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METHODOLOGICAL NOTES

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Localization of a stationary electromagnetic field by a planar boundary of a metamaterial

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Contents

Abstract. Methods and results of research on the focusing properties of planar boundaries of some metamaterials modeled by a negative electromagnetic medium (i.e., one with both a negative permittivity and a negative permeability) are presented. The properties of flat focusing lenses made of isotropic, anisotropic, or chiral negative media are described in terms of ray theory. The wave theory of an isotropic flat negative-material lens is developed, which shows that such a lens allows localizing a stationary electromagnetic field, in principle, onto an area of the effective linear size either larger or smaller (or even much smaller) than the radiation wavelength in a homogeneous medium at some distance from the source.

1. Introduction

The unusual physical phenomenon of localization (concentration) of a stationary electromagnetic field in a homogeneous medium and, in particular, in free space can be realized using a planar boundary of a metamaterial: inside, if a source is outside the metamaterial, and outside, if a source is inside the metamaterial. Using two boundaries, i.e., a metamaterial slab (plate), the divergent field of an outside source located at one side of the plate can be localized in space on the other side of the plate [1, 2].

From the methodological standpoint, this effect can be separated into two, generally speaking different, but related physical processes: the first corresponds to the focusing of an electromagnetic field diverging from the radiation source (described as a divergent beam of rays) and the second can be interpreted as a transfer of the source near field, concentrated around the source, to a new location with the field localization preserved. The first process is related to the so-called negative refraction of beams (plane waves) at the planar metamaterial boundary, and both processes are related to the notion of backward waves (see [3–21] for metamaterials, negative refraction, and backward waves).

The models of metamaterials, i.e., artificial electromagnetic media currently being considered can be separated into two types according to their investigation methods: continuous electromagnetic media (isotropic, anisotropic, and chiral [1, 2, 12, 13, 22–31], described by the model of negative medium, i.e., a medium with negative values of both the permittivity and permeability) and structured media, which cannot be described by the above electromagnetic parameters: these are cases where the sizes of artificial media elements (artificial molecules) are of the order of or larger than the radiation wavelength, while the 'media' themselves are actually given by spatial structures in the form of periodic gratings [2, 14–18, 32–41].

In this paper, we consider models of metamaterials as continuous media, i.e., in the case where the linear size of medium elements and the distances between them are significantly smaller than the radiation wavelength. The structure, deterministic or random, of such a medium does not matter here. It is assumed that the medium can be described in terms of averaged macroscopic electromagnetic parameters — a scalar or tensor (in an anisotropic case) permittivity/permeability. First, we present the ray (geometrical optics) theory of focusing of a divergent ray beam transmitted through a planar metamaterial (negative medium) boundary and transformed into a convergent ray beam. Then we consider the wave theory for a divergent spherical wave (which would be a cylindrical wave in two dimensions) transmitted through such a boundary and turned into a convergent spherical (cylindrical) wave or a similar wave. It

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is then possible to describe the effect of the transfer of the source near-field (without the source itself) to a new location with the field localization preserved, and, when wave losses are taken into account, with the localization preserved approximately.

These theoretical problems are considered here on the basis of analytic methods of mathematical physics.

2. Ray theory of flat homogeneous focusing lenses

Flat homogeneous focusing lenses — isotropic [1], anisotropic [2], and chiral [31] — are plates (slabs) of metamaterial, i.e., structures with two parallel plane boundaries. When considering the ray (plane wave) focusing process by such lenses, the key issue is to describe this process for a ray beam diverging from a source and transmitted through one boundary of the lens, for example, inside the metamaterial. We emphasize that just the planar interface (between the ambient medium and the metamaterial) has the focusing properties. At the second interface (metamaterial–ambient medium), the process actually repeats itself.

2.1 Isotropic lens

An isotropic homogeneous flat lens was first considered in Ref. [1]. It is a plate of a metamaterial — a negative electromagnetic medium described by a negative scalar permittivity and a negative scalar permeability. The ray theory for such a lens is especially simple.

An isotropic lens, which includes the planar interface at $z = a$ between the positive and negative media, is sketched in Fig. 1. The relative (with respect to free space) permittivity (permeability) of the positive medium is given by ε_+ (μ_+), and that of the negative medium is ε_- (μ_-). In order that all rays propagating from a point source located in the positive medium be focused inside the negative medium at the point $\rho = 0$, $z = 2a$, where $\rho = (x^2 + y^2)^{1/2}$, it suffices to set

$$\varepsilon_+ = \varepsilon\,, \quad \mu_+ = \mu\,, \quad \varepsilon_- = -\varepsilon\,, \quad \mu_- = -\mu\,. \tag{1}$$

Indeed, in this case, the refractive index for rays (plane waves) at the interface of the media is

$$n = \frac{m_-}{m_+} = -1\,, \tag{2}$$

where

$$m_\pm = (\varepsilon_\pm \mu_\pm)^{1/2} = \pm(\varepsilon\mu)^{1/2} \tag{3}$$

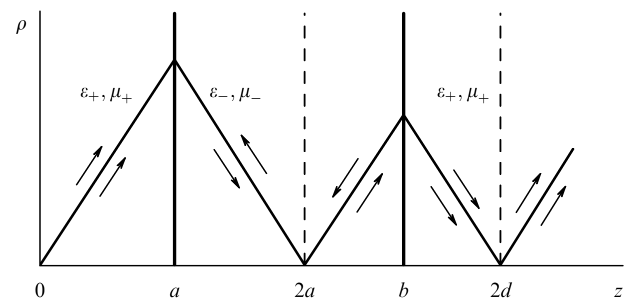

Figure 1. Isotropic flat lens: a plate of the thickness $d = b - a$. The upper arrows along the rays indicate the direction of the ray (plane wave) phase velocity, and the lower arrows show the power flux (Umov–Poynting vector) direction.

are the deceleration factors for forward and backward plane waves in the media relative to free space [1, 19]. This leads to a convergent bundle of rays transmitted through the boundary. This beam is symmetric with respect to the boundary to the divergent bundle of rays incident on the boundary. The ratio of the wave impedances of the media is given by [1, 19]

$$\frac{Z_-}{Z_+} = 1\,, \qquad Z_\pm = \zeta^0 \zeta_\pm\,, \tag{4}$$

where ζ^0 is the wave impedance of free space, and

$$\zeta_\pm = \left(\frac{\mu_\pm}{\varepsilon_\pm}\right)^{1/2} = \frac{m_\pm}{\varepsilon_\pm} = \frac{\mu_\pm}{m_\pm} = \left(\frac{\mu}{\varepsilon}\right)^{1/2} = \zeta\,, \tag{5}$$

which results in the absence of rays (plane waves) reflected from the boundary [1, 42, 43].

Based on these results, we can make an important generalization and formulate it as the following statement. To focus radiation from a point-like source at a point in the region of its image after it is transmitted through a planar interface between positive and negative media, it is not sufficient to have equal optical paths from the source to its image for all rays; a stronger condition is necessary: the full optical path for each ray must be equal to zero. We call this condition the principle of zero optical paths for rays from a source to its image.

This principle can be written as

$$s_+ + s_- = 0\,, \tag{6}$$

where, for homogeneous isotropic media, the optical paths are

$$s_\pm = \mathbf{m}_\pm \mathbf{L}_\pm = m_\pm L_\pm\,, \tag{7}$$

$\mathbf{m}_+$ and $\mathbf{m}_-$ are the wave vectors, $\mathbf{L}_+$ and $\mathbf{L}_-$ are the vector coordinates of the ray geometrical paths before and after the boundary, $m_\pm = |\mathbf{m}_\pm|$, and $L_\pm = |\mathbf{L}_\pm|$. For homogeneous anisotropic media,

$$s_\pm = \frac{L_\pm}{p_\pm}\,, \tag{8}$$

where $p_\pm = |\mathbf{p}_\pm|$, and $\mathbf{p}_+$ and $\mathbf{p}_-$ are the ray vectors related to the wave vectors as $\mathbf{m}_\pm \mathbf{p}_\pm = \pm 1$ [44].

2.2 Anisotropic lens

The idea to make an anisotropic homogeneous flat lens from a metamaterial was suggested in Ref. [2]. The focusing theory for such a lens, presented below, differs from that in Ref. [2], but gives the same result. Here, we use the above principle of zero optical paths for rays transmitted through a metamaterial boundary from a point-like source to its image.

