successful theory of neutrino masses should be able to explain this fact (see also Section 4.3).

3. Astrophysical and cosmological indications in favor of new physics

While laboratory experiments in particle physics give only limited indications of the incompleteness of the SM (neutrino oscillations being the main one), most scientists are confident that a more complete theory should be invented. The main reason for this confidence comes from astrophysics and cosmology. In recent decades, intense development of observational astronomy in various energy ranges has forced cosmology (i.e., the branch of science studying the Universe as a whole) to become an accurate quantitative discipline based on precise observational data (see, e.g., textbooks [5, 82]).

Today, cosmology has its own 'standard model,' which is in good agreement with most observational data. The basis of the model is the concept of the expanding Universe that, long ago, was very dense and so hot that the energy of thermal motion of elementary particles did not allow them to compose bound states. As a result, the particle interactions determined all processes and eventually influenced the development of the Universe and the state of the world as we observe it today. The expanding Universe cooled down and particles were unified into bound states: first atomic nuclei from nucleons, then atoms from nuclei and electrons. Unstable particles decayed and the Universe arrived at its present appearance. As we see below, the Universe is presently expanding with acceleration and mainly consists of unknown particles.

Even a dedicated book would be insufficient to describe all aspects of interrelations between cosmology and particle physics (the readers of *Physics–Uspekhi* might be interested in reviews [83, 84]). Here, we briefly consider three principal observational indications in favor of physics beyond the SM: the baryon asymmetry of the Universe, dark matter, and the accelerated expansion of the Universe (both the associated notion of dark energy and the physical reasons for inflation).

3.1 Baryon asymmetry

Quark–antiquark pairs had to be created intensively in the hot early Universe. The Universe then expanded and cooled down, and quarks and antiquarks annihilated, with the surviving ones forming baryons (protons and neutrons). Notably, there are very few antibaryons in the present-day Universe, which means that there were more quarks than antiquarks at the early stages. The predominance can be quantified: the number of quark–antiquark pairs was of the same order as the number of photons, while the baryon–photon ratio can be determined from an analysis of the cosmic microwave background anisotropy and from studies of primordial nucleosynthesis. The ratio of the excess of the quark number n_q over the antiquark number $n_{\bar{q}}$ is of the order of

$$\frac{n_q - n_{\bar{q}}}{n_q + n_{\bar{q}}} \sim 10^{-10},$$

that is, a single 'unpaired' quark was present for each ten billion quark–antiquark pairs. It is hard to imagine that this tiny excess of matter over antimatter was present in the Universe from the very beginning; moreover, a number of quantitative cosmological models predict the exact baryon symmetry of the very early Universe. It looks as though the asymmetry appeared in the course of the evolution of the Universe. For this to happen, the following Sakharov conditions [85] have to be satisfied:

(1) baryon number nonconservation;

(2) *CP* violation;

(3) breaking of thermodynamic equilibrium.

Although the classical SM Lagrangian preserves the baryon number, nonperturbative quantum corrections may break it, i.e., condition 1 may be fulfilled in the SM. The source of *CP*

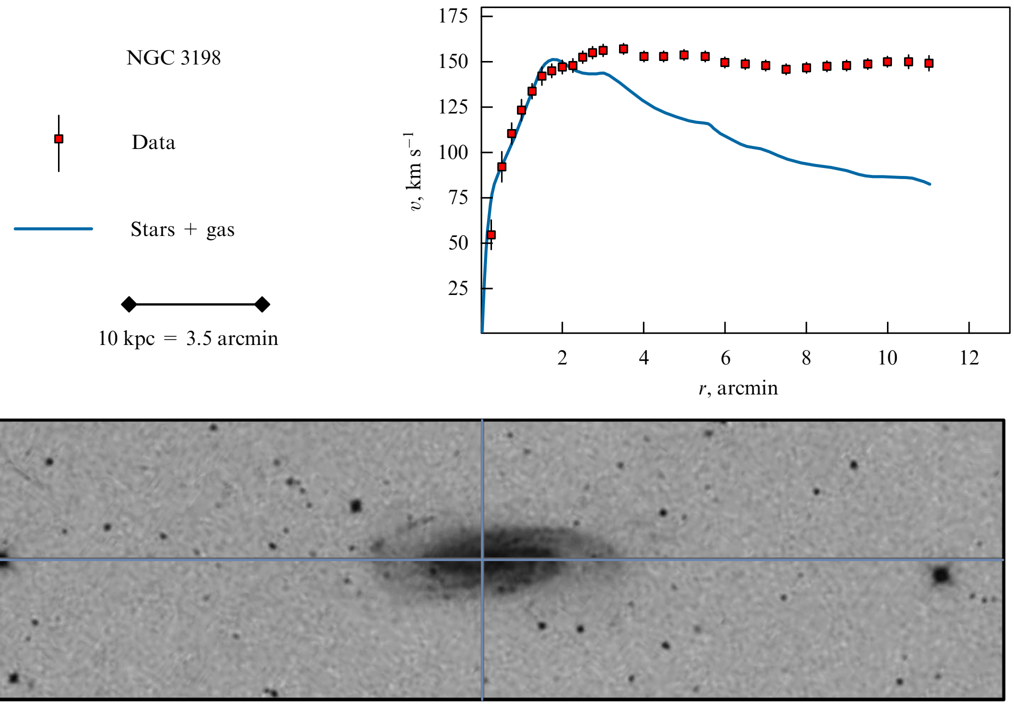

Figure 7. Some of the first indications of the existence of dark matter were obtained from the analysis of the rotation curves of galaxies. Observational data on the rotation velocity as a function of the distance to the axis, given here for the galaxy NGC 3198 (boxes), are not described by the curve that represents the expected velocity calculated from the distribution of luminous matter (lower line; data and calculation from [88]). At distances $\gtrsim$ 10 kpc, the luminous matter is practically absent (as can be seen in the lower photograph taken from the digitalized Palomar sky atlas [89]), but the rotation velocities of gas clouds seen in the radio band are almost constant. This indicates that a significant concentration of mass (the so-called halo) exists at the periphery of the galaxy.

violation (condition 2) is also present in the SM: it is the phase in the quark mixing matrix. Finally, in the course of the evolution of the Universe, the state with the zero vacuum expectation value of the Higgs field (high temperature) has been replaced by the present state. It can be shown (see, e.g., [86] and the references therein) that if this were a first-order phase transition, then the thermodynamic equilibrium would be severely broken at that instant. Therefore, in principle, all three conditions might be satisfied in the SM. However, it has been shown that a first-order electroweak phase transition in the SM is possible only for the Higgs boson mass $M_{\mathrm{H}} \lesssim 50$ GeV, which was excluded from direct searches long ago. Also, the amount of *CP* violation in the CKM matrix is insufficient. We conclude that the observed baryon asymmetry of the Universe is an indication of the incompleteness of the SM. A particular mechanism of generation of the baryon asymmetry is still unknown (it should also explain the smallness of the asymmetry amount, $\sim 10^{-10}$).

3.2 Dark matter

Studying the dynamics of astrophysical objects (galaxies and galaxy clusters) and of the Universe as a whole allows determining the distribution of mass, which may be subsequently compared with the distribution of visible matter. Various independent observational data point to the estimate that the contribution of visible matter (mostly baryons) to the energy density of the Universe is five times smaller than the contribution of invisible matter. We first briefly discuss modern observational evidence for the existence of dark matter and then proceed to a discussion of the implications of these observations for particle physics.

3.2.1 Rotation curves of galaxies. Much attention has been attracted to the question of invisible matter since the analysis of rotation curves of galaxies (see, e.g., [87]) (Fig. 7). For nearby galaxies, using the Doppler effect, it is possible to measure the velocities of stars and gas clouds at different distances from the galaxy center, that is, from the rotation axis. The Newtonian law of gravitation allows estimating the distribution of mass as a function of the distance from the center; it was found that in the outer parts of galaxies, where luminous matter is practically absent, there is a significant mass density, such that the visible part of a galaxy is embedded into a much larger invisible massive halo. These measurements have been performed for many galaxies, and for our Milky Way in particular.

3.2.2 Dynamics of galaxy clusters. In a similar way (although based on completely different observations), it is possible to determine the mass distribution in galaxy clusters. This provided the historically first argument in favor of dark matter [90]. Modern observations have demonstrated that the main part of baryonic matter lies not in star systems — galaxies — but in hot gas clouds in the intergalactic space. This gas emits X rays, and therefore observations allow reconstructing the distribution of the electron density and temperature. From this, using the conditions of hydrostatic equilibrium, the mass distribution can be determined. A comparison with the distribution of luminous matter (that is, mostly of gas) points again to the existence of some hidden mass. A similar, although less precise, conclusion can be obtained from the analysis of velocities of galaxies inside a cluster.

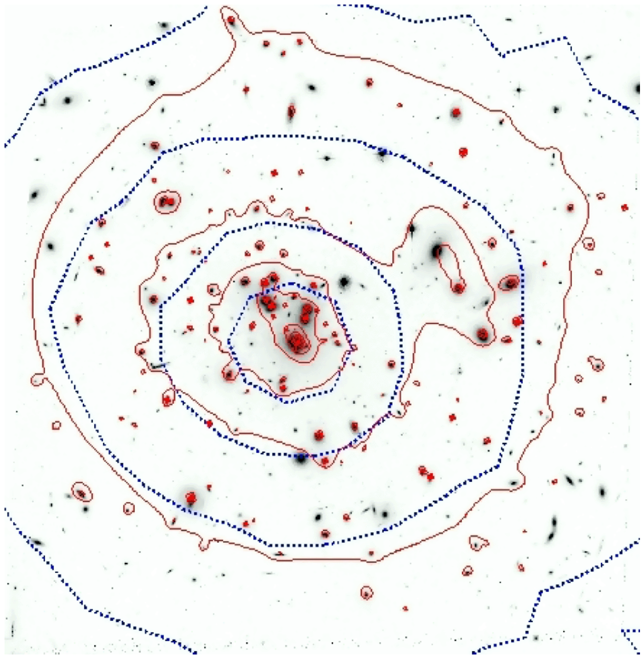

Figure 8. The galaxy cluster Abell 1689. The background image of the cluster in the optical band was obtained by the Hubble Space Telescope (image from archive [92]). The contours describe the model of mass distribution (solid curves, Ref. [91]) based on gravitational lensing and the distribution of luminous gas observed in X-rays (dotted curves based on data from the Chandra X-ray telescope archive, Ref. [93]). Currently, this mass model is one of the most precise ones.

3.2.3 Gravitational lensing. It may happen that a massive object (e.g., a galaxy cluster) is located between a distant source (e.g., a galaxy) and the observer. According to general relativity, the light from the source is deflected by the massive object, which then serves as a gravitational lens producing several distorted images of the source. A joint analysis of images of several sources allows modeling the mass distribution in the lens and comparing it with the distribution of visible matter (see, e.g., [91]). The baryon distribution is reconstructed from X-ray observations of luminous gas, which contains about 90% of the cluster mass (Fig. 8). The full mass of the cluster calculated in this way greatly exceeds the mass of the baryons obtained from observations.

3.2.4 Colliding clusters of galaxies. One of the most beautiful observational proofs of the existence of dark matter is the observation of colliding clusters of galaxies [94] (Fig. 9). Contrary to the case of a usual cluster (Fig. 8), there is no need to calculate the mass in this case: a comparison of the mass distribution and the gas distribution demonstrates that the main part of the mass of the clusters and that of luminous matter are located in different places. The reason for this dramatic difference, not seen in normal, noninteracting clusters, is related to the fact that dark matter, constituting the dominant part of the mass, behaves like a nearly collisionless gas. During the collision of clusters, the dark matter of one cluster, together with rare — and therefore also collisionless — galaxies held by its gravitational potential, went through another one, while the gas clouds collided, stopped, and were left behind.

