The same components of the Abraham tensor are as follows:

$$T_{\alpha\beta} = \frac{1}{8\pi}(E_\alpha D_\beta + E_\beta D_\alpha + H_\alpha B_\beta + H_\beta B_\alpha) - \frac{1}{8\pi}\delta_{\alpha\beta}(\mathbf{ED} + \mathbf{HB}),$$

$$\mathbf{S} = \frac{c}{4\pi}[\mathbf{EH}], \quad \mathbf{g} = \frac{1}{4\pi c}[\mathbf{EH}], \quad W = \frac{1}{8\pi}(\mathbf{ED} + \mathbf{HB}).$$

The formula for the energy density W given above can be written out as $W = (1/8\pi)(\varepsilon E^2 + \mu H^2)$, but using this form implies that the two permeabilities ε and μ are both essentially positive. If ε and μ are negative, Ref. [23] gives the following expression for the energy density:

$$W = \frac{1}{8\pi}\left[\frac{\partial(\varepsilon\omega)}{\partial\omega}E^2 + \frac{\partial(\mu\omega)}{\partial\omega}H^2\right]. \tag{21}$$

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Acoustic waves in metamaterials, crystals, and anomalously refracting structures

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1. Introduction

At present, much attention in the literature is being paid to media in which the propagation of waves occurs in the 'unusual' fashion. In particular, such media include so-called left-handed media in electrodynamics. The concept of these media was first proposed in paper [1], where they were introduced as media in which both the permittivity ε and permeability μ are negative. Interest in such media was rekindled after the publication of a number of papers (for example, Ref. [2]) reporting their experimental realization based on metamaterials — artificial structures with characteristic sizes of elements that are well below the wavelength of propagating radiation.

One of the unusual peculiarities of the propagation of plane waves in such media is that the Umov–Poynting vector is antiparallel to the wave vector [1]. This specific feature of the wave propagation is of a general character and is inherent in metamaterials studied not only in electrodynamics and optics, but also in acoustics. Because the phase and group velocity vectors of bulk waves in metamaterials are antiparallel, it is interesting to analyze the possibility of other spatial orientations of these vectors. It is also important to know the angles between the phase and group velocity vectors of a plane wave that can appear whatsoever in optical and acoustic anisotropic materials.

2. Double negative acoustic media

It has been shown in a number of theoretical and experimental papers [3–6] that double negative media (below, for brevity, we will call them simply negative) with negative effective dynamic characteristics (density ρ and compressibility η) are suited for playing the role of media in acoustics in which wave processes proceed similarly to those in left-handed media in electrodynamics. The characteristics of these media are dynamic in the sense that each element of such a medium within a certain frequency band can behave as an element of a homogeneous medium with negative parameters (for example, due to the presence of resonance structures in it).

In negative media, both in electrodynamics and acoustics, negative refraction can be observed, which is often described by using the wave equation or Helmholtz equation [7]. In this case, a principal difficulty appears because these equations contain the square of the refractive index, and to determine its sign, it is necessary to invoke additional considerations which can be based on the choice of one branch or another of the

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square root, on the causality principle, etc. For this reason, it was proposed to use first-order equations for describing processes in negative media [3]. In electrodynamics, these are Maxwell's equations, while in acoustics, these are the linearized hydrodynamic equations

$$\frac{\partial}{\partial t}(\hat{\eta} p) + \nabla \mathbf{v} = \varphi\,, \qquad \frac{\partial}{\partial t}(\hat{\rho}\mathbf{v}) + \nabla p = \mathbf{f}\,, \tag{1}$$

where p is the acoustic pressure, $\mathbf{v}$ is the vibrational velocity, and φ and $\mathbf{f}$ are scalar and vector primary acoustic-field sources, respectively. In the absence of dispersion, the parameters $\hat{\rho}$ and $\hat{\eta}$ are scalars; otherwise, they can be treated as time-convolution type integral operators. These parameters enter equations (1) separately, and therefore no problems appear with the choice of one sign or another.

