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Equilibrium models of economics in the period of a global financial crisis

I G Pospelov

1. Economy as an example of a complex creative system

The economy is a subsystem of society that controls the production, distribution, and consumption of resources, goods, and services. The task of modern economics is extremely difficult. We speak here about producing several billion types of goods and distributing them among several billion individuals and entities. This is the reason why economics as a management system is always fairly decentralized. We recall that the USSR Planning Committee operated with approximately 2000 types of goods, while the actual list of types of goods exceeded a hundred million. We believe that the glaring mismatch of management and the growing complexity of economic links constituted the main reason for the demise of the idea of centralized planning.

To build a model of the economy, we need to face the *complexity of the system*. Complex systems are special, not just in consisting of a large number of elements but, above all, in their *uniqueness* and, perhaps most importantly, their *ability to undergo qualitative changes*. Consequently, when we study complex systems, we are actually following a single trajectory, which does not display statistical reproducibility and does not reveal the full potential of the system. The study of complex systems takes us beyond the empirical method, which underlies the triumph of science over the past 300 years. Models of physical systems are expected to explain the results of experiments and to predict the results of the planned ones; in contrast, models of complex systems are essentially built to replace the experiment. As a result, for a complex system, we obtain many models that cannot be derived as special cases of a universal 'supermodel.' Partial models describe different perspectives of the system. They operate with different sets of concepts and neglect by no means small deviations from the dependences that they take into account [1].

However, experience shows that models can go a long way in describing a complex system. A good model not only describes the behavior of the system under a current structure of relations but also contains a description of its own applicability limits and the limits of stability of the described structure. However, we have to be reconciled with the following facts:

• different models of a complex system cannot add up to a full adequate model, as happens with computer simulators of engineering systems;

• it is unlikely that we will someday be able to predict which structure arises in a complex system after the loss of stability by the previous structure. The model can reveal the real threat to the existing order of things, but it cannot predict how the crisis will be resolved.

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The oil crisis of 1975 is an example of an abrupt break in trends. Both history and physics claimed that production cannot grow faster than the consumption of energy. In fact, however, the living standards in the West grew by a factor of 1.5–2, while energy consumption per capita did not increase at all!

Another example is the current global economic crisis: everyone expected the crisis of possibilities (depletion of resources), but the crisis that actually occurred was that of needs (depletion of growth stimuli). Physically, there is no constraint on the economy of the 'golden billion,' but it nonetheless refuses to grow! In addition, the virtual economy proved more stable than the real one — foodstuffs, fuel, metals, and gold proved to be in excess and lost in price much more than did services and information. Even in the financial field, the item that for centuries was considered the most reliable investment — the mortgage business, i.e., credits to a real person pledging real property — has collapsed; at the same time, such credits as needed for launching web sites with advertising continue to be financed. The farther production is located in the technological chain from base industries, the less if suffers from a crisis.

This happens for the same reason that the doubling and tripling of energy and metal prices hardly affect car prices. The cost of design, quality control, and promotion make up a considerably greater part of the net cost of a modern automobile than the cost of materials. The new product for which people are prepared to pay (i.e., the value added) is currently created not so much on the field or in a factory as in the design office, in the department of technical control, and in the shops [2]. It is difficult to say now where all this may end up, especially if we consider S P Kapitsa's important observation that the current crisis coincides with a phenomenon unprecedented in human history: population growth is slowing down without famines and epidemics! We do not rule out the possibility that humanity is on the path toward zero growth and a purely 'spiritual' life. Not to the life, of course, that ecologists and moralists painted for us. What we are witnessing is the stabilization of aggregate material consumption plus the loss of interest that humankind has in the world outside, focusing its attention on inexhaustible variations of problems of interpersonal relations.[1]

This, however, is a remote future, at least because a well-settled system of economic mechanisms is operational only if the prospects of economic growth materialize. Therefore, either growth will be restored in a foreseeable future or we are in for a series of unsuccessful attempts to restore it. As long as new mechanisms adapted to zero growth have not been shaped and started to work, only processes of decline and restoration of growth can be modeled in seriousness.

