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Nonclassical random walks and the phenomenology of fluctuations of securities returns in the stock market

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1. Introduction. Experimental facts observed in the fluctuations of securities returns

The logarithmic return of shares and stock indices, $S(\Delta t)$, measured over a time interval Δt is defined as

$$S(\Delta t) = \ln \frac{Y(t+\Delta t)}{Y(t)}\,, \tag{1}$$

where $Y(t)$ is the price of a share or the value of an index at time t. It was the subject of systematic study already at the time of L Bachelier [1]. Several facts have been established by experimental studies of share return in international financial markets.

First, for shares of the largest U.S. companies on the time interval from 1994 to 1995, the cumulative distribution function of probability of a fluctuation greater than x, and also smaller than $-x$, is well described by a power-law function of the form [2]

$$\Phi(x) \approx \begin{cases} x^{-3}\,, & S(\Delta t) > x\,, \\ -x^{-3}\,, & S(\Delta t) < -x\,. \end{cases} \tag{2}$$

Similar results were obtained for the shares of German [3], Norwegian [4], French, Japanese, Swiss, and British [5] companies, as well as stock indices [6].

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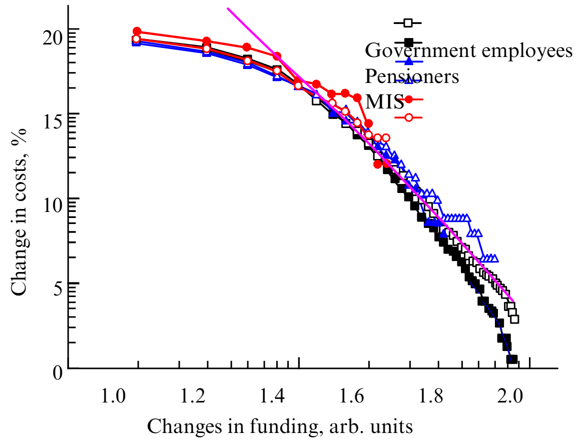

Figure 1. Cumulative distributions of normalized returns (see text) of Sberbank ordinary shares for various Δt. One-minute positive fluctuations — light squares, one-minute negative fluctuations — shaded squares; one-hour positive fluctuations — light triangles, one-hour negative fluctuations — shaded triangles; daily positive fluctuations — light circles, daily negative fluctuations — shaded circles. Bold straight line shows the dependence x^{-3}. One-minute, one-hour and daily data were obtained on trading days 10.01.2009–10.02.2009, 01.09.2008–30.09.2009, 23.01.2006–30.09.2009 (MMVB stock exchange).

Russian stocks ('blue chips') exhibit similar behavior (2). Figure 1 plots the cumulative distribution of returns for positive (black symbols) and negative (white symbols) fluctuations in Sberbank shares. The straight line in Fig. 1 plots the law x^{-3}. The ordinate is the cumulative distribution function, while the abscissa is the return normalized to the appropriate experimentally calculated root-mean-square return. We obtained similar curves for shares of other Russian companies, too. Figure 2 plots the distribution function of fluctuations of the Russian RTS stock index. It is clearly seen that all the plots of cumulative distributions resemble one another. At the same time, return curves for larger Δt lie somewhat higher than return curves for lower Δt (see also paper [5]).

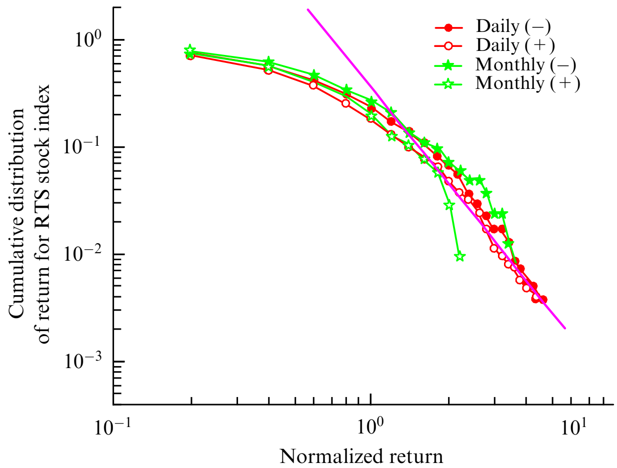

Figure 2. Cumulative distributions of normalized returns (see text) for the RTS stock index at various Δt: daily positive fluctuations — light circles, daily negative fluctuations — shaded circles; monthly positive fluctuations — light stars, monthly negative fluctuations — shaded stars. Bold straight line shows the dependence x^{-3}. Daily data were obtained on trading days 09.01.1995–27.06.2007 (RTS stock exchange), monthly data — on trading days 09.01.1995–20.10.2010.

