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In memory of V L Ginzburg
(Scientific session of the Physical Sciences Division of the Russian Academy of Sciences, 27 October 2010)

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The scientific session of the Physical Sciences Division (PSD) of the Russian Academy of Sciences (RAS), dedicated to the memory of V L Ginzburg, took place in the conference hall of the Lebedev Physical Institute, RAS on 27 October 2010.

The agenda of the session announced on the website www.gpad.ac.ru of the RAS Physical Sciences Division listed the following reports:

(1) **Mesyats G A** (Lebedev Physical Institute, RAS, Moscow) "Introductory word";

(2) **Rubakov V A** (Institute for Nuclear Research, RAS, Moscow) "Cosmology and the Large Hadron Collider";

(3) **Gurevich A V** (Lebedev Physical Institute, RAS, Moscow), **Zelenyi L M** (Space Research Institute, RAS, Moscow) "Intense gamma bursts in Earth's atmosphere (TGE) and the mission 'Chibis'";

(4) **Vasiliev M A** (Lebedev Physical Institute, RAS, Moscow) "Higher-spin theory";

(5) **Maksimov E G** (Lebedev Physical Institute, RAS, Moscow) "What is and what is not known about HTSC";

(6) **Pudalov V M** (Lebedev Physical Institute, RAS, Moscow, and Moscow Institute of Physics and Technology) "V L Ginzburg and the development of experimental work on high-temperature superconductivity at LPI: 'iron superconductors.'"

Papers based on talks 2, 4, and 6 are published below.

For several reasons, L P Pitaevskii was unable to attend the session. He presented a paper dedicated to the memory of V L Ginzburg, which is published in this issue of *Physics–Uspekhi* (p. 625).

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Cosmology and the Large Hadron Collider

V A Rubakov

1. Introduction

It is conceivable that the Large Hadron Collider, LHC, will give solutions to, or at least shed light on, major cosmological problems, such as the nature and origin of dark matter and generation of matter–antimatter asymmetry. We give several examples showing the LHC potential to uncover the history of the early Universe: WIMPs as cold dark matter, gravitinos as warm dark matter, and electroweak baryogenesis as a mechanism for generating matter–antimatter asymmetry.

The startup of the LHC is a major event, not only in particle physics but also in cosmology. The Universe we know is full of mysteries. It hosts matter but not antimatter, and 40 years since it was understood that this presents a problem, we do not have an established theory explaining this asymmetry. The Universe hosts dark matter, and we do not know what it is made of. There is dark energy in the Universe whose nature is also obscure. The LHC may well shed light at least on some of these mysteries, which are important items in V L Ginzburg's famous list. Optimistically, the LHC experiments may discover dark matter particles and their companions, and establish the physics underlying the matter–antimatter asymmetry. Otherwise, they will rule out some very plausible scenarios; this will also have profound consequences for our understanding of the early Universe. There are also exotic hypotheses on the physics beyond the Standard Model, like the TeV-scale gravity; their support by the LHC will have a dramatic effect for cosmology and is hard to overestimate.

Here, we concentrate on a few examples highlighting the LHC cosmological potential. We first turn to dark matter, and present the WIMP scenario for cold dark matter, which is currently the most popular, and for good reason. We also consider a light gravitino scenario for warm dark matter. Both are to be probed by the LHC, as they require a rather particular new physics in the LHC energy range. We then discuss electroweak baryogenesis — a mechanism for the generation of matter–antimatter asymmetry that may have operated at a temperature of the order of 100 GeV in the early Universe. This mechanism also needs new physics at energies 100 – 300 GeV, and it will be definitely confirmed or ruled out by the LHC.

In our presentation, we necessarily omit numerous details, while trying to make the basic ideas and results clear. More complete accounts of the particle physics aspects of cosmology may be found in book [1] and reviews [2–7]. Dark matter, including various hypotheses about its particles, is reviewed in [8–11]. Electroweak baryogenesis is discussed in detail in reviews [12–15].

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Before coming to our main topics, we overview the present energy balance in the Universe. It is characterized by the parameters $\Omega_i = \rho_{i,0}/\rho_c$, where $\rho_{i,0}$ is the present energy density of the type of matter i, and

$$\rho_c = 5 \times 10^{-6}\ \text{GeV cm}^{-3}$$

is the total energy density at the present epoch. This value is obtained from the measurement of the Hubble constant, assuming the validity of General Relativity and spatial flatness of the Universe (the precision at which the spatial flatness is established experimentally is less than 2 per cent deviation of the total energy density from ρ_c [16]). Clearly, in a spatially flat universe, $\sum_i \Omega_i = 1$, where the sum ranges over all forms of energy. The known forms of matter in the Universe are mostly photons of the Cosmic Microwave Background (CMB), whose temperature is $T_0 = 2.726$ K, plus baryons and neutrinos. The present number density of CMB photons is $n_\gamma = 410\ \text{cm}^{-3}$, and their energy density is $\rho_{\gamma,0} = 2.7 \times 10^{-10}\ \text{GeV cm}^{-3}$. We see that the energy density of photons is very small today compared to the total energy density. We note in passing that an important characteristic of the early Universe is the entropy density. It is of the order of the number density of photons. More precisely, in thermal equilibrium at temperature T, it is given by

$$s = \frac{2\pi^2}{45}\, g_* T^3 , \tag{1}$$

where g_* is the number of the degrees of freedom with $m \lesssim T$, that is, the degrees of freedom that are relativistic at the temperature T (fermions contribute with a factor of 7/8). The entropy stays *exactly* constant in a comoving volume, unless there are fairly exotic processes of entropy production. The present value of the entropy density (taking neutrinos into account as if they were massless) is

$$s_0 \approx 3000\ \text{cm}^{-3} . \tag{2}$$

There are two ways of measuring the mass density of baryons. One is related to the Big Bang Nucleosynthesis, the epoch of thermonuclear reactions ($T \sim 10^9$ K). The resulting light element abundances depend on the baryon-to-photon ratio in that epoch, which has stayed constant since then. Comparing theory with observations of light element abundances yields

$$\eta \equiv \frac{n_B}{n_\gamma} \approx 6 \times 10^{-10} . \tag{3}$$

The energy density of baryons in the present Universe is therefore equal to

$$\rho_{B,0} = m_B\, n_{B,0} \approx 2.5 \times 10^{-7}\ \text{GeV cm}^{-3} , \tag{4}$$

or, in terms of the proportion of the total energy density,

$$\Omega_B = 0.045 .$$

The same value is independently obtained by the analysis of the CMB temperature anisotropy. By electric neutrality, the number density of electrons is nearly the same as that of baryons, and they therefore contribute a negligible portion of the total energy.

The remaining known stable particles are neutrinos. Their number density is calculable in the Hot Big Bang theory, and these calculations are nicely confirmed by Big Bang Nucleosynthesis. The present number density of each neutrino type is $n_{\nu_\alpha} = 115\ \text{cm}^{-3}$, where $\nu_\alpha = \nu_e, \nu_\mu, \nu_\tau$. The direct bound on the mass of the electron neutrino, $m_{\nu_e} < 2.6$ eV, along with the observations of neutrino oscillations, suggest that every type of neutrino has a mass smaller than 2.6 eV (neutrinos with masses above 0.05 eV must be degenerate, according to the neutrino oscillation data). The present energy density of all types of neutrinos is therefore smaller than ρ_c:

$$\rho_\nu^{\text{total}} = \sum_\alpha m_{\nu_\alpha} n_{\nu_\alpha} \lesssim 8 \times 10^{-7}\ \text{GeV cm}^{-3} ,$$

which means that $\Omega_\nu^{\text{total}} < 0.16$. This estimate does not use any cosmological data. In fact, cosmological observations give a stronger bound:

$$\Omega_\nu^{\text{total}} \lesssim 0.014 . \tag{5}$$

In terms of the neutrino masses, bound (5) is given by $\sum m_{\nu_\alpha} \lesssim 0.6$ eV [17–19], and hence every neutrino must be lighter than 0.2 eV. On the other hand, atmospheric neutrino data, as well as the K2K and MINOS experiments, show that the mass of at least one neutrino must be larger than 0.05 eV. Comparing these numbers, we see that it may be feasible to measure neutrino masses by cosmological observations (!) in the future.

We conclude that most of the energy density in the present Universe is not in the form of known particles; most energy in the present Universe must be in 'something unknown.' Furthermore, there is strong evidence that this something has two components: clustered (dark matter) and unclustered (dark energy).

Clustered dark matter presumably consists of new stable massive particles. These form clumps of energy density that constitutes most of the mass of galaxies and clusters of galaxies. There are a number of ways to estimate the contribution of nonbaryonic dark matter to the total energy density of the Universe (see Refs [8–11, 20] for details):

— The composition of the Universe affects the angular anisotropy of the cosmic microwave background. Quite accurate measurements of the CMB anisotropy, available today, allow estimating the total mass density of dark matter.

— Nonbaryonic dark matter is crucial for the structure formation of the Universe (see below). Comparison of the results of numerical simulations of structure formation with observational data gives a reliable estimate of the mass density of nonbaryonic clustered dark matter.

The bottom line is that the nonrelativistic component constitutes about 28 per cent of the total present energy density, which means that nonbaryonic dark matter has

$$\Omega_{\text{DM}} \approx 0.23 ; \tag{6}$$

the rest is due to baryons.

There is direct evidence that dark matter exists in the largest gravitationally bound objects, clusters of galaxies. It comes from the determination of gravitational potentials in clusters via measuring the velocities of galaxies, the X-ray temperature of the intracluster gas, gravitational lensing effects, and so on. These methods enable directly determining the mass-to-light ratio in clusters of galaxies. Assuming that this ratio applies to all matter in the Universe,[1] we arrive at the estimate for the mass density of clumped matter.

[1] This is a strong assumption, because only about 10 percent of galaxies are in clusters.

Remarkably, this estimate coincides with (6). Finally, dark matter also exists in galaxies. Its distribution is measured by observations of the rotation velocities of distant stars and gas clouds around a galaxy.

Nonbaryonic clustered dark matter is not the whole story. The above estimates yield an estimate for the energy density of all particles, $\Omega_\gamma + \Omega_B + \Omega_\nu^{\rm total} + \Omega_{\rm DM} \approx 0.3$. This implies that 70 percent of the energy density is unclustered. This component is called dark energy; it is responsible for the present accelerated expansion of the Universe. One candidate is the vacuum energy density, or the cosmological constant (see, e.g., Refs [20–26] for the reviews).

All this fits all cosmological observations nicely, but does not fit the Standard Model of particle physics. It is our hope that the LHC will shed light at least on some of the properties of the Universe.

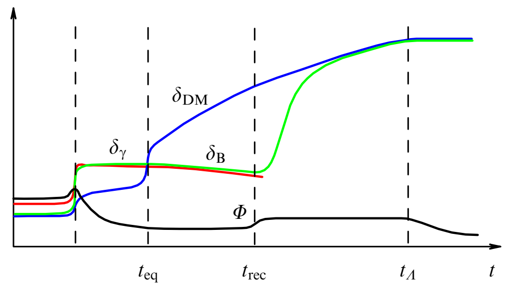

Figure 1. Time dependence, in the linearized theory, of density contrasts of dark matter, baryons, and photons, $\delta_{\rm DM} \equiv \delta\rho_{\rm DM}/\rho_{\rm DM}$, $\delta_{\rm B}$ and δ_γ, as well as the Newtonian potential Φ. $t_{\rm eq}$ and t_Λ correspond to the transitions from radiation domination to matter domination, and from decelerated expansion to accelerated expansion, and $t_{\rm rec}$ refers to the recombination epoch.

2. Dark matter

Dark matter is characterized by the mass-to-entropy ratio

$$\left(\frac{\rho_{\rm DM}}{s}\right)_0 = \frac{\Omega_{\rm DM}\rho_{\rm c}}{s_0} \approx \frac{0.23 \times 5 \times 10^{-6}\ {\rm GeV\ cm^{-3}}}{3000\ {\rm cm^{-3}}} = 4 \times 10^{-10}\ {\rm GeV}\,. \quad (7)$$

This ratio has been constant in time since the freeze out of dark matter density: both the number of dark matter particles (and hence their mass) and the entropy are constant in a comoving volume.

Dark matter is crucial for our existence, for the following reason. Density perturbations in the baryon–electron–photon plasma before recombination do not grow because of high pressure, which is mostly due to photons; instead, perturbations produce sound waves of constant amplitude that propagate in the plasma. Hence, in a universe without dark matter, density perturbations in the baryonic component would start to grow only after baryons decouple from photons, i.e., after recombination. The physics behind the growth is quite simple: an overdense region gravitationally attracts surrounding matter; this matter falls into the overdense region, and the density contrast increases. In an expanding matter-dominated universe, this gravitational instability results in the density contrast growing as $(\delta\rho/\rho)(t) \propto t^{2/3} \propto T^{-1}$. Hence, in a universe without dark matter, the growth factor for baryon density perturbations is at most [2]

$$\frac{a(t_0)}{a(t_{\rm rec})} = 1 + z_{\rm rec} = \frac{T_{\rm rec}}{T_0} \approx 10^3\,. \quad (8)$$

The initial amplitude of density perturbations is very well known from CMB anisotropy measurements: $(\delta\rho/\rho)_{\rm i} = 1.5\times10^{-4}$. Hence, a universe without dark matter would still be rather homogeneous: the density contrast would be in the range of ten percent. No structure would have been formed, no galaxies, no life. No structure would be formed in the future either, because the accelerated expansion due to dark energy would soon terminate the growth of perturbations.

[2] Because of the presence of dark energy, the growth factor is even somewhat smaller.

Because dark matter particles decoupled from plasma much earlier than baryons did, perturbations in dark matter started to grow much earlier. The corresponding growth factor is larger than (8), and hence the dark matter density contrast at galactic scales reaches a value close to unity and perturbations enter the nonlinear regime and form dense dark matter clumps at the redshift $z \simeq 5$ and even earlier. After recombination, baryons fall into potential wells formed by dark matter, and then dark matter and baryon perturbations develop together. Galaxies form in those regions where dark matter was overdense originally. The development of perturbations in our Universe is shown schematically in Fig. 1. For this picture to hold, dark matter particles must be nonrelativistic early enough, because relativistic particles travel through gravitational wells instead of being trapped there. This means, inter alia, that neutrinos cannot make up a considerable part of dark matter, whence bound (5) follows.

