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Spin wave acoustics of antiferromagnetic structures as magnetoacoustic metamaterials

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Contents

Abstract. This is a review of research results on conditions under which spatially restricted low-temperature antiferromagnets and their composites can be considered as a special class of acoustic magnetic metamaterials (magnetoacoustic metamaterials). In these, the dynamic magnetoacoustic interaction produces a number of effects that are acoustic analogs of polariton effects and which are currently intensively studied in nonmagnetic acoustic metamaterials. It is shown that the elastostatic approach to the analysis of the magnetoelastic dynamics of spatially restricted compensated magnetics is an effective tool in the search for new types of resonance acoustic anomalies, part of which are typical of the magnetostatic spin wave physics (elastostatic bulk and surface spin waves, nonuniform spin–spin resonances with their participation, etc.).

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1. Introduction

In recent years, much attention has been paid to so-called metamaterials — composite media whose dynamic properties in the long-wavelength limit prove to be qualitatively different from those of the constituent resonance structural elements [1–5]. The number of such composite structures, which demonstrate unique electrodynamic characteristics, is steadily increasing. Such structures, inter alia, comprise artificial dielectrics and artificial magnets [6–8], chiral and omega media [9, 10], photonic crystals [11, 12], and single-negative and double-negative media [13–15]. This, in turn, has stimulated the search for acoustic analogs of a whole number of fundamentally new electrodynamic effects that are quite numerous in the physics of metamaterials. The appropriate class of composite media involving acoustically resonant spatially ordered structural elements is called acoustic metamaterials.

Because of the large potential of acoustic metamaterials for practical applications [16], the analysis of their wave properties has become one of the most dynamically developing areas of modern physical acoustics of composite media. In particular, the opportunity to apply acoustic metamaterials for the creation of highly efficient acoustic insulators in certain frequency ranges [17–20], the realization of the negative acoustic refraction effect [21–24], focusing acoustic beams and acoustic superlenses [25–31], acoustic cloaking coatings [32–36], etc. have been intensely studied in recent years. Intensive work is also being performed in the field of creation of single- and double-negative acoustic metamaterials [37–39]. In spite of numerous publications devoted to these issues, however, the overwhelming majority of them are related to studying exclusively nonmagnetic structures [40–42].

At the same time, much attention has been paid in recent years to photonic crystals involving magnetic materials, and specifically to magnetic photonic materials [43–45]. Since the overwhelming majority of photonic crystals are acoustically continuous media [43], they can also be considered to be magnetic acoustic metamaterials [more precisely, magnetic phononic crystals (MPCs)]. Based on the magnetoacoustics of spatially homogeneous magnets [46], we should expect that, through the action of a constant external field and by changing the temperature or pressure, we can substantially change the degree of acoustic contrast between magnetic and nonmagnetic components of such a composite medium. Furthermore, a smooth variation of the frequency of acoustic resonances of magnetic inclusions in a nonmagnetic matrix becomes possible.

Another distinctive feature of the acoustic properties of magnets is the existence of additional internal degrees of freedom in them, which are related to the spin subsystem of a magnetic crystal (spin waves). Of special interest is the realization of conditions for magnetoacoustic resonance (MAR) — a fundamental rearrangement of the spectra of elastic and spin waves in a magnet if their frequencies and wave vectors are equal to one another. In this case, for the wave vectors $k \gg k_{\rm mph}$ ($k_{\rm mph}$ is determined from the existence condition for the MAR: $\omega_{\rm s}(k_{\rm mph}) = \omega_{\rm ph}(k_{\rm mph})$, where $\omega_{\rm s}(k)$ and $\omega_{\rm ph}(k)$ are the respective dispersion relations for the spectrum of normal spin and elastic waves of an unbounded magnet without allowance for the magnetoelastic interaction [47]), we should search for an analogy between the spectrum of magnetoelastic excitations and the spectrum of polaritons.

Thus, the range of wavelengths with $k \gg k_{\rm mph}$ can represent a significant interest for the creation of new acoustic metamaterials based on magnetic structural elements with a certain characteristic dimension d_* [e.g., the film thickness or the diameter of the filament (particle)]. If $k \approx 2\pi/d$, the condition $k \gg k_{\rm mph}$ will be fulfilled already for $d_* \gg d \gg a$, where a is the lattice parameter.

To date, a number of monographs (see, for example, Refs [47–49]) and reviews [46, 50–54] devoted to the analysis (within the continuum model) of different aspects of the interaction of the spin subsystem of a magnetically ordered crystal and the lattice have been published. Thus, the specific features of the resonance interaction of spin and acoustic waves ($k \approx k_{\rm mph}$) in unbounded magnets were considered in monograph [47]. The specificity of propagation of soft magnetoacoustic waves in infinite ferromagnetic (FM) or antiferromagnetic (AFM) crystals in the region of magnetic phase transitions for $k \ll k_{\rm mph}$ has been analyzed in the reviews [46, 50, 51]. The problems of the resonance interaction of magnetostatic waves with normal elastic waves in ferromagnetic plates (fast magnetoelastic waves) are discussed in paper [52]. Review [53] is devoted to the specific features of the nonlinear magnetoelastic dynamics of an unbounded antiferromagnet in the vicinity of magnetic phase transitions for $k \ll k_{\rm mph}$. The rotational invariance effects and the surface acoustic waves in bounded ferromagnets and antiferromagnets are considered in Ref. [54].

As to the specific features of the interaction between bulk spin and elastic waves with wave vectors $k \gg k_{\rm mph}$ in bounded nongyrotropic magnets, they have not been represented in the review literature. At the same time, if, following Ref. [55], we describe the magnetoelastic dynamics of a spatially homogeneous magnet in terms of effective elastic moduli, it is precisely in this range of wave vectors that we can expect the appearance of frequency intervals in which some effective elastic moduli become negative. However, we should take into account that a magnetically ordered medium is characterized by a sufficiently strong intrinsic nonlocality related to an inhomogeneous exchange interaction. This means that for the inequality $k \gg k_{\rm mph}$ to be fulfilled, it is necessary that already in an unbounded magnetic medium the phase velocity of the elastic wave be much greater than the propagation velocity of normal spin oscillations. Apart from ferromagnets, such a property is, as is known, characteristic of low-temperature antiferromagnets, for which, according to review [53], $T_{\rm N} < T_{\rm D}$, where $T_{\rm N}$ and $T_{\rm D}$ are the Néel and Debye temperatures, respectively. It should be emphasized that, in contrast to the spin-wave dynamics of FMs, the spin-wave dynamics of exchange-collinear AFMs is characterized by the simultaneous exchange enhancement of effects related to the magnetoelastic interaction and by an exchange weakening of the role of the magnetodipole interaction [46, 50, 53]. In addition, since the magnetodipole interaction effects in antiferromagnetic crystals are proportional to the equilibrium magnetization, their contribution can be additionally reduced in comparison with the magnetoelastic contribution if, in the equilibrium state, the total magnetization of the antiferromagnet sublattices is equal to zero. Such a magnetic structure, according to monograph [56], corresponds to a compensated antiferromagnet. This definition corresponds not only to spatially homogeneous exchange-collinear AFMs, in which the ferromagnetism vector is equal to zero in the ground state, but also to composite magnetic materials produced on their basis. One-dimensional MPCs consisting of identical ferromagnetic or ferrimagnetic layers can also be related to the compensated AFM structures under the condition that the orientations of the equilibrium magnetizations of any two neighboring layers be mutually opposite.

In the case of magnet–ideal-diamagnet structures, the phononic mechanism of the interlayer interaction should be much more efficient than the magnetodipole one. As a rule, these structures are studied from the viewpoint of the conditions for the coexistence of magnetic and superconducting phases [57]. In addition, the photonic crystals consisting of superconducting and nonsuperconducting components have been intensely studied recently in connection with the exploration of the terahertz frequency range [58].

The fulfillment of the inequality $k \gg k_{\rm mph}$ means that, to describe the dynamics of the elastic subsystem of a magnet in all the above-mentioned structures, we can use, instead of the total equations of the mechanics of continuum medium, their elastostatic limit [59]. As a result, by analogy with the spectrum of magnetostatic excitations [60, 61], it can be expected that for $k_{\rm mph}d \ll 1$ in a bounded magnet with a characteristic dimension d there will simultaneously act, apart from the mechanism of inhomogeneous exchange interaction, also two mechanisms of indirect spin–spin interaction, namely, magnetodipole and phononic. In the phononic mechanism, the indirect spin–spin interaction is implemented through the long-range field of quasistatic magnetoelastic deformations.

This review aims to present the results of investigations into acoustic analogs of polariton effects induced by dynamic magnetoelastic interaction through the example of bounded compensated AFM structures, in particular, easy-axis (EA) AFMs.

The paper outline is as follows. In Section 2, the principal relations are given that make it possible to describe the magnetoelastic dynamics of finite compensated antiferromagnets and related layered structures [in particular, one-dimensional (1D) MPCs] in terms of the continuum model.

In Section 3, we discuss the results of investigations of the magnetoelastic dynamics of plates of compensated AFMs in the framework of the elastostatic approximation, from which it follows that in some cases it is expedient to speak of the existence in such systems of a special class of nonexchange propagating spin-wave excitations — elastostatic spin waves. These waves can be considered to be an acoustic analog of well-known magnetostatic spin waves. Discussed also are the anomalies of the bulk spin-wave dynamics of bounded magnets induced by the hybridization of the considered mechanism of the indirect spin–spin interaction through the long-range field of elastostatic magnetoelastic deformations and the inhomogeneous exchange interaction.

Possible mechanisms of the localization of shear elastic waves near the external surface of a compensated antiferromagnetic structure in the cases of both mechanically free surfaces and acoustically continuous interfaces between the antiferromagnetic and nonmagnetic media are considered (in the elastostatic approximation) in Section 4.

Section 5 is devoted to the discussion of possible acoustic analogs of Otto and Kretschmann configurations for the excitation of shear surface acoustic waves (SAWs) considered in Section 4; conditions of the reflectionless propagation of a shear elastic wave through a finite 1D MPC are also presented. Based on an analysis of the local geometry of the wave-vector surface (refraction surface), the problems of the interrelation between the specific features of the refraction of a shear bulk wave at the compensated AFM–nonmagnetic dielectric interface, as well as the anomalies of the bulk and surface magnetoelastic dynamics of a bounded compensated AFM that were considered in Sections 3 and 4, are discussed.

In Section 6 (Conclusions), some opportunities for applying the results obtained are discussed, and a number of questions concerning the further development of the investigations presented in the review are formulated.

2. Energy, equations of motion, and boundary conditions

Consider the most frequently encountered models of compensated antiferromagnetic structures: a spatially homogeneous two-sublattice exchange-collinear AFM; an EA-AFM–ideal-diamagnet-type 1D MPC, and an EA-FM–ideal-diamagnet-type one-dimensional MPC with an antiferromagnetic type of ordering of neighboring tangentially magnetized ferromagnetic layers that form the elementary period of the 1D MPC.

2.1 Models of spatially homogeneous magnetic (two-sublattice exchange-collinear AFM) and nonmagnetic media

In terms of the vectors of ferromagnetism ($\mathbf{m}$) and antiferromagnetism ($\mathbf{l}$), the energy density of the two-sublattice model of an orthorhombic antiferromagnet (medium 1) with isotropic elastic and magnetoelastic interaction can be represented as [60, 62]

$$W = W_{\rm m} + W_{\rm me} + W_{\rm e}\,, \tag{2.1}$$

where the densities of the magnetic ($W_{\rm m}$), magnetoelastic ($W_{\rm me}$), and elastic ($W_{\rm e}$) energies are written out as

$$W_{\rm m} = M_0^2\left[\frac{\delta}{2}\,\mathbf{m}^2 + \frac{\alpha}{2}(\nabla\mathbf{l})^2 + \frac{\beta_1}{2}\,l_z^2 + \frac{\beta_2}{2}\,l_y^2 - \mathbf{m}\mathbf{H}_{\rm m}\right], \tag{2.2}$$

$$W_{\rm me} = M_0^2(bl_il_ku_{ik})\,, \tag{2.3}$$

$$W_{\rm e} = \frac{\lambda}{2}\,u_{ii}^2 + \mu u_{ik}^2\,. \tag{2.4}$$

Here, δ and α are the homogeneous and inhomogeneous exchange interaction constants; β_1 and β_2 are the anisotropy constants; $\mathbf{H}_{\rm m}$ is the magnetodipole field strength; $u_{ik} = (\partial u_i/\partial x_k + \partial u_k/\partial x_i)/2$ is the deformation tensor; $\mathbf{u}$ is the elastic displacement vector; λ and μ are the Lamé coefficients; b is the magnetoelastic interaction constant, and $\mathbf{m} = (\mathbf{M}_1 + \mathbf{M}_2)/2M_0$ and $\mathbf{l} = (\mathbf{M}_1 - \mathbf{M}_2)/2M_0$, where $\mathbf{M}_1$ and $\mathbf{M}_2$ are the sublattice magnetizations, with $|\mathbf{M}_1| = |\mathbf{M}_2| = M_0$. When writing out formulas (2.1)–(2.4), it was assumed that

$$|\mathbf{m}| \ll |\mathbf{l}|\,. \tag{2.5}$$

As a result, the set of dynamic equations for this phenomenological model of a magnet should include not only the Landau–Lifshitz equations for the vectors of ferromagnetism and antiferromagnetism [60, 62]:

$$\begin{aligned}\frac{1}{gM_0}\,\frac{\partial\mathbf{m}}{\partial t} &= \left[\mathbf{m}\,\frac{\partial W}{\partial\mathbf{m}}\right] + \left[\mathbf{l}\,\frac{\partial W}{\partial\mathbf{l}}\right],\\ \frac{1}{gM_0}\,\frac{\partial\mathbf{l}}{\partial t} &= \left[\mathbf{m}\,\frac{\partial W}{\partial\mathbf{l}}\right] + \left[\mathbf{l}\,\frac{\partial W}{\partial\mathbf{m}}\right],\end{aligned} \tag{2.6}$$

where g is the magnetomechanical ratio, but also the equations of magnetostatics

$$\operatorname{div}\mathbf{B} = 0\,, \qquad \operatorname{rot}\mathbf{H}_{\rm m} = 0 \tag{2.7}$$

and elastodynamics [59, 63]

$$\rho_1\,\frac{\partial^2 u_i}{\partial t^2} = \frac{\partial^2 W}{\partial x_k\,\partial u_{ik}}\,, \tag{2.8}$$

where $\mathbf{B}$ is the magnetic induction vector, and ρ is the density of the medium (of medium 1 in this case).

It follows from expressions (2.1)–(2.8) that if conditions $\beta_1 < 0$ and $\beta_2 = 0$ are fulfilled simultaneously, then in the equilibrium state vector $\mathbf{l}$ is collinear to the z-axis: $\mathbf{l}\,\|\,z$. In that case, equations (2.6) and (2.8) in the approximation linear in the amplitudes of small oscillations of the antiferromagnetic vector $\mathbf{l}$ ($\tilde{\mathbf{l}}$) and lattice displacements $\mathbf{u}$ ($\tilde{\mathbf{u}}$) can be written out as follows:

$$\left(c_{\rm m}^2\Delta - \frac{\partial^2}{\partial t^2} - \omega_0^2 - \omega_{\rm me}^2\right)\tilde{l}_x = \frac{c_{\rm m}^2}{\alpha}\, bl_0\left(\frac{\partial \tilde{u}_z}{\partial x} + \frac{\partial \tilde{u}_x}{\partial z}\right),$$
$$\left(c_{\rm m}^2\Delta - \frac{\partial^2}{\partial t^2} - \omega_0^2 - \omega_{\rm me}^2\right)\tilde{l}_y = \frac{c_{\rm m}^2}{\alpha}\, bl_0\left(\frac{\partial \tilde{u}_z}{\partial y} + \frac{\partial \tilde{u}_y}{\partial z}\right), \quad (2.9)$$
$$\rho_1\,\frac{\partial^2 \tilde{u}_i}{\partial t^2} = \frac{\partial \sigma_{ik}}{\partial x_k}\,, \qquad \sigma_{ik} = \frac{\partial(W_{\rm e} + W_{\rm me})}{\partial u_{ik}}\,,$$

where $\omega_0^2 = g^2 M_0^2 \delta|\beta_1|$ is the frequency of the homogeneous antiferromagnetic resonance, $\omega_{\rm me}^2 = g^2 M_0^2 b^2 \delta/\mu_1$ is the magnetoelastic gap, $c_{\rm m}^2 = g^2 M_0^2 \delta\alpha$ is the limiting velocity of the spin wave propagation in an unbounded magnet [53], and σ_{ik} is the elastic stress tensor.

We will seek the solution to the set of equations (2.9) in the form of plane waves with a frequency ω and a wave vector $\mathbf{k}$. From the set of equations obtained, which are linear with respect to the unknown oscillation amplitudes of the vectors $\tilde{\mathbf{l}}$ and $\tilde{\mathbf{u}}$, using the first and second equations (2.9), we eliminate the components of $\tilde{\mathbf{l}}$ that enter into the elastic stress tensor σ_{ik}. As a result, we will find that, without allowance for the magnetodipole interaction (in compensated antiferromagnets its influence is weakened owing to the exchange interaction), the spectrum of linear magnetoacoustic oscillations of the model of the unbounded magnet under consideration, just like the spectrum of the nonmagnetic crystal, can formally be described by an equation of the form

$$(\bar{c}_{ijkl}\, k_j k_k - \rho\omega^2 \delta_{il})\,\tilde{u}_{0l} = 0\,, \qquad (2.10)$$

where $\tilde{u}_{0j}$ is the amplitude of small oscillations of the vector $\tilde{\mathbf{u}}$ that are polarized along the j-axis.

In a magnetic medium, however, part of the moduli of elasticity $\bar{c}_{ijkl}$ entering into equation (2.10), in contrast to those in a nonmagnetic medium, are only effective because of the dynamic magnetoelastic interaction, since they can possess both time and spatial (for $c_{\rm m} \neq 0$) dispersions. Notably, the nonzero effective moduli entering into equation (2.10) for the model of an antiferromagnet under consideration have the form

$$\bar{c}_{11} = \bar{c}_{22} = \bar{c}_{33} = \lambda_1 + 2\mu_1\,,$$
$$\bar{c}_{12} = \bar{c}_{21} = \bar{c}_{23} = \bar{c}_{32} = \bar{c}_{13} = \bar{c}_{31} = \lambda_1\,, \qquad (2.11)$$
$$\bar{c}_{44} = \bar{c}_{55} = \mu_1\,\frac{\omega_0^2 + c_{\rm m}^2\mathbf{k}^2 - \omega^2}{\omega_0^2 + \omega_{\rm me}^2 + c_{\rm m}^2\mathbf{k}^2 - \omega^2}\,, \quad \bar{c}_{66} = \mu_1\,.$$

If we neglect the magnetoelastic interaction (to this end, we should formally pass in Eqn (2.11) to the limit of $\omega_{\rm me}^2 \to 0$), the relationships (2.10) and (2.11) will describe the spectrum of elastic waves of an unbounded elastically isotropic medium [59].

Note at once that in this case it is expedient to refrain from drawing strong analogies to the problem of dispersion properties of exciton polaritons that is considered in crystal optics [64, 65]. In the exchange-collinear magnets considered in this review, the manifestation of the spatial dispersion effect in the effective elastic moduli (2.11) (just as in the components of the magnetic susceptibility tensor [47]) is due to the inhomogeneous exchange interaction. As a result, the role of waves that are additional for acoustics or optics is played in this case by exchange spin waves which for this class of magnetic media are sufficiently correctly described by the Landau–Lifshitz equations. This circumstance cardinally changes the approach to the analysis of the spatial dispersion effects in the class of magnets under consideration as compared to that developed in the theory of excitons [64, 65].

It follows from expressions (2.10) and (2.11) that in the unbounded easy-axis elastically isotropic two-sublattice antiferromagnet ($\mathbf{l}\,\|\,z$) the factorization of the spectrum of normal magnetoelastic waves (independent propagation of elastic waves polarized in the plane of propagation and perpendicularly to it) without allowance for the magnetodipole interaction is possible for the following geometries: $\mathbf{u}\,\|\,\mathbf{l}\,\|\,z$ and $\mathbf{k} \in xy$, $\mathbf{u}\,\|\,y$ and $\mathbf{k} \in xz$, or $\mathbf{u}\,\|\,x$ and $\mathbf{k} \in yz$. In these cases, the spectrum of the magnetoacoustic waves of the unbounded EA AFM under consideration when neglecting magnetodipole interaction can be presented, using the above-introduced effective moduli of elasticity (2.11), in the following form [46, 50, 51]

at $\mathbf{k} \in xy$, as

$$(\rho\omega^2 - \bar{c}_{11}k_x^2 - \bar{c}_{66}k_y^2)(\rho\omega^2 - \bar{c}_{11}k_y^2 - \bar{c}_{66}k_x^2)$$
$$- (\bar{c}_{12} + \bar{c}_{66})^2 k_x^2 k_y^2 = 0\,, \quad \mathbf{u} \in xy\,, \qquad (2.12)$$
$$\rho\omega^2 - \bar{c}_{44}(k_x^2 + k_y^2) = 0\,, \quad \mathbf{u}\,\|\,z;$$

at $\mathbf{k} \in xz$, as

$$(\rho\omega^2 - \bar{c}_{11}k_x^2 - \bar{c}_{44}k_z^2)(\rho\omega^2 - \bar{c}_{11}k_z^2 - \bar{c}_{44}k_x^2)$$
$$- (\bar{c}_{12} + \bar{c}_{44})^2 k_x^2 k_z^2 = 0\,, \quad \mathbf{u} \in xz\,, \qquad (2.13)$$
$$\rho\omega^2 - \bar{c}_{66}\,k_x^2 - \bar{c}_{44}k_z^2 = 0\,, \quad \mathbf{u}\,\|\,y;$$

and at $\mathbf{k} \in yz$, as

$$(\rho\omega^2 - \bar{c}_{11}k_y^2 - \bar{c}_{44}k_z^2)(\rho\omega^2 - \bar{c}_{11}k_z^2 - \bar{c}_{44}k_y^2)$$
$$- (\bar{c}_{12} + \bar{c}_{44})^2 k_y^2 k_z^2 = 0\,, \quad \mathbf{u} \in yz\,, \qquad (2.14)$$
$$\rho\omega^2 - \bar{c}_{66}\,k_y^2 - \bar{c}_{44}k_z^2 = 0\,, \quad \mathbf{u}\,\|\,x\,.$$

Let us analyze the relationships (2.11)–(2.14) for the spectrum of magnetoacoustic waves in the long-wavelength and short-wavelength limits. As is known, in the long-wavelength limit ($|\mathbf{k}| \to 0$) the spin subsystem effects in the dynamics of elastic oscillations of a magnetically compensated nongyrotropic magnet can approximately be taken into account if we formally pass to a static limit ($\partial\mathbf{m}/\partial t = \partial\mathbf{l}/\partial t \approx 0$ for $\partial\mathbf{u}/\partial t \neq 0$) in the Landau–Lifshitz equations in formula (2.9). Then, we can eliminate the amplitudes of spin oscillations from the third equation in formula (2.9) using the first and second equations of this set. As a result, the spectrum of elastic waves in the magnet ('quasiphonons') will be described by the equations of elastodynamics under the condition that they contain effective moduli of elasticity (2.11) with $\omega = 0$ [46, 50, 51]:

$$\rho\,\frac{\partial^2 u}{\partial t^2} = \frac{\partial \sigma_{ik}}{\partial x_k}\,, \qquad \sigma_{ik} = \bar{c}_{ik}\,(\omega = 0)\, u_{ik}\,. \qquad (2.15)$$

If without allowance for the magnetoelastic interaction the limiting phase velocity of propagation of spin waves ($c_{\rm m}$) is much lower than the phase velocity $v_{\rm s}$ of propagation of elastic waves, then it follows from Eqns (2.9)–(2.14) that in the short-wavelength range of the spectrum of normal magnetoacoustic waves ($k \gg k_{\rm mph}$) the elastic subsystem effects in the dynamics of spin oscillations can be taken into account by formally moving in the equations of elastodynamics in formula (2.9) to a static limit ($\partial \mathbf{u}/\partial t \approx 0$ for $\partial \mathbf{m}/\partial t \neq 0$, $\partial \mathbf{l}/\partial t \neq 0$). Next, we can eliminate the amplitudes of spin oscillations from the third equation in formula (2.9) using the first and second equations of this set. The spectrum of spin-wave excitations in the magnet ('quasimagnons') is then described by the equations of elastostatics under the condition that the relationships for the stress tensor contain the effective moduli of elasticity (2.11) with $\omega \neq 0$:

$$\frac{\partial \sigma_{ik}}{\partial x_k} \approx 0\,, \qquad \sigma_{ik} = \bar{c}_{ik}(\omega)\, u_{ik}\,. \tag{2.16}$$

A similar approach to the analysis of low-lying spin-wave excitations is realized, as is known, in the electrodynamics of unbounded magnets. In the short-wavelength (Coulomb) limit, equations of magnetostatics are studied, instead of simultaneously solving the Landau–Lifshitz and Maxwell equations, and the components of the magnetic susceptibility tensor in the material relations are assumed to be frequency-dependent (they are calculated separately from the Landau–Lifshitz equations [47]).

2.2 Variants of boundary conditions

In terms of the phenomenological theory, the existence of a contact between two media is taken into account by introducing appropriate boundary conditions; in this case, elastic and magnetodipole.