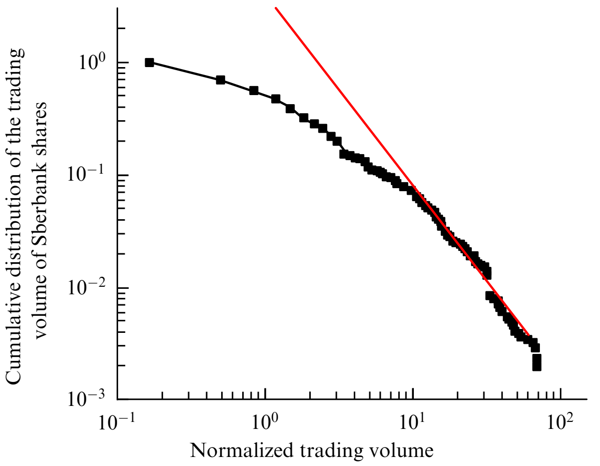

Figure 3. Cumulative distributions of the trading volume in one tick for Sberbank shares on 21.11.2007. The straight line traces the 'tail' curve $x^{-\zeta}$, where $\zeta = 1.7$.

Second, the distribution $Q(x)$ of the number of shares traded in one transaction (one tick) definitely falls within the Lévy range, i.e., the asymptotic ('tail') part of the distribution is well described by a $x^{-\zeta}$ curve, where $0 < \zeta < 2$ and we are dealing with the cumulative distribution function (see paper [7] and the discussion in Refs [8, 9]). The parameters $1.45 < \zeta < 1.63$ were obtained using a number of statistical methods applied to the same sample of shares of major U.S. market caps (see also Ref. [10]), $\zeta \approx 1.58$ for the shares of the 85 largest companies trading on the London Stock Exchange (LSE) in 2001–2002, and $\zeta \approx 1.53$ for the shares of 13 highest market caps included on the CAC 40 (Paris stock market) index.

For Russian blue chips we have obtained indices in the range $1.6 < \zeta < 1.7$, depending on the particular security. Figure 3 plots the cumulative distributions of the trading volume in one tick for Sberbank shares on 21.11.2007.

Obviously, these correlations are valid for shares only. The situation with index returns is somewhat more complicated. The returns of the indices can naturally depend on the volume of trading in shares included in the index. However, it would be difficult to verify this assumption experimentally.

Finally, it is well known that the process $S(t)$ is delta-correlated in time:

$$B(\Delta t) = \big\langle S(t)\, S(t+\Delta t)\big\rangle \sim \delta(\Delta t)\,, \tag{3}$$

for all shares [11]. This statement was tested for Russian blue chips for various values of Δt, including the smallest interval available to us, namely 1 min. The following result was obtained in all cases: the value of the correlation function (3) tends to zero at the first nonzero measurement point Δt. A similar correlation function for the indices takes the form $\sim \exp(-t/\tau_{\mathrm{corr}})$ [6], where the correlation time for the S&P 500 index (one of the most popular indices of the U.S. stock market) is about 4 min [6], and for the Russian RTS index it is 0.85 min [12]. Therefore, the behavior of share and index returns resembles a random process with independent increments.

2. Brownian motion and Gaussian random walk

Random walk is an attractive visually convincing model of a random process with independent increments. Formally, the problem of a random walk is posed as follows. Find the probability density that a particle, after N jumps from the starting point (for this point we can choose, without loss of generality, the point of origin of the coordinates) in space of some dimensionality G, will find itself at a distance in the range from $\mathbf{R}$ to $\mathbf{R} + \Delta\mathbf{R}$. Each ith jump can be made in an interval of length (in the model G-dimensional space) from $\mathbf{r}_i$ to $\mathbf{r}_i + \Delta\mathbf{r}_i$ with probability $\tau(\mathbf{r}_i)$. All jumps are independent random variables.

