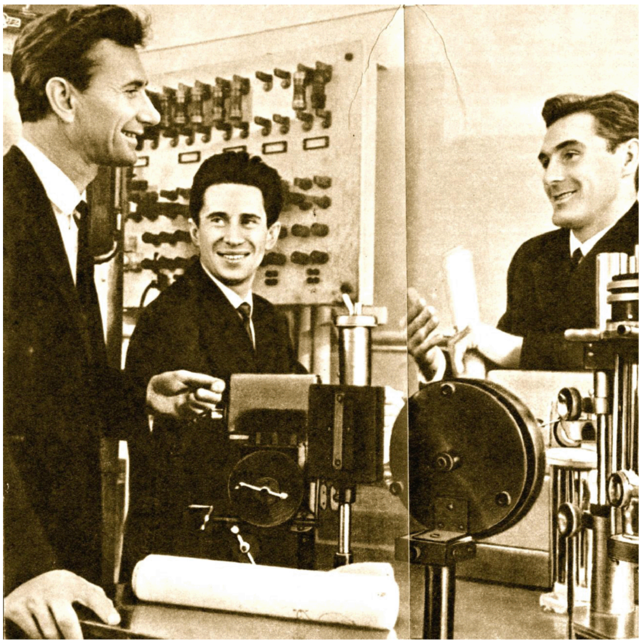

A P Shotov, Yu M Popov, and O N Krokhin (from left to right), 1964 Lenin Prize Laureates for fundamental studies which led to the creation of semiconductor lasers.

and development of semiconductor lasers by researchers working under the scientific leadership of N G Basov and Zh I Alferov at FIAN and LFTI, respectively.

The role of injection lasers in our lifetimes is well known. They are used in fiberoptic communications, laser printers, high-capacity memory on optical discs, numerous medical devices, and technological devices for laser processing of various materials. It should be expected that their importance will further increase in the near future, especially in the production of light sources and displays, including high-quality three-dimensional televisions.

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Self-focusing of laser pulses: current state and future prospects

A A Manenkov

1. Introduction

The self-focusing of laser beams in their propagation through nonlinear media is among the fundamental phenomena of nonlinear optics. At the heart of this phenomenon lies the variation of the spatial beam profile owing to the dependence of the refractive index of a medium on the radiation intensity. The character of this variation depends on the amplitude-temporal beam parameters and the optical properties of the medium. The significance of the self-focusing effect is underlain by its strong influence on the interaction of high-power laser radiation with optical media (ionization, damage) and on other nonlinear effects (stimulated scattering, harmonic generation, phase self-modulation, etc.). In relation to the discovery of the self-focusing of femtosecond (fs) laser beams in the air, considerable recent interest has been generated by the prospect of practical applications of this phenomenon (remote sensing of the atmosphere, control of electric discharges, etc.).

In connection with the foregoing, investigation of the mechanisms of self-focusing in different media and different frequency and pulse-duration ranges, as well as elucidation of adequate models of the phenomenon, are among the most important areas of laser physics and nonlinear optics.

The aim of this report is to outline the main results of investigations into the self-focusing effect obtained to date, and to analyze the prospects of further research. It should be emphasized that the self-focusing effect, since its prediction in 1962, has been the subject of a wealth of investigations, which have been discussed in numerous reviews, monographs, and other publications. Published in 2009, for instance, was a book [1] containing a vast collection of 24 chapters, which were written by well-known experts in this area, covering different theoretical and experimental aspects of the problem. In what follows we shall discuss only the main — in our view, fundamental — aspects of the problem.

2. History of self-focusing research: main stages

The following main stages may be distinguished in the development of investigations into the phenomenon of self-focusing of laser beams.

• Prediction of the effect, introduction of the term 'self-focusing', a qualitative analysis of self-channeling (diffraction-free beam propagation) (Askar'yan [2], 1962).

• First observations of self-focusing: discovery of filamentary damages in solids (Hercher [3], 1964), and self-focusing in liquids (Pilipetskii, Rustamov [4], 1965).

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• First detailed investigation, introduction of the notion of critical power, formulation of the theoretical model of self-channeling (Chiao, Garmire, Townes [5], and Talanov [6, 7], 1964).