An acoustically continuous interface between the two media, namely, magnetic medium 1 and nonmagnetic medium 2, with the normal $\mathbf{n}$ to the interface, can be described using the relationships

$$\sigma^1_{ik} n^1_k = \sigma^2_{ik} n^2_k \quad \text{at} \quad \xi = 0\,, \tag{2.17}$$

$$u_{1i} = u_{2i} \qquad \text{at} \quad \xi = 0\,, \tag{2.18}$$

where σ^{α}_{ik} is the elastic stress tensor, $\mathbf{u}_\alpha$ is the vector of elastic displacements in the medium α ($\alpha = 1, 2$), and ξ is the current coordinate along the normal to the interface between the two media.

The mechanically free or rigidly fixed surface of the medium α at $\xi = 0$ with the normal $\mathbf{n}$ is determined by one of the following conditions

$$\sigma^{\alpha}_{ik} n_k = 0 \quad \text{at} \quad \xi = 0\,, \tag{2.19}$$

$$\mathbf{u}_\alpha = 0 \qquad \text{at} \quad \xi = 0\,, \tag{2.20}$$

respectively. If at $\xi = 0$ there exists a slip-boundary condition between media 1 and 2, then, according to book [66], we have

$$(u_{1i} - u_{2i})\, n_i = 0\,, \quad [\mathbf{s}_\alpha \mathbf{n}] = 0\,, \quad \xi = 0\,, \tag{2.21}$$

$$s_{\alpha i} \equiv \sigma^{\alpha}_{ik} n_k\,, \quad \alpha = 1, 2\,. \tag{2.22}$$

In the magnetostatic limit, the electrodynamic interfaces between media 1 and 2 with a normal $\mathbf{n}$ to the interface are described by the relations [60]

$$B_{1i} = B_{2i}\,, \quad \varphi_1 = \varphi_2\,, \quad \xi = 0\,. \tag{2.23}$$

If medium 2 is an ideal diamagnet, then

$$\mathbf{B}_1 \mathbf{n} = 0\,, \quad \xi = 0\,. \tag{2.24}$$

If, simultaneously, the spatial dispersion effects are taken into account (which are due to the inhomogeneous exchange interaction in the magnetoelastic medium model considered), it is also necessary to specify additional (exchange) boundary conditions. For the class of magnetic media under consideration, the exchange boundary conditions, which also take into account the effects of uniaxial surface anisotropy with a constant κ at the magnet–nonmagnet interface, can be presented as [47, 60]

$$\frac{\partial \tilde{l}_i}{\partial \xi} + \kappa \tilde{l}_i = 0\,, \quad \xi = 0\,. \tag{2.25}$$

If the magnetodipole-active elastic wave is localized in the semibounded medium 1 occupying the lower half-space, then, apart from the realization of the above boundary conditions, it is necessary to require the fulfillment of the following relations far from the interface:

$$\varphi_1 \to 0\,, \quad \tilde{u}_{1i} \to 0\,, \quad \tilde{l}_i \to 0 \quad \text{as} \quad \xi \to -\infty\,. \tag{2.26}$$

2.3 Models of one-dimensional compensated antiferromagnetic structures. Effective-medium approximation

As was noted in the Introduction, a magnetic photonic crystal can be considered in many situations to be a particular case of an acoustic magnetic metamaterial, since not only the electrodynamic but also the elastic properties of the composite magnetic medium prove to be spatially modulated. In other words, the real acoustically continuous magnetic photonic crystals represent, in fact, magnetic photonic–phononic crystals.

In the general case, the dispersion properties of the magnetic medium itself that enters into the composite material are determined not only by the magnetodipole and magnetoelastic interactions but also by the spatial dispersion effect (for a magnet this effect is caused by the nonlocal exchange spin–spin interaction [60]). As a result, an analysis of only the linear dynamics of an MPC on the basis of the T-matrix method (see, e.g., paper [67]) with allowance for all three above-mentioned interactions would require extremely cumbersome analytical calculations even in the case of a one-dimensional two-component (media 1 and 2) acoustic magnetic superlattice.

If a magnetic photonic crystal contains a nonmagnetic-medium–ideal-superconductor-type structure (London penetration depth $\lambda_{\rm L} = 0$), no magnetodipole mechanism of interlayer interaction is possible in such a structure. In the presence of acoustic interlayer contact, the elastic interlayer interaction can determine the spectrum of collective magnetoelastic excitations of a given composite magnetic medium. Thus, an acoustically continuous magnet–superconductor-type superlattice with a thickness of the superconducting layers of more than $2\lambda_{\rm L}$ can already be considered to be the simplest example of a one-dimensional MPC.

Some acoustic properties of such structures were studied in Refs [68–70] by the example of a one-dimensional piezomagnet–ideal-superconductor-type superlattice, for which the spectrum of shear normal bulk oscillations and the Bragg type amplitude and phase resonances for the shear bulk wave propagating through such bounded superlattice have been investigated. However, the magnetoelastic interaction was not taken into account in these calculations, which, in contrast to piezomagnetic interaction [59], exists at an arbitrary symmetry of a magnetic crystal. An analytical investigation of the resonance properties of a composite structure is simplified substantially if we restrict ourselves to the long-wavelength limit in the spectrum of elastic oscillations, i.e., if we assume that the component $k_{\alpha\parallel}$ of the wave vector that is normal to the interface between the layers in each of the media α ($\alpha = 1, 2$) satisfies the condition

$$k_{\alpha\parallel} d_\alpha \ll 1\,, \tag{2.27}$$

where d_α is the thickness of an αth medium layer incorporated into the 1D MPC. As a result, the spectrum of collective excitations of such a one-dimensional photonic–phononic crystal can be studied using the effective-medium method [71, 72], since in the range of frequencies ω and wave numbers $k_\perp$ that satisfy the above inequality the superlattice can be considered to be a certain hypothetical spatially homogeneous medium. The latter is characterized, with allowance made for interlayer electrodynamic, elastic, and exchange boundary conditions, by quantities averaged over the superlattice period $D = d_1 + d_2$. Let A_α be some physical quantity that refers to the medium α, and $\langle A\rangle$ be its magnitude averaged over the superlattice period D. In the case of a two-component ($\alpha = 1, 2$) fine-layered superlattice, one finds $\langle A\rangle \equiv (A_1 d_1 + A_2 d_2)/D$.

Let us consider an EA-AFM–ideal-superconductor (diamagnet)-type two-component 1D MPC, assuming that the superlattice axis (normal $\mathbf{n}$ to the interface between the layers) and the easy magnetic axis are directed along the axes of the Cartesian coordinate system [73].

The requirement of acoustic continuity of this superlattice and of ideality ($\lambda_{\rm L} = 0$) of the diamagnetic properties of the medium 2 that enters into its composition leads to the following relationships at the interface between the magnetic and nonmagnetic layers:

$$u_{1i} = u_{2i}\,, \qquad \xi = d_1 + ND\,, \quad \xi = ND\,, \tag{2.28}$$

$$\sigma^1_{ik} n^1_k = \sigma^2_{ik} n^2_k\,, \quad \xi = d_1 + ND\,, \quad \xi = ND\,, \tag{2.29}$$

$$\mathbf{B}_1 \mathbf{n} = 0\,, \qquad \xi = d_1 + ND\,, \quad \xi = ND\,. \tag{2.30}$$

Here, ξ is the coordinate along the normal $\mathbf{n}$ to the interface between the layers (along the superlattice axis), and $N = 0, 1, 2, \ldots$. In the range of frequencies and wave numbers for which inequality (2.27) is fulfilled, the rigorous allowance for the boundary conditions (2.28)–(2.30) makes it possible to regard the fine-layered superlattice under consideration as a hypothetical spatially homogeneous medium (see paper [71]). Its elastic dynamics can be described in this case using effective moduli of elasticity $\bar{c}_{ik}$, which possess both spatial and time dispersion. For the case of an EA-AFM–ideal-superconductor-type one-dimensional MPC that is of interest in this section, three main configurations permitting the propagation of a shear elastic wave in the unbounded easy-axis (z-axis) antiferromagnet (as well as in a nonmagnetic elastically isotropic medium 2) are possible at $\mathbf{l}\parallel z$ and $|\mathbf{m}| = |\mathbf{H}| = 0$ with allowance for the cylindrical symmetry of the EA AFM model in question [see also relationships (2.9)–(2.11)]:

$$\begin{aligned}
&(1)\quad \mathbf{n}\parallel x\,,\quad \mathbf{k}_\perp \parallel y\,,\quad \mathbf{u}\parallel \mathbf{l}\parallel z\,;\\
&(2)\quad \mathbf{n}\parallel x\,,\quad \mathbf{k}_\perp \parallel \mathbf{l}\parallel z\,,\quad \mathbf{u}\parallel y\,;\\
&(3)\quad \mathbf{n}\parallel \mathbf{l}\parallel z\,,\quad \mathbf{k}_\perp \parallel x\,,\quad \mathbf{u}\parallel y\,.
\end{aligned} \tag{2.31}$$

If $\mu_1 = \mu_2 = \mu$ and $\rho_1 = \rho_2 = \rho$, then for all these relative orientations of the vectors $\mathbf{n}$, $\mathbf{k}_\perp$, $\mathbf{l}$, and $\mathbf{u}$, the corresponding effective moduli, which determine the dispersion law of the normal magnetoelastic SH wave propagating in the fine-layered one-dimensional MPC under consideration, can be represented in the nonexchange approximation [with $c_{\rm m} \to 0$ in Eqn (2.11)] in the following form [73]

at $\mathbf{n}\parallel x$, $\mathbf{k}_\perp \parallel y$, and $\mathbf{u}\parallel\mathbf{l}\parallel z$, as

$$\bar{c}_{55} \equiv \mu\,\frac{\omega_0^2 - \omega^2}{\omega_0^2 + \omega_{\rm me}^2 f_1 - \omega^2}\,, \qquad \bar{c}_{44} \equiv \mu\,\frac{\omega_0^2 + \omega_{\rm me}^2 f_2 - \omega^2}{\omega_0^2 + \omega_{\rm me}^2 - \omega^2}\,; \tag{2.32}$$

at $\mathbf{n}\parallel x$, $\mathbf{k}_\perp \parallel \mathbf{l}\parallel z$, and $\mathbf{u}\parallel y$, as

$$\bar{c}_{44} \equiv \mu\,\frac{\omega_0^2 + \omega_{\rm me}^2 f_2 - \omega^2}{\omega_0^2 + \omega_{\rm me}^2 - \omega^2}\,, \qquad \bar{c}_{66} = \mu\,; \tag{2.33}$$

and at $\mathbf{n}\parallel\mathbf{l}\parallel z$, $\mathbf{k}_\perp \parallel x$, and $\mathbf{u}\parallel y$, as

$$\bar{c}_{44} \equiv \mu\,\frac{\omega_0^2 - \omega^2}{\omega_0^2 + \omega_{\rm me}^2 f_1 - \omega^2}\,, \qquad \bar{c}_{66} = \mu\,. \tag{2.34}$$

Here, $f_1 = d_1/D$, and $f_2 = d_2/D$. In the limiting case of the absence of the nonmagnetic medium 2 ($f_2 = 0$), the relationships (2.32)–(2.34) coincide with the corresponding relationships (2.9) and (2.10) derived for the spatially homogeneous EA AFM.

As a result, for all these magnetoacoustic configurations the dispersion law for the normal SH wave propagating in an unbounded fine-layered 1D MPC like that under consideration can be represented, with allowance made for equation (2.15), as follows:

$$\omega^2 = s_{\rm t}^2 (c_\parallel k_\parallel^2 + c_\perp k_\perp^2)\,, \qquad s_{\rm t}^2 \equiv \frac{\mu}{\rho}\,, \tag{2.35}$$

where, at $\mathbf{k} \in xy$ and $\mathbf{n}\parallel x$, we have

$$c_\parallel = \frac{\bar{c}_{55}}{\mu}\,, \qquad c_\perp = \frac{\bar{c}_{44}}{\mu}\,, \qquad k_\parallel = k_x\,, \qquad k_\perp = k_y\,;$$

at $\mathbf{k} \in xz$ and $\mathbf{n}\parallel x$,

$$c_\parallel = \frac{\bar{c}_{66}}{\mu}\,, \qquad c_\perp = \frac{\bar{c}_{44}}{\mu}\,, \qquad k_\parallel = k_x\,, \qquad k_\perp = k_z\,,$$

and at $\mathbf{k} \in xz$ and $\mathbf{n}\parallel z$,

$$c_\parallel = \frac{\bar{c}_{44}}{\mu}\,, \qquad c_\perp = \frac{\bar{c}_{66}}{\mu}\,, \qquad k_\parallel = k_z\,, \qquad k_\perp = k_x\,.$$

Another variant of a magnetically compensated, i.e., acoustically nongyrotropic, structure can be a magnet–ideal-superconductor-type 1D MPC composed of tangentially magnetized equivalent ferromagnetic or ferrimagnetic layers with an antiferromagnetic type of interlayer ordering [74, 75].

As an example of a magnetic medium entering into the composition of the superlattice under study, let us consider a one-sublattice model of an easy-axis (EA coincides with the z-axis) ferromagnet, assuming that its magnetoelastic and elastic properties are isotropic. In this case, with due regard for the interaction between the spin and elastic subsystems and neglecting the inhomogeneous exchange interaction, the energy density W of the one-sublattice model of a uniaxial ferromagnetic crystal (medium 1) is determined by the following expression:

$$W = -\frac{\beta}{2} M_z^2 - \mathbf{M}\mathbf{h}_{\mathrm{m}} + bM_iM_ku_{ik} + \lambda_1 u_{ii}^2 + \mu_1 u_{ik}^2 , \quad (2.36)$$

where β and b are the constants of the easy-axis anisotropy and isotropic magnetoelastic interaction, respectively; λ_1 and μ_1 are the Lamé coefficients of the magnetic medium; u_{ik} is the elastic deformation tensor, and $\mathbf{h}_{\mathrm{m}}$ is the magnetodipole field strength. The dynamic properties of this model of the magnet are described by a set of equations consisting of the basic equation of the mechanics of continua, magnetostatics equations, and the Landau–Lifshitz equation:

$$\rho_1 \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial^2 W}{\partial x_k \, \partial u_{ik}} , \quad (2.37)$$

$$\operatorname{div} \mathbf{h}_{\mathrm{m}} = -4\pi \operatorname{div} \mathbf{M} , \qquad \operatorname{rot} \mathbf{h}_{\mathrm{m}} = 0 , \quad (2.38)$$

$$-\frac{1}{g}\frac{\partial \mathbf{M}}{\partial t} = [\mathbf{M}\mathbf{h}_{\mathrm{m}}] . \quad (2.39)$$

It follows from Eqns (2.36)–(2.39) that, since for $\beta > 0$ in the equilibrium state we have $\mathbf{M} \| z$, in the unbounded easy-axis one-sublattice ferromagnet under consideration the dispersion law for the normal SH wave with $\mathbf{u} \| \mathbf{M} \| z$ and $\mathbf{k} \in xy$ can be written out as

$$\omega^2 = s_{\mathrm{t}}^2 \eta (k_x^2 + k_y^2) , \qquad \eta = 1 - \frac{\omega_0 \omega_{\mathrm{me}}}{\Delta} , \qquad \Delta \equiv \omega_0^2 - \omega^2 , \quad (2.40)$$

where $\omega_0 = g\beta M_0 + gb^2 M_0^2/\mu_1$, and $\omega_{\mathrm{me}} = gb^2 M_0^2/\mu_1$.

Notice that, for a given geometry of the shear wave propagation, the EA FM can be considered as a nonmagnetic acoustically gyrotropic medium with the following effective moduli of elasticity:

$$\bar{c}_{11} = \bar{c}_{22} = \bar{c}_{33} = \lambda_1 + 2\mu_1 ,$$

$$\bar{c}_{12} = \bar{c}_{21} = \bar{c}_{23} = \bar{c}_{32} = \bar{c}_{13} = \bar{c}_{31} = \lambda_1 , \quad \bar{c}_{44} = \bar{c}_{55} = \eta\mu_1 ,$$

$$\bar{c}_{45} = -\bar{c}_{54} = \mathrm{i}\eta_* \mu_1 , \quad \eta_* \equiv \frac{\omega_{\mathrm{me}}\omega}{\Delta} , \quad \bar{c}_{66} = \mu_1 . \quad (2.41)$$

As a result, if, as before, $\mu_1 = \mu_2 = \mu$ and $\rho_1 = \rho_2 = \rho$, then in terms of the effective-medium method at $\mathbf{k} \in xy$, $\mathbf{u} \| z$, and $\mathbf{n} \| x$ the propagation of an elastic SH wave in an unbounded compensated EA-FM–ideal-superconductor 1D MPC with an antiparallel ordering of neighboring identical tangentially magnetized layers is described by relationship (2.35), but with the following effective elastic moduli:

$$c_{\|} = \frac{\bar{c}_{55}}{\mu} = \frac{\omega_2^2 - \omega^2}{\omega_1^2 - \omega^2} , \qquad c_{\perp} = \frac{\bar{c}_{44}}{\mu} = \frac{\omega_3^2 - \omega^2}{\omega_2^2 - \omega^2} . \quad (2.42)$$

Here, $\omega_1^2 \equiv \omega_0^2 - f_2\omega_0\omega_{\mathrm{me}}$, $\omega_2^2 \equiv \omega_0^2 - \omega_0\omega_{\mathrm{me}}$, and $\omega_3^2 \equiv \omega_0^2 - \omega_{\mathrm{me}}\omega_0(1 + f_1) + \omega_{\mathrm{me}}^2 f_1$.

3. Elastostatic magnons — a special class of nonexchange spin waves

3.1 Nonexchange bulk elastostatic s- and p-type spin waves

When studying spin waves in different magnetically ordered crystals, the main focus is on two formation mechanisms of the spectrum: exchange and magnetodipole interactions (see, e.g., Refs [60, 76, 77]). In the limiting cases, when one of these interactions is dominating in the magnet, we are talking about exchange or magnetostatic spin waves. As to one more type of interaction — magnetoelastic — its influence on the character of the spin wave spectrum is usually considered either in the region of the magnetoacoustic resonance [47, 52] or at small values of the wave vector $\mathbf{k}$, at which the frequency of the quasiacoustic branch is much lower than the frequency of the quasispin branch, $\omega \gg s_{\mathrm{t}} k_{\perp}$ (where s_{t} is the velocity of the shear wave) [46, 50, 51, 53]. In the last case, the magnetoelastic interaction leads to the appearance of a magnetoelastic gap in the spectrum of spin waves, whose existence manifests itself most clearly upon spin-reorientation phase transitions. As this takes place, the transverse quasiacoustic branch of the spectrum with a distinct polarization can change its character from linear to quadratic at the very point of transition even as $|\mathbf{k}| \to 0$ (i.e., the velocity of the transverse quasisound in the theoretical limit will decrease to zero when approaching the stability limit of a given magnetic state).

As said in the Introduction, we are mainly interested in the other part of the spectrum of magnetoelastic waves, namely, the range of sufficiently large wave vectors, $k \gg k_{\mathrm{mph}}$, in which the frequency of the quasispin branch satisfies the inequality

$$\omega^2 \ll s_{\mathrm{t}}^2 \mathbf{k}^2 . \quad (3.1)$$

This part of the spectrum of magnetoelastic (ME) waves is mainly analogous to the spectrum of magnetostatic waves (MSWs) [60, 61, 76–78], but now the role of the electromagnetic subsystem is played by the elastic subsystem of the crystal. In the description of the region of the spectrum of ME waves that is of interest for us, we can use equations of elastostatics [59, 63] instead of the dynamic equations of the elasticity theory, just as in the description of MSWs the magnetostatics equations are utilized instead of the general Maxwell equations. Therefore, by analogy with the case of MSWs, we will call these branches of the spectrum of ME waves elastostatic spin waves (ESSWs). The 'fast' subsystem for ESSWs is the elastic subsystem (in contrast, the 'fast' subsystem for the range of small wave vectors is the spin subsystem of the crystal). In this case, the role of the elastic subsystem reduces to the formation of an indirect (non-Heisenberg) exchange interaction between the spins through a field of quasistatic phonons. The long-range character of this interaction, as we will see below, leads to a quasinonanalytical dependence of the frequency of ESSWs on the components of the wave vector (the term quasinonanalytical dependence is used here because the wave vector $\mathbf{k}$ can tend to zero only formally, since ESSWs exist only at sufficiently large $|\mathbf{k}|$), just as the long-range character of magnetostatic interactions with $|\mathbf{k}| \to 0$ leads to a nonanalytical dispersion law for MSWs [79].

It is natural that in any magnet there simultaneously exist both magnetoelastic and magnetodipole interactions. In the antiferromagnets considered in this review, however, the magnetoelastic interaction is exchange-enhanced, whereas the magnetodipole interaction is exchange-weakened and, therefore, can be neglected in the first approximation.

In addition, the ESSWs manifest themselves most vividly when the effect of the exchange interaction on the spectrum, in spite of the condition $k \gg k_{\mathrm{mph}}$, does not yet suppress other contributions to the dispersion relation for ESSWs, for which the fulfillment of the inequality $c_{\rm m} \ll s_{\rm t}$ is required (where $c_{\rm m}$ is the characteristic velocity of spin waves), which leads to the condition $T_{\rm N} < T_{\rm D}$ that takes place in many AFMs [53].

The condition $k \gg k_{\rm mph}$ is sufficiently rigid; for usual ESSWs in magnets of finite dimensions, however, it can be fulfilled in view of the presence of a component of the wave vector that is normal to the sample surface. This component, because of the size-quantization effect, is equal to $\pi\nu/d$ (where d is the sample thickness, and ν is an integer) and, at sufficiently small thicknesses (and for $\nu \neq 0$) can exceed $k_{\rm mph}$.

Let us find those necessary conditions whose fulfillment, together with a rigorous allowance for the interaction between the spin and elastic subsystems in a bounded magnet, leads to the possibility of the formation of a new class of propagating nonexchange spin oscillations whose dispersion properties and conditions of localization are completely determined by the magnetoelastic and elastic properties of the crystal.

Let us start with the case of an elastically isotropic model of a two-sublattice tetragonal antiferromagnet with an easy-axis anisotropy (with the EA coinciding with the z-axis). Below, we will assume that the external magnetic field is zero, and the magnetic sample comprises an infinite uniformly magnetized plate of width d. If in this case the surface of the crystal at $\xi = 0, d$ is free of elastic stresses, the corresponding boundary condition can be written out as [63, 66]

$$\sigma_{ik} n_k = 0\,, \qquad \xi = 0, d\,. \tag{3.2}$$

The dynamics of this crystal can be described using a set of equations including Landau–Lifshitz equations for the vectors of the ferromagnetism and antiferromagnetism and the equations of elastostatics (2.16). In the equilibrium state, $\mathbf{l} \parallel z$ and $|\mathbf{m}| = 0$; thus, an unbounded magnet exhibits a cylindrical symmetry with the rotation z-axis. As before, we restrict ourselves to magnetoacoustic configurations that allow (without regard for magnetodipole interaction) an independent propagation of SH-type elastic waves (see Section 2). We also assume that the vector $\mathbf{n}$ of the normal to the surface of the plate is aligned with one of the Cartesian coordinate axes.