2. System analysis of a developing economy

Numerous models of an economy and a plethora of methodological approaches to its modeling are available. The most popular models since the 1990s are the *Computable General Equilibrium models*, CGE (see, e.g., [4]), because it has been understood that taking only technological limitations (*equilibrium models* [5]), extrapolation of past trends (*econo-*

[1] It was already mentioned in [3] that the transition of civilization to the introvert phase could be an explanation for the phenomenon of 'silent cosmos,' which appears gradually stranger in view of the recent discoveries that seem to demonstrate a wide distribution of conditions suitable for life in outer space.

metric models [6]), and direct superposition of external constraints (*models of global dynamics* [7]) into account is insufficient for adequately describing the present-day economy. We need to remark here that in modern science, the term 'equilibrium' is given three originally different interpretations:

• *dynamic equilibrium — the balance of forces* acting on a system;

• *statistical equilibrium — the balance of probabilities of transitions* between states of a system;

• *economic equilibrium* — the balance of interests of the parties to a conflict.

There is something that these concepts share, but ignoring the differences among them would be an inadmissible vulgarization. We see below that economic equilibrium implies neither static behavior nor simple dynamics.

A new avenue of research emerged in 1975 at the Computing Center of the USSR Academy of Sciences (later, Russian Academy of Sciences): the *system analysis of an evolving economy* (SAEE), in which the methodology of mathematical modeling of complex systems, developed in the natural sciences, was synthesized with the achievements of modern economic theory [8]. The ideas of SAEE models are close to those of CGE models, but they pay more attention to the specifics of current economic relations; furthermore, we began our research some 15 years before the appearance of the first CGE models.

The study began with a model of a market economy, and in 1988, we constructed a model that reproduced the main qualitative features of the evolution of a planned economy. Therefore, an approach to analyzing the changes occurring in the economy had already been developed by the time the economy started to change in the USSR, and later in Russia. The short-term consequences were correctly predicted two years before the reform of 1992. Each of the subsequent models — the model of the high-inflation period 1992–1995, the model of the 'financial stabilization' period 1995–1998, which predicted the 1998 crisis, and the model of assessment of the prospects for economic development after the 1998 crisis — was based on a set of hypotheses concerning the nature of economic relations that dominated the corresponding period in Russia. It can be said that we have produced a real 'chronicle' of economic reforms in Russia that was written in the language of mathematical models. These models are described in detail in [9], and a detailed overview is given in [10].

These models allow understanding the internal logic of the evolution of economic processes. The experience accumulated in applying the models showed that they provide a reliable tool for analyzing macroeconomic regularities, as well as for predicting the effects of macroeconomic decisions *under the assumption of preservation of entrenched economic links*.

3. Model of intertemporal equilibrium of Russia's economy

In 2004, we pushed the CGE methodology aside and turned to the theoretically more consistent but conceptually and technically much more complicated construction of the *general intertemporal equilibrium with control of capital* (GEC). The GEC model is constructed not by the sequential addition of descriptions of individual economic processes but by specialization of the general scheme of operation of an ideal market economy. This scheme seems to be the most important discovery of mathematical economics over the entire 150 years of its existence. If we formally consider the task of economic planning in the interests of consumers subject to technological constraints of the individual production units and constraints on material balances over the entire economy, then the problem of *central planning* using the saddle-point theorem can be written as the *equilibrium problem in a game of independent macroagents each of which pursues its own interests* (equilibrium of perfect competition). The top five agents from the list below emerge in the process, as do *money and prices* [10]. Each of the GEC macroagent models solves the problem of optimally controlling the material and financial flows that the microagent manages.[2] We assumed that the macroagent knows the *correct prognosis of economic trends*, and these prognoses are determined by the conditions of approval of plans of agents within the complete system of material and financial balances. This constitutes the *Principle of Rational Expectations* — a strange principle that nevertheless works well in practical situations (Rational Expectations [11]).[3] Possible reasons for the applicability of this principle are discussed in Section 4.

An applied model of the GEC is obtained from this ideal construction by aggregating wealth, differentiating the money, introducing agents 'external' to the economy (see the last four items in the list given below), and, most of all, subjecting the actions of the agents to additional restrictions that reflect the 'observed rules of the game' in the economy.