• Further theoretical investigations:

— establishment of the main features of the phenomenon: beam collapse, determination of the base characteristics: critical power, the self-focusing length, phase self-modulation, and spatial instability (Kelley [8], Bespalov, Talanov [9], Akhmanov, Sukhorukov, Khokhlov [10, 11], Marburger et al. [12, 13]);

— formulation of multifocus structure models (Dyshko, Lugovoi, Prokhorov [14]) and moving nonlinear foci models (Lugovoi, Prokhorov [15]). Since these two models turned out to be the most adequate and theoretically substantiated, reliably confirmed by experiments (especially in the nanosecond range of pulse durations), and provide the basis for further self-focusing research in the range of ultrashort pulses, we shall enlarge on their analysis in Section 3.

• New stage: investigations into the self-focusing of ultrashort (femtosecond) pulses:

— the first observation of the self-focusing of femtosecond pulses in the air, discovery of the 'superlong' filaments of laser radiation and plasma formations (Braun et al. [16], 1995);

— further experimental investigations of the self-focusing of femtosecond pulses in the air: elucidation of the main characteristics of the filamentation phenomenon (length and diameter, energy, spectrum of the filaments, conical emission, supercontinuum) (Nibbering et al. [17], 1996; Brodeur et al. [18], 1997);

— theoretical investigations, analysis of different models of femtosecond pulse filamentation in the air (Chien et al. [19], 1968; Mlejnek et al. [20], 1998).

In recent years, investigations into the filamentation of femtosecond pulses in the air, other gases, and condensed media are the main avenue of inquiry into the self-focusing effect. Some of the findings of these investigations are discussed below.

3. Lugovoi–Prokhorov theory: multifocus structure model and moving nonlinear foci model

In this section we briefly set out the main findings of the theoretical research through which the models of multifocus structure (MFS) [14] and moving nonlinear foci (MNF) were developed [15].

Dyshko et al. [14] considered the propagation of a light beam with a Gaussian initial intensity profile through a medium with an inertialess Kerr nonlinearity of the refractive index. Proceeding from the numerical solution of the wave equation

$$\frac{\partial^2 E}{\partial r^2} + \frac{1}{r}\frac{\partial E}{\partial r} + 2ik\frac{\partial E}{\partial z} + n_2 k^2 |E|^2 E = 0\,, \tag{1}$$

where E is the electric field strength of the light beam, r and z are the radial and longitudinal coordinates (z coincides with the direction of beam propagation in the medium), k is the wave number, n_0 is the initial refractive index of the medium, and n_2 is the nonlinearity coefficient of the refractive index, namely

$$n = n_0 + n_2 |E|^2\,, \tag{2}$$

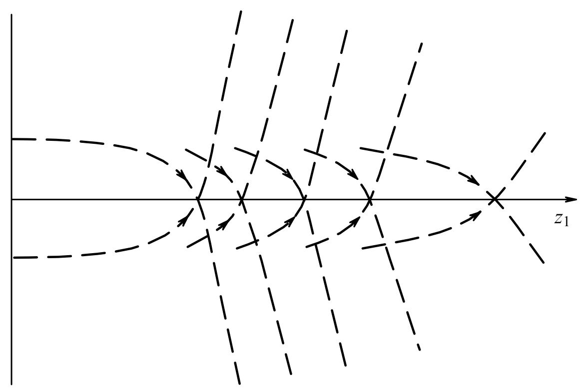

Figure 1. Schematic representation of a longitudinal section of a light beam propagating through a Kerr nonlinear medium for $P > P_{\rm cr}$. Here, $z_1 = z/l_{\rm sf}$, where $l_{\rm sf}$ is the self-focusing length.

they found that the *waveguide propagation* hypothesized in several earlier studies (see review Ref. [21] and references cited therein) did not take place when the incident beam power P exceeded some critical power $P_{\rm cr}^{(1)} = cn_0 N_1^2/8n_2k^2$, where c is the speed of light in a vacuum, and N_1 is the numerical coefficient equal to about 2. Instead, a *multifocus structure* formed. Figure 1 is a schematic representation of such a structure for a stationary light beam (P is time-independent). The mechanism of multifocus structure formation consists in the fact that the first focus in the paraxial beam region accumulates only a fraction of the initial beam power (this fraction is close to the critical power $P_{\rm cr}^{(1)}$). This fraction is partly absorbed in the nonlinear focus and partly diffracts (upon transit through the focus) at relatively large angles to the beam axis. In a similar way, a fraction of the remaining part of the light beam, which passed by the first focus, detaches to produce the second nonlinear focus, etc. Therefore, the sequential formation of nonlinear foci is seemingly recurring.