These results, both by themselves and in combination with other results of quantitative cosmology (first and foremost, those obtained from the analysis of the cosmic microwave background and the large-scale structure of the Universe; see, e.g., [5]), point unequivocally to the existence of nonluminous matter.

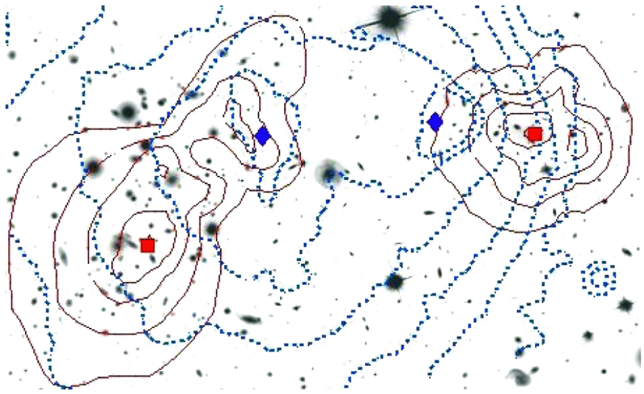

Figure 9. The same as in Fig. 8, but for colliding clusters 1E 0657–558 (the mass distribution model from [95], optical and X-ray images from archives [92, 93], respectively). Squares denote the positions of maxima of mass distributions; diamonds denote the positions of gas emission maxima.

We note that the term 'dark', or 'nonluminous', means that the matter does not interact with electromagnetic radiation and does not just happen to be in a nonemitting state. Indeed, it should not also absorb electromagnetic waves, because otherwise the induced radiation would inevitably appear. Usual matter, that is, baryons and electrons, may be put in this state only if packed into compact, very dense objects (neutron stars, brown dwarfs, etc.), which should be located in the halo of our Galaxy, as well as in other galaxies and in the intergalactic space within clusters. We can estimate the number of these objects that is required to explain the observational results concerning nonluminous matter. This number turns out to be so large that these compact objects should often pass between the observer and some distant sources, which should result in temporal distortion of the source image because of the gravitational lensing (the so-called microlensing effect). These events have indeed been observed, but at a very low rate, which allows firmly ruling out this explanation for dark matter [96, 97].

We are forced to say that dark matter probably consists of new, yet unknown, particles, and therefore explaining it requires extending the SM. Dark-matter particles should be (almost) stable in order not to decay during the lifetime of the Universe (~ 14 billion years). These particles should also interact with ordinary matter only very weakly to avoid direct experimental detection (direct searches for dark matter, which should exist everywhere, in particular in laboratories, have already gone on for decades).

A number of theoretical models of the origin of dark matter predict the mass of the new particle to be between ~ 1 GeV and ~ 1 TeV and the cross section of their interaction with ordinary particles of the order of a typical weak-interaction cross section. Particles with these properties are called WIMPs (weakly interacting massive particles); they are absent in the SM but exist in some of its extensions.

One of the most popular candidates for the WIMP is the lightest superpartner (LSP) in supersymmetric extensions of the SM with conserved R parity (see Section 4.2 below). The

LSP cannot decay because the R parity conservation requires that at least one supersymmetric particle be present among decay products, while all other supersymmetric particles are heavier by definition (in the same way, the electric charge conservation ensures the electron stability and the baryon number conservation ensures the stability of the proton). In a wide class of models, the LSP is an electrically neutral particle (neutralino), which is considered a good candidate for a dark matter particle.

We note that there is a plethora of other scenarios in which dark matter particles have very different masses, from 10^{-5} eV (axion) to 10^{22} eV (superheavy dark matter). Also, in principle, dark matter may consist of large composite particles (solitons).

3.3 Accelerated expansion of the Universe

In this section, we briefly discuss several technically interrelated problems that concern one of the least understandable, from the particle physics standpoint, parts of modern cosmology. The problems are:

(1) observation of the accelerated expansion of the Universe ('dark energy');

(2) weakness of the effect of accelerated expansion compared to typical scales of particle physics (the cosmological constant problem);

(3) indications of intense accelerated expansion of the Universe at one of the early stages of its evolution (inflation).

We start with the observational evidence in favor of (recent and present) accelerated expansion of the Universe.

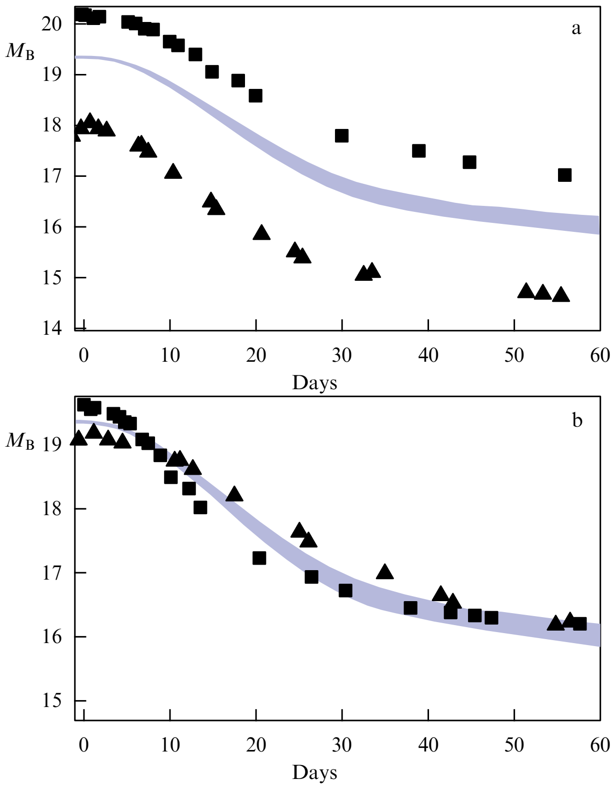

Figure 10. Time dependence of the absolute magnitude of type-Ia supernovae. (a) Light curves of 68% of supernovae are contained within the shaded band; however, there are very rare outsiders (for example, the light curves of an unusually bright supernova SN 1991T (squares) and an unusually weak supernova SN 1986G (triangles) are presented); light curves and the band are taken from [101]. (b) The same curves but scaled simultaneously in the horizontal (time) and vertical (luminosity) axes in accordance with the rules described in [102]. Introducing this correction shifts all 'exclusive' curves to the band. Therefore, to know the absolute value of the luminosity, it suffices to measure the shape of the light curve.

3.3.1 The Hubble diagram. The first practical instrument of quantitative cosmology, the Hubble diagram, plots distances to remote objects as a function of the cosmological redshift of their spectral lines, and was the way to discover the expansion of the Universe and to measure its rate, the Hubble constant. When methods to measure distances to objects located extremely far away became available to astronomers, they found deviations (see, e.g., [98, 99]) from the simple Hubble law that indicate that the expansion rate of the Universe is changing with time, namely, the expansion is accelerating. The method of distance determination we are speaking about (Nobel Prize, 2011) is based on the study of type-Ia supernovae and deserves a brief discussion (see also [100]).

The probable mechanism of the type-Ia supernova explosion is as follows. A white dwarf (a star at the latest stage of its evolution in which nuclear reactions have stopped) is rotating in a dense double system with a normal star. The matter from the normal star flows to the white dwarf and increases its mass. When the mass exceeds the so-called Chandrasekhar limit (the limit of stability of a white dwarf, whose value depends, in practice, on the chemical composition of the star only), intense thermonuclear reactions start and the white dwarf explodes.

It is interesting and useful to note that the exploding stars have roughly the same mass and constitution in all cases (up to the details of the chemical composition). As a consequence, all type-Ia supernova explosions resemble each other not only qualitatively but also quantitatively: the energy release is roughly the same, and the time dependence of the luminosity is similar.

Even more remarkable is the fact that even for rare outsiders (which differ from most supernovae either by the chemical composition or by some random circumstances), all curves representing the luminosity as a function of time are homothetic (Fig. 10), that is, map one to another under a simultaneous scaling of both time and luminosity. This means that by measuring a lightcurve of any type-Ia supernova, we can determine its absolute luminosity with good precision. Comparison with the observed magnitude then allows determining the distance to the object. In this way, it is possible to construct the Hubble diagram (Fig. 11) that demonstrates statistically significant deviations from the law that corresponds to the uniform (or decelerated) expansion of the Universe.

3.3.2 Gravitational lensing. The method of gravitational lensing discussed above allows not only the reconstruction of the mass distribution in the lensing cluster of galaxies but also the determination of geometrical distances between sources, the lens, and the observer. If the redshifts of the source and the lens are known, they can be compared with the derived distances and deviations from the Hubble law can be found with high precision [104].

3.3.3 Flatness of the Universe and the energy balance. A number of measurements point to the spatial flatness of the Universe, that is, to the fact that its three-dimensional curvature is zero. The main argument here is based on the

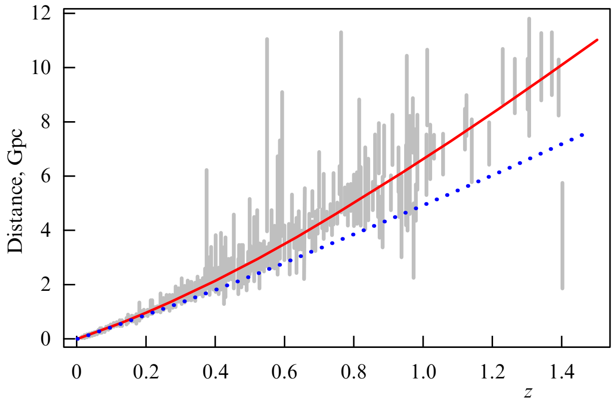

Figure 11. The Hubble diagram, presenting the dependence of the distance from the redshift z of spectral lines of distant galaxies, obtained from observations of type-Ia supernovae. Gray lines correspond to data on individual supernovae with experimental error bars [103]. The uniform expansion of the Universe corresponds to the lower (dotted) line, the accelerated expansion is the upper (solid) line. (Data taken from http://supernova.lbl.gov/Union/.)

analysis of the cosmic microwave background anisotropy [105]. In the past, the Universe was denser and hotter than now. Various particles (photons in particular) were in thermodynamic equilibrium, and hence the distribution of photons over energies was Planckian, corresponding to the temperature of the surrounding plasma. The Universe cooled while expanding, and at some instant, electrons and protons started to join into hydrogen atoms. Compared to plasma, the gas of neutral atoms is practically transparent to radiation; since then, photons born in the primordial plasma have propagated almost freely. We see them as the cosmic microwave background (CMB) now. At the instant when the Universe became transparent, the size of the causally connected region (that is, the region that a light signal had time to cross after the Big Bang), called the horizon, was only ~ 300 kpc. This quantity can be related to a typical scale of the CMB angular anisotropy; the present Universe is much older and we simultaneously see many regions that were not causally connected in the early Universe. This angular scale has been directly measured from the CMB anisotropy. The theoretical relation between this scale and the size of the horizon at the instant when the Universe became transparent is very sensitive to the value of the spatial curvature; the analysis of the data from the WMAP satellite points to a flat Universe with a very high accuracy.

Other methods exist to test the flatness of the Universe. One of the most beautiful among them is the geometric Alcock–Paczinski criterion. If it is known that an object has a purely spherical shape, we can try to measure its dimensions along the line of sight and in the transverse direction. Taking distortions related to the expansion of the Universe into account, we can compare the two sizes and constrain the cosmological parameters, first and foremost, deviations from flatness. Clearly, it is not an easy task to find an object whose longitudinal and transverse dimensions are certainly equal; but it is possible to measure characteristic dimensions of some astrophysical structures which should be isotropic when averaged over large samples. The most precise measurement of this kind [106] uses double galaxies whose orbits are randomly oriented in space, while the orbital motion is described by the Newtonian dynamics.