Figure 2 shows the boundary between an isotropic external medium with the parameters

$$\begin{gathered} \varepsilon_+ = \varepsilon > 0\,, \qquad \mu_+ = \mu > 0\,, \\ m_+ = m = (\varepsilon\mu)^{1/2} > 0 \end{gathered} \tag{9}$$

and an anisotropic uniaxial metamaterial with the parameters

$$\begin{gathered} \hat{\varepsilon}_- = \begin{pmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix} = \begin{pmatrix} \varepsilon_\perp & 0 & 0 \\ 0 & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_\parallel \end{pmatrix} = \varepsilon_- \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & q^2 \end{pmatrix}, \\ \varepsilon_- < 0\,, \qquad q = \left(\frac{\varepsilon_\parallel}{\varepsilon_\perp}\right)^{1/2}, \qquad \hat{\mu}_- = \mu_- < 0\,, \\ m_\perp = (\varepsilon_\perp \mu_-)^{1/2} = (\varepsilon_- \mu_-)^{1/2} < 0\,, \qquad m_\parallel = q m_\perp\,. \end{gathered} \tag{10}$$

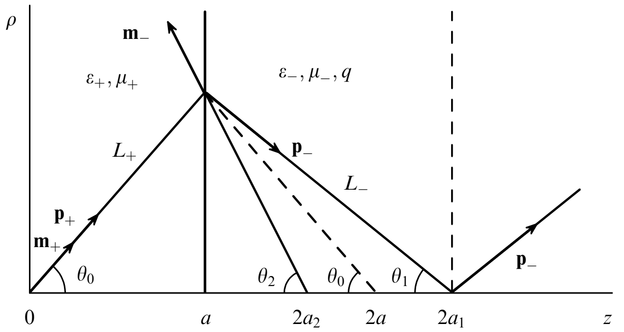

Figure 2. The interface $\rho \geqslant 0,\ z = a$ between isotropic positive and anisotropic negative media.

The optical path of a ray in the isotropic external medium from the source to the metamaterial boundary is

$$s_+ = m_+ L_+ = m(\rho^2 + a^2)^{1/2} = m\rho(1 + \tan^{-2}\theta_0)^{1/2}\,,$$
$$\rho = (x^2 + y^2)^{1/2}\,, \tag{11}$$

where θ_0 is the angle of incidence of the ray at the boundary, and the optical path of an extraordinary ray in the metamaterial with polarization corresponding to an extraordinary plane wave [44–46] from the boundary to the image of the source is given by

$$s_- = \frac{L_-}{p_-} = \frac{\rho}{\sin\theta_1}\,(m_\parallel^2 \sin^2\theta_1 + m_\perp^2 \cos^2\theta_1)^{1/2}$$
$$= m_\parallel \rho(1 + q^{-2}\tan^{-2}\theta_1)^{1/2}\,, \tag{12}$$

where θ_1 is the refraction angle of the extraordinary ray, corresponding in this case to the backward extraordinary plane wave.

Hence, based on the condition of zero optical path, i.e., zero total optical path from the source to its image (6), we obtain

$$m\rho(1 + \tan^{-2}\theta_0)^{1/2} + m_\parallel \rho(1 + q^{-2}\tan^{-2}\theta_1)^{1/2} = 0\,. \tag{13}$$

We see that Eqn (13) is satisfied when the relations

$$m_\parallel = -m\,, \tag{14}$$
$$\tan\theta_1 = q^{-1}\tan\theta_0 \tag{15}$$

hold; in this case, all rays from a point-like source transmitted through the boundary are focused at one point of its image, because it follows from Eqn (15) that projections of the paths of transmitted rays on the optical axis are equal to

$$L_- \cos\theta_1 = 2a_1 - a = qa\,, \tag{16}$$

and are therefore the same for all rays.

From the same equation (15) and the known relation [44]

$$\tan\theta_2 = \frac{\varepsilon_\parallel}{\varepsilon_\perp}\tan\theta_1 = q^2\tan\theta_1\,, \tag{17}$$

where θ_2 is the refraction angle of a plane wave in the metamaterial in the direction of the wave vector $\mathbf{m}_-$ (see Fig. 2), it follows that

$$\tan\theta_2 = q\tan\theta_0\,. \tag{18}$$

In the particular case of an isotropic metamaterial, i.e., for $q = 1$, we reproduce the results in Section 2.1.

2.3 Chiral lens

The theory of a chiral isotropic lens is given in Ref. [31]. The lens is a double-layer plate (Fig. 3) in which one layer consists of a right-handed and the other of a left-handed chiral negative medium. To describe propagation of electromagnetic waves in chiral media, another parameter that describes the medium chirality is added to the permittivity ε and the permeability μ. The matter equations, for example, in the Drude–Born–Fedorov form [22, 23, 47, 48], are then given by

$$\mathbf{D} = \varepsilon^0\varepsilon(\mathbf{E} + \rho_c \operatorname{rot}\mathbf{E})\,,$$
$$\mathbf{B} = \mu^0\mu(\mathbf{H} + \rho_c \operatorname{rot}\mathbf{H})\,, \tag{19}$$

where $\mathbf{D}$ and $\mathbf{B}$ are the field inductions, $\mathbf{E}$ and $\mathbf{H}$ are the field strengths, ε^0 and μ^0 are the dimensional parameters of free space (the vacuum), ε and μ are the dimensionless permittivity and permeability of the medium, and ρ_c is the chiral parameter measured in the units of length and proportional to the linear size of particles (artificial molecules) that constitute the medium. The right- and left-handed chiral media differ by the sign of ρ_c, which can be positive or negative, respectively.

In an isotropic chiral medium, either right-handed or left-handed, only circularly polarized waves can propagate, right-handed and left-handed in each medium. Because the right-hand polarized and left-hand polarized plane waves have different phase velocities, the splitting (bifurcation) of an incident ray into two parts, i.e., birefringence, occurs at the boundary of the chiral medium (in our case, the metamaterial). In the lens as a whole, when the ray from the source propagates through it, the birefringence occurs at the first boundary, the bireflection at the second boundary, and both effects at the interface between the layers. In Fig. 3, we show only the trajectories of the main rays, carrying most of the power of rays originating from the source in low-chirality layers and, consequently, with low reflection from the boundaries. These last are not considered here.

The chiral lens layers have equal thicknesses and consist of right-handed (r) and left-handed (l) chirally symmetric media, with an arbitrary arrangement (sequence) of layers. The deceleration factors (refractive indices) for right-hand and left-hand circularly polarized rays (denoted by superscripts

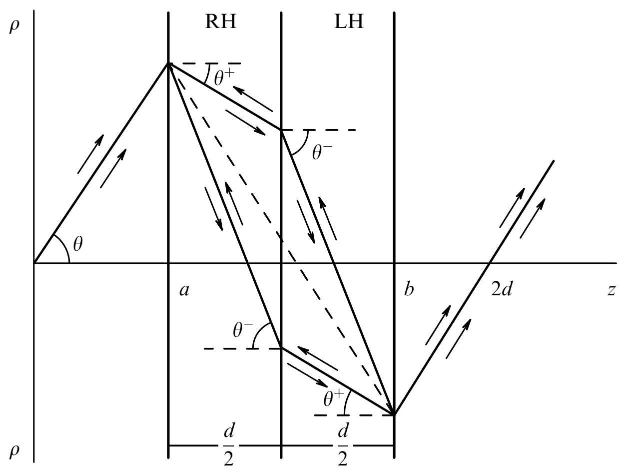

Figure 3. Chiral flat isotropic lens.

'+' and '−' below) in the layers are

$$m^{\pm}_{\text{rh, lh}} = \frac{m_-}{1 \pm \delta_{\text{rh, lh}}}\,, \tag{20}$$

where

$$m_- = (\varepsilon_- \mu_-)^{1/2} < 0\,, \tag{21}$$

$\varepsilon_- = -\varepsilon$ and $\mu_- = -\mu$ are the permittivity and permeability of the lens layers, ε and μ are the permittivity and permeability of the external nonchiral positive medium, $\varepsilon, \mu > 0$, $m = (\varepsilon\mu)^{1/2} > 0$, $m_- = -m$, and

$$\delta_{\text{rh, lh}} = m_- k^0 \rho_{\text{rh, lh}}\,. \tag{22}$$

For the right- and left-handed chirally symmetric layers, the parameters are $\rho_{\text{rh}} = \rho_{\text{c}} > 0$, $\rho_{\text{lh}} = -\rho_{\text{c}}$, $\delta_{\text{rh}} = -\delta_{\text{c}}$, and $\delta_{\text{lh}} = \delta_{\text{c}}$, where

$$\delta_{\text{c}} = mk^0 \rho_{\text{c}} \ll 1\,, \tag{23}$$

$$k^0 = \omega(\varepsilon^0 \mu^0)^{1/2} = \frac{\omega}{c}\,,$$

and c is the speed of light in the vacuum.

It is not difficult to demonstrate that in this lens, despite the ray bifurcation, the principle of zero optical paths is satisfied for each ray coming out from the source and reaching the point of its image [31].