The main characteristics of the MFS model are the positions of nonlinear foci and the critical powers at which they are produced [21]:

$$\xi_{{\rm f}m} = \frac{\chi_m}{N_m}\,\frac{k\bar{a}_0^2}{\sqrt{P_0/P_{\rm cr}^{(m)}} - 1}\,, \tag{3}$$

$$P_{\rm cr}^{(m)} \approx m P_{\rm cr}^{(1)}\,, \tag{4}$$

where m is the number of a nonlinear focus, $P_{\rm cr}^{(m)}$ is the critical power of the mth focus, χ_m and N_m are numerical parameters, and $\bar{a}_0$ is the radius of the incident beam. The dimensions of the foci and their relative arrangement on the longitudinal coordinate may depend on additional physical effects (nonlinear absorption, ionization, etc.) which limit the light energy density in the nonlinear focal regions. Theoretical investigations of many of these effects have revealed, however, that the multifocus structure persists, i.e., that the MFS model is universal and may be observed under different physical conditions [21].

It is evident that nonlinear foci positions for pulsed beams with a smooth power variation in time will vary in accordance with relation (3), in which $\xi_{{\rm f}m}$ and P_0 are now functions of time: $\xi_{{\rm f}m} = \xi_{{\rm f}m}(t)$, $P_0 = P_0(t)$. Hence, it follows that the case of nonstationary light beams will see the realization of the moving nonlinear foci model. The total number of foci in this structure at the point in time t is defined by the condition

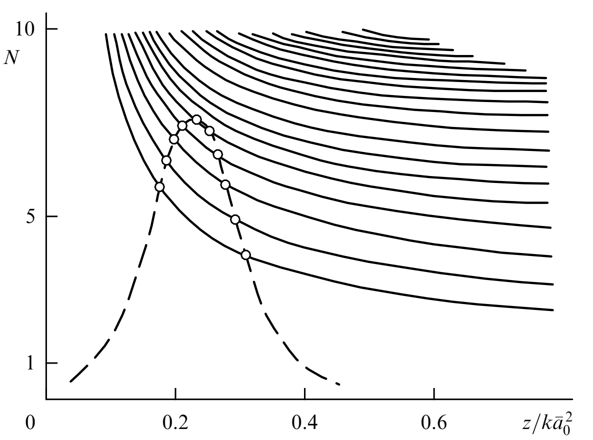

Figure 2. Positions of nonlinear foci on the $z/k\bar{a}_0$-axis (solid curves) and the shape of incident radiation pulse (dashed curve), which illustrate the formation of the structure of moving nonlinear foci in a Kerr nonlinear medium.

$P_0(t) > P_{cr}^{(m)}$. Figure 2 illustrates the production of the multifocus structure of moving nonlinear foci in a Kerr medium. The positions of nonlinear foci on the beam axis (solid curves) and the shape $N(t, z)$ of the incident light pulse (dashed curve) are depicted on the $(z/k\bar{a}_0^2, N)$ coordinates. The pulse shape $N(t)$ is given by the expression [21]

$$N(t, z) = \frac{1}{E_{cr}} \left| E_0 \left(t - \frac{k\bar{a}_0^2}{v} \frac{z}{k\bar{a}_0^2} \right) \right| , \qquad (5)$$

where E_{cr} is the critical field strength corresponding to the critical self-focusing power P_{cr}, and v is the speed of light in the medium. The quantities $z/k\bar{a}_0^2$ corresponding to the intersection of solid and dashed curves define the position of nonlinear foci on the beam axis at the time t. The model of moving nonlinear foci was first proposed by Lugovoi and Prokhorov in 1968 [15]. Subsequently, a comprehensive investigation was made of its characteristics (including the structure and velocity of movement of nonlinear foci in the case of ultrashort laser pulses) (see review [21]).

The multifocus structure model and moving nonlinear foci model were reliably borne out in purposeful experimental investigations of self-focusing in different media employing different approaches and recording techniques [22–24].

Loy and Shen [22] investigated the self-focusing of a ruby laser beam with a pulse length of 8 ns in toluene and CS_2. As they changed the power of incident radiation, which exceeded the critical power P_{cr}, they observed a beam evolution inside and at the output end of the cell with the liquid being studied. An analysis of observed data was clearly indicative of the motion of nonlinear foci (a detailed investigation was made of the behavior of the first nonlinear focus).