The current version of the model of the Russian economy describes the real sector, which produces domestic and export products and consumes imported and domestic products, and the financial sector. The financial flows that accompany production, distribution, and consumption of products are described as the turnover of six financial instruments: *cash, payment accounts, correspondent accounts at the Central Bank* (*CB*), *bank loans, bank deposits, and bank deposits/loans in securities and foreign currency*. Products, labor, and financial instruments form a set of additive variables for which a complete system of balances is written in the model with the flows of financial instruments divided into legal and shadow ones. The movements of additive quantities are described as produced by nine macroagents:

(1) the *producer*, which represents nonfinancial commercial organizations;

(2) the *bank*, which represents financial commercial organizations;

(3) the *households*, which represent individuals — consumers and employed personnel;

(4) the *owner*, representing the physical and legal persons who control the movement of capital among domestic sectors of the economy and its flows outside the country;

(5) the *seller*, acting as a pure intermediary between consumers, producer, exporter, and importer;

[2] Both the experience of economic theory and a large amount of empirical data suggest that the rationality of behavior is typical of large groups of subjects that play a similar role in the economy, while the behavior of individuals, including the state, is chaotic and difficult to predict.

[3] Any alternative to this principle means that simulation must be executed separately for the economy evolving the way it does in reality, and separately for the picture of this evolution in the minds of subjects. This attitude appears to be not only hardly realizable, but equally too self-assured. The Principle of Rational Expectations assigns equal rights to the researcher and to the subjects under study: the model agents use the same model for their forecasts that we are constructing!

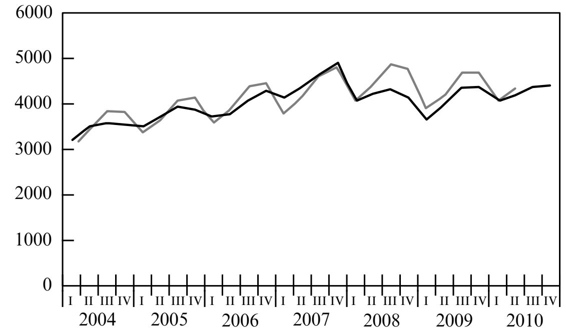

Figure 1. Real GDP, billion rubles in 2004 prices, per quarter.

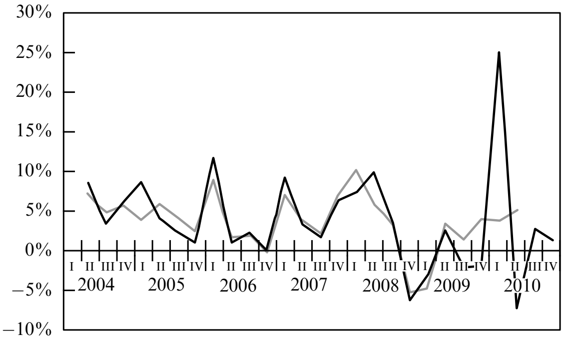

Figure 2. Quarterly rate of inflation in GDP, per quarter.

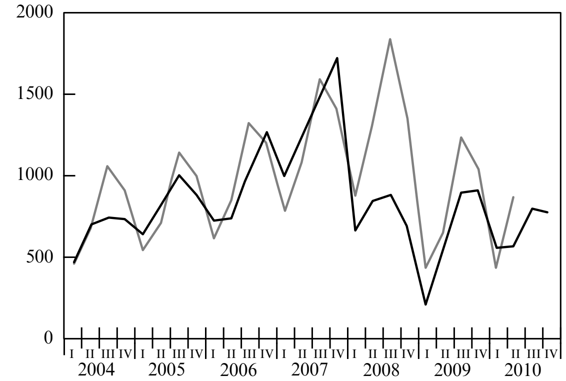

Figure 3. Real investment, billion rubles in 2004 prices.

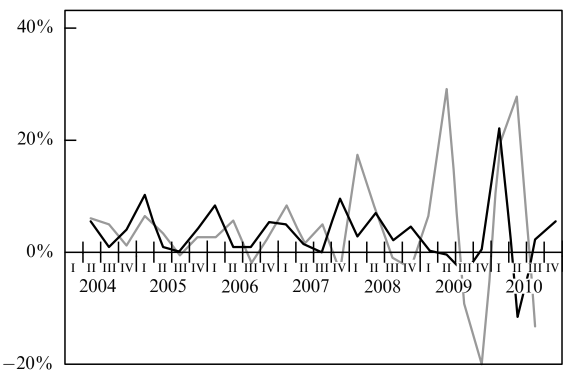

Figure 4. Quarterly inflation rate in capital investments.