If the sagittal plane coincides with the xz plane, it follows from Eqn (2.13) that at $\mathbf{u} \parallel y$ for plane waves with a frequency ω and wave vector $\mathbf{k} \in xz$ the spectrum of magnetoelastic oscillations in the elastostatic limit (3.1) in an unbounded magnet without allowance for boundary conditions (3.2) can be written as

$$\bar{c}_{44} k_z^2 + \bar{c}_{66} k_x^2 \approx 0\,, \quad \bar{c}_{44} \equiv \mu\, \frac{\omega_0^2 - \omega^2}{\omega_0^2 + \omega_{\rm me}^2 - \omega^2}\,, \quad \bar{c}_{66} = \mu\,. \tag{3.3}$$

Regarding the frequency ω and the component of the wave vector $\mathbf{k}$ that is tangential to the sample surface as specified external parameters, we can affirm, based on Eqn (3.3), that the magnetoelastic wave with $\mathbf{u} \parallel y$ that propagates in the sagittal plane xz is one-partial. In this case, it follows from the solution to equations of elastostatics (using the standard method of calculation [63]) that in a given geometry in the spectrum of bulk ESSWs propagating along the antiferromagnetic plate at $\mathbf{n} \parallel z$ only the direct waves ($\mathbf{k}_\perp \partial\omega/\partial\mathbf{k}_\perp > 0$, $k_\perp$ is the wave number) are formed (depending on the orientation of the normal $\mathbf{n}$ to the surface

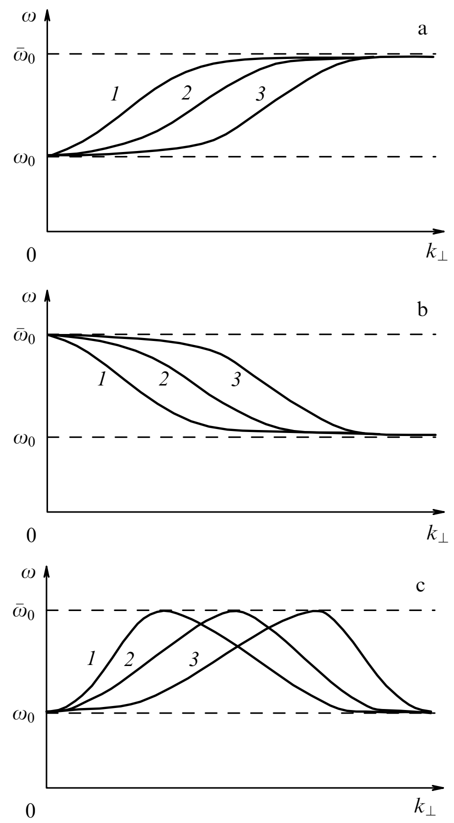

Figure 1. Structure of the spectrum of isotropic bulk ESSWs in an EA AFM plate (with EA along the z-axis): (a) $\mathbf{l} \parallel z$, $\mathbf{n} \parallel z$, s-type waves with $\mathbf{k} \in xz$; (b) $\mathbf{l} \parallel z$, $\mathbf{n} \parallel x$, s-type waves with $\mathbf{k} \in xz$, and (c) $\mathbf{l} \parallel z$, $\mathbf{n} \parallel z$, or $\mathbf{n} \parallel x$, p-type waves with $\mathbf{k} \in xz$. Curves *1–3* correspond to spectrum modes with $\nu = 1, 2, 3$, respectively.

of the AFM plate) [80]:

$$\Omega_\nu^2 = \omega_0^2 + \omega_{\rm me}^2\, \frac{k_\perp^2}{k_\perp^2 + (\pi\nu/d)^2}\,, \qquad \nu = 1, 2, \ldots, \tag{3.4}$$

whereas at $\mathbf{n} \parallel x$, $\mathbf{k} \in xz$, only waves of the reverse type ($\mathbf{k}_\perp \partial\omega/\partial\mathbf{k}_\perp < 0$) arise [80]:

$$\Omega_\nu^2 = \omega_0^2 + \omega_{\rm me}^2\, \frac{(\pi\nu/d)^2}{k_\perp^2 + (\pi\nu/d)^2}\,, \qquad \nu = 1, 2, \ldots. \tag{3.5}$$

In either case, the spectrum of the nonexchange bulk ESSWs under consideration possesses both long-wavelength and short-wavelength crowding points and these points are nondegenerate in frequency between themselves (Figs 1a, b). In addition, for any values of $k_\perp$ and mode-order numbers ν and ρ ($\rho > \nu$) in the case of Eqn (3.4), the inequality $\Omega_\nu > \Omega_\rho$ is fulfilled, whereas the relation $\Omega_\nu < \Omega_\rho$ is observed in the case of Eqn (3.5).

It should be noted that the physical mechanism responsible for the formation of such propagating spin-wave excitations is, as follows from an analysis, the existence in a bounded magnet of an indirect spin–spin interaction through the long-range field of quasistatic magnetoelastic deformations, whose displacement vector $\mathbf{u}$ is polarized along the normal to the sagittal plane (in this case, $\mathbf{u} \parallel y$).

However, the dispersion characteristics of the class of nonexchange spin waves under discussion are only partly similar to those already known in the physics of magnetostatic oscillations, since in the case of ESSWs the indirect spin–spin exchange, in contrast to that for MSWs, is realized through the tensor field of elastic deformations rather than through the vector field (for MSWs, the magnetodipole field). As a result, for a given magnetoacoustic configuration a change in the direction of the polarization of the field of electrostatic elastic displacements with respect to the plane of propagation of spin oscillations can qualitatively change the dispersion properties of propagating ESSWs. As an example, we consider, in the same elastostatic limit (3.1) for the magnetoacoustic configurations corresponding to Eqn (3.4) or (3.5), the dispersion relation for a propagating bulk ESSW with $\mathbf{k} \in xz$, whose vector $\mathbf{u}$ of elastic lattice displacements lies in the sagittal plane.

Since it is known from the general theory of wave processes in layered media that the spectrum of bulk waves nonuniformly distributed over the plate thickness is only weakly sensitive to the character of boundary conditions, we will assume, for convenience and to facilitate calculations, that the following boundary conditions are fulfilled on both sides of the antiferromagnetic plate of thickness d at $\mathbf{n} \parallel z$:

$$\sigma_{ik} n_k = 0\,, \qquad \mathbf{un} = 0\,, \qquad z = 0, d\,, \tag{3.6}$$

which, from the physical point of view, describe a slip boundary between elastic and absolutely rigid bodies [66, 81].

In an unbounded magnet, the spectrum of the magnetoelastic waves with $\mathbf{k} \in xz$ and $\mathbf{u} \in xz$ can be represented in the elastostatic limit (3.1), following Eqn (2.13), as

$$(\bar{c}_{11} k_x^2 + \bar{c}_{44} k_z^2)(\bar{c}_{11} k_z^2 + \bar{c}_{44} k_x^2) - (\bar{c}_{12} + \bar{c}_{44})\, k_x^2 k_z^2 \approx 0\,,$$

$$\bar{c}_{11} = \lambda + 2\mu\,, \qquad \bar{c}_{12} = \lambda\,, \tag{3.7}$$

$$\bar{c}_{44} \equiv \mu\, \frac{\omega_0^2 - \omega^2}{\omega_0^2 + \omega_{\mathrm{me}}^2 - \omega^2}\,, \qquad \bar{c}_{66} = \mu\,.$$

If, as before, we consider the frequency ω and the component of the wave vector $\mathbf{k}$ tangential to the sample surface as given external parameters, then, based on formulas (3.7), we can state that in this case the magnetoelastic wave with $\mathbf{u} \in xz$ propagating in the sagittal xz plane is two-partial. Then, by solving equations of elastostatics using the standard method [63], we can show that both at $\mathbf{n} \parallel z$ and at $\mathbf{n} \parallel x$ the spectrum of propagating nonexchange bulk ESSWs satisfying conditions (3.1), (3.6) and (3.7) takes on the form

$$\Omega_\nu^2 \approx \omega_0^2 + \omega_{\mathrm{me}}^2 \left(1 - \frac{s_{\mathrm{t}}^2}{s_{\mathrm{l}}^2}\right) \frac{4\pi^2 \nu^2 d^2 k_\perp^2}{(k_\perp^2 d^2 + \pi^2 \nu^2)^2}\,, \quad \nu = 1, 2, \ldots\,. \tag{3.8}$$

Here, $s_{\mathrm{t}} \equiv \mu/\rho$ and $s_{\mathrm{l}} \equiv (\lambda + 2\mu)/\rho$.

It follows from expression (3.8) that in this case the long-wavelength and short-wavelength crowding points in the spectrum of the nonexchange bulk spin waves under consideration are degenerate in frequency. As a result, for any given mode number ν of the spectrum of bulk ESSWs, the corresponding dispersion curve is characterized by the existence of an extremum (maximum) (or minimum if $\mathbf{n} \parallel [101]$) for $k_\perp = k_* \neq 0$, so that for $k_\perp < k_*$ the corresponding dispersion curve refers to a direct-type wave, and for $k_\perp > k_*$, to a reverse-type wave (Fig. 1c). In addition, in contrast to waves (3.4) and (3.5), now for any mode numbers ν and any ρ ($\rho > \nu$) there always exists such a wave number $k_\perp \neq 0$ at which a crossover of dispersion curves belonging to various modes of bulk ESSWs like those defined by formula (3.8) is possible: $\Omega_\nu = \Omega_\rho$. If $\mathbf{k} \in xz$, the spectrum (3.8) of propagating bulk ESSWs remains unchanged at $\mathbf{n} \parallel x$, as well.

An analysis shows that the physical mechanism responsible for the formation of spin-wave excitations of such a kind is the existence in a bounded magnet of an indirect spin–spin interaction through a long-range field of quasistatic magnetoelastic deformations whose displacement vector $\mathbf{u}$ lies in the sagittal plane.

As is seen from the above, the dispersion properties of a given class of spin-wave excitations depend substantially on the orientation (relative to the sagittal plane) of the vector $\mathbf{u}$ of elastic displacements of the field of quasistatic elastic deformations responsible for the formation of a given ESSW. If we restrict ourselves to the consideration of only the above-mentioned geometries, then, following the analogy with the spectrum of polariton excitations [82], we will call the elastostatic spin wave in which the vectors $\mathbf{u}$, $\mathbf{n}$, and $\mathbf{k}$ are coplanar a p-type ESSW [see, e.g., formula (3.8)], and the ESSW for which conditions $\mathbf{u} \perp \mathbf{n}$ and $\mathbf{u} \perp \mathbf{k}$ are fulfilled simultaneously an s-type ESSW [see, e.g., formulas (3.4), (3.5)].

The structure of the spectrum of nonexchange bulk s- and p-type ESSWs propagating along the antiferromagnetic plate under consideration in the case of the sagittal yz plane and $\mathbf{u} \parallel x$ or $\mathbf{u} \in yz$ at $\mathbf{n} \parallel z$ or $\mathbf{n} \parallel y$ coincides with the above-considered one.

Thus, we can state that, in contrast to the properties of MSWs, the dispersion properties of a given class of propagating nonexchange spin excitations are determined first of all by the elastic and magnetoelastic parameters of the crystal, and the type of wave (direct or reverse) is determined by the relative orientation of the normal $\mathbf{n}$ to the surface of the film, the equilibrium orientation of the vector of antiferromagnetism, and the wave vector $k_\perp$ in the film plane.

This means that in a bounded magnet the indirect spin–spin exchange through the long-range field of quasistatic magnetoelastic deformations exhibits an additional (with respect to the magnetodipole and inhomogeneous exchange interactions) formation mechanism of the dispersion of propagating spin-wave excitations.

Notice that for the sagittal plane containing the normal $\mathbf{n}$ to the interface between the media conditions (3.6) allow the simultaneous propagation of bulk ESSWs of both s and p types in an EA AFM in one and the same range of frequencies and wave numbers. As a result, an additional effect becomes possible in the spectrum of propagating bulk ESSWs, which consists in the formation of additional, as compared to expression (3.8), crossover points (Fig. 2). The appearance of such crossover points is caused in this case by the possibility of the simultaneous propagation of bulk ESSWs of both s and p types with the same frequency and the same wave number. Thus, if the sagittal plane coincides with xz, then these crossover points are determined from the condition of the equality among the right-hand sides of formulas (3.4) and (3.8) at $\mathbf{n} \parallel z$ (Fig. 2b):

$$\frac{1}{k_\perp^2 d^2 + \pi^2 \nu^2} = \left(1 - \frac{s_{\mathrm{t}}^2}{s_{\mathrm{l}}^2}\right) \frac{4\pi^2 \rho^2}{(k_\perp^2 d^2 + \pi^2 \rho^2)^2}\,, \quad \nu, \rho = 1, 2, \ldots \tag{3.9}$$

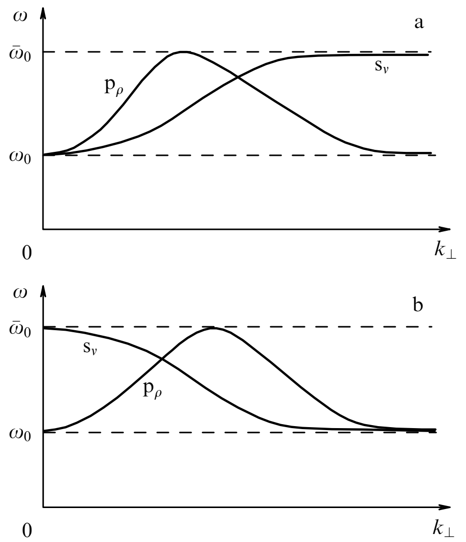

Figure 2. Inhomogeneous spin–spin resonance with the participation of isotropic bulk s- and p-type ESSWs in an EA AFM plate (with EA along the z-axis): (a) $\mathbf{l}\parallel z$, $\mathbf{n}\parallel x$, $\mathbf{k}\in xz$; (b) $\mathbf{l}\parallel z$, $\mathbf{n}\parallel z$, $\mathbf{k}\in xz$; p_ρ and s_ν denote dispersion curves of the spectrum of bulk p- and s-type ESSWs with mode numbers ρ and ν.

or from the condition of the equality between the right-hand parts of formulas (3.5) and (3.8) at $\mathbf{n}\parallel x$ (Fig. 2a):

$$\frac{\nu^2}{k_\perp^2 d^2+\pi^2\nu^2}=\left(1-\frac{s_t^2}{s_l^2}\right)\frac{4k_\perp^2 d^2}{(k_\perp^2 d^2+\pi^2\rho^2)^2}\,,\quad \nu,\rho=1,2,\ldots\,. \tag{3.10}$$

It can easily be shown that the above types of propagating nonexchange bulk ESSWs are also possible in the case of a plate of a cubic AFM in the phase for which $\mathbf{l}\parallel[001]$ ($|\mathbf{m}|=0$) in the equilibrium state.

Up to now, we have restricted ourselves to the case in which the equilibrium vector of antiferromagnetism in the tetragonal AFM was collinear to the high-symmetry direction and was located in the sagittal plane. This makes it possible, following the analogy with the MSWs [61, 78], to assume that the bulk ESSWs discussed above in this section are isotropic bulk elastostatic spin waves.

The derivation of formulas (3.2)–(3.10) was mainly based on the following assumptions: (1) the equilibrium vector of antiferromagnetism is collinear to the high-symmetry direction and lies in the sagittal plane; (2) the vector $\mathbf{l}$ and the normal $\mathbf{n}$ to the plate surface are collinear or orthogonal, and (3) the elastic and magnetoelastic properties of the magnet are isotropic.

Let us now consider how the rejection of any of these assumptions under condition (3.1) will change the spectrum of nonexchange bulk s-type ESSWs propagating in a plate of a two-sublattice AFM, as compared to the conditions (3.2)–(3.10).

3.2 Formation mechanisms of anisotropic bulk elastostatic spin waves

Let us consider now (using the example of nonexchange bulk s-type ESSWs) the effects of the orthorhombic and magnetoelastic anisotropies of the magnetic medium.

3.2.1 Role of magnetic anisotropy. It was shown in Refs [83–86] that if the indirect spin–spin exchange in a thin magnetic film occurs through the magnetodipole interaction, the rigorous allowance for the magnetocrystalline anisotropy can lead to the formation of new types of propagating magnetic excitations — anisotropic-dipole spin waves. However, because of the exchange weakening of the magnetodipole interaction, the efficiency of this formation mechanism of nonexchange spin-wave excitations is reduced sharply in the films of antiferromagnetic materials. At the same time, it was shown in Section 3.1 that if the frequency ω of spin oscillations and the projection $k_\perp$ of its wave vector $\mathbf{k}$ onto the plane of the magnetic film satisfies the elastostatic criterion (3.1), a mechanism that is an alternative to the magnetodipole mechanism of the formation of nonexchange spin waves can be the exchange-enhanced (in antiferromagnets) indirect spin–spin exchange through the long-range field of quasistatic magnetoelastic deformations.

Let us determine the necessary conditions under which the influence of magnetocrystalline anisotropy on the character of spin–spin exchange through the field of 'elastostatic' phonons can lead to the formation in bounded magnets of a new class of propagating nonexchange bulk spin-wave excitations — anisotropic ESSWs. As an example, let us consider the two-sublattice model (2.1)–(2.4) of an antiferromagnet [60, 62], assuming that the magnetoelastic and elastic properties of the crystal are isotropic. If $\beta_{1,2}>0$ [in the equilibrium state, $\mathbf{l}\parallel x$ ($|\mathbf{m}|=0$)] and, moreover, the inequality

$$|\beta_2|\ll|\beta_1| \tag{3.11}$$

holds true, then condition (3.1) can be satisfied by only the low-frequency mode of the spectrum of spin waves in an unbounded easy-plane (EP) AFM. We will restrict ourselves to the analysis of situations in which the normal to the sagittal plane coincides with the direction of one of the axes of the Cartesian coordinate system. The calculations performed have shown that in this case, for a given sagittal plane, the simultaneous propagation of nonexchange bulk s- and p-type ESSWs in the same ranges of frequencies and wave numbers is impossible.

In particular, when both surfaces of the EP AFM plate under consideration are mechanically free [see boundary condition (3.2)], direct nonexchange bulk s-type ESSWs with the dispersion relation analogous to formula (3.4) are formed under conditions (3.1) at $\mathbf{n}\parallel x$, $\mathbf{k}\in xz$ or at $\mathbf{n}\parallel y$, $\mathbf{k}\in yz$. Under the same boundary conditions, the formation of reverse nonexchange bulk s-type ESSWs with the dispersion relation analogous to formula (3.5) takes place at $\mathbf{n}\parallel z$ and $\mathbf{k}\in yz$ or $\mathbf{k}\in xz$. If the conditions analogous to formula (3.6) are satisfied on both surfaces of the plate, then nonexchange bulk p-type ESSWs with the dispersion law analogous to formula (3.8) can propagate along the plate at $\mathbf{k}\in xy$ and $\mathbf{n}\parallel y$ or $\mathbf{n}\parallel x$.

Thus, in the case of an EP AFM plate we can, by analogy with anisotropic dipole MSWs, say that the s-type ESSWs exemplify anisotropic bulk ESSWs, since their formation in the sagittal plane with the normal along the equilibrium direction of the vector $\mathbf{l}$ is related exclusively to the presence of a hard easy axis orthogonal to $\mathbf{l}$ in the AFM medium.

The calculations reveal that if the elastostatic criterion (3.1) is simultaneously satisfied for both branches of the spectrum of the orthorhombic AFM with $\mathbf{l}\parallel x$ ($|\mathbf{m}|=0$) under consideration, then for a plate with boundary conditions (3.2) the spectrum of propagating anisotropic bulk s-type ESSWs can be written out in the case of $\mathbf{k} \in yz$ and $\mathbf{n} \parallel y$ or $\mathbf{n} \parallel z$ as follows:

$$(\omega_{\parallel}^2 + \omega_{\rm me}^2 - \omega^2)(\omega_{\perp}^2 - \omega^2) + (\omega_{\parallel}^2 - \omega_{\perp}^2)\omega_{\rm me}^2 \frac{(\pi v/d)^2}{k_{\perp}^2 + (\pi v/d)^2} = 0\,, \tag{3.12}$$

$$\omega_{\parallel}^2 = \frac{\beta_1 c^2}{\alpha}\,, \quad \omega_{\perp}^2 = \frac{\beta_2 c^2}{\alpha}\,, \quad \mathbf{n} \parallel z\,, \tag{3.13}$$

$$\omega_{\parallel}^2 = \frac{\beta_2 c^2}{\alpha}\,, \quad \omega_{\perp}^2 = \frac{\beta_1 c^2}{\alpha}\,, \quad \mathbf{n} \parallel y\,. \tag{3.14}$$

When deriving expressions (3.12)–(3.14), the magnetoelastic and elastic properties of the magnet were assumed to be isotropic. An analysis of these relationships gives evidence that a characteristic feature of the spectrum of this type of nonexchange bulk s-type ESSWs is that the dispersion curves of its constituent modes for any fixed mode number v and any wave number $k_{\perp}$ lie on the (v, k) plane of external parameters in two nonintersecting bands (let us arbitrarily call them high-frequency and low-frequency bands). Let us denote the frequencies of these excitations with a mode number v as $\Omega_{+v}(k_{\perp})$ and $\Omega_{-v}(k_{\perp})$, respectively. The magnitude of the 'forbidden' gap between these bands is determined by the extent of the difference from unity of the magnetic anisotropy effective parameter $\eta \equiv \beta_1/\beta_2$. If $\eta = 1$ or $b \to 0$, this type of propagating nonexchange spin-wave excitations in such a magnetoacoustic configuration is not realized. This gives grounds to assume them to be anisotropic bulk s-type ESSWs (acoustic analogs of anisotropic bulk MSWs). As follows from expressions (3.12)–(3.14), in contrast to the spectrum of bulk isotropic s-type ESSWs (or bulk anisotropic s-type ESSWs in the plate of an EP AFM), the spectrum of this type of bulk anisotropic ESSW features two long-wavelength $(k_{\perp} \to 0\ (\omega_{\parallel}, \bar{\omega}_{\perp}))$ and two short-wavelength $(k_{\perp} \to \infty\ (\bar{\omega}_{\parallel}, \omega_{\perp}))$ crowding points:

$$\bar{\omega}_{\parallel} \equiv \sqrt{\omega_{\parallel}^2 + \omega_{\rm me}^2}\,, \qquad \bar{\omega}_{\perp} \equiv \sqrt{\omega_{\perp}^2 + \omega_{\rm me}^2}\,.$$

In the model of the magnet under consideration, one of the following systems of inequalities can be realized for the magnetoacoustic configurations considered in formulas (3.12)–(3.14), depending on the magnitudes of the magnetic anisotropy constants (the magnetoelastic and elastic properties were assumed to be isotropic):

$$\begin{aligned}
&(1)\ \ \omega_{\parallel} < \Omega_{-v}(k_{\perp}) < \bar{\omega}_{\parallel} < \omega_{\perp} < \Omega_{+v}(k_{\perp}) < \bar{\omega}_{\perp}\,,\\
&(2)\ \ \omega_{\parallel} < \Omega_{-v}(k_{\perp}) < \omega_{\perp} < \bar{\omega}_{\parallel} < \Omega_{+v}(k_{\perp}) < \bar{\omega}_{\perp}\,,\\
&(3)\ \ \omega_{\perp} < \Omega_{-v}(k_{\perp}) < \bar{\omega}_{\perp} < \omega_{\parallel} < \Omega_{+v}(k_{\perp}) < \bar{\omega}_{\parallel}\,,\\
&(4)\ \ \omega_{\perp} < \Omega_{-v}(k_{\perp}) < \omega_{\parallel} < \bar{\omega}_{\perp} < \Omega_{+v}(k_{\perp}) < \bar{\omega}_{\parallel}\,.
\end{aligned} \tag{3.15}$$

Notice that the behavior of the dispersion curve of each of the branches $\Omega_{\pm v}$ ($\mathbf{k} \in yz$) depends on the relative orientation of the normal $\mathbf{n}$ to the film surface and on the direction of the equilibrium antiferromagnetic vector $\mathbf{l}$.

In particular, the 'high-frequency' branch of the spectrum of anisotropic bulk ESSWs (3.12)–(3.14) for variants 1 or 2 in Eqn (3.15) is represented by a reverse wave $(\mathbf{k}_{\perp}\partial\Omega_{-v}/\partial\mathbf{k}_{\perp} < 0)$, and the 'low-frequency' branch of the spectrum (3.12)–(3.14) refers to direct-type waves $(\mathbf{k}_{\perp}\partial\Omega_{+v}/\partial\mathbf{k}_{\perp} > 0)$. An opposite situation for the spectrum of anisotropic bulk s-type ESSWs is realized in the case of variants 3 or 4 in Eqn (3.15). Variants 1 and 3 are illustrated in Fig. 3.

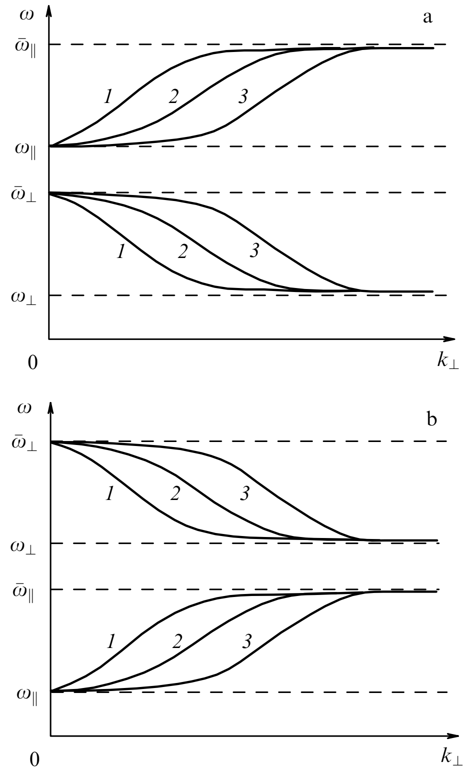

Figure 3. Structure of the spectrum of anisotropic bulk s-type ESSWs in a plate of an orthorhombic AFM (with EA along the x-axis) at $\mathbf{k} \in yz, \mathbf{l} \parallel x$, and $\mathbf{n} \parallel z$ or $\mathbf{n} \parallel y$: (a) variant 3 from Eqn (3.15); (b) variant 1 from Eqn (3.15); curves *1–3* correspond to spectrum modes with $v = 1, 2, 3$, respectively.

In addition, the magnetocrystalline anisotropy effect leads in the case of $\mathbf{k} \in xy$ and $\mathbf{n} \parallel y$ or $\mathbf{n} \parallel x$ to the possibility of the formation, instead of a single point as in formulas (3.9) and (3.10), of two crossover points for any two dispersion curves belonging to the spectrum of propagating nonexchange bulk s- and p-type ESSWs, respectively (see also Fig. 4).

3.2.2 Effect of oblique geometry. As follows from an analysis of expressions (3.4) and (3.5), the coincidence of the crowding points of the spectrum with the upper and lower boundaries of the domain of existence of isotropic nonexchange bulk s-type ESSWs occurs because in these cases either the direction of the normal to the surface of the EA AFM plate [in the case of spectrum (3.4)] or the direction of the propagation of ESSWs [in the case of spectrum (3.5)] coincides with the direction along which in the unbounded elastoisotropic EA AFM a maximum decrease occurs in the phase velocity of the shear elastic wave at the boundary of the loss of stability of a given magnetic state. This means that we can expect the formation of anisotropic bulk s-type ESSWs already in the case of an elastically isotropic EA AFM plate with boundary conditions (3.2), when the direction of the normal $\mathbf{n}$ to the film surface lies, as before, in the sagittal xz plane but makes an angle ψ with the z-axis $(0 < \psi < \pi/2)$. The structure of the spectrum of anisotropic bulk s-type ESSWs for this case is illustrated in Fig. 5.

