CONFERENCES AND SYMPOSIA

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Electromagnetic and acoustic waves in metamaterials and structures (Scientific session of the Physical Sciences Division of the Russian Academy of Sciences, 24 February 2011)

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The scientific session, titled "Electromagnetic and acoustic waves in metamaterials and structures", of the Physical Sciences Division of the Russian Academy of Sciences (RAS) was held on February 24, 2011 in the conference hall of the Lebedev Physical Institute, RAS.

The agenda of the session announced on the website www.gpad.ac.ru of the RAS Physical Sciences Division featured presentation of the following reports:

(1) **Veselago V G** (A M Prokhorov General Physics Institute, RAS, Moscow, and Moscow Institute of Physics and Technology, Dolgoprudnyi, Moscow region) "Waves in metamaterials: their role in modern physics";

(2) **Burov V A, Voloshinov V B, Dmitriev K V, Polikarpova N V** (Lomonosov Moscow State University, Moscow) "Acoustic waves in metamaterials, anisotropic crystals and anomalously refracting structures";

(3) **Shvartsburg A B** (Joint Institute for High Temperatures, RAS, Moscow), **Erokhin N S** (Space Research Institute, RAS, Moscow) "Resonant tunneling of ultrashort electromagnetic pulses in gradient metamaterials: paradoxes and prospects";

(4) **Petnikov V G** (A M Prokhorov General Physics Institute, RAS, Moscow), **Stromkov A A** (Institute of Applied Physics, RAS, Nizhny Novgorod) "Focusing of low-frequency sound fields on the ocean shelf";

(5) **Luchinin A G, Khil'ko A I** (Institute of Applied Physics, RAS, Nizhny Novgorod) "Low-mode acoustics of shallow water waveguides";

(6) **Esipov I B** (RAS Research Council on Acoustics, Moscow) "Basic results for 2010 in the field of acoustics as presented at a RAS Council session".

Papers written on the basis of these oral presentations are published below.

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Waves in metamaterials: their role in modern physics

V G Veselago

The present report discusses both the already-known parameters of metamaterials and those their properties that have to date not received due research attention but which, it turns out, are fundamental for understanding some aspects of both nonrelativistic and relativistic physics.

A 'metamaterial' as currently understood is an artificial composite crystal made of macroscopic structural elements immersed in a homogeneous medium weakly absorbing electromagnetic radiation. The properties mentioned above (and to be discussed below) are those with respect to electromagnetic radiation with wavelength $\lambda > d$, where d is the characteristic crystal lattice parameter. The opposite case, $d > \lambda$, is that of so-called photonic crystals, and is not considered here.

Among other things, the reason for the interest in metamaterials is that their dielectric permittivity ε, magnetic permeability μ, and refractive index $n = \pm\sqrt{\varepsilon\mu}$ can be varied over sufficiently wide ranges by varying the size, shape, and concentration of their constituent macroscopic elements. Of particular interest is the fact that ε and μ can often be made negative, thus leading to a negative n. The electrodynamic properties of such materials with ε, μ, and $n < 0$ were described in most general terms in Ref. [1], at which time neither such materials nor indeed the term 'metamaterial' was known. Because there was no background section in that first paper, it is worthwhile to refer the reader to Fig. 1, which illustrates the logic of how the field has developed historically.

D V Sivukhin, apparently the first to point out that ε and μ can be simultaneously negative [2], was himself in doubt as to the actual existence of such materials — he did not even mention this issue in his well-known course of physics. As for Pafomov [3], his primary concern was the Cherenkov effect in

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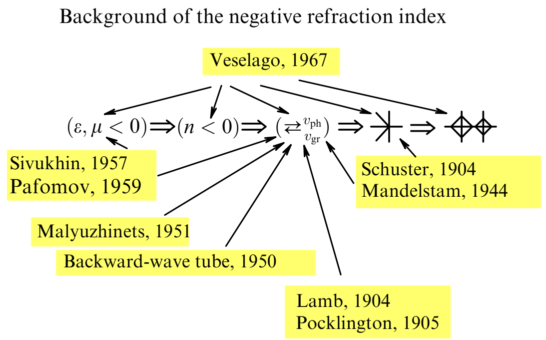

Figure 1. Short history and the development logic of introducing the concept of the negative refractive index — from negative ε and μ to a flat lens.

materials with negative ε and μ. Both Sivukhin and Pafomov argued, quite justifiably, that the phase and group velocities are antiparallel in media with negative ε and μ. Notice that, although electronic devices using such waves were already known at the time of these publications (as exemplified by long transmission lines [4] and backward-wave tubes [5]), they were, of course, not amenable in principle to a description in terms of anything like ε, μ, or n.