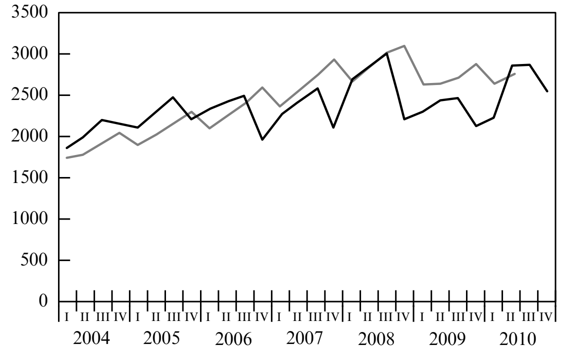

Figure 5. Quarterly real consumption, billion rubles in 2004 prices.

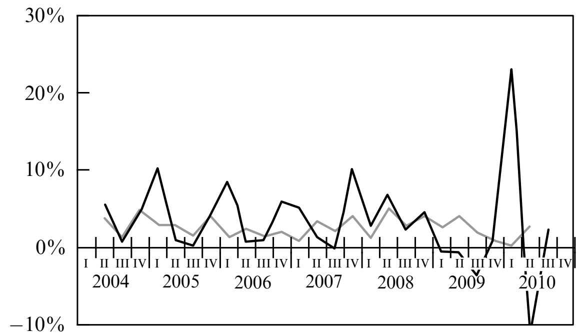

Figure 6. Quarterly rate of inflation in consumption.

(6) the *government*, whose work is represented in the model explicitly by an aggregated description of the Ministry of Finance and implicitly by establishing various parameters of economic policy (tax rates, government spending, regulations for reserves, etc.);

(7) the *Central Bank*, represented in the model by its functions of the issuer of national currency, the holder of currency reserves, the settlement center, and the lender to commercial banks;

(8) *exporter*;

(9) *importer*.

The initial structured representation of the model consists of 162 dynamic and finite nonlinear equations for which a *boundary value problem* is posed. *Indices of export and import prices, numbers of employees*, as well as the national economic policy (described using *government consumption, subsidies to the households, exchange rate, and the Central Bank discount rate* as variables) are treated as external variables. The set of equations of the model contains 50 constant parameters, of which 30 are identified regardless of the model (tax rates, the parameters of production functions, etc.), and the other 20 are used as fitting parameters. The model is identified using the official *unsmoothed quarterly statistics*.

Some results of calculations are shown in Figs 1–12. The lighter curves plot series of statistical data and the darker ones are calculated curves from the first quarter of 2004 to the fourth quarter of 2010. We note that this entire set of trajectories represents *a single economic equilibrium*.

It is obvious that the model reproduces the statistics satisfactorily, including the phases of fluctuations, the decline as a result of the crisis, and the difference in inflation rates for different products. The model *describes the phenomenon that distinguishes the Russian crisis from the crisis in all other countries*: everywhere, a decline in production is accompanied by deflation, whereas in our case, by inflation!

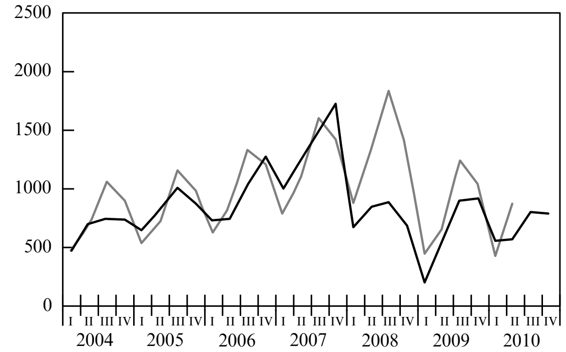

Figure 7. Quarterly real exports, billion rubles in 2004 prices.