From the general relativity standpoint, the flat Universe represents a rather special solution, which is characterized by a particular total energy density (the so-called critical density, $\rho_c \sim 5 \times 10^{-6}$ GeV cm^{-3}). At the same time, estimates of the energy density related to matter contribute $\sim 0.25\rho_c$, that is, the remaining three fourths of the energy density of the Universe are due to something else. This contribution, whose primary difference from the contribution of matter is in the absence of clustering (i.e., of concentration in stars, galaxies, clusters, etc.), carries the not fully successful name ‘dark energy’.

The question about the nature of dark energy is presently open. The technically simplest explanation is that the accelerated expansion of the Universe results from a nonzero vacuum energy (in general relativity, the reference point on the energy axis is significant!), the so-called cosmological constant. From the particle physics standpoint, the dark energy problem is, in this case, twofold. In the absence of special cancelations, the vacuum energy density should be of the order of the characteristic scale Λ of relevant interactions, that is

$$\rho \sim \frac{\Lambda^4}{c^3 \hbar^3} .$$

The observed value of ρ corresponds to $\Lambda \sim 10^{-3}$ eV, while characteristic scales of the strong ($\Lambda_{\rm QCD} \sim 10^8$ eV) and electroweak ($v \sim 10^{11}$ eV) interactions are many orders of magnitude higher.

One side of the problem (known for a long time as the cosmological constant problem) is to explain how the contributions of all these interactions to the vacuum energy cancel. In principle, some symmetry may be responsible for this cancelation: for instance, the energy of a supersymmetric vacuum in field theory is always zero. Unfortunately, the supersymmetry, even if it has some relation to the real world, should (as discussed in Section 4) be broken at a scale not smaller than $\sim v$, and the contributions to the vacuum energy should then have the same order.

On the other hand, the observed accelerated expansion of the Universe tells us that the cancelation is not complete, and hence there is a new energy scale in the Universe $\sim 10^{-3}$ eV. The explanation of this scale is a task that cannot be completed within the SM, where all parameters of the dimension of energy are orders of magnitude higher. If this scale is set by the mass of some particle, the properties of that particle should be very exotic in order both to solve the problem of the accelerated expansion of the Universe and not to be found experimentally. For instance, one of the proposed explanations [107] introduces a scalar particle whose effective mass depends on the density of the medium (this particle is called the chameleon). By itself, the dependence of the effective mass on the properties of the medium is well known (for instance, the dispersion relation of a photon in plasma is modified such that it acquires a nonzero effective mass). In our case, due to the interaction with the external gravitational field, the chameleon has a short-range potential in a relatively dense medium (e.g., on Earth), which prohibits its laboratory detection, but at large scales of the (almost empty) Universe, the effect of this particle becomes important.

We also note that a solution to the problem of the accelerated expansion of the Universe might have nothing to do with particle physics at all but be entirely based on peculiar properties of the gravitational interaction (for instance, on deviations from general relativity at large distances).

However, the problem of the accelerated expansion of the Universe is not exhausted by the analysis of the modern state. There are serious indications that at an early stage of its evolution, the Universe experienced a period of intense exponential expansion, called inflation (see, e.g., [82, 108]). Although the inflation theory is currently not a part of the standard cosmological model (it awaits more precise experimental tests), it solves a number of problems in standard cosmology and, presently, has no elaborated alternative. We briefly list some problems that are solved by the inflationary model.

(1) As has already been noted, various parts of the presently observed Universe were causally disconnected from each other in the past, if we extrapolate the present expansion of the Universe backwards in time. Information between the regions that are now observed in different directions could not be transmitted, for instance, at the instant when the Universe became transparent to the CMB. At the same time, the CMB is isotropic up to a high level of accuracy (relative variations of its temperature do not exceed 10^{-4}), a fact that indicates the causal connection between all currently observed regions.

(2) The zero curvature of the Universe, from the theoretical standpoint, is not singled out by any condition: the Universe should be flat from the very beginning, but nobody knows why.

(3) The modern Universe is not precisely homogeneous: matter is distributed inhomogeneously, being concentrated in galaxies, clusters, and superclusters of galaxies; a weak anisotropy is also observed in the CMB. Most probably, these structures were developed from tiny primordial inhomogeneities, whose existence should be assumed as the initial condition.

These and some other arguments point to the fact that the initial conditions for the theory of a hot expanding Universe had to be very specific. A simultaneous solution of all these problems is provided by the inflationary model, which is based on the assumption of an exponential expansion of the Universe that occurred before the hot stage. From the theoretical standpoint, this situation is fully analogous to the present accelerated expansion, but the energy density, which determines the acceleration rate, was much higher. It may be related to the presence of a new scalar field, inflaton, which is absent in the SM. If it has a relatively flat potential (weakly depending on the field value) and the value itself slowly changes with time, then the energy density of the inflaton provides the required exponential expansion. For a particle physicist, at least two questions arise: first, what is the nature of the inflaton? And second, what was the reason for the inflation to stop and not to continue until now?

To summarize, we note that a large number of observations related to the structure and evolution of the Universe cannot be explained if the particle physics is described by the SM only: new particles and interactions have to be introduced. Jointly with the observation of neutrino oscillations, these facts constitute the experimental basis for the confidence in the incompleteness of the SM. At the same time, none of these experimental results presently points to a specific model of new physics, and therefore the guidance in constructing hypothetical models is sought in purely theoretical arguments.

4. Aesthetic difficulties: the origin of parameters

4.1 Electroweak interaction and the Higgs boson

The results of high-precision measurements of the electroweak theory parameters, in the LEP accelerator in particular, confirm the SM predictions based on the Higgs mechanism. At the same time, the only SM particle that has not been discovered experimentally is the Higgs boson. Its mass is a free parameter of the model and is not directly related to any measurable parameter, and therefore the lack of signs of the Higgs boson in data may simply be explained by its mass: the energies and luminosities of available accelerators might be insufficient to create this particle with a significant probability.

At the same time, purely theoretical concerns suggest that the Higgs boson should not be too heavy. This is related to the fact that without taking the Higgs scalar into account, the scattering amplitudes of massive W bosons increase as E^2 with the energy E. As a result, at energies somewhat higher than the W mass, the perturbation theory fails and all model predictions start to depend on unknown higher-order contributions; the theory enters a strong coupling regime and loses predictability. The contribution of the Higgs boson, however, cancels the part of the amplitude that increases with energy, and hence only the constant term $\sim g^2 M_{\mathrm{H}}^2/(4M_{\mathrm{W}}^2)$ remains, where M_{H} and M_{W} are masses of the Higgs and W bosons and g is the $\mathrm{SU}(2)_{\mathrm{L}}$ gauge coupling constant. Therefore, to preserve the calculability, M_{H} should not be too large; a quite reliable limit is $M_{\mathrm{H}} \lesssim 800$ GeV. Even more restrictive limits come from the radiative corrections to the potential of the Higgs boson itself. In the leading order of the perturbation theory, the self-coupling constant of the Higgs boson has a pole at the energy scale

$$Q \sim v \exp \frac{4\pi^2 v^2}{3M_{\mathrm{H}}^2} \,.$$

This means that at energies $\Lambda \leqslant Q$, the contributions of new particles or interactions should change the coupling behavior to avoid a divergence. The requirement $\Lambda \geqslant 1$ TeV results in the limit $M_{\mathrm{H}} \lesssim 550$ GeV. We note that this means that the SM Higgs boson should be discovered at the LHC.

A possibly even more interesting situation is related to the experimental data on the search for the Higgs particle. It may reveal itself not only directly, being produced at colliders, but also indirectly, through the influence of virtual Higgs bosons on numerous observables. Although this influence is not large, a number of electroweak observables are known with very high precision, and their joint analysis may constrain the mass of the yet undiscovered Higgs particle.

We consider Fig. 12, which is based on the analysis of indirect experimental data as of September 2011.[2] The horizontal axis gives the possible Higgs boson mass; the shaded regions of M_{H} are excluded, as of December 2011, from the direct experimental search for the Higgs boson in colliders at the 95% confidence level (the light band $M_{\mathrm{H}} < 114$ GeV is from the LEP [109], the light bands $114 < M_{\mathrm{H}} < 115.5$ GeV and $127 < M_{\mathrm{H}} < 600$ GeV, the LHC [110, 111], and the dark bands $100 < M_{\mathrm{H}} < 109$ GeV and $156 < M_{\mathrm{H}} < 177$ GeV, the Tevatron [112]). The curve demonstrates [113] how well a given value of M_{H} agrees with a

[2] See also the regularly updated webpage at http://gfitter.desy.de/GSM.

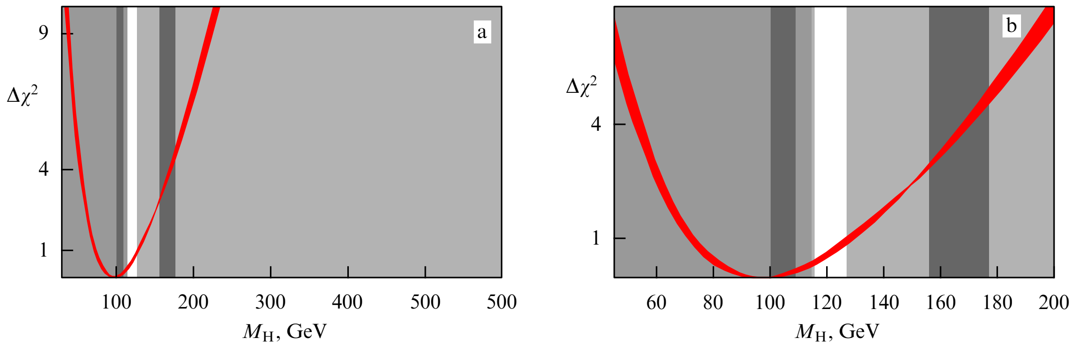

Figure 12. The Higgs boson mass expected from indirect data and constrained from direct searches: (a) all experimental limits; (b) a blowup of the most interesting region, $M_H < 200$ GeV.

combination of all *other* experiments carried out in summer of 2011 (except direct search experiments) (the lower $\Delta\chi^2$ is, the better the agreement; the curve width represents the uncertainty in theoretical predictions). We can see that the most preferable value of M_H is already experimentally excluded! Clearly, this does not imply a catastrophe, because a narrow range of slightly less preferable values is allowed, but it motivates theoretical physicists to think about possible alternative explanations of the electroweak symmetry breaking [114]. We note that it is rather difficult to discover a light, $115 < M_H < 127$ GeV, Higgs boson at the LHC: unlike for a heavy one, several years of work might be required.

The absence of the Higgs boson with the expected mass and the prospect of further restriction of the allowed mass region at the LHC are important, but are far from being the principal arguments in favor of alternative theoretical models of electroweak symmetry breaking, whose history goes back for decades. The Higgs boson is the only scalar SM particle (all others are either fermions or vectors). A scalar particle brings a number of unfavored properties to a theory, some of which we have mentioned above, while others are discussed below. That is why alternative mechanisms of the electroweak symmetry breaking typically use only fermionic and vector fields.

A class of hypothetical models in which the vacuum expectation value of the Higgs particle is replaced by the vacuum expectation value of a two-fermion operator with the same quantum numbers is called technicolor models (see, e.g., [115]). The replacement of the scalar by a fermion condensate looks quite natural if we recall that in the historically first example of the Higgs mechanism surely realized in Nature, the Ginzburg–Landau superconductor, the condensate of the Cooper pairs of electrons plays the role of the Higgs boson.