L I Mandelstam's lectures [6], while providing, with reference to Lamb [7], a clear physical picture of how electromagnetic waves with opposite phase and group velocities propagate in media, did not use the concepts of dielectric and magnetic permeabilities, and refractive index, let alone that of a negative refractive index. At the same time, Mandelstam discussed in detail a somewhat unconventional version of this general problem, in which a ray passes through the boundary of a medium with antiparallel group and phase velocities. In this case, the incident and refracted rays are on the same side of the interface normal. Note, though, that it was probably Schuster [8] who first pointed to this opportunity (illustrating it, parenthetically, by a schematic diagram), and that at about the same time Pocklington's work [9] was also published.

It should be noted that the very concept of a 'negative refractive index' had not been in use prior to our work [1], and that this paper was the first to trace the logical chain from negative ε and μ, to negative refractive index n, to antiparallel phase and group velocities, to the realization of Snell's law for negative n, and to a flat lens. The point to note here is that the terms 'negative refraction' and 'negative refraction index' are sometimes not distinguished from each other in the literature. The former term relates to the situation in which the refracted and incident rays are on the same side of the normal to the interface between two transparent bodies: a well-known phenomenon readily observable in, for example, transparent birefringent materials. In this case, however, referring to Snell's law

$$\frac{\sin\varphi}{\sin\phi} = n\,, \tag{1}$$

we cannot think of a universal refractive index n independent of the angle φ of incidence. It is when n is negative and independent of φ that a negative refractive index can reasonably be spoken of.

As already noted, metamaterials have offered new opportunities in the synthesis and design of new materials with novel, never-before-seen properties that can be used for very interesting and promising applications. Two such potential applications have generated a vast literature.

In the pioneering work by Pendry [10], it was shown that using a flat lens made of $n = -1$ metamaterial, super-resolution imaging can be achieved, which is impossible in the limit of a geometric optics and, according to Pendry, arises here due to the propagation of so-called evanescent modes, or more precisely due to the presence of a near field.

Another major possibility that arose with the advent of metamaterials is that of creating an 'invisibility cloak', i.e. a metamaterial coating that makes the coated region invisible [11]. This possibility has already been realized and gave rise to so-called transformation optics, a field which is concerned, in effect, with changing the geometric properties of space by placing in it metamaterials with prescribed parameters [12].

Importantly, in our view, the advent of metamaterials has triggered the statement and solution of a number of fundamental problems in physics. One of these is, on the face of it, very simple: if in a negative-refraction medium the wave vector $\mathbf{k}$ is negative (because it is directed opposite to the Umov–Poynting vector $\mathbf{S}$), does it mean that the momentum of the field in such a medium will also be negative and directed opposite to the wave propagation direction which we consider to coincide with the direction of $\mathbf{S}$? Stated in more concise and simple terms, the question is: given the wave–particle duality principle, can the relation

$$p = hk \tag{2}$$

be considered valid for negative k as well? A positive answer will imply that in the absorption and reflection of waves with negative k propagating in a medium, light pressure will be replaced by light attraction. Strange as it may seem, our paper [1] was the first to address this question; a more detailed treatment was given in Ref. [13].

Most surprisingly, an even simpler question has not until recently had a clear answer: to what degree does relation (1) determine the momentum of electromagnetic radiation for positive k? Furthermore, doubts are often cast on whether the term 'electromagnetic field momentum' itself can justifiably be introduced for radiation propagating through a substance. The reason is that the field propagating through a medium sets its particles in motion (albeit on a microscopic scale), making it generally impossible, or at least difficult, to distinguish between the momentum of the field proper and that of the substance it propagates through. This point is readily interpreted by using the relativistic four-dimensional formulation to determine the forces the field exerts on the substance. For any closed system, we can introduce the energy–momentum four-dimensional tensor T_{ik} such that its four-dimensional divergence is zero:

$$\frac{\partial T_{ik}}{\partial x_k} = 0\,. \tag{3}$$

We next proceed by dividing the system into two subsystems: the electromagnetic field in some region, whose tensor T^{f}_{ik} depends only on the field strengths and inductions, E, H, D, and B, and the matter proper, with the tensor T^{m}_{ik}; we have the relationship $T_{ik} = T^{\mathrm{f}}_{ik} + T^{\mathrm{m}}_{ik}$. In that case, one can write

$$f_i = \frac{\partial T^{\mathrm{f}}_{ik}}{\partial x_k} = -\frac{\partial T^{\mathrm{m}}_{ik}}{\partial x_k}\,. \tag{4}$$

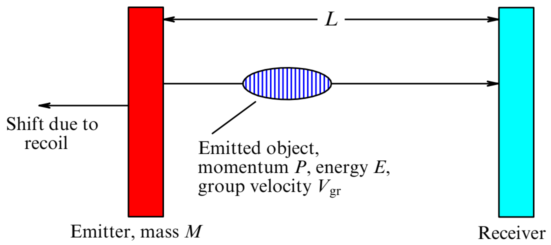

Figure 2. Schematic of how an object is ejected by the emitter and absorbed by the receiver.