It follows, thence, that in this case there exist two short-wavelength crowding points in the spectrum of nonexchange

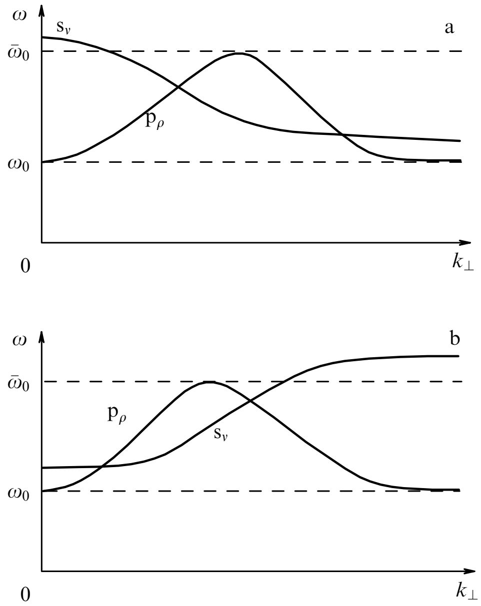

Figure 4. Inhomogeneous spin–spin resonance with the participation of anisotropic bulk s- and p-type ESSWs in an EA AFM plate (with EA along the z-axis): (a) $\mathbf{l}\parallel z$, $\mathbf{n}\parallel z$, $\mathbf{k}\in xz$, and (b) $\mathbf{l}\parallel z$, $\mathbf{n}\parallel x$, and s-type waves with $\mathbf{k}\in xz$.

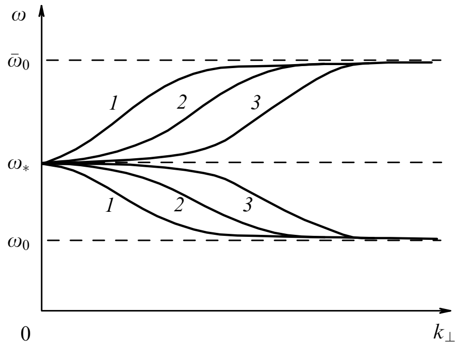

Figure 5. Structure of the spectrum of anisotropic bulk s-type ESSWs in an EA AFM plate (with EA along the z-axis) at $\mathbf{k}\in xz$, $\mathbf{l}\parallel z$, and $\mathbf{n}\in xz$. The normal $\mathbf{n}$ makes an angle ψ with the z-axis. Curves *1–3* correspond to spectrum modes with $\nu = 1, 2, 3$, respectively.

bulk anisotropic ESSWs lying at the boundary of the domain of existence of bulk spin-wave excitations of a given type, and one (noncoincident with them) long-wavelength crowding point ω_* ($\omega_*^2 \equiv \omega_0^2 + \omega_{\rm me}^2 \sin^2\psi$). For a specified mode number $\nu \neq 0$ and any fixed value of $k_\perp$ and a value of the angle ψ from the interval $0 < \psi < \pi/2$, there are two branches in the spectrum of bulk ESSWs, one of which corresponds to a direct wave, and the other to a reverse wave [87].

In the limiting cases of $\psi = 0$ or $\psi = \pi/2$, the spectrum of anisotropic bulk ESSWs passes into the spectrum of isotropic bulk s-type ESSWs considered in Section 3.2.1.

3.2.3 Magnetoelastic mechanism. An alternative mechanism of formation of bulk anisotropic s-type ESSWs with a spectrum structure analogous to that shown in Fig. 5 can also be related in the case of an EA AFM plate with mechanically free conditions (3.2) to the anisotropy of the magnetoelastic and elastic properties of the magnet even at $\psi = 0$ or $\psi = \pi/2$ ($\mathbf{k}\in xz$). Notably, this is possible for a cubic AFM at $\mathbf{l}\parallel[111]\parallel z$ and $\mathbf{u}\parallel[\bar{1}10]\parallel y$ [88, 89]. In addition, the allowance made for the magnetoelastic and elastic anisotropy in the anisotropic bulk s-type ESSWs considered in Section 3.2.1 can lead to a situation where at any mode number ν and an arbitrary magnitude of the wave number the high-frequency and low-frequency branches $\Omega_{\pm\nu}$ of the spectrum can simultaneously be waves of the same type (direct or reverse).

3.3 Elastoexchange spin dynamics of bounded compensated antiferromagnets

3.3.1 Classification of propagating elastoexchange spin waves. Traditionally, the theoretical description of spin-wave excitation in bounded ferromagnets and antiferromagnets (a plate of thickness $2d$) is constructed on the basis of a rigorous allowance for only magnetodipole and inhomogeneous exchange interactions [60, 76]. If any one of these mechanisms dominates in a bounded magnet, the spin wave is called magnetostatic or exchange. Although the number of modes ν forms an infinite countable set ($\nu = 1, 2, \ldots$) in both cases, the dispersion curves $\omega = \Omega_\nu(k_\perp)$ that are characteristic of each type of magnons differ substantially — they depend on both the polarization of the spin wave and the magnitude of the wave number $k_\perp$, and also on the relative orientation of the vectors $\mathbf{n}$, $\mathbf{l}$, $\mathbf{m}$, and $\mathbf{k}_\perp/|\mathbf{k}_\perp|$. At sufficiently large values of ν/d, the dispersion properties of the bulk spin wave are formed first of all due to the inhomogeneous exchange interaction (the exchange interaction is considered in the nearest-neighbor approximation); for this reason, inequalities $\partial\Omega_\nu(k_\perp)/\partial k_\perp > 0$ and $\partial^2\Omega_\nu(k_\perp)/\partial k_\perp^2 > 0$ simultaneously take place.

As the magnitude of ν/d decreases, the structure of the spectrum of bulk magnons becomes progressively more dependent on the hybridization effect for both types of spin–spin interaction. Therefore, in the dispersion curve $\Omega_\nu(k_\perp)$ of the mode with the number ν in the spectrum of bulk dipole–exchange spin waves there can arise a whole number of anomalies for $k_\perp \neq 0$. We can relate inflection points ($\partial^2\Omega_\nu(k_\perp)/\partial k_\perp^2 = 0$), crossover points $\Omega_\nu(k_\perp) = \Omega_\rho(k_\perp)$ ($\nu \neq \rho$), and extremum points to the specific features of the spectrum of bulk magnons induced by the dipole-exchange interaction. The character of the formation of these features at a given ω, $k_\perp$, ν, and d depends on the relative orientation of $\mathbf{n}$, $\mathbf{k}_\perp/|\mathbf{k}_\perp|$, $\mathbf{m}$, and $\mathbf{l}$. At arbitrary boundary conditions, the degeneracy $\Omega_\nu(k_\perp) = \Omega_\rho(k_\perp)$ is removed and the corresponding dispersion curves (without regard for dissipation) experience mutual repulsion. Such a situation corresponds to an inhomogeneous dipole–exchange spin–spin resonance.

The necessary condition for the formation of the above anomalies in the spectrum of the bulk dipole-exchange magnon mode with a mode number ν, frequency ω, and wave number $k_\perp$ is the fulfillment of the magnetostatic criterion $\omega \ll c\pi\nu/d$ for $\nu \neq 0$ and $\omega \ll ck_\perp$ at $\nu = 0$ (c is the speed of light in vacuum) for a magnetic plate of thickness $2d$. In view of the relativistic nature of the magnetodipole interaction, the optimum conditions for the realization of these anomalies exist primarily in those magnetic crystals in which the spectrum of normal spin-wave oscillations contains branches with a sufficiently low activation energy. Inter alia, this takes place in weakly anisotropic magnets (e.g., cubic or easy-plane ones) or in the vicinity of soft-mode-type magnetic phase transitions. In all the above cases, as is known, the dimensionless parameter of a linear magnon–phonon interaction becomes of the order of unity [46, 50, 51], and a correct description of the spectrum of a low-frequency magnon in the model of an unbounded crystal is only possible with allowance made for the effect of the elastic subsystem, even beyond the conditions of magnetoacoustic resonance.

With the above in mind, we can affirm that a rigorous theoretical description of the low-frequency spin dynamics of a real magnetic crystal requires simultaneously taking into account at least three factors: (1) the finite dimension of a real magnetic sample; (2) nonlocal spin–spin interactions (magnetodipole, inhomogeneous exchange, etc.), and (3) the interaction between the spin and elastic subsystems.

As noted above, with increasing wave number, an increase is observed in both the contribution of the inhomogeneous exchange interaction to the formation of the character of dispersion and the localization of linear spin-wave excitations propagating along the bounded magnet. As a result, a rigorous description of the spin-wave dynamics of a bounded magnet in the elastostatic limit should simultaneously take into account both the elastostatic and the exchange mechanisms of the development of dispersion even in the case of exchange-reduced magnetodipole interaction. Let us perform an appropriate analysis by the example of the two-sublattice model of an elastically isotropic EA AFM (see Section 3.1), which now additionally takes into account an inhomogeneous exchange interaction.

If we restrict ourselves to the case where in the equilibrium state $\mathbf{l} \parallel z$ and $|\mathbf{m}| = 0$, then the total spectrum of magnetoelastic excitations defined in terms of this model will consist of five branches (see Section 2). Since in the elastostatic approximation (3.1) that is of interest for us we will take into account only the phononic and inhomogeneous exchange mechanisms of spin–spin interaction, then, by analogy with dipole-exchange spin waves, we can assume in this case that the above shortened description of the dynamics of the spin system of a magnet corresponds to the elastoexchange approximation [the corresponding spin-wave excitations will be called elastoexchange spin waves (EESWs)]. Because in terms of the magnet model under consideration the inhomogeneous exchange interaction is isotropic, then without regard for the shape of the magnetic sample the problem will, as before, exhibit a cylindrical symmetry relative to the z-axis. This permits us, upon analyzing the peculiarities of hybridization of the above two types of spin–spin interactions, just as in Section 3.1, to restrict ourselves to the case of $\mathbf{k} \in xz$. From formula (2.13) it follows that without consideration for the boundary conditions in this magnetoacoustic configuration the simultaneous and independent propagation of elastoexchange spin waves of both s type,

$$\omega^2 \approx \omega_0^2 + \omega_{\mathrm{me}}^2 \frac{k_x^2}{k_x^2 + k_z^2} + c_{\mathrm{m}}^2 (k_x^2 + k_z^2)\,, \qquad c_{\mathrm{m}}^2 = g^2 M_0^2 \delta\alpha\,, \tag{3.16}$$

and p type [90],

$$\omega^2 \approx \omega_0^2 + \omega_{\mathrm{me}}^2 \left(1 - \frac{s_{\mathrm{t}}^2}{s_{\mathrm{l}}^2}\right) \frac{4 k_x^2 k_z^2}{(k_x^2 + k_z^2)^2} + c_{\mathrm{m}}^2 (k_x^2 + k_z^2)\,, \tag{3.17}$$

is now possible. Under real conditions, we always deal with bounded crystals in which the presence of the surface, as is known, can lead to the appearance of surface (or quasi-surface) excitations of various natures with an amplitude that decreases when moving away from the surface. In such oscillations, the component of the wave vector that is normal to the sample surface is no longer independent but, in view of the boundary conditions, is determined by the wave frequency ω (or by the component $k_\perp$ of its wave vector along the surface).

Thus, under conditions of the simultaneous allowance for the phononic and inhomogeneous exchange mechanisms of spin–spin interactions, we can perform a classification of the possible types of EESWs at given values of ω and $k_\perp$ for various orientations of the wave vector $\mathbf{k}$ and normal $\mathbf{n}$ to the surface (an analogous classification of MSWs was performed in Refs [91, 92].

As is seen from the dispersion relations (3.16) and (3.17), the wave-vector components k_x and k_z enter into them in a different way. Therefore, the classification of the possible types of EESWs proves to be different at different orientations of the normal $\mathbf{n}$ [90].

Let us consider first the variant with $\mathbf{n} \parallel x$. Assuming that $k_z = k_\perp$ and $k_x = \mathrm{i}\kappa$, let us rewrite the dispersion relation (3.16) using dimensionless variables:

$$\Omega^2 = \Omega_0^2 + r^2 (1 - q^2) - \frac{q^2}{1 - q^2}\,, \tag{3.18}$$

where the following designations were introduced:

$$\Omega^2 = \frac{\omega^2}{\omega_{\mathrm{me}}^2}\,, \qquad \Omega_0^2 = \frac{\omega_0^2}{\omega_{\mathrm{me}}^2}\,, \qquad r = \frac{c_{\mathrm{m}} k_\perp}{\omega_{\mathrm{me}}}\,, \qquad q = \frac{\kappa}{k_\perp}\,.$$

If we construct the graph of the dependence $\Omega^2(r)$, the section of this graph with straight lines corresponding to a certain value of Ω yields roots q_i^2 of equation (3.18). And it is the number and signs of these roots that determine the type of EESWs: if all the roots are positive ($q_i^2 > 0$), the EESW represents purely surface wave (two-partial evanescent EESW); if at least one of the roots is negative ($q_i^2 < 0$), this corresponds to a bulk wave; if the intersection is absent at a given Ω^2, this means that all the roots are complex and the EESW represents a generalized two-partial evanescent EESW. In the case under consideration, we can easily obtain from formula (3.18) that for $\Omega^2 < \Omega_0^2 + r^2$ there are two real roots $q_{1,2}^2 > 0$, which corresponds to a two-partial evanescent EESW. For $\Omega^2 > \Omega_0^2 + r^2$, one of the roots becomes negative, and the EESW acquires a bulk character. Consequently, on the (r, Ω) plane there are two ranges of parameter values in which the types of possible EESWs are different (Fig. 6a, $f_1(r) > \Omega_0^2 + r^2$).

A substantially different classification of the possible types of EESWs takes place when $\mathbf{n} \parallel z$. Suppose now that $k_x = k_\perp$ and $k_z = \mathrm{i}\kappa$; then, in the dimensionless variables that were introduced above, Eqn (3.16) takes on the form

$$\Omega^2 \approx \Omega_0^2 + r^2 (1 - q^2) + \frac{1}{1 - q^2}\,. \tag{3.19}$$

Although last expression, just as Eqn (3.18), is quadratic with respect to q^2, the partition of the (r, Ω^2) plane of parameters becomes more complex. It can be shown that on the (r, Ω^2) plane there are three characteristic curves

$$f_1(r) = \Omega_0^2 + r^2 + 1\,, \qquad f_{2,3}(r) = \Omega_0^2 \pm 2r\,, \tag{3.20}$$

which divide this plane into four regions (Fig. 6b). In region I, $q_{1,2}^2 > 0$, and it is the region of a two-partial evanescent EESW; in region II, all the roots are complex, which corresponds to a generalized two-partial evanescent EESW;

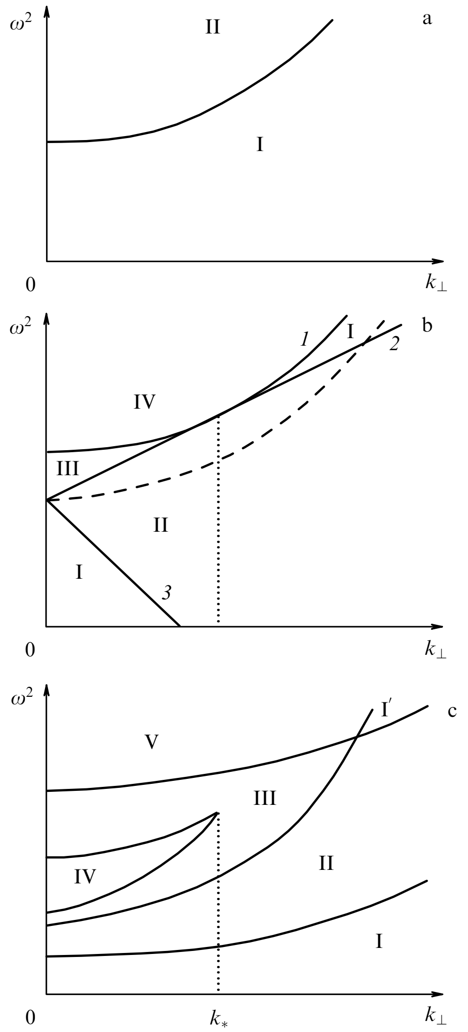

Figure 6. Classification of the possible types of EESWs in a bounded EA AFM ($\mathbf{l} \| z$, $\mathbf{k} \in xz$): (a) $\mathbf{n} \| x$, s-type waves; (b) $\mathbf{n} \| z$, s-type waves, and (c) $\mathbf{n} \| z$, p-type waves. In the dimensionless variables introduced in formula (3.18), the equation of the curve in figure (a) is $\Omega^2 = \Omega_0^2 + r^2$; the equation of curve *1* in (b), $\Omega^2 = \Omega_0^2 + r^2 + 1$; of curve *2* in (b), $\Omega^2 = \Omega_0^2 + 2r$; of curve *3* in (b), $\Omega^2 = \Omega_0^2 - 2r$ (the dashed line is discussed at the end of Section 4.1).

in region III, $q_{1,2}^2 < 0$, and in region IV, $q_1^2 > 0$ and $q_2^2 < 0$, i.e., these are regions where bulk EESWs exist.

The case of a p-type EESW (3.17) is 'symmetrical' with respect to the components k_x and k_z; therefore, the classification of the EESWs at $\mathbf{n} \| x$ and $\mathbf{n} \| z$ is the same. Assuming, without the loss of generality, that $\mathbf{n} \| x$, we rewrite relation (3.17) in the form

$$\Omega^2 = \Omega_0^2 + r^2(1 - q^2) - \frac{q^2}{(1 - q^2)^2}\,, \qquad (3.21)$$

where

$$\Omega^2 \equiv \frac{\omega^2}{4\omega_{\mathrm{me}}^2(1 - s_{\mathrm{t}}^2/s_{\mathrm{l}}^2)}\,, \qquad \Omega_0^2 \equiv \frac{\omega_0^2}{4\omega_{\mathrm{me}}^2(1 - s_{\mathrm{t}}^2/s_{\mathrm{l}}^2)}\,,$$

$$r \equiv \frac{c_{\mathrm{m}} k_\perp}{4\omega_{\mathrm{me}}\sqrt{1 - s_{\mathrm{t}}^2/s_{\mathrm{l}}^2}}\,, \qquad q \equiv \frac{\kappa}{k_\perp}\,.$$

Thus, Eqn (3.21) is cubic with respect to q^2. Nevertheless, in this case as well, it is possible to classify the types of EESWs depending on the values of $k_\perp$ and ω. The corresponding partition of the (r, Ω^2) plane is shown in Fig. 6c.

In regions I and I′, all three roots of the bicubic equation (3.21) are real and positive ($q_{1,2,3}^2 > 0$), i.e., this is the region of three-partial evanescent EESWs. In region II, there is one positive root, $q_1^2 > 0$, and two complex roots, which corresponds to three-partial generalized evanescent EESWs. In region III, $q_1^2 < 0$, and $q_{2,3}^2$ are complex-conjugate roots; in region IV, $q_{1,2,3}^2 < 0$; in region V, $q_1^2 < 0$, and $q_{2,3}^2$ are positive; consequently, regions III, IV, and V correspond to different cases of three-partial bulk EESWs.

The analytical expressions for the curves that divide the (r, Ω^2) plane in the case under consideration are quite unwieldy: we therefore do not give them here. Notice only that $r_*^2 = 3^{-3/2}$.

The effect of the allowance for the anisotropy of the magnetoelastic and elastic interactions on the localization of EESWs near the surface of an orthorhombic antiferromagnet was considered in detail in Ref. [90].

3.3.2 Dispersion properties of elastoexchange bulk spin waves in a thin film.

The above classification of possible types of EESWs that can be realized in crystals at given $k_\perp$ and ω does not answer the question of which type of EESW exists in a bounded AFM at a fixed $k_\perp$ (or ω), since the dispersion relation $\omega = \omega(k_\perp)$ for a wave is determined both by the geometry of the problem and by a particular type of boundary conditions.

In Sections 3.1 and 3.2 we have analyzed the influence of the lattice type on the bulk spin-wave dynamics of thin magnetic films not only in the elastostatic ($\omega/(s_{\mathrm{t}} k_\perp) \to 0$) but also in the nonexchange ($c_{\mathrm{m}} \to 0$) approximations. The first corresponds to disregarding the acoustic delay effects and, consequently, is valid in the case of sufficiently thin films at arbitrary values of the wave number $|\mathbf{k}_\perp|$ of the propagating bulk spin oscillations. As to the nonexchange approximation, it can easily be shown that its application leads to more serious restrictions, since it prevents the extension of the results of the above analysis of the elastostatic dynamics of thin magnetic films to both the case of sufficiently large wave numbers $|\mathbf{k}_\perp|$ satisfying the condition $c_{\mathrm{m}} k_\perp \approx \omega_0$ and the case of sufficiently thin films, $c_{\mathrm{m}}/d \approx \omega_0$. The more so, since, as follows from the results of the classification of the possible types of surface and bulk spin-wave excitations performed in Section 3.3.1 under the conditions of $\omega \ll s_{\mathrm{t}} k_\perp$ and an arbitrary value of $|\mathbf{k}_\perp|$, a rigorous analysis of the spin dynamics of bounded magnets should be based on the simultaneous allowance for both the above-studied indirect spin–spin exchange interaction through the long-range field of 'elastostatic phonons' and the Heisenberg mechanism of spin–spin interaction.

Thus, this section is aimed at an analysis of anomalies in spin-wave dynamics of thin magnetic films under the condition of $\omega \ll s_{\mathrm{t}} k_\perp$ in the framework of the elastoexchange approach [90].

Note at once that, in contrast to the calculations of the spectrum in terms of crystallooptics involving exciton-type excitations [82], in this case the calculation [in conditions specified by inequality (3.1)] of the spectrum of magnetoelastic excitations with allowance for the spatial dispersion effects is performed not based on equations of elastostatics (2.16) with effective elastic moduli (2.11) for $c_{\mathrm{m}} \neq 0$, but on the basis of a simultaneous solution to the set of dynamic equations that consists of equations of elastostatics and Landau–

Lifshitz equations with consideration for the elastic and exchange boundary conditions on the surface of the magnet. A similar approach is used (with the replacement of equations of elastostatics by equations of magnetostatics) in the calculations of the spectrum of dipole–exchange spin waves in magnetic plates and semibounded magnets [76, 91, 92].

Let us start from the case of a propagating s-type EESW under the condition $\mathbf{n}\parallel z$, $\mathbf{k}\in xz$, and such values of the wave number and frequency that correspond to regions III and IV in Fig. 6b. In these regions, as noted in Section 3.3.1, there are two negative roots to equation (3.19), $q_{1,2}^2<0$, i.e., four purely imaginary values of q. In a semiinfinite magnet, such a situation permits us to pose the problem of a multipath reflection of a spin wave from the surface of the magnet without a change in its polarization. We here consider an EESW propagating in a plate of finite thickness, where a problem of eigenvalues exists.

Let the magnet occupy the region $|z|<d$. If on both mechanically free surfaces of an EA AFM plate the spins are completely unpinned, viz.

$$\sigma_{yz}\big|_{z=\pm d}=0\,,\qquad \frac{\partial\tilde{\mathbf{l}}}{\partial z}\bigg|_{z=\pm d}=0\,, \tag{3.22}$$

then the solution of the equations of motion corresponding to the bulk branch of the spectrum of EESWs should be sought in the form of a four-partial wave. However, it can easily be shown that in the situation under consideration there exist independent symmetrical and antisymmetrical (with respect to the $z=0$ plane) solutions of the form

$$\begin{cases}\tilde{u}_y(x,z,t)=\big(A_1\cos(p_1z)+A_2\cos(p_2z)\big)\exp(\mathrm{i}k_\perp x-\mathrm{i}\omega t)\,,\\ \tilde{l}_y(x,z,t)=\big(B_1\cos(p_1z)+B_2\cos(p_2z)\big)\exp(\mathrm{i}k_\perp x-\mathrm{i}\omega t)\,,\end{cases}$$
$$\begin{cases}\tilde{u}_y(x,z,t)=\big(A_1\sin(p_1z)+A_2\sin(p_2z)\big)\exp(\mathrm{i}k_\perp x-\mathrm{i}\omega t)\,,\\ \tilde{l}_y(x,z,t)=\big(B_1\sin(p_1z)+B_2\sin(p_2z)\big)\exp(\mathrm{i}k_\perp x-\mathrm{i}\omega t)\,,\end{cases} \tag{3.23}$$

where $p_{1,2}^2=-q_{1,2}^2k_\perp^2>0$ and $q_{1,2}^2$ are the roots of equation (3.19).

The corresponding dispersion equations take the form

$$p_1(p_1^2+k_\perp^2)\tan(2p_1d)=p_2(p_2^2+k_\perp^2)\tan(2p_2d) \tag{3.24}$$

for the symmetrical mode, and

$$p_1(p_1^2+k_\perp^2)\cot(2p_1d)=p_2(p_2^2+k_\perp^2)\cot(2p_2d) \tag{3.25}$$

for the antisymmetrical mode described by formulas (3.23).

