time derivatives). This means that in the former case, the absolute scales of quantities are essential, while in the latter case they are not. As a result, in physics, the 'favorite' (i.e., the most illustrative) solutions are uniform motions at constant speed, while in economics, these are self-similar solutions in which extensive quantities increase exponentially, i.e., at a constant rate. The vast majority of the conclusions of economic theory have been obtained by comparative analysis of self-similar solutions of simple models.[4]

The variational principle in physics 'controls' *the system as a whole*, while in economic models, *each agent has its own variational principle*. Even more important is the significant difference among the topological structures of these principles. Application of the variational principle always leads to a Hamiltonian system of equations of motion, and this motion is on the surface of a constant Hamiltonian function.

In physics, the Hamiltonian function is, roughly speaking, downward convex. Therefore, its stable *critical points* are essentially *centers* around energy minima, and typical motions reduce to rotations, vibrations, and windings on tori. In general, these movements demonstrate neutral stability, i.e., they shift on the whole by a distance of the order of the increment in the initial conditions.

With the characteristic orientation of the *economy to maximization of capitalization*, utility, profit, etc., the Hamiltonian function is convex downward in 'momenta' and convex upward in 'coordinates,' and all its *critical points* are *saddle-shaped*. As a result, any economically meaningful movements of the system are close to stable separatrices of saddles. These solutions are unstable with respect to the initial values of the momenta (which in addition are unobservable) but depend weakly on the initial conditions for coordinates and on perturbations in the distant future. For a Hamiltonian system, we have to solve not a Cauchy problem but a *boundary value problem*. The ensuing results are known as *turnpike theorems*. They give us hope that models like intertemporal equilibrium models will be true in the mid-term, regardless of the accuracy of predictions for the distant future.

Our main result obtained in recent years was the discovery of a strong turnpike property: *even though we allow agents in the model to know the future, this knowledge proves useless for them as regards choosing the optimal behavior*. Because this property holds, it removes all objections to the application of the principle of rational expectations. In other words, the model reduces to a conventional dynamic system. However, the property itself requires an explanation.

The key here is that a strong turnpike effect is observed not in the model in general, at the level of formulas, but only if the parameters are correctly identified [12]. We need to recall here that economics as a management system is meant not only to coordinate the actions of billions of people but also to do it in a way that allows people to make a reasonable choice in most cases, without complex calculations. Therefore, even widely familiar economic mechanisms may not work, owing to complexity and risk. We can assume that at any given time, the economic system selects and engages a set of mechanisms that do not require detailed calculations for reasonable solutions. Consequently, by describing the mechanisms in the model 'true to life,' not true to textbooks, we arrive at a model with a strong turnpike property.

All this somewhat resembles the anthropic principle well familiar to physicists: the Universe appears to the observer as harmonious and 'adapted' to him or her because no observer could appear in a differently arranged universe.

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[4] Incidentally, in a historical perspective, the 'economic exponent' of the industrial society continues to break through seemingly fairly objective external constraints, while forecasts found by using models that attempt to include specific limits to growth — from the predictions of T Malthus to those of D Meadows — proved unsuccessful.

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On econophysics and its place in modern theoretical economics

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1. Introduction

Theoretical economics has the same goals as other theoretical fields:

(1) Description of an object (system) in the language of mathematical methods.

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(2) Description of the system responses to external stimuli.

(3) Prognosis of the system behavior under constant external conditions.

Even though economics is traditionally regarded as belonging to the humanities, theoretical economics operates with mathematical and physical concepts and methods. In this sense, theoretical economics is an interdisciplinary field. As a rule, cross-disciplinary areas (biophysics, physical chemistry, etc.) succeed in not breaking the ties to their progenitors or other sciences. There are exceptions to this rule, however.

We are currently facing a crisis in theoretical economics [1]. There are several (at least two) different avenues of research that start with different basic assumptions (axioms), use different mathematical methods, and in the end arrive at different solutions of the problems formulated above.

Nevertheless, the fates of people, countries, and perhaps the world may depend on the solutions chosen in the current period of crisis. This, in the nutshell, is what makes the situation critical.

We now discuss these fields of research.

I. The approach that is known best (especially outside Russia) is the so-called neoclassical approach, considered to be 'mainstream' [2]. The principal features of the mainstream are:

(1) an individual is an elementary unit of society. An individual fabricates products with a view to gain maximum profit. Individuals consume products with a view to extract maximum benefit for themselves. The main function considered by classical economics is the utility function. It is assumed that the individual is 'rational' and is capable of identifying the most useful set of benefits (material ones first and foremost). Differences in customs ('mentalities') in different countries and societies are not taken into consideration;

(2) self-organization of individuals in a society leads to the occurrence of a stationary (equilibrium) state in which demand (for goods, labor, money, etc.) is counterbalanced by supply. It is assumed that the society (state) is formed as a result of the assembly of individuals and that this does not generate new qualities that are not deducible from the qualities of individuals.