The method of solving this problem is well known (Chandrasekhar's scheme [13]). Let us assume that

$$\mathbf{R} = \sum_{i=1}^{N} \mathbf{r}_i\,. \tag{4}$$

Given that the probability density function $\tau(\mathbf{r}_i)$ possesses moments of all orders, we have

$$W_1(R) \to \frac{1}{\sqrt{2\pi N\langle r^2\rangle}} \exp\left(-\frac{R^2}{2N\langle r^2\rangle}\right). \tag{5}$$

Putting now $N\langle r^2\rangle = Dt$ (with D being the diffusion coefficient), we obtain the standard solution for the classical one-dimensional diffusion of a Brownian particle, in which its mean square displacement (variance) from the starting point is proportional to $t^{1/2}$.

The most important requirement in the Chandrasekhar scheme [13] is the existence of all moments of the jump law, even though only the second moment appears in expression (5). It seems likely that the jump law which is the 'slowest' in falling off to infinity and has all-orders finite moments is the Subbotin distribution [14]: $p(x) \sim \exp(-x^{\alpha})$, $\alpha > 0$ (in fact, only slightly greater than 0).

3. The Lévy walk

We shall analyze the one-dimensional random walk with the law of the elementary jump $\tau(\mathbf{r}_i)$, which allows normalization even though it does not have all finite moments. The simplest approximation is provided by the power law which assumes boundedness and smoothness for small jumps (at the zero point):

$$\tau(\mathbf{r}_i) = \frac{C_1}{(z^2 + r_i^2)^{\beta}}\,. \tag{6}$$

Here, C_1 is a constant determined by the normalization condition, $C_1 = 2\Gamma(\beta) z^{2\beta-1}/\pi^{1/2}\Gamma(\beta - 1/2)$, where $\Gamma(\beta)$ is Euler's gamma function, $\beta > 1/2$, and z is a constant interpretable as the characteristic length of the jump. Law (6) is therefore scaleless only for big jumps with $r \gg z$; in this case, it is reducible to a Pareto type law [15]: $\tau(r_i) \sim C_1/r_i^{2\beta}$. The distribution function for the law of the jump (6) reduces to the Lévy function

$$W_1(R) = \frac{1}{\pi}\int_0^{\infty} \cos(KR) \times \exp\left[-N(Kz)^{2\beta-1}\,\frac{\Gamma(3/2-\beta)}{2^{2\beta-1}\Gamma(\beta+1/2)}\right] \mathrm{d}K\,. \tag{7}$$

In principle, there is no need to demand that law (6) be identical for all jumps — the values of z can all be different (z_i); in this case, the quantity $Nz^{2\beta-1}$ in formula (7) should be replaced with the expression $\sum_i z_i^{2\beta-1}$.

The distribution law of Lévy random walk is characterized by a slowly decaying asymptotics, i.e., by a significant number of large fluctuations. Indeed, the asymptotics of function (7) is

$$W_1(R \to \infty) \approx \frac{\Gamma(2\beta)\sin\left[(\pi/2)(2\beta-1)\right]}{\pi\rho^{2\beta}}\,, \qquad \rho = \frac{R}{R_0}\,,$$
$$R_0 = \frac{z}{2}\left[N\,\frac{\Gamma(3/2-\beta)}{\Gamma(\beta+1/2)}\right]^{1/(2\beta-1)}, \tag{8}$$

i.e., the asymptotics of the Lévy distribution falls within the range from $1/\rho$ to $1/\rho^3$. The Lévy distribution has one very interesting property. By dividing asymptotics (8) by the asymptotics of the jump law (6), we obtain

$$\frac{W_1(R \to \infty)}{\tau(r \to \infty)} = \frac{Nr^{2\beta}}{R^{2\beta}}\,. \tag{9}$$

This expression means that large fluctuations may occur as a result of a single jump ($R = r$ at $N = 1$).