Indeed, the total optical path length from the source to the image, for example, for the upper ray of those bifurcated inside the lens (see Fig. 3), is given by

$$s^+ = \frac{ma}{\cos\theta} + \frac{m^+_{\text{rh}} d}{2\cos\theta^+} + \frac{m^+_{\text{lh}} d}{2\cos\theta^-} + \frac{m(2d - b)}{\cos\theta}\,, \tag{24}$$

where a is the distance from the source to the first boundary of the lens, d is the lens thickness, and $2d - b$ is the distance from the second boundary of the lens to the image of the source. From Snell's law for the rays shown in Fig. 3,

$$m \sin\theta = m^{\pm}_{\text{rh}} \sin\theta^{\pm} = m^{\pm}_{\text{lh}} \sin\theta^{\mp}\,, \tag{25}$$

it follows that

$$\sin\theta^{\pm} = -(1 \mp \delta_{\text{c}}) \sin\theta\,. \tag{26}$$

Hence, for small δ in (23), we obtain

$$\cos\theta^{\pm} = (1 \pm 2\delta_{\text{c}} \tan^2\theta)^{1/2} \cos\theta\,. \tag{27}$$

Substitution of this relation in (24) gives

$$s^+ = 0\,. \tag{28}$$

It can be shown similarly that the full optical path length for the lower ray is

$$s^- = 0\,. \tag{29}$$

In Ref. [31], a calculation was made for a particular disk-shaped chiral lens, bounded by the radius $R = d$, with the reduced chirality parameter $\delta_{\text{c}} = 0.1$ of the metamaterial (Fig. 4). Under the condition

$$\frac{d}{2} \leqslant a < d \tag{30}$$

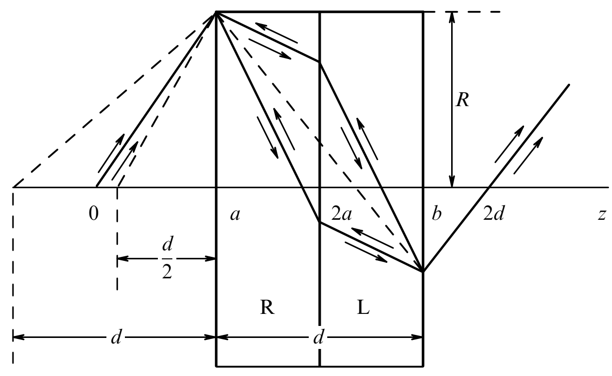

Figure 4. Disk-shaped chiral lens of thickness d and finite size radius R.

for the distance a from the field source to the first boundary of the lens, all rays incident on this boundary are focused by both front and back boundaries of the lens. If the distance a is less than $d/2$, then some rays are not focused by the second (back) boundary, i.e., only partial ray focusing is realized.

3. Wave theory of field localization

We consider the wave theory of the simplest homogeneous flat lens: an isotropic nonchiral lens. We formulate the key problem as follows. A divergent spherical wave excited by a point electric dipole oriented normal to the boundary is incident on a planar interface between positive and negative media. The stationary dipole field depends harmonically on time, $\exp(\mathrm{i}\omega t)$. The function of the wave field transmitted through the boundary has to be found. This problem is first solved without taking wave losses in the media into account, and then qualitative and quantitative effects of the wave losses in the media are considered.

Two approaches are used to solve this key problem. The first directly considers spherical waves incident on and transmitted through the boundary. The second approach is to first expand these waves with respect to spectral plane-cylindrical (waveguide) eigenfunctions (a spatially spectral expansion with continuous spectrum, i.e., an integral decomposition) [53–55]. The problem is solved analytically in both approaches when wave losses in the media are not taken into account, and can be solved approximately when the losses are taken into account. Here, the known concept of a point dipole source as well as the notions of point 'sink' and 'sinksource' introduced by this author are used.

3.1 Hertz vector potentials for point source, sink, and sinksource fields

An electromagnetic field excited in a medium by an elementary (point) dipole source is well known [56–59]. The point-like source and the function of its excited field, the so-called Green's function, are idealized mathematical models convenient for simplified descriptions of physical field radiation processes.

For the field of a stationary electric dipole with the dipole moment

$$\mathbf{D}_0 = D_0 \mathbf{z}_0 \exp(\mathrm{i}\omega t)\,, \tag{31}$$

which satisfies the equation

$$\Delta\mathbf{\Pi} + k^2\mathbf{\Pi} = -\frac{4\pi}{k} Z\mathbf{D}\delta(r)\,, \tag{32}$$

where $\mathbf{z}_0$ is the z-axis unit vector, $k = mk^0$, $m = (\varepsilon\mu)^{1/2} > 0$, $k^0 = \omega/c$, c is the speed of light in free space, the wave impedance Z of the medium is given by (4), and $\mathbf{D} = \omega\mathbf{D}_0$. The Hertz vector potential, or simply the Hertz vector $\mathbf{\Pi}$ can be written as

$$\mathbf{\Pi} = \Pi\mathbf{z}_0 \exp(\mathrm{i}\omega t)\,. \qquad (33)$$

Here, the only z-component of the Hertz vector of the emitted wave is given by

$$\Pi^{\mathrm{s}} = \frac{1}{kr}\,ZD\exp(-\mathrm{i}kr)\,, \qquad (34)$$

$$r = (\rho^2 + z^2)^{1/2}\,, \qquad \rho = (x^2 + y^2)^{1/2} \qquad (35)$$

and describes a divergent spherical wave.

The nonzero spherical components of the electric and magnetic field strengths in this case are

$$E_r^{\mathrm{s}} = \frac{2\mathrm{i}}{kr^2}\,Zf(kr)\cos\theta\,,$$
$$H_\varphi^{\mathrm{s}} = \frac{-1}{r}\,f(kr)\sin\theta\,, \qquad (36)$$
$$E_\theta^{\mathrm{s}} = \frac{-1}{r}\,Zg(kr)\sin\theta\,,$$

where

$$f(u) = 1 - \frac{\mathrm{i}}{u}\,, \qquad g(u) = 1 - \frac{\mathrm{i}}{u}\,f(u)\,, \qquad (37)$$

and the cylindrical components are

$$E_\rho^{\mathrm{s}} = E_r\sin\theta + E_\theta\cos\theta = \frac{-1}{2r}\,Z\left[g(kr) - \frac{2\mathrm{i}}{kr}\,f(kr)\right]\sin 2\theta\,,$$
$$H_\varphi^{\mathrm{s}} = \frac{-1}{r}\,f(kr)\sin\theta\,, \qquad (38)$$
$$E_z^{\mathrm{s}} = E_r\cos\theta - E_\theta\sin\theta = \frac{1}{r}\,Z\left[g(kr)\sin^2\theta + \frac{2\mathrm{i}}{kr}\,f(kr)\cos^2\theta\right].$$

For brevity, the common factor $k^2 D\exp(-\mathrm{i}kr)$ is omitted in these formulas.

The notions of a 'sink' (radiation-receiving sink), in contrast to the radiation source (emitting the field), and the 'sinksource', i.e., coupled sink and source, receiving and re-emitting the field, were introduced by the author in the Anniversary Talk "Integral and spectral representations (expansions) of wave fields. A review" at the Moscow Ya N Feld Electrodynamics Seminar on November 6, 2007.

The notion of a point dipole field sink was introduced using the second linearly independent solution of Eqn (32) for the Hertz vector component as

$$\Pi^{\mathrm{a}}(k, Z) = \Pi^{\mathrm{s}}(-k, -Z) = \frac{1}{kr}\,ZD\exp(\mathrm{i}kr)\,, \qquad (39)$$

which describes the field of a converging spherical wave. In this case, the dipole in Eqn (32) does not emit but does absorb the incident wave field.

According to (34) and (39), the spherical and cylindrical components of the point dipole sink field are obtained from Eqns (36) and (38) by replacing k with $-k$ and Z with $-Z$.

When using the term 'field source', either an external device that emits a field or a medium inhomogeneity that scatters an incident wave is physically understood. The sink should be understood similarly, but in contrast to the source, it does not emit: it receives (absorbs) the incident wave energy.

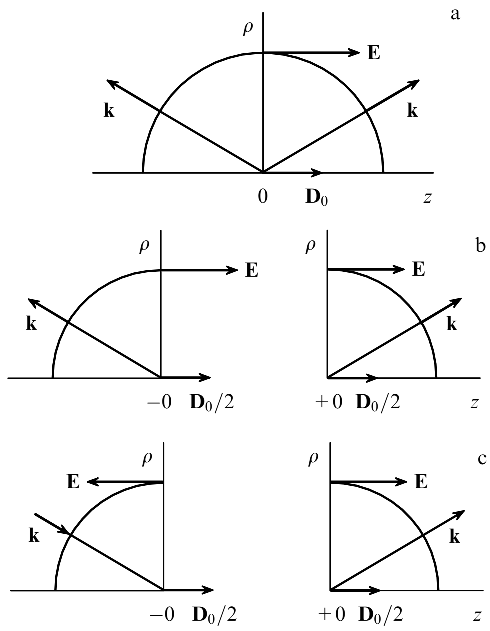

Figure 5. Sketch of: (a) a dipole point source, (b) coupled sources, and (c) a sinksource; for their fields, the boundary condition is $E_\rho = 0$ for $\rho > 0$, $z = 0$.