Korobkin et al. [23] studied the self-focusing of the beam of a ruby laser with a 15-ns-long pulse in nitrobenzene and CS_2 to observe temporal beam evolution in the medium. In this case, they employed the techniques of recording light intensity with a high temporal resolution. For high values of incident radiation power exceeding the critical one, the authors observed the moving structure of nonlinear foci in the liquids under investigation.

The investigations reported in Refs [22, 23] were conducted using Q-switched lasers which generated conventional bell-shaped temporal pulses. This corresponded to the case of *nonstationary* self-focusing.

Lipatov et al. [24] investigated the self-focusing of a ruby laser beam with a variable temporal pulse shape (bell-shaped, saw-tooth, and rectangular) in TF-105 glass. This approach made it possible to investigate the character of self-focusing both in the *nonstationary* (the first two pulse shapes) and in the *stationary* (rectangular pulses) modes. Laser-induced damage was observed in the samples, and a study was conducted of its morphology in relation to the pulse shapes. The data of this research were unambiguously interpreted in the framework of the moving nonlinear foci and stationary multifocus structure models. It is pertinent to note that the approach involving the employment of laser pulses with a varied temporal shape in self-focusing research was first proposed and implemented in Ref. [24]. The conceptual development of this approach is discussed in the analysis of the prospects of further research into the self-focusing of ultrashort laser pulses (Section 5).

4. Self-focusing of femtosecond laser pulses

In this section we outline the main findings that have emerged from experimental and theoretical research into the self-focusing of laser beams with a femtosecond pulse duration.

4.1 Experimental data

As noted in Section 2, the self-focusing of laser beams with femtosecond-long pulses was initially observed and comprehensively studied in the air, and subsequently in other gases and condensed media. As for radiation sources, in experimental research advantage was primarily taken of Ti:Al_2O_3 crystal-based laser systems (quite frequently referred to as titanium–sapphire lasers), which lase in the $\sim$ 800-nm wavelength range and produce pulses several tens to hundreds of femtoseconds long.

The typical characteristics of the filaments observed in such experiments in the air are as follows (see Ref. [25] and references cited therein): critical power $P_{cr} \approx 3$ GW, filament length (for $P_0 \leqslant 10\,P_{cr}$) ≈ 10 m, filament diameter ≈ 100 μm, the energy fraction in the filament amounts to 6–10% of the total beam energy, multiple filamentation for $P_0 \geqslant 10\,P_{cr}$, filament lengths for $P_0 \gg P_{cr}$ may range up to 2 km, the filament emission spectrum experiences radical changes: a strong broadening (a supercontinuum in the range from 230 nm to 4 μm) and a conical emission are observed (Fig. 3).

The typical characteristics of the filaments observed in condensed media (crystals, glasses, liquids) are as follows (see Ref. [25] and references cited therein): the critical power P_{cr} is

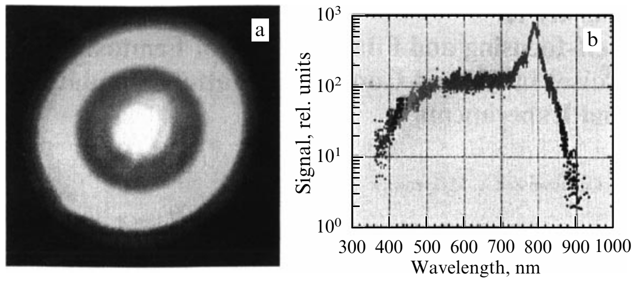

Figure 3. (a) Conical radiation accompanying the self-focusing effect in the air: the central white spot (filament) is colored with Newton rings with a divergence of ≈ 1 mrad. (b) Spectrum of a laser beam ($\lambda_0 = 800$ nm, $\Delta t = 70$ fs, $P = 3$ TW) after propagation through a distance of more than 10 m in the air. (The picture was borrowed from Ref. [25].)

by an order of magnitude lower than in gases, the filament lengths are on the order of several centimeters, and the filament diameter is about 2 μm. The diameter of a filament becomes smaller and its energy rises in the propagation of a beam through an amplifying medium (Ti:Al_2O_3 crystals, etc.).

4.2 Results of theoretical research

A large number of theoretical studies on the self-focusing of femtosecond pulses in the air have been published to date (see Refs [17–20, 25–28] and references cited therein).