4. Truncated Lévy walk

The main difference between the truncated Lévy random walk [16, 17] and the Gaussian random walk lies in the thick tails, i.e., a great number of large fluctuations R. The law of the jump for truncated Lévy distribution is the same law (6), where now $\beta > 3/2$ (we continue to consider one-dimensional random walk). Under these conditions, the law has at least a second moment. For small fluctuations, up to $R \sim 10z$, these distributions are well approximated by a corresponding Gaussian function:

$$W_1^{\mathrm{G}}(R) = \sqrt{\frac{\beta-3/2}{\pi N z^2}}\,\exp\left(-\frac{\beta-3/2}{Nz^2}\,R^2\right). \tag{10}$$

This fact is an expression of the central limit theorem (CLT) for such random processes [18]: the Gaussian function describes fluctuations up to the magnitudes of $(N\ln N)^{1/2}$ greater than the characteristic average value z [19]. Sometimes, the result is referred to as the Chebyshev theorem; it holds true for any $\beta \geqslant 2$ [20].

To determine the behavior of truncated Lévy distributions in the range of large fluctuations $R \geqslant (N\ln N)^{1/2}z$, we need to find the exact form, so far unknown, of the asymptotics of the distribution function. It can be shown exactly that the asymptotic behavior of the density distribution of truncated Lévy walk can be described for any β by the law (Fig. 4)

$$W_1(R) \underset{R\to\infty}{\longrightarrow} \frac{2^{\beta}z^{2\beta-1}N}{\pi(2\beta-3)R^{2\beta}}\,. \tag{11}$$

Furthermore, distribution function (11) describes not only an infinitely divisible process [21], but also a stable one. Large R fluctuations after a single jump (9) are possible for truncated distributions only at $\beta = 2$, unlike fluctuations described by the Lévy function, which are possible for any $1/2 < \beta < 3/2$.

Let us trace now how the root-mean-square deviation changes with time. We obtain

$$\langle R^2\rangle = \frac{Nz^2}{2\beta-3}\,. \tag{12}$$

The law of truncated Lévy walk [asymptotics (10), (11)] can be normalized to the mean square of R (12). In this case, all the Gaussian asymptotes (for small R) for every β become identical. At the same time, the asymptotes (11) change to

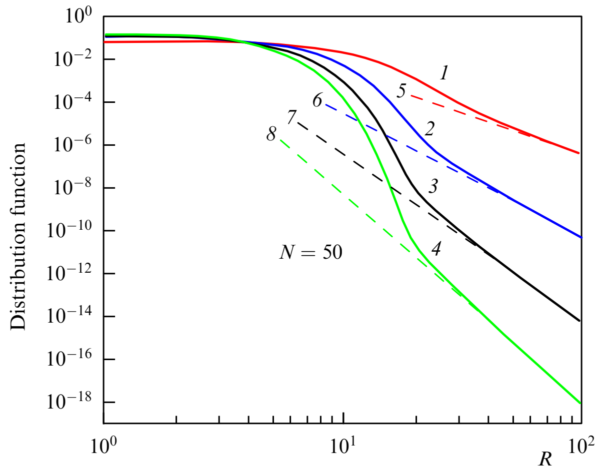

Figure 4. Exact normalized distribution functions for $\beta = 2$ (curve *1*), $\beta = 3$ (curve *2*), $\beta = 4$ (curve *3*), and $\beta = 5$ (curve *4*) as a function of jump length R normalized to z. Dashed lines *5–8* are the corresponding asymptotes for large R.