By a sinksource, we here mean a singularity of the field in a homogeneous medium, induced by an incident convergent wave. The point dipole sinksource can be modeled on the basis of the following considerations. According to (38), the field of a point emitting dipole (Fig. 5a) satisfies the condition

$$E_\rho = 0 \quad \text{for} \quad \rho > 0\,, \quad z = 0 \qquad (40)$$

and, consequently, $\theta = \pi/2$, except at the point $\rho = z = 0$. This condition allows considering such a field separately in semi-infinite spaces $z < 0$ and $z > 0$, as the field excited by two point sources with half dipole moments (Fig. 5b) located at the points $\rho = 0$, $z = -0$ and $\rho = 0$, $z = +0$. The same condition (40) is satisfied in the description of a converging field absorbed by the sink at the point $\rho = 0$, $z = -0$, and in the description of a diverging field emitted by the source at the point $\rho = 0$, $z = +0$. Such a sink–source pair (Fig. 5c) is the sinksource under the assumption of a consistent relation between the sink and the source, i.e., if the electromagnetic energy (power) received by the sink is fully transferred to the source. Here, it is not necessary to introduce an external device or a medium inhomogeneity to understand the physical meaning of a sinksource, and the presence of a point-like singularity of the field at the point of convergence and divergence of spherical waves, i.e., at the focal point, can be explained by the absence of the field diffraction effect in the model of the sinksource field. The diffraction should be taken into account only in the case of a finite incident wave beam, in particular, to account for wave losses in a medium (see Section 3.5 below).

The sinksource field (Hertz vector) therefore satisfies Eqn (32); for $z > 0$, it satisfies the Sommerfeld radiation condition, and for $z < 0$, the analogous condition for counter radiation.

By using the sinksource, the problem of transmission of convergent spherical waves through a focal point can be solved easily. The field after the focal point is obtained based on the formulas known for a point-like source.

The above results for fields in an ordinary positive medium with $m_+ = m > 0$ can be easily generalized to the case of a metamaterial, i.e., a negative medium with $m_- = -m < 0$ (3). To do so, in expressions (32), (34), (36), and (38), we replace $k = k_+ = m_+ k^0$ with $-k = k_- = m_- k^0$ while keeping $Z = Z_+ = Z_-$ in (4) and (5), or, alternatively, replace Z with $-Z$ while keeping k for counter backward waves [19].

To conclude this section, we note that we have considered the so-called elementary (point) electric dipole field source and the corresponding sink, sinksource, and Hertz vector. The obtained results can also be written for an elementary magnetic dipole source and the corresponding sink, sinksource, and Hertz vector [which, in turn, correspond to the electromagnetic field that is orthogonal to the field in Eqns (36)–(38)] by using the known procedure: replacing ε with μ, μ with ε, Z with $1/Z$, $\mathbf{E}$ with $\mathbf{H}$, and $\mathbf{H}$ with $-\mathbf{E}$.

3.2 Spherical waves at the interface between positive and negative media

Figure 6 schematically shows a convergent spherical wave incident on the planar interface between the positive and negative media and excited by a point-like source — the electric dipole with normal orientation to the boundary. Using the above notation, we write the Hertz vector z-component of the incident wave (34) as

$$\Pi_1^{\rm s} = \frac{1}{k_+ r_1} Z_+ D \exp(-{\rm i} k_+ r)\,, \tag{41}$$

where

$$k_+ = m_+ k^0\,, \qquad r_1 = (\rho^2 + z^2)^{1/2}\,, \tag{42}$$

$z \leqslant a$, and D is the factor that includes the absolute value of the dipole moment. According to (38), the cylindrical components needed in what follows are given by

$$\begin{aligned} E_{\rho 1}^{\rm s} &= \frac{-1}{2 r_1} Z_+ \left(1 - \frac{3{\rm i}}{k_+ r_1} - \frac{3}{k_+^2 r^2}\right) \sin 2\theta_1\,,\\ H_{\varphi 1}^{\rm s} &= \frac{-1}{r_1}\left(1 - \frac{\rm i}{k_+ r_1}\right) \sin\theta_1\,, \end{aligned} \tag{43}$$

where $\theta_1 = \arctan(\rho/z)$ and the common factor $k_+^2 D \exp(-{\rm i} k_+ r_1)$ is omitted.

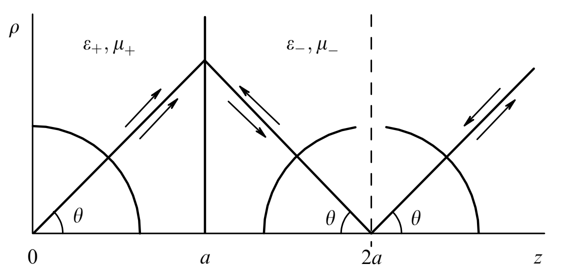

Figure 6. Transmission of spherical waves through the interface between positive and negative media for $\rho \geqslant 0$, $z = a$ and through the focal point $\rho = 0$, $z = 2a$ in the negative medium.

We seek the corresponding Hertz vector and field strength components of the wave transmitted through the boundary $z = a$ in the range $a \leqslant z < 2a$ in the negative medium in the form of a backward spherical wave [21]:

$$\Pi_2^{\rm a} = \frac{T}{k_- r_2} Z_- D \exp({\rm i} k_- r_2)\,, \tag{44}$$

$$\begin{aligned} E_{\rho 2}^{\rm a} &= \frac{T}{2 r_2} Z_- \left(1 + \frac{3{\rm i}}{k_- r_2} - \frac{3}{k_-^2 r_2^2}\right) \sin 2\theta_2\,,\\ H_{\varphi 2}^{\rm a} &= \frac{-T}{r_2}\left(1 + \frac{\rm i}{k_- r_2}\right) \sin\theta_2\,, \end{aligned} \tag{45}$$

where

$$k_- = m_- k^0\,, \quad r_2^2 = \rho^2 + (z - 2a)^2\,, \quad \tan\theta_2 = \frac{\rho}{z - 2a}\,, \tag{46}$$

and we omit the common factor $k_-^2 D \exp({\rm i} k_- r_2)$ in (45). The transmitted wave field in the range $2a < z < \infty$, i.e., behind the focal point $\rho = 0$, $z = 2a$, is to be given below.

The components of the reflected (forward divergent) wave for $z \leqslant a$ can be written similarly:

$$\Pi_2^{\rm s} = \frac{R}{k r_2} Z_+ D \exp(-{\rm i} k_+ r_2)\,, \tag{47}$$

$$\begin{aligned} E_{\rho 2}^{\rm s} &= \frac{-R}{2 r_2} Z_+ \left(1 - \frac{3{\rm i}}{k_+ r_2} - \frac{3}{k_+^2 r_2^2}\right) \sin 2\theta_2\,,\\ H_{\varphi 2}^{\rm s} &= \frac{-R}{r_2}\left(1 - \frac{\rm i}{k_+ r_2}\right) \sin\theta_2\,, \end{aligned} \tag{48}$$

where the common factor $k_+^2 D \exp(-{\rm i} k_+ r_2)$ is omitted in (48).

Because the continuity conditions must be satisfied at the interface $z = a$ $(r_2 = r_1,\ \theta_2 = \pi - \theta_1)$ for the tangential components,

$$E_{\rho 1}^{\rm s} + E_{\rho 2}^{\rm s} = E_{\rho 2}^{\rm a}\,, \qquad H_{\varphi 1}^{\rm s} + H_{\varphi 2}^{\rm s} = H_{\varphi 2}^{\rm a}\,, \tag{49}$$

after substitution of expressions (43), (45), and (48) in (49) and taking (3)–(5), (42), and (46) into account, we obtain

$$1 - R = T\,, \qquad 1 + R = T\,, \tag{50}$$

whence it follows that

$$R = 0\,, \qquad T = 1\,. \tag{51}$$

Thus, spherical waves, as well as plane waves [1, 42, 43], are transmitted without reflection through an interface between isotropic positive and negative media if conditions (1) are satisfied. These conditions are the field focusing $(m_- = -m_+ = -m)$ and consistency $(Z_- = Z_+ = Z)$ conditions at such an interface.

3.3 Transmission of a convergent wave through the focal-point region

It is noted in Ref. [60] that this problem is not related to the specific properties of the metamaterial (negative medium) or to the properties of backward waves. In Section 3.1, based on the sinksource concept, we actually considered the problem of transmission of a convergent spherical wave through the focal point in an ordinary (positive) medium. From those results, by properly changing the notation, it is not difficult to obtain a solution of the problem considered in Section 3.2 for a transmitted backward wave in a negative medium after the focal point, i.e., in the range $(2a, \infty)$, or, equivalently, in the half-space $\rho > 0$, $z > 2a$ occupied by the metamaterial.

In this half-space, the field components of a backward spherical wave diverging from a sinksource (located at the point $\rho = 0$, $z = 2a$) that complement solutions (44) and (45) of the problem considered above are given by

$$\Pi_2^{\mathrm{s}} = \frac{1}{k_- r_2}\, Z_- D \exp\left(-\mathrm{i}k_- r_2\right), \tag{52}$$

$$E_{\rho 2}^{\mathrm{s}} = \frac{-1}{2r_2}\, Z_- \left(1 - \frac{3\mathrm{i}}{k_- r_2} - \frac{3}{k_-^2 r_2^2}\right) \sin 2\theta_2\,, \qquad H_{\varphi 2}^{\mathrm{s}} = \frac{-1}{r_2}\left(1 - \frac{\mathrm{i}}{k_- r_2}\right) \sin\theta_2 \tag{53}$$

in the range $(2a, \infty)$; the common factor $k_-^2 D \exp\left(-\mathrm{i}k_- r_2\right)$ is omitted in (53).