Depending on the mass of dark matter particles and the mechanism of their production in the early Universe, dark matter may be *cold* (CDM) or *warm* (WDM). If dark matter particles were in thermal equilibrium with cosmic plasma in the early cosmological epoch, the CDM and WDM cases respectively correspond to heavy and light new particles:

$$m_{\rm DM} \gtrsim 100\ {\rm keV} \quad {\rm for} \quad {\rm CDM}\,, \quad (9)$$

$$m_{\rm DM} \lesssim 100\ {\rm keV} \quad {\rm for} \quad {\rm WDM}\,. \quad (10)$$

We discuss the warm dark matter option later, and now proceed with CDM.

2.1 WIMPS: Best guess for cold dark matter

There is a simple mechanism for the generation of dark matter in the early Universe. It applies to *cold* dark matter. Because of its simplicity and robustness, it is regarded by many as very likely, and the corresponding dark matter candidates — weakly interacting massive particles, WIMPs — are considered the best candidates. We describe this mechanism in general terms.

We assume that a heavy stable neutral particle Y exists and that Y particles can only be destroyed or created via their pair annihilation or creation, with annihilation products being the particles of the Standard Model. If the annihilation cross section is large enough, the overall cosmological behavior of the Y particles is as follows. At high temperatures, $T \gg m_{\rm Y}$, the Y particles are in thermal equilibrium with the rest of the cosmic plasma; there are many Y particles in the plasma, and they are continuously being created and annihilated. As the temperature decreases below $m_{\rm Y}$, the equilibrium number density decreases. At some 'freeze-out' temperature $T_{\rm f}$, the number density becomes so small that the Y particles can no longer meet each other during the Hubble time, and their annihilation terminates. After that, the number of surviving Ys remains time independent in the comoving volume, and these relic particles contribute to the mass density in the present Universe.

It is straightforward to calculate the present mass density of Y particles in this scenario. The result is [3]

$$\frac{\rho_{\rm Y,0}}{s_0} = \frac{m_{\rm Y} n_{\rm Y,0}}{s_0} \simeq 7.6\,\frac{\ln\left(\sqrt{g_*}\,M_{\rm Pl} m_{\rm Y}\langle\sigma v\rangle\right)}{\sqrt{g_*}\,M_{\rm Pl}\langle\sigma v\rangle}\,, \qquad (11)$$

where $g_* = g_*(T_{\rm f})$ is the number of degrees of freedom upon freeze-out of the Y particles, σ is the annihilation cross section, v is the velocity of Y particles, and angle brackets denote thermal average, also at the freeze-out temperature:

$$T_{\rm f} \simeq \frac{m_{\rm Y}}{\ln\left(\sqrt{g_*}\,M_{\rm Pl} m_{\rm Y}\langle\sigma v\rangle\right)}\,. \qquad (12)$$

Formula (11) is quite remarkable. The mass density depends mostly on one parameter, the annihilation rate per particle, $\langle\sigma v\rangle$. The dependence on the Y-particle mass is through the logarithm and through $g_*(T_{\rm f})$; it is very weak. The value of the logarithm here is between 20 and 40, depending on the parameters (this means, in particular, that freeze-out occurs when the temperature decreases 20 to 40 times below the Y-particle mass). Substituting the numerical value $g_*(T_{\rm f}) \sim 100$ characteristic of the Standard Model and comparing with (7), we obtain the estimate

$$\langle\sigma v\rangle = (1-2)\times 10^{-36}\ {\rm cm}^2\,. \qquad (13)$$

This is a weak-scale cross section, which tells us that the relevant energy scale is the TeV scale. We note in passing that estimate (13) is quite precise and robust.

The annihilation rate can be parameterized as $\langle\sigma v\rangle = \alpha^2/M^2$, where α is some coupling constant and M is the mass scale (which may be higher than $m_{\rm Y}$). This parameterization is particularly appropriate for s-wave annihilation; it is suggested by the picture of pair annihilation of nonrelativistic Y particles via the exchange with another particle of mass M. With $\alpha \sim 10^{-2}$, the estimate for the mass scale is $M \sim 1$ TeV.

Thus, under very general assumptions, we find that the nonbaryonic dark matter may naturally originate from the TeV-scale physics. In fact, what we have found can be understood as an approximate equality between the cosmological parameter, the mass-to-entropy ratio of dark matter, and the particle physics parameters,

$$\left(\frac{\rho_{\rm DM}}{s}\right)_0 \simeq \frac{1}{M_{\rm Pl}}\left(\frac{\rm TeV}{\alpha_{\rm W}}\right)^2,$$

where $\alpha_{\rm W}$ is the electroweak gauge constant. The quantities in both sides of the above relations are both of the order of 10^{-10} GeV, and it is very tempting to think that this is not a mere coincidence. If it is not, the dark matter particle should be found at the LHC.

Of course, the most prominent candidate for the WIMP is the neutralino of the supersymmetric extensions of the Standard Model [27, 28]. The situation with the neutralino is somewhat difficult, however: pair annihilation of neutralinos often occurs in the p-wave, rather than in the s-wave. This results in a suppression factor in the annihilation rate proportional to $v^2 \sim T_{\rm f}/m_{\rm Y} \sim 1/30$. Hence, neutralinos tend to be overproduced in most of the parameter space of the MSSM and other models. Yet the neutralino remains a good candidate, especially at high $\tan\beta$.

2.2 Warm dark matter: light gravitinos

The cold dark matter scenario successfully describes the bulk of the cosmological data. Yet there are clouds above it. First, according to numerical simulations, the CDM scenario tends to overproduce small objects — dwarf galaxies: it predicts hundreds of dwarf galaxies in the vicinity of a large galaxy like the Milky Way, whereas only dozens of dwarfs have been observed so far (see, e.g., Ref. [30]). Second, again according to simulations, CDM tends to produce densities in galactic centers that are too high (cusps in density profiles); this feature is not confirmed by observations, either (see, e.g., Ref. [30] and the references therein). There is no crisis yet, but one may be motivated to analyze the possibility that dark matter is not that cold.

An alternative to CDM is warm dark matter, whose particles have energies of the order of T, where $T \gtrsim m$ after decoupling (m is their mass). Then their spatial momenta decrease linearly with temperature, i.e., the momenta are approximately equal to T all the time after decoupling. WDM particles become nonrelativistic at $T \sim m$. Only after that do the WDM perturbations start to grow: [4] as we mentioned above, relativistic particles escape from gravitational potentials, and hence the gravitational potentials are smeared out instead of becoming deeper. Before becoming nonrelativistic, WDM particles travel a distance of the order of the horizon size; the WDM perturbations are therefore suppressed at those scales. The horizon size at the time $t_{\rm nr}$ when $T \sim m$ is of the order of

$$l(t_{\rm nr}) \simeq H^{-1}(T\sim m) = \frac{M_{\rm Pl}}{\sqrt{g_*}T^2} \sim \frac{M_{\rm Pl}}{\sqrt{g_*}m^2}\,,$$

where $H(T)$ is the Hubble parameter at the temperature T. Due to the expansion of the Universe, the corresponding length at present is

$$l_0 = l(t_{\rm nr})\,\frac{a_0}{a(t_{\rm nr})} \sim l(t_{\rm nr})\,\frac{T}{T_0} \sim \frac{M_{\rm Pl}}{mT_0}\,, \qquad (14)$$

where we neglected the (rather weak) dependence on g_*. Hence, in the WDM scenario, objects smaller than l_0 in size are less abundant than in the CDM case. We point out that l_0 refers to the size of the perturbation as if it were in the linear regime; in other words, this is the size of the region from which matter clumps into a compact object.

The present size of a dwarf galaxy is a few kpc, and the density is about 10^6 of the average density in the Universe. Hence, the size l_0 for these objects is around

[3] We omit irrelevant numerical factors in the arguments of the logarithm here and in (12).

[4] The situation is in fact somewhat more complicated, but this is irrelevant for our estimates.

100 kpc $\simeq 3 \times 10^{23}$ cm. From (14), requiring that perturbations of this size, but not much larger, be suppressed, we obtain the estimate for m of a few keV for the mass of WDM particles. In fact, such a small mass of WDM particles is most likely inconsistent with the data. Hydrogen in the Universe was reionized at the redshift $z \sim 10$; this property leaves imprints on the CMB temperature anisotropy and, most notably, CMB polarization, and these imprints have been detected [16]. The reionization is attributed to the formation, burning, and explosions of the first stars, which are believed to have formed in dark matter halos with a mass around $10^5 M_{\odot}$. The initial comoving size of these halos is roughly $l_0 \sim 10$ kpc, and hence perturbations of this spatial size must not be strongly suppressed. According to (14), this implies a bound on the WDM particle mass around (a few) $\times 10$ keV.

Among WDM candidates, the light gravitino is probably the best. The gravitino mass is of the order of

$$m_{3/2} \simeq \frac{F}{M_{\mathrm{Pl}}} ,$$

where $\sqrt{F}$ is the supersymmetry breaking scale. Hence, gravitino masses are in the right ballpark for rather low supersymmetry breaking scales, $\sqrt{F} \sim 10^7$ GeV. This is the case, for instance, in the gauge mediation scenario [31]. With such a low mass, the gravitino lifetime is much longer than the age of the Universe, and from this standpoint, gravitinos can indeed serve as dark matter particles. For what follows, the important parameters are the widths of decays of other superpartners into gravitino and Standard Model particles. These are approximately

$$\Gamma_{\tilde{S}} \simeq \frac{M_{\tilde{S}}^5}{F^2} \simeq \frac{M_{\tilde{S}}^5}{m_{3/2}^2 M_{\mathrm{Pl}}^2} , \qquad (15)$$

where $M_{\tilde{S}}$ is the superpartner mass.

One mechanism of gravitino production in the early Universe is decays of other superpartners. The gravitino interacts with everything else so weakly that, once produced, it moves freely, without interacting with the cosmic plasma. At production, gravitinos are relativistic; hence, they are indeed *warm* dark matter candidates. We assume that production in decays is the dominant mechanism and consider under what circumstances the present mass density of gravitinos coincides with that of dark matter (see, e.g., Ref. [32] for the details).

The rate of gravitino production in decays of $\tilde{S}$-type superpartners in the early Universe is

$$\frac{\mathrm{d}(n_{3/2}/s)}{\mathrm{d}t} = \frac{n_{\tilde{S}}}{s} \Gamma_{\tilde{S}} ,$$

where $n_{3/2}$ and $n_{\tilde{S}}$ are number densities of gravitinos and superpartners and s is the entropy density. For superpartners in thermal equilibrium, we have $n_{\tilde{S}}/s = const \sim g_*^{-1}$ for $T \gtrsim M_{\tilde{S}}$, and $n_{\tilde{S}}/s \propto \exp(-M_{\tilde{S}}/T)$ at $T \ll M_{\tilde{S}}$. Hence, the production is most efficient at $T \sim M_{\tilde{S}}$, when the number density of the superpartners is still large, while the Universe expands most slowly. The density of gravitinos produced in decays of $\tilde{S}$s is therefore given by

$$\frac{n_{3/2}}{s} \simeq \left(\frac{\mathrm{d}(n_{3/2}/s)}{\mathrm{d}t} H^{-1} \right)_{T \sim M_{\tilde{S}}} \simeq \frac{\Gamma_{\tilde{S}}}{g_*} H^{-1}(T \sim M_{\tilde{S}})$$
$$\simeq \frac{1}{g_*} \frac{M_{\tilde{S}}^5}{m_{3/2}^2 M_{\mathrm{Pl}}^2} \frac{M_{\mathrm{Pl}}}{g_*^{1/2} M_{\tilde{S}}^2} .$$

This gives the present mass-to-entropy ratio

$$\frac{m_{3/2} n_{3/2}}{s} \simeq \sum_{\tilde{S}} \frac{M_{\tilde{S}}^3}{g_*^{3/2} M_{\mathrm{Pl}} m_{3/2}} , \qquad (16)$$

where the sum ranges over all superpartner species *that have ever been relativistic in thermal equilibrium*. The correct value (7) is obtained for gravitino masses in range (10) at

$$M_{\tilde{S}} = 100-300 \text{ GeV} . \qquad (17)$$

Hence, the scenario with the gravitino as the warm dark matter particle requires light superpartners, which are to be discovered at the LHC.

A few comments are in order. First, the decay of superpartners is not the only mechanism of gravitino production: gravitinos may also be produced in the scattering of superpartners. To avoid overproduction of gravitinos in the latter processes, we have to assume that the maximum temperature in the Universe (reached, say, after the post-inflationary reheating stage) is quite low, $T_{\max} \sim 1-10$ TeV. This is not a particularly plausible assumption, but it is consistent with everything else in cosmology and can indeed be realized in some models of inflation. Second, existing constraints on masses of strongly interacting superpartners (gluinos and squarks of the first and second generations) suggest that their masses exceed (17). Hence, these particles should not contribute to the sum in (16); otherwise, WDM gravitinos would be overproduced. This is possible if the masses of squarks and gluinos are larger than $T_{\max}$, such that they were never abundant in the early Universe. Third, the decay into gravitino and Standard Model particles is the only decay channel for the next-to-lightest superpartner (NLSP). Hence, with the estimate for the total width of the NLSP given by (15), we have

$$c\tau_{\mathrm{NLSP}} = 1 \text{ mm} - 100 \text{ m}$$

for $m_{3/2} = 1-10$ keV and $M_{\mathrm{NLSP}} = 100-300$ GeV. The NLSP should therefore either be visible in a detector or fly through it.

Needless to say, the outlined scenario is much more contrived than the WIMP option. It is reassuring, however, that it will be ruled out or confirmed at the LHC.

Finally, gravitinos can be much heavier than 100 keV, and still be the lightest supersymmetric particles. Then they serve as CDM candidates. Obviously, direct detection of CDM particles is hopeless in that case.