An arbitrary distribution of parameters $\rho(\mathbf{r})$ and $\eta(\mathbf{r})$ can be represented as a sum of constant positive background values ρ_0 and η_0 with arbitrarily large additions $\rho'(\mathbf{r})$ and $\eta'(\mathbf{r})$: $\rho(\mathbf{r}) \equiv \rho_0 + \rho'(\mathbf{r})$, $\eta(\mathbf{r}) \equiv \eta_0 + \eta'(\mathbf{r})$. In the monochromatic case, system of equations (1) for the time dependence of fields fitted by $\sim \exp(-\mathrm{i}\omega t)$ can be represented in the form of the matrix analogue of the Lippmann–Schwinger equation [3]:

$$\breve{u}(\mathbf{r}) = \breve{u}_0(\mathbf{r}) + \int \hat{G}(\mathbf{r}-\mathbf{r}')\left[\hat{A}_1(\mathbf{r}')\,\breve{u}(\mathbf{r}')\right]\mathrm{d}\mathbf{r}'\,, \tag{2}$$

where $\breve{u}_0$ and $\breve{u}$ are column vectors characterizing the incident and perturbed fields, viz.

$$\breve{u}_0 \equiv \begin{pmatrix} \mathbf{v}_0 \\ p_0 \end{pmatrix}, \qquad \breve{u} \equiv \begin{pmatrix} \mathbf{v} \\ p \end{pmatrix},$$

$\hat{A}_1$ is the addition operator defined as

$$\hat{A}_1 \equiv \begin{pmatrix} \mathrm{i}\omega\rho'(\mathbf{r}) & \mathbf{0} \\ \mathbf{0} & \mathrm{i}\omega\eta'(\mathbf{r}) \end{pmatrix},$$

and $\hat{G}(\mathbf{r}-\mathbf{r}')$ is the matrix Green function expressed in terms of the Green function $G(\mathbf{r}-\mathbf{r}')$ in the Helmholtz equation:

$$\hat{G}(\mathbf{r}-\mathbf{r}') \equiv \begin{pmatrix} \mathrm{i}\omega\eta_0 & \nabla \\ \nabla & \mathrm{i}\omega\rho_0 \end{pmatrix} G(\mathbf{r}-\mathbf{r}')\,.$$

Equation (2) can be solved numerically for any configuration of the incident field $\breve{u}_0$ and arbitrary medium density and compressibility distributions.

To observe negative refraction phenomenon, we simulated the incidence of a plane monochromatic wave with the wavelength λ_0 at an angle of 18° to the normal on a rectangular plate made of a negative-index material with the parameters $\rho = -\rho_0$ and $\eta = -\eta_0$ (Fig. 1). The beam in the plate lies on the same side of the normal to its boundaries as beams in the environment. The wave fronts prove to be mirror-symmetric with respect to the plate boundaries. These properties demonstrate the negative refraction of sound in this plate. In this case, the phase velocity in the plate is directed antiparallel to the phase velocity in the environment (background medium), thereby being negative. Because the energy flux is continuous and waves are not reflected from plate boundaries, the energy flux in the system under study is oriented in the positive direction.

The following fact is noteworthy. At two points located in the environment on both sides of the plate in a straight line perpendicular to its boundaries and separated by a distance equal to the doubled thickness of the plate, the wave phases are coincident with each other (for example, points A and B in Fig. 1). This is the consequence of the phase velocity negativity in the negative medium, i.e., the phase incursion in the positive environment is exactly compensated by the phase incursion in the negative medium of the plate. This circumstance may lead to the conclusion that the causality principle is violated in negative media, and therefore they cannot exist in reality, which was discussed, for example, in review [8].

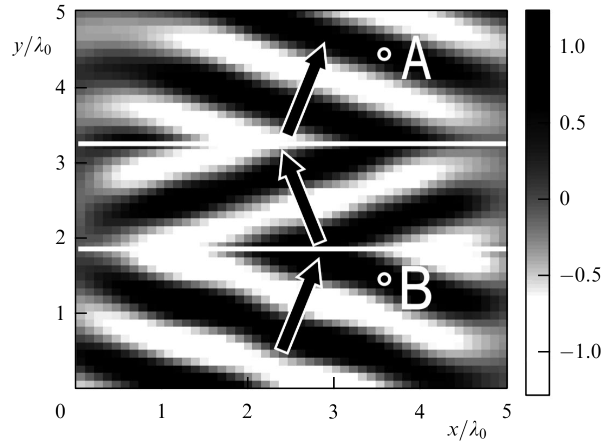

Figure 1. Real part of the calculated acoustic pressure field $p(\mathbf{r})$ during the incidence of a plane monochromatic wave on a negative-material plate. The plate boundaries are indicated by light horizontal straight lines. The arrows show the propagation directions of the wave energy in the plate and environment.