It seems that the purpose of the mainstream is to build (even if only in one's mind) a stable ideal society in which the interests of everyone would be satisfied to the maximum, and that this society would exist forever (like a utopian 'City of the Sun').

Mainstream systems often use dogmas such as:

(1) the state should not intervene in the economy. This provision has not been proved (hence, 'dogma') and cannot be proved because neither total noninterference nor total intervention are possible. The important factor is the degree of intervention, which depends on the situation;

(2) the stationary and equilibrium state of the market economy is unique. This position is also an unproved dogma and has been recently thrown into doubt. Nevertheless, practically no study is available in the mainstream on the dynamics of transitions from state to state.

The strength of the mainstream is that it involves large teams of high-level professionals. The main achievement of the mainstream is the analysis of the optimal steady state of the market economy.

The weakness of the mainstream is that it is isolated from other natural sciences and has in fact degenerated into Herman Hesse's 'glass beads game.' Internal consistency and conformity with the axioms are regarded as the quality criteria for assessing a solution of a problem. The condition of the results matching reality are typically not discussed. Solutions of the above-mentioned urgent problems are not considered within the mainstream, and these problems are not posed.

II. A new approach that appeared in theoretical economics relatively recently is econophysics (or physical economics). This field attacks a group of various problems.

(1) Evolutionary economics emerged in the early twentieth century in the work of Schumpeter [3]. It was further developed in many papers (see [4, 5] and the references therein). In fact, it was Schumpeter who drew attention to the fact that economies are not static but evolving systems. For a while, this approach evolved without using mathematical tools. Currently, it does have a mathematical apparatus to work with — the same as in other currently evolving systems (and in synergetics).

(2) Mathematical modeling (based on the theory of dynamic systems) of both macroeconomic and microeconomic processes [this field is often called economy synergetics (Zhang [6])].

(3) Analysis of stock series and of their properties.

The last three approaches to economics are grouped together by similarity of the systems of the main concepts, the nature of the tasks, and the methods of solving them. The economy is treated in all three as an evolving system, and all descriptions work with the same tool, the theory of dynamical dissipative systems. In other words, theoretical economics is not treated as an isolated science; rather quite the opposite is true: it joins the family of natural sciences. At the same time, econophysics is a fully defined field of science in this family.

The problem that plays a special role here is the dualism of evolution: on the one hand, society must preserve the stored information, while on the other hand, it must create new information. These two problems are complementary (in the sense of complementarity defined by Niels Bohr). In physics, complementary problems have been solved in many cases, while in economics this is a task for the future.

We thus see that econophysics does not offer anything very new (in comparison with other natural sciences). The important side of physics in this context is that it deals both with nature and with modern mathematics, and as a result it exercises disciplined thinking and a critical attitude toward dogmas. For example, physics both accepts the emergence (and disappearance) of several stationary states and widely uses this scenario [7, 8].

From this perspective, economics is a very interesting and important application area for physics.

In reality, the boundary between neoclassical economics and econophysics is somewhat blurred. Many economists use both of these approaches in different papers.

The purpose of this talk is to select examples of mathematical models in economics and discuss their results.

2. Basic concepts and models. The demand function

Demand plays an important role in the economy. Demand depends on the needs of individuals; but the behavior of others (including the media) plays at least as important a role. In other words, demand is a collective behavioral response of

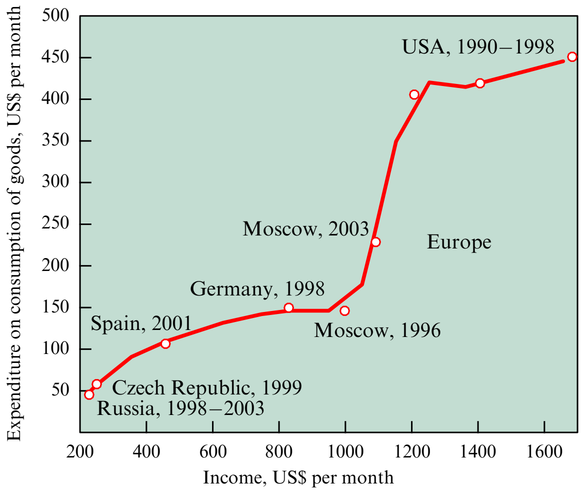

Figure 1. Empirical plot of demand function (empirical data) [9].

Figure 2. Analytic representation of the demand function.

human society. Demand is described by the demand function, of which there are several versions.

(1) The demand function for a given commodity (or a group of similar commodities) is the amount of product Q consumed per unit time as a function of the amount of money U available to the consumer or their income D and the commodity price p. Because both of these are arbitrary, the demand function depends on the ratio U/p (or D/p). The quantity Q can be expressed in natural units (items, kilograms), but more often the quantity Qp is used, which is expressed in monetary units. As an example, Fig. 1 plots an empirical demand function [9] based on the data published by statistics offices of various countries. Nonmonotonicity (the presence of a so-called 'beak') is an important property of this function. We see below that the beak plays an important role.