2.3 Discussion

If dark matter particles are indeed WIMPs, and the relevant energy scale is about 1 TeV, then the Hot Big Bang theory will be probed experimentally up to temperatures of $(10-100)$ GeV and down to the age of $10^{-9}-10^{-11}$s in the relatively near future (which is to be compared with 1 MeV and 1 s accessible today through Big Bang Nucleosynthesis). With the microscopic physics to become known from collider experiments, the WIMP density will be reliably calculated and checked against data from observational cosmology. Thus, the WIMP scenario offers a window to a very early stage of the evolution of the Universe.

If dark matter is warm and its particles are gravitinos, then the prospect of quantitatively accessing such an early stage of the cosmological evolution is not so bright: it would be very difficult, if at all possible, to assess the gravitino mass experimentally; furthermore, the present gravitino mass density depends on an unknown reheat temperature $T_{\max}$. On the other hand, if this scenario is realized in Nature, the whole picture of the early Universe will be quite different from what we think today is the most likely early cosmology. Indeed, the gravitino scenario requires a low reheat temperature, which in turn calls for rather exotic mechanisms of inflation, etc.

The mechanisms discussed here are by no means the only ones capable of producing dark matter, and WIMPs and gravitinos are by no means the only candidates for dark matter particles. Other dark matter candidates include axions, sterile neutrinos, Q-balls, very heavy relics produced towards the end of inflation, and so on. Hence, even though there are grounds to hope that the dark matter problem will be solved by the LHC, there is no guarantee at all.

3. Baryon asymmetry of the Universe

In the present Universe, there are baryons and almost no antibaryons. The number density of baryons today is characterized by the ratio η [see Eqn (3)]. In the early Universe, the appropriate quantity is

$$\Delta_{\mathrm{B}} = \frac{n_{\mathrm{B}} - n_{\bar{\mathrm{B}}}}{s} ,$$

where $n_{\bar{\mathrm{B}}}$ is the number density of antibaryons and s is the entropy density. If the baryon number is conserved and the Universe expands adiabatically, then Δ_{B} is constant and its value is equal to η up to a numerical factor,

$$\Delta_{\mathrm{B}} \approx 0.8 \times 10^{-10} .$$

At early times, at temperatures well above 100 MeV, the cosmic plasma contained many quark–antiquark pairs, whose number density was of the order of the entropy density, $n_{\mathrm{q}} + n_{\bar{\mathrm{q}}} \sim s$, while the baryon number density was related to the densities of quarks and antiquarks as $n_{\mathrm{B}} - n_{\bar{\mathrm{B}}} = (1/3)(n_{\mathrm{q}} - n_{\bar{\mathrm{q}}})$ (the baryon number of a quark is 1/3). Hence, in terms of quantities characterizing the very early epoch, the baryon asymmetry can be expressed as

$$\Delta_{\mathrm{B}} \sim \frac{n_{\mathrm{q}} - n_{\bar{\mathrm{q}}}}{n_{\mathrm{q}} + n_{\bar{\mathrm{q}}}} .$$

We see that there was one extra quark for about every 10 billion quark–antiquark pairs. It is this tiny excess that is responsible for all the baryonic matter in the present Universe.

There is no logical contradiction in supposing that the tiny excess of quarks over antiquarks was built in as an initial condition. But this is not at all satisfactory for a physicist. Furthermore, the inflationary scenario does not provide such an initial condition for the hot stage; rather, inflation theory predicts that the Universe was baryon-symmetric immediately after inflation. Hence, we would like to explain the baryon asymmetry dynamically.

The baryon asymmetry can be generated from the initially symmetric state only if three necessary conditions, called Sakharov's conditions, are satisfied. These are:

(i) baryon number nonconservation;

(ii) *C*- and *CP*-violation; and

(iii) deviation from thermal equilibrium.

All three conditions are easily understood. (i) If the baryon number were conserved, and the initial net baryon number in the Universe was zero, the Universe today would still be symmetric. (ii) If C or CP were conserved, the rates of reactions with particles would be the same as the rates of reactions with antiparticles, and no excess of quarks over antiquarks would be generated. (iii) Thermal equilibrium is the most symmetric state of a system. If the baryon number were the only relevant quantum number, it would be washed out, rather than generated, as the system approaches thermal equilibrium. In fact, the baryon number is *not* the only relevant quantum number, and hence the last point requires qualification, which is not of importance for us.

There are two well-understood mechanisms of baryon number nonconservation. One of them emerges in hypothetical Grand Unified Theories and is due to the exchange by supermassive particles. It is very similar, e.g., to the mechanism of charm nonconservation in weak interactions, which occurs via the exchange by heavy W-bosons. The scale of these new, baryon-number-violating interactions is the Grand Unification scale, presumably of the order of 10^{16} GeV.

Another mechanism is nonperturbative [33] and is related to the triangle anomaly in the baryonic current. It already exists in the Standard Model and, possibly with slight modifications, operates in all its extensions. The two main features of this mechanism, as applied to the early Universe, are that it is effective over a wide range of temperatures [34], $100\,\mathrm{GeV} < T < 10^{11}$ GeV, and that it preserves $B - L$.

We pause here to discuss the physics behind electroweak baryon and lepton number nonconservation in a little more detail, although still at a qualitative level. The first object to consider is the baryonic current

$$B^{\mu} = \frac{1}{3} \sum_i \bar{q}_i \gamma^{\mu} q_i ,$$

where the sum ranges over quark flavors. Naively, the baryonic current is conserved, but at the quantum level, its divergence is nonzero due to the triangle anomaly (similar effects go under the name of axial anomaly in the context of QED and QCD),

$$\partial_{\mu} B^{\mu} = \frac{1}{3} \times 3_{\mathrm{colors}} \times 3_{\mathrm{generations}} \times \frac{g^2}{32\pi^2}\, \epsilon^{\mu\nu\lambda\rho} F^{\mathrm{a}}_{\mu\nu} F^{\mathrm{a}}_{\lambda\rho} ,$$

where $F^{\mathrm{a}}_{\mu\nu}$ and g are the field strength of the $\mathrm{SU(2)_W}$ gauge field and the $\mathrm{SU(2)_W}$ coupling. Likewise, each leptonic current ($\alpha = \mathrm{e}, \mu, \tau$) is anomalous,

$$\partial_{\mu} L^{\mu}_{\alpha} = \frac{g^2}{32\pi^2}\, \epsilon^{\mu\nu\lambda\rho} F^{\mathrm{a}}_{\mu\nu} F^{\mathrm{a}}_{\lambda\rho} .$$

A nontrivial fact [35] is that there exist large field fluctuations, $F^{\mathrm{a}}_{\mu\nu}(\mathbf{x}, t) \propto g^{-1}$, for which

$$Q \equiv \int \mathrm{d}^3 x\, \mathrm{d}t\, \frac{g^2}{32\pi^2}\, \epsilon^{\mu\nu\lambda\rho} F^{\mathrm{a}}_{\mu\nu} F^{\mathrm{a}}_{\lambda\rho} \neq 0 .$$

Furthermore, for any such fluctuation, the value of Q is an integer.

We now suppose that a fluctuation with a nonvanishing Q has occurred. Then the baryon numbers at the end and beginning of the process are different:

$$B_{\mathrm{fin}} - B_{\mathrm{in}} = \int \mathrm{d}^3 x\, \mathrm{d}t\, \partial_{\mu} B^{\mu} = 3Q . \qquad (18)$$

Similarly,

$$L_{n,\,\mathrm{fin}} - L_{n,\,\mathrm{in}} = Q\,. \tag{19}$$

This explains the selection rule mentioned above: B is violated and $B - L$ is not.

At zero temperature, the large field fluctuations that induce baryon and lepton number violation are vacuum fluctuations, called instantons [35], which to a certain extent are similar to virtual fields that emerge and disappear in the vacuum of quantum field theory at the perturbative level. The difference is that instantons are *large* field fluctuations. This property results in a suppression of the corresponding probability, and hence the rate of baryon-number-violating processes, by the factor $\exp(-16\pi^2/g^2) \sim 10^{-165}$. On the other hand, at high temperatures, there are large *thermal* fluctuations, 'sphalerons' [36], whose rate is not necessarily small. And, indeed, B-violation in the early Universe is rapid compared to the cosmological expansion at sufficiently high temperatures, when

$$\langle\phi\rangle_T < T\,, \tag{20}$$

where $\langle\phi\rangle_T$ is the Higgs expectation value at temperature T.

One may wonder how the baryon number may be not conserved even though there are no baryon-number-violating terms in the Lagrangian of the Standard Model. This is discussed in detail, for instance, in Ref. [37]. In any case, it is tempting to use this mechanism of baryon number nonconservation for explaining the baryon asymmetry of the Universe. There are two problems, however. One is that CP-violation in the Standard Model is too weak: the CKM mechanism alone is insufficient to generate the realistic value of baryon asymmetry. Hence, we need extra sources of CP-violation. Another problem has to do with departure from thermal equilibrium, which is necessary for the generation of baryon asymmetry. At temperatures well above 100 GeV, the electroweak symmetry is restored, the expectation value of ϕ is zero,[5] relation (20) holds, and the baryon number nonconservation is rapid compared to the cosmological expansion. At temperatures around 100 GeV, relation (20) may be violated, but the Universe expands very slowly: the cosmological time scale at these temperatures is

$$H^{-1} \sim \frac{M_{\mathrm{Pl}}}{\sqrt{g_*}\,T^2} \simeq \frac{10^{19}\ \mathrm{GeV}}{10 \times (100\ \mathrm{GeV})^2} \sim 10^{-10}\ \mathrm{s}\,, \tag{21}$$

which is very large by the electroweak physics standards. The only way in which a strong departure from thermal equilibrium at these temperatures may occur is through a first-order phase transition.

The property (valid within the perturbation theory only) that the expectation value of the Higgs field is zero at temperatures well above 100 GeV, while it is nonzero in the vacuum, suggests that there may be a phase transition from, crudely speaking, the phase with $\langle\phi\rangle = 0$ to the phase with $\langle\phi\rangle \neq 0$. The situation is pretty subtle here, because ϕ is not gauge invariant, and hence cannot serve as an order parameter, and therefore the notion of phases with $\langle\phi\rangle = 0$ and $\langle\phi\rangle \neq 0$ is vague. In fact, neither electroweak theory nor most of its extensions have a gauge-invariant order parameter, and hence there is no real distinction between these 'phases.' This situation is very similar to that in a liquid–vapor system, which does not have an order parameter and may or may not experience the vapor–liquid phase transition as the temperature decreases, depending on other parameters characterizing this system, e.g., pressure. In the Standard Model, the role of such a parameter is played by the Higgs self-coupling λ or, in other words, the Higgs boson mass.

Continuing to use the somewhat inexact terminology, the interesting case for us is the first-order phase transition. Here, the effective potential $V_{\mathrm{eff}}(\phi)$ (free energy density as a function of ϕ) has one minimum at $\phi = 0$ at high temperatures, and the expectation value of the Higgs field is zero. As the temperature decreases, another minimum appears at a finite ϕ, and then becomes lower than the minimum at $\phi = 0$. However, the probability of the transition from the phase $\phi = 0$ to the phase $\phi \neq 0$ is very small for some time, and the system becomes overcooled.

The first-order phase transition occurs via spontaneous creation of bubbles of the new phase inside the old phase. These bubbles then grow, their walls eventually collide, and the new phase eventually occupies the entire space. The Universe boils. In the cosmological context, this process occurs when the bubble nucleation rate per Hubble time per Hubble volume is approximately 1. The velocity of the bubble wall in the relativistic cosmic plasma is roughly of the order of the speed of light (in fact, it is somewhat smaller, from $0.1c$ to $0.01c$), simply because there are no relevant dimensionless parameters characterizing the system. Hence, the bubbles grow large before their walls collide: their size at collision is roughly comparable to the Hubble size. While the bubble is microscopic at nucleation — its size is dictated by the electroweak scale and is roughly $(100\ \mathrm{GeV})^{-1} \approx 10^{-16}$ cm — its size is macroscopic at the time the walls collide, $0.1H^{-1} \sim 1$ mm, as follows from (21). Clearly, boiling is a highly nonequilibrium process, and it may be hoped that the baryon asymmetry may be generated at that time. And, indeed, there exist mechanisms for the generation of baryon asymmetry, which have to do with interactions of quarks and leptons with moving bubble walls. The value of the resulting baryon asymmetry may well be around 10^{-10}, as required by observations, if there is enough CP-violation in the theory.

A necessary condition for the electroweak generation of the baryon asymmetry is that inequality (20) be violated *immediately after* the phase transition. Indeed, in the opposite case, electroweak baryon number violating processes are fast after the transition, and the baryon asymmetry generated during the transition is washed out afterwards. Hence, the phase transition must be of a strong enough first order. This is *not* the case in the Standard Model. To understand why, and to see in which extensions of the Standard Model the transition may be strong enough, we consider the effective potential in some detail. At zero temperature, the Higgs potential has the standard form

$$V(\phi) = -\frac{m^2}{2}\,|\phi|^2 + \frac{\lambda}{4}\,|\phi|^4\,,$$

where

$$|\phi| \equiv \left(\phi^\dagger\phi\right)^{1/2} \tag{22}$$

is the magnitude of the Higgs doublet ϕ, $m^2 = \lambda v^2$, and $v = 247$ GeV is the Higgs expectation value in the vacuum.

[5] There are subtleties here; see below.