In analyzing the radiation field transmission through the entire flat focusing lens, an additional problem is to describe the transmission process of a divergent backward spherical wave (52) and (53) through the second interface between the negative and positive media. The description of this process fully replicates the results obtained for wave transmission through the first boundary, up to notational differences. We note the special case where the focal point is on the second boundary, i.e., at $2a = b$ (see Fig. 1). In the case $2a < b$, a convergent backward wave on the left side of the plane $z = 2a$ and a divergent backward wave on the right side of this plane have opposite directions of phase motion but identical directions of energy fluxes (Umov–Poynting vectors), but in this special case, the wave phase at $z = b - 0$ and the wave phase at $z = b + 0$ propagate in one direction, while the directions of energy fluxes remain the same. We recall that the direction of backward wave propagation is the direction of the energy flux (the Umov–Poynting vector) and not the direction of the wave phase motion [19].

3.4 Spectral expansion of divergent and convergent spherical wave fields

The problem of spherical wave transmission through a planar interface between positive and negative media was solved by using the spectral expansion of the transmitted field with respect to plane or plane-cylindrical eigenwaves [60–68] (see, e.g., [53–59] for integral and spectral or, more accurately, spatially spectral expansions of wave fields). In Refs [61–63, 65–67], the spectral expansion used for divergent waves was also applied to transmitted convergent waves. But it turned out [63] (see also [54, 55]) that when conditions (1) for media are satisfied, an improper integral for the transmitted wave field expansion diverges, and hence the expansion procedure cannot be used here. The cases where conditions (1) are not strictly satisfied, in particular in the presence of wave losses in media were considered in Refs [63, 65–67] (see Section 3.5 below). In Refs [60, 64, 68], another spectral expansion, which can be used to analyze convergent waves when conditions (1) are satisfied, was applied to the transmitted wave. Below, the solution of the key problem of spherical wave transmission through an interface between positive and negative media with the use of this spectral expansion is reproduced from Ref. [64]; this not only is of methodological interest but also gives better physical understanding of the wave transmission process through the interface.

Following Ref. [64], we write the Hertz vector component (31) of a divergent spherical wave incident on the boundary (see Fig. 6) as a spectral expansion with respect to plane-cylindrical (waveguide) eigenwaves [53–55], i.e., the waves with a plane phase front and the cylindrical coordinate dependence in the plane orthogonal to the z axis:

$$\Pi_1^{\mathrm{s}} = \frac{1}{k_+ r_1}\, Z_+ D \exp\left(-\mathrm{i}k_+ r_1\right) = Z_+ D \int_0^\infty \frac{\mathrm{i}k_+}{\gamma_+}\, J_0(\kappa k_+ \rho) \exp\left(-\mathrm{i}\gamma_+ z\right) \kappa\, \mathrm{d}\kappa\,, \tag{54}$$

where $J_0(\kappa k \rho)$ is the Bessel function, $0 < z < a$, $k_+ = k$, and

$$\gamma_+ = k_+ \begin{cases} (1-\kappa^2)^{1/2} & \text{for } \kappa \leqslant 1\,, \\ -\mathrm{i}(\kappa^2 - 1)^{1/2} & \text{for } \kappa > 1\,. \end{cases} \tag{55}$$

The corresponding component of the wave field reflected from the boundary $z = a$ can be written similarly:

$$\Pi_2^{\mathrm{s}} = Z_+ D \int_0^\infty \frac{\mathrm{i}k_+}{\gamma_+}\, R(\kappa)\, J_0(\kappa k_+ \rho) \exp\left[\mathrm{i}\gamma_+ (z - 2a)\right] \kappa\, \mathrm{d}\kappa\,, \tag{56}$$

where $R(\kappa)$ is the reflection coefficient and $z \leqslant a$.

The solution for the convergent backward wave transmitted through the boundary can be written as

$$\Pi_2^{\mathrm{a}} = Z_- D \int_0^\infty \frac{\mathrm{i}k_-}{\gamma_-}\, T(\kappa)\, J_0(\kappa k_- \rho) \exp\left[-\mathrm{i}\gamma_- (z - 2a)\right] \kappa\, \mathrm{d}\kappa\,, \tag{57}$$

where $T(\kappa)$ is the transmission coefficient, $a \leqslant z < 2a$, $k_- = -k$, and

$$\gamma_- = k_- \begin{cases} (1-\kappa^2)^{1/2}\,, & \kappa \leqslant 1\,, \\ -\mathrm{i}(\kappa^2 - 1)^{1/2}\,, & \kappa > 1\,. \end{cases} \tag{58}$$

When dividing the integration interval $(0, \infty)$ in (54) into two ranges, $(0, 1)$ and $(1, \infty)$, the following physical interpretation for partial integrals resulting from this procedure is usually given. The first integral describes the field emitted by the source and the second describes the field localized near the source. To correctly describe the localized field around the source, the square root value γ_+ is chosen for $\kappa > 1$ as indicated in (55). This leads to the integrand in (54) that exponentially decreases as z increases, i.e., decreases in the direction of the wave propagation; this ensures the integral convergence. Similar considerations can be applied to representation (56) for the reflected wave field with decreasing z. The same considerations were used in Refs [61–63, 65–67] to find the field of a convergent wave transmitted through the boundary; this leads, as we noted already, to a divergent integral in the spectral expansion of the field.

In our spectral expansion (57), the integral converges in the considered z range for medium parameters satisfying conditions (1); here, the exponentially increasing (with increasing z) integrand correctly describes the singularity of the integral, i.e., the field, at the focal point of spherical wave convergence $\rho = 0$, $z = 2a$ (see [64] for the details).

Applying the field matching conditions at the boundary,

$$E_{\rho 1}^{\mathrm{s}} + E_{\rho 2}^{\mathrm{s}} = E_{\rho 2}^{\mathrm{a}}\,, \qquad H_{\varphi 1}^{\mathrm{s}} + H_{\varphi 2}^{\mathrm{s}} = H_{\varphi 2}^{\mathrm{a}}\,,$$

where

$$E_\rho^{\mathrm{s,a}} = \frac{\partial^2 \Pi^{\mathrm{s,a}}}{\partial\rho\,\partial z}\,, \qquad H_\varphi^{\mathrm{s,a}} = \frac{-\mathrm{i}k_{+,-}}{Z_{+,-}}\,\frac{\partial \Pi^{\mathrm{s,a}}}{\partial\rho}\,, \tag{59}$$

and using the orthogonality property of the Bessel functions,

$$k^2\int_0^\infty J_1(\kappa k\rho)\,J_1(\tilde\kappa k\rho)\,\rho\,\mathrm{d}\rho = \kappa^{-1}\delta(\kappa-\tilde\kappa)\,,$$

we obtain the set of equations

$$1 - R(\kappa) = T(\kappa)\,, \qquad 1 + R(\kappa) = T(\kappa) \tag{60}$$

similar to (50), with solution (51): $R(\kappa) = 0$, $T(\kappa) = T = 1$.

Substitution of the obtained result in expansion (57) for $T = 1$ allows reducing it to form (44) [64].

3.5 Accounting for wave losses in media

To take wave losses in media into account, we have to generalize the above notation for the positive and negative media parameters as follows:

$$\begin{aligned}
&\varepsilon_+ = \varepsilon(1-\mathrm{i}\delta_{\varepsilon+})\,, \qquad \mu_+ = \mu(1-\mathrm{i}\delta_{\mu+})\,,\\
&k_+ = k^0(\varepsilon_+\mu_+)^{1/2} \cong k(1-\mathrm{i}\delta_+)\,,\\
&Z_+ = \zeta^0\left(\frac{\mu_+}{\varepsilon_+}\right)^{1/2} \cong Z(1+\mathrm{i}\varDelta_+)\,,
\end{aligned} \tag{61}$$

$$\begin{aligned}
&\varepsilon_- = -\varepsilon(1+\mathrm{i}\delta_{\varepsilon-})\,, \qquad \mu_- = -\mu(1+\mathrm{i}\delta_{\mu-})\,,\\
&k_- = k^0(\varepsilon_-\mu_-)^{1/2} \cong -k(1+\mathrm{i}\delta_-)\,,\\
&Z_- = \zeta^0\left(\frac{\mu_-}{\varepsilon_-}\right)^{1/2} \cong Z(1-\mathrm{i}\varDelta_-)\,,
\end{aligned} \tag{62}$$

where

$$\begin{aligned}
&\varepsilon, \mu > 0\,, \qquad 0 \leqslant \delta_{\varepsilon\pm} \ll 1\,, \qquad 0 \leqslant \delta_{\mu\pm} \ll 1\,,\\
&k = k^0(\varepsilon\mu)^{1/2} > 0\,, \qquad 2\delta_\pm = \delta_{\varepsilon\pm} + \delta_{\mu\pm}\,,\\
&Z = \zeta^0\left(\frac{\mu}{\varepsilon}\right)^{1/2} > 0\,, \qquad 2\varDelta_\pm = \delta_{\varepsilon\pm} - \delta_{\mu\pm}\,.
\end{aligned} \tag{63}$$