(2) Macroeconomics operates with a joint aggregation, which incorporates first-priority goods (food, clothing, housing), durable goods (cars, television sets, etc.), and elitist goods. In this case, it is more convenient to use the demand function $Q(U/p)$.

The demand function $Q(U/p)$ (Fig. 2) can be represented in an analytic form as [10]

$$Q(r) = Q_1 \frac{r}{r + r_1} + \Theta(r - r_{\min}) \left[Q_2 \frac{r - r_{\min}}{r - r_{\min} + r_2} + \varepsilon (r - r_{\min}) \right], \quad (1)$$

where

$$\Theta(x) = \begin{cases} 0, & x < 0, \\ 1, & x > 0. \end{cases}$$

The parameters Q_1, Q_2, r_1, r_2, $r_{\min}$, and ε have the following meaning. The parameter Q_1 corresponds to the total supply of goods of vital importance; r_1 is the value of purchasing power at which these needs are half-satisfied; $r_{\min}$ is the critical level of purchasing power: at a purchasing power less than $r_{\min}$, the consumer does not buy durable goods. The magnitude of $r_{\min}$ depends on the consumer psychology. Large $r_{\min}$ signify that people are more inclined to satisfy their essential needs, i.e., to live a simpler life. Small $r_{\min}$ signal that consumers prefer to live in a 'modern way,' even by cutting their food consumption. The parameter Q_2 corresponds to complete satisfaction in durable goods, i.e., to having bought the entire 'gentleman's set of goods': the full set of household conveniences, a car, a country house, etc. The parameter r_2 characterizes humanity's desire to look worthy of the title of gentleman. If r_2 is small, the individual who succeeded in accumulating the amount $U \approx r_{\min}/p$ seeks to immediately spend the money on acquiring the gentleman's set. If r_2 is large, the individual behaves in the opposite way: modestly and frugally even if the accumulations are $U > r_{\min}/p$. The parameter ε reflects the syndrome of 'ever-growing needs of an individual,' i.e., the inability to stop the spending spree for luxury items when the money is there to spend.

To summarize, the parameters of the demand function reflect the human factor, i.e., the consumer psychology.

The factor important for the model is the collective behavior of a large group of consumers, and, in this sense, the parameters of the demand function are of a social and psychological nature, i.e., they reflect the customs and rules of conduct that have evolved in the community.

In general, these parameters are different in different countries; demand functions may also differ significantly. The sigmoid form of the demand function plays a very important role for the model. Its effect depends on the parameters $r_{\min}$ and r_2: if $r_{\min}$ is small while r_2 is large, the demand function becomes smooth and everywhere convex. The sigmoid form (the beak in Fig. 2) disappears. The parameters of the demand function may change over time, albeit slowly, for example, when one generation is replaced by another (we discuss the role of these changes later).

3. Production function

The production function $F(r)$ is the amount of commodities produced (per unit time) depending on invested funds (current assets). These last variables are more conveniently expressed not in monetary units but as the ratio of the amount of money to the weighted average price of the product p: $r = U/p$. The role of inflation is excluded from consideration. Current assets include variable costs (proportional to the volume of production) and fixed costs (the costs of sustaining and modernizing production, i.e., of research and development). Three segments can be identified in the production function:

(1) segment of constant returns — investment is proportional to revenue;

(2) segment of decreasing returns — society displays no demand for the excess of produced commodities;

(3) segment of increasing returns — the demand for a commodity (typically an innovative one) keeps increasing and the enterprise receives superprofit.

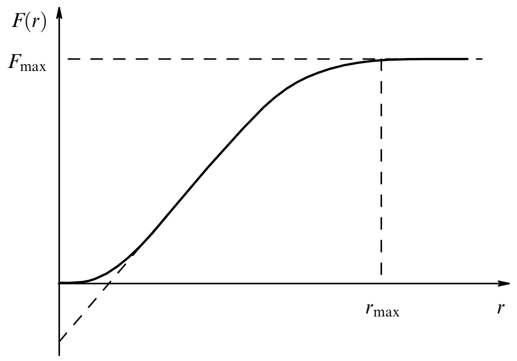

Figure 3. The production function.

The production function and the above segments are plotted in Fig. 3.

4. Basic models

In every science, there are two stages of model building [11]: first, the so-called basic (elementary) models are formulated. They operate with a small number of variables and equations and describe the essential features of the phenomenon (oscillations, crises, etc.). Next, basic models become more complex to reflect specific conditions, are modified or combined, and transform into imitation models. Physical economics has developed a set of basic models. We discuss them briefly.