Because the discussion of the causality in the purely monochromatic case is impossible, we analyzed the following situation: nine monochromatic beams with the Gaussian amplitude distribution in the front plane are incident on a $3\,\lambda_0$-thick plate $10\,\lambda_0$ in length at an angle of 18° to the normal. The field frequency ω in the beam was varied from $\omega_0 = 2\pi c_0/\lambda_0$ to $1.4\omega_0$ in increments of $0.05\omega_0$, where $c_0 = 1/\sqrt{\rho_0\eta_0}$ is the speed of sound in the environment. The amplitudes of the beams on the axis were different and described by a Gaussian distribution over ω:

$$P(\omega) = P_0 \exp\left(-\frac{(\omega - 1.2\omega_0)^2}{(\Delta\omega)^2}\right),$$

where $\Delta\omega \approx \omega_0/6.3$. The fields calculated for each frequency were then summed up. As a result, by solving several monochromatic problems, we obtained the solution to a polychromatic problem corresponding to the propagation of an infinite series of pulses through a plate made of a negative material.

The results of calculations for a fixed instant of time are presented in Fig. 2. At the moment when the incident pulse is located at a distance equal to the plate thickness from the plate boundary, a perturbation representing a pair of pulses appears on the opposite boundary. One of the pulses continues to propagate in the environment behind the plate in the direction parallel to that of the incident pulse. Another pulse, which is mirror-symmetric to it with respect to the plate boundary, moves to the mirror-symmetric side. This pulse reaches the plate boundary simultaneously with the incident pulse, and they quench each other.

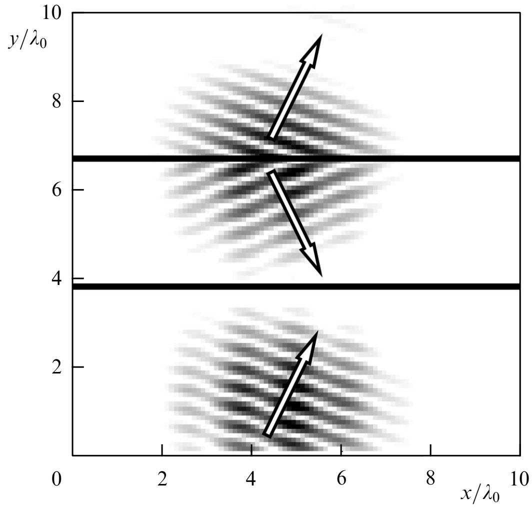

Figure 2. Real part of the acoustic pressure field $p(\mathbf{r}, t)$ calculated at a fixed instant of time during the incidence of pulses, representing a superposition of nine plane monochromatic waves, on a negative-material plate. The plate boundaries are indicated by dark horizontal straight lines. The arrows show the propagation directions of pulses in the plate and environment.

Such a consideration brings up a number of questions. First, a pulse leaving the plate appears before the moment when the incident pulse touches the plate, i.e., the causality principle is violated. Second, the mechanism of the appearance of additional energy on one side of the plate and its absorption on the other side is not quite clear. Third, a pulse propagates in the plate mirror-symmetrically to the traveling direction of a pulse in the environment, i.e., not only the phase but also the group velocities (if we relate the latter to the envelope maximum velocity) are negative. On the other hand, it was pointed out [3] that the phase and group velocities in the negative medium are antiparallel.

The possible answer to the first question is that a periodic time-infinite process is considered and therefore there exist an infinite number of pulses that have propagated through the plate earlier. Then, the observed pulse 'leaving' the plate is the consequence of their propagation rather than the precursor of the pulse incident on the plate. The creation of the pulse is caused by the energy of propagated pulses stored in the plate.