With these relations, we seek a solution for a wave transmitted through the interface between the positive and negative media; this can be done approximately in the same form (44), (45), with r_2 replaced with

$$\bar r_2 = r_2 - \mathrm{i}\sigma r_a\,, \tag{64}$$

where

$$r_a = (\rho^2 + a^2)^{1/2}\,. \tag{65}$$

To satisfy matching conditions (49) for wave fields, we suppose that the condition

$$k_+ r_1 = -k_-\bar r_2 \tag{66}$$

is satisfied at the boundary $z = a$; we then substitute expressions (43), (48), and (45) in (49), taking (66) into account. As a result, instead of (50), we obtain the set of equations

$$\begin{aligned}
&k_+^3 Z_+(1-R) = -k_-^3 Z_- T\,,\\
&k_+^3(1+R) = -k_-^3 T\,,
\end{aligned} \tag{67}$$

which has the solution

$$R = \frac{Z_+ - Z_-}{Z_+ + Z_-}\,, \qquad T = -\frac{k_+^3}{k_-^3}\,\frac{2Z_+}{Z_+ + Z_-}\,. \tag{68}$$

With (61) and (62) used here, this solution can be written as

$$\begin{aligned}
&R = \frac{\mathrm{i}}{2}(\varDelta_+ + \varDelta_-) \neq 0\,,\\
&T = 1 - \mathrm{i}\left(3\delta_+ + 3\delta_- - \frac{\varDelta_+ + \varDelta_-}{2}\right) \neq 1\,,
\end{aligned} \tag{69}$$

where we have taken into account that $\delta_\pm$ and $\varDelta_\pm$ are small, and therefore their squares can be omitted; however, these squares have to be taken into account when calculating the reflected and transmitted wave power.

Furthermore, relation (66) (for $z = a$) implies that $k_+ r_a = -k_- r_a(1-\mathrm{i}\sigma)$; hence follows the equality

$$\sigma = \delta_+ + \delta_-\,. \tag{70}$$

To summarize the results, we note that if the Hertz vector component of the wave incident on the media interface has the same form (41) in this case, i.e.,

$$\Pi_1^{\mathrm{s}} = \frac{1}{k_+ r_1}\,Z_+ D\exp(-\mathrm{i}k_+ r_1)\,, \tag{71}$$

but with a different value of k_+ in (61), then the transmitted spherical wave, instead of (44), is given by

$$\Pi_2^{\mathrm{a}} = \frac{T}{k_-\bar r_2}\,Z_- D\exp(\mathrm{i}k_-\bar r_2)\,, \tag{72}$$

where k_- is given by (62), $\bar r_2$ is given by (64) and (70), and the transmission coefficient T in (69) now differs from unity. The Hertz vector component of the reflected wave then retains its form (47) but with a new value of k_+ in (61) (as for the incident wave) and with a nonzero reflection coefficient R in (69).

It follows from relations (63)–(65) and (70), that in the vicinity of the focal point $\rho = 0$, $z = 2a$, i.e., for $\rho \ll a$, we approximately have

$$\bar r_2 = r_2 - \mathrm{i}\sigma a\,. \tag{73}$$

This expression allows analyzing the field structure near the focal point, where

$$|\Pi_2^{\mathrm{a}}| = \frac{|TZ_- D|}{\sigma k a}\exp(-\sigma k a)\,, \tag{74}$$

i.e., in contrast to (44), the field function does not diverge at this point.

To quantitatively estimate the size of the field concentration (localization) region around the former focal point, we can introduce the effective radius $r_2 = r_0$ of the '3D diffraction spot'. For this, from the condition of a twofold decrease in the squared absolute value of function (72) compared to its value at focal point (74), we derive the equation

$$\left[1 + \left(\frac{r_0}{\sigma a}\right)^2\right]^{-1}\exp(2\delta_- k r_0) = \frac{1}{2}\,. \tag{75}$$

Whence the spot radius is given by

$$r_0 = \sigma a(1 + 2\delta_{-}\sigma ka) . \quad (76)$$

This result is correct for the values of ka satisfying the condition $(2\delta_{-}\sigma ka)^2 \ll 1$.

For the transverse planar diffraction spot, it follows from (76) that

$$\rho_0 = \sigma a(1 + 2\delta_{-}\sigma ka) . \quad (77)$$

The problem of wave transmission through an interface between positive and negative media with wave losses in the media taken into account was considered in Refs [60, 68] using a generalized spectral expansion of a divergent spherical wave similar to (57), where a slightly different but also approximate estimate of the radius of the transverse spot was introduced, calculated, and plotted:

$$\rho_0 = \delta a \ln \frac{4}{\delta} . \quad (78)$$

Here, $\delta = \delta_+ = \delta_-$, meaning that the calculation in Ref. [60] was performed in the case of identical wave losses in the media.

As we noted already, the results of solution of the considered problem with the standard spectral (integral) expansion (used for divergent waves) applied to convergent waves were published in Refs [63, 65–67]. It was demonstrated that if conditions (1) are not satisfied, but losses (61)–(63) are introduced, then there is a case where the integral in the standard spectral expansion still converges (although poorly). The estimates for the diffraction spot radius given in Refs [65, 66] differ from those in (77) and (78). For example, the estimate obtained from condition (1) in Ref. [65] in the model of a two-dimensional field transmitted through a flat lens with a small aggregate deviation of media parameters (with losses taken into account) can be approximately written as

$$\rho_0 = a\left(\ln \frac{2}{\delta}\right)^{-1} \quad (79)$$

(see also [63] for real deviation values).

Figure 7 shows plots of functions (77)–(79). We can see that the presented results describe the tendency whereby the field spot vanishes for decreasing wave losses in the media in a

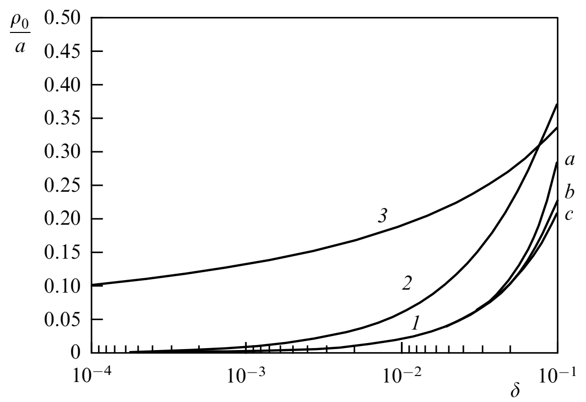

Figure 7. Effective transverse length (radius) of the field spot in the former focal point region depending on the parameter of losses: (*1*), according to (77) for (*a*) $ka = 10$, (*b*) 3.0, (*c*) 1.0; (*2*) and (*3*), according to respective expressions (78) and (79).

qualitatively similar, albeit quantitatively different, ways. For $\delta < 10^{-2}$, the values of ρ_0/a for dependence *3* differ from those for dependences *1* and *2* by orders of magnitude, while those for dependences *1* and *2* are approximately of the same order. Overall, it follows from the results presented in Fig. 7 that the size of the diffraction spot can be greater as well as less (and even much less) than the radiation wavelength.

The following should be added to what was stated above.

(1) The mathematical model of an elementary dipole field source that was used here adequately describes the divergent field of a physical (not point-like) source model only in the case where the radiated power that remains finite in the limit of a point-like source. The local field at the source point is then infinite, with a nonintegrable field energy density in the source domain [56–59]. The power flux density for the wave propagating from the source is expressed via the squared absolute value of the field Hertz vector near the source, or more precisely, the power flux density is proportional to this square (see, e.g., vol. 2 of Ref. [59]).

Everything said about the source field can also be applied to the divergent field of a sink. Therefore, in Refs [60, 68], as well as here, the effective field spot through which the power flux passes when the wave losses in the media are taken into account was estimated based on local values of the Hertz vector in the converging field region around the former focal point (the one calculated without losses).

(2) We also note that the authors of Refs [63, 65] related the size estimate in (79) for the field spot in the focal point region to the presence of surface waves excited by the source on the interface between positive and negative media. Such near-boundary surface waves can indeed be formed if complex values of the media parameter deviations from conditions (1) include real parts [62, 69, 70]. When only wave losses in the media are taken into account, such deviations are purely imaginary. In this case, the near-boundary surface waves are not formed, and we therefore disregard them here.

On the other hand, because the field spot size estimate in (79) indicates a significantly lesser degree of field localization at the focal point compared to estimates (77) and (78), the latter can also be more accurate in the presence of surface waves.

As concerns numerical results in Ref. [66] that showed that the spot size does not vanish as $\delta \to 0$, their invalidity can be explained as follows. In numerical calculations of the field in [66], the wave was actually assumed to propagate not through the entire plane normal to the z axis but only through its limited part (limited aperture), with the size of the order of $2a$. That resulted in a bound on the field and the diffraction broadening of the spot, complementary to the diffraction effect due to a limited field extent in this plane because of wave losses in the media. This complementary effect is independent of δ.