An analysis of relationships (3.24) and (3.25) gives evidence that, in the regions of the wave vectors $\mathbf{k}_\perp$ and excitation frequencies ω under consideration, the simultaneous allowance for the inhomogeneous exchange interaction and indirect spin–spin interaction through the field of virtual phonons leads to the formation of an additional (with respect to the exchange bulk spin wave) bulk ($p_{1,2}^2>0$) elastostatic-type spin wave with the same polarization. The analytical expression for the dispersion relation for the regions that are determined by expressions (3.24) and (3.25) could not be obtained explicitly and numerical calculations are required. The most important features of the dispersion law can easily be analyzed, however, in the limit of sufficiently small values of the wave vector ($\omega_{\mathrm{me}}/c_{\mathrm{m}}\gg|\mathbf{k}_\perp|$). In this case, it follows from Eqns (3.24) and (3.25) that they describe two qualitatively different types of bulk spin waves with the same polarization. In the limit under consideration, the dispersion law of one of the waves is mainly formed by the indirect exchange through the field of elastostatic phonons; this wave is direct ($\mathbf{k}_\perp\,\partial\omega/\partial\mathbf{k}_\perp>0$), and the following dispersion relation corresponds to such a wave:

$$\omega^2\approx\omega_0^2+\omega_{\mathrm{me}}^2\frac{k_\perp^2}{k_\perp^2+p_1^2}\,,\quad p_1\approx\frac{\pi\nu}{2d}\,,\quad \nu=1,2,\ldots\,. \tag{3.26}$$

Under the same conditions, an ordinary bulk spin wave with the dispersion law

$$\omega^2\approx\omega_0^2+c_{\mathrm{m}}^2(k_\perp^2+p_2^2)\,,\quad p_2\approx\frac{\pi\nu}{2d}\,,\quad \nu=1,2,\ldots, \tag{3.27}$$

which is mainly formed due to the inhomogeneous exchange interaction, corresponds to the second solution. Comparing relationships (3.26) and (3.27), we can conclude that in the vicinity of the values of the wave vector that are determined by the condition

$$\omega_{\mathrm{me}}^2\frac{k_\perp^2}{k_\perp^2+\big(\pi\nu/(2d)\big)^2}=c_{\mathrm{m}}^2\left[k_\perp^2+\left(\frac{\pi\rho}{2d}\right)^2\right],\quad \nu\neq\rho \tag{3.28}$$

a resonance interaction between the above two types of spin excitations takes place, i.e., an inhomogeneous spin–spin resonance. In this regard, the structure of the spin-wave spectrum is determined by the general relationships (3.24) and (3.25), and the simultaneous allowance for both above-considered mechanisms of spin–spin exchange is of fundamental importance for their formation. Here, as usual, a 'repulsion' of the interacting modes shows its worth and, as a consequence, 'windows of nontransparency' (in frequency) are formed for the propagating spin-wave oscillations with a specified $k_\perp$. Notice that the above-described elastoexchange mechanism of the inhomogeneous spin–spin resonance is a magnetoelastic analog of the well-known dipole–exchange resonance (see, e.g., paper [93]), and the windows of nontransparency represent an analog of the so-called 'dipole gaps' [94]. The magnitude of such a gap depends on the character of the boundary conditions. In particular, if on both surfaces of the plate the conditions

$$\sigma_{yz}=0\,,\qquad \tilde{l}=0\,,\qquad z=\pm d \tag{3.29}$$

(where $\tilde{l}$ is the amplitude of small oscillations of the vector of antiferromagnetism near the equilibrium orientation) are fulfilled simultaneously, i.e., both surfaces of the plate are mechanically free and the spins are completely pinned (Kittel conditions), then the spectrum of the elastoexchange bulk spin wave takes on the form

$$\Omega_\nu^2=\omega_0^2+\omega_{\mathrm{me}}^2\frac{k_\perp^2}{k_\perp^2+p_1^2}+c_{\mathrm{m}}^2(k_\perp^2+p_1^2)\,,\qquad p_1=\frac{\pi\nu}{2d}\,,\quad \nu=1,2,\ldots\,. \tag{3.30}$$

Thus, the gap will be equal to zero in this particular case, since for $k_\perp\neq0$ a crossover of the modes of the spectrum of elastoexchange bulk waves becomes possible: $\Omega_\nu=\Omega_\rho$ ($\nu\neq\rho$) (Fig. 7a).

Let us now consider the features of the AFM-crystal spin dynamics in the case where the indirect spin–spin exchange through the field of virtual phonons leads to the formation of

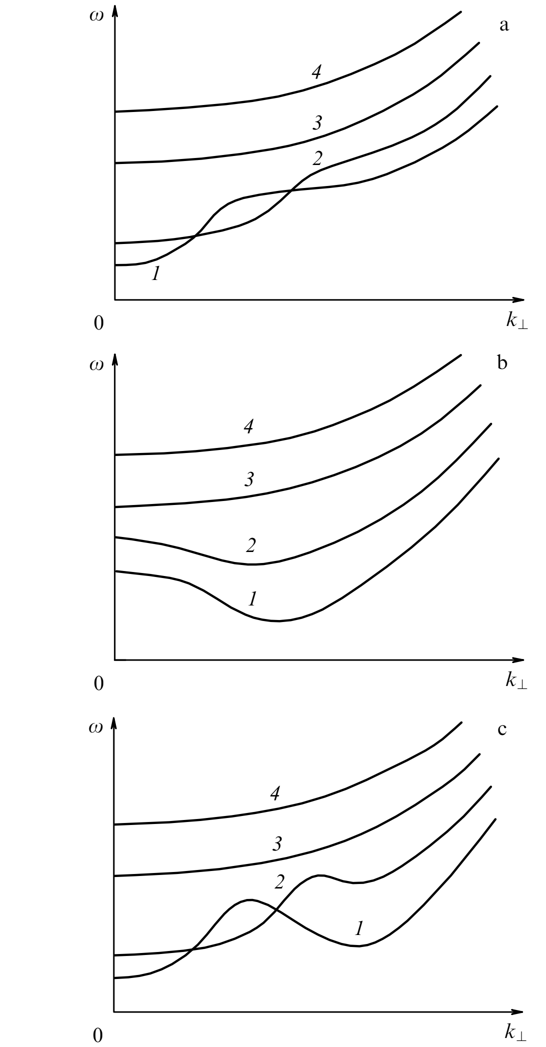

Figure 7. Structure of the spectrum of isotropic bulk EESWs with $\mathbf{k} \in xz$ in an EA AFM plate: (a) $\mathbf{l} \parallel z$, $\mathbf{n} \parallel z$, s-type waves, (b) $\mathbf{l} \parallel z$, $\mathbf{n} \parallel x$, s-type waves, and (c) $\mathbf{l} \parallel z$, $\mathbf{n} \parallel z$, or $\mathbf{n} \parallel x$, p-type waves. Curves *1–4* correspond to spectrum modes with $\nu = 1, 2, 3, 4$, respectively.

an associated surface elastostatic-type spin wave, which accompanies the exchange bulk spin wave propagating over the crystal. Let, as before, $\mathbf{k} \in xz$, but now $\mathbf{n} \parallel x$. Assuming that the magnet is bounded by $x = -d$ and $x = d$ planes, let us write down boundary conditions analogous to those used in formulas (3.22)–(3.25):

$$\sigma_{yx}\Big|_{x=\pm d} = 0\,, \qquad \frac{\partial \mathbf{l}}{\partial x}\Big|_{x=\pm d} = 0\,. \tag{3.31}$$

In a finite plate, the solutions to the equations of motion in a given geometry are four-partial:

$$p_1 = \frac{\pi \nu}{2d}\,, \quad p_2 = \pm \mathrm{i}\left(k_\perp^2 + p_1^2 + \frac{\eta}{k_\perp^2 + p_1^2}\right)^{1/2}, \quad \nu = 1, 2, \ldots, \tag{3.32}$$

where η is a positive constant, and the dispersion relation for EESWs is written out as

$$\Omega_\nu^2 = \omega_0^2 + \omega_{\mathrm{me}}^2 \frac{p_1^2}{k_\perp^2 + p_1^2} + c_{\mathrm{m}}^2 (k_\perp^2 + p_1^2)\,, \quad \nu = 1, 2, \ldots. \tag{3.33}$$

Since without regard for the phononic mechanism of exchange the spectrum of the exchange spin wave has the form $\omega^2 = \omega_0^2 + \omega_{\mathrm{me}}^2 + c_{\mathrm{m}}^2(k_\perp^2 + p_1^2)$, we can conclude that in a magnetic film the interference of the exchange bulk ($p_1^2 > 0$) and elastostatic accompanying surface ($p_2^2 < 0$) partial spin waves (3.32) qualitatively changes the character of the dispersion curve of the propagating bulk spin wave (Fig. 7b). More specifically, the competition between the inhomogeneous exchange ($c_{\mathrm{m}}^2 k_\perp^2$) and phononic mechanism of non-Heisenberg exchange $[\omega_{\mathrm{me}}^2 p_1^2/(p_1^2 + k_\perp^2)]$ leads to the appearance of an extremum in the dispersion curve (3.33). For $k_\perp < k_*$ ($k_*^2 = \omega_{\mathrm{me}} p_1 / c_{\mathrm{m}} - p_1^2$), the dispersion law for a bulk EESW corresponds to the reverse spin wave ($\mathbf{k}_\perp \partial\omega/\partial\mathbf{k}_\perp < 0$); for $k_\perp > k_*$, it corresponds to the direct wave ($\mathbf{k}_\perp \partial\omega/\partial\mathbf{k}_\perp > 0$). It should be noted that the existence of an extremum point in the dispersion curve at a fixed mode number ν of the propagating bulk ESSW is determined by the condition $d > \pi\nu c_{\mathrm{m}}/2\omega_{\mathrm{me}}$, $\nu = 1, 2, \ldots$. If $d < \pi\nu c_{\mathrm{m}}/2\omega_{\mathrm{me}}$, then $k_* = 0$ and, consequently, the corresponding bulk ESSW mode represents a direct wave ($\mathbf{k}_\perp \partial\omega/\partial\mathbf{k}_\perp > 0$) at any magnitude of the wave vector $\mathbf{k}_\perp$.

It should be noted that the above dispersion relations for propagating bulk s-type EESWs remain valid even when on both surfaces of an AFM plate slip boundary conditions (3.6) are fulfilled instead of the conditions for a mechanically free surface.

Let us now consider the second class of elastoexchange waves propagating in the plate of the AFM considered, whose dispersion properties result from the hybridization of the inhomogeneous exchange interaction and indirect spin–spin exchange through the field of quasistatic elastic deformations polarized in the sagittal plane (p-type EESWs). Since, as is known, the spectrum of bulk waves distributed inhomogeneously over the plate thickness only weakly depends on the character of the boundary conditions, we restrict ourselves to the consideration of the case in which the solution can be obtained explicitly. To this end, we assume that the following conditions are fulfilled simultaneously on both surfaces of the plate with $\mathbf{n} \parallel x$:

$$\tilde{l}\Big|_{x=\pm d} = 0\,, \tag{3.34}$$

$$\tilde{u}_x = 0\,, \qquad \sigma_{xz} = 0\,, \qquad x = \pm d\,. \tag{3.35}$$

If the sagittal plane is the xz plane, then, as follows from formulas (2.13), for $\mathbf{u} \in xz$ the solution to the Landau–Lifshitz equations and equations of elastostatics, corresponding to this type of bulk EESWs, should be sought in the form of a six-partial wave. The calculations show that, as a result, the dispersion law of the propagating bulk p-type EESW can be written out as follows:

$$\Omega_\nu^2 \approx \omega_0^2 + \omega_{\mathrm{me}}^2 \left(1 - \frac{s_{\mathrm{t}}^2}{s_{\mathrm{l}}^2}\right) \frac{4 p_1^2 k_\perp^2}{(p_1^2 + k_\perp^2)^2} + c_{\mathrm{m}}^2 (k_\perp^2 + p_1^2)\,,$$
$$\nu = 1, 2, \ldots. \tag{3.36}$$

A comparison of formula (3.36) with analogous formula (3.8) obtained in the nonexchange limit ($c_{\mathrm{m}} \to 0$) shows that the hybridization of the above two mechanisms of spin–spin exchange can lead to the formation, in the dispersion curves of the corresponding elastostatic bulk modes with a sufficiently small number, of not only a point of maximum but also of a point of minimum (Fig. 7c). Moreover, for given two mode numbers, two crossover points can arise rather than one, as for $c_{\rm m} \to 0$. In contrast to the dispersion relation for the above-considered elastoexchange bulk s-type waves, the dispersion law of this class of elastoexchange excitations does not change qualitatively if the orientation of the normal to the plate surface swings in the sagittal plane (xz plane) by 90° ($\mathbf{n} \| z$).

3.4 Relation between the spectrum of elastoexchange spin waves and the spectrum of bulk magnetoelastic waves in the plate of a compensated antiferromagnet

It follows from the results of paper [95] that the various combinations of elastic and exchange boundary conditions used in Sections 3.1–3.3 make it possible to obtain, for the same magnetoacoustic configurations, the dispersion equations for the spectrum of bulk magnetoelastic waves in the AFM plate of thickness $2d$ explicitly, without imposing the restriction on the relative magnitude of the phase velocities of elastic and spin waves in an unbounded magnet.

In the AFM plate being considered, let xz be the sagittal plane, $\mathbf{n}$ be parallel to z, and conditions

$$\tilde{l} = 0\,, \quad \tilde{u}_z = 0\,, \quad \sigma_{xz} = \sigma_{yz} = 0\,, \quad z = \pm d \qquad (3.37)$$

be fulfilled. In this case, the solution to the set of equations (2.9) with the simultaneous allowance for the magnetoelastic and inhomogeneous exchange interactions and arbitrary magnitude of the wave number $k_\perp$ (satisfying requirements of the phenomenological theory of elasticity [63, 66]) makes it possible to represent the spectrum of bulk magnetoelastic waves as

$$\begin{aligned}
&(\rho\omega^2 - \bar{c}_{11} k_\perp^2 - \bar{c}_{44} p_1^2)(\rho\omega^2 - \bar{c}_{11} p_1^2 - \bar{c}_{44} k_\perp^2) - \\
&\quad - (\bar{c}_{12} + \bar{c}_{44}) k_\perp^2 p_1^2 = 0\,, \quad \mathbf{u} \in xz\,, \\
&\rho\omega^2 - \bar{c}_{66} k_\perp^2 - \bar{c}_{44} p_1^2 = 0\,, \quad \mathbf{u} \| y\,, \\
&\bar{c}_{11} = \lambda + 2\mu\,, \quad \bar{c}_{12} = \lambda\,, \\
&\bar{c}_{44} = \mu\,\frac{\omega_0^2 + c_{\rm m}^2 (k_\perp^2 + p_1^2) - \omega^2}{\omega_0^2 + \omega_{\rm me}^2 + c_{\rm m}^2 (k_\perp^2 + p_1^2) - \omega^2}\,, \quad \bar{c}_{66} = \mu\,.
\end{aligned} \qquad (3.38)$$

If for the same sagittal xz plane the condition $\mathbf{n} \| x$ is fulfilled and boundary conditions (3.31) are specified on both surfaces of the AFM plate under consideration, then for the spectrum of bulk magnetoelastic waves with $\mathbf{u} \| y$ we have

$$\rho\omega^2 - \bar{c}_{66} p_1^2 - \bar{c}_{44} k_\perp^2 = 0\,. \qquad (3.39)$$

From a comparison of Eqn (3.38) with Eqns (3.30) and (3.36), and of Eqn (3.39) with Eqn (3.33), it follows that, to obtain the spectrum of the above-considered elastoexchange bulk spin-wave excitations, it is necessary in the dispersion relation (3.38) for bulk magnetoelastic waves in a bounded magnet to formally pass to the limit of $\rho/\mu \to 0$. In other words, the bulk elastoexchange spin waves considered in Section 3.1 describe in the frequency range (3.1) the dispersion properties of the low-frequency branch of bulk magnetoelastic excitations in zero order in the parameter $\rho\omega^2/[\mu(p_1^2 + k_\perp^2)]$, whose smallness just corresponds to the elastostatic criterion (3.1). Notice that the elastostaticity criterion (3.1) for the bulk waves under consideration can be represented in the form

$$\omega^2 \ll s_{\rm t}^2 (p_1^2 + k_\perp^2)\,. \qquad (3.40)$$

Thus, the elastostaticity criterion for an AFM with $c_{\rm m} < s_{\rm t}$ can be satisfied for the low-frequency branch of the spectrum of magnetoelastic waves with a fixed mode number ν in the entire range of the variability of the wave number $k_\perp$, starting from zero if the plate thickness satisfies the condition

$$\omega^2 \ll \frac{s_{\rm t}^2 \pi^2 \nu^2}{4d^2}\,, \quad \nu = 1, 2, \ldots\,. \qquad (3.41)$$

In this case, the elastostaticity criterion will be associated with the standing (over the plate thickness) bulk magnetoelastic wave. It follows from Eqns (3.37)–(3.41) that under the condition (3.41) at $k_\perp = 0$ we have

$$\Omega_\nu^2 \approx \omega_0^2 + c_{\rm m}^2 p_1^2\,, \quad \mathbf{u} \in xz\,, \quad \mathbf{l} \| \mathbf{n} \| z\,, \qquad (3.42)$$

$$\Omega_\nu^2 \approx \omega_0^2 + c_{\rm m}^2 p_1^2\,, \quad \mathbf{u} \| y\,, \quad \mathbf{n} \| \mathbf{l} \| z\,, \qquad (3.43)$$

$$\Omega_\nu^2 \approx \omega_0^2 + \omega_{\rm me}^2 + c_{\rm m}^2 p_1^2\,, \quad \mathbf{u} \| y\,, \quad \mathbf{l} \| z\,, \quad \mathbf{n} \| x\,, \qquad (3.44)$$

where $\nu = 1, 2, \ldots$.

Thus, the fulfillment of the existence criterion (discussed in Refs [46, 50, 51]) for a magnetoelastic gap in the spectrum of spin-wave excitations of a bounded magnetic sample with a linear dimension L, namely

$$\omega_{\rm me} > s_{\rm t} L^{-1} \qquad (3.45)$$

(for the AFM plate under consideration, $L = 2d$), depends to a significant extent on a concrete magnetoacoustic configuration [96]. For $\omega_0 \ll \omega_{\rm me}$ and $c_{\rm m} \ll s_{\rm t}$, condition (3.41) is in fact opposite to criterion (3.45); nevertheless, in the case of spectrum (3.44), the magnetoelastic gap in the spectrum of the standing bulk quasimagnon wave in an AFM plate does exist.

4. Shear surface acoustic waves (SAWs) at the interface between magnetic and nonmagnetic media that are nonvanishing in the elastostatic limit

4.1 A shear SAW in a nonpiezomagnetic antiferromagnet caused by the hybridization of magnetoelastic and inhomogeneous exchange interactions

It is well-known that, from the viewpoint of both crystal-lattice dynamics and the elasticity theory of a continuous medium, the mechanically free surface of an elastic half-space can be considered to be a specific local disturbance in an unbounded ideal half-space. In this case, the surface acoustic wave propagating near the crystal boundary can be represented as a localized oscillation in a crystal with a planar defect [97].

At present, the problem of the existence and the uniqueness of the SAW solutions in the elasticity theory is being well studied analytically for both a mechanically free surface [98–100] and a loaded boundary of a nonmagnetic crystal [101, 102]. Namely, it has been shown that for a mechanically free boundary of a crystal, an SAW can exist at arbitrary directions of propagation of elastic oscillations, except for some preferred orientations. The problem of the existence of SAWs should be solved for them separately, since the boundary conditions in this geometry are satisfied in the case of a purely shear bulk wave. This circumstance makes such a bulk elastic wave unstable with respect to the transformation into an SAW already in the presence of small changes in the elastic boundary conditions. The formation of a shear Gulyaev–Bleustein wave can serve as an example in the presence of piezoelectric [103–105] or piezomagnetic [106–108] interactions in a crystal, as can the appearance of a Love wave [101, 109] when the surface of a semibounded crystal (medium 1) has a rigid acoustic contact with the surface of a layer (medium 2), and the relationship between the elastic parameters of the layer and the half-space is such that

$$s_1 > s_2 \,, \tag{4.1}$$

where s_1 (s_2) is the phase velocity of propagation of a shear elastic wave in an unbounded medium 1 (2). In the case of an antiferromagnet, additional mechanisms of localization of a shear elastic wave arise, namely, piezomagnetic and magnetoelectric [99, 106–108, 110]; for this to occur, however, the magnetic structure should satisfy certain symmetry requirements [59].

At the same time, in spite of intense investigations of various aspects of the formation and propagation of SAWs in magnetically ordered crystals, the corresponding calculations were traditionally performed neglecting the inhomogeneous exchange interaction in the spin system of the magnet ('nonexchange approximation'). In a few studies devoted to the investigation of the effect of the inhomogeneous spin–spin exchange on the conditions of the localization and propagation of SAWs, the role of the nonlocality of the Heisenberg spin–spin exchange in the phonon dynamics of the crystal was reduced to the transformation of an SAW into a pseudosurface (leaky) acoustic wave, i.e., to the implementation of the delocalization of the SAW [111].

Let us show that the effect of the nonlocality of the Heisenberg mechanism of spin–spin interaction on the propagation of a limiting wave even without allowance for the magnetodipole interaction leads to the formation of a shear SAW of a new type near the mechanically free surface of a semibounded magnet [90, 112]. Since an exchange enhancement of magnetoelastic effects and an exchange weakening of magnetodipole effects simultaneously occur in antiferromagnets [46, 50, 51], we consider, as an example, the magnetoelastic dynamics of a two-sublattice model of an easy-axis (with the EA coincident with the z-axis) antiferromagnet, assuming, to simplify the calculations and make them more demonstrable, that the magnetoelastic and elastic properties of the magnet are isotropic. If the antiferromagnetic medium occupies the upper half-space ($z > 0$), whose surface ($z = 0$) is mechanically free and the spins are completely unpinned, the corresponding set of boundary conditions can be written down as follows:

$$\frac{\partial \tilde{\mathbf{l}}}{\partial z} = 0 \,, \quad \sigma_{ik} n_k = 0 \quad \text{at} \quad z = 0 \,, \qquad |\tilde{\mathbf{l}}| \to 0 \quad \text{as} \quad z \to \infty \,. \tag{4.2}$$

According to the standard technique of solving boundary-value problems, the solution to the above dynamic set of equations localized near the free surface of a magnet, e.g., for u, should be sought in the form

$$u \approx \sum_j A_j \exp\left(\mathrm{i}\omega t - q_j z - \mathrm{i} k_\perp r_\perp\right) , \tag{4.3}$$

where $1 < j < N$, A_j are arbitrary constants to be determined; q_j ($q^2 \equiv -(\mathbf{kn})^2$) are the roots of the dispersion (characteristic) equation determining the spectrum of magnetoelastic oscillations of an unbounded magnet, and $k_\perp$ and $r_\perp$ are the projection of the wave vector and the current coordinate along the direction of the propagation of magnetoelastic oscillations in the boundary plane, respectively.

Since in this review we are interested only in shear acoustic oscillations propagating along high-symmetry directions (i.e., in this case, in planes with the normals along the Cartesian coordinate axes), we consider the case of $\mathbf{n} \parallel z$ as an example. Since the problem exhibits cylindrical symmetry, the y-axis was chosen without the loss of generality in the yz plane of the wave propagation. Thus, $\mathbf{k} \in yz$ and $\mathbf{u} \parallel x$.