Therefore, because the medium possesses the internal stored energy, this energy storage can decrease during the pulse propagation in it. In this sense, the pulse in a negative medium can have the negative energy (with respect to the environment). Taking this fact into account, the energy conservation law on both boundaries of the plate remains valid: as a pair perturbation is produced on the far side (from a radiation source) of the plate, a pulse with the positive energy propagates in the environment, while in the plate, i.e., in the negative medium, a pulse with the negative energy propagates. These pulses merge on the front side of the plate to produce the zero total energy.

The negativity of both phase and group velocities is due to the fact that simulations were performed assuming that the medium was nondispersing. Negative metamaterials realized in practice, which often contain resonance elements, are strongly dispersing. To understand whether a nondispersing negative medium can exist in principle, we will analyze the propagation of a single pulse (packet) through it, i.e., consider the problem in a broad frequency range. The consideration should be based on the system of equations (1). Then, the field in the medium in the one-dimensional case is described by the Lippmann–Schwinger type equation

$$\begin{pmatrix} p(x,t) \\ v(x,t) \end{pmatrix} = \begin{pmatrix} p_0(x,t) \\ v_0(x,t) \end{pmatrix} - \frac{1}{2}\iint \mathrm{d}x'\,\mathrm{d}t'\,\delta\left(t - t' - \frac{|x-x'|}{c}\right)$$
$$\times \begin{pmatrix} \sqrt{\dfrac{\rho_0}{\eta_0}} & \operatorname{sgn}(x-x') \\ \operatorname{sgn}(x-x') & \sqrt{\dfrac{\eta_0}{\rho_0}} \end{pmatrix} \begin{pmatrix} \eta'(x') & 0 \\ 0 & \rho'(x') \end{pmatrix}$$
$$\times \frac{\partial}{\partial t'}\begin{pmatrix} p(x',t') \\ v(x',t') \end{pmatrix}, \qquad (3)$$

where δ is the Dirac delta function. (The derivation of this equation will be presented in detail elsewhere.) Equation (3) allows one to study the propagation of a single Gaussian packet through a plate made of a negative material.

The results of such a simulation are presented in Fig. 3.The packet width is 1.5 times larger than the plate thickness which is indicated by double vertical straight lines. The coordinate x is plotted on the abscissa, and the quantity $c_0 t$ is plotted along the ordinate. To obtain the field distribution at a certain instant of time, it is necessary to draw a horizontal straight line (section) across the figure. A packet incident on the plate in the positive direction of the x-axis is located in region I. Packets propagating in the plate and environment behind the plate are located in regions II and III, respectively. One of the possible sections, shown by the dashed straight line, corresponds to the instant of time when the center of the primary packet resides at a distance on the order of the plate thickness from the plate. In this case, the field perturbation begins to appear on the opposite side of the plate. Thus, the packet transmitted through the plate appears earlier than the primary incident packet reaches the plate and, therefore, the causality principle is violated in such a medium. It seems that this means that nondispersing negative media cannot exist. In the presence of dispersion in a medium (for example, if a

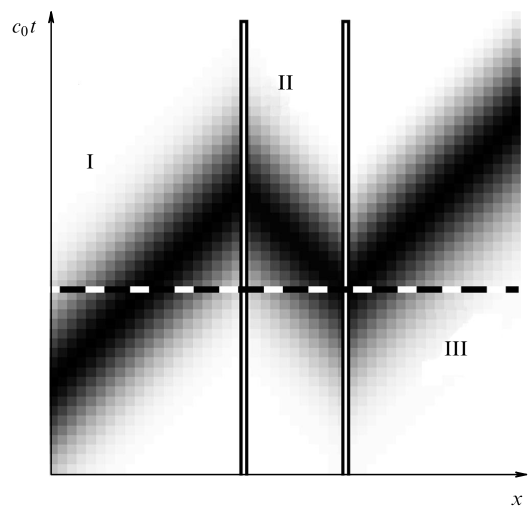

Figure 3. Real part of the acoustic pressure field $p(x, t)$ calculated during the normal incidence of a Gaussian pulse on a negative-material plate. The thick vertical straight lines indicate the plate boundary coordinates. The dashed horizontal straight line corresponds to the instant of time at which the packet center is located at a distance from the plate equal to its thickness, and the field perturbation appears on the far side of the plate.

negative medium is constructed of resonance elements), scalar quantities $\eta'(x)$ and $\rho'(x)$ in equation (3) should be replaced by operators in the form of the time convolution of response functions of resonators. The simulation of this case showed that a precursor packet does not appear, i.e., the causality principle is not violated. Simultaneously, negative refraction is observed in the steady-state regime, which is not accompanied by considerable absorption in a broad frequency band.