We stress in this regard that in the presence of wave losses in the media, the effective apertures are bounded for radiation transmitted through the planes $z = a$ and $z = b$; this leads to the appearance of a diffraction spot in the plane $z = 2a$ and its expansion in the plane $z = 2d$ (see Fig. 1). This problem was considered in detail in Ref. [60]. If the losses are absent in an infinite lens plate and conditions (1) are satisfied, then the wave diffraction is also absent, and therefore the waves are localized at the focal point.

When wave losses in the media are taken into account, the whole problem of radiation transmission through a flat lens,

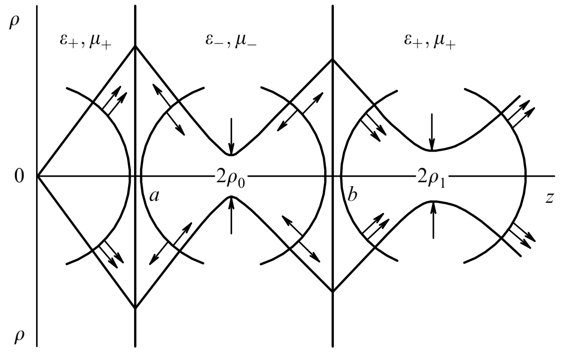

Figure 8. Structure of a quasi-spherical wave beam propagating through a plane isotropic lens with losses ($\rho_1 > \rho_0$). For coupled arrows at the beam wave front, the arrows beginning in the upper front points indicate the direction of wave phase motion while the arrows beginning in the lower points indicate the power flux (Umov–Poynting vector) direction.

which also includes the solution of the transmission problem for a convergent quasispherical backward wave through the region of the former focal point and then such a divergent wave transmission through the interface between negative and positive media, requires special consideration. This can be done using the following two-stage method for solving the problem.

First, from the above solution of the first key wave transmission problem (for a divergent spherical wave excited by a point source) through the first interface between the positive and negative media, we derive (by changing the notation) the solution of the similar second wave transmission problem for a divergent backward spherical wave also excited by a point-like source located in the plane $z = 2a$ in the negative medium and propagating through the interface between the negative and positive media. Next, this solution, regarded as the Green's function, can be used to solve (by the known method [56–59]) the wave emission problem for a source with distributed current density in the plane $z = 2a$. The current density in this plane can be obtained from the tangential field components calculated in the first problem. Such a method to solve the problem of wave transmission through a flat lens taking wave losses in the media into account is still waiting for its application, which is to involve numerical computations.

However, even now we can already qualitatively predict the structure of the wave field transmitted through the lens. It must be a Gaussian wave beam resembling a Gaussian beam [71, 72], damped along the direction of its propagation and having an increased spot size in the beam neck (Fig. 8).

4. Conclusion

There are still insufficiently studied and incorrectly understood problems, somehow or other related to the problem of field localization in a homogeneous medium in metamaterial applications. We mention the following three.

First, there is the question of how to realize metamaterials as negative (continuous) media. Previously, this question was raised in Ref. [31]. Even a 'simple' flat lens made of a homogeneous isotropic negative medium [1] has not been realized yet because the means to create such a medium has not been implemented, although 15 years have passed since the first publications related to the problem of the realization of isotropic negative media [24–28]. In these publications, the results of theoretical investigations were given for the model of a negative medium made of small chiral elements: chirally conducting dipole particles — artificial molecules [73–76]; by this example, the physical mechanism of possible realization of a negative medium has been demonstrated, in essence, for the first time. It was shown that such a medium can have simultaneous negative values of the dielectric permittivity and magnetic permeability in the resonance frequency range of dipole particles for sizes significantly less than the radiation wavelength. In that case, the medium structure, deterministic or random, is not essential.

The models of structured media published somewhat later [32, 33, 35, 37] are not directly related to the negative electromagnetic media [1, 12, 13]. An incorrect interpretation of experimental data given in these studies has caused only a great deal of confusion for further studies. The proper explanation of the effect of negative wave refraction at the boundary of structured media was given in theoretical and experimental studies [34, 36] based on the isofrequency method and theory developed previously (see [14–18]) and devoted to the investigation of artificial structured media for which the use of dielectric permittivity and/or magnetic permeability notions is not correct.

The second question is related to the incorrect wave interpretation presented in Ref. [61] for the physical process of radiation transmission through a homogeneous isotropic flat lens made of a negative medium [1]. This interpretation has had a detrimental effect on subsequent studies of the problem as well, although the suggestion given in [61] of the conceptual possibility of obtaining the resolution effect for behind-the-lens images of objects separated by small distances and small compared to the radiation wavelength, i.e., the effect of superresolution, turned out to be valid (see [64], the other references given above related to that problem, as well as the results presented in this paper). There is still an open question how to realize this effect technologically and to obtain a substantial superresolution.

The third question is related to the terminology used in the metamaterial theory. In various studies, various different terms are used for metamaterials as electromagnetic media: (1) left-handed media (not to be confused with left-handed chiral media: see [22, 23, 31] and Section 2.3 above); (2) negative media or double negative media, which is the same; (3) media with a negative frequency dispersion of parameters; and (4) backward-wave media. However, all these are actually the same type of metamaterials, which can be universally called the negative electromagnetic media. For metamaterials that are described by the model of a continuous isotropic electromagnetic medium and can be called by the above terms, this follows from papers [19, 21].

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References

1. Veselago V G *Usp. Fiz. Nauk* **92** 517 (1967) [*Sov. Phys. Usp.* **10** 509 (1968)]
2. Silin R A *Opt. Spektrosk.* **44** 189 (1978) [*Opt. Spectrosc.* **44** 109 (1978)]
3. Caloz C, Itoh T *Electromagnetic Metamaterials: Transmission Line Theory and Microwave Applications* (Hoboken, N.J.: John Wiley and Sons, 2006)
4. Sihvola A *Metamaterials* **1** 2 (2007)