(1) The logistic model, which contains a single equation of the form

$$\frac{dx}{dt} = a(x - x^2)\,. \tag{2}$$

This model is used to describe the development of a company, the evolution of a species (in biology), demographic processes, etc. It includes two stages: the first is exponential growth (or an even steeper growth with a sharper peak [12]) and the second is reaching the steady-state mode reflecting external constraints.

(2) The basic model of competition among arbitrary portions of information, which contains two (or more) equations of the form [13, 14]

$$\frac{du_i}{dt} = \frac{1}{\tau_i}\,u_i - \sum_{j\neq i}^{n} b_{i,j}\,u_j u_i - a_i u_i^2 + D_i\,\Delta u_i\,, \quad i, j = 1, 2, \ldots, n\,, \tag{3}$$

where u_i is the number of carriers of the ith portion of information; the first term in the right-hand side is the reproduction of the ith portion of information, the second is the interaction among carriers of different portions of information, the third represents external constraints, and the last term represents the migration of information carriers in space.

It is assumed that the interaction is antagonistic, i.e., all carriers attempt to keep their information, force it on the other carriers, and create new (proprietary) information.

Model (3) was used to describe the emergence of a universal genetic code in biology [13, 14]. In economics, it was used to describe the competition among firms (in particular, between innovators and conservatives) [15], the role of advertising [16], and the interaction among the major currencies on the external trade market [17]. In addition, the same model was used to describe processes in history (the formation of large states) [18]: it was demonstrated that the importance of ideological (informational) factors is on a par with economic factors.

(3) The covert bankruptcy model, which has the form [19, 20]

$$\frac{dM}{dt} = -\frac{M}{\tau} + p_m Q_0\,\frac{P}{P_0 + P} - \frac{p}{\tau_p}\,P - \kappa\,,$$

$$\frac{dP}{dt} = -pQ_0\,\frac{P}{P_0 + P} + \frac{M}{p\tau}\,,$$

where M are current assets and P is the quantity of goods in stock. The parameters are as follows: Q_0 is the maximum demand for the product, p_m is the market price, p is the production cost, τ is the length of the production cycle, τ_p is the duration of storage of goods, and κ is the fixed cost.

The model describes both the stable state of a company and the bifurcation, i.e., the transition to the state of bankruptcy. Simulated bankruptcy develops slowly at first, but then quickly reaches the critical phase. This model was used to discuss ecological and economic problems on a global scale. Especially important is the point at which resources are close to running out while the cost of their regeneration by waste reprocessing increases [20].

(4) The base model of the transition from the high production (HP) to the low production (LP) state; in fact, a model of the crisis, which is akin to the model of a phase transition. In econophysics, this model was made possible by dropping the dogma of the uniqueness of the state of the market. A macroeconomic imitation model of modern Russia has been created using this base model as a basis; we discuss it in more detail in Section 5.

5. Macroeconomic model of modern Russia

The purpose of the model is to describe what happened as a result of price liberalization and what the possible options are for further development. The model was built in the 1990s and was published in *Physics–Uspekhi* [7]. The crisis of the 1990s was then interpreted as a phase transition from the HP state to the LP state as a result of price liberalization.

The model has been improved and extended in recent years [10]. In this period, it has become clear that the results depend strongly on the external situation and the decisions of the government of Russia. At the present moment, the model is not intended to provide a long-term prognosis. We go even further: such a prognosis is simply impossible because the economy of Russia (and of the rest of the world, too) has moved close to an unstable state. As a consequence, the decisions taken by governments (of every country) produce important effects; however, neither those who take those decisions nor those who are trying to implement them understand what specific consequences to expect.

The proposed model was created to serve as a tool to support decision making. A model can do that if it is capable of answering questions like: what would happen if...? In addition, the model should be capable of short-term forecasting (like answering 'what would happen if we do nothing at all').

We do not reproduce the model here in full (it has been published); we only note some recent results.

(1) Visual representation of the state of the economy is shown in Fig. 4. Plotted along the ordinate axis are the demand function $Q(gr)$ and $F(r)\mu$, where g is the fraction of

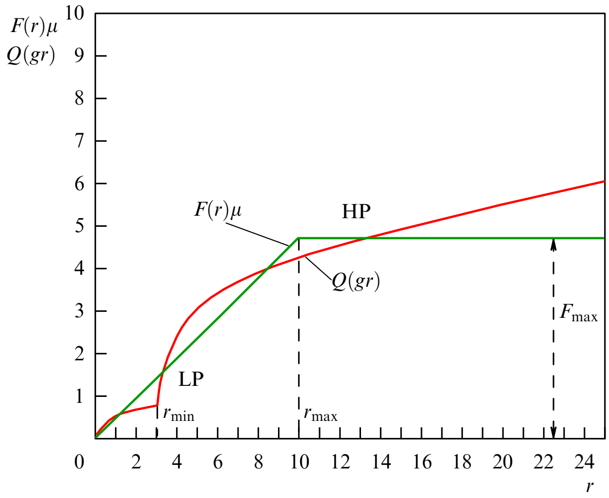

Figure 4. Balance diagram.

funds assigned by property owners to their personal needs, $F(r)$ is the production function, and μ is the level of costs [7]. Their intersections are stationary states. There are three of them: HP, LP, and the collapse of production. As these functions (their parameters) change, transitions (of the phase-transition type) become possible.