All of the studies rely on the numerical solution of the wave equation with due regard for Kerr nonlinearity (responsible for self-focusing) and plasma (responsible for defocusing) produced owing to multiphoton absorption in the air. The typical system of equations used in such investigations (see, for instance, Ref. [25]) is written out as

$$
\begin{aligned}
&2\mathrm{i}\,\frac{\partial E}{\partial z}+\frac{1}{k_0}\,\Delta_{\perp}E-k''\,\frac{\partial^2 E}{\partial t^2}\\
&\quad+k_0 n_2\left(|E|^2+\tau_{\mathrm{K}}^{-1}\int_{-\infty}^{t}\exp\left(-\frac{t-t'}{\tau_{\mathrm{K}}}\right)|E(t')|^2\,\mathrm{d}t'\right)E\\
&\quad-k_0\,\frac{\omega_{\mathrm{pe}}^2(\rho)}{\omega_0^2}\,E+\mathrm{i}\beta^{(K)}|E|^{2K-2}E=0\,, \qquad (6)
\end{aligned}
$$

$$
\frac{\partial\rho}{\partial\tau}=\frac{\beta^{(K)}}{K\hbar\omega_0}\,|E|^{2K}\left(1-\frac{\rho}{\rho_{\mathrm{at}}}\right). \qquad (7)
$$

The first two terms in equation (6) describe the beam propagation through the medium with the inclusion of diffraction, and the third term with the inclusion of group velocity dispersion; the third and fourth terms account for the Kerr nonlinearity of the medium (the fourth term for inertialess nonlinearity, and the fifth one for the retarded part of the nonlinearity with the time characteristics τ_{K}), and the sixth and seventh terms take into account the production of plasma with a density ρ and multiphoton absorption with a probability $\beta^{(K)}$ [here, the superscript (K) indicates the number of photons in the multiphoton process].

Equation (7) describes the kinetics of the plasma produced owing to multiphoton ionization (ρ is the plasma density, and ρ_{at} is the density of neutral atoms).

The data of numerical simulations were interpreted by different authors on the basis of various models: dynamic spatial replenishment [20], moving foci and refocusing, and slice-by-slice self-focusing [18, 27].

An analysis [29] of these models revealed that they are *inherently* similar to the multifocus structure model and moving nonlinear foci model, and that they are only terminologically different from them. Indeed, the terminology employed by Mlejnek et al. [20] (for instance, 'self-guided light strings') may be misleading and create the illusion of some new concept of nonlinear light propagation. In reality, this is nothing more than another name for the trajectories of the moving nonlinear foci of the multifocus structure (involving plasma production). Similarly, there is nothing new about 'dynamic spatial replenishment' — this is just the mechanism of sequential formation of the nonlinear foci of the multifocus structure described in Section 3. The same is true of the 'slice-by-slice self-focusing' model [27], which is inherently the reformulated moving nonlinear foci model.

In an analysis of the presently available papers concerned with the self-focusing of femtosecond laser beams in the air, several of their drawbacks were noted [29], which made difficult an adequate comparison of numerical simulation results with experimental data. In particular, it was noted that experimental data were not informative enough for gaining a complete understanding of the mechanisms and processes involved in laser beam filamentation and plasma production. Specifically, the observations of filaments were time- and space-integrated, with a resolution insufficiently high to reveal the filament structure. As regards numerical simulations, it was noted, in particular, that they did not take into full account the group velocity dispersion (neglect of the contribution from the plasma component) and inadequately interpreted the retarded Kerr nonlinearity term (it was groundlessly ascribed to stimulated Raman scattering).

The imperfections of experimental works and theoretical calculations mentioned above, as well as the complexity of taking into account the strong variation of the laser radiation spectrum in the course of filamentation and the dissimilarity of the processes occurring at the leading and trailing edges of a laser pulse, call for further investigation into the self-focusing of femtosecond laser beams in different media and invite the use of new approaches. One such approach proposed in Ref. [30] — the employment of laser pulses with a varied temporal shape — will be discussed in Section 5.

5. Variation of the temporal shape of laser pulses — a promising approach to the investigation of self-focusing of ultrashort laser pulses

The conception of this approach relies on the dependence of the spatial and spectral self-focusing characteristics on the temporal pulse shape $P_0(t)$, predicted by the theory of self-focusing (the moving nonlinear foci model). For media with the Kerr nonlinearity of the refractive index, in particular, the velocity of nonlinear foci movement on the beam propagation axis is defined by the derivative

$$
V=\frac{\mathrm{d}\xi}{\mathrm{d}t}\,, \qquad (8)
$$

which is a function of the pulse shape $P_0(t)$ [see formula (3)]. The spectrum broadening due to phase self-modulation also depends on the pulse shape:

$$
\Delta\omega=-\frac{\mathrm{d}\varphi_{\mathrm{nl}}}{\mathrm{d}t}=-\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\omega}{c}\,\frac{n_2 I(t)}{L}\right). \qquad (9)
$$

Here, φ_{nl} is the radiation phase change arising from the nonlinear variation of the refractive index $n_2 I(t)$ over the propagation length L, and $I(t)$ is the intensity.