Here, $f_i = \partial T^{\mathrm{f}}_{ik}/\partial x_k$ is precisely what we are looking for — the force the field exerts on the matter. Unfortunately, there is a serious problem involved in the approach being developed: it is not yet clearly understood how the tensor T^{f}_{ik} depends on field variables. The two possible forms of T^{f}_{ik} that are most often discussed were proposed by Minkowski [14] and Abraham [15] (see Appendix). The distinction between the two, which in English literature is generally identified as a 'Minkowski–Abraham controversy', consists only in the fact that the field momentum density $\mathbf{g}$ is given by $\mathbf{g} = [\mathbf{BD}]/(4\pi c)$ in the former, and by $\mathbf{g} = [\mathbf{EH}]/(4\pi c)$ in the latter. However, this seemingly small difference becomes fundamentally important when discussing forces that arise in a transparent body through which an electromagnetic wave propagates. In particular, different $\mathbf{g}$'s give rise to different magnitudes for the force that acts on the radiation absorber, and for the recoil force that acts on the field source. Estimating the latter force is key when discussing the idea proposed long ago by Einstein [16] and currently often referred to as the 'Einstein box' thought experiment. It is this thought experiment which demonstrates most clearly that the transfer of energy E in a vacuum from the emitter to the receiver involves the transfer of mass m equal to

$$m = \frac{E}{c^2}\,. \tag{5}$$

Referring to Fig. 2, the following most general considerations apply to the process in which a certain object with energy E and momentum P is transferred from the emitter to the receiver (note that the relation between the energy and momentum is not initially specified, nor is even the nature of the object transferring them discussed).

After ejecting an object with momentum P, the emitter acquires velocity

$$V = \frac{P}{M}\,, \tag{6}$$

and the object reaches the receiver in time

$$t = \frac{L}{V_{\mathrm{gr}}}\,. \tag{7}$$

In this time, the emitter shifts to the left through a distance

$$\Delta x = Vt = \frac{PL}{MV_{\mathrm{gr}}}\,. \tag{8}$$

This relation can be conveniently rewritten as

$$\Delta x M = \frac{P}{V_{\mathrm{gr}}}\,L\,. \tag{9}$$

Noting that the system as a whole has its center of inertia in the same position all the time it moves, Eqn (9) clearly implies that the mass

$$m = \frac{P}{V_{\mathrm{gr}}} \tag{10}$$

will move a distance L to the right in this time. At this point, we would do well to turn to the relation between the object's momentum P and energy E. Very importantly, this relation varies with the nature of the object. If the object is a field pulse (wave packet, photon), then the P versus E relation has the form

$$P = \frac{E}{V_{\mathrm{ph}}}\,. \tag{11}$$

This expression is a direct consequence of the particle–wave duality principle, i.e., directly follows from the relations $E = \hbar\omega$ and $P = \hbar k$. Substituting formula (11) into formula (10), we obtain

$$m = \frac{E}{V_{\mathrm{ph}} V_{\mathrm{gr}}}\,. \tag{12}$$

If the ejected object is a material body (a thrown stone, a speeding bullet, an elementary particle), then instead of formula (11) we can write (following Ref. [17])

$$P = \frac{E}{c^2}\,V_{\mathrm{gr}}\,, \tag{13}$$

which, when substituted into formula (10), yields the commonly known expression

$$m = \frac{E}{c^2}\,. \tag{14}$$

The considerations above are primarily based on the fact, seen from expressions (11) and (13), that the relation between energy and momentum is essentially different for the electromagnetic field and material particles. The reason is the Lorentz transformation: it applies to material particles, and this is exactly what ensures the validity of relation (13) and, as a consequence, of equality (14), a fundamental relation for the emitter-to-receiver mass transfer. If mass and energy are transferred by electromagnetic field, they should be taken to be related by equation (11), yielding formula (12) for the mass transfer.