The following remarks are in order:

— the situation in Russia is truly variegated. Production functions in different regions and different companies are different, and therefore a range of functions can be shown in the diagram, covering companies both in their HP state (the so-called gazelles [21]) and approaching bankruptcy;

— generally, the demand function changes with the change of generation.

Figure 5a plots the demand function corresponding to the 'frugal' generation, which spends money to buy long-life commodities under the condition that they are available. The curve clearly shows that only the HP state is implemented in this situation. The demand function in Fig. 5b represents generations born into a prosperous life and wanting to have everything — all at once. Clearly, the economy then plunges into the LP state.

Long-term periodic changes (of the order of 10–20 years) are known in economics as the Kondratieff cycles. It is possible that one of the causes of these cycles is the process outlined above. In fact, Kondratieff himself connected cycles with the innovative activity of people (leaving aside the psychology and the demand function, although these factors also have a bearing on the issue.)

6. Crisis of 2008

The crisis engulfed the whole world, but its causes were different in the United States and in Russia.

In Russia, prices for raw materials and products of natural monopolies (NMs) started to grow in 2004. Prices of goods manufactured domestically grew more slowly (they followed inflation). Profitability fell and approached the critical value (the beak). The price equation showed decreasing stability and pointed to an Andronov–Hopf bifurcation (i.e., to an oscillation mode). If this tendency had continued, an industrial crisis in Russia would have been inevitable (this has been mentioned at conferences on numerous occasions [21] and also appeared in publications).

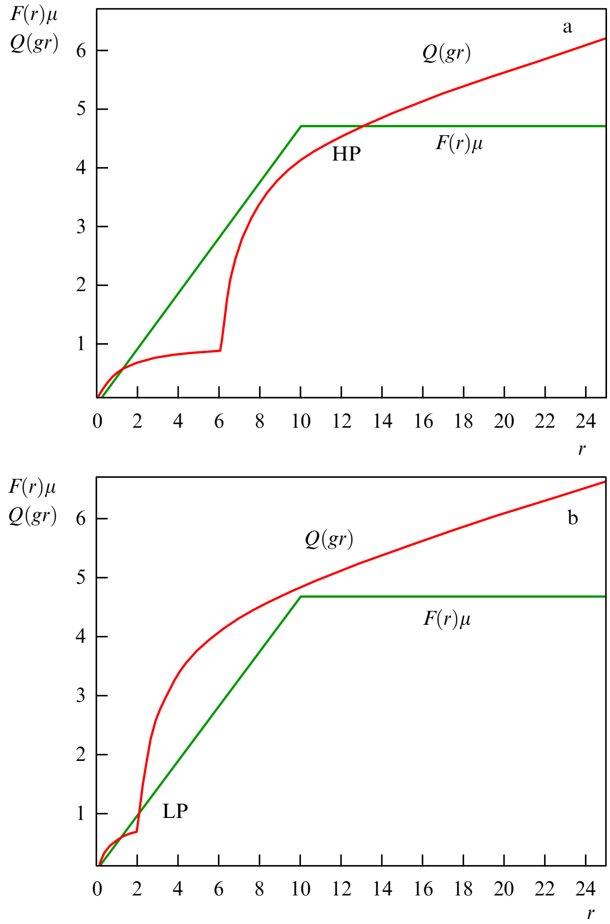

Figure 5. The 'phase transition' in response to a change in the demand function in the case of (a) a frugal generation and (b) a generation that grew up in prosperity.

In the U.S., the crisis was financial in nature (falling stocks and a crisis of commercial banks). The first and foremost to suffer from it were Russian financiers: U.S. banks demanded that they return loans and refused to offer new loans to repay debts. Russia's administration supported Russian financiers at the expense of funds withdrawn from the real sector. As a result, Russian industrialists were deprived of working capital for six months (the second half of 2008). Profitability being low, this led to the industrial crisis [22]. Figure 6 plots official data from 2003 to 2009 (quarterly, in rubles).

We see that the general trend of the gross domestic product (GDP) was positive prior to August 2008 (against the background of seasonal fluctuations). This does not contradict the assertion that the profitability of the manufacturing sector declined over these years; the aggregate GDP also includes services and the production of raw materials (where prices had risen), and this component is responsible for the overall positive trend (as we see from Fig. 6). The dashed curve represents the calculated data without the withdrawal of working capital in 2009. It is clear that in this case, there would have been no crisis in Russia (i.e., it would have happened a little later). As a result, the crisis did hit Russia 'unexpectedly' at the end of 2008, and the total GDP declined by nearly half. The GDP has increased subsequently, but has not reached the previous level.

