

LETTERS TO THE EDITORS

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Can even monochromatic radiation ensure ideal invisibility?

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Abstract. Discusses conclusions reached by A E Dubinov and D A Mytareva in their paper "Invisible cloaking of material bodies using the wave flow method" (*Usp. Fiz. Nauk* 180 475 (2010) [*Phys. Usp.* 53 455 (2010)]) on whether the perfect optical cloaking of material objects is possible.

A recent analysis [1] of a wide range of publications on the optical cloaking of material objects has drawn conclusions which differ in some essential aspects from those in the more previous review [2], possibly due to the latter's limited volume and popular exposition. These conclusions seem to be in need of refinement.

Let us clarify terminology first. Cloaking using the wave flow method consists in surrounding the object to be cloaked by a shell of matter with electrodynamic characteristics varying in a certain specific way in space. For ideal cloaking, the optical radiation incident on the object and the shell should, whatever its incidence direction and frequency, be scattered such that the total radiation outside the shell is the same as in the absence of the object and the surrounding shell of matter. We will reduce requirements by limiting ourselves to monochromatic (fixed-frequency) radiation, thus eliminating the hard-to-solve problem of frequency dispersion. We will also consider that this frequency falls into the transparency range of the matter because the presence of absorption will inevitably make the object detectable. In the ideal case, the shell of matter is universal, meaning that its electrodynamic characteristics are independent of the properties of the object. Reference [1] considers how to choose shell characteristics using the transformation optics method, which is based on the invariance of Maxwell's equations under a continuous and one-to-one transformation of coordinates. (For example, a one-to-one correspondence is set up between the free propagation of radiation in an isotropic uniform medium in one coordinate system and its propagation in an anisotropic medium with a certain spatial dependence of the dielectric constant (ε) and magnetic permeability (μ) tensor elements in the other.)

A word on the priority for the transformation optics approach is in order here. According to Sections 1 and 3 of Ref. [1], the approach was first proposed in 2006 by the British researchers J Pendry et al. [3, 4] and U Leonhardt [5] (the hypotheses mentioned in Appendix III [1] will not be commented on here). As shown in Ref. [2], however, it is the 1961 paper by the Russian physicist L S Dolin [6] which was the first in the literature to suggest the idea.

Furthermore, a fundamental question is that of whether the wave flow method allows ideal cloaking to be achieved. We will not repeat here the arguments of Ref. [2] that ideal invisibility is in contradiction to the uniqueness property of the solution to the inverse scattering problem (this property was given a mathematically rigorous proof for optically isotropic media and can hardly be affected by introducing anisotropy). Notice the concluding phrase of Ulf Leonhardt's paper [5]: "One can never completely hide from waves, but can from rays," which means that only approximate cloaking — namely. cloaking within geometrical optics — is possible. For us, however, even this statement is overoptimistic for the following reasons.

The application of the transformation optics method to the problem of invisibility fundamentally relies on a transformation of coordinates (mapping) in which real space minus the volume containing the cloaked object is transformed into a simply connected region. Thus, in an example given in Ref. [1], the region of a sphere of radius R_2 is transformed into a spherical layer $R_1 < r < R_2$, where r is the distance of the point from the origin on which the sphere and shell are centered. Once this transformation is found, the spatial dependence of the ε and μ tensor elements is fairly easy to calculate.

Now, is such a transformation possible? Let us quote from monograph [7]: "... a multiply connected region cannot be mapped onto a simply connected region in a continuous one-to-one way." Reference [7] uses the following reasoning to explain this by the example of a two-dimensional space. Suppose a multiple region D *can* be mapped in this way onto a simply connected region D^*. Consider a closed contour C within D, which contains in itself the excluded region with the cloaked object. The mapping under consideration will map C onto a closed contour C^* lying within D^*. If contour C^* is shrunk in a continuous way to a certain point in D^*, then, because the mapping is continuous, contour C , while remaining within D, will have to shrink in a continuous way to a certain point in D. But this is impossible because there are points within contour C that do not belong to D.

When moving to three dimensions, it suffices to replace the word 'contour' by the word 'surface' (meaning the one that fully embraces the cloaked object) to again come to the conclusion that the required transformation of coordinates is impossible. In other words, such a transformation will inevitably be singular — which compromises the invariance proof of Ref. [1] (Appendix I) for Maxwell's equations. This conclusion also applies to the geometrical optics approxima-

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tion and indicates, in accordance with Ref. [2], that the ε and/or μ tensor elements must inevitably have singularities (become infinite) for ideal cloaking to be achieved.

There is another argument against the possibility of a 'universal' cloaking cover capable of hiding objects with arbitrary electrodynamic characteristics. For an electromagnetic field, this possibility exists if the object is ideally screened from arbitrary external radiation, i.e., if field strengths are identically zero within the object. But Maxwell's equations for monochromatic radiation and media with finite values and a piecewise-smooth spatial profile of ε and μ tensor elements will have as their solution identical zero everywhere in space, which is inconsistent with the presence of a radiation source. In this case as well, we see that ideal invisibility can only be achieved if ε and/or μ are allowed either to vary in a physically inconsistent nonanalytical way in space or to take physically inadmissible infinite values. Of course, for real media a treatment in the framework of the full (nonstationary) Maxwell equations can produce a zero field behind the interface (the region into which the radiation did not penetrate while the source was on). But this boundary cannot be stationary and must move at the speed of light [8], implying that, in time, the radiation from a continuous source will fill the whole of space, flowing, in particular, into the volume of the object being cloaked.

To conclude, then, a macroscopic object of finite size cannot be made ideally invisible even with monochromatic radiation if the media used have physically admissible (finite) values of the dielectric constant and magnetic permeability tensor elements. This, though, does not reduce the significance of the elegant method of transformation optics, proposed in Ref. [6] and later developed in papers cited in Refs [1, 2]. Because ideal invisibility requires infinite ε and/or μ, it follows that real invisibility will be more complete the closer the properties of the shell of matter (metamaterial) are to ideal, in particular, the wider their working range of change. If the diagnostic radiation is allowed to have more than one frequency, the spatial profiles of ε and μ in the shell will have to be frequency-dependent, and the shell can no longer be universal. In particular, the absorption bands in its material must be located in a higher-frequency region than those in the material of the object, because at sufficiently high frequencies the cloaking shell effectively disappears (i.e., becomes electrodynamically similar to a vacuum), not providing due refraction of the radiation. It appears that knowing natural limitations can only facilitate the solution of the nontrivial problems of real invisibility.

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