3. Angle between the phase and group wave velocities in crystals

It is known that birefringent crystalline media with specifically combined optical and acoustic characteristics are widely used in modern optics, acoustics, acoustooptics, and acoustoelectronics [9–11]. For example, quartz (α-SiO_2), lithium niobate ($LiNbO_3$), calcite ($CaCO_3$), paratellurite (TeO_2) crystals and some others are extensively applied in optical, acoustooptic, and acoustoelectronic devices [9–12]. However, recently crystal materials with a large anisotropy of acoustic and optical properties have found applications in optics, acoustooptics, and acoustics. Such materials include mercury (Hg)- and tellurium (Te)-based single crystals having extremely large birefringence and a strong dependence of the phase velocity of sound on the propagation direction [12].

4. Optical birefringent media

The birefringence Δn of crystalline materials is determined by the difference $\Delta n = n_e - n_o$ of the refractive indices for extraordinarily (n_e) and ordinarily (n_o) polarized light [9, 10]. This difference in some materials can be rather large. For example, the relative birefringence (the anisotropy coefficient) $\delta = \Delta n / n_o$ in tellurium crystals reaches $\delta = 0.3$, and in calomel (Hg_2Cl_2) $\delta = 0.35$. For comparison, birefringence in quartz is two orders of magnitude lower: $\delta = 0.006$ [12, 13].

It is known from optics and electrodynamics that extraordinary waves in materials with large birefringence propagate with angles ψ between the phase and group velocity vectors exceeding 10° [9, 10]. The 'walk-off' angle ψ for the extraordinary optical wave is found from the analysis of the surface of wave vectors in a crystal. The wave surface in a uniaxial optical material represents the ellipsoid of revolution. The value of the wave vector k_e of light in a crystal depends on the light propagation direction with respect to the optical axis Z and changes in a positive crystal within the limits $2\pi n_o/\lambda \leqslant k_e \leqslant 2\pi n_e/\lambda$, where λ is the light wavelength [9, 10]. It is known that the group velocity direction of an electromagnetic wave coincides with the direction of the normal to the wave-vector surface erected at a point in which the wave vector $\mathbf{k}_e$ touches the normal surface. It can be shown that the maximum walk-off angle in a uniaxial crystal is described by the expression [12]

$$\psi_{max} = \arctan \frac{\delta(1 + 0.5\delta)}{1 + \delta} \,. \tag{4}$$

Calculations by means of formula (4) show that the angle between the wave phase velocity vector and the Umov–Poynting vector in a negative calcite crystal with the birefringent coefficient $\delta = -0.1$, widely used in polarization devices, is $\psi_{max} = -6°$. The maximum optical walk-off angles in paratellurite and quartz crystals with $\delta = 0.07$ and 0.006 do not exceed $\psi_{max} = 4°$ and $\psi_{max} = 0.4°$, respectively. On the other hand, the maximum walk-off angle in a tellurium crystal with $\delta = 0.3$ is $\psi_{max} = 15°$, while this angle in a mercury bromide (Hg_2Br_2) crystal with $\delta = 0.36$ already reaches a considerable value of $\psi_{max} = 19°$ [12]. Finally, the maximum angle between the Umov–Poynting vector and the wave vector of light in one of the base planes of a biaxial antimony sulfoiodide (SbSI) crystal with the record-high birefringence $\delta = 0.6$ [13] is $\psi_{max} \approx 25°$.