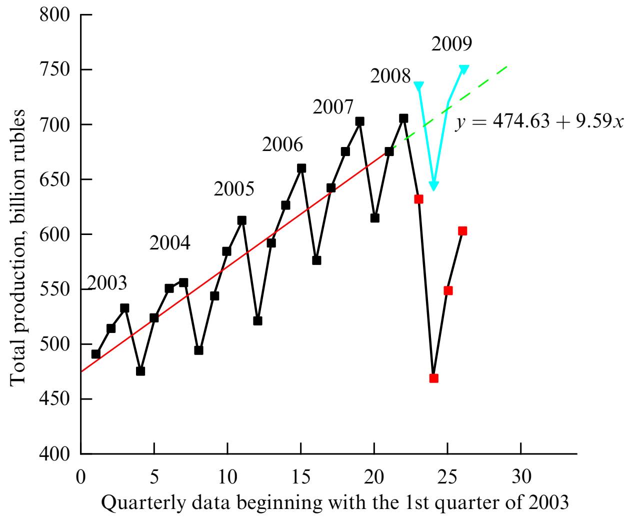

Figure 6. GDP dynamics in the interval 2003–2009: ■, ● — statistical data, ▼ — trend for 2008 (IV quarter) and 2009 (I–III quarters), straight line is the trend, with seasonal variations not taken into account.

7. Post-crisis measures

Anti-crisis measures have been proposed since the crisis (both by the government and by other organizations). We now discuss anti-crisis measures in the framework of our model.

(1) Partial state regulation of prices of raw materials, goods, and services of the NM. This factor enters our model as the parameter of profitability, and depends on inflation, global prices, and state-imposed stabilization of prices for a certain part of production. Figure 7 shows the results of GDP calculations for the manufacturing industries, for different levels of state regulation of prices. The parameter a is the fraction of the output of the NM whose prices are frozen at this (current) level and do not grow. The lower curve corresponds to no regulation, and the upper curve corresponds to the total freezing of prices. The starting point was August 2008.

The real GDP declined by about 30% and has stayed at this level ever since. Corresponding to this in the model is the freezing of prices at 75%. In reality, the falling of global prices also played its role and price freezing need not have been so harsh.

Figure 8 plots the results of model calculations and post-crisis statistical data. It is evident that the country has not yet emerged from the crisis (is still in depression) and, as we noted above, the process is oscillatory.

(2) The challenge of modifying the taxation system (lowering the value-added tax (VAT) and income tax). In our model, this is the parameter κ, determining profitability. Figure 9 plots model calculations of GDP growth resulting from tax cuts.

The figure shows that reducing the tax by 3–4% is already sufficient for leaving the crisis behind. A bifurcation is the moment when the taxation system changes.

(3) Implications of the increasing costs of the military-industrial sector (MIS). Most of the output of the MIS never reaches the internal market; hence, it generates no profit. Bearing these costs is nevertheless necessary due to non-economic reasons. The issue is: what damage will they do to the economy and what inflation will they cause? Figure 10 plots the calculated inflation as a function of the investment in the MIS, the increase in pensions, and the increase in government-paid salaries.

Figure 10 shows that inflation caused by MIS costs is considerably smaller than could be expected. The reason is that not all investments in the MIS take part in the monetary market (in the form of salaries). They are partly channeled into the industrial sector as payment for parts for weaponry. Additionally, the amount of MIS costs is a small fraction of the budget in comparison with the amounts

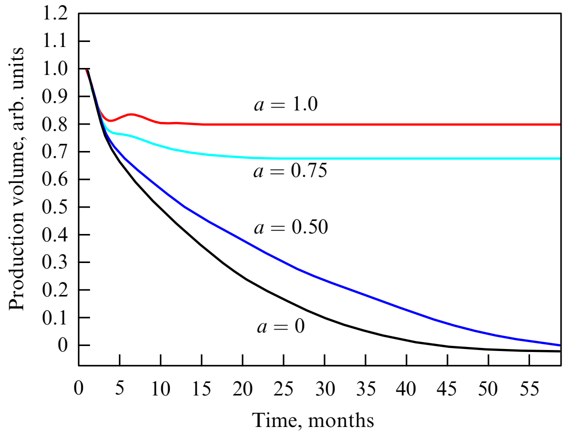

Figure 7. GDP dynamics for varying degrees of regulation of basic prices a.