The Higgs boson mass is related to it as

$$m_{\rm H} = \sqrt{2\lambda}\,v\,. \tag{23}$$

In the leading order of the perturbation theory, finite-temperature effects modify the effective potential into

$$V_{\rm eff}(\phi, T) = \frac{\alpha}{2}|\phi|^2 - \frac{\beta}{3}\,T|\phi|^3 + \frac{\lambda}{4}\,|\phi|^4\,, \tag{24}$$

with $\alpha(T) = -m^2 + \hat{g}^2T^2$ and $\beta = \tilde{g}^3/(2\pi)$, where $\hat{g}^2$ is a positive linear combination of squares of the coupling constants of all fields to the Higgs field (in the Standard Model, a linear combination of g^2, g'^2, and y_i^2, where g and g' are gauge couplings and y_i are Yukawa couplings), while $\tilde{g}^3$ is a positive linear combination of cubes of coupling constants of all *bosonic* fields to the Higgs field. In the Standard Model, β is a linear combination of g^3 and g'^3, i.e., a linear combination of $M_{\rm W}^3/v^3$ and $M_{\rm Z}^3/v^3$,

$$\beta = \frac{1}{2\pi}\,\frac{2M_{\rm W}^3 + M_{\rm Z}^3}{v^3}\,. \tag{25}$$

The cubic term in (24) is rather peculiar: in view of (22), it is not analytic in the original Higgs field ϕ. Yet this term is crucial for the first-order phase transition: for $\beta = 0$, the phase transition would be of the second order. The origin of the nonanalytic cubic term can be traced back to the enhancement of the Bose–Einstein thermal distribution at low momenta, $p, m \ll T$,

$$f_{\rm Bose}(p) = \frac{1}{\exp\left(\sqrt{p^2 + m_{\rm b}^2}/T\right) - 1} \simeq \frac{T}{\sqrt{p^2 + m_{\rm b}^2}}\,,$$

where $m_{\rm b} \simeq g_{\rm b}|\phi|$ is the mass of the boson b that is generated due to the nonvanishing Higgs field and $g_{\rm b}$ is the coupling constant of the field b to the Higgs field. Clearly, at $p \ll g_{\rm b}|\phi|$ the distribution function is nonanalytic in ϕ,

$$f_{\rm Bose}(p) \simeq \frac{T}{g_{\rm b}|\phi|}\,.$$

This nonanalyticity gives rise to the nonanalytic cubic term in the effective potential. Importantly, the Fermi–Dirac distribution

$$f_{\rm Fermi}(p) = \frac{1}{\exp\left(\sqrt{p^2 + m_{\rm f}^2}/T\right) + 1}\,,$$

is analytic in $m_{\rm f}^2$, and hence in $\phi^\dagger\phi$, and therefore fermions do not contribute to the cubic term.

With the cubic term in the effective potential, the phase transition is indeed of the first order (within the approximation considered here): at high temperatures, the coefficient α is positive and large, and there is one minimum of the effective potential at $\phi = 0$, while at intermediate temperatures, α is small but still positive, and therefore there are two minima. The phase transition occurs at $\alpha \approx 0$; at that instant,

$$V_{\rm eff}(\phi, T) \approx -\frac{\beta}{3}\,T|\phi|^3 + \frac{\lambda}{4}\,|\phi|^4\,.$$

We find from this expression that immediately after the phase transition, the minimum of $V_{\rm eff}$ is at

$$\phi \simeq \frac{\beta}{\lambda} = \frac{\tilde{g}^3 T}{\lambda}\,.$$

Hence, the necessary condition for successful electroweak baryogenesis, $\phi > T$, translates into

$$\beta > \lambda\,. \tag{26}$$

According to (23), λ is proportional to $m_{\rm H}^2$, whereas in the Standard Model, β is proportional to $2M_{\rm W}^3 + M_Z^3$. Therefore, relation (26) holds for small Higgs boson masses only; in the Standard Model, using (23) and (25) shows that that would happen for $m_{\rm H} < 50$ GeV, which is ruled out.[6]

This discussion indicates a possible way to make the electroweak phase transition strong. We need the existence of new bosonic fields that have large enough couplings to the Higgs field(s), and hence make large contributions to β. To produce an effect on the dynamics of the transition, the new bosons must be present in the cosmic plasma at the transition temperature, $T \sim 100$ GeV, and therefore their masses should not be too high, $M \lesssim 300$ GeV. In supersymmetric extensions of the Standard Model, the natural candidate is the scalar partner of the top quark, whose Yukawa coupling to the Higgs field is the same as that of the top quark, that is, large. The light stop scenario for electroweak baryogenesis indeed works, as has been shown by the detailed analysis in Refs [41–43].

Yet another issue is *CP*-violation, which has to be strong enough for the successful electroweak baryogenesis. Because the asymmetry is generated in the interactions of quarks and leptons (and their superpartners in supersymmetric extensions) with the bubble walls, *CP*-violation must occur at the walls. We now recall that the walls are made of the Higgs field(s). This points towards the necessity of *CP*-violation in the Higgs sector, which may only be the case in a theory with more than one Higgs fields.

To summarize, electroweak baryogenesis requires a considerable extension of the Standard Model, with masses of new particles in the range $100-300$ GeV. Hence, this mechanism will definitely be ruled out or confirmed by the LHC. We emphasize, however, that electroweak baryogenesis is not the only option: an elegant and well-motivated competitor is leptogenesis, and there are several other mechanisms that have been proposed in the literature.

4. Concluding remarks

The ideas we have discussed may not be the right ones: we can only hypothesize on physics beyond the Standard Model and its role in the early Universe. The TeV-scale physics may be dramatically different from the physics we are used to. As an example, it cannot be ruled out that TeV is not only an electroweak scale but also a gravitational scale. This is the case in models with large extra dimensions, in which the Planck scale is related to the fundamental gravity scale in a way that involves the volume of extra dimensions, and hence the fundamental scale can be much below $M_{\rm Pl}$ (for a review, see, e.g., Ref. [44]).

If the LHC finds that, indeed, the fundamental gravity scale is in the TeV range, this will have very profound

[6] In fact, in the Standard Model with $m_{\rm H} > 114$ GeV, there is no phase transition at all [38–40]; the electroweak transition is a smooth crossover instead. This fact is not visible from expression (24), but that expression is the lowest-order perturbative result, while the perturbation theory is not applicable in describing the transition in the Standard Model with large $m_{\rm H}$.

consequences for both microscopic physics and cosmology. On the microscopic physics side, this would allow studying quantum gravity and its high-energy extension (possibly string theory) at colliders, while on the cosmological side, the entire picture of the early Universe would have to be revised. Inflation, if any, would have to occur either at low energy density or in the state of strong quantum gravity effects. The highest temperatures in the usual expansion history would be at most in the TeV range, such that dark matter and baryon asymmetry would have to be generated either below TeV temperatures or in the quantum gravity mode. Even more intriguing would be the study of the quantum gravity cosmological epoch, with hints from colliders gradually coming in. This, probably, is too optimistic an outlook to be realistic.

It is more likely that the LHC will find something entirely new, something theorists have not thought about, or, conversely, find so little that we will have to get serious about the anthropic principle. In any case, the LHC results will definitely change the landscape of fundamental physics, cosmology included.

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V L Ginzburg and higher-spin fields

M A Vasiliev

1. Higher-spin fields yesterday and today

Relativistic fields are characterized by two types of quantum numbers: mass $m \geqslant 0$, and spin $s = 0, 1/2, 1, 3/2, \ldots, \infty$. To date, only two types of particles have been observed experimentally: particles of spin $s = 1/2$ — that is, $\mathrm{e}, \nu, \mu, \mathrm{u}, \mathrm{d}\ldots$, which describe matter fields, and those of spin $s = 1$, like photons, gluons, and W and Z bosons, which serve as mediators of interactions.

The main goal of the LHC is to find the hypothetical particle of spin 0, the Higgs boson H. Massless particles of spin 2 (graviton) and spin 3/2 (gravitino) remain to be discovered, although gauge theories related to them, namely gravity and supergravity, are well known, at least at the classical level.

The theory of free fields of any spin and mass is perfectly defined at the Lagrangian level. A nontrivial and highly interesting problem arises once the question of the structure of the theory of interacting fields of spins $s > 2$ is raised.

The foundation of the theory of free higher-spin fields was laid in the classical work of Dirac [1] and Fierz and Pauli [2]. The history of the development of higher-spin theory can be roughly split into two stages. Before the creation of supergravity [3], i.e., approximately from 1936 till 1976, the main goal was to describe higher-spin resonances. During this period, the main focus was on the study of massive particles in four dimensional spacetime. After the creation of super-

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gravity, i.e., since 1976 up to now, interest has shifted to the study of fundamental interactions based on the gauge symmetry principle, which requires first of all the study of massless higher-spin fields. In addition, the development of superstring theory and supergravity led to the necessity of studying gauge theories in higher dimensions $d > 4$.

Vitaly Lazarevich Ginzburg actively worked on higher-spin fields at the beginning of the 1940s, i.e., at a quite early stage of the development of the theory. By his own account and published memoirs [4–6], as well as from remarks in his early papers (see, e.g., the paper on the theory of spin 3/2 [7]), we know that many fundamental results of the theory were obtained independently by Tamm and his collaborators at approximately the same time as analogous results by Western authors, although they turned out to be published somewhat later (it should not be forgotten that these were years of war). In particular, this concerns the work of Davydov [8], published only in 1943, where the Lagrangian for spin-3/2 particles was constructed, found independently by Rarita and Schwinger in 1941 [9]. Tamm acquainted Ginzburg with the results of his joint investigations with Davydov, and with the permission of the authors, Ginzburg used them in his paper [7]. Ginzburg's Habilitation thesis, "On the theory of elementary particles" [10], was defended in 1942 and published (with some abridgment) in Refs [11, 12].[1]

The key idea of Ginzburg's studies was that using systems of fields of different spins [10–13] may provide a means for overcoming the difficulties arising in the quantum theory of interacting fields. Interestingly, this idea also remains of key importance at present, with the only distinction that appropriate spectra of fields are dictated by one symmetry principle or another. For instance, the addition of the spin-0 Higgs field makes it possible to construct a consistent quantum theory of a massive spin-1 field. Details of the unification of fields of spins 1 and 0 in the Standard Model are dictated by gauge symmetry.

Ginzburg's favorite system consisted of fields of spins 1/2 and 3/2. As a matter of fact, the theories he considered [10–13] were prototypes of the theory of supergravity with spontaneously broken supersymmetry. The study of the systems of fields of different spins drove Ginzburg to the idea phrased in his habilitation work that most natural relativistic models should probably describe systems of fields of all integer spins $s = 0, 1, 2, \ldots$ and/or half-integer spins $s = 1/2, 3/2, 5/2, \ldots$. Later on, this idea was fully confirmed. Thus, superstring theory, which resolves many of the problems of local field theory, indeed describes infinite systems of fields of all spins with the Regge character of the dependence of mass on spin. Higher-spin gauge theories also necessarily contain infinite systems of fields of unbounded spins whose pattern is dictated by higher-spin symmetries.

Due to Ginzburg's interest in higher-spin theory, Efim Samoilovich Fradkin got a position in the Theory Department of the Lebedev Physical Institute (FIAN in *Russ. abbr.*). Being on the front lines of the army and having no systematic education, Fradkin was able soon after the end of the war to understand Ginzburg's papers on the theory of spin-3/2 fields and generalized them to the case of spin 5/2 [14]. When Fradkin approached Vitaly Lazarevich with his work on the theory of a massive spin-5/2 field, Ginzburg was so impressed that soon after Efim Samoilovich found his place in the Theory Department of FIAN. In this way, the interest in the theory of higher-spin fields propagated from Ginzburg to Fradkin, then to his pupils, and then to their pupils. During this period, priorities in higher-spin theory changed significantly. The main modification in the ideology of the development of the theory occurred in the last quarter of the 20th century, becoming the principle of gauge invariance.

Let us focus in some more detail on the paper by Ginzburg and Tamm, "To the theory of spin" [15], where an attempt was undertaken at a unified description of particles of different spins and masses. The main subject of this work was the field $\Psi(x, u)$, which depends not only on space–time coordinates x^n, $n = 0, 1, 2, 3$, but also on auxiliary variables u_n subjected to the condition $u_n u^n = 1$. The equations for $\Psi(x, u)$ have the form

$$\left(\Box - m^2 + \frac{\beta}{2}\,\alpha M^{mn} M_{mn}\right)\Psi(x, u) = 0\,, \qquad (1.1)$$

$$\left(\alpha M^{il} M_l^{\ j}\,\partial_i \partial_j - \Box\right)\Psi(x, u) = 0\,, \qquad (1.2)$$

where

$$M_{ij} = u_i\,\frac{\partial}{\partial u^j} - u_j\,\frac{\partial}{\partial u^i}\,.$$

An alternative version of the theory is related to the introduction of auxiliary variables, which form an antisymmetric tensor $u_n \to u_{nm} = -u_{mn}$. In both cases, this model faces difficulties. Without additional condition (1.2) it leads to a nonphysical spectrum with experimentally unacceptable points of condensation of states at finite masses.[2] Introduction of the additional condition (1.2) resolves this problem, but obstructs the introduction of interactions.

Although the original Ginzburg–Tamm model itself was not successful, it is quite interesting as possessing many features of modern theories. Indeed, states of string theory are described by a vector in the space of states of string:

$$\left|\Psi(x)\right\rangle = \sum \psi_{m_1\ldots m_{s_1},\, n_1\ldots n_{s_2},\,\ldots}(x)\, a_{-1}^{m_1}\ldots a_{-1}^{m_{s_1}}\, a_{-2}^{n_1}\ldots a_{-2}^{n_{s_2}}\ldots|0\rangle$$

that satisfies the condition

$$Q\left|\Psi(x)\right\rangle = 0\,, \qquad (1.3)$$

where Q is the Beckey–Rue–Stora–Tyutin (BRST) string operator satisfying the condition $Q^2 = 0$, which implies invariance of the theory under the gauge transformations $\delta|\psi\rangle = Q|\varepsilon\rangle$. The analogy between the variables u^n and string creation operators a_i^n which, however, form an infinite set in the case of string ($i = 0, 1, 2, \ldots \infty$), is obvious.

The mass scale of string theory is provided by the string tension parameter $m^2 \sim 1/\alpha'$. In the tensionless limit $\alpha' \to \infty$, all string excitations become massless and one can expect the appearance of additional symmetries of string theory in the high-energy limit as was discussed, for instance, in paper [16]. Whatever construction underlies string theory, if in a certain limit it exhibits higher-spin symmetries, it can be interpreted in this limit as a higher-spin gauge theory.