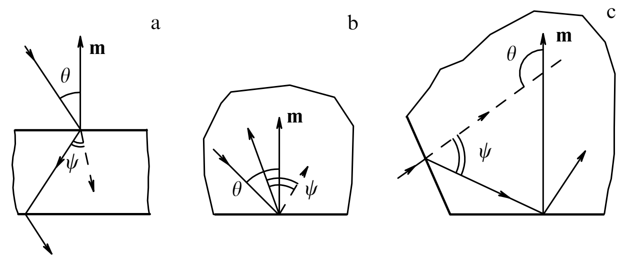

Figure 4. Cases of the unusual propagation and refraction of light waves in birefringent crystals. The direction of the phase velocity of light is shown by dashed straight lines and arrows, and the energy flux direction is indicated by solid straight lines and arrows: (a) the propagation of a light wave through a plane-parallel plate; (b) anomalous internal reflection of light from a crystal–vacuum interface, and (c) the incidence of light on an interface at an angle θ exceeding 90°.

The anisotropy of physical properties resulting in large angles between the phase and group velocities of optical waves can lead to unusual cases of propagation and reflection of these waves from the crystal–vacuum interface. Some of these cases of wave propagation are illustrated in Fig. 4. Figure 4a demonstrates the oblique incidence at an angle of θ and propagation of a light beam through a birefringent crystal plane-parallel plate. The angle of incidence is traditionally measured between the wave vector and the normal **m** to the interface. The directions of the electromagnetic energy flux and the group velocity vector in the beam are shown by the arrows and solid straight lines, while the direction of the phase velocity of light is indicated by the dashed arrow and the straight line. One can see from the figure that the energy flux refracted in the plate is directed anomalously, i.e., just as this occurs in metamaterials. The refraction angle for the energy flux proves to be negative, although the wave vector of light is oriented with respect to the normal strictly in accordance with Snell's law. Therein lies the difference between the classical case of light propagation in a crystal and the propagation of light in a metamaterial. Obviously, the anomalous refraction of the optical beam in Fig. 4a is caused by a large optical walk-off angle ψ.

Figure 4b demonstrates the unusual reflection of a light beam from the interface during the propagation of a light wave through a birefringent crystal. One can see from this figure that, because of the large optical anisotropy of the material, the energy fluxes of the incident and reflected optical beams are located on the one side of the normal to the interface. Finally, Fig. 4c illustrates the propagation of light in the crystal and its incidence at an angle of θ to the crystal–vacuum interface exceeding 90°. The wave vector of light at such an anomalously large angle of incidence is found directed not to the interface side but from it, while the energy flux of the optical wave hits the interface. Obviously, these unusual effects are caused by the considerable optical anisotropy of the crystal.

5. Acoustic anisotropic media

Acoustic media are known to possess even the more pronounced anisotropy of physical properties compared to that in optical media. An acoustic crystal with the large anisotropy of elastic properties is characterized by a strong dependence of the phase velocity v of ultrasound on its propagation direction [11–16]. For example, a slow shear acoustic wave in a calomel (Hg_2Cl_2) crystal propagates along the [110] axis with the anomalously slow velocity $v = 347$ m s^{-1}, whereas the same acoustic mode propagating along the [100] and [010] axes has the velocity $v = 1305$ m s^{-1} [13–15]. Thus, the ratio of the maximal and minimal phase velocities of sound in calomel for this acoustic mode is $r = 3.76$. In mercury bromide (Hg_2Br_2) and iodide (Hg_2I_2) crystals, the ratio of acoustic velocities reaches $r = 4.39$ and 4.89, respectively. Finally, the acoustic anisotropy coefficient in a paratellurite (TeO_2) crystal is $r = 4.95$. The large spatial dispersion of acoustic velocities leads to extremely large acoustic walk-off angles ψ, i.e., angles between the wave vector and the Umov–Poynting vector. Thus, for example, the acoustic walk-off angle in a calomel crystal is equal to $\psi = 70°$. Similarly, the acoustic walk-off angle in a mercury bromide crystal reaches $\psi = 72°$, while the wave vectors of the phase and group velocities in Hg_2I_2 and TeO_2 crystals are separated by a very large angle $\psi = 74°$. It should be noted that rather large acoustic walk-off angles are also typical for many other acoustic materials. For example, the walk-off angles in a double lead molybdate (Pb_2MoO_5) crystal and a tellurium crystal are $\psi = 69°$ and 56°, respectively [17].

The unusually large angles between the phase and group velocities of acoustic waves lead to many unusual wave phenomena observed at crystal–vacuum or crystal–isotropic medium interfaces. One such unusual phenomenon is the nearly backward reflection of the acoustic energy in a paratellurite crystal during the glancing incidence of a wave on the free crystal–vacuum interface [15, 16, 18, 19].