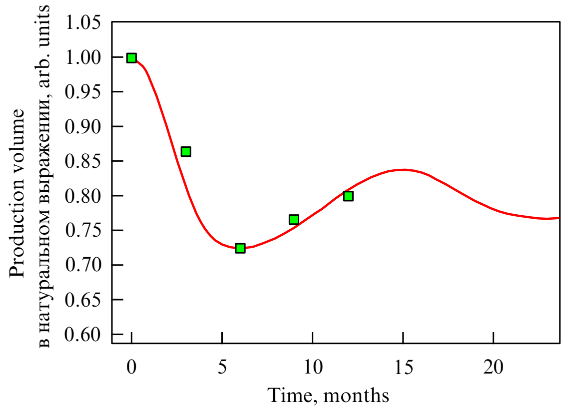

Figure 8. GDP dynamics (ignoring the trend): ■ — empirical data, the curve — model calculation taking price regulation into account for $a = 0.45$.

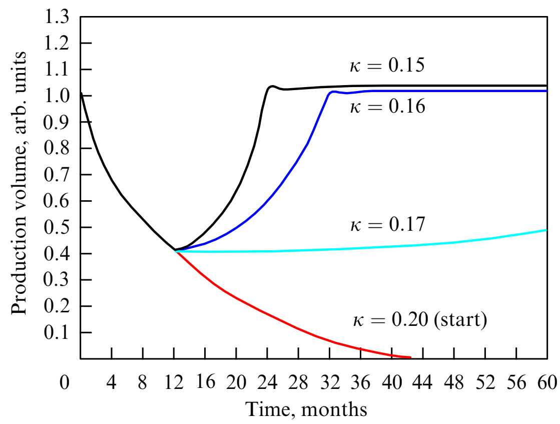

Figure 9. Response of GDP dynamics during crisis to change in taxation.

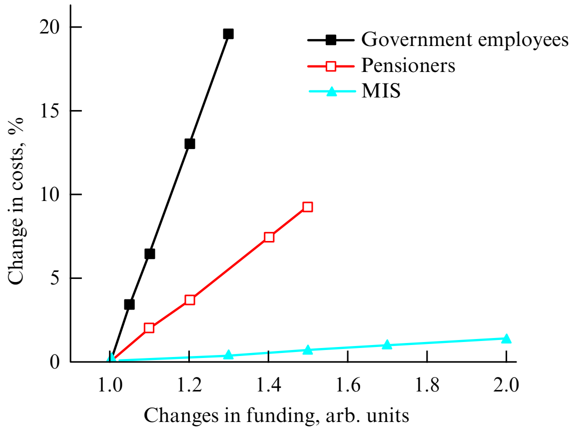

Figure 10. Changes in price inflation affected by additional funding of: ■ — government employees, □ — pensioners, ▲ — MIS industrial plants.

required to pay pensions and the salaries of government employees.

8. Conclusion

The study outlined in this paper belongs to a broad field of research known under several titles: 'self-organization' (I R Prigozhin), 'synergetics' (H Haken), and 'complexity.' The basic models mentioned above have in fact been investigated and developed in more detail than can be concluded from the present paper. Not all of them have been elaborated to the status of imitation models capable of addressing real current problems for specific conditions.

The need to reach this stage is obvious now, and research in this direction is being actively pursued. For example, papers [23, 24] present models of global dynamics. The model of 'strife of currencies' has been developed and refined for three or more participants. This is especially important for planning the introduction of a single trading currency for mutual settlements among the BRIC countries (Brazil, Russia, India, and China).

The basic model implies that the situation is far from simple and requires model calculations.

The basic model of covert failure is directly related to the global situation. The fact is that we are now supporting our lives not by producing commodities but by utilizing 'stored' materials. In other words, we are already in a state of covert bankruptcy, but have not noticed it yet. The basic model implies that the transition to an open bankruptcy may be 'unexpected' and very abrupt. It must be anticipated in time, which is why a mathematical model is needed.

The examples above do not exhaust all the problems of the economics of our time. But the basic models outlined above (and their combinations) provide a basis for modeling practically any of the pressing problems.

We have presented the model of modern macroeconomics of Russia, not as a basis model but already as an imitation one. Consequently, it claims to yield a description and a forecast of events to come. The reader will judge how successful the prognosis proves to be.

It is important that the model describes not so much a prediction as the system response to one specific external factor or another (events that have not yet occurred but may occur). In other words, the model can serve as a tool to support decision making by leaders guiding the economy of the country.

Moreover, the model is sufficiently complete, i.e., it provides information on the income and savings distribution among the population (both of these are very polarized in today's Russia), the demand for goods of different categories, etc. In other words, it allows piecing together an economic portrait of Russia as it is. It goes without saying that the model is upgraded each year (by taking the varying parameters into account). Special attention is paid to the question of how close (or how far) the country is from a bifurcation point.

This paper provides answers to only some of the possible "what if" questions. Nevertheless, econophysics has a mathematical basis for answering any such question.