[1] For convenience of readers, the chapter 'Higher spins' from book [5] will be placed on *PU*'s site www.ufn.ru as an Appendix to this paper by M A Vasiliev along with a number of rather inaccessible at present papers by V L Ginzburg covering the subject matter of interest. (*Editor's note.*)

[2] Notice that this property is a consequence of the condition imposed in paper [15] that Ψ has to form a unitary representation of the Lorentz group, the necessity of which raises serious questions today.

The central role in higher-spin gauge theory is played by gauge symmetries. The case of symmetric massless fields of any spin was considered by Fronsdal in 1978 [17]. In the Fronsdal formulation, a symmetric massless field of spin s is described by a rank s symmetric tensor $\varphi_{n_1\ldots n_s}$ subjected to the double tracelessness condition $\eta^{n_1 n_2}\eta^{n_3 n_4}\varphi_{n_1\ldots n_s}=0$. The gauge transformation takes the form

$$\delta\varphi_{k_1\ldots k_s}=\partial_{(k_1}\varepsilon_{k_2\ldots k_s)}\,, \qquad \varepsilon^{m}{}_{mk_3\ldots k_{s-1}}=0\,.$$

In the spin-1 case, the gauge transformations with the parameter $\varepsilon(x)$ describe inner symmetries, while the corresponding nonlinear gauge theory gives rise to electrodynamics and Yang–Mills theory.

Spin 2 is related to vector gauge parameters $\varepsilon_n(x)$, which correspond to changes in coordinates $x^n \to x^n+\varepsilon^n(x)$ in the nonlinear theory of gravity.

The gauge parameter $\psi_{n\alpha}$ for the fermion field of spin 3/2 considered in Refs [8, 9] turns out to be a spinor $\varepsilon_\alpha(x)$ and corresponds to supersymmetry transformations, while the corresponding nonlinear gauge theory is called supergravity [3].

The key question in the theory of higher-spin fields, to which fields with spins $s>2$ are attributed after the creation of supergravity, is what the structure of the corresponding nonlinear theories is. The answer to this question is closely related to the fundamental question of the structure of non-Abelian higher-spin symmetries. For instance, a pattern of fields in a consistent model is determined to a large extent by representations of its symmetry group. At least as important is that symmetries of the theory determine the structure of the space where they can be realized. For instance, the symmetries of the Poincaré group, which contain spacetime translations and Lorentz rotations, are realized geometrically in Minkowski spacetime. Supersymmetry is naturally realized in superspace. The 'nongeometricity' of higher-spin symmetries in Minkowski space suggests the necessity of revising conventional conceptions of spacetime.

Up to the end of the 1970s, dominating statements in the literature on the possibility of the existence of interacting higher-spin theories were negative. They were mostly based on two kinds of arguments. The first-kind arguments were in the spirit of the Coleman–Mandula theorem [18], which states that a nontrivial S-matrix in Minkowski space does not admit higher-spin symmetries. Negative arguments of the second kind were based on the direct analysis of the compatibility of higher-spin symmetries with the symmetries of gravity (diffeomorphisms), as in the paper by Aragone and Deser [19].

The proper way started to become clear in the mid-eighties of the last century. By the example of a scalar field, it was found out that there exist conserved higher-spin currents [20–22] that contain higher derivatives:

$$J_s \sim \sum_{n=0}^{s} a_n \partial^n\phi\,\partial^{s-n}\phi\,.$$

The number of derivatives increases with spin. An important conclusion on the structure of interactions of higher-spin fields that agreed with the results of the earlier analysis in the framework of the light-cone gauge [23] was that gauge invariant interactions of higher spins contain higher derivatives:

$$L^3=\sum_{p,q,r}(D^p\varphi)(D^q\varphi)(D^r\varphi)\,\rho^{p+q+r+1/2d-3}\,.$$

The appearance of interactions with higher derivatives requires the introduction of a dimensionful constant ρ, which compensates for extra dimensions carried by higher derivatives. In string theory, the parameter ρ is expressed via string tension: $\rho^2\sim\alpha'$. In the theory of higher-spin gauge fields, which describes massless fields, an independent mass scale is absent. An unexpected way out of this situation is to consider the theory in the space with nonzero curvature $M\sim\lambda=\rho^{-1}$, which sets a nontrivial scale unrelated to the mass scale of the theory.

As a result, choosing de Sitter (dS) or anti-de Sitter (AdS) space as the most symmetric one with a nonzero curvature tensor (for definiteness we will talk about anti-de Sitter space), it can be shown that, while not admitting a consistent formulation in Minkowski space, higher-spin gauge theory admits a formulation in AdS space [24]. This generalization not only allowed avoiding the no-go statements valid in flat space, but also turned out to be preparation for what was at that time an unknown conjecture on the correspondence between conformal theories in d dimensions and a theory in the $(d+1)$ dimensional space of nonzero curvature (AdS/CFT) [25–27].

2. Frame-like formulation as the key to symmetry

Symmetries can be conveniently studied by describing gauge fields as differential forms valued in one symmetry algebra or another. For instance, a spin-1 field is described by a 1-form $A_\nu{}^i{}_j\,(\mu,\nu=0,1,2,3)$ valued in a Yang–Mills algebra g. Spin 2 in the Cartan–Weyl formulation is described by the vierbein $e_\nu{}^a$ and Lorentz connection $\omega_\nu{}^{ab}$. It is useful to identify the $e_\nu{}^a$ and $\omega_\nu{}^{ab}$ fields with the gauge fields of the Lie algebras of the groups of Poincaré iso(1,3), de Sitter SO$(d,1)$, or anti-de Sitter SO$(d-1,2)$. A spin-3/2 field $\psi_\nu{}^\alpha$ admits natural interpretation as a gauge field associated with the generators Q_α of supersymmetry in the supersymmetric extension of the Poincaré or AdS symmetry algebras. (Notice that the dS algebra SO$(d,1)$ allows no consistent supersymmetric extension.)

The frame-like formulation for free fields of an arbitrary spin [28–30] requires the introduction of the following set of fields:

$$e_\nu^{a_1\ldots a_{s-1}}\,,\ \omega_\nu^{a_1\ldots a_{s-1},\,b}\,,\ \ldots,\ \omega_\nu^{a_1\ldots a_{s-1},\,b_1\ldots b_t}\,,\ 0\leqslant t\leqslant s-1\,,$$

which, in turn, dictates the pattern of symmetry parameters associated with the field of a fixed spin s:

$$\varepsilon^{a_1\ldots a_{s-1}}\,,\ \varepsilon^{a_1\ldots a_{s-1},\,b}\,,\ \ldots,\ \varepsilon^{a_1\ldots a_{s-1},\,b_1\ldots b_t}\,,\ 0\leqslant t\leqslant s-1\,.$$

(Both the fields and the symmetry parameters are symmetric traceless tensors with respect to the Lorentz indices a and b, subject to the condition that the symmetrization of any of the indices b with all indices a gives zero.)

The simplest higher-spin algebra with such a set of parameters was originally found for the case of a four-dimensional theory [31]. The spectrum of spins in the higher-spin gauge theory, which possesses such a symmetry, contains fields of all integer spins $s=0,1,2,3,\ldots,\infty$, precisely corresponding to the spin spectrum conceived as most natural to Ginzburg.

One of the important properties of higher-spin symmetries is that fields of lower-spins $s=0,1,2$ do transform under the higher-spin symmetry transformations. In particular, the metric tensor loses covariant meaning in the framework of higher-spin gauge theory. Implying that the notion of a distance between infinitesimally closed points of spacetime has no invariant sense in the higher-spin gauge theory, this property itself indicates the nonlocality of the latter. Finite-dimensional subalgebras of higher-spin algebras correspond to the sets of fields of lower-spins $s \leqslant 2$ associated with supergravity. One can expect that these fields can remain massless (light) after the spontaneous breaking of higher-spin symmetries to their finite-dimensional subgroups, precisely corresponding to the class of field-theoretical models considered in the modern theories of fundamental interactions. This scenario precisely corresponds to a picture where present-day field-theoretical models should correspond to a low-energy approximation of some complete nonlocal theory.

The fact that higher-spin symmetries mix fields of all spins means that the spin-2 field should not play a preferred role in phase with unbroken higher-spin symmetries. Nevertheless, we assume that, as any other theory in the framework of gravity, higher-spin gauge theory should be formulated in a coordinate-independent form in agreement with the Einstein equivalence principle. To preserve the independence from the coordinate choice without the explicit use of a metric, it is very useful to apply the Cartan formalism of exterior forms. The key property of this formalism is that antisymmetrized derivatives of antisymmetric tensors

$$\partial_{[\nu_1} A_{\nu_2 \ldots \nu_{p+1}]} \tag{2.1}$$

turn out to be automatically covariant without introducing Christoffel symbols, because the latters can always be chosen symmetric with respect to lower indices, hence dropping out of the expressions fully antisymmetrized with respect to the world indices ν_i. The compact form of formula (2.1), viz.

$$dA\,, \quad d = \mathrm{d}x^{\nu}\,\frac{\partial}{\partial x^{\nu}}\,, \quad A = \mathrm{d}x^{\nu_1} \wedge \ldots \wedge \mathrm{d}x^{\nu_p} A_{\nu_1 \ldots \nu_p}\,,$$

is achieved by virtue of introducing anticommutative symbols

$$\mathrm{d}x^{\mu} \wedge \mathrm{d}x^{\nu} = -\,\mathrm{d}x^{\nu} \wedge \mathrm{d}x^{\mu}\,.$$

The central fact expressing the symmetry of second derivatives consists in the following:

$$d^2 = 0\,. \tag{2.2}$$

As a consequence of formula (2.2), the Abelian field strength $F = dA$ turns out to be gauge invariant:

$$\delta A(x) = d\varepsilon(x)\,, \qquad \delta F = 0\,.$$

The non-Abelian generalization is achieved via covariant derivative extension

$$d \to D = d + \omega\,, \qquad \omega(x) = \mathrm{d}x^{\nu}\omega_{\nu}(x)\,,$$

where the 1-form[3] ω is valued in some matrix or operator algebra (higher-spin algebra in the case under consideration).

Higher-spin gauge fields in four dimensions take values in the algebra of functions of oscillators:

$$\omega(\hat{Y}|x)\,, \quad [\hat{Y}_A\,, \hat{Y}_B] = 2\mathrm{i}C_{AB}\,, \quad C_{AB} = -C_{BA}\,,$$

where $\hat{Y}_A$ is a noncommutative spinor, and C_{AB} is the charge conjugation matrix. $A, B = 1, \ldots 4$ are Majorana indices in four dimensions.

Spin-s fields are described by homogeneous polynomials of $\hat{Y}$:

$$\omega(\mu\hat{Y}|x) = \mu^{2(s-1)}\omega(\hat{Y}|x)\,.$$

This construction is analogous in many respects to the Ginzburg–Tamm construction. The difference is that $\omega(\hat{Y}|x) = \mathrm{d}x^{\nu}\omega_{\nu}(\hat{Y}|x)$ depends on the auxiliary spinor $\hat{Y}_A$, rather than on the vector, and carries the differential form index ν. The latter circumstance is, however, quite significant, providing natural realization of higher-spin symmetries with 0-forms $\epsilon(\hat{Y}|x)$ as gauge parameters.

3. Unfolded dynamics

The formulation in terms of differential forms has a number of advantages, allowing, in particular, a representation of equations in partial derivatives in the so-called unfolded form. This formulation is based on the direct generalization of the well-known trick allowing one to represent ordinary differential equations in the form of first-order equations:

$$\dot{q}^{\,i}(t) = \varphi^{\,i}\big(q(t)\big)\,,$$

by virtue of introducing new variables for all those derivatives of the dynamical variables that are not determined by the original equations. Such a formulation has a number of advantages, allowing, in particular, the control of a number of degrees of freedom, which coincides with the number of dynamical variables.

The field theory studies systems with an infinite number of degrees of freedom, described by functional spaces. In the Hamiltonian formulation of the Maxwell theory, for example, generalized coordinates are identified with the space components of the vector potential $\mathbf{A}(x)$, while generalized momenta are identified with components of the electric field $\mathbf{E}(x)$. Given all merits of the Hamiltonian approach to the field theory, its substantial disadvantage is the loss of covariance with respect to both Lorentz symmetry and the ambiguity in the coordinate choice.

Unfolded dynamics represents a multidimensional covariant generalization of the first-order formulation of ordinary differential equations, achieved by virtue of the replacement of the time derivative by the exterior differentiation, and a set of variables $q^{\,i}(t)$ by a set of differential forms $W^{\Omega}(x)$ which play the role of dynamical variables:

$$\frac{\partial}{\partial t} \to d\,, \quad q^{\,i}(t) \to W^{\Omega}(x) = \mathrm{d}x^{\nu_1} \wedge \ldots \wedge \mathrm{d}x^{\nu_p} W^{\Omega}_{\nu_1 \ldots \nu_p}(x)\,.$$

Unfolded equations have the form [32]

$$dW^{\Omega}(x) = G^{\Omega}\big(W(x)\big)\,, \quad d = \mathrm{d}x^{\nu}\partial_{\nu}\,, \tag{3.1}$$

where $G^{\Omega}(W)$ is some function of dynamical differential forms $W^{\Omega}(x)$:

$$G^{\Omega}(W) = \sum_{n=1}^{\infty} f^{\Omega}{}_{A_1 \ldots A_n} W^{A_1} \wedge \ldots \wedge W^{A_n}\,.$$

Due to the use of the language of differential forms, equations (3.1) turn out to be coordinate-independent, i.e., are insensitive to the coordinate choice.

For $d > 1$, compatibility conditions with the property (2.2) impose nontrivial restrictions on the form of functions

[3] A polynomial of degree p of $\mathrm{d}x^{\nu}$ or, equivalently, an antisymmetric tensor of rank p is called p-form.

$G^{\Omega}(W)$:

$$G^{A}(W)\wedge\frac{\partial G^{\Omega}(W)}{\partial W^{A}}\equiv 0\,, \qquad (3.2)$$

equivalent to generalized Jacobi identities

$$\sum_{n=0}^{m}(n+1)f^{\Gamma}{}_{[A_1\ldots A_{m-n}}f^{\Omega}{}_{\Gamma A_{m-n+1}\ldots A_m]}=0\,.$$

The problem is to find such functions $G^{A}(W)$ that satisfy (3.2) for any W^{Ω}.