The base for the construction of technicolor models is provided by the analogy to QCD. Indeed, an unbroken non-Abelian gauge symmetry, similar to $SU(3)_c$, may result in the confinement of fermions and in the formation of bound states (in QCD, these are hadrons, bound states of quarks).

In fact, in QCD, a nonzero vacuum expectation value of the quark condensate also appears, but its value, of the order of $\Lambda_{QCD} \sim 200$ MeV, is much less than the required electroweak symmetry breaking scale ($v \approx 246$ GeV). It is therefore postulated that another gauge interaction exists, in a way resembling QCD, but with a characteristic scale of the order of v. The corresponding gauge group G_{TC} is called the technicolor group. The bound states, technihadrons, are composed from fundamental fermions, T techniquarks, which feel this interaction. The techniquarks carry the same quantum numbers as quarks, but instead of $SU(3)_c$, they transform under the fundamental representation of G_{TC}. Then the vacuum expectation value $\langle \bar{T}T \rangle$ breaks $SU(2)_L \times U(1)_Y \to U(1)_{EM}$ such that the correct relation between masses of the W and Z bosons is fulfilled automatically.

A practical implementation of this beautiful idea faces a number of difficulties, however, which result in a complication of the model. First, the role of the Higgs boson in the SM is not only to break the electroweak symmetry: its vacuum expectation value also gives masses to all charged fermions. Attempts to explain the origin of fermion masses in technicolor models result in a significant complication of the model and, in many cases, in a contradiction with experimental constraints on flavor-changing processes. Second, many electroweak theory parameters are known with very high precision (and agree with the usual Higgs breaking), while even a minor deviation from the standard mechanism destroys this well-tuned picture. To construct an elegant and viable technicolor model is a task for the future, which will become relevant if the Higgs scalar is not found at the LHC.

In another class of models (suggested in [116] and further developed in numerous studies reviewed, e.g., in [114]), the Higgs scalar appears as a component of a vector field. Because the vacuum expectation value of a vector component breaks the Lorentz invariance, this mechanism works exclusively in models with extra spatial dimensions. For instance, from the four-dimensional standpoint, the fifth component of a five-dimensional gauge field behaves as a scalar, and giving a vacuum expectation value to it breaks only the five-dimensional Lorentz invariance, while keeping the observed four-dimensional one intact. Symmetries of the five-dimensional model, projected onto the four-dimensional world, protect the effective theory from unwanted features related to the existence of a fundamental scalar particle. These models also have a number of phenomenological problems, which can be solved at the price of significantly complicating the theory.

The so-called higgsless models [117] (also see [114]) are rather close to these multidimensional models, but differ from them in some principal points. The higgsless models are based on the analogy between the mass and the fifth component of momentum in extra dimensions: both appear in the effective four-dimensional equations of motion similarly. In higgsless models, the nonzero momentum appears due to imposing some particular boundary conditions in a compact fifth dimension. These boundary conditions are eventually responsible for breaking the electroweak symmetry. Unlike in five-dimensional models, where the Higgs particle is a component of a vector field, the physical spectrum of the effective theory in higgsless models does not contain the corresponding degree of freedom. These models have some phenomenological difficulties (related, e.g., to precise electroweak measurements). Another shortcoming of this class of models is a considerable arbitrariness in the choice of the boundary conditions, which are not derived from the model but are crucial for the electroweak breaking.

Finally, we note that a composite Higgs boson may be even more complex than just a fermion condensate: it may be a bound state that includes strongly coupled gauge fields. Describing these bound states requires a quantitative understanding of nonperturbative gauge dynamics. In view of the analogy between strongly coupled four-dimensional theories and weakly coupled five-dimensional ones (which is discussed in Section 5.3), these models may even happen to be equivalent to the multidimensional models described above.

4.2 The gauge hierarchy

Each of the main interactions of particles has its own characteristic energy scale. For the strong interaction, it is $\Lambda_{\mathrm{QCD}} \sim 200$ MeV, the scale at which the QCD running coupling becomes strong; this scale determines the masses of hadrons made of light quarks. The electroweak theory scale is determined by the vacuum expectation value of the Higgs boson, $v \approx 246$ GeV, which determines the masses of the W and Z bosons and of SM matter fields through the corresponding coupling constants. For gravity, the characteristic scale is the Planck scale $M_{\mathrm{Pl}} \sim 10^{19}$ GeV, determined by the Newton constant of the classical gravitational interactions.

These three scales are related to known forces. Extensions of the SM give motivation to some other interactions and, consequently, to other scales. Most important is $M_{\mathrm{GUT}} \sim 10^{16}$ GeV, the scale of the suggested Grand Unification of interactions. In several models explaining neutrino masses, a scale M_{ν} exists; sometimes, the scale M_{PQ} related to the *CP* invariance of the strong interaction is also introduced. Values of these two scales are model dependent but roughly $M_{\mathrm{PQ}} \sim M_{\nu} \sim 10^{14}$ GeV.

The gauge hierarchy problem (also see [3, 83, 118]) consists in the disproportionality of these scales:

$$(\Lambda_{\mathrm{QCD}}, v) \ll (M_{\mathrm{Pl}}, M_{\mathrm{GUT}}, M_{\mathrm{PQ}}, M_{\nu}),$$

and in a range of related questions that may be divided into three groups.

4.2.1 The origin of the hierarchy. Why are the scales of the strong and electroweak interactions smaller than others by many orders of magnitude? That is, why, for instance, are all SM particles practically massless at the gravity scales? In the framework of the Grand Unification hypothesis, it is possible to obtain a reasonable explanation of the relation $\Lambda_{\mathrm{QCD}} \ll M_{\mathrm{GUT}}$. It is based on the logarithmic renormalization-group dependence of the gauge coupling constant on the energy *E*. In the leading approximation, this dependence for the strong interaction coupling α_3 is given by

$$\alpha_3(E) = \frac{\alpha_{\mathrm{GUT}}}{1 + \beta_3 \alpha_{\mathrm{GUT}} \ln(E/M_{\mathrm{GUT}})},$$

where β_3 is a positive coefficient that depends on the set of strongly interacting matter fields [in the SM, $\beta_3 = 11/(12\pi)$], while $\alpha_{\mathrm{GUT}} \sim 1/30$ is the value of the coupling constant of the unified gauge theory at the energy scale $\sim M_{\mathrm{GUT}}$. The scale Λ_{QCD} where α_3 becomes large can be determined in this approximation as

$$\Lambda_{\mathrm{QCD}} = M_{\mathrm{GUT}} \exp\left(-\frac{1}{\beta_3 \alpha_{\mathrm{GUT}}}\right),$$

and the exponential provides the required hierarchy. However, a similar analysis is not successful for the electroweak interaction, whose coupling constants are small at the scale v. It is unrelated to any dynamical scale and is introduced into the theory as a free parameter.

4.2.2 The stability of the hierarchy. In the standard electroweak breaking mechanism, the characteristic scale is $v = M_{\mathrm{H}}/\sqrt{2\lambda}$, where λ is the self-coupling constant of the Higgs boson. Together with M_{H}, the scale v acquires quadratically divergent radiative corrections in the SM,

$$\delta v^2 \sim \delta M_{\mathrm{H}}^2 = f(g)\Lambda_{\mathrm{UV}}^2,$$

where $f(g)$ is a symbolic notation for some known combination of the coupling constants [in the SM, $f(g) \approx 0.1$], and Λ_{UV} is the ultraviolet cutoff, which can be interpreted as the energy scale above which the SM cannot give a good approximation to reality. This scale can be related to one of the scales $M_{\mathrm{Pl}}, M_{\mathrm{GUT}}$, etc. discussed above; under the assumption of the absence of 'new physics', that is, of particle interactions other than those already discovered (the SM and gravity), we should set $\Lambda_{\mathrm{UV}} \sim M_{\mathrm{Pl}}$. Therefore, because $v^2 = v_0^2 - \delta v^2$, where v_0 is the parameter of the tree-level Lagrangian, the hierarchy $v^2 \ll M_{\mathrm{Pl}}^2$ appears as a result of cancelation between two huge contributions, v_0^2 and δv^2. Each of them is of the order of $f(g)M_{\mathrm{Pl}}^2 \sim 10^{33} v^2$, i.e., the cancelation has to be precise up to 10^{-33} in each order of the perturbation theory. This *fine tuning* of parameters of the model, although technically possible, does not look natural. We can revert this logic and say that to avoid fine tuning in the SM, we should have

$$f(g)\Lambda_{\mathrm{UV}}^2 \sim v^2. \tag{3}$$

Relation (3) provides a basis for optimism to researchers who expect the discovery of not only the Higgs boson but also some new physics beyond the SM at the LHC.

4.2.3 The gauge desert. The third aspect of the same problem is related to the presumed absence of particles with masses (and of interactions with scales) between 'small' $(\Lambda_{\mathrm{QCD}}, v)$ and 'large' $(M_{\nu}, M_{\mathrm{GUT}}, M_{\mathrm{Pl}})$ energetic scales (Fig. 13).

All known particles are settled in a relatively narrow range of masses $\lesssim v$, beyond which, for many orders of magnitude, lies the so-called *gauge desert*. Clearly, we may suppose that the heavier particles simply cannot be discovered due to the insufficient energy of accelerators, but this suggestion is not easy to accommodate within the standard approach. Indeed, new, relatively light $(\sim v)$ particles that carry the SM quantum numbers are constrained by electroweak precision measurements. Also, the latest Tevatron and first LHC results on the direct search for new quarks strongly constrain the range of their allowed masses (see [119] and the references therein). In particular, for the fourth generation of matter fields similar to the known three, the mass of its up quark should exceed 338 GeV, while that of the down quark should exceed 311 GeV. The mass of the corresponding charged

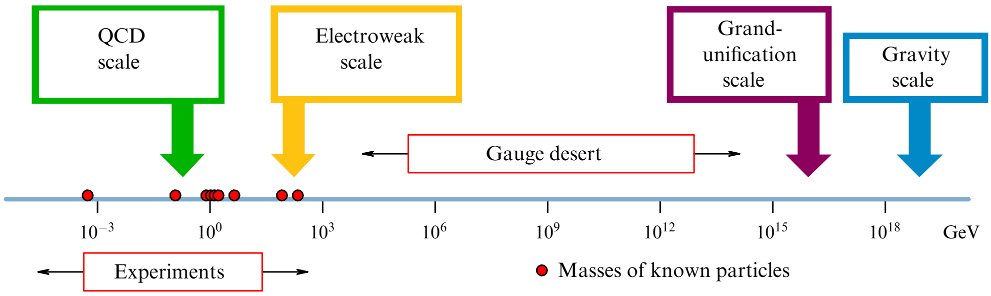

Figure 13. Hierarchy of scales of gauge interactions.

lepton cannot be lower than 101 GeV [7]. The mass of the fourth-generation standard neutrino should exceed half the Z-boson mass, as has been already discussed above. At the same time, these values of masses of the fourth-generation charged fermions cannot have the same origin as those of the first three generations because generating masses much larger than v requires Yukawa constants much larger than unity. Because the methods to calculate nonperturbative corrections to masses are yet unknown, we cannot be sure that these masses can be obtained in the usual way at all. Moreover, the SM fermion masses exceeding the electroweak breaking scale are forbidden by the $SU(2)_L \times U(1)_Y$ gauge symmetry: a mechanism generating these masses would also break the electroweak symmetry at a scale $> v$. The addition of matter fields that do not constitute full generations may be considered an essential extension of the SM. Finally, the addition of new matter affects the energy dependence of gauge coupling constants and spoils their perturbative unification (unless we add either full generations or other very special sets of particles of roughly the same mass that constitute full multiplets of a unified gauge group). We see that attempts 'to inhabit the gauge desert' inevitably result in significant steps beyond the SM, while the desert itself does not look natural.