The temporal shapes of laser pulses that are of interest in the investigation of spatial and spectral characteristics of self-focusing are illustrated in Fig. 4.

The following effects are expected in self-focusing phenomenon, when the indicated pulse shapes are employed:

— for a rectangular pulse ($P_0=\mathrm{const}$): $\xi_m=\mathrm{const}$, $\mathrm{d}\xi_m/\mathrm{d}t=0$, $\Delta\omega=0$ — a stationary nonlinear foci structure (no filamentation) and the absence of spectrum broadening should be observable;

— for all other pulse shapes: $\mathrm{d}\xi_m/\mathrm{d}t\neq 0$ — a moving nonlinear foci structure and spectrum broadening whose character depends on the specific shape $P_0(t)$ should be observable.

The foregoing brief analysis shows the promise of the proposed conception for further investigations into self-

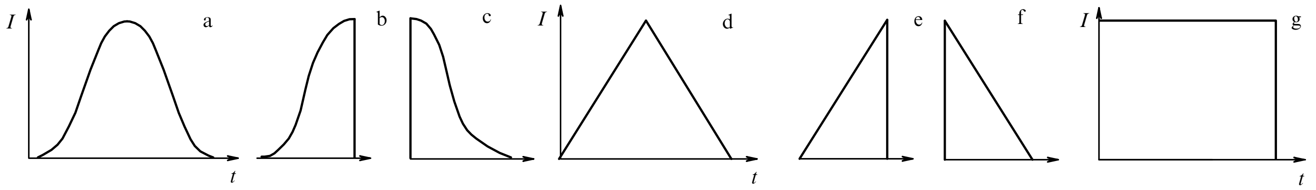

Figure 4. Temporal shapes of laser pulses: (a) symmetric bell-shaped, (b, c) asymmetric (truncated) bell-shaped, (d) symmetric triangular, (e, f) asymmetric (truncated) triangular, and (g) rectangular.

focusing and its related laser–matter interaction effects (ionization, damage, etc.) in the area of ultrashort pulse durations. To implement this approach requires solving several practical problems, namely, developing:

— methods for generating laser pulses with a prescribed temporal shape;

— methods for recording laser pulses with a high (femtosecond) temporal resolution;

— efficient techniques for recording self-focusing processes and several related effects with a high temporal resolution.

One of the promising methods for producing ultrashort laser pulses with a prescribed temporal shape — the use of volume Bragg diffraction gratings — was proposed at the A M Prokhorov General Physics Institute of the RAS. This method is presently being developed in the RAS GPI in collaboration with OptiGrate (USA).

6. Conclusions

The main results of investigations into the self-focusing effect of laser beams, performed to date, may be summarized as follows.

• Experimental research (especially comprehensive investigations in the nanosecond range of laser pulse durations) has revealed diverse effects caused by self-focusing in optical media of different kinds, which testifies to the universal nature of the phenomenon.

• Theoretical investigations have resulted in the determination of the main features and characteristics of self-focusing phenomenon common to different optical media. Different self-focusing models have been proposed; the most appropriate and best substantiated of them are the multifocus structure (MFS) and moving nonlinear foci (MNF) models.

• The MFS and MNF models have been reliably borne out by experimental data.

• The phenomenon of the self-focusing of ultrashort (femtosecond) pulses was discovered and has been comprehensively investigated in the air and other gases and in condensed media, and the features of the effect have been determined: the formation of long thin light and plasma filaments and a dramatic spectrum transformation (supercontinuum and conical emission).

• Proceeding from the data of theoretical research (numerical simulations), different ultrashort-pulse filamentation models have been proposed. An analysis of these models suggests that they correspond in essence to the MFS and MNF models, the difference being merely terminological.

• A more comprehensive explanation of the features of ultrashort-pulse self-focusing invites further experimental and theoretical investigations. A method involving laser pulses of a varied temporal shape has been proposed as a promising experimental research method.

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