The case of a grazing incidence of the elastic energy on a free face of a paratellurite crystal is illustrated in Fig. 5. A piezoelectric transducer excites an acoustic wave in the crystal with the phase velocity and wave vector directed horizontally, i.e., at an angle of incidence $\theta = 90°$ with respect to the normal **m** to the upper boundary of the crystal. One can see from the figure that the group velocity of the initial wave is directed towards the upper face of the crystal, and therefore the acoustic energy is incident on the interface obliquely, as shown in Fig. 5a. The group velocity vector of the acoustic wave reflected from the upper face of the crystal is, in fact, antiparallel to the incident wave energy flux. Calculations predict that the spatial angle γ between the energy fluxes of the incident and reflected waves does not exceed 6°.

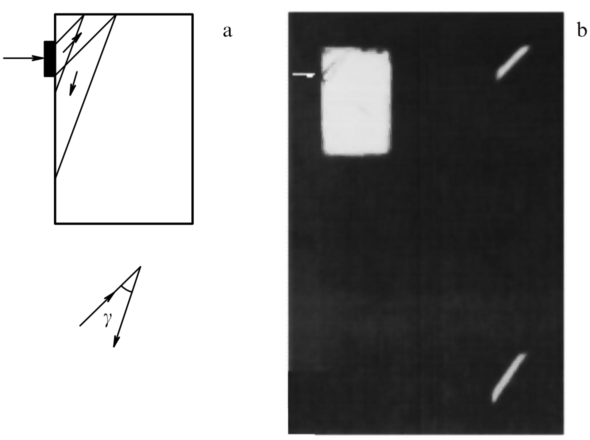

Figure 5. Nearly backward reflection of the acoustic energy from a crystal–vacuum interface during the glancing incidence of an acoustic wave front on the interface: (a) the general view of the crystal and the propagation directions of energy fluxes shown by the arrows, and (b) the visualization of acoustic fields in a paratellurite crystal by the acoustooptic method.

The wave vector of the reflected acoustic wave is spatially oriented in strict accord with the known condition for the equality between tangential projections of the wave vectors of incident and reflected waves onto the interface [14, 15]. The unusual backward propagation of the reflected acoustic energy is caused by the large acoustic walk-off angle in paratellurite. It turns out that, in the case of unusual reflection, almost all the energy of the incident acoustic wave can be converted without losses to the energy of the backward reflected wave.

6. Observation of the unusual reflection of acoustic waves

The unusual reflection of acoustic waves was experimentally confirmed by the acoustooptic method via visualization of acoustic fields and illumination of a TeO_2 crystal by a broad collimated 633-nm beam from an He–Ne laser [18, 19]. Longitudinal acoustic waves were excited by a lithium niobate piezoelectric transducer at the ultrasonic frequency $f = 150$ MHz. The linear dimensions of the transducer, 0.3×0.5 cm, exceeded the ultrasonic wavelength in the crystal by a few orders of magnitude. Therefore, the acoustic waves were treated in the experiments as plane waves.

Figure 5b displays the general view of the diffraction pattern observed at the crystal output. The presence of a visualized acoustic column in the lower right corner of the photograph, which was oriented parallel to the reflected acoustic column shown in Fig. 5a, proves that it was the backward reflection of acoustic waves that was detected in experiments [18, 19]. The angle γ between the group velocity vectors of the incident and reflected acoustic waves was measured to be 9°, with the calculated angle being 6°.

Our analysis has evidenced that the unusual cases of propagation, reflection, and refraction of light waves shown in Fig. 4 can also be easily realized in acoustic crystals. Thus, unusual wave phenomena with anomalously directed waves can be observed in both optics and acoustics. Although the physical nature of these phenomena and effects differs from that in metamaterials, they are manifested similarly to those in double negative media. Notice that the unusual propagation and reflection of waves occur not only in anisotropic media known in optics and acoustics. For example, the propagation of electromagnetic waves in thin magnetic films is also characterized by large angles between the phase velocity vector and the Umov–Poynting vector [20].