Attempts to solve the gauge hierarchy problem may also be divided into several large groups.

I. The most radical approach, rather popular in recent years, is to assume that the high-energy scales are simply absent in Nature. For a theoretical physicist, the easiest scales to give up are M_ν and M_{PQ}, because they do not appear in all models respectively explaining neutrino masses and *CP* conservation in strong interactions.

M_{GUT} is somewhat more difficult: the Grand Unification of interactions receives support not only from aesthetic expectations (electricity and magnetism unified to electrodynamics, electrodynamics and weak interactions unified to the electroweak theory, etc.) and arguments related to the electric charge quantization (see, e.g., [4]) but also from the analysis of the renormalization group running of the three SM gauge coupling constants, which take approximately the same value at the M_{GUT} scale (see, e.g., [3, 4]).

It is worth noting that on the plot of $\alpha_{1,2,3}(E)$ as functions of energy in the SM (Fig. 14), the three lines do not intersect strictly at one point; however, for the evolution over many orders of magnitude in the energy scale, already an approximate unification is a surprise. To make three lines intersect at one point precisely, we need a free parameter, which may be introduced into the theory with some new particles, e.g., with masses ~ 1 TeV (this happens, notably, in models with the low-energy supersymmetry; see below).

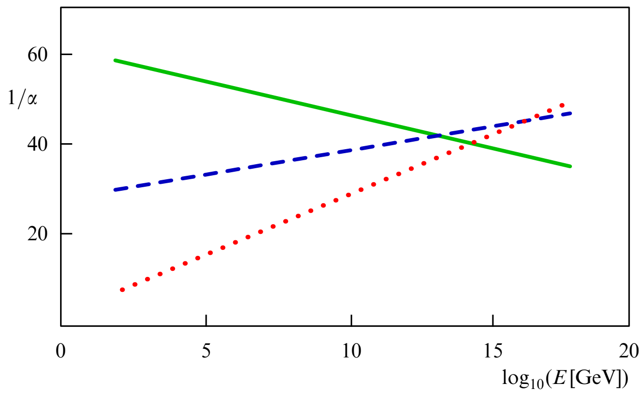

Figure 14. The energy scale dependence of coupling constants of the SM gauge interactions $U(1)_Y$ (the solid line), $SU(2)_W$ (the dashed line), and $SU(3)_c$ (the dotted line) in the leading order.

Therefore, the most surprising fact is not the precise unification of couplings in an extended theory with additional parameters but the approximate unification already in the SM. It is not that easy to keep this miraculous property and at the same time to decrease the M_{GUT} scale in order to avoid the hierarchy $v \ll M_{GUT}$. Indeed, the addition of new particles that affect the renormalization group evolution either spoils the unification or, in the leading order, does not change the M_{GUT} scale (we note that in the SM, the unification occurs in the perturbative regime and higher corrections do not change the picture significantly). The only option is to give up the perturbativity (so-called 'strong unification' [120, 121]).

In that approach, the addition of a large number of new fields that belong to the full multiplets of a certain unified gauge group results in increasing the coupling constants at high energies; QCD stops being asymptotically free at energies higher than the masses of the new particles. In the leading order, all three coupling constants have poles at high energies; the unification of SM couplings guarantees that the three poles coincide and are located at M_{GUT}. But this leading-order approximation has nothing to do with the real behavior of constants in the strong coupling regime, and the theory may generate a new scale M_s at which $\alpha_{1,2,3}$ become strong, this scale being an ultraviolet analog of Λ_{QCD}. For a sufficiently large number of additional matter fields, M_s can be sufficiently close to the electroweak scale v: in certain cases, it might be that $M_s \ll M_{GUT}$ (a nonperturbative fixed point). In this scenario, low-energy observable values of the coupling constants appear as infrared fixed points and do not depend on unknown details of the strong dynamics. We note that the

Grand Unified theory may have degrees of freedom very different from the SM ones in this case.

In the recent decades, models in which the hierarchy problem is solved by giving up the large parameter $M_{\rm Pl}$ have become quite popular. This parameter is related to the gravitational law, and any attempt to change the parameter requires a change in the Newtonian gravity. This may be achieved, for instance, if the number of spatial dimensions exceeds three but the extra dimensions remain unseen for some reason (see, e.g., review [122]). Indeed, if we assume that the extra dimensions are compact and have a characteristic size $\sim R$, where R is sufficiently small, then it is easy to obtain the relation

$$M_{\rm Pl}^2 \sim R^{\delta} M_{{\rm Pl},\,4+\delta}^{2+\delta}\,, \qquad (4)$$

where δ is the number of extra spatial dimensions and $M_{{\rm Pl},\,4+\delta}$ is the fundamental parameter of the $(4+\delta)$-dimensional theory of gravity, while $M_{\rm Pl}$ is now the effective four-dimensional Planck mass.

Already at the beginning of the past century, in studies by Kaluza [123], subsequently developed by Klein [124], the possible existence of these extra dimensions, unobservable because of small R, was discussed. This approach assumed that $R \sim 1/M_{\rm Pl}$ (and therefore $M_{\rm Pl} \sim M_{{\rm Pl},\,4+\delta}$) and became well known and popular in the second half of the 20th century in the context of various models of string theory, which, however, has not resulted in successful phenomenological applications to date. We discuss, in a little more detail, another approach, which allows making R larger but avoids problems with phenomenology. It is based on the idea of localization of observed particles and interactions on a 4-dimensional manifold of a $(4+\delta)$-dimensional spacetime [125–127].

From the field theory standpoint, the localization of a $(4+\delta)$-dimensional particle means that the field describing this particle satisfies an equation of motion with the variables related to the observed four dimensions (to be denoted as x_μ, $\mu = 0, 1, 2, 3$) separated from those related to the δ extra dimensions (z_A, $A = 1, \ldots \delta$), and the solution for the z-dependent part is nonzero only in the vicinity (of the size $\sim \varDelta$) of a given point in the δ-dimensional space (without loss of generality, we can consider the point $z = 0$), while the x-dependent part satisfies the usual four-dimensional equations of motion for this field. As a result, the particles described by the field move along the four-dimensional hypersurface corresponding to our world and do not move away from it to the extra dimensions for distances exceeding $\varDelta$. This may happen if the particles are kept on the four-dimensional hypersurface by a force from some extended object that coincides with the hypersurface. This soliton-like object is often called a brane; hence the expression 'brane-world'. The readers of *Physics–Uspekhi* may find a more detailed description of this mechanism in [122].

Based on topological properties of the brane, localization of light (massless in the first approximation) scalars and fermions in four dimensions[3] implies that many direct experimental bounds on the size of extra dimensions in a Kaluza–Klein-like model now restrict the region $\varDelta$ accessible to the observed particles, instead of the size R of the extra dimension. In [133], it has been suggested to use this possibility, for $R \gg \varDelta$, to remove a large fundamental scale $M_{\rm Pl}$ and the corresponding hierarchy in accordance with Eqn (4). It has been pointed out that in this class of models, R is bound from above mostly by the nonobservation of deviations from the Newtonian gravity at short distances; experiments now exclude the deviations at scales of the order of 50 μm only [134] (it was $\sim$ 1 mm at the moment when the model was suggested). According to Eqn (4), this allows having $M_{{\rm Pl},\,4+\delta} \sim 1$ TeV, which is almost of the same order as v.

Models of this class are well studied from the phenomenological standpoint but have two essential theoretical drawbacks. The first is related to the apparent absence of a reliable mechanism of localization of *gauge* fields in four dimensions. The only known field theory mechanism for that is based on some assumptions about the behavior of a multidimensional gauge theory in the strong coupling regime [135]. Although these assumptions look realistic, they currently cannot be considered well justified. The second difficulty is aesthetic and is related to the appearance of a new dimensional parameter R: the hierarchy $v \ll M_{\rm Pl}$ turns out to be simply reformulated in terms of a new unexplained hierarchy $1/R \ll M_{{\rm Pl},\,4+\delta}$.

To a large extent, these difficulties are overcome in somewhat more complicated models, in which the spacetime cannot be presented as a direct product of our four-dimensional Minkowski space and compactified extra dimensions [136–138]. The principal difference of this approach from the one discussed above is that the gravitational field of the brane in extra dimensions is not neglected. For $\delta = 1$ and in the limit of a thin brane, we then obtain the usual five-dimensional general relativity equations. In particular, these equations have solutions with the four-dimensional Poincaré invariance. The metric in these solutions is exponential in the extra-dimensional coordinate (so-called anti-de Sitter space),

$$\mathrm{d}s^2 = \exp\left(-2k|z|\right)\mathrm{d}x^2 - \mathrm{d}z^2\,, \qquad (5)$$

where $\mathrm{d}s^2$ and $\mathrm{d}x^2$ are the respective squares of the five-dimensional and usual four-dimensional (Minkowski) intervals. For a finite size $z_{\rm c}$ of the fifth dimension, the relation between the fundamental scales then becomes

$$M_{\rm Pl} \sim \exp\left(kz_{\rm c}\right) M_{{\rm Pl},\,5}\,.$$

If the fundamental dimensional parameters of the five-dimensional gravity satisfy $M_{{\rm Pl},\,5} \sim k \sim v$, it is possible [138] explain the hierarchy $v/M_{\rm Pl}$ for $z_{\rm c} \approx 37/k$; that is, instead of fine tuning with the precision of 10^{-16}, the parameters now have to be tuned up to ~ 0.1. It is interesting that in models of this kind with two or more extra dimensions, it is possible [139] to localize gauge fields on the brane in the weak coupling regime, contrary to the case of factorable geometry.

II. A totally different approach to the problem of stabilization of the gauge hierarchy is to add new fields that cancel quadratic divergences in expressions for the running SM parameters. The best-known realization of this approach is based on supersymmetry (see, e.g., reviews [140–142]), which allows the cancelation of divergences due to opposite signs of fermionic and bosonic loops in Feynman diagrams.

The requirement of supersymmetry is very restrictive for the mass spectrum of particles described by the theory. Namely, together with the observed particles, their super-

[3] We note that a fully analogous mechanism of localization in one- or two-dimensional manifolds was recently tested *experimentally* for a number of solid-state systems (the quatum Hall effect, topological superconductors and topological insulators, graphene) (see, e.g., [128–132]).

partners, i.e., particles with the same masses and different spins, should be present. The absence of scalar particles with the masses of leptons and quarks and of fermions with the masses of gauge bosons means that unbroken supersymmetry does not exist in Nature. It has been shown, however, that it is possible to break supersymmetry while keeping the cancelation of quadratic divergencies. This breaking is called 'soft' and naturally results in massive superpartners.

In the minimal supersymmetric extension of the SM (MSSM; see, e.g., [142]), each of the SM fields has a superpartner with a different spin: the Higgs boson corresponds to a fermion, higgsino; matter-field fermions correspond to scalar squarks and sleptons; gauge bosons correspond to fermions that transform in the adjoint representation of the gauge group and are called gauginos [in particular, gluino for $SU(3)_c$, wino and zino for the W and Z bosons, bino for the hypercharge $U(1)_Y$, and photino for the electromagnetic gauge group $U(1)_{EM}$].

For the theory to be self-consistent (the absence of anomalies being related to the higgsino loops) and to generate fermion masses in a supersymmetric way, the second Higgs doublet is introduced, which is absent in the SM. The cancelation of quadratic divergences can be easily seen in Feynman diagrams: in the leading order, closed fermion loops have the overall minus sign and cancel the contributions from loops of their superpartner bosons. This cancelation is precise as long as the masses of particles and their superpartners are equal; otherwise, the contributions differ by an amount proportional to the difference between squared masses of superpartners, Δm^2. The condition of stability of the gauge hierarchy then requires that $g^2/(16\pi^2)\Delta m^2 \lesssim v^2$, where g is the coupling constant in the vertex of the corresponding loop (the maximal, $g \sim 1$, coupling constant is that of the top quark).