As follows from the calculated results, the characteristic equation of the magnetoelastic boundary-value problem (4.2) represents a reduced equation biquadratic in q [at $N = 2$ in formula (4.3)], whose coefficients are functions of external parameters specified in experiment, i.e., of the frequency ω, and of the component $k_\perp$ of the wave vector of magnetoelastic oscillations perpendicular to $\mathbf{n}$ [112]:

$$\begin{aligned} & q^4 - P_1 q^2 + P_2 = 0 \,, \\ & P_1 = \frac{\omega_0^2 + 2c_{\mathrm{m}}^2 k_\perp^2 - \omega^2 (1 + c_{\mathrm{m}}^2/s_{\mathrm{t}}^2)}{c_{\mathrm{m}}^2} \,, \\ & P_2 = \frac{\omega_0^2 + c_{\mathrm{m}}^2 k_\perp^2 + \omega_{\mathrm{me}}^2 - \omega^2}{c_{\mathrm{m}}^2} \left(k_\perp^2 - \frac{\omega^2}{s_{\mathrm{t}}^2} \right) . \end{aligned} \tag{4.4}$$

Equations (4.3) and (4.4) permit us to classify the possible types of propagating magnetoelastic oscillations depending on the character of their spatial localization near the surface of the magnet. An analysis showed that, depending on the magnitude of the frequency ω and of the projection $k_\perp$ of the wave vector onto the film plane, four fundamentally different types of propagating two-partial magnetoelastic normal oscillations differing in the character of their spatial localization along the normal to the surface of the magnet are possible. For ω and $k_\perp$ satisfying the conditions

$$\begin{aligned} & k_\perp > k_3 \,, \qquad \omega_+^2(k_\perp) < \omega^2 < \omega_0^2 + \omega_{\mathrm{me}}^2 + c_{\mathrm{m}}^2 k_\perp^2 \,, \\ & 0 < k_\perp < k_1 \,, \qquad \omega^2 < s_{\mathrm{t}}^2 k_\perp^2 \,, \\ & k_1 < k_\perp < k_4 \,, \qquad \omega^2 < \omega_-^2(k_\perp) \end{aligned} \tag{4.5}$$

[where $\omega_\pm^2(k_\perp)$ are the roots of the equation $P_1^2 = 4P_2$ quadratic in ω^2], a two-partial magnetoelastic shear evanescent wave with $q_{1,2}^2 > 0$ can exist, whereas in the region of

$$k_\perp > k_1 \,, \qquad \omega_-^2(k_\perp) < \omega^2 < \omega_+^2(k_\perp) \tag{4.6}$$

a generalized two-partial magnetoelastic shear evanescent wave with $\mathrm{Re}\, q_{1,2}^2 \neq 0$, $\mathrm{Im}\, q_{1,2}^2 \neq 0$, and $\mathrm{Im}\, q_{1,2}^2 > 0$ can be formed. Bulk magnetoelastic waves of the first type ($q_1^2 < 0$, $q_2^2 > 0$) arise for

$$\begin{aligned} & 0 < k_\perp < k_2 \,, \quad s_{\mathrm{t}}^2 k_\perp^2 < \omega^2 < \omega_0^2 + \omega_{\mathrm{me}}^2 + c_{\mathrm{m}}^2 k_\perp^2 \,, \\ & k_\perp > k_2 \,, \qquad s_{\mathrm{t}}^2 k_\perp^2 > \omega^2 < \omega_0^2 + \omega_{\mathrm{me}}^2 + c_{\mathrm{m}}^2 k_\perp^2 \,, \end{aligned} \tag{4.7}$$

where

$$k_1^2 \equiv \frac{\omega_0^2}{s_{\mathrm{t}}^2 - c_{\mathrm{m}}^2} \,, \qquad k_2^2 \equiv \frac{\omega_0^2 + \omega_{\mathrm{me}}^2}{s_{\mathrm{t}}^2 - c_{\mathrm{m}}^2} \,, \qquad \omega_-(k_4) = 0 \,,$$

$$k_3^2 \equiv \frac{\omega_0^2 + \omega_{\mathrm{me}}^2 (1 + s_{\mathrm{t}}^2/c_{\mathrm{m}}^2)}{s_{\mathrm{t}}^2 - c_{\mathrm{m}}^2} \,.$$

Outside the regions specified by formulas (4.5)–(4.7), bulk magnetoelastic waves of the second type ($q_{1,2}^2 < 0$) can form. The condition for the existence of a localized solution to the boundary-value problem under consideration is the existence of a nontrivial solution to the set of equations (4.2) with respect to the partial amplitudes A_j from expansion (4.3) in the case where the corresponding ω and $k_\perp$ belong to the regions (4.5) or (4.6). An analysis shows that in the case of completely free spins the solution to the boundary-value problem (4.2), which describes the dispersion law of the two-partial shear magnetoelastic SAW, can be found exactly, and at an arbitrary value of the wave number $|k_\perp|$ can be represented explicitly as [112]

$$\omega^2 = \frac{N_1}{2} + \sqrt{\left(\frac{N_1}{2}\right)^2 - N_2}\,, \tag{4.8}$$

$$N_1 = \frac{2\omega_0^2 + c_{\rm m}^2 k_\perp^2 - c_{\rm m}^2(\omega_0^2 + \omega_{\rm me}^2 + c_{\rm m}^2 k_\perp^2)/s_{\rm t}^2}{1 - c_{\rm m}^2/s_{\rm t}^2}\,,$$

$$N_2 = \frac{(\omega_0^2 + c_{\rm m}^2 k_\perp^2)^2 - (\omega_0^2 + \omega_{\rm me}^2 + c_{\rm m}^2 k_\perp^2)c_{\rm m}^2 k_\perp^2}{1 - c_{\rm m}^2/s_{\rm t}^2}\,.$$

The amplitudes of oscillations of both the elastic displacement vector **u** and the vector of antiferromagnetism **l** are linearly polarized and are directed along the normal to the sagittal plane (in this case, to the *yz* plane).

A comparison of Eqn (4.8) with Eqns (4.5)–(4.7) indicates that a SAW of this type at $k_\perp = k_1$ becomes delocalized, transforming into a bulk elastic wave. If $k_\perp < k_1 < k_*$ ($k_* \equiv 6\omega_{\rm me}/5c_{\rm m}$), the shear SAW under consideration is a two-partial generalized shear surface elastic wave, whose dispersion curve at $k_\perp = k_*$ is transformed into a two-partial surface shear elastic wave [with $q_{1,2}^2 > 0$ in expression (4.3)]. It can easily be shown that without regard for the inhomogeneous exchange interaction ($\alpha \to 0$) this type of localized magnetoelastic excitations is not realized. In this connection, the above solution (4.8) of the boundary-value magnetoelastic problem (4.2) can be considered as a magnetoelastic shear exchange-type SAW.

It is easily comprehended that in the elastostatic limit ($\omega^2/(s_{\rm t}^2 k_\perp^2) \to 0$) the dispersion relation for a shear SAW of this type describes a surface elastoexchange spin wave whose dispersion curve is shown by a dashed line in Fig. 6b [90].

In the limit of $p_{1,2}d \to \infty$, this dispersion law is obtained from expression (3.24) under the condition that $p_{1,2}^2 < 0$. Notice that a shear SAW of this type can be regarded as an acoustic analog of surface exciton polaritons [113] or generalized surface magnetic polaritons [91, 92].

4.2 Shear first- and second-type SAWs in a fine-layered one-dimensional magnetic phononic crystal of the antiferromagnet–ideal-diamagnet type

Up to now, we have considered only the case of a mechanically free surface of a magnet, for which reason the shear SAWs formed near the surface of a compensated EA AFM represented acoustic analogs of the corresponding types of surface magnetic TE polaritons. In this regard, the localization effect appeared as a result of a hybridization of the inhomogeneous exchange interaction and phononic mechanism of the indirect spin–spin interaction due to the long-range field of elastostatic elastic deformations. However, if the magnetic medium investigated has a continuous acoustic contact with a more rigid nonmagnetic medium, a lateral spin–spin interaction via the long-range field of elastostatic elastic deformations in the nonmagnetic medium also becomes possible. Thus, we can expect in the case of an acoustically continuous interface between magnetic and nonmagnetic media that the localization effect of a shear elastic wave near the surface of a magnet can be realized as a result of hybridization of only phononic mechanisms of indirect spin–spin interaction through a long-range field of elastostatic elastic deformations in the magnet and in its nonmagnetic coating.

Such a shear SAW will represent an acoustic analog of surface magnetic TE polaritons existing near the antiferromagnet–nonmagnetic-dielectric interface.

As said in Section 2, the magnetodipole interaction effects for a superlattice of the magnet–ideal-diamagnet (superconductor) type can be neglected in terms of the effective medium method.

However, the problem of the spectrum of collective elastic excitations of a magnetic phononic crystal of the nongyrotropic-magnet–ideal-diamagnet type with allowance for its finite dimensions has remained unsolved to date.

Let us determine, in terms of the effective medium method, the necessary conditions for the localization of a shear elastic SH wave propagating along the external surface of a bounded one-dimensional magnetic phononic crystal of an EA-AFM (medium 1)–ideal-superconductor (medium 2) type, which has an acoustically continuous contact with the nonmagnetic coating [114]. We will consider only such magnetoacoustic configurations which allow the propagation of SH-type waves (see Section 2).

Let the magnetic superlattice considered have a limited number of periods N_* and represent a strip ($-t \leqslant \xi \leqslant t$) of thickness $2t = DN_*$ which is infinite in the most developed plane. Assume that at $\xi = \pm t$ this strip is acoustically rigidly connected with the semibounded ideal diamagnetic layers of thickness t_+ and t_-, respectively, whose external surfaces are mechanically free.

If we restrict ourselves to a long-wavelength limit, then the effective elastic moduli introduced in Section 2 can be used to describe the elastic dynamics of the superlattice. We will also assume that the nonmagnetic coating of the superlattice is an elastically isotropic medium and that its elastic properties (shear modulus $\tilde{\mu}$, and density $\tilde{\rho}$) are identical for $\xi < -t$ and $\xi > t$. As a result, with allowance made for expressions (2.32)–(2.35), the dispersion equation for the spectrum of SH-type shear waves propagating along such a structure for all three geometries considered can be represented in the form [114]

$$c_\parallel^2\alpha^2 + ac_\parallel \alpha q\left[\tanh(qk_\perp t_+) + q\tanh(qk_\perp t_-)\right]\coth(2\alpha k_\perp t) + a^2q^2\tanh(qk_\perp t_+)\tanh(qk_\perp t_-) = 0\,, \tag{4.9}$$

where

$$\alpha^2 \equiv \frac{c_\perp - \omega^2/(s_{\rm t}^2 k_\perp^2)}{c_\parallel} > 0\,, \qquad q^2 \equiv 1 - \frac{\omega^2}{(\tilde{s}k_\perp)^2}\,,$$

$$\tilde{s}^2 \equiv \frac{\tilde{\mu}}{\tilde{\rho}}\,, \qquad a \equiv \frac{\tilde{\mu}}{\mu_1}\,.$$

If we formally assume that $t_+ = 0$ and $t_- = 0$, the relationships (2.32)–(2.35) and (4.9) will describe the spectrum of a shear elastic wave propagating along a bounded one-dimensional fine-layered magnetic phononic crystal of the EA-AFM–ideal-superconductor type of thickness $2t$, one surface of which is mechanically free and the other has an acoustically continuous contact with the nonmagnetic layer. The external surface of the nonmagnetic layer is mechanically free. An analysis of dispersion equation (4.9) with regard for expressions (2.32)–(2.35) indicates that the formation of an SH-type SAW occurs under the condition that the inequalities $c_{\parallel} < 0$ and $q^2 > 0$ are satisfied simultaneously. For the magnetoacoustic configurations (2.32)–(2.35) this is possible if $\mathbf{k}_{\perp} \parallel x$, $\mathbf{u} \parallel y$, $\mathbf{l} \parallel \mathbf{n} \parallel z$ or $\mathbf{k}_{\perp} \parallel y$, $\mathbf{u} \parallel \mathbf{l} \parallel z$, and $\mathbf{n} \parallel x$. Let us consider the interface between two half-spaces, one of which is occupied by the above-mentioned fine-layered superlattice ($\xi < 0$), and the other by the nonmagnetic medium ($\xi > 0$) under the condition of $\tilde{s} > s_{\rm t}$. Then, relationships (4.9) with allowance made for expressions (2.32)–(2.35) take on the form

$$c_{\parallel}\alpha = -aq\,. \tag{4.10}$$

Thus, the long-wavelength point of the termination of the dispersion curve for a shear SAW is determined for both the above geometries of propagation of a surface SH wave from expressions (2.32)–(2.35) and dispersion equation (4.9) at $q = 0$, $c_{\parallel} = 0$. A joint analysis of expressions (2.32)–(2.35) and (4.9) also shows that if $\mathbf{k}_{\perp} \parallel x$, $\mathbf{u} \parallel y$, and $\mathbf{l} \parallel \mathbf{n} \parallel z$, then the shear SAW for any ratio $d_1/d_2 \neq 0$ also has a short-wavelength ending point of the spectrum, whose position on the (frequency, wave number) plane of the external parameters is determined from the relationships (2.32)–(2.35) and (4.10) at $\alpha = 0$.

According to the terminology accepted in the dynamics of polaritons [113], such an SH-type SAW, which possesses a short-wavelength ending point of the spectrum, can be called a virtual shear SAW, or a shear SAW of the second type.

Let us now consider the geometry where $\mathbf{k}_{\perp} \parallel y$, $\mathbf{u} \parallel \mathbf{l} \parallel z$, and $\mathbf{n} \parallel x$. It follows from relationships (2.32)–(2.35) and (4.10) that in this case the structure of the spectrum of the shear SAW propagating along the semibounded-fine-layered-1D-MPC–semibounded-nonmagnetic-medium interface will substantially depend on the ratio between magnetic layer and nonmagnetic layer thicknesses which determine the elementary period of the given MPC. If the inequality

$$d_1 < d_2 \tag{4.11}$$

is satisfied, the dispersion curve for the shear SAW described by expressions (2.32)–(2.35) and (4.10) will have a short-wavelength ending point of the spectrum (i.e., will be characteristic of a virtual shear SAW). Its position on the (frequency, wave number) plane of the external parameters is determined from the relationships (2.32)–(2.35) and (4.10) at $\alpha = 0$.

Thus, under the condition (4.11), it is a virtual shear SAW that propagates along the interface between the semibounded fine-layered 1D MPC and semibounded nonmagnetic medium. If condition (4.11) is not fulfilled, the relationships (2.32)–(2.35) and (4.10) describe the spectrum of a shear SAW whose dispersion curve has no short-wavelength ending point (in terms of the phenomenological approach).

According to the terminology accepted in the dynamics of polaritons [113], such an SH-type SAW, possessing no ending point of the spectrum, can be called a shear SAW of the first type.

From expressions (2.32)–(2.35) and (4.9), (4.10), it follows that already in the elastostatic limit (3.1) nonexchange surface elastostatic s-type spin waves can propagate in the bounded one-dimensional fine-layered MPC with a nonmagnetic coating under consideration at $\mathbf{k}_{\perp} \parallel y$, $\mathbf{u} \parallel \mathbf{l} \parallel z$, and $\mathbf{n} \parallel x$. The corresponding dispersion relations follow from formulas (2.32)–(2.35) and (4.9), (4.10) under the condition that

$$\alpha^2 \equiv \frac{c_{\perp}}{c_{\parallel}} > 0\,, \qquad q^2 \equiv 1\,, \qquad c_{\parallel} < 0\,. \tag{4.12}$$

In particular, in the limit of $d_2/d_1 \to 0$ we understand from dispersion equation (4.9) that the spectrum of such s-type ESSWs has the form

$$(\omega^2 - \Omega_{+}^2)(\omega^2 - \Omega_{-}^2) = 0\,, \tag{4.13}$$

where

$$\Omega_{\pm}^2 = \omega_0^2 + \omega_{\rm me}^2\,\frac{R_1 \pm \sqrt{R_1^2 - R_2}}{1 + R_1 \pm \sqrt{R_1^2 - R_2}}\,,$$
$$R_1 \equiv \frac{a}{2}\left[\tanh(k_{\perp}t_{+}) + \tanh(k_{\perp}t_{-})\right]\coth(2k_{\perp}t)\,, \tag{4.14}$$
$$R_2 \equiv a^2\tanh(k_{\perp}t_{+})\tanh(k_{\perp}t_{-})\,.$$

The physical mechanism responsible for the formation of this type of nonexchange surface spin-wave excitations is the hybridization of phononic mechanisms of indirect spin–spin interaction through the field of long-range quasistatic shear elastic deformations both in the magnet itself and in its nonmagnetic coating. This, in particular, leads to the possibility of the formation of an extremum (maximum) in the dispersion curve for the shear SAW, corresponding to formulas (4.13), (4.14). The position of this maximum on the (frequency, wave number) plane of external parameters can be changed by varying the thicknesses of the nonmagnetic coating and magnetic layer (1D MPC layer).

Thus, the presence of an acoustic contact ($a \neq 0$) between the nonmagnetic coating ($\zeta < 0$) and the surface of the superlattice considered ($\xi \geqslant 0$) is of fundamental importance for the formation of a surface elastic SH wave of the type under examination and of an s-type ESSW.

The structure of the spectrum of the shear SAW being studied in the case of a bounded 1D PMC, both of whose external surfaces have continuous acoustic contact with identical nonmagnetic half-spaces, is shown in Fig. 8 for the geometry of $\mathbf{k}_{\perp} \parallel y$, $\mathbf{u} \parallel \mathbf{l} \parallel z$, and $\mathbf{n} \parallel x$ for $d_1/d_2 > 1$ and $d_1/d_2 < 1$. It should also be noted that any one of relationships (4.9)–(4.12) and the conclusions made on their basis (in particular, Fig. 8) remain valid for the spectrum of the shear SAW of a fine-layered compensated 1D MPC of the ferromagnet–ideal-diamagnet type with an antiparallel ordering of equilibrium magnetizations of any neighboring tangentially magnetized ferromagnetic layers. In this regard, it is necessary to assume that the effective elastic moduli in formulas (4.9)–(4.12) are determined by relationships (2.42).

In addition, as follows from the results of calculations, the dispersion properties of shear SAWs of this type with the dispersion relation (4.9) can also be governed by changing the character of elastic boundary conditions on the external surfaces of the nonmagnetic coatings bounding the layer of the 1D MPC at hand. More specifically, if the boundary conditions correspond to a rigidly fixed, rather than mechanically free, boundary, then $\coth(qk_{\perp}t_{\pm})$ should be substituted for $\tanh(qk_{\perp}t_{\pm})$ in the dispersion relation (4.9).

It should also be noted that as $d_1/d_2 \to 0$ the spectrum of shear SAWs propagating along the EA AFM plate, both of

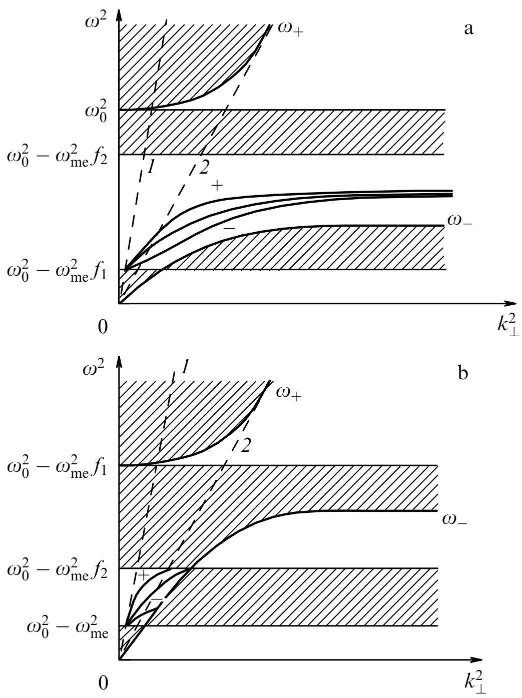

Figure 8. Structure of the spectrum of a shear SAW in the layer of a 1D MPC of the EA-AFM–ideal-diamagnet type at $\mathbf{k} \in xy$, $\mathbf{l} \parallel z$, $\mathbf{n} \perp \mathbf{l}$: (a) $f_1 > f_2$, and (b) $f_1 < f_2$. Solid lines lying between the lines marked by plus and minus signs correspond to the dispersion law of the shear SAW in a semibounded 1D MPC. Dashed line *1* complies with $\omega = \tilde{s} k_\perp$, and dashed line *2* corresponds to $\omega = s_t k_\perp$.

whose surfaces have a continuous acoustic contact with the nonmagnetic medium, represents an acoustic analog of the spectrum of surface TM polaritons in a plate [115].

5. Refraction of bulk elastic waves at the interface between the magnetic and nonmagnetic media induced by dynamic magnetoelastic interaction

As follows from the general theory of wave processes in layered media [59, 66, 116], the local geometry of the wave-vector surface (surface of refraction) of a normal wave propagating in an unbounded medium should be closely connected with the conditions of refraction and localization of a given normal oscillation with allowance for the finite dimensions of an actual sample. In this section, we will consider some examples indicating the validity of these statements for the case of magnetoelastic waves in layered media [74].

Let us start from an analysis of the possibility of excitation of shear SAWs of types discussed in Section 4 by the example of already considered one-dimensional fine-layered MPC.

5.1 Excitation of shear SAWs using acoustic analogs of Otto and Kretschmann configurations

Since the above-described types of shear SAWs are acoustic analogs of various types of surface polaritons, they can be studied using elastic analogs of the Otto and Kretschmann configurations (the nontotal internal reflection method), which are widely applied in the analysis of the spectra of surface polaritons in both nonmagnetic and magnetic media [82, 113, 117, 118]. As is known, an electromagnetic wave with polarization similar to that of the surface polariton investigated falls in both configurations from a more dense half-space onto an optically less dense two-layer structure of the magnetic-layer–nonmagnetic-half-space type (Kretschmann configuration) or nonmagnetic-layer–magnetic-half-space type (Otto configuration), on whose internal boundary a surface magnetic polariton can be formed. Thus, the surface magnetic polariton traveling along the interface between a magnetic and nonmagnetic media generates in the optically denser half-space a bulk electromagnetic wave with the same polarization as that of the surface wave; this means that it is a leaky-type wave.

It follows, thence that if we choose a fine-layered 1D MPC as an example of a magnetic medium, then an elastic analog of the Otto configuration for the excitation of a shear SAW near the MPC external surface will be an acoustically continuous sandwich structure of the 'semiinfinite soft nonmagnetic medium (medium I)–rigid nonmagnetic layer of thickness L (medium II)–semiinfinite fine-layered 1D MPC (medium III)' type. An elastic analog of the Kretschmann configuration is an acoustically continuous sandwich structure of the 'semi-infinite soft nonmagnetic medium (medium I)–layer of a fine-layered 1D MPC of thickness L (medium III)–semiinfinite rigid nonmagnetic medium (medium II)' type. In both cases, the structures indicated permit one to resonantly excite a shear SAW near the 1D-fine-layered-MPC–rigid-nonmagnetic-medium interface at the expense of a bulk SH wave incident from the soft nonmagnetic medium I.

The structure of the sections of the wave-vector surfaces (WVSs) that ensure the excitation of a virtual shear SAW in a three-component acoustically continuous structure in the case of the Otto and Kretschmann configurations is demonstrated in Fig. 9 (see also paper [119]). The region of wave numbers in which the excitation of the considered shear SAW of the first or second type proves to be possible is limited in Fig. 9 by vertical dotted lines, while the region of the wave numbers is designated by two-sided arrows at that interface between the media where a surface wave is formed.

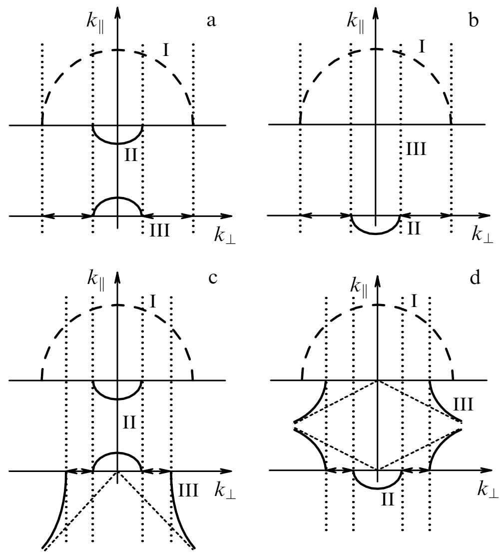

Figure 9. Structure of the sections of the surface of refraction for the excitation of a shear SAW at the interface between magnetic and nonmagnetic media: (a, b) shear SAW of the first type; (c, d) shear SAW of the second type excited using the acoustic analog of the Otto configuration (a, c) and the Kretschmann configuration (b, d); I (II) corresponds to a soft (rigid) nonmagnetic medium, and III corresponds to a magnetic medium (1D MPC).

As to the reflection coefficient R at the interface between the layer and the upper half-space, the absolute value $|R|$ is rigorously equal to unity without allowance for the dissipation in the lower half-space, since in both the layer and the lower half-space only evanescent waves are excited. Otherwise, $|R|$ will pass through a minimum at the same values of the frequency and wave number (angle of inclination), which simultaneously satisfy the dispersion law of the shear SAW localized at the layer–lower-half-space interface.

As shown in paper [120], the allowance made for the dissipation can qualitatively change the character of the reflection of an elastic wave incident on the surface of a defect near which there is a localized mode—from the complete reflection to virtually total transmission. At the same time, under certain conditions a nondissipative mechanism of the realization of the reflectionless propagation effect of a shear elastic wave is also possible in a fine-layered one-dimensional MPC of a finite thickness.

5.2 Reflectionless propagation of a bulk shear elastic wave through a bounded fine-layered one-dimensional MPC with a nonmagnetic coating

Using the transition matrix $\hat{M}$ for a layer of thickness d with a normal $\mathbf{n}$ to the surface and a surface impedance Z, let us derive an expression for the reflection coefficient R of the shear bulk wave for a structure consisting of a layer of thickness d, both surfaces of which have continuous acoustic contact with nonidentical (in terms of elastic properties) half-spaces [121] [see formula (2.22)].

If u is the component of the polarization vector and $k_\parallel$ is the component of the wave vector of the bulk SH wave normal to the layer surface, then one obtains

$$\begin{pmatrix} u \\ s \end{pmatrix}_0 = M_{ik} \begin{pmatrix} u \\ s \end{pmatrix}_{-d} ,$$
$$M_{ik}(k_\parallel, d) = \begin{pmatrix} \cos(k_\parallel d) & Z^{-1}\sin(k_\parallel d) \\ -Z\sin(k_\parallel d) & \cos(k_\parallel d) \end{pmatrix} . \qquad (5.1)$$

For a shear bulk SH wave incident from the upper half-space ($\xi > 0$) on the layer surface, we then have

$$R = \frac{Z_+ Z_- M_{12} + M_{21} + \mathrm{i}(Z_+ M_{11} - Z_- M_{22})}{Z_+ Z_- M_{12} - M_{21} + \mathrm{i}(Z_+ M_{11} + Z_- M_{22})} , \qquad (5.2)$$

where Z_+ and Z_- are the surface impedances of the media for $\xi > 0$ and $\xi < -d$, respectively, and M_{ik} are the elements of the transition matrix (5.1).

Thus, in the case of a half-wave layer, we obtain

$$M_{11} = M_{22} = 1 , \quad M_{21} = M_{12} = 0 , \quad R = \frac{Z_+ - Z_-}{Z_+ + Z_-} . \qquad (5.3)$$

If the shear wave in the layer is evanescent ($k_\parallel^2 < 0$), we have $|R| < 1$ at $Z_+ = Z_-$ in expression (5.2).