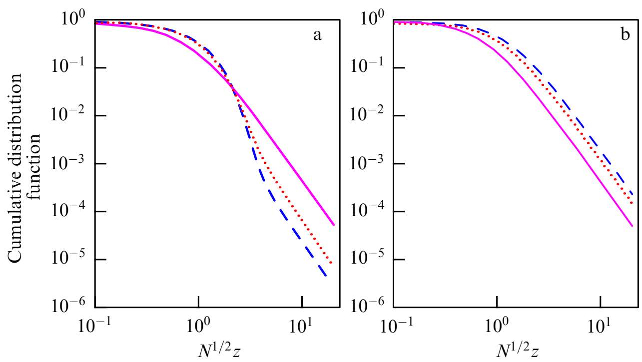

Figure 5. (a) Cumulative distribution function of truncated Lévy walk at $\beta = 2$, normalized to $R = N^{1/2}z$. Solid curve — $N = 1$, dashed curve — $N = 60$, and dotted curve — $N = 450$. The number of jumps corresponds to the ratio between 10-minute, one-hour, and one-day returns. (b) Cumulative distribution function at $\beta = 2$, normalized to R with $\delta = 2.7$. Solid curve — $N = 1$, dashed curve — $N = 60$, and dotted curve — $N = 450$. The number of jumps corresponds to the ratio between 10-minute, one-hour and one-day returns.

$\sim N^{-1/2}$. Figure 5 plots the cumulative distribution function of truncated Lévy walk at $\beta = 2$. The difference among the curves for different values of N is clearly seen. Cumulative distributions behave similarly for all values of β.

5. Comparison with experimental data

The form of the distribution of the truncated Lévy walk obtained as a result of implementing the scheme with the law of single jump (6) corresponds, therefore, at $\beta = 2$ to expression (1); however, in this case there are differences for various values of N — something we do not observe in real situations (see Introduction). To eliminate this discrepancy, the walk scheme needs to be corrected. First we ask a question: What in the experiment corresponds to a single transaction — to the so-called tick. Now we need to answer the following question: Is a tick a single jump in the random walk scheme?

At $\beta = 2$, the variance simply equals $N^{1/2}z$. Experimentally, $N \sim t$, where t is the frequency of fixing the values of the return. Hence, it should be possible to determine the

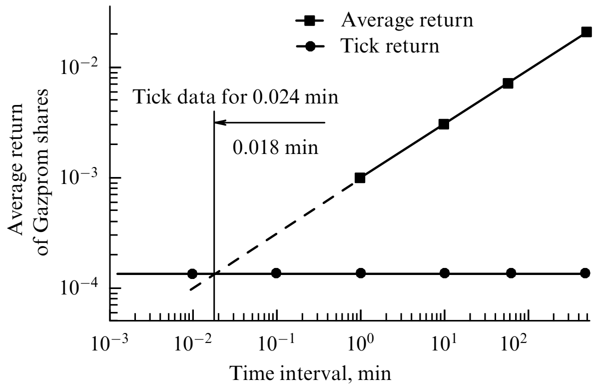

Figure 6. The average return of Gazprom shares for several t (squares connected by the solid line). Marked on the abscissa are time intervals at which the values of return are fixed. Dashed line is an extrapolation of data to small t. Horizontal solid line marks the level of root-mean-square tick return. Intersection of the two lines gives $t = 0.018$ min, the average time between two consecutive ticks is 0.024 min, and the difference comprises 33%. The tick data were obtained from the results of trading on 15.01.2008 on the MMVB stock exchange; 1-minute, 10-minute, one-hour, and daily data are the results of trading on 10.01–10.02.2009, 07.01–30.09.2009, 01.09.2008–30.09.2009, and 23.01.2006–30.09.2009, respectively.

minimum time t that corresponds to the least possible interval between instants of fixing the return, i.e., the interval between two consecutive ticks. On the one hand, this interval is a random quantity. Experimentally, it should not be difficult to find its mean value. On the other hand, in terms of the model this mean value should correspond to the average return of a tick, i.e., to the value of z (see Section 4). It is possible to plot mean returns for different time intervals t. Theoretically, by virtue of formula (12) this curve should exhibit the form $\sim t^{1/2}$. The plot of this function should definitely start from the level of tick return. We can experimentally compare the theoretical minimal time interval, dictated by the point of intersection of the curve of the root-mean-square returns with the level of the root-mean-square tick return (Fig. 6), with the average time interval between two consecutive ticks. The difference between the theoretically predicted minimum time interval and the experimentally obtained average time between two ticks for Gazprom shares is large compared with the difference for the stocks of other companies on the Russian market — 33%. The minimum difference between these values is observed for the shares of Sberbank — only 3%.