We arrive at an important conclusion that partly motivates the current interest in phenomenological supersymmetry: if the problem of stabilization of the gauge hierarchy is solved by supersymmetry, then the superpartner masses cannot exceed a few TeV, which means that they might be experimentally found in the nearest future.

The MSSM Lagrangian, in the limit of unbroken supersymmetry, satisfies all symmetry requirements of the SM, including the conservation of the lepton and baryon numbers. At the same time, for this set of fields, the SM gauge symmetries do not forbid certain interaction terms that violate the lepton and baryon numbers. The coefficients at these terms should be very small in order to satisfy experimental constraints, for instance, those related to the proton lifetime. It is usually assumed that these terms are forbidden by an additional global symmetry $U(1)_R$. When supersymmetry is broken, this $U(1)_R$ breaks down to a discreet Z_2 symmetry called the R parity. With respect to the R parity, all SM particles carry charges $+1$, while all their superpartners carry charges -1. The R-parity conservation leads to the stability of the lightest superpartner (see Section 3.2).

The soft supersymmetry breaking terms are introduced in the MSSM Lagrangian explicitly. They include the usual mass terms for gaugino and scalars, as well as trilinear couplings of the scalar fields. In addition to the SM parameters, about 100 independent real parameters are thus introduced. In general, these new couplings with arbitrary parameters may result in nontrivial flavor physics. The absence of flavor-changing neutral currents and of processes with nonconservation of leptonic quantum numbers, as well as limits from CP violation, narrow the allowed region of the parameter space significantly.

We note the following characteristic features of the phenomenological supersymmetry.

(1) The coupling constant unification at a high energy scale becomes more precise than in the SM if superpartners have masses $\sim v$ as required for the stability of the gauge hierarchy.

(2) In the same regime, the gauge desert between $\sim 10^3$ GeV and $\sim 10^{16}$ GeV is still present.

(3) In the MSSM, there is a rather restrictive bound on the mass of the lightest Higgs boson. In the leading approximation of the perturbation theory, it is $M_H < M_Z$. Taking loop corrections into account allows relaxing it slightly, but in most realistic models $M_H < 150$ GeV is predicted. The absence of the light Higgs boson discussed in Section 4.1 is a much more serious problem for supersymmetric theories than for the SM.

(4) The phenomenological model described above explains the stability of the gauge hierarchy but not its origin. The small parameter v/M, where $M = M_{GUT}$ or $M = M_{Pl}$, does not require tuning in every order of the perturbation theory but should be introduced into the model by hand, that is, cannot be derived or expressed through a combination of numbers of the order of unity. At the same time, if the supersymmetry breaking is moderate, as is required to solve the quadratic divergency problem, it may be explained dynamically in terms of nonperturbative effects, which become important at the characteristic scale

$$\Lambda \sim \exp\left[-O\left(\frac{1}{g^2}\right)\right] M,$$

where g is some coupling constant. If g is small, the supersymmetry breaking scale is also small, $\Lambda \ll M$. In a number of realistic models, it is possible to obtain $v \sim \Lambda$ dynamically up to powers of the coupling constants (by means of radiative corrections) and thus to explain the origin of the gauge hierarchy. But in the MSSM framework, there is no place for nonperturbative effects of the required scale: these effects are relevant only for QCD and with $\Lambda \sim \Lambda_{QCD} \ll v$. The dynamical supersymmetry breaking should occur in a new sector, introduced expressly for this purpose and containing a new strongly coupled gauge theory with its own set of matter fields. No sign of this sector is seen in experiments, and it is therefore assumed that the interaction between the SM (or MSSM) fields and this sector is rather weak and becomes significant only at high energies, unreachable in the present experiments. This interaction is responsible for soft terms, that is, for mediation of the supersymmetry breaking from the invisible sector to the MSSM sector. The gravity mediation (at Planck energies) and the gauge mediation of supersymmetry breaking are distinguished here. Gravity-mediated and gauge-mediated models have quite different phenomenologies.

We see that the MSSM, with the addition of a sector that breaks supersymmetry dynamically and of a certain interaction between this hidden sector and the observable fields, may explain the origin and stability of the gauge hierarchy if the masses of superpartners are not very high ($\lesssim$ TeV). We note that searches for supersymmetry in accelerator experiments put serious constraints on the low-energy supersymmetry. Already the fact that superpartners have not been seen at LEP implies that a significant part of the theoretically allowed MSSM parameter space is excluded experimentally. Subse-

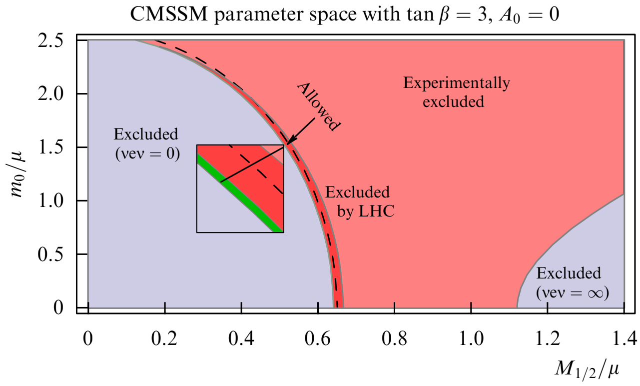

Figure 15. Constraints on the MSSM parameters [143] in one popular scenario. The allowed region is the narrow strip that can be seen in the blowup.

quent results of the Tevatron and especially the first LHC data squeeze the allowed region of parameters significantly, such that only a very narrow and not fully natural region of possible superpartner masses remains allowed for 'canonical' supersymmetry.

In Fig. 15, theoretical and experimental constraints (as of summer 2011) on the MSSM parameters are plotted for one rather natural and popular scenario of gravity-mediated supersymmetry breaking. The masses of all scalar superpartners at the M_{GUT} energy scale are equal to m_0 in this scenario, while masses of all fermionic superpartners are $M_{1/2}$. Their ratios to the supersymmetric mixing matrix of the Higgs scalars, μ, are given in the plot. In a scenario that explains the gauge hierarchy, the MSSM parameters and the Z-boson mass should be of the same order; for instance, in the model that corresponds to the illustration, the relation

$$M_Z^2 \simeq 0.2m_0^2 + 1.8M_{1/2}^2 - 2\mu^2$$

holds. The LHC bound $M_{1/2} \gtrsim 420$ GeV results in the requirement of not fully natural cancelations because $1.8M_{1/2}^2 \gtrsim 40M_Z^2$. Together with the absence of the light Higgs boson discussed in Section 4.1, this 'little hierarchy' problem makes the approach based on supersymmetry less reasonable than it looked some time ago, although versions of supersymmetric models exist where this difficulty is overcome.

III. The Higgs field may be a pseudo-Goldstone boson. The Goldstone theorem guarantees a massless scalar particle (even with radiative corrections taken into account!) for each generator of a broken global symmetry. A weak explicit violation of this symmetry allows giving a small mass to this scalar to obtain the so-called pseudo-Goldstone boson. The same mechanism results in a low but nonzero mass of some composite particles in a strongly interacting theory (for instance, of the π meson). A direct application of this approach to the Higgs boson is not possible because the coupling of a pseudo-Goldstone particle to other fields contains derivatives and is very different from the SM couplings. Realistic models of this kind with large coupling constants and with interactions without derivatives, and at the same time free from quadratic divergencies, are called 'Little Higgs models' (see, e.g., [144] and the references therein). Diagram by diagram, the absence of quadratic divergencies occurs due to complicated cancelations of the contributions of a number of particles with masses of the order of 1 TeV, in particular of additional massive scalars. We note that to reconcile a large number of new particles with experimental constraints, in particular with those from the precision electroweak measurements, the model requires significant complications.

IV. Composite models: besides the Little Higgs models, a composite Higgs scalar is considered in a number of other constructions (see, e.g., [145]). In some rather popular models with composite quarks and leptons, the SM matter fields, together with the Higgs boson (or even without it), represent low-energy degrees of freedom of a strongly coupled theory, similarly to hadrons, which can be considered low-energy degrees of freedom of QCD. The mass scales of the theory, v in particular, are determined by the scale Λ at which the running coupling of the strongly coupled theory becomes large, analogously to Λ_{QCD}. The hierarchy $\Lambda \ll M_{\mathrm{Pl}}$ is now determined by the evolution of couplings in the fundamental theory. These models generalize the technicolor models to some extent, having more freedom in its construction at the price of even greater complications in the quantitative analysis. We note that (at least) in some supersymmetric gauge theories, low-energy degrees of freedom may also include gauge fields; hence, in principle, models in which all SM particles are composite can be considered (see, e.g., [121, 145]).

On the other hand, the correspondence between strongly coupled four-dimensional models and weakly coupled five-dimensional theories (see Section 5.3) may open prospects for a quantitative study of composite models. It might even happen that the approaches to the gauge hierarchy problem based on the assumptions of extra space dimensions are equivalent to the approaches that invoke strongly coupled composite models. As in other approaches, to explain the hierarchy, the scale Λ should not significantly exceed the electroweak scale v, and hence the LHC constraints on the compositeness of quark and leptons [roughly, $\Lambda \gtrsim (4-5)$ TeV] may again be problematic.

4.2.4 Conclusions. All known scenarios that explain the origin and stability of the gauge hierarchy without extremely fine tuning predict new particles and/or interactions at the energy scale not far above the electroweak scale. The absence of experimental signs of these particles, especially in the first LHC data, puts the ability of these scenarios to solve the hierarchy problem into question. If the LHC finds the Higgs scalar but does not confirm the predictions of any of the models discussed above (and finds no signs of some other, not yet invented mechanism), then we would have to reconsider the question of the naturalness of fine tuning. A principally different position, based on the anthropic principle, is seriously being discussed but lies beyond the scope of our consideration.

4.3 The fermion mass hierarchy

As has already been pointed out, the SM fermionic fields, quarks and leptons, comprise three generations, that is, three sets of particles with identical interactions but with very different masses (see Fig. 16 for a pictorial illustration). The hierarchy of these masses is one of the biggest puzzles of particle physics. Indeed, for instance, the electron ($m_{\mathrm{e}} = 0.511$ MeV), the muon ($m_{\mu} = 105.7$ MeV), and the tau lepton ($m_{\tau} = 1777$ MeV) carry identical gauge quantum numbers. For quarks, it is convenient to define the mass matrix whose diagonal elements determine the masses of the

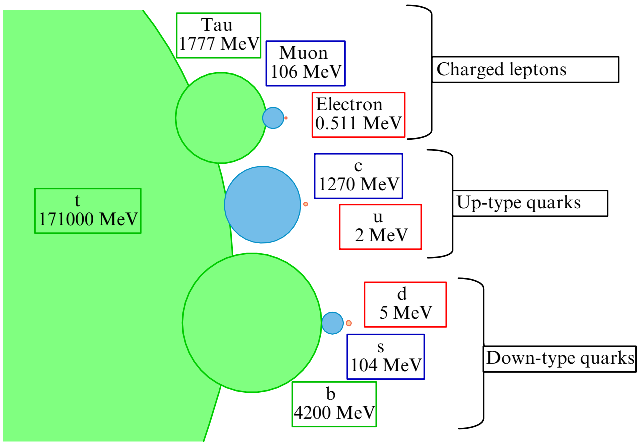

Figure 16. Masses of the charged SM fermions. The area of each circle is proportional to the mass of the corresponding particle.