Now let the intermediate layer that connects the upper and lower half-spaces in the structure under consideration be an acoustically continuous system consisting of a layer of an elastically isotropic nonmagnetic medium (medium II) and a 1D MPC layer described by relationships (2.32)–(2.35) (medium III), with thicknesses t_{II} and t_{III}, respectively. If Z_{II} (Z_{III}) is the surface acoustic impedance of medium II (III), the expression for the reflection coefficient R of the shear bulk elastic wave will, as before, have the form (5.2), but now

$$M_{11} = \cos(k_{\mathrm{II}} t_{\mathrm{II}}) \cos(k_{\mathrm{III}} t_{\mathrm{III}}) - \frac{Z_{\mathrm{III}}}{Z_{\mathrm{II}}} \sin(k_{\mathrm{II}} t_{\mathrm{II}}) \sin(k_{\mathrm{III}} t_{\mathrm{III}}) ,$$
$$M_{12} = Z_{\mathrm{III}}^{-1} \cos(k_{\mathrm{II}} t_{\mathrm{II}}) \sin(k_{\mathrm{III}} t_{\mathrm{III}}) + Z_{\mathrm{II}}^{-1} \cos(k_{\mathrm{III}} t_{\mathrm{III}}) \sin(k_{\mathrm{II}} t_{\mathrm{II}}) , \qquad (5.4)$$
$$M_{21} = -Z_{\mathrm{II}} \sin(k_{\mathrm{II}} t_{\mathrm{II}}) \cos(k_{\mathrm{III}} t_{\mathrm{III}}) - Z_{\mathrm{III}} \sin(k_{\mathrm{III}} t_{\mathrm{III}}) \cos(k_{\mathrm{II}} t_{\mathrm{II}}) ,$$
$$M_{22} = \cos(k_{\mathrm{II}} t_{\mathrm{II}}) \cos(k_{\mathrm{III}} t_{\mathrm{III}}) - \frac{Z_{\mathrm{II}}}{Z_{\mathrm{III}}} \sin(k_{\mathrm{II}} t_{\mathrm{II}}) \sin(k_{\mathrm{III}} t_{\mathrm{III}}) ,$$

where $k_{\mathrm{II}}^2 \equiv \omega^2 \rho_{\mathrm{II}}/\mu_{\mathrm{II}} - k_\perp^2$, ρ_{II} and μ_{II} are the density and shear modulus of the medium II, respectively, and k_{III} is the component [determined from expressions (2.32)–(2.35)] of the wave vector of the shear bulk wave in the MPC that is normal to the 1D MPC surface.

An analysis of Eqn (5.4) shows that, as before, relationships (5.3) can be valid in this case again under the condition that the frequency and the angle of incidence of the bulk SH wave in the upper half-space simultaneously satisfy the following conditions

$$Z_{\mathrm{II}} = -Z_{\mathrm{III}} , \qquad q_{\mathrm{II}} t_{\mathrm{II}} = q_{\mathrm{III}} t_{\mathrm{III}} , \qquad (5.5)$$

where $q_{\mathrm{II}}^2 \equiv -k_{\mathrm{II}}^2$, and $q_{\mathrm{III}}^2 \equiv -k_{\mathrm{III}}^2$. When the upper and lower half-spaces are identical in terms of elastic properties, $Z_+ \equiv Z_-$, these properties correspond to the total transmission ($R = 0$) of the shear bulk wave through the two-layered structure under consideration. In this case, in particular, it is necessary that relationships $Z_{\mathrm{II}} Z_{\mathrm{III}} < 0$ and $t_{\mathrm{II}} t_{\mathrm{III}} \neq 0$ be fulfilled simultaneously.

An analysis of conditions (5.5) shows that, in spite of the existence of a field of evanescent shear elastic waves in the nonmagnetic layer of medium II, the presence of a layer of a composite magnetic material (fine-layered MPC) makes possible a reflectionless transmission of a shear bulk wave through the layered structure under examination.

From the physical viewpoint, the first of the relationships in Eqn (5.5) defines the dispersion equation of the shear SAW propagating along the interface between two half-spaces, one of which is occupied by the nonmagnetic medium II, and the other by the fine-layered 1D MPC (4.10). The validity of these relationships is possible upon the excitation of a shear SAW of both the first and second types on the surface of the 1D MPC.

The structure of the sections of the WVS of a shear wave in the four-component structure of interest is shown in Fig. 10 for the case of reflectionless transmission with the participation of a shear SAW of the first or second type.

It should also be noted that the conditions (5.5) for the reflectionless transmission of a shear bulk wave remain valid for $t_{\mathrm{III}} < t_{\mathrm{II}}$, even when the layer of medium III is located inside rather than on the surface of the nonmagnetic layer (medium II). If $t_{\mathrm{III}} > t_{\mathrm{II}}$, relationships (5.5) determine the conditions for the reflectionless transmission of the bulk SH wave also for the structure in which the layer of nonmagnetic medium II is inside rather than on the surface of medium III.

The anomalies found in the propagation of a shear elastic wave through the layered acoustically continuous structure containing a layer of a composite magnetic material represent an acoustic analog of the enhancement effect of photon

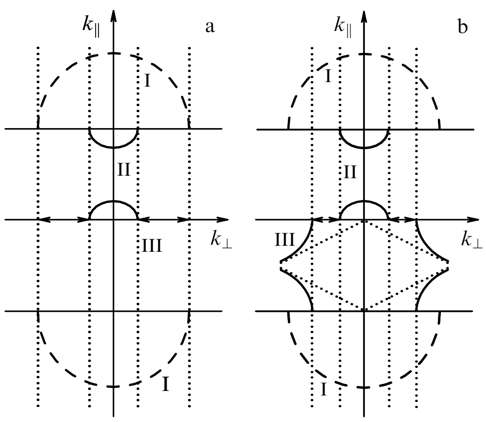

Figure 10. Structure of the sections of the surface of refraction for a layered magnet–nonmagnet-type structure that is responsible for the reflectionless transmission of a bulk SH wave due to the excitation of a shear magnetoelastic SAW of the (a) first or (b) second type. I (II) corresponds to a soft (rigid) nonmagnetic medium, and III corresponds to a magnetic medium (1D MPC).

tunneling by a layer of a uniaxial anisotropic left medium [122].

5.3 Negative and anomalous acoustic refraction in a one-dimensional fine-layered phononic crystal

As known from crystal optics and crystal acoustics [59, 123, 124], in the analysis of conditions for reflection and refraction of bulk normal vibrations at the interface between media, the shape of the refraction surface of such a normal wave is of great importance.

In this section, using the example of a fine-layered 1D MPC of the EA-AFM–ideal-diamagnet type (see Section 2), we will show (see also paper [125]) that under certain conditions in such a composite medium some new acoustic effects can manifest themselves, whose electromagnetic analogs have actively been studied in the dynamics of photonic crystals possessing negative refraction indices (so-called left media), but which have not been known for magnetic phononic crystals. The case in point is negative acoustic refraction (group velocities of the incident and refracted elastic waves lie on the same side from the normal to the interface between the media, i.e., the tangential components of the group velocities of the incident and refracted waves differ in sign). If, as before, we restrict ourselves to the same magnetoacoustic configurations as in Section 2, then the corresponding effective elastic moduli will be determined in terms of the effective-medium model by relationships (2.32)–(2.35).

Let the 1D MPC under study occupy the lower half-space ($\xi < 0$) and its external surface ($\xi = 0$) have continuous acoustic contact with a homogeneous elastically isotropic half-space ($\xi > 0$), from which an elastic bulk wave with a frequency ω and a wave number $k_\perp$ is incident on the surface of the superlattice.

Below, we restrict ourselves to the case where the normal to the incidence plane coincides with one of the coordinate axes; then, the elastic normal modes excited in the superlattice will be split into waves polarized in the plane of incidence (quasilongitudinal and quasitransverse modes) and those polarized perpendicular to it (SH-type waves). If $\mathbf{k} \in xy$, the quasilongitudinal and quasitransverse waves transform into longitudinal and transverse (P- and SV-type) waves, respectively [63]. Then, the kinematics of the reflected (refracted) normal wave with a given frequency ω and projection $k_\perp$ of the wave vector onto the surface of the medium is determined by the structure of the section of its WVS by the plane of reflection (refraction) in the k space [59, 123, 124]. The radius vector of a point in such a section is collinear to the direction of the phase velocity, and the external normal coincides with the direction of the group velocity of the normal mode being excited with $k_\parallel = k_\parallel(k_\perp, \omega)$, where $k_\parallel$ is the projection of the wave vector of the excited wave onto the normal to the interface between the media.

Thus, we can distinguish four types of points in the WVS section, in which the projections of the wave and group velocities onto the interface between the media and onto the normal to it have different signs (we restrict ourselves to the case where both the phase and group velocities of the refracted wave lie in the same plane):

$$\begin{aligned} &\mathrm{A}\ \left(\mathbf{k}_\perp \frac{\partial\omega}{\partial\mathbf{k}_\perp} > 0\,,\quad \mathbf{k}_\parallel \frac{\partial\omega}{\partial\mathbf{k}_\parallel} > 0\right),\\ &\mathrm{B}\ \left(\mathbf{k}_\perp \frac{\partial\omega}{\partial\mathbf{k}_\perp} > 0\,,\quad \mathbf{k}_\parallel \frac{\partial\omega}{\partial\mathbf{k}_\parallel} < 0\right),\\ &\mathrm{C}\ \left(\mathbf{k}_\perp \frac{\partial\omega}{\partial\mathbf{k}_\perp} < 0\,,\quad \mathbf{k}_\parallel \frac{\partial\omega}{\partial\mathbf{k}_\parallel} > 0\right),\\ &\mathrm{D}\ \left(\mathbf{k}_\perp \frac{\partial\omega}{\partial\mathbf{k}_\perp} < 0\,,\quad \mathbf{k}_\parallel \frac{\partial\omega}{\partial\mathbf{k}_\parallel} < 0\right). \end{aligned} \tag{5.6}$$

For the refracted wave, two conditions should always be fulfilled: (1) tangential projections of the wave vectors of the incident ($\mathbf{k}_{\perp\mathrm{i}}$) and refracted ($\mathbf{k}_{\perp\mathrm{r}}$) waves onto the interface between the media should be equal, and (2) the vector of the group velocity $\mathbf{v}_\mathrm{g}$ of the refracted wave should make an acute angle with the internal normal to the interface between the media. Thus, negative refraction ($\mathbf{k}_{\perp\mathrm{r}}\,\partial\omega/\partial\mathbf{k}_{\perp\mathrm{r}} < 0$) is, in principle, possible only for points of C and D types (5.6) on the WVS of the refracted wave.

With allowance made for relationships (2.32)–(2.35), the shape of the WVS section due to the incident plane for the bulk SH-type wave refracted in a 1D MPC takes the form

$$\begin{aligned} &k^2 = \frac{k_0^2}{c_\parallel \cos^2\phi + c_\perp \sin^2\phi}\,, \qquad k_0 \equiv \frac{\omega}{s_\mathrm{t}}\,,\\ &\tan\phi = \frac{k_\perp}{k_\parallel}\,, \qquad k_\parallel^2 + k_\perp^2 = k^2\,. \end{aligned} \tag{5.7}$$

Here, the following notations were introduced:

$$\begin{aligned} &c_\parallel = \bar{c}_{44}\,,\ \ c_\perp = \bar{c}_{66}\,,\ \ \mathbf{k} \in xz\,,\ \ \mathbf{n} \parallel z\,,\ \ \tan\phi = \frac{k_x}{k_z}\,,\\ &c_\parallel = \bar{c}_{66}\,,\ \ c_\perp = \bar{c}_{44}\,,\ \ \mathbf{k} \in xz\,,\ \ \mathbf{n} \parallel x\,,\ \ \tan\phi = \frac{k_z}{k_x}\,. \end{aligned} \tag{5.8}$$

Thus, negative acoustic refraction is possible if

$$\bar{c}_{44} < 0\,,\quad \mathbf{k} \in xz\,,\quad \mathbf{n} \parallel x\,, \tag{5.9}$$

whereas the anomalous acoustic refraction effect (phase velocities of the incident and refracted elastic waves lie on the same side of the normal to the interface between the media, i.e., the normal components of phase velocities of the incident and refracted waves have different signs) shows itself for

$$\bar{c}_{44} < 0\,,\quad \mathbf{k} \in xz\,,\quad \mathbf{n} \parallel z\,. \tag{5.10}$$

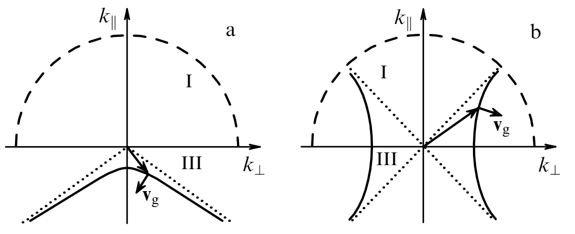

Figure 11. Structure of the sections of the surface of refraction in the case of a bulk SH wave ($\mathbf{k} \in xz$) incident from the elastically isotropic nonmagnetic medium (upper half-space, dashed curve) on the surface of a semibounded 1D MPC: (a) negative acoustic refraction ($\mathbf{n} \parallel x$), and (b) anomalous acoustic refraction ($\mathbf{n} \parallel z$).

The structure of the WVS sections for the bulk shear wave in the cases under consideration is illustrated in Fig. 11.

For the above-considered magnetoacoustic configurations, the revealed anomalies of acoustic refraction also exist in the limiting case of $d_2/d_1 \to 0$, which corresponds to a spatially uniform EA AFM.

A qualitatively different situation is observed in the case where the sagittal plane coincides with the xy plane in the fine-layered 1D MPC [see formulas (2.32)–(2.35)]. For such a magnetoacoustic configuration, the WVS section of a normal shear wave under the condition $d_1 d_2 = 0$ [infinite spatially uniform magnetic (at $d_2 = 0$) or nonmagnetic (at $d_1 = 0$) medium] has the shape of a circle, which strongly contradicts the conditions for the fine-layered MPC ($d_1 d_2 \neq 0$). The shape of the WVS section due to the incidence plane for the bulk SH-type wave refracted in the MPC is, as before, determined by Eqn (5.7), but now

$$c_\parallel = \bar{c}_{55}\,, \quad c_\perp = \bar{c}_{44}\,, \quad \mathbf{k} \in xy\,, \quad \mathbf{n} \parallel x\,, \quad \tan\phi = \frac{k_y}{k_x}\,. \tag{5.11}$$

An analysis shows that in this magnetoacoustic configuration the negative acoustic refraction effect for the shear bulk wave is realized if the following inequalities hold true simultaneously:

$$c_\parallel = \bar{c}_{55} > 0\,, \qquad c_\perp = \bar{c}_{44} < 0\,, \tag{5.12}$$

whereas the anomalous acoustic refraction effect in the same magnetoacoustic configuration is possible if the following inequalities are satisfied simultaneously:

$$c_\parallel = \bar{c}_{55} < 0\,, \quad c_\perp = \bar{c}_{44} > 0\,. \tag{5.13}$$

Thus, we can govern the character of refraction even at fixed values of the frequency and angle of incidence of the bulk shear wave on the external surface of the superlattice by changing the relative fraction of the magnetic and nonmagnetic media comprising the period of the 1D MPC. In this case, both the negative and anomalous acoustic refraction effects are unattainable, in principle, for a given magnetoacoustic configuration, when, instead of the fine-layered 1D MPC, there is a spatially homogeneous magnetic or nonmagnetic medium forming the elementary period of the phononic crystal under examination (i.e., at $d_1 d_2 = 0$). Thus, the 1D MPC can be regarded in this case as a magnetoacoustic metamaterial.

It follows from expressions (2.35) and (2.42) that the result analogous to inequalities (5.13) can also be obtained for a fine-layered compensated 1D MPC of the ferromagnet–ideal-diamagnet type with an antiparallel ordering of equilibrium magnetizations of any neighboring tangentially magnetized ferromagnetic layers.

The negative acoustic refraction can arise not only in the case of a refracted elastic wave polarized orthogonally to the plane of incidence (sagittal plane), but also when the refracted elastic wave is polarized in the incidence plane. In the model of a fine-layered magnetic superlattice under consideration, this is possible if the sagittal plane coincides with xz or yz.

Let, for example, $\mathbf{n} \parallel z$, $\mathbf{k} \in xz$; then, the calculations with allowance made for expressions (2.32)–(2.35) show that in the fine-layered superlattice considered at a given magnetoacoustic configuration an acoustic birefringence effect is possible without a change in the branch, and moreover both branches belong to the same mode (possess the same frequency ω, polarization, and wave number $k_\perp$ of the spectrum of normal elastic oscillations of the quasitransverse mode).

The section of the corresponding part of the WVS by the incidence plane (xz) is described by the following relationships:

$$k^2 = \frac{A + \sqrt{A^2 - 4B}}{2B}\,\omega^2\,, \quad A = \lambda + 2\mu + \bar{c}_{55}\,,$$
$$\tan\theta = \frac{k_x}{k_z}\,, \qquad \bar{c}_{55} \equiv \mu_1\,\frac{\omega_0^2 - \omega^2}{\omega_0^2 + \omega_{\mathrm{me}}^2 f_1 - \omega^2}\,, \tag{5.14}$$
$$B = (\lambda + 2\mu)\bar{c}_{55} + (\lambda + \mu)(\mu - \bar{c}_{55})\sin^2 2\theta\,.$$

It follows from the last formulas that negative acoustic refraction is possible for the specified values of $k_\perp$ and ω only for one of the branches of the refracted quasibulk wave under the condition that the inequality $k_\perp > k_*$ and one of the following inequalities

$$-(\lambda + \mu) < \bar{c}_{55} < 0\,,$$
$$\bar{c}_{55}(\omega) > \sqrt{\mu(\lambda + \mu) + 0.25(3\lambda + 2\mu)^2} - 0.5(3\lambda + 2\mu) \tag{5.15}$$

are fulfilled. Here, k_* is determined as the value of $k_\perp$ satisfying the relationship $\partial k_\perp / \partial k_\parallel = 0$.

As to the second branch of the refracted quasitransverse wave from Eqn (5.14), possessing the same polarization, frequency ω, and wave number $k_\perp$, the negative acoustic refraction effect is absent for this branch ($\mathbf{k}_\perp \partial\omega/\partial\mathbf{k}_\perp > 0$) and the phase velocities of the incident and refracted waves lie on the same side from the external normal to the interface between the media (anomalous acoustic refraction effect).

The structure of the WVS sections of the quasitransverse wave in this case is shown in Fig. 12.

Similar to the case of photonic crystals [126], the above-considered negative acoustic refraction effect can also manifest itself in the nonlinear magnetoelastic dynamics of a one-dimensional MPC, for instance, upon the generation of a refracted wave with harmonics being multiples of the principal frequency.

It should also be noted that, apart from striction, there are also possible other mechanisms of formation of the negative curvature on the section of the WVS by the sagittal plane and, consequently, of the realization of both the negative and anomalous acoustic refraction effects, e.g., piezoelectric and piezomagnetic effects for SH-type waves and the anisotropy of elastic properties for quasitransverse elastic waves.

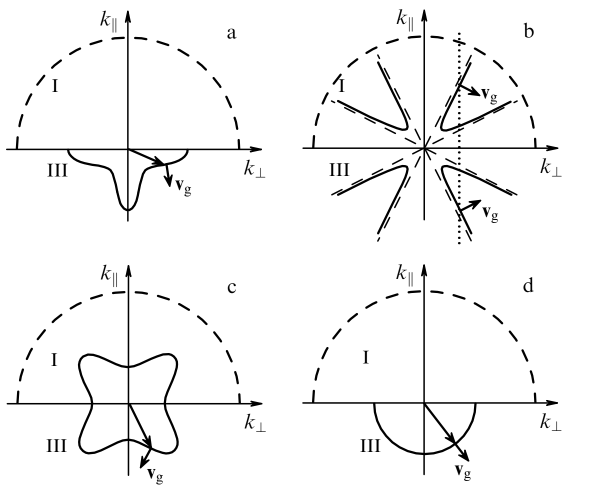

Figure 12. Frequency dependence of the sections of the surface of refraction in the case of the incidence of a bulk SV wave ($\mathbf{k} \in xz$) from an elastically isotropic nonmagnetic medium (upper half-space, dashed line) on the surface of a semibounded 1D MPC: (a) $0 < \bar{c}_{55}(\omega) \ll \mu$; (b) $-(\lambda + \mu) < \bar{c}_{55}(\omega) < 0$; (c) second inequality in Eqn (5.15), and (d) $0 < \bar{c}_{55}(\omega) \approx \mu$.

5.4 Manifestation of the topology of the wave-vector surface of normal magnetoelastic excitations in the spectrum of a bounded magnet

Since for the given values of the wave number $k_\perp$ and frequency ω the limiting bulk wave propagating along the surface of the magnet can prove to be unstable with respect to a transformation into a surface wave with the same polarization [98, 109], it can be expected that the necessary conditions for the formation of such a surface wave will be connected with the shape of the section of the WVS of the corresponding normal bulk wave by the sagittal plane (this was first demonstrated by the example of Rayleigh waves propagating in nonmagnetic media, and by the example of shear SAWs in piezomagnets in Refs [127, 128] and [108], respectively).

Let us consider now how the effects in the surface dynamics of the semibounded magnet are connected with a local geometry of the WVS of the normal magnetoelastic wave propagating in an unbounded magnet. Let us start with the case of the nonexchange limit.

5.4.1 Relation between the local geometry of the WVS section and the conditions for the formation of a shear SAW on the surface of a magnet. As an example, we will use relationships (2.32)–(2.35). For a fixed value of the frequency, these relationships determine, inter alia, the shape of the WVS section of a normal SH wave of an unbounded fine-layered 1D MPC of the EA-AFM–superconductor type at $\mathbf{n} \parallel x$, $\mathbf{k}_\perp \parallel y$, and $\mathbf{u} \parallel \mathbf{l} \parallel z$. Let us compare relationships (2.32)–(2.35) with the conditions for the existence of a shear surface one-partial SAW near the external surface of a semibounded 1D MPC with $d_1 < d_2$ in the same magnetoacoustic configuration [see dispersion relation (4.10)]. The analysis gives evidence that, for the formation of a virtual shear SAW of the second type, it is necessary that the maximum of the negative curvature on the WVS section of the corresponding normal wave in the unbounded medium be coincident with the direction of the propagation of the nonexchange SAW under consideration (Fig. 11b).

It also follows from the relationships obtained in Section 4.2 that this criterion remains, in principle, valid also in the case of a shear two-partial SAW forming near the mechanically free surface of the EA AFM due to the hybridization of the magnetoelastic and inhomogeneous exchange interactions. In particular, it follows from dispersion relation (3.16) that in the case of $\mathbf{u} \parallel y$, $\mathbf{k} \in xz$, and $\mathbf{n} \parallel z \parallel \mathbf{l}$, while neglecting the acoustic delay ($\omega / s_t k_\perp \to 0$), the shape of the section, in the k space, of the isofrequency ($\omega = \text{const}$) surface of the normal bulk elastoexchange spin wave in an unbounded antiferromagnet crystal by the sagittal xz plane is determined by an equation of the form

$$c_m^2 k^2 \approx \omega^2 - \omega_0^2 - \omega_{me}^2 \sin^2\theta \,. \tag{5.16}$$

The calculated results testify that the conditions under which portions with a negative Gaussian curvature are formed in dispersion curve (5.16) can be represented in the form

$$\omega^2 < \omega_0^2 + 2\omega_{me}^2 \,, \tag{5.17}$$

$$k_\perp^2 < \frac{\omega_{me}^2}{c_m^2} \,. \tag{5.18}$$

As follows from equation (5.16), the portion of the curve with the maximum negative Gaussian curvature in the WVS section of interest is realized at $\theta = \pi/2$ (Fig. 13). In addition, it is required that the normal $\mathbf{n}$ to the surface of the magnet be perpendicular to the direction in which the above-mentioned portion of the curve with the maximum negative curvature is formed.

Since the spatial distribution of the amplitudes of the normal bulk oscillations is a result of the interference of waves incident on the sample and reflected from its boundaries, it can be expected that the local geometry of the surface of wave vectors of normal oscillations under consideration should also substantially affect the structure of bulk oscillations with the same polarization as in the case of a crystal of finite dimensions.

5.4.2 Relation between the local geometry of the WVS section and the structure of the spectrum of bulk magnetoelastic excitations in a bounded magnet (elastostatic limit). As an

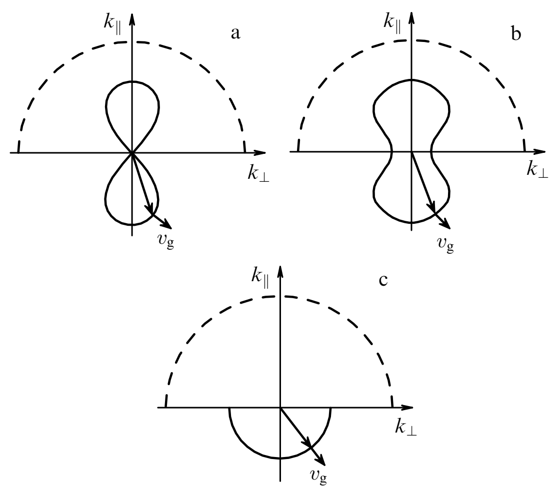

Figure 13. Frequency dependence of the sections of the surface of refraction in the case of the incidence of a bulk SH wave from the elastically isotropic nonmagnetic medium (upper half-space, dashed line) on the surface of a semibounded EA AFM ($\mathbf{l} \parallel z$, $\mathbf{n} \parallel z$, $\mathbf{k} \in xz$) in the elastoexchange limit: (a) $\omega_0^2 < \omega^2 < \omega_{me}^2 + \omega_0^2$; (b) $\omega_0^2 + \omega_{me}^2 < \omega^2 < 2\omega_{me}^2 + \omega_0^2$, and (c) $\omega^2 > 2\omega_{me}^2 + \omega_0^2$.