It is nearly certain that a tick is a single jump in the random walk scheme. The first obvious possibility of modifying the model boils down to an attempt of applying the random walk scheme with continuous time (Continuous Time Random Walk, CTRW) [22]. Indeed, the time intervals between two successive ticks can vary in a wide range. The distribution of these intervals for the U.S. stock market is known [23], with appropriate distribution function falling off with a decrease in Δt as $(\Delta t)^{4.4}$. Presumably, no new results can be obtained by taking into account the time interval between transactions due to the presence of mathematical expectation of the time interval between ticks.

Another possibility of modification of the truncated Lévy random walk scheme is the use of the power-law correlation of standard deviation of z and the average volume of one transaction. Our modification is limited to the assumption that each standard deviation of z in the walk scheme [see the law of jump (6)] is a random variable z_i proportional to the volume of the ith transaction (ith tick). We are returning to the second experimentally identified property described in the Introduction. We actually utilize the quite familiar, widely used stock exchange rule: 'the volume of trading drives the price' [24, 25].

This modification signifies that we are introducing the dependence of the probability distribution function of single fluctuations $\tau_i(r_i)$ on another random variable z_i. In this case, the model again resembles the CTRW model. The problem of direct application of CTRW scheme lies in the fact that the final distribution function for R will depend on the set of random variables $\{z_i\}$. For example, the distribution function of truncated Lévy random walks at $\beta = 2$ is found in the form

$$W_{\beta=2,\,z_i}(R) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{d}K \exp(\mathrm{i}KR) \prod_{i=1}^{N} \exp(-Kz_i)(1+Kz_i)\,. \tag{13}$$

Since all variables $\{z_i\}$ have the distribution function $\sim x^{-\delta}$ for large z_i, where $\delta \sim 2.5-2.7$ ($\delta = \zeta - 1$), formula (13) can be averaged over each z_i. This result will nevertheless be wrong since the final expression for function (13) averaged in this manner will disagree with the experimentally observed data, namely it will not be proportional to R^{-4} for large R.

It seems that using the simple CTRW scheme is possible, at least for asymptotic values of function (13), because for large R we have

$$W_{\beta=2,\,z_i}(R) = \frac{4\sum z_i^3}{\pi R^4}\,, \tag{14}$$

and the set of random variables $\{z_i\}$ only returns a single random variable: $\sum z_i^3$. Unfortunately, the distribution function of the probability density of this variable on the tails assumes the form $\sim x^{-2/3-\delta/3}$. There is no mathematical expectation for this function, which is necessary for the application of the CTRW scheme [22]. In fact, the variables $\{z_i\}$ are included explicitly in expression (13) in various combinations: $\sum z_i^2$, $\sum z_i^3$, etc. Each of these combinations is in itself a random variable. Because the asymptotic distribution function is a function of the sum of cubed z_i, we can conclude that the sum converges to the Lévy distribution (see Ref. [13]). Only this condition ensures the stability of $\sum z_i^3$ as new terms are added to the sum.

Consequently, the CTRW method must be generalized to the case of the absence of the conditional mean [of the random variable $\sum z_i^3$ in formula (14)] (see Ref. [26]). As in the case of the Lévy distribution (7), expression (13) can be examined for dependence on N, i.e., on renormalization. If the quantity R in formula (13), or the corresponding asymptotic cumulative distribution

$$\Phi_{\beta=2,\,z_i}(R) \simeq \frac{4\sum z_i^3}{3\pi R^3} \tag{15}$$

is renormalized to the standard deviation $\left(\sum z_i^2\right)^{1/2}$, as we do in all experiments, the result is the scaling dependence of expression (15) in the form $N^{-1/2}$ for $\sum z_i^3 \sim N$ [see formula (11) and Fig. 5a). At the same time, the dependence of $\sum z_i^3$ on N looks different because the distribution function of the random variable $\sum z_i^3$ converges to the Lévy distribution (see above). The end result for the Lévy distribution function is $\sum z_i^3 \sim N^{3/(\delta-1)}$, and the final observed dependence of distribution (15) on N after renormalization of the actual profitability R to the experimentally obtained standard deviation takes on the form

$$\Phi^{\mathrm{renorm}}_{\beta=2,\, z_i}\left(\frac{R}{\sqrt{\sum z_i^2}}\right) \sim N^{3/(\delta-1)-3/2}\,. \qquad (16)$$