The unfolded form of equations possesses a number of remarkable properties.

First of all, being coordinate independent, unfolded equations are ideally adjusted for the description of gravity. The use of the formalism of differential forms also guarantees gauge invariance of equations (3.1) under the gauge transformations

$$\delta W^{\Omega}=d\varepsilon^{\Omega}+\varepsilon^{A}\,\frac{\partial G^{\Omega}(W)}{\partial W^{A}}\,,$$

where the gauge parameter $\varepsilon^{\Omega}(x)$ is a $(p_{\Omega}-1)$-form if the corresponding gauge field W^{Ω} is a p_{Ω}-form (0-forms W^{Ω} have no gauge parameters).

An important property of the unfolded formulation is its general applicability. Any system of partial differential equations can be represented in the unfolded form by virtue of introducing an appropriate set of auxiliary variables. Interactions are described as nonlinear deformations of the function $G^{\Omega}(W)$ in Eqn (3.1).

Unfolded equations allow a fruitful interpretation in terms of Lie algebras and their cohomology that, in particular, provide the possibility of systematical classification of g-invariant equations in terms of g-modules (see, e.g., Refs [33, 34]).

The degrees of freedom of a dynamical system formulated in the unfolded form are described by a subset of 0-forms $C^{i}(x)$ of the full set of forms $W^{\Omega}(x)$. p-forms W^{Ω} of nonzero degrees $p^{\Omega}>0$ are determined by 0-forms up to the gauge transformations. Values of $C^{i}(x_0)$ at any $x=x_0$ determine the local evolution of the system, similarly to how $q(t_0)$ determines the local evolution for ordinary differential equations rewritten in the first-order form. The space of fields C^{i} is analogous (dual) to the space of single-particle states of the corresponding field theory.

A surprising property of the unfolded formulation is that space–time coordinates x play a secondary role. In this language, spacetime geometry turns out to be encoded by the function $G^{\Omega}(W)$.

If the set of functions $W^{\Omega}(x)$ can be described by a finite set of functions $W(Y|x)$ of auxiliary variables Y_A, unfolding acquires the meaning of the covariant Penrose transform (i.e., the twistor transform).

4. Nonlinear higher-spin equations

The nonlinear higher-spin dynamics is formulated in terms of star-product

$$(f\star g)(Z,Y)=\int \mathrm{d}S\,\mathrm{d}T f(Z+S,Y+S)\times g(Z-T,Y+T)\exp\left(-\mathrm{i}S_A T^{A}\right), \qquad (4.1)$$

which describes the associative algebra of oscillators that satisfy the relations

$$[Y_A,Y_B]_\star=-[Z_A,Z_B]_\star=2\mathrm{i}C_{AB}\,,\quad Y^{A}=C^{AB}Y_B\,,$$

$$Z_A=(z_\alpha,\bar z_{\dot\alpha})\,,\quad Y_A=(y_\alpha,\bar y_{\dot\alpha})\,,\quad \alpha,\ \dot\alpha=1,2\,.$$

More precisely, product (4.1) describes the normal ordering of the oscillators $Z-Y$ and $Z+Y$. The star-product (4.1) admits the introduction of inner Klein operators

$$\kappa=\exp\left(\mathrm{i}z_\alpha y^{\alpha}\right),\quad \bar\kappa=\exp\left(\mathrm{i}z_{\dot\alpha}\,y^{\dot\alpha}\right)$$

possessing the following properties

$$\kappa\star f(Z,Y)=f(\widetilde Z,\widetilde Y)\star\kappa\,,\qquad \kappa\star\kappa=1\,,$$

where $(\tilde a_\alpha,\tilde{\bar a}_{\dot\alpha})=(-a_\alpha,\bar a_{\dot\alpha})$.

The full system of higher-spin equations can be written out in the form [35, 36]

$$dW+W\star W=0\,, \qquad (4.2)$$

$$dB+W\star B-B\star W=0\,, \qquad (4.3)$$

$$dS+W\star S+S\star W=0\,, \qquad (4.4)$$

$$S\star B-B\star S=0\,, \qquad (4.5)$$

$$S\star S=\mathrm{i}(\mathrm{d}z^{\alpha}\,\mathrm{d}z_\alpha+\mathrm{d}\bar z^{\dot\alpha}\,\mathrm{d}\bar z_{\dot\alpha}+\mathrm{d}z^{\alpha}\,\mathrm{d}z_\alpha F(B)\star k\star\kappa + \mathrm{d}\bar z^{\dot\alpha}\,\mathrm{d}\bar z_{\dot\alpha}\bar F(B)\star\bar k\star\bar\kappa)\,, \qquad (4.6)$$

where $W=\mathrm{d}x^{n}W_n(Z;Y;K|x)$ and $S=\mathrm{d}z^{\alpha}s_\alpha(Z;Y;K|x)+\mathrm{d}\bar z^{\dot\alpha}\,\bar s_{\dot\alpha}(Z;Y;K|x)$ describe 1-form connections in the space-time with coordinates x and in the noncommutative space with coordinates Z. The 0-form $B(Z;Y;K|x)$ serves as a generating function for the curvatures of higher-spin gauge fields and for lower-spin fields. $f(B)$ is an arbitrary star-product function of the 0-form B:

$$f(B)=\sum_{n=1}^{\infty}f_n\,\underbrace{B\star\ldots\star B}_{n}\,.$$

The Klein operators $K=(k,\bar k)$ generate chirality transformations

$$k\star f(A)=f(\tilde A)\star k\,,\quad \bar k\star f(A)=f(-\tilde A)\star\bar k\,,$$

$$A=(A_\alpha,\bar A_{\dot\alpha}):\quad \tilde A=(-A_\alpha,\bar A_{\dot\alpha})\,,$$

which act not only on the functions of Y and Z, as the operators κ and $\bar\kappa$ do, but also on the differentials of noncommutative coordinates $\mathrm{d}Z$. It should be noted that $k\bar k$ is the generator of total boson–fermion parity.

Equations (4.2)–(4.6) are invariant under the gauge transformations

$$\delta W=\varepsilon\star W-W\star\varepsilon\,,\quad \delta S=\varepsilon\star S-S\star\varepsilon\,,$$

$$\delta B=\varepsilon\star B-B\star\varepsilon\,,$$

where the gauge parameter $\varepsilon=\varepsilon(Z;Y;K|x)$ is an arbitrary function of its arguments.

A remarkable feature of equations (4.2)–(4.6) is that all those equations that contain derivatives with respect to space-time coordinates via exterior differential d, i.e., equations (4.2)–(4.4), have the form of zero-curvature equations and covariant constancy equations. It is not hard to write down their explicit local solution in the pure gauge form. As a result, it turns out that all the information about solutions of the nonlinear system of higher-spin equations is encoded in equations (4.5) and (4.6), which describe the expression for the curvature of the noncommutative space of the variables Z in terms of the 0-form B. These equations admit an interesting interpretation: they describe a two-dimensional quantum hyperboloid of radius $B(x)$ in the noncommutative space of Y_A and Z_A.

On the other hand, solving equations (4.5) and (4.6) order-by-order and substituting the result into equations (4.2)–(4.4) gives the unfolded form of massless equations [32] with all nonlinear corrections. Although unfolded equations have the form of first-order equations, because the fields W, S, and B contain an infinite number of auxiliary fields, interaction terms contain fields of all spins, along with all their derivatives.

In contrast to most of the known problems of classical field theory, nonlinear higher-spin equations contain no low-energy expansion parameter. Indeed, the dimensionless combination composed from the derivative and space–time curvature ρD_{ν}, where ρ is a typical radius scale of the background spacetime, while D_{ν} is the background covariant derivative, cannot be regarded as small because, when acting on one tensor or another, $\rho^2[D_{\mu}, D_{\nu}]$ turns out to be a dimensionless matrix of order one.

In other words, the higher-spin equations (4.2)–(4.6) describe interaction vertices with all degrees of derivatives of dynamic fields, which can make a competing contribution. The structure of interactions is determined by the higher-spin symmetries. Thus, on the one hand, higher-spin symmetries lead to the nonlocality of the interacting higher-spin theory, while, on the other hand, they fully determine the structure of this nonlocality. The nonlocal character of nonlinear higher-spin equations does not allow using for their analysis many standard means of field theory and general relativity (GR), such as, e.g., low-energy expansion or geodesic motion, demanding the development of alternative approaches.

5. Recent progress and prospects

At present, higher-spin theory is going through the stage of impetuous development. Let us list here some of the latest results and avenues of investigations, in most cases confining ourselves merely to references to recent publications where one can find a more comprehensive review of the available literature.

Nonlinear equations (4.2)–(4.6) for symmetric higher-spin fields in four dimensions were generalized to any number of dimensions in Ref. [37].

Extension of the higher-spin theory to gauge fields of any symmetry type is not yet completed, even at the level of free fields. One surprising phenomenon related to this problem is that the very notions of a free field in Minkowski and AdS spaces differ for massless fields of any symmetry type: in most cases, an irreducible field in the AdS space reduces in the flat limit to a number of elementary fields in Minkowski space [38, 39]. During recent years, much attention has been paid to the analysis of free fields with an arbitrary type of symmetry (see, e.g., papers [40–51] and references therein).

An interesting area is related to the study of conformal higher-spin fields [33, 34, 52–56]. Although in most cases these systems turn out to be nonunitary, their study is of considerable interest because they allow analyzing unitary field-theoretical models as conformal models with spontaneously broken conformal symmetry, which not only leads to technical simplifications but also may help to find fundamental symmetries of the theory.

One more area of research is related to the construction of cubic interaction vertices of massive and massless fields of arbitrary spin for any number of dimensions, both in Minkowski and in AdS spaces [57–71]. Of great interest is work on the derivation of scattering amplitudes of higher-spin fields from string theory [72–74].

Considerable efforts are aimed at the development of the general theory of unfolded equations and clarification of their relation with other approaches to dynamical systems and applications [34, 49 – 51, 75 – 80].

A distinguished position is occupied by the problem of finding exact solutions to complete nonlinear higher-spin equations, which essentially differs from most known problems of classical field theory in that the equations under consideration possess no low-energy expansion parameter. At present, very few exact solutions of nonlinear higher-spin equations are known in three and four dimensions [81–84]. One of the most interesting ones is a spherically symmetric exact solution of the four-dimensional higher-spin theory [85], which in the weak field regime behaves as the black hole solution of GR. It is a very interesting problem to analyze strong-field phenomena related to this solution.

One more important and interesting avenue of investigations is related to the Sp(8)-invariant description of four-dimensional massless fields in a ten-dimensional space $x_{\alpha\dot{\alpha}} \to X_{AB} = X_{BA}$ $(A, B = 1, 2, 3, 4)$ [34, 87–90]. Possessing a number of remarkable properties, this formulation allows, in particular, new insight into such a fundamental concept underlying Einstein's approach to spacetime as a local event, i.e., a point of spacetime.

One of the most unusual features of the theory of higher-spin gauge fields is that they admit a nontrivial interaction only in a curved space such as AdS space [24]. This property, which seemed strange at the first stage, later on acquired deep meaning in the context of the AdS/CFT-correspondence conjecture [25–27]. A possible interpretation of higher-spin theories in terms of AdS/CFT correspondence has been discussed by various authors. In the context of the four-dimensional higher-spin theory described in this paper, Klebanov and Polyakov [91] (see also paper [92]) put forward a hypothesis for its duality to the three-dimensional O(N)-sigma model in the limit $N \to \infty$. An explicit verification of this hypothesis turned out to be quite laborious and was performed only recently [93, 94].

Apart from the work on AdS_4/CFT_3 correspondence, there are a number of interesting papers on establishing AdS_3/CFT_2 correspondence between three-dimensional higher-spin theories and two-dimensional conformal theories [95–98], and even on the analysis of the AdS_{d+1}/CFT_d correspondence for higher-spin theories for any number of dimensions [99, 100]. At the free-field level, important results in this area were also obtained by Metsaev [101, 102].

Despite the considerable progress achieved over recent years, a number of interesting questions in higher-spin theory remain to be solved. We can mention the construction of nonlinear equations for mixed-symmetry fields, the construction of a complete nonlinear action in higher-spin theory, [4] a deeper understanding of higher-spin geometry, an accurate

[4] Interesting suggestions proposed recently in the papers [103–105], which generalize the old remark in the paper [32], unlikely close this problem.

definition of the notion of (non)locality in higher-spin theory, clarification of the relation with string theory, and many others.

In conclusion, I would like to reproduce a statement by Vitaly Lazarevich Ginzburg made about five years ago. In his speech to newcoming students at the Chair of Problems of Physics and Astrophysics at MIPT, Ginzburg mentioned his passion for higher-spin theory in a somewhat unexpected context, saying that anyone has to realistically estimate the limits of their abilities, in time leaving behind too hard problems, as he himself at the time quit his work on the higher-spin theory (see also Ref. [5]). This is the edifying example of the sober assessment, not so much of his own abilities, but of the state of science at the time the decision was taken. Indeed, at the time Vitaly Lazarevich was talking about, there remained a quarter of a century till the discovery of supergravity, leading to the perception of the fundamental role of the principle of gauge invariance in the higher-spin theory. Before this had happened, the chances of real progress in the theory were as little as the chances of constructing the theory of electroweak interactions before the development of Yang–Mills theory.

Turning out to be remarkably deep and promising, the higher-spin theory is now going through a true renaissance, probably leading to a new understanding of a number of fundamental physical concepts. Still, knowing too little about the higher-spin theory as a whole, we already know enough to claim that today is probably the best time for research on this extremely interesting and quickly developing area.

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V L Ginzburg and the development of experimental work on high-temperature superconductivity at LPI: 'iron superconductors'

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1. Introduction

One day in 2006, one of the authors of this paper (VMP) received a surprise phone call with an exciting proposal. The caller was V L Ginzburg, and the proposal was to undertake research into high-temperature superconductivity (HTSC) to develop superconductors with the critical temperature above room temperature: a worthwhile effort, Ginzburg convincingly explained, because it was of exceptional practical importance and because no theoretical reason was known to forbid room-temperature superconductivity (RTSC).