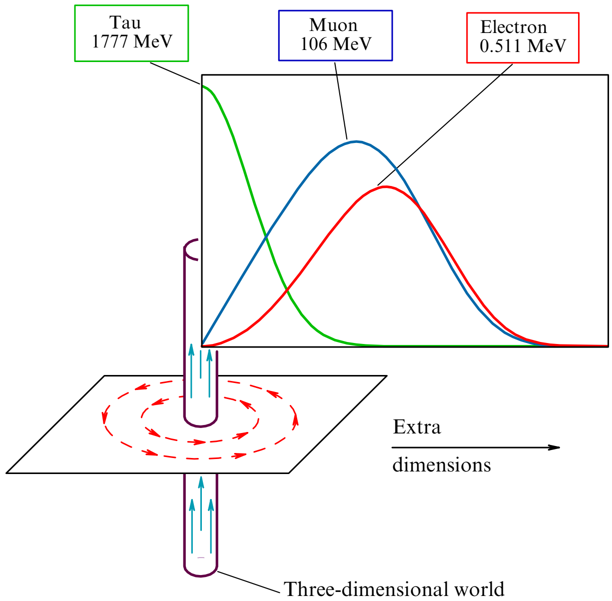

Figure 17. A model with extra spatial dimensions that explains the mass hierarchy.

quarks of three generations with identical interactions, while combinations of off-diagonal elements provide the possibility of mixing between generations. The hierarchical structure appears both in the diagonal elements (which differ by orders of magnitude) and in the off-diagonal ones (the mixing is suppressed). In the SM framework, neutrinos are strictly massless, and the mixing of charged leptons is absent, but the same hierarchical structure is seen in the set of masses of charged leptons.

As we have discussed in Section 2, experiments over the past decade have not only confidently established the existence of neutrino oscillations (therefore pointing to nonzero neutrino masses and giving the first laboratory indication of the incompleteness of the SM) but also opened the possibility of a quantitative study of neutrino masses and of the mixing in the leptonic sector. It is interesting that the neutrino masses and the leptonic mixings also have a hierarchical structure, but it is very different from the corresponding hierarchy in the quark sector: contrary to the suppressed quark mixings, the leptonic mixing is maximal; the hierarchy of neutrino masses is at the same time moderate. A modern theory that would successfully explain the fermion masses should describe both hierarchical structures and explain why they are different.

Meanwhile, even outside the neutrino sector, the intergeneration mass hierarchy is very difficult to explain. A natural idea is to suppose that there is an extra global symmetry that relates the fermionic generations to each other and that is spontaneously broken; however, this approach is not successful because it implies the existence of a massless Goldstone boson, the so-called familon, whose parameters are strictly constrained by experiments [7].

A model of fermion masses should explain only the origin of the hierarchy: its stability is provided automatically by the fact that all radiative corrections to the fermion–Higgs Yukawa constants, to which the fermion masses are proportional, depend on the energy logarithmically, that is, weakly; this does not make the issue significantly less complicated, however.

An explanation of the hierarchy may be obtained in a model with extra spatial dimensions (Fig. 17), in which a single generation of particles in a six-dimensional spacetime effectively describes three generations in four dimensions [146, 147]. Each multidimensional fermionic field has three linearly independent solutions that are localized on a four-dimensional hypersurface and have different behaviors close to the brane. Denoting the polar coordinates in two extra dimensions as r, θ and considering the brane at $r = 0$, we obtain

$$u_0 \sim \text{const} = r^0 \exp(\mathrm{i}0\theta)\,, \qquad u_1 \sim r^1 \exp(\mathrm{i}1\theta)\,,$$
$$u_2 \sim r^2 \exp(\mathrm{i}2\theta)$$

for the three solutions as $r \to 0$. The Higgs scalar has a vacuum expectation value $v(r)$ that depends on r and is nonzero only in the immediate vicinity of the brane. The effective observable fermion masses are proportional to the overlap integrals

$$m_i \propto \int \mathrm{d}r\,\mathrm{d}\theta\, v(r) \left| u_i(r, \theta) \right|^2$$

of the coordinate-dependent vacuum expectation value v and extra-dimensional parts of the fermionic wave functions that correspond to the three localized solutions ($i = 0, 1, 2$ enumerates the three generation of fermions). We can see from Fig. 17 that the resulting m_i are hierarchically different. Therefore, the mass hierarchy follows in this model from the linear independence of eigenfunctions of the Dirac operator in a particular external field. The same model automatically describes the required structure of neutrino masses and mixings [148]. Presently, this model is the only one known in which the hierarchy of families of both charged fermions and neutrinos is explained through one and the same concept. We note that contrary to other multidimensional models (see, e.g., [149]), the number of free parameters in this model is smaller than the number of parameters it describes.

Compared to the hierarchy of masses of particles with identical interactions from different generations, the question of the difference of masses of particles within a generation is much easier. For instance, the difference between masses of the τ lepton and the b and t quarks can be explained by different (because of different quantum numbers) renormalization-group evolutions of the Yukawa couplings, such that these constants are equal at the Grand Unification scale, but are different at low energies.

5. Theoretical challenges in the description of hadrons

5.1 Problems of the perturbative QCD

In this section, we discuss the question of the practical applicability of quantum field theory to the description of interactions with large coupling constants, and to the low-energy limit of QCD in particular. It would not be an exaggeration to say that most of the theoretical achievements in quantum field theory in the past two decades have been related to this problem. Before proceeding to the discussion of these achievements, we note that despite significant progress, the problem of describing strong interactions at low energies in terms of QCD has not been solved, and the development of the corresponding methods remains one of the basic tasks of quantum field theory.

We recall that QCD, which describes the strong interaction at high energies, is a gauge theory with the gauge group $SU(3)_c$ and $N_f = 6$ fermions, quarks that transform under its fundamental representation, and the same number of antiquarks transforming under the conjugate representation. A peculiarity of the model is that the asymptotic states in terms of which the quantum theory is constructed do not coincide with the fundamental fields in terms of which the Lagrangian is written, that is, with fermions (quarks) and gauge bosons (gluons). Conversely, the observable particles do not carry $SU(3)_c$ quantum numbers (this phenomenon is called confinement).

The observable strongly interacting particles are hadrons, whose classification and interactions allow interpreting them as bound states of quarks. At the same time, the theory that describes the interaction of quarks, QCD, is unable to calculate the properties of these bound states.

Intuitively, it seems possible to relate confinement and the formation of hadrons to the energy dependence of the QCD gauge coupling constant, which increases with the decrease in energy (i.e., with the increase in distance; the so-called asymptotic freedom) and becomes large, $\alpha_s \sim 1$, at the scale $\Lambda_{QCD} \sim 150$ MeV: as the distance between quarks increases, the force between them also increases, and maybe this force binds them to hadrons. But this picture is not fully consistent because the perturbative expansion stops working at $\alpha_s \gtrsim 1$ and the true energy dependence of the coupling constant is unknown. Indeed, examples exist of theories with asymptotic freedom but without confinement [150].

To understand the nature of confinement and to describe the properties of hadrons from first principles (and eventually to answer whether QCD is applicable to the description of hadrons), we require field theory methods that do not use the expansion in powers of the coupling constants (nonperturbative methods).

It is natural to assume (and it was assumed for a long time) that the perturbative QCD has to describe well the physics of strong interactions at characteristic energies above a few hundred MeV, because the coupling constant becomes large at ~ 150 MeV. A number of recent experimental results related to the measurement of the form factors of π mesons question the applicability of perturbative methods at a considerably higher momentum transfer (a few GeV). In general, form factors are the coefficients by which the true amplitude of a process involving composite or extended particles differs from the same amplitude calculated for point-like particles with the same interaction. These coefficients are determined by the internal structure of particles (for instance, by the electric charge distribution); their particular form depends on the process considered and on the value of the square of the momentum transfer, Q^2. A full theory describing the interaction that keeps the particles in a bound state should allow the derivation of form factors from first principles. The results of the experimental determination of form factors of π mesons related to various processes are given in Figs 18 and 19. We can see that the perturbative QCD experiences some difficulties in explaining the experiment at the momentum transfer $\lesssim 4$ GeV.

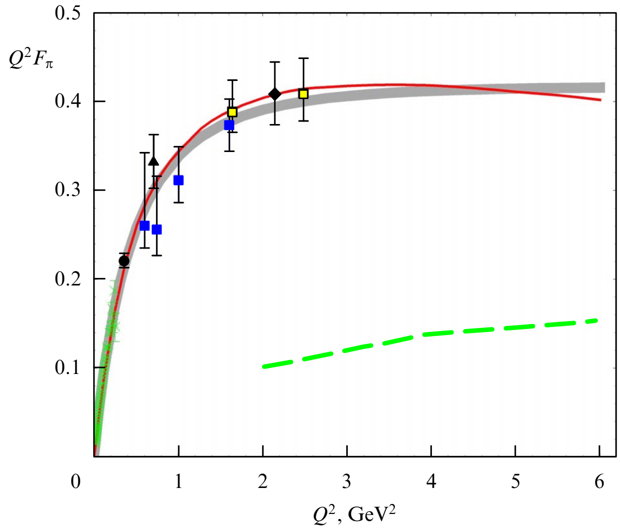

Figure 18. Electromagnetic pion form factor [151]: experimental data versus theoretical calculations, perturbative (QCD, the dashed line) and nonperturbative (solid lines representing working models that are not derived from QCD). Up to the energy scale ~ 2 GeV, there are no signs of approaching the perturbative state.

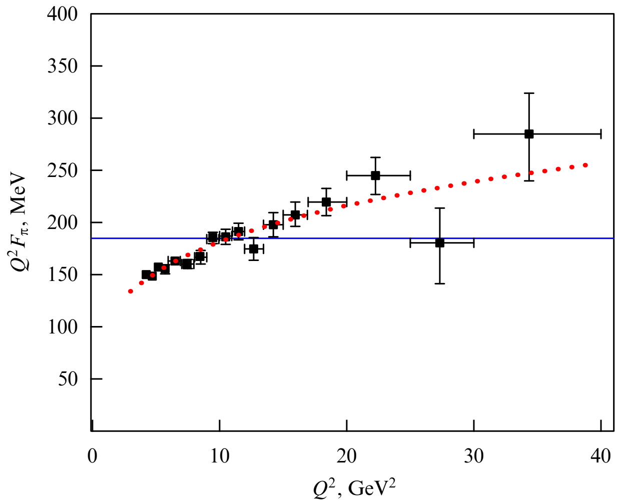

Figure 19. The transitional form factor of the π meson that describes the $\pi^0 \to \gamma\gamma$ process: experimental data [152] versus perturbative QCD calculations. QCD predicts the behavior $Q^2F(Q^2) \sim$ const (the horizontal solid line); at least up to $\sqrt{Q^2} \sim 4$ GeV, the experiment suggests $Q^2F(Q^2) \sim (Q^2)^{0.5}$ (the dotted line).

Approaches to the nonperturbative description of QCD can be divided into two classes: (1) calculations in QCD beyond the perturbation theory (the only available method here is the numerical calculation of the functional integral on a lattice); and (2) construction of an effective theory in terms of degrees of freedom that correspond to observable particles. In the latter case, the main unsolved question is typically how to justify the connection of the effective theory to QCD. To some extent, progress in this direction became possible within the concept of dual theories, discussed below.

5.2 Lattice results

The Feynman functional integral is a formally rigorous approach to quantization of fields, equivalent to other approaches in the domain of applicability of the perturbation theory. It is natural to suppose that in the nonperturbative domain, this method also reproduces the results that would be obtained within the standard framework if the means to derive them existed. Numerical calculation of the functional integral is possible in lattice calculations in which the continuous and infinite spacetime is replaced by a finite discrete lattice (see, e.g., [153]). In current calculations, $32^3 \times 64$ lattices (32 sites in each of the space coordinates and 64 sites in time) are used. For physics applications, it is very important that the gauge invariance be rigorously defined in the lattice theory.