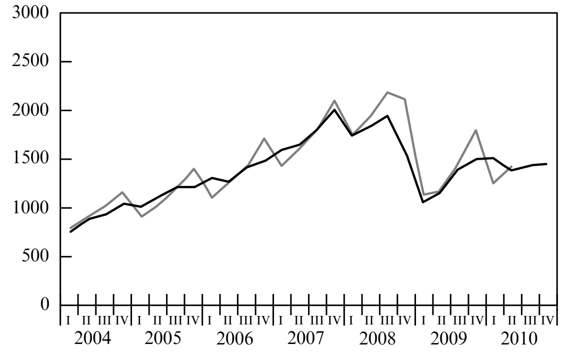

Figure 8. Quarterly real imports, billion rubles in 2004 prices.

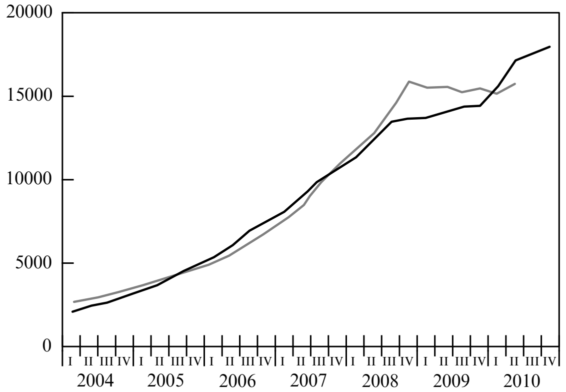

Figure 9. Credits to legal entities, billion rubles.

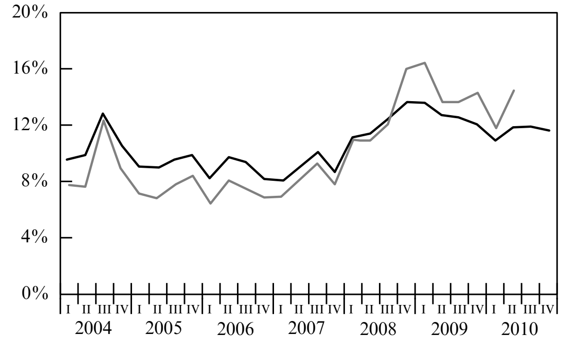

Figure 10. Annual interest on loans.

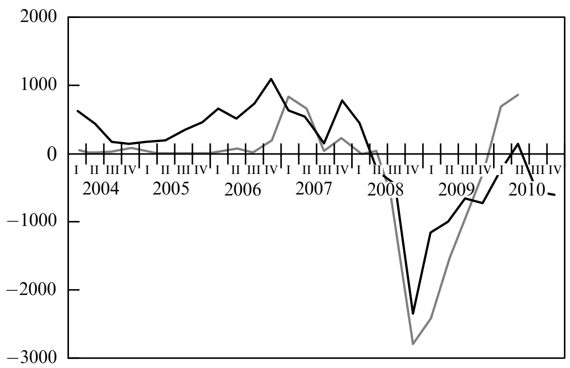

Figure 11. Net deposits of banks at the Central Bank, billion rubles.

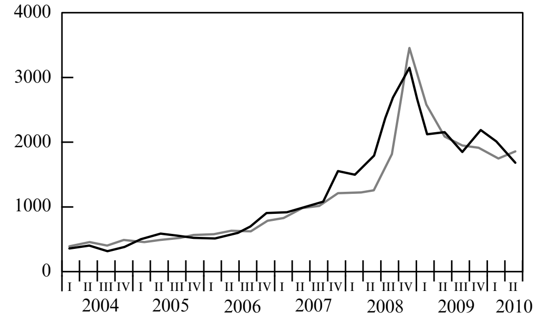

Figure 12. Liquid assets of banks, billion rubles.

4. Economics and physics: why similar approaches yield different results

In modeling the economy, we can successfully use the approaches established in theoretical physics long ago: variational principles, principles of symmetry (both exact and broken), separation of variables into intensive and extensive, etc. Nevertheless, our experience shows that the similarity in approaches does not imply a qualitative similarity in the behavior of models of physical and economic systems.

In *physics*, the extensive (additive) quantities are the *masses* of different forms of matter, different types of *charges*, all forms of *energy, entropy, momentum, angular momentum*, and so on. Their changes are described by transfer equations or reaction–diffusion equations.