What does changing the subject mean for an experimentalist? First, the scale of the proposed research ruled out small group work and required the effort of most, if not all, of the laboratory, thus necessitating that the personnel be freshly trained and undergraduate programs be set up to prepare specialists in the new field. Second, the project needed to be financed and equipment and materials had to be purchased. Finally, it was necessary to find funds for refitting the building for different experimental work and to develop a redesign project for the existing building, as a whole and in parts, to optimize the operation of the new equipment. It was not until after three years [1] of these types of concerns that the first experiments were carried out to synthesize and study

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high-temperature superconductors. Ginzburg showed keen interest in how the work was progressing and was constantly briefed on it [2]. This talk is, in fact, a regular progress report which should but alas cannot be submitted to Ginzburg.

At about the same time, in 2008, a new class of high-temperature superconductors, based on iron arsenides and iron selenides rather than cuprate compounds, was discovered [3–6], providing a logical starting point for the new experimental project. Because 'broad front' research was planned — and this with no experience in fields such as material science and analytical diagnostics — the cooperation of the Lebedev Physical Institute, Russian Academy of Sciences (LPI), with research bodies possessing the necessary experience and technology was absolutely necessary. Accordingly, a number of laboratories of Moscow State University's chemical and physical departments became involved in the project, as did the Institute for High Pressure Physics (HPPI), Russian Academy of Sciences. The first physical results on 'iron' superconductors from the collaboration of the LPI high-temperature superconductivity department with these institutions are summarized in this talk.

2. Briefly on the properties of 'iron' superconductors

There are some similarities and numerous differences found in comparing the properties of cuprate oxides and the new, 'iron' superconductors [7]. To date, a number of types of such materials have been synthesized and studied, which are classified as 1111 (50 K), 122 (40 K), 111 (18 K), 11 (8 K), and 22438 (40 K). Their typical representatives are REFeAsO(F) (where $RE =$ Sm, La, Dy, Eu, Th, Gd, and so on) [8, 9], Ba(K)Fe_2As_2 [10–12], LiFeAs [13, 14], FeSe(Te) [15], and $Fe_2As_2Ca_4(Sc,Ti)_3O_8$ [16]. Detailed state-of-the-art reviews of the subject were given in Refs [7, 17–19]. Similarly to the cuprates, the new compounds are layered, with layers of Fe, where electrons condense into superconducting pairs, spatially separated from oxygen layers that supply charge carriers when the composition deviates from the stoichiometry.

Unlike the cuprates, stoichiometric (undoped) FeAs materials are not insulators above T_c but show band conduction, and hence are metals, if poor ones. As the temperature is decreased, they undergo a structural transition from tetragonal to orthorhombic spin ordering (at $T \approx 150$ K) and then, at an even lower temperature ≈ 130 K, a magnetic transition leading to the antiferromagnetic (AF) spin ordering in the Fe sublattice. The anomalously strong magnetostructural coupling — or, more precisely, the coupling between the spin state of Fe and the lattice structure (the displacement of As atoms) — was discovered by comparing inelastic neutron scattering data [20] for the normal state with the calculated phonon spectrum [21, 22]. It turned out that the calculated and measured [20] positions of peaks in the phonon spectrum were in marked disagreement (up to 14%), which could not be removed unless the magnetic moment of Fe was taken into account. For the same reason, the calculated positions of As atoms in the lattice turn out to be strongly dependent on the magnetic moment of Fe atoms, and the calculated lattice constant along the c-axis is 10% (!) less than measured if the calculations ignore the magnetic moment of Fe.

A deviation from stoichiometry, due either to an induced oxygen deficiency or to oxygen being in part substituted by fluorine, suppresses the antiferromagnetic ordering of the Fe sublattice and gives rise to a superconducting (SC) state [7, 17, 18, 19, 23, 24]. The facts that the magnetic and structural transitions have their temperatures close to the critical temperature $T_c \approx 50$ K and that magnetostructural coupling is anomalously strong suggest a dominant role of phonons and possibly of spin fluctuations in superconducting pairing [25–28].

The key issues addressed in the study of these materials include the mechanisms of various types of doping, the carrier pairing mechanism, the order parameter symmetry, the quasiparticle energy spectrum, the possible existence of a pseudogap state, and the superconducting gap values. The following intriguing properties of this class of superconductors have stimulated great interest in their study: the possibility (currently being discussed) of an unusual order parameter symmetry, an unusually strong coupling between spin fluctuations and phonons, the emergence of an SC state regardless of the Mott insulator (in contrast to cuprates), the competition between AF ordering and SC pairing, and the existence of spin ordering of rare earth elements in the SC phase below T_c [23].

The essential point is that the isotope effect due to the substitution of ^{16}O by the ^{18}O isotope turned out to be much weaker that the iron isotope effect (i.e., substitution of ^{58}Fe for ^{56}Fe), confirming that electron pairing predominantly occurs in Fe layers. The isotope effect exponent $\alpha = 0.4$ is close to the standard Bardeen–Cooper–Schrieffer (BCS) theory prediction 0.5. Band structure calculations show that the total density of states at the Fermi level is formed from 3d atomic states of Fe [29, 30] and that the critical temperature correlates with the density of states [30] (evidence of superconducting pairing assisted by phonons in these compounds).

Knight shift measurements in 122 and 1111 compounds [31–33] show conclusively that the superconducting pairing is singlet and hence the coordinate wave function of the condensate must be antisymmetric. For the superconducting gap, s and d symmetries are possible. Data from different experiments are still inconsistent and can be interpreted as favoring multiband superconductivity with either the s^{++} [34] or $s^{\pm}$ [32, 33, 35] or d symmetry (see Refs [7, 17] for a detailed review of available experimental data); this problem remains unsolved experimentally. If experimentally confirmed, the theoretically proposed $s^{\pm}$ symmetry [25–28] would imply the existence of a previously unencountered type of multiband superconductivity, with the sign of the order parameter different for two different condensates, at the Γ and M points. It has also been predicted theoretically that for the order parameter with this type of symmetry, the superconducting condensate can coexist with antiferromagnetic order [36].

The magnitude Δ and the structure of the superconducting gap are closely related to the pairing mechanism. ARPES measurements, even when performed at the temperature 0.3 K, do not yet provide a sufficient (≈ 0.01 meV) resolution to reveal the fine structure of the superconducting gap. In addition, ARPES spectra are difficult to interpret because measuring photoemission involves only a thin near-surface layer of the material and because the surface itself undergoes a rearrangement [37] that changes the spectrum of the near-surface layer. As a result, the information on the gap comes almost exclusively from microcontact spectroscopy in either the tunneling (T) or the Andreev reflection (AR) regime. It turns out, however, that different experiments give different results [38–41], even when using similar measurement methods and performed on similar 1111-class materials (including the most studied compound, SmO(F)FeAs). Such differences may be not only quantitative but also qualitative: some experiments report on the d-wave symmetry, on single-band superconductivity, and, finally, on two-band superconductivity.

These experimental inconsistencies are partly due to the fact that the quaternary 1111 compounds are difficult to synthesize: they are currently available almost exclusively in a polycrystalline form and are usually not homogeneous. Moreover, in a number of experiments, surfaces not cleaned in advance or not cleaved under high-vacuum cryogenic conditions were studied by microcontact spectroscopy.

3. Sample synthesis and preparation

In recent collaborative work with LPI, Moscow State University (MSU), and the Institute of High Pressure Physics, type-1111 HTSC compounds CdFeAsO(F), DyFeAsO(F), CeFeAsO(F), and EuFeAsO(F) were synthesized, for which microcontact spectroscopic studies in the superconducting state were carried out for the first time, the presence of two superconducting gaps in the superconducting state was revealed, and the gap values were measured. The maximum critical temperature, $T_c = 52 - 53$ K, was observed for $GdFeAsO_{1-x}F_x$ samples with the doping level $x = 0.12$ [42].

Polycrystalline samples of $GdFeAsO_{1-x}(F_x)$ were prepared by high-pressure synthesis [42, 43]. The chips of high-purity (99%) Gd and As, and powders of FeF_3, Fe, and Fe_2O_3 (99.9%) were used as starting materials. The precursors GdAs and FeF_3 produced in the first stage were mixed in a nominal proportion with Fe and Fe_2O_3 and pressed into pellets 3 mm in diameter and 3 mm in height. These were then put into a boron nitride crucible and a synthesis process was carried at about 50 kbar and at 1350 °C for 60 min.

Figure 1 presents the results of quantitative X-ray analysis of the powder diffraction data ($CuK_{\alpha 1}$-radiation, reflection geometry) obtained with Bruker-D8 Advance diffractometer using the Rietveld full-profile refinement method. The X-ray diffraction patterns indicate that the materials studied have a fine polycrystalline structure dominated by the 1111-type phase and contains an admixture of secondary phases (in particular, FeAs and Gd_2O_3). The primary phase, GdFeAsO(F), has the space group symmetry $P4/nmm$ and unit cell constants $a = 3.8982(3)$ Å and $c = 8.4059(9)$ Å. Because admixture phases decrease in content inward from the surface, the surface layer was polished away. The subsequent elemental analysis of sample surfaces using the JSM-700IFA scanning microscope with an EDX (Energy-Dispersive X-ray) extension shows that the Gd-excess regions were distributed randomly in the form of grains about 1 μm in size. Figure 2 demonstrates the results of the local elemental composition analysis over a region $\approx 1\ \mu m^2$ in area ('spectrum 1'). The table shows the percentage of each of Gd, Fe, and As in five randomly chosen regions of the sample (the oxygen content failed to be reliably measured) and average values over an area of 1.175 mm^2.

As confirmed by magnetic measurements [42], a bulk superconducting phase is present in synthesized samples. Shown in Fig. 2 is the temperature dependence of the resistance $R(T)$ for the approximately optimal composition $x = 0.12$ for a number of magnetic field values; the sharp peak of the derivative dR/dT (Fig. 3b) indicates the narrow width

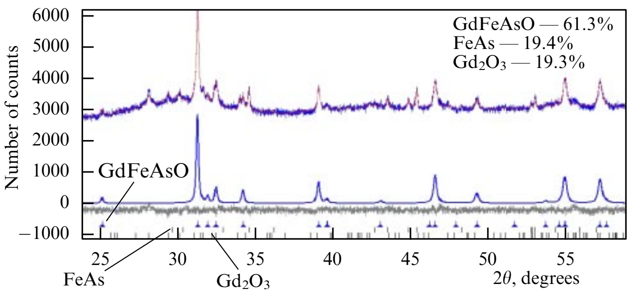

Figure 1. Measured profile (upper curve), refined results, and peak positions of the Gd-1111 phase (middle curve) and the difference between the measured and fitted spectra (lower curve).

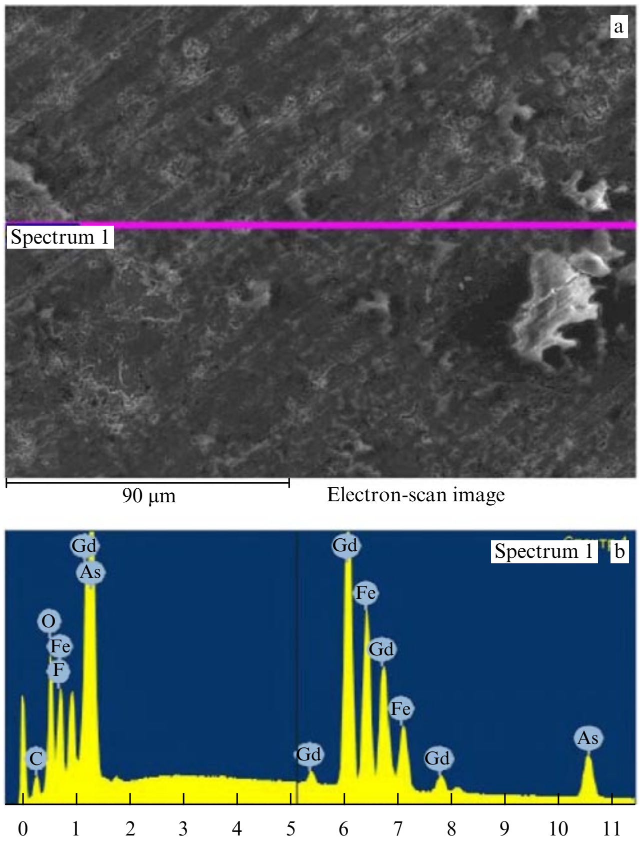

Figure 2. (a) Scanning electron microscope image of the sample surface and (b) the local spectrum of the characteristic lines of elements for the surface region designated as 'Spectrum 1.' Abscissa: energy in keV.

Table. Percentage content of Fe, As, and Gd in different regions of the sample surface (in atomic percent).