One of the first major achievements of lattice field theory was the discovery that the lattice model with symmetries and field content of QCD exhibits confinement [154]. Subsequent work allowed refining which particular field configurations are responsible for confinement; the work on this problem continues.

The lattice approach allows calculating the values of masses and decay constants of hadrons, and significant progress in this area has been achieved in recent years (see Fig. 20). Currently, the most precise results [155] have been obtained for the so-called '2 + 1' parameterization, in which masses of u and s quarks are free parameters, the d-quark mass is assumed to be equal to that of the u quark, and the contributions of heavy c, b, and t quarks are neglected. Besides the two parameters $m_{\mathrm{u}} = m_{\mathrm{d}}$ and m_{s}, there is one more, the physical length that corresponds to a unit step of the lattice. To determine the masses of hadrons, these three parameters should be specified, and it is assumed in real calculations that, e.g., the masses of π, K, and Ω mesons are known, while all other masses and decay constants are expressed through them. One might try to fix masses of heavier particles and to calculate those of the lightest ones, but a large lattice is required for a reliable calculation of masses of light hadrons. Currently, the π meson mass can be calculated in this way only up to an order of magnitude.

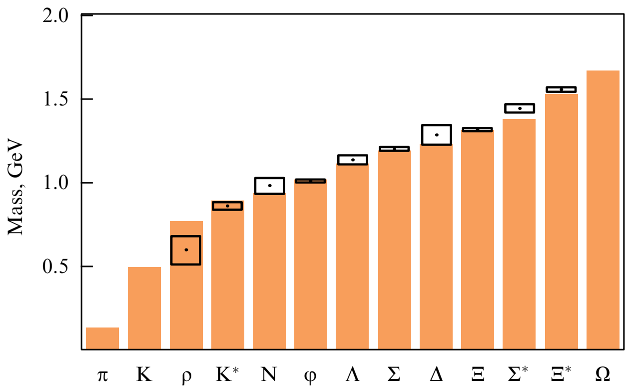

Figure 20. Results of lattice calculations of the hadron masses. The masses of π, K, and Ω mesons are taken as input parameters. The calculations have been performed in the three-quark approximation, $m_{\mathrm{u}} = m_{\mathrm{d}} \neq m_{\mathrm{s}}$. The histogram gives the experimentally measured values of masses [7]; the points (with the error-bar rectangles) represent the results of calculations [155].

At high temperatures, a transition to the state in which quarks cannot be confined in hadrons, i.e., a phase transition is expected. In reality, these conditions appear in collisions of nuclei in high-energy colliders; they probably also occurred in the very early Universe. By means of lattice methods, the existence of this phase transition has been demonstrated, its temperature has been determined, and the dependence of the phase transition order on the quark masses has been studied [156, 157].

Proving that the continuum limit of lattice field theory exists (that is, the physical results are independent of the way in which the lattice size tends to infinity and the lattice step tends to zero) and coincides with QCD is an open theoretical question. It may happen that this proof is impossible in principle, unless an alternative way to work with QCD at a strong coupling is found. However, a series of arguments exist suggesting that the lattice theory indeed describes QCD (first of all, it is the fact that lattice calculations reproduce experimental results). At the same time, theoretically, the difference between lattice and continuum theories is large; for instance, topologically stable configurations in the continuum theory, instantons, which determine the structure of the vacuum in non-Abelian gauge theories, are not always stable on the lattice; the lattice description of chiral fermions (automatic in the continuum) requires complicated constructions, etc.

5.3 Dual theories: supersymmetric duality and holography

In the past two decades, in attempts to relate low-energy models of strong interactions to QCD, theorists have created a number of successful descriptions of the dynamics of theories with large coupling constants in terms of other theories, in which the perturbation theory works. Such theories, called dual to each other, have coupling constants g_1 and g_2 for which $g_1 \sim 1/g_2$; the knowledge of the Green's functions of one theory allows calculating the Green's functions of the other following some known rules. We note that the theory dual to QCD has not yet been constructed.

The simplest example of duality (see, e.g., [158]) is the theory of the electromagnetic field with magnetic charges. The Maxwell equations in the vacuum are invariant under the exchange of the electric field **E** and the magnetic field **B**:

$$\mathbf{E} \mapsto \mathbf{B}\,, \qquad \mathbf{B} \mapsto -\mathbf{E}\,. \tag{6}$$

This duality breaks down in the presence of electric charges and currents. But it can be restored if we assume that sources of the other kind exist in Nature, namely, magnetic charges and currents that correspond to their motion. The self-consistency of the theory requires the Dirac quantization condition: the unit electric charge e and the unit magnetic charge $\tilde{e}$ have to satisfy the relation $e\tilde{e} = 2\pi$. The charge e is the coupling constant of the usual electrodynamics, while the magnetic charge $\tilde{e}$ is the coupling constant of the theory of interaction of magnetic charges, which is obtained from electrodynamics by duality transformation

(6). Therefore, the weak coupling of electric charges, $e \ll 1$, corresponds to the strong coupling of magnetic ones, $\tilde{e} = 2\pi/e \gg 1$.

The electromagnetic duality is based on the geometric properties of Abelian gauge fields, which cannot be directly transferred to the non-Abelian case, which is the most interesting phenomenologically. Somewhat similar but much more complicated dualities appear in supersymmetric non-Abelian gauge theories. The best known one is the 'electromagnetic' duality in the SU(2) supersymmetric theory with two supercharges ($N = 2$), which is related to the names of Seiberg and Witten [159]. From the standpoint of particle physics, this model is an SU(2) gauge theory with scalar and fermionic fields transforming under the adjoint representation of the gauge group, whose couplings are invariant under a special symmetry. For this model, an effective theory has been calculated that describes the interaction of light composite particles at low energies and a correspondence has been given between the effective low-energy and fundamental degrees of freedom. Like QCD, the fundamental theory is asymptotically free and is in the strong coupling regime at low energies; the effective theory describes weakly interacting composite particles.

The success of the Seiberg–Witten model gave rise to the hope that the low-energy effective theory for a nonsupersymmetric gauge model with strong coupling (for instance, for QCD) may be obtained from the problem already solved by means of the addition of supersymmetry-breaking terms to the Lagrangians of both the fundamental and the dual theories. The first step in this direction was to consider $N = 1$ supersymmetric gauge theories. Earlier, starting in the mid-1980s, a number of exact results were obtained in these theories by using analytic properties (governed by supersymmetry) of the effective action [160]. In contrast to the case of the $N = 2$ supersymmetry, this is insufficient for the reconstruction of the full effective theory, but models dual to supersymmetric gauge theories with different gauge groups and matter content have been suggested [161]. In contrast to the $N = 2$ case, it is impossible to prove the duality here, but the conjecture withstood all the checks carried out. Moreover, it has been shown that the addition of small soft breaking terms to the Lagrangians of $N = 1$ theories corresponds to a controllable soft supersymmetry breaking in dual models [162]. Unfortunately, it can be proved that with the increase in the supersymmetry-breaking parameters (for instance, when the superpartner masses tend to infinity, such that the $N = 1$ theory becomes QCD), a phase transition occurs and the dual description stops working, and hence the straightforward application of this approach to QCD is not possible [163]. Also, it is worth noting that the approach does not allow a quantitative description of the dynamics at intermediate energies, when the coupling constants of the dual theories are both large. Nevertheless, these methods themselves, as well as physics intuition based on their application, have played an important role in the development of other modern approaches to the study of the dynamics of strongly coupled theories.

One of the theoretically most beautiful and practically most promising approaches to the analysis of the dynamics of strong interactions at low and intermediate energies is the so-called holographic approach. Its idea is that the dual theories can be formulated in spacetimes of different dimensions such that, for instance, the four-dimensional dynamics of a theory with large coupling constants is equivalent to the five-dimensional dynamics of another theory, which is weakly coupled (similarly to the two-dimensional description of a three-dimensional object with a hologram). The best-known realization of this approach is based on the AdS/CFT correspondence [164, 165], a practical realization of the duality between a strongly coupled gauge theory with a four-dimensional conformal invariance (CFT = conformal field theory) and multidimensional supergravity with a weak coupling constant. The four-dimensional conformal symmetry includes the Poincaré invariance supplemented by dilatations and inversions.

An example of a nontrivial four-dimensional conformal theory with a large coupling constant g is the $N = 4$ supersymmetric Yang–Mills theory with the gauge group $SU(N_c)$ which, in the limit $N_c \to \infty$, $g^2 N_c \gg 1$, appears to be dynamically equivalent to a certain supergravity theory living on the 10-dimensional manifold $AdS_5 \times S_5$, where AdS_5 is the $(4+1)$-dimensional space with anti-de Sitter metric (5) and S_5 is the five-dimensional sphere (the S_5 factor is almost irrelevant in applications; hence the name, AdS/CFT correspondence). In the limit considered, these two models are equivalent. To proceed with phenomenological applications, the conformal invariance must be broken. As a result, the theory has fewer symmetries, and hence the results proved by making use (direct or indirect) of these symmetries are downgraded to conjectures. Nevertheless, this not fully rigorous approach (sometimes called AdS/QCD) brings interesting phenomenological results.

An example is provided by a five-dimensional gauge theory defined on a finite interval in the z coordinate of the AdS_5 space (other geometries of the extra dimensions are also considered). For the $SU(2) \times SU(2)$ gauge group and a special set of matter fields, the effective theory with QCD symmetries then follows. The series of Kaluza–Klein states corresponds to a sequence of mesons whose masses and decay constants can therefore be calculated directly in the five-dimensional theory.

This approach has been successful; it allows calculating various physical observables (in particular, the π-meson form factor discussed above) that agree reasonably with data. A disadvantage of the method is that the duality between QCD and the five-dimensional effective theory is not proved. As a result, the choice of the latter is somewhat arbitrary. An indisputable advantage of this approach is its phenomenological success achieved without a large number of fitting parameters, as well as the possibility of calculating observables for intermediate energies, and not only in the zero-energy limit. It can be hoped that in the future, a low-energy effective theory for QCD might be *derived* in the framework of this approach.

6. Conclusions

The Standard Model of particle physics gives an excellent description of almost all data obtained in accelerators already for several decades. At the same time, the results of both a number of nonaccelerator experiments (neutrino oscillations) and astrophysical observations cannot be explained in the SM framework and undoubtedly point to its incompleteness. A more complete theory, yet to be constructed, should allow a derivation of the SM parameters and an explanation of their values, which are theoretically not fully natural. The main unsolved problem of the SM itself is how to describe the dynamics of gauge theories at strong coupling, which would allow QCD to be applied to the description of hadrons at low and intermediate energies.

We can hope that in the next few years, the particle theory will acquire additional experimental information both from the Large Hadron Collider, a powerful accelerator which is bound to explore the entire range of energies related to the electroweak symmetry breaking, and from numerous experiments of smaller scales (in particular, those studying neutrino oscillations and rare processes) and astrophysical observations. Possibly, this information will allow constructing a successful extension of the Standard Model already in the coming decade.

This work was born (and grew up) from a review lecture given by the author at the Physics Department of Moscow State University. I am indebted to V Belokurov, who suggested converting this lecture into a printed text, read the manuscript carefully, and discussed many points. I thank V Rubakov and V Troitsky for attentively reading the manuscript and the numerous discussions, to M Vysotsky and M Chernodub for the useful discussions related to particular topics, and to A Strumia for his kind permission to use Fig. 15. The work was supported in part by the RFBR grants 10-02-01406 and 11-02-01528, by the FASI state contract 02.740.11.0244, by the grant of the President of the Russian Federation NS-5525.2010.2, and by the Dynasty Foundation.

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