5. Vinogradov A P, Nikitov S A *Radiotekh. Elektron.* **52** 1029 (2007) [*J. Commun. Technol. Electron.* **52** 951 (2007)]
6. Gulyaev Yu V, Lagar'kov A N, Nikitov S A *Vestn. Ross. Akad. Nauk* **78** 438 (2008) [*Herald Russ. Acad. Sci.* **78** 268 (2008)]
7. Marqués R, Martin F, Sorolla M *Metamaterials with Negative Parameters: Theory, Design, and Microwave Applications* (Hoboken, N.J.: Wiley-Intersci., 2008)
8. Solymar L, Shamonina E *Waves in Metamaterials* (Oxford: Oxford Univ. Press, 2009)
9. Mandel'shtam L I *Polnoe Sobranie Trudov* (Complete Collected Works) Vol. 2 (Ed. S M Rytov) (Moscow: Izd. AN SSSR, 1947) p. 334
10. Mandel'shtam L I *Polnoe Sobranie Trudov* (Complete Collected Works) Vol. 5 (Ed. S M Rytov) (Moscow: Izd. AN SSSR, 1950) p. 461
11. Mandel'shtam L I *Lektsii po Optike, Teorii Otnositel'nosti i Kvantovoi Mekhanike* (Lectures on Optics, Relativity Theory, and Quantum Mechanics) (Moscow: Nauka, 1972) pp. 392, 431
12. Sivukhin D V *Opt. Spektrosk.* **3** 308 (1957)
13. Pafomov V E *Zh. Eksp. Teor. Fiz.* **36** 1853 (1959) [*Sov. Phys. JETP* **9** 1321 (1959)]
14. Silin R A, Sazonov V P *Zamedlyayushchie Sistemy* (Decelerating Systems) (Moscow: Sov. Radio, 1966)
15. Silin R A *Neobychnye Zakony Prelomleniya i Otrazheniya* (Unusual Refraction and Reflection Laws) (Moscow: FAZIS, 1999)
16. Silin R A *Periodicheskie Volnovody* (Periodic Waveguides) (Moscow: FAZIS, 2002)
17. Silin R A *Radiotekh. Elektron.* **47** 186 (2002) [*J. Commun. Technol. Electron.* **47** 169 (2002)]
18. Silin R A *Usp. Fiz. Nauk* **176** 562 (2006) [*Phys. Usp.* **49** 542 (2006)]
19. Shevchenko V V *Radiotekh. Elektron.* **48** 1202 (2003) [*J. Commun. Technol. Electron.* **48** 1102 (2003)]
20. Byrdin V M *Radiotekh. Elektron.* **50** 1413 (2005) [*J. Commun. Technol. Electron.* **50** 1307 (2005)]
21. Shevchenko V V *Usp. Fiz. Nauk* **177** 301 (2007) [*Phys. Usp.* **50** 287 (2007)]
22. Lindell I V et al. *Electromagnetic Waves in Chiral and Bi-isotropic Media* (Boston: Artech House, 1994)
23. Shevchenko V V *Sorosovsk. Obrazovat. Zh.* (2) 109 (1998)
24. Kostin M V, Shevchenko V V, in *Proc. of 4th Intern. Conf. on Chiral, Bi-Isotropic and Bianisotropic Media* (Eds A Sihvola et al.) (State College: The Pennsylvania State Univ., 1995) p. 64
25. Kostin M V, Shevchenko V V, in *Advances in Complex Electromagnetic Materials* (NATO ASI Series, Partnership Sub-series 3, Vol. 28, Eds A Priou et al.) (Dordtecht: Kluwer Acad. Publ., 1997) p. 261
26. Kostin M V, Shevchenko V V, in *Proc. of Bianisotropics'97: Intern. Conf. and Workshop on Electromagnetics of Complex Media, Great Britain, 5–7 June 1997* (Eds W S Weiglhofer) (Glasgow, Scotland: The Department, 1997) p. 249
27. Kostin M V, Shevchenko V V *Radiotekh. Elektron.* **43** 921 (1998) [*J. Commun. Technol. Electron.* **43** 858 (1998)]
28. Kostin M V, Shevchenko V V, in *Bianisotropics'98: Proc. of 7th Intern. Conf. on Complex Media, Braunschweig, Germany, 1998* (Eds A F Jacob, J Reinert) (Braunschweig: Techn. Univ., 1998) p. 277
29. Vinogradov A P, Dorofeenko A V *Opt. Commun.* **256** 333 (2005)
30. Vinogradov A P, Dorofeenko A V, Zouhdi S *Usp. Fiz. Nauk* **178** 511 (2008) [*Phys. Usp.* **51** 485 (2008)]
31. Shevchenko V V *Radiotekh. Elektron.* **54** 696 (2009) [*J. Commun. Technol. Electron.* **54** 662 (2009)]
32. Pendry J B et al. *IEEE Trans. Microwave Theory Tech.* **47** 2075 (1999)
33. Smith D R et al. *Phys. Rev. Lett.* **84** 4184 (2000)
34. Silin R A, Chepurnykh I P *Radiotekh. Elektron.* **46** 1212 (2001) [*J. Comm. Techol. Electron.* **46** 1121 (2001)]
35. Shelby R A, Smith D R, Schultz S *Science* **292** 77 (2001)
36. Parkhomenko M P, Silin R A, Chepurnykh I P *Radiotekh. Elektron.* **49** 624 (2004) [*J. Comm. Techol. Electron.* **49** 585 (2004)]
37. Smith D R, Pendry J B, Wiltshire M C K *Science* **305** 788 (2004)
38. Lagarkov A N, Kissel V N *Phys. Rev. Lett.* **92** 077401 (2004)
39. Lagarkov A N, Kisel' V N *Dokl. Ross. Akad. Nauk* **394** 40 (2004) [*Dokl. Phys.* **49** 5 (2004)]
40. Kisel' V N, Lagar'kov A N *Radiotekh. Elektron.* **54** 531 (2009) [*J. Comm. Techol. Electron.* **54** 506 (2009)]
41. Lagarkov A N et al. *Usp. Fiz. Nauk* **179** 1018 (2009) [*Phys. Usp.* **52** 959 (2009)]
42. Shevchenko V V *Radiotekh. Elektron.* **49** 683 (2004) [*J. Comm. Techol. Electron.* **49** 639 (2004)]
43. Shevchenko V V *Radiotekh. Elektron.* **49** 1048 (2004) [*J. Comm. Techol. Electron.* **49** 981 (2004)]
44. Landau L D, Lifshitz E M, Pitaevskii L P *Elektrodinamika Sploshnykh Sred* (Electrodynamics of Continuous Media) (Moscow: Nauka, 1982) [Translated into English (Oxford: Pergamon Press, 1984)]
45. Fedorov F I, Filippov V V *Otrazhenie i Prelomlenie Sveta Prozrachnymi Kristallami* (Light Reflection and Refraction by Transparent Crystals) (Minsk: Nauka i Tekhnika, 1976)
46. Fedorov F I *Optika Anizotropnykh Sred* (Optics of Anisotropic Media) (Moscow: URSS, 2004)
47. Fedorov F I *Teoriya Girotropii* (Gyrotropy Theory) (Minsk: Nauka i Tekhnika, 1976)
48. Turov E A *Material'nye Uravneniya Elektrodinamiki* (Matter Equations in Electrodynamics) (Moscow: Nauka, 1983)
49. Holliger R, Varadan V V, Varadan V K *Radio Sci.* **26** 1150 (1991)
50. Kluskens M S, Newman E H *IEEE Trans. Microwave Theory Tech.* **39** 1889 (1991)
51. Shevchenko V V *Telecommun. Radio Eng.* **48** 120 (1993)
52. Shevchenko V V *Radiotekh.* (2) 80 (1994)
53. Shevchenko V V *Plavnye Perekhody v Otkrytykh Volnovodakh. Vvedenie v Teoriyu* (Continuous Transitions in Open Waveguides; Introduction to the Theory) (Moscow: Nauka, 1969) [Translated into English (Boulder, Colo.: Golem Press, 1971)]
54. Shevchenko V V "Metod spektral'nogo razlozheniya polei v teorii otkrytykh volnovodov" ("The method of spectral expansion of fields in the theory of open waveguides"), Doctor of Physical and Mathematical Sciences, Degree Thesis (Moscow: Institute for Radio Technology and Electronics, the USSR Academy of Sciences, 1976)
55. Shevchenko V V *Differents. Uravn.* **15** 2004 (1979) [*Different. Equat.* **15** 1431 (1980)]
56. Katsenelenbaum B Z *Vysokochastotnaya Elektrodinamika* (Moscow: Nauka, 1966); *High-frequency Electrodynamics* (Weinheim: Wiley-VCH, 2006)
57. Vaganov R B, Katsenelenbaum B Z *Osnovy Teorii Difraktsii* (Foundations of Diffraction Theory) (Moscow: Nauka, 1982)
58. Markov G T, Chaplin A F *Vozbuzhdenie Elektromagnitnykh Voln* (Excitation of Electromagnetic Waves) (Moscow – Leningrad: Energiya, 1967)
59. Felsen L B, Marcuvitz N *Radiation and Scattering of Waves* (Englewood Cliffs, N.J.: Prentice-Hall, 1973) [Translated into Russian (Moscow: Mir, 1978)]
60. Shevchenko V V *Radiotekh. Elektron.* **53** 1121 (2008) [*J. Commun. Technol. Electron.* **53** 1060 (2008)]
61. Pendry J B *Phys. Rev. Lett.* **85** 3966 (2000)
62. Shatrov A D *Radiotekh. Elektron.* **52** 909 (2007) [*J. Commun. Technol. Electron.* **52** 842 (2007)]
63. Shatrov A D *Radiotekh. Elektron.* **52** 1430 (2007) [*J. Commun. Technol. Electron.* **52** 1324 (2007)]
64. Shevchenko V V *Zh. Radioelektron.* (6) (2007); http://jre.cplire.ru/jre/jun07/5/text.html
65. Bankov S E *Elektromagnitnye Kristally* (Electromagnetic Crystals) (Moscow: Fizmatlit, 2010)
66. Petrin A B *Pis'ma Zh. Eksp. Teor. Fiz.* **87** 550 (2008) [*JETP Lett.* **87** 464 (2008)]
67. Petrin A B *Zh. Eksp. Teor. Fiz.* **134** 436 (2008) [*JETP* **107** 364 (2008)]
68. Grachev G G, Shevchenko V V *Radiotekh. Elektron.* **55** 432 (2010) [*J. Commun. Technol. Electron.* **55** 403 (2010)]
69. Shevchenko V V *Radiotekh. Elektron.* **50** 1363 (2005) [*J. Commun. Technol. Electron.* **50** 1260 (2005)]
70. Shevchenko V V *Radiotekh. Elektron.* **55** 1052 (2010) [*J. Commun. Technol. Electron.* **55** 986 (2010)]
71. Deschamps G A, Mast P E, in *Proc. of Symp. on Quasi-Optics, New York, USA, 1964* (Microwave Research Institute Sympo. Ser., Vol. 14, Ed. J Fox) (Brooklyn: Polytechnic Press, 1964) p. 177 [Translated into Russian: in *Kvazioptika* (Quasioptics) (Eds B Z Katsenelenbaum, V V Shevchenko) (Moscow: Mir, 1966) p. 189]

72. Marcuse D *Light Transmission Optics* (New York: Van Nostrand Reinhold, 1972) [Translated into Russian (Moscow: Mir, 1974)]
73. Shevchenko V V *Radiotekh. Elektron.* **40** 1777 (1995)
74. Shevchenko V V, in *Proc. of 4th Intern. Conf. on Chiral, Bi-Isotropic and Bianisotropic Media* (Eds A Sihvola et al.) (State College: The Pennsylvania State Univ., 1995) p. 42
75. Shevchenko V V *Radiotekh. Elektron.* **43** 1090 (1998) [*J. Commun. Technol. Electron.* **43** 1014 (1998)]
76. Shevchenko V V, in *Bianisotropics'98: Proc. of 7th Intern. Conf. on Complex Media, Braunschweig, Germany, 1998* (Eds A F Jacob, J Reinert) (Braunschweig: Techn. Univ., 1998) p. 279