Sample regions	Fe	As	Gd
2	11.74	11.36	20.55
4	14.64	15.43	19.16
5	16.11	15.29	17.19
6	16.17	13.57	18.70
7	14.44	15.30	18.12
Average content over the area 350 × 500 μm^2	14.1±2.2	15.5±1.9	18±11

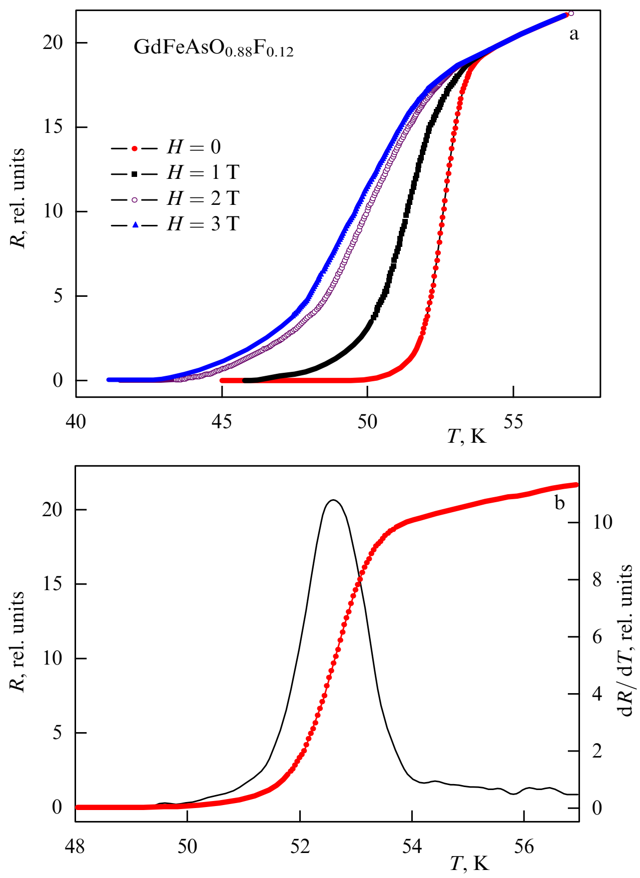

Figure 3. (a) Temperature dependence of resistance for three magnetic field values for a GdFeAsO(F) sample of an approximately optimal composition [37]. (b) Derivative $\mathrm{d}R(T)/\mathrm{d}T$ for $H = 0$.

of the superconducting transition. The superconducting transition temperature $T_c \approx 52.5$ K, determined from the maximum of the derivative $\mathrm{d}R(T)/\mathrm{d}T$, is slightly less than the maximum value $T_c = 55.4$ K that has been achieved for this class of compounds (specifically, for SmFeAsO(F) [8]). In a magnetic field, the superconducting transition is broadened and T_c decreases monotonically (Fig. 3a), a behavior typical of FeAs superconductors [7, 17, 43]. Measurements [42] of $T_c(H)$ yielded the estimate $|\mathrm{d}H_{c2}/\mathrm{d}T| \approx 3$ T K^{-1}, comparable to the optimally doped superconductor SmFeAsO(F) [43] with close values of T_c. Using the above value of the derivative, the critical field is estimated as $H_{c2} \sim 130$ T, implying good prospects for high magnetic field applications of FeAs superconductors.

Along with polycrystalline 1111-system samples, we also studied variably doped single crystals of the 122 system; in these, the optimal doping T_c reached a value about 34 K.

4. Measuring the specific heat jump at the superconducting transition

The specific heat jump at the superconducting transition has been measured in the single crystals of 122 compounds $Ba_{1-x}K_xFe_2As_2$, $Ba(Fe_{1-x}Co_x)_2As_2$, and $Ba(Fe_{1-x}Ni_x)_2As_2$, varying in the type and level of doping. In ordinary superconductors, measurements of the jump provide information on the electron density of states at the Fermi level, the electron–electron coupling constant, and the volume of the superconducting phase. In the BCS model, the jump in the

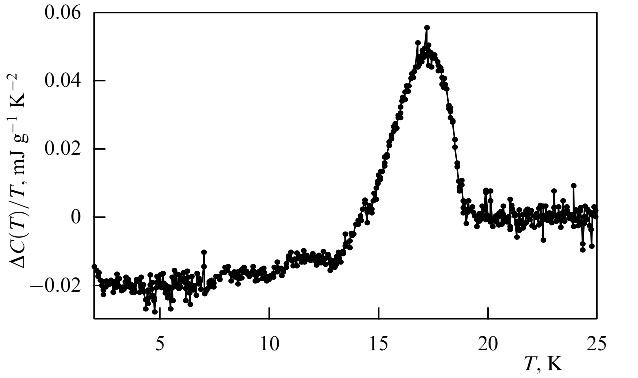

Figure 4. $\Delta C(T)/T = C(T, H = 0)/T - C(T, H = 9\ \mathrm{T})/T$ is plotted for an electron-doped sample of $Ba(Fe_{1-x}Ni_x)As_2$ [44].

electronic specific heat is given by $\Delta C = 1.43\gamma T_c$, where γ is the electronic specific heat coefficient. This relation holds well for low-temperature superconductors.

For high-temperature superconductors, the situation is much more complex. In particular, underdoped phases of HTSC cuprates show no specific heat jump at the superconducting transition point at all [44]. Still, such samples can achieve the critical temperature above 60 K. Importantly, because such samples are structurally perfect, there is no reason to speak of their crystal inhomogeneity as a possible mechanism behind the blurring of the transition.

In [44], specific heat was measured in magnetic fields from 0 to 9 T to determine the jump $\Delta C/T_c$ at $T = T_c$. The field $H = 9$ T shifted the transition to lower temperatures, and the magnitude of the jump was estimated from the difference curve $\Delta C(T)/T = C(T, H=0)/T - C(T, H=9\ \mathrm{T})/T$ (Fig. 4).

An important result of that work was to establish an empirical relation between the transition temperature T_c and the magnitude of the specific heat jump in 122 iron pnictides. It was found that the specific heat jump is given by $\Delta C/T_c \propto T_c^2$, showing that it is determined only by the transition temperature T_c and not by whether the material is hole-doped or electron-doped or by the dopant concentration (Fig. 5). This agrees well with the results in Ref. [45]. The data obtained lend indirect support to the assumption that similarly to the cuprates, the 122 iron pnictides have both the specific heat jump and T_c determined by a single parameter. The reason for this unusual ΔC versus T_c dependence is not yet clear. It is only to be hoped that future

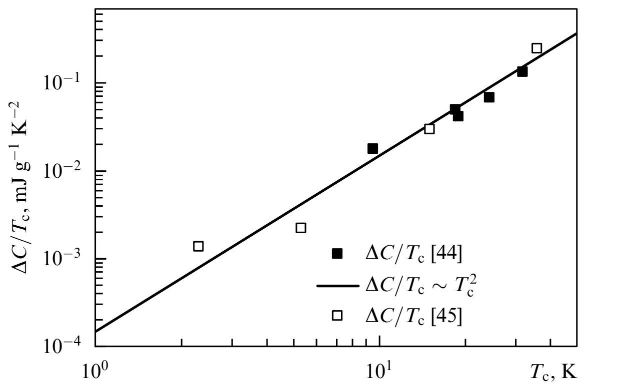

Figure 5. Specific heat jump $\Delta C/T_c$ as a function of T_c for type 122 iron pnictides. Black squares: results in Ref. [44], white squares: data in Ref. [45].

research will elucidate the physical mechanism behind this relation.

5. Measuring the characteristics of a 'superconductor–normal metal–superconductor' (SNS) microcontact

The microcontact spectroscopy studies of Gd-1111 either used the multiple Andreev reflection spectroscopy [46] of individual 'superconductor–constriction–superconductor' microjunctions [47] (with the constriction acting as the normal metal) or relied on the intrinsic multiple Andreev reflection spectroscopy of stacked contacts, which often arise due to the presence of steps and terraces on pure cryogenic cleaves of a crystal. In these studies, synthesized pellet-shaped samples were cut into thin rectangular plates ($2 \times 1 \times 0.1$ mm^3), which were mounted on the microjunction spectrometer stage. The spectrometer holder was subjected to controlled precision bending at the temperature 4.2 K to create a crack in the sample. The details of this 'break junction' measuring technique can be found in Refs [48, 49].

Figure 6 shows the measured current–voltage characteristic $I(V)$ and its derivatives $\mathrm{d}I(V)/\mathrm{d}V$ and $\mathrm{d}^2I(V)/\mathrm{d}V^2$ for a single SNS microcontact on a polycrystalline sample of $GdO_{0.88}F_{0.12}FeAs$ [50]. The symmetric $I(V)$ characteristic seen in the figure is typical of 'pure' SNS microcontacts with a moderate excess current [47, 51]. The differential conductance $\mathrm{d}I(V)/\mathrm{d}V$ exhibits a series of dips, at $V = 22$, 11, and 5 mV. In the case of multiple Andreev reflection, dips should appear at the SNS contact voltages $V_n = 2\Delta/en$ with integer $n = 1, 2, \ldots$. Therefore, the first two features can be associated with $n = 1, 2$ and the local value of the superconducting gap can be estimated as $2\Delta = 22$ meV. For a two-gap

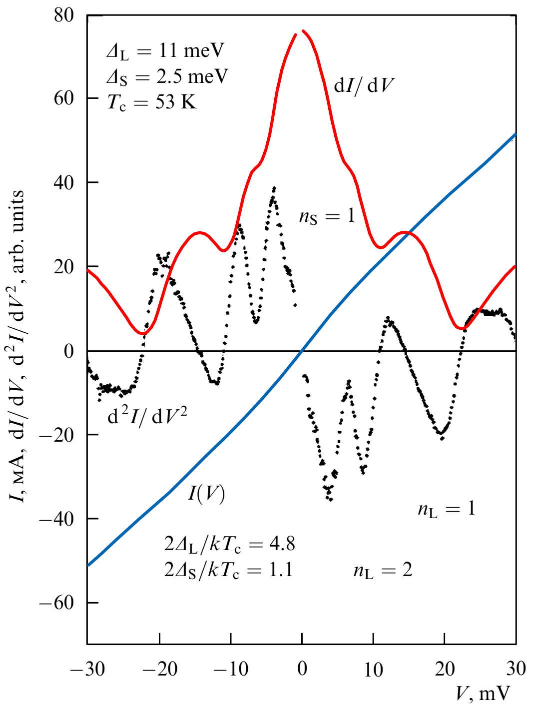

Figure 6. Current–voltage characteristic $I(V)$ and its derivatives $\mathrm{d}I(V)/\mathrm{d}V$ and $\mathrm{d}^2I(V)/\mathrm{d}V^2$ for a single SNS contact at $T = 4.2$ K for a $GdO_{0.88}(F_{0.12})FeAs$ sample [50].

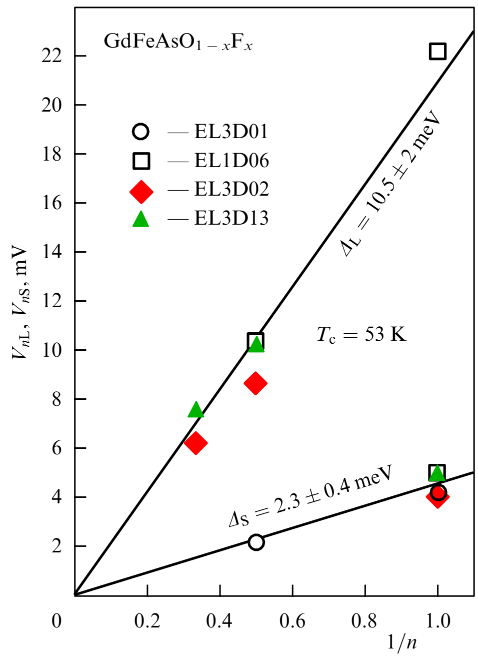

Figure 7. Measured characteristic voltages $V_n = 2\Delta_{L,S}/en$ versus $1/n_{L,S}$ for four SNS contacts prepared by the break-junction method at helium temperature [50].

superconductor, it can be expected that there are two independent subharmonic sequences corresponding to the large (Δ_L) and small (Δ_S) gaps in the spectrum. Supporting this view is the fact that the feature at $V \approx 5$ meV in Fig. 6 does not correspond to the expected voltage 7.3 meV for the third harmonic from the large gap and hence indicates the presence of a smaller gap ≈ 2.5 meV.

By re-adjusting a contact in the same sample during the same low-temperature experiment, it was possible to observe various $I(V)$ characteristics with a series of features that correspond to a large or small gap, or even to both of them. Because of the fine crystalline structure of the sample, different microcontact representations gave uncorrelated gap values and characterized the local properties of the superconducting phase at different points. Figure 7 summarizes the results from a large number of microcontacts. It can be seen that the measured values of the voltages V_n fit fairly well into the two linear dependences on $1/n_{L,S}$. The obtained data therefore suggest that superconducting $GdFeAsO_{0.88}(F_{0.12})$ has two gaps with the energies $\Delta_L = (10.5 \pm 2)$ and $\Delta_S = (2.3 \pm 0.4)$ meV at $T = 4.2$ K.

Using the measured gap values and $T_c = (52.5 \pm 1)$ K, we obtain an estimate for the ratio $2\Delta/k_B T_c$. For the large gap, it follows from our data that $2\Delta_L/k_B T_c = (4.8 \pm 1.0)$, which is larger than the standard weak-coupling one-band BCS value 3.52; but this result is not inconsistent with the strong coupling regime in the BCS model. As regards the smaller gap, our measured ratio $2\Delta_S/k_B T_c$ is ≈ 1, much less than the standard 3.52. This small value indicates that 'weaker' superconductivity at $T > T_c^*$ (where T_c^* is the 'intrinsic' critical temperature of the weaker condensate in the absence of interband interaction) may be due to an internal reason, namely, the fact that two condensates in two regions of the Brillouin zone are close to one another in k-space, with the larger-gap condensate playing the 'leadership' role. It is commonly held that such a situation occurs, in particular, in MgB_2 [48, 49] and $LaO_{0.9}F_{0.1}FeAs$ [52].

The existence of a large superconducting gap on the scale of $2\Delta_L/k_B T_c > 3.52$ in 1111 compounds *RE*OFeAs (*RE* = La, Sm, Nd) is confirmed by tunneling spectroscopy break-junction (BJ) measurements [53, 54], point contact Andreev reflection (PCAR) spectroscopy [38, 40, 55–60], scanning tunneling spectroscopy (STS) [39, 51], and angle-resolved photoelectron spectroscopy [61]. To date, the only reported measurement of the gap in Gd-1111 is that in Ref. [50]. Comparing gap measurements on Gd-1111 and other similar-T_c same-class superconductors (Sm-1111, Nd-1111, and Tb-1111) displays a fairly good agreement of the values of $2\Delta_L/k_B T_c$ measured in Ref. [50] with STS data [39], BJ spectroscopy data [51], and with most PCAR measurements [40, 58–60]. There are, however, reports of gaps about twice as large as this [55, 56]. As regards the small gap, its published experimental values disagree even more strongly, by a factor of about three, as a comparison [50] showed. Moreover, a number of studies do not report the small gap at all, and Ref. [50] argues that a third gap, with an even smaller value about 1 meV, may exist. Therefore, both the symmetry of the order parameter and the width of the superconducting gaps in class *RE*-1111 superconductors remain questions for further experimental verification.

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