7. Orientation of the phase and group velocity vectors in anisotropic media and metamaterials

Generalizing results of studies into the propagation of acoustic waves in optical and acoustic media [12, 15–20], we can propose a unified view of wave processes proceeding in crystals, anisotropic media, and artificial periodic structures in electrodynamics, optics, and acoustics. The conditional direction of the phase velocity of a plane bulk wave propagating in an anisotropic medium is indicated in Fig. 6 along the horizontal direction (the x-axis). According to the conclusions presented in Section 4, angles between the wave

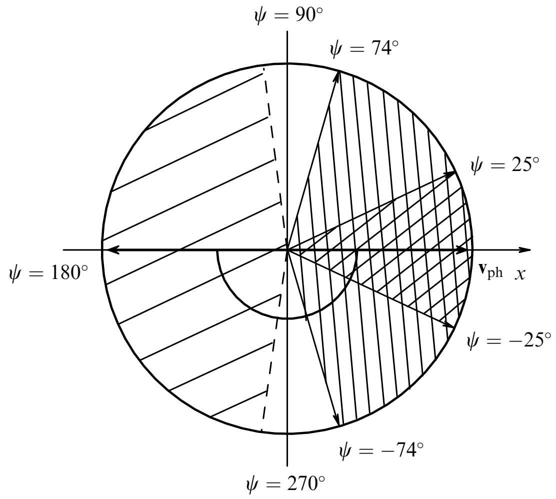

Figure 6. Orientation of the group-velocity vector with respect to the phase velocity vector of optical and acoustic waves in crystals and metamaterials.

vector of the extraordinary light wave and the Umov–Poynting vector in birefringent crystals can lie within the limits $-25° < \psi < 25°$, as shown in Fig. 6. In acoustic anisotropic media, angles between the phase and group velocities of the waves lie in the range $-74° < \psi < 74°$. Thus, almost the entire right half of the diagram in Fig. 6 describes wave processes typical of optics and acoustics.

The propagation of waves in metamaterials and double negative acoustic media with the Umov–Poynting vector antiparallel to the wave vector corresponds in Fig. 6 to the horizontal direction oriented at an angle of $\psi = 180°$ relative to the initial wave vector. In this connection, it is interesting to determine the waves corresponding to the energy walk-off angles lying in the ranges $90° < \psi < 180°$ and $180° < \psi < 270°$ in the diagram. Our analysis suggests that the left part of the diagram describes waves propagating in artificial media, i.e., metamaterials. However, these media should have the anisotropy of physical properties providing the propagation of waves not only with the antiparallel phase and group velocity vectors but also at angles different from $\psi = 180°$. Obviously, such artificial media should also possess a strong spatial anisotropy of physical properties.

By analyzing the data presented in Fig. 6, we should consider separately the cases of the group velocity directed at angles of $\psi = 90°$ and 270° to the phase velocity vector. It is well known from optics and acoustics that the group velocity of a wave in a crystal exceeds in absolute value the phase velocity, these velocities being related by the expression $v_{ph} = v_g \cos\psi$, where ψ is the angle between the phase and group velocities [9, 10, 14]. It follows from this relation that for walk-off angles $\psi = 90°$ and 270° and the finite group velocity of the wave, its phase velocity vanishes. In other words, when the phase and group velocities are mutually orthogonal, the concept of a 'wave' becomes completely meaningless. Similarly, when the phase velocity is finite, it follows from the last relation that the energy transfer rate becomes infinite, which also contradicts the physical sense. Thus, we can assume that wave processes with the orthogonal phase and group velocities of volume waves are absent in optical and acoustic media.

8. Conclusion

The description of wave acoustic processes directly based on hydrodynamic equations and an analogue of the Lippmann–Schwinger equation is correct both for classical (positive) and negative media. The propagation of a wave packet in a plate made of a negative material, assuming the absence of dispersion, gives rise to a precursor packet behind the plate, which contradicts the causality principle. This does not occur when the response of the metamaterial is resonant in character.

The strong anisotropy of the optical and elastic properties of crystals leads to the unusual propagation and 'negative' reflection of waves in these materials. The reflection of elastic waves from a free surface separating a crystal and a vacuum can be accompanied by the propagation of the reflected-wave energy almost antiparallel to the energy flux of the incident wave.

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