Extensive quantities in *economics* are the *accumulated* wealth and financial instruments. Their movements are described by equilibrium equations, not in space but in a set of economic agents.

The most important for *physics* are conserved extensive quantities. In modern formal financial systems, a conserved quantity is the algebraic sum of reserves of any financial instrument. The aggregate turnover of a financial instrument grows only at the expense of the so-called credit emission — simultaneous growth in assets (positive stocks) and liabilities (negative stocks). Consequently, formal laws of conservation of financial instruments are less useful than conservation laws in physics. For example, it would be naive to expect that with the collapse of one market the money would flow out and transfer to another. In a crash, the liabilities and assets cancel each other, and turnovers slide downwards on all markets.

The symmetries typical of natural systems are translational and rotational, while the economy reveals scale symmetries. The best proof of this is the fact that we usually characterize changes in the physical world by their speed and changes in economic indicators by their rates (logarithmic time derivatives). This means that in the former case, the absolute scales of quantities are essential, while in the latter case they are not. As a result, in physics, the 'favorite' (i.e., the most illustrative) solutions are uniform motions at constant speed, while in economics, these are self-similar solutions in which extensive quantities increase exponentially, i.e., at a constant rate. The vast majority of the conclusions of economic theory have been obtained by comparative analysis of self-similar solutions of simple models.[4]

The variational principle in physics 'controls' *the system as a whole*, while in economic models, *each agent has its own variational principle*. Even more important is the significant difference among the topological structures of these principles. Application of the variational principle always leads to a Hamiltonian system of equations of motion, and this motion is on the surface of a constant Hamiltonian function.

In physics, the Hamiltonian function is, roughly speaking, downward convex. Therefore, its stable *critical points* are essentially *centers* around energy minima, and typical motions reduce to rotations, vibrations, and windings on tori. In general, these movements demonstrate neutral stability, i.e., they shift on the whole by a distance of the order of the increment in the initial conditions.

With the characteristic orientation of the *economy to maximization of capitalization*, utility, profit, etc., the Hamiltonian function is convex downward in 'momenta' and convex upward in 'coordinates,' and all its *critical points* are *saddle-shaped*. As a result, any economically meaningful movements of the system are close to stable separatrices of saddles. These solutions are unstable with respect to the initial values of the momenta (which in addition are unobservable) but depend weakly on the initial conditions for coordinates and on perturbations in the distant future. For a Hamiltonian system, we have to solve not a Cauchy problem but a *boundary value problem*. The ensuing results are known as *turnpike theorems*. They give us hope that models like intertemporal equilibrium models will be true in the mid-term, regardless of the accuracy of predictions for the distant future.

Our main result obtained in recent years was the discovery of a strong turnpike property: *even though we allow agents in the model to know the future, this knowledge proves useless for them as regards choosing the optimal behavior*. Because this property holds, it removes all objections to the application of the principle of rational expectations. In other words, the model reduces to a conventional dynamic system. However, the property itself requires an explanation.

The key here is that a strong turnpike effect is observed not in the model in general, at the level of formulas, but only if the parameters are correctly identified [12]. We need to recall here that economics as a management system is meant not only to coordinate the actions of billions of people but also to do it in a way that allows people to make a reasonable choice in most cases, without complex calculations. Therefore, even widely familiar economic mechanisms may not work, owing to complexity and risk. We can assume that at any given time, the economic system selects and engages a set of mechanisms that do not require detailed calculations for reasonable solutions. Consequently, by describing the mechanisms in the model 'true to life,' not true to textbooks, we arrive at a model with a strong turnpike property.

All this somewhat resembles the anthropic principle well familiar to physicists: the Universe appears to the observer as harmonious and 'adapted' to him or her because no observer could appear in a differently arranged universe.

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[4] Incidentally, in a historical perspective, the 'economic exponent' of the industrial society continues to break through seemingly fairly objective external constraints, while forecasts found by using models that attempt to include specific limits to growth — from the predictions of T Malthus to those of D Meadows — proved unsuccessful.

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On econophysics and its place in modern theoretical economics

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1. Introduction

Theoretical economics has the same goals as other theoretical fields:

(1) Description of an object (system) in the language of mathematical methods.

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