For light propagation in a vacuum, when

$$V_{\mathrm{ph}} = V_{\mathrm{gr}} = c\,, \tag{15}$$

it is easily shown that both Eqn (11) and Eqn (13) yield for the mass transfer the 'standard' result, Eqn (14). Besides an electromagnetic wave in a vacuum, at least three objects can be identified for which the relation $V_{\mathrm{ph}} V_{\mathrm{gr}} = c^2$ — and, as a consequence, equality (14) — hold true:

(a) an electromagnetic wave in a plasma;

(b) an electromagnetic wave in a hollow waveguide;

(c) de Broglie waves related to the energy and momentum of material bodies by the relations $E = \hbar\omega$ and $P = \hbar k$.

Thus, it can be argued that formula (12) is the most general expression for the relation between the energy transferred and mass transferred, and that equality (14) is only its special case for $V_{\rm ph} V_{\rm gr} = c^2$. The interesting point to note is that the derivation of the well-known formula (14), which is given above and which is based on Einstein's work [16], has itself no relation whatsoever to the theory of relativity: it makes no mention of the Lorentz transformations, nor of the postulate of the threshold value of the speed of light, nor of the postulate that all physical laws are the same in all inertial reference frames. It is only on the basis of the laws of classical physics — in particular, conservation laws — that this derivation was made.

From relations (11) and (12), somewhat weird implications follow, in particular, that in the negative-index material light pressure is replaced by light attraction [1], and that the radiation transfer from the emitter to the receiver involves mass transfer in the opposite direction, from the receiver to the emitter [13]. It should be noted here that mass transfer from the emitter to the receiver (or the other way around) does not imply that the radiation itself possesses a certain mass [18].

Let us now return to the Minkowski–Abraham controversy.

The Minkowski tensor [14] can be obtained by direct calculation from Maxwell's equations, the way it is done in, for example, book [22]. In the opinion of many, the Minkowski tensor is disadvantageous because of its lack of symmetry. It is because of this fact, and because of a desire to make the tensor symmetric, that Abraham [15] changed the magnitude of field momentum g. This, though, led to the result that the tensor ceased to be covariant, as shown by direct calculation in Ref. [19], and hence of no use in calculating the forces that the field exerts on the substance in accordance with expression (4). It should be noted that the symmetry of the tensor is not itself a necessary requirement, as shown, inter alia, in Møller's book [20].

In the books [20, 22], the components of the Minkowski tensor are presented in the form of functions of **E**, **H**, **D**, and **B**. They can, however, be expressed in an equivalent form in terms of the energy density W and the components of the four-dimensional wave vector $K_i = (\mathbf{k}, \omega/c)$ and the four-dimensional group velocity

$$U_k = \left(\frac{\mathbf{u}}{\sqrt{1-u^2/c^2}} \, , \frac{c}{\sqrt{1-u^2/c^2}} \right),$$

leading to the result for the components T_{ik} in the form [21]

$$T_{ik} = \frac{W}{\omega} \sqrt{1-\frac{u^2}{c^2}} K_i U_k \, . \tag{16}$$

Taking into account that the momentum density $g_\alpha = T_{\alpha 4}/c$, from the last formula follows that W and $\mathbf{g}$ are related by

$$g_\alpha = \frac{W k_\alpha}{\omega} \, , \tag{17}$$

which, apart from the notation, is equivalent to formula (11).

As for the Abraham tensor, the requirement that it be symmetric leads, notably, to the equality

$$g = \frac{S}{c^2} \, , \tag{18}$$

which, in turn, implies, bearing in mind the well-known relation $S = W V_{\rm gr}$, that

$$g = \frac{W V_{\rm gr}}{c^2} \, , \tag{19}$$

in complete equivalence to formula (13).

To summarize, then, the definition of the Minkowski tensor treats the momentum of the field as that of a wave, whereas for the Abraham tensor this is, in fact, the momentum of a material particle.

One further comment is in order on the Abraham tensor or, more specifically, on its spatial components. These are, in fact, identical to those of the Minkowski tensor. On the other hand, each spatial component — say, indexed by ik — equals the product of momentum density in the ith direction and the kth component of the group velocity or, in other words, to the flux of the ith component of momentum in the kth direction. It is clear that if the Minkowski and Abraham momentum densities differ, so should the spatial components of these two tensors. This is actually not the case, and it is precisely this fact which deprives the Abraham tensor of relativistic invariance. It may well be that having changed the temporal components of the tensor, Abraham just 'forgot' to change the spatial ones.

We would claim that the above discussion of the Minkowski–Abraham controversy is totally in favor of the Minkowski tensor.

In conclusion, the following points serve to summarize:

(1) The mass transferred from the emitter to the receiver by an electromagnetic field in a substance is given by

$$m = \frac{E}{V_{\rm ph} V_{\rm gr}} \, .$$

The formula $E = mc^2$ is a special case of the above relation.