If $\delta \sim 2.5-2.7$, we obtain dependences (16) in the range from $N^{0.5}$ to $N^{0.27}$ (Fig. 5b). It is seen that the standard experimental renormalization provides a weak dependence of all return distribution functions on the number of jumps (ticks) N. Notice that such dependences (16) as a function of N are similar to the experimental results [6] and to our results, too, obtained for the Russian stock market, where we observed weak dependences on N: the returns increase as N increases, in contrast to what we observe in Fig. 5a for the simple scheme of truncated Lévy random walks.

Notice that the established dependence on N occurs only for the profitability of the stock. The possible dependence of the index returns on the volume of trading in shares listed in the index may have a different form (from the law $x^{-\zeta}$). If this law does not fall within the Lévy range $0 < \zeta < 2$ and ζ is greater than 2 (by only a little), then the cumulative distribution of the index returns on long time intervals (16 days as in Ref. [6], positive monthly returns of the RTS index in Fig. 2: the last two points) can converge to Gaussian one (see formula (16) for $\delta \sim 3-4$). These distributions will look similar to those shown in Fig. 5a for large N, not like the same curves in Fig. 5b.

6. Conclusion

The introduction of the law of jump of (6) type allows one to consider in a unified analytical manner both the ordinary and the truncated Lévy walks. The truncated Lévy walk asymptotically manifested the same properties of stability and scalability as the ordinary random walk. Analytical asymptotes were obtained for the truncated random walk and scaling laws were established. It turned out that the asymptotic truncated Lévy walk possesses characteristic scaleless distribution $\sim R^{-2\beta}$, which is also typical of the asymptotes of the 'pure' random Lévy walk but, in contrast to the latter, decays faster with increasing R. Therefore, the truncated Lévy walk, together with the pure random walk, covers the entire class of Pareto distributions [15].

We can assume that the law $\sim 1/x^3$ for the cumulative distribution function of share and index fluctuations is universal. Such a distribution can be obtained by using the scheme of random walks (jumps) with the law of single jump (6) only at $\beta = 2$. This means that the law of jumps at such a value of β, namely

$$\tau_i(r) = \frac{4z_i^3}{\pi(z_i^2 + r^2)^2}\,, \qquad (17)$$

is also universal. Here, the value of z_i represents some characteristic return used for normalization. This result can be considered as proof of the existence of a microscopic law of return fluctuations on the stock market. Therefore, the prices of all shares (indices act essentially as baskets of shares and their behavior is similar) perform 'jumps' for different 'distances' at constant probabilities. The microscopic law (17) explains the phenomenology of the law $\sim 1/x^3$ [2].

Apparently, the existence of strict laws of a single jump (16) is possible for two reasons. First, the probability distributions of fluctuations of returns should have a second moment, i.e., have variance. In the final analysis, this requirement is a reflection of the limited amount of money available. Second, the distribution function must have the same asymptotics as the law of the jump, i.e., a nonzero probability of large fluctuations resulting from a single jump must exist. All Lévy functions meet the second requirement but not the first. Only distribution function (12) at $\beta = 2$ satisfies both conditions.

A simple definition of the z_i variable as a characteristic length of a jump cannot provide an exact explanation for the dependence of the normalized distribution functions and cumulative distributions on N. A modification of the random walk scheme is provided through the introduction of the dependence of $\{z_i\}$ on the number of shares traded in one transaction (tick), because the correspondence of one tick to one jump is an experimentally verified fact (see also Ref. [12]). In this case, the distribution function of the quantity $\sum z_i^3$ converges to the Lévy function with the Lévy index $(\beta - 1)/3$. The final dependence of the cumulative distribution functions on the number of ticks (jumps) falls into the range from $N^{0.5}$ to $N^{0.27}$. Russian stocks exhibit weaker dependence than shares from the USA and shares traded in the LSE and the Paris Stock Exchange. We conclude that the presented final random walk scheme looks like the CTRW scheme lacking the conditional expectation (for the quantity $\sum z_i^3$).

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