We avoided touching on the issues concerned with the behavior of share prices in stock exchanges. These aspects are the subject of two other papers [25, 26] presented in this issue of *Physics–Uspekhi*.

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Nonclassical random walks and the phenomenology of fluctuations of securities returns in the stock market

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1. Introduction. Experimental facts observed in the fluctuations of securities returns

The logarithmic return of shares and stock indices, $S(\Delta t)$, measured over a time interval Δt is defined as

$$S(\Delta t) = \ln \frac{Y(t+\Delta t)}{Y(t)}\,, \tag{1}$$

where $Y(t)$ is the price of a share or the value of an index at time t. It was the subject of systematic study already at the time of L Bachelier [1]. Several facts have been established by experimental studies of share return in international financial markets.

First, for shares of the largest U.S. companies on the time interval from 1994 to 1995, the cumulative distribution function of probability of a fluctuation greater than x, and also smaller than $-x$, is well described by a power-law function of the form [2]

$$\Phi(x) \approx \begin{cases} x^{-3}\,, & S(\Delta t) > x\,, \\ -x^{-3}\,, & S(\Delta t) < -x\,. \end{cases} \tag{2}$$

Similar results were obtained for the shares of German [3], Norwegian [4], French, Japanese, Swiss, and British [5] companies, as well as stock indices [6].

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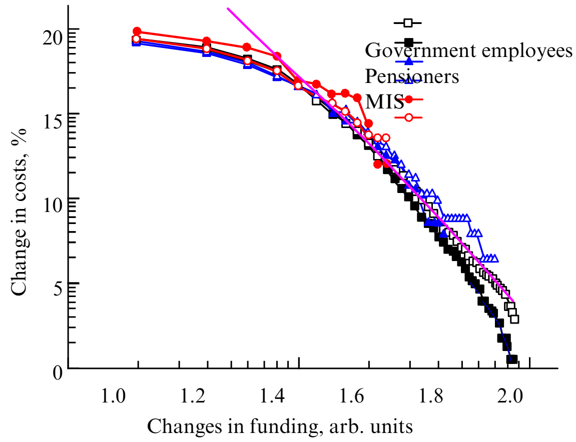

Figure 1. Cumulative distributions of normalized returns (see text) of Sberbank ordinary shares for various Δt. One-minute positive fluctuations — light squares, one-minute negative fluctuations — shaded squares; one-hour positive fluctuations — light triangles, one-hour negative fluctuations — shaded triangles; daily positive fluctuations — light circles, daily negative fluctuations — shaded circles. Bold straight line shows the dependence x^{-3}. One-minute, one-hour and daily data were obtained on trading days 10.01.2009–10.02.2009, 01.09.2008–30.09.2009, 23.01.2006–30.09.2009 (MMVB stock exchange).

Russian stocks ('blue chips') exhibit similar behavior (2). Figure 1 plots the cumulative distribution of returns for positive (black symbols) and negative (white symbols) fluctuations in Sberbank shares. The straight line in Fig. 1 plots the law x^{-3}. The ordinate is the cumulative distribution function, while the abscissa is the return normalized to the appropriate experimentally calculated root-mean-square return. We obtained similar curves for shares of other Russian companies, too. Figure 2 plots the distribution function of fluctuations of the Russian RTS stock index. It is clearly seen that all the plots of cumulative distributions resemble one another. At the same time, return curves for larger Δt lie somewhat higher than return curves for lower Δt (see also paper [5]).

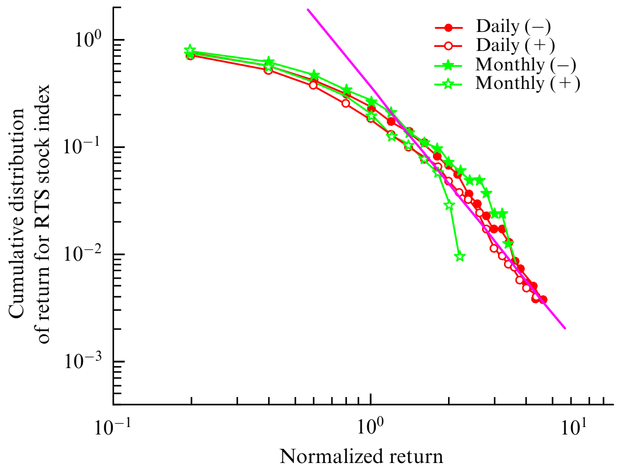

Figure 2. Cumulative distributions of normalized returns (see text) for the RTS stock index at various Δt: daily positive fluctuations — light circles, daily negative fluctuations — shaded circles; monthly positive fluctuations — light stars, monthly negative fluctuations — shaded stars. Bold straight line shows the dependence x^{-3}. Daily data were obtained on trading days 09.01.1995–27.06.2007 (RTS stock exchange), monthly data — on trading days 09.01.1995–20.10.2010.