(2) Inside a negative refraction material, light pressure is replaced by light attraction, and mass is transferred by light, not from the emitter to the receiver, but the other way round, from the receiver to the emitter.

(3) The Abraham tensor is not relativistically covariant and cannot generally be used to calculate forces exerted by the force on matter. For this purpose, the Minkowski tensor should be applied.

Appendix

The energy–momentum tensor can be written out in the most general form as

$$T_{ik} = \begin{bmatrix} T_{\alpha\beta} & -\mathrm{i}c\mathbf{g} \\ -\dfrac{\mathrm{i}}{c}\mathbf{S} & W \end{bmatrix} . \tag{20}$$

Here, $T_{\alpha\beta}$ are the spatial components of the tensor, with $\alpha, \beta = x, y, z$; $\mathbf{g}$ is the field momentum density; $\mathbf{S}$ is the Umov–Poynting vector, and W is the field energy density.

The individual components of the Minkowski energy–momentum tensor take the form

$$T_{\alpha\beta} = \frac{1}{4\pi}(E_\alpha D_\beta + H_\alpha B_\beta) - \frac{1}{8\pi}\,\delta_{\alpha\beta}(\mathbf{ED} + \mathbf{HB}) \, ,$$

$$\mathbf{S} = \frac{c}{4\pi}\,[\mathbf{EH}] \, , \qquad \mathbf{g} = \frac{1}{4\pi c}\,[\mathbf{DB}] \, , \qquad W = \frac{1}{8\pi}(\mathbf{ED} + \mathbf{HB}) \, .$$

The same components of the Abraham tensor are as follows:

$$T_{\alpha\beta} = \frac{1}{8\pi}(E_\alpha D_\beta + E_\beta D_\alpha + H_\alpha B_\beta + H_\beta B_\alpha)$$
$$- \frac{1}{8\pi}\delta_{\alpha\beta}(\mathbf{ED} + \mathbf{HB}),$$
$$\mathbf{S} = \frac{c}{4\pi}[\mathbf{EH}], \quad \mathbf{g} = \frac{1}{4\pi c}[\mathbf{EH}], \quad W = \frac{1}{8\pi}(\mathbf{ED} + \mathbf{HB}).$$

The formula for the energy density W given above can be written out as $W = (1/8\pi)(\varepsilon E^2 + \mu H^2)$, but using this form implies that the two permeabilities ε and μ are both essentially positive. If ε and μ are negative, Ref. [23] gives the following expression for the energy density:

$$W = \frac{1}{8\pi}\left[\frac{\partial(\varepsilon\omega)}{\partial\omega}E^2 + \frac{\partial(\mu\omega)}{\partial\omega}H^2\right]. \tag{21}$$

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Acoustic waves in metamaterials, crystals, and anomalously refracting structures

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1. Introduction

At present, much attention in the literature is being paid to media in which the propagation of waves occurs in the 'unusual' fashion. In particular, such media include so-called left-handed media in electrodynamics. The concept of these media was first proposed in paper [1], where they were introduced as media in which both the permittivity ε and permeability μ are negative. Interest in such media was rekindled after the publication of a number of papers (for example, Ref. [2]) reporting their experimental realization based on metamaterials — artificial structures with characteristic sizes of elements that are well below the wavelength of propagating radiation.

One of the unusual peculiarities of the propagation of plane waves in such media is that the Umov–Poynting vector is antiparallel to the wave vector [1]. This specific feature of the wave propagation is of a general character and is inherent in metamaterials studied not only in electrodynamics and optics, but also in acoustics. Because the phase and group velocity vectors of bulk waves in metamaterials are antiparallel, it is interesting to analyze the possibility of other spatial orientations of these vectors. It is also important to know the angles between the phase and group velocity vectors of a plane wave that can appear whatsoever in optical and acoustic anisotropic materials.

2. Double negative acoustic media

It has been shown in a number of theoretical and experimental papers [3–6] that double negative media (below, for brevity, we will call them simply negative) with negative effective dynamic characteristics (density ρ and compressibility η) are suited for playing the role of media in acoustics in which wave processes proceed similarly to those in left-handed media in electrodynamics. The characteristics of these media are dynamic in the sense that each element of such a medium within a certain frequency band can behave as an element of a homogeneous medium with negative parameters (for example, due to the presence of resonance structures in it).

In negative media, both in electrodynamics and acoustics, negative refraction can be observed, which is often described by using the wave equation or Helmholtz equation [7]. In this case, a principal difficulty appears because these equations contain the square of the refractive index, and to determine its sign, it is necessary to invoke additional considerations which can be based on the choice of one branch or another of the

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