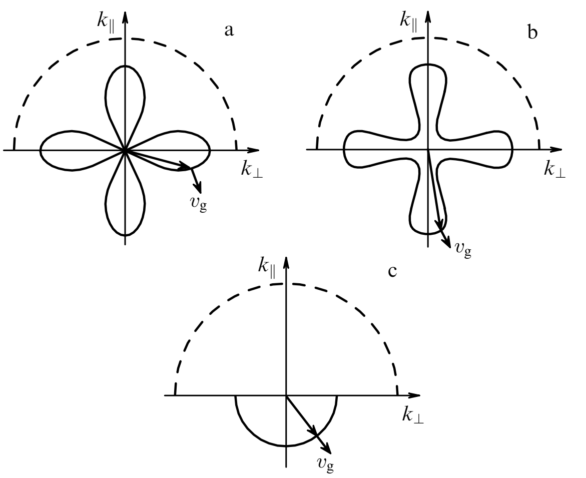

Figure 14. Frequency dependence of the sections of the surface of refraction in the case of the incidence of a bulk SV wave from an elastically isotropic nonmagnetic medium (upper half-space, dashed line) on the surface of a semibounded EA AFM ($\mathbf{l}\parallel z$, $\mathbf{n}\parallel z$, $\mathbf{k}\in xz$) in the elastoexchange limit ($\tilde{\omega}_{\rm me}^2 \equiv \omega_{\rm me}^2(1 - s_t^2/s_l^2)$): (a) $\omega_0^2 < \omega^2 < \tilde{\omega}_{\rm me}^2 + \omega_0^2$; (b) $\omega_0^2 + \tilde{\omega}_{\rm me}^2 < \omega^2 < 5\tilde{\omega}_{\rm me}^2 + \omega_0^2$, and (c) $\omega^2 > 5\tilde{\omega}_{\rm me}^2 + \omega_0^2$.

example, we will consider a plate of an EA AFM (in the equilibrium state, with $\mathbf{l}\parallel z$ and $|\mathbf{m}| = 0$), whose spectrum of elastoexchange excitations was discussed in Section 3.3. The results of calculations reveal that in the elastostatic approximation (3.1), with allowance made for the magnetoelastic and inhomogeneous exchange interactions in an unbounded EA AFM, the shape of the section of the corresponding WVS by the sagittal xz plane for s-type EESWs is determined by equation (5.16), whereas for p-type EESWs, it is determined by the following expression:

$$c_{\rm m}^2 k^2 \approx \omega^2 - \omega_0^2 - \omega_{\rm me}^2\left(1 - \frac{s_t^2}{s_l^2}\right)\sin^2 2\theta\,. \tag{5.19}$$

The structure of these curves depending on frequency is illustrated in Figs 13 and 14.

In the nonexchange approximation ($c_{\rm m} \to 0$), the WVS sections of both bulk s- and p-type ESSWs in the k space, which are described by relationships (5.16) and (5.19), will represent two [in the case of an s-type wave (5.16)] or four [in the case of a p-type wave (5.19)] straight lines that intersect at the origin (see Figs 11 and 12b, respectively). It is known that the normal to the surface of refraction (WVS) coincides with the direction of the group velocity of the wave [59, 123]. In this regard, it can be expected that the investigation of the local geometry of the section of the isofrequency surface [expressions (5.16), (5.19)] by the sagittal plane will make it possible to decide which type of wave (direct or reverse) the portion of the dispersion curve of the corresponding bulk wave in the plate refers to, since any point of sections (5.16) and (5.19) will be described by two numbers from the following set characterizing our bulk wave: ω, ν, d, and $k_\perp$ (in choosing two numbers, the remaining two should be considered to be given).

A comparison of relationships (5.16) and (5.19) [both in the nonexchange approximation ($c_{\rm m} \to 0$) and with allowance made for the inhomogeneous exchange interaction] with the corresponding dispersion relations for the elastostatic or elastoexchange bulk s- or p-type spin-wave excitations propagating along an AFM plate (see Sections 3.1, 3.3) allows the following conclusions.

As an example, let us consider the case of bulk s-type ESSWs and EESWs [see relationships (3.4), (3.5), Figs 1a, b; and relationships (3.30), (3.33), Figs 7a, b, respectively] and section (5.16) by the sagittal xz plane (see Fig. 13).

If the projection of the group velocity onto the interface between the media is negative for a given $\omega = {\rm const}$ and the surface of refraction (5.16) at the point of intersection of this surface with the straight line $k_\perp = {\rm const}$, then a bulk ESSW with a mode number ν, which propagates along a film of thickness d in the $\mathbf{k}_\perp/|\mathbf{k}_\perp|$ direction, will be a reverse wave [see, e.g., relationships (3.5), (3.33)]. If this projection is positive, the corresponding bulk wave propagating along the plate in the $\mathbf{k}_\perp/|\mathbf{k}_\perp|$ direction will be a direct wave [see, e.g., relationships (3.4), (3.30)]. Thus, just as in the case of bulk polaritons, the negative refraction and the formation of a reverse wave are accompanying effects. If at a given $\omega = {\rm const}$ the straight line $k_\perp = {\rm const}$ intersects the curve determining the section of the WVS by the sagittal plane several times [two times in the case of section (5.16)], this means that in this magnetoacoustic configuration the necessary conditions for the realization of crossover between the dispersion curves belonging to the modes of the spectrum of bulk waves with the same polarization that propagate along the plate in the $\mathbf{k}_\perp/|\mathbf{k}_\perp|$ direction exist. The mode numbers correspond to the ordinates of the points of intersection of the straight line $k_\perp = {\rm const}$ with the section (5.16).

In the section of the WVS by the sagittal plane, the existence of points at which $\partial\omega/\partial k_\perp = 0$ for some $k_\perp \neq 0$ (the normal to the surface is orthogonal to the interface between the media) corresponds to the case where, in the dispersion curve of the mode with a number ν belonging to the spectrum of bulk oscillations traveling along the surface of a film of thickness $2d$ in the $\mathbf{k}_\perp/|\mathbf{k}_\perp|$ direction, there is an extremum at this value of the wave number $k_\perp$. Whether this point is a maximum or a minimum depends on whether the Gaussian curvature of the WVS section of this type of normal wave is negative or positive, respectively.

The validity of this approach is retained in the case of bulk p-type ESSWs and EESWs as well [see relationships (3.8), Fig. 1c and relationships (3.36), Fig. 7c, respectively, and also the WVS section (5.19) by the sagittal xz plane in Fig. 14]. Notice that the above-mentioned linkage between the local symmetry of the WVS section and the structure of the spectrum of both surface and bulk normal modes remains valid for waves of other physical natures, too [129–132].

Up to now, when considering the relation between the shape of the WVS and the magnetoelastic dynamics of a bounded magnet by the example of a plate of an EA AFM in the collinear phase, we have restricted ourselves to an analysis of the effects connected with the topology of the section of the WVS with a given polarization of spin (elastic) oscillations. But, as follows from expressions (5.16) and (5.19), already in the unbounded EA AFM a closed line can exist, along which the above-mentioned wave vector surfaces corresponding to different polarizations of normal oscillations intersect. Each point of such a line determines some special direction in the k space, along which any spin wave with a frequency $\omega = {\rm const}$ has one and the same phase velocity, irrespective of the polarization. If the polarization of the wave in a plane with the normal along such a special direction describes an elliptic curve, the group velocity of the spin wave will determine a conical surface with an apex lying at the point of the intersection of the surfaces from Eqns (5.16) and (5.19)

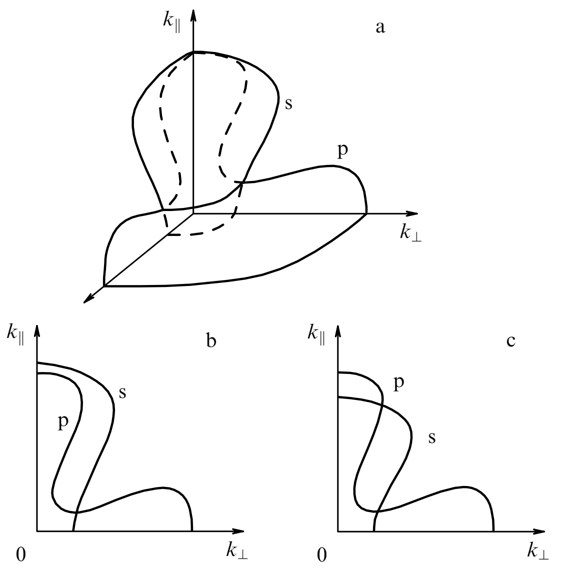

Figure 15. Elastoexchange mechanism of the formation of binormals of spin waves in an orthorhombic AFM (2.2) at $\mathbf{l}\,\|\,z$, $\mathbf{k}_{\|}\,\|\,z$ ($\beta_1 < 0$): (a) $\beta_2 = 0$, $\mathbf{k}_{\perp} \in xy$; (b) $\beta_1 < \beta_2 < 0$, $k_{\perp} = k_x$, and (c) $\beta_2 > 0$, $k_{\perp} = k_x$. The dashed line in (a) delineates the shape of the sections of the internal cavity of a two-cavity WVS (figure of rotation about the z-axis) by the coordinate planes.

[133]. In this case, the section of the WVS by the plane in which the easy axis lies is shown in Fig. 15a.

Analogous special directions of the propagation of normal oscillations are well known in the acoustics (optics) of anisotropic media — they represent such directions of the wave normal $\mathbf{n} \equiv \mathbf{k}/|\mathbf{k}|$, along which two normal waves propagate with coincident phase velocities [59, 134–136]. These directions of $\mathbf{n}$ are called acoustic (optical) axes [AAs (OAs)], respectively. The main physical cause of the existence of AAs (OAs) is the anisotropy of acoustic (optical) properties of crystals, which induces the anisotropy of the dispersion law of the corresponding normal oscillations depending on their polarization and propagation direction. As to magnetic oscillations, the analogous effects upon the propagation of spin waves have never been studied to date. In the case under consideration, the physical factor responsible for the appearance of such special directions of propagation of normal elastoexchange waves is the indirect spin–spin exchange through the long-range field of quasistatic magnetoelastic deformations, which leads to the removal of the degeneracy of the spectrum of spin oscillations in the EA AFM for virtually all directions of propagation in the k space, except for the above-indicated magnetic axes (they can also be called the binormals of spin waves).

If the sagittal plane contains such a magnetic axis (spin-wave binormal), it is just in the case of a magnetic plate with the above magnetoacoustic configuration that the discussed type of inhomogeneous spin–spin resonance induced by the phonon mechanism of the indirect spin–spin interaction under consideration will be observed. Notice that the existence of orthorhombic anisotropy can substantially affect the character of the orientation of the spin wave binormals in the k space (see Figs 15b, c). In particular, if the sagittal plane is the xz plane, the existence of a hard y-axis will lead to the formation of an additional binormal of spin waves in this sagittal plane, which corresponds to the formation of an additional point of an inhomogeneous spin–spin resonance between s- and p-type ESSWs (and EESWs), whose existence was discussed at the end of Section 3.3.2 (see also Fig. 4).

As is well known, the optical and acoustic axes (optical and acoustic binormals) are related to a whole class of polarization effects [e.g., optical or acoustic internal conical refraction (ICR)] which are undoubtedly of practical interest. Correspondingly, in the case under consideration, the phononic mechanism of indirect spin–spin interaction can induce the spin-wave internal conical refraction effect in the EA AFM [133]. In addition, upon the propagation of bulk magnetoelastic waves along the spin-wave binormal tangent to the surface of the magnet under the conditions of spin-wave ICR, the possibility of the formation of additional polarization anomalies (analogous to those considered in paper [137] for the conditions of an acoustic ICR) appears.

6. Conclusions

It follows from the results presented in this review that if a magnetic material possesses properties necessary for the realization of magnetoacoustic resonance, there exists for it a certain critical dimension d_* of the magnetic sample that determines the changeover of the behavior of the resonance characteristics of the magnet: for the dimensions d of the magnetic sample that are much less than d_* ($d \ll d_*$) the resonance characteristics of the magnet can change qualitatively, compared to those for a macroscopic sample ($d \gg d_*$). The physical factor responsible for this effect is the appearance in such a bounded magnet of an indirect spin–spin interaction through the long-range field of quasistatic magnetoelastic deformations. As a consequence, in such a bounded magnet there arises a new, special class of propagating nonexchange spin excitations — elastostatic spin waves (the acoustic counterpart of well-known MSWs — slow electromagnetic waves in magnets).

These excitations can be resonantly induced by an external elastic field acting on the magnetic plate (magnetic switching) whose thickness is far less than the length of an acoustic wave with a given frequency. As a result, an assemblage of such thin magnetic plates in a nonmagnetic medium (in particular, magnetic photonic crystals) can be considered to be a special type of acoustic metamaterial in which the local resonances have a spin-wave nature. The characteristics of nonexchange spin-wave excitations of this type (both bulk and surface), in contrast to the characteristics of MSWs, are mainly determined by the elastic and magnetoelastic properties of the magnet. Therefore, the presence of a nonmagnetic coating (or substrate) in the case of acoustically continuous contact between the magnetic and nonmagnetic media makes it possible to substantially affect the dispersion properties of both nonexchange surface and bulk ESSWs that are quasihomogeneous over the thickness of the magnetic plate.

The simultaneous allowance for some other earlier known mechanisms (first and foremost exchange and magnetodipole), apart from elastostatic ones, of the formation of the dispersion of spin oscillations in a bounded magnet leads to a number of additional anomalies in both the bulk and surface spin-wave dynamics of such bounded magnetic media. In monograph [78], the physics of MSWs is considered to be spin-wave electrodynamics; the results discussed in this review make it possible to regard the physics of ESSWs as spin-wave acoustics of bounded magnetic structures.

Going beyond the framework of elastostatic approximation (the employment of the total equations of the mechanics of continua, instead of their elastostatic variant, for describing the dynamics of the elastic subsystem of a magnet) corresponds to the allowance for the acoustic delay effects. This permits us to indicate a number of new localization mechanisms of shear elastic waves, both near the mechanically free surface of the magnet and near the acoustically continuous interface between the magnetic and nonmagnetic media.

Of undoubted interest can also be the linkage between the local geometry of the wave vector surface (WVS) of normal magnetoelastic oscillations and the specific features of the refraction and propagation of the waves of this type in bounded magnetic media. Notice that a number of problems concerning the relation between the local geometry of the wave vector surfaces of normal elastic waves and the propagation of acoustic waves in nonmagnetic spatially periodic structures have been discussed in review [138].

A significant part of the above effects (related to s-type elastostatic spin waves) represent acoustic analogs of well-known polariton effects in the dynamics of composite materials, including not only slow surface and bulk electromagnetic waves (MSWs), inhomogeneous spin–spin resonance, and dipole-exchange spin waves, but also exciton-type surface polaritons, the negative and anomalous refraction effects, the enhancement of evanescent waves, reflectionless transmission, acoustic analogs of the Fröhlich mode, Otto and Kretschmann configurations, binormals of spin waves, and so forth.

As to the analysis in the elastostatic limit (3.40) of the dynamics of one-dimensional MPCs without the assumption about their fine-layered structure, the corresponding calculation in terms of the T-matrix method [139–142], just as in the case of MSWs [143–145], leads to quite unwieldy relationships even in a nonexchange approximation. However, if following the approach developed in paper [146] for an analysis of the band structure of the MSW spectra in one-dimensional MPCs, it can be stated that the explicit expressions found in Section 3 for the spectra of bulk elastostatic [see Eqns (3.4), (3.5), (3.8)] and elastoexchange [see Eqns (3.30), (3.33), (3.36)] spin waves determine [in the elastostatic approximation (3.41) for corresponding magnetoacoustic configurations] the band structure of the spectrum of spin-wave excitations of the one-dimensional MPC of the AFM–ideal-diamagnet type with very narrow allowed bands. To ensure this, it is necessary that the interlayer elastic boundary conditions only quite insignificantly differ from those used in the calculations in Section 3. It should be noted that in the same approach relationships (3.38) and (3.39) determine for the same magnetoacoustic configurations the band structure of the spectrum of magnetoelastic excitations of the one-dimensional MPC of the AFM–ideal-diamagnet type with very narrow allowed bands and with an arbitrary relationship between the phase velocities of propagation of spin and elastic waves.

On the whole, this review can be considered to be a supplement to Maynard's review [147] devoted to the discussion of acoustic analogs of some effects in condensed state physics, and to a review by Lu et al. [148] concerning nonmagnetic phononic crystals and acoustic metamaterials.

As follows from the results of calculations, all effects that were discussed in this review are determined to a significant extent by a concrete magnetoacoustic configuration which substantially depends on external parameters. This permits us to expect the possibility of smoothly governing the dynamic characteristics of an acoustic metamaterial, including magnetic components of the above-mentioned characteristic dimension.

Notice that the results given in this review were obtained without allowance for the magnetodipole interaction, since, as was noted in the Introduction, in the spectrum of low-frequency spin-wave excitations of exchange-collinear antiferromagnets the exchange-related weakening of the magnetodipole interaction and exchange-induced enhancement of the magnetoelastic interaction occur simultaneously. At the same time, a rigorous theoretical description of the spin-wave dynamics of real bounded magnets for $d_* \gg d \gg a$ should simultaneously take into account the phononic, magnetodipole, and inhomogeneous exchange mechanisms of the formation of the dispersion of propagating spin waves.

In our opinion, in order to solve such a boundary-value problem in the elastostatic limit (3.1), (3.41) under consideration it is suitable to use an extension of the approach that was developed earlier for an analysis of the effect of the magnetodipole interaction on the spectrum of exchange magnons in a thin ferromagnetic film (see, e.g., Ref. [149]). Now, however, with allowance made for the electrodynamic and elastic boundary conditions in the elastostatic limit, it is necessary to exclude from consideration not only the magnetostatic potential ϕ but also the vector $\mathbf{u}$ of elastic lattice displacements. Then, it is necessary to solve the corresponding set of integro-differential equations for the components of magnetization with only exchange boundary conditions. The solution, as before, can be sought in the form of an expansion in terms of the eigenfunctions of the exchange boundary-value problem. In separate particular cases of the combination of exchange elastic and electromagnetic boundary conditions, such a problem admits an explicit solution, for instance, when the following relationships are fulfilled simultaneously:

$$\frac{\partial \tilde{\mathbf{l}}}{\partial z} = 0 , \quad \tilde{\mathbf{u}} = 0 , \quad B_z = 0 , \quad z = \pm d . \tag{6.1}$$

Physically, this corresponds to an EA AFM plate of thickness $2d$ $(\mathbf{l} \| \mathbf{n} \| z)$, on both surfaces of which the spins are completely free and the very surfaces have a continuous acoustic contact with an ideal superconductor. If the superconductor is much more rigid in its elastic characteristics than the magnetic medium, this can approximately be described as the attachment of both surfaces of the antiferromagnetic plate. As a result, the spectrum of a bulk elastoexchange s-type spin wave for the sagittal xz plane in the elastostatic limit (3.40) with allowance for the magnetodipole and inhomogeneous exchange interactions takes on the form

$$\Omega_\nu^2 = \left[\omega_0^2 + \omega_{\rm me}^2 \frac{k_\perp^2}{k_\perp^2 + p_1^2} + c_{\rm m}^2 (k_\perp^2 + p_1^2)\right] \left(1 + \frac{4\pi}{\delta}\, \frac{k_\perp^2}{k_\perp^2 + p_1^2}\right), \tag{6.2}$$
$$p_1 = \frac{\pi \nu}{2d} , \quad \nu = 1, 2, \ldots .$$

As to the spectrum of an elastoexchange p-type spin wave with $\mathbf{u} \in xz$ (see Section 3), this wave is not magnetodipole-active in the given magnetoacoustic configuration.

Thus, it can be said that the relationships for the spectrum of elastoexchange bulk spin waves represent a zero-th approximation in the small parameter $4\pi/\delta \ll 1$. Since the influence of the magnetodipole interaction on the dispersion characteristics of spin-wave excitations can be substantial only in the long-wavelength range, it can easily be shown that the magnetodipole mechanism of the indirect spin–spin interaction can be efficient, most of all, in a bounded compensated low-temperature AFM for bulk magnetization waves that are quasihomogeneous over the thickness of the magnetic plate residing in a vacuum and on both surfaces of which the spins are fully free. If both surfaces of the plate have continuous acoustic contact with an ideal diamagnet, then the dispersion properties of bulk spin oscillations that are quasihomogeneous over the thickness of the magnetic plate will be determined in the elastostatic limit (3.1) by the phononic and inhomogeneous exchange mechanisms of the indirect spin–spin interaction.

Much more substantial can be the influence of magnetodipole interaction on the localization conditions of the shear elastic wave near a mechanically free surface of an easy-axis nonpiezomagnetic antiferromagnet [150].

It can easily be shown that the phononic mechanism of the indirect spin–spin interaction in the bounded magnets, considered in this review, is also valid for the uncompensated magnetic structures, e.g., in the case of a Parekh wave propagating in the Voigt geometry along a mechanically free EA-FM–vacuum interface [54, 151]. In this case, it is necessary to perform a formal limiting transition $4\pi \to 0$ in the relationships that describe the dispersion law and the spatial distribution of the elastic displacement vector of the surface shear SAW considered. An analysis of the relationships obtained shows that the formation of a shear SAW on the mechanically free boundary of a magnet, even without allowance for the magnetodipole interaction, occurs because the magnetic medium studied possesses acoustic activity.

Thus, the rigorous allowance for the above-discussed phononic mechanism of the spin–spin interaction in the case of uncompensated magnetic structures leads to a whole number of additional effects in the dynamics of bulk and surface magnetoelastic excitations already in the elastostatic approximation both by itself and with simultaneous regard for the magnetodipole and inhomogeneous exchange mechanisms of the formation of the dispersion of propagating spin waves [87, 152]. However, the discussion of these effects is beyond the scope of this review.

It should also be noted that, according to the performed investigations of the role of phonons in semiconducting nanostructures (see monograph [153]), the description of the spectrum of acoustic phonons in terms of the continuum model remains valid even in the case of nanoparticles. This permits us to hope that the elastostatic mechanism of the indirect spin–spin interaction that is discussed in this review can prove to be efficient also in the case of other magnetic heterostructures (see also paper [154]). Notice that the elastostatic mechanism of the formation of an additional class of nonexchange spin waves in bounded magnets can be considered as a particular case of the manifestation of the non-Heisenberg spin–spin interaction in a magnetic sample of finite dimensions, discussed in Nagaev's monograph [155].

In our opinion, of undoubted interest is the development of a similar approach in the following areas:

(1) analysis of the dynamic properties of composite multiferroics representing structures of various connectivity and consisting of ferromagnetic and piezoelectric or ferroelectric components in which the determining role belongs to the elastic system [156, 157];

(2) searching in magnetic heterostructures induced by striction for bulk and surface spatially inhomogeneous states like those discussed in Refs [158–161] on the basis of an analysis of the spectrum of magnetoelastic excitations;

(3) allowance for the existence of a mismatch in the elastic parameters of the contacting media in the dynamics of composite magnetic media [162];

(4) analysis (with allowance for the elastic system) of the dynamics of magnetic heterostructures differing in the connectivity, conductivity, and shape of inclusions [163];

(5) studying additional features under the conditions of the formation and propagation of elastoexchange-type spin-wave excitations in bounded low-temperature antiferromagnets induced by the symmetry of the magnetic structure (first and foremost, those caused by piezomagnetic or magnetoelectric interactions [164–166];

(6) studying the manifestation of nonlinear magnetoacoustic effects (the generation of higher harmonics, formation of soliton regimes of the propagation of surface and bulk waves) in such materials [167];

(7) allowance made for the possibility of a qualitative rearrangement of the character of the propagation and localization of magnetoelastic waves in composite structures under the dissipation effect [120, 168].

It should be noted that a major part of the above acoustic counterparts of polariton effects in bounded magnets can, in principle, be also realized in other dipole-active media with a long-range order, e.g., in ferroelectrics, whose parameters, as is known [169], under certain conditions permit the appearance of an effect analogous to magnetoacoustic resonance. More specifically, as follows from monograph [170], it is precisely the elastostatic mechanism that determines the specific features of the dynamics of the order parameter upon phase transitions with a linear striction. Consequently, a part of the above-mentioned effects can also be observed in the bounded samples of materials in the vicinity of a given phase transition.

Most effects considered in this review require experimental verification. The materials that are promising in this field are cubic or easy-plane magnetic low-temperature antiferromagnets, various magnetic heterostructures, in particular, compensated antiferromagnetic structures such as one-dimensional magnetic photonic crystals based on ferro-ferrimagnetic layers of magnets with an antiferromagnetic mechanism of interlayer ordering.

On the whole, the use of the above-discussed acoustic analogs of polariton effects in the production of acoustic magnetic metamaterials can substantially widen and vary the functional opportunities of this promising class of composite media.

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