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V A Fock and gauge symmetry †

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Contents

Abstract. V A Fock, in 1926, was the first to have the idea of an Abelian gradient transformation and to discover that the electromagnetic interaction of charged particles has a gradient invariance in the framework of quantum mechanics. These transformation and invariance were respectively named Eichtransformation and Eichinvarianz by H Weyl in 1929 (the German verb *zu eichen* means *to gauge*). The first non-Abelian gauge theory was suggested by O Klein in 1938; and in 1954, C N Yang and R L Mills rediscovered the non-Abelian gauge symmetry. Gauge invariance is the underlying principle of the current Standard Model of strong and electroweak interactions.

1. From H Weyl to V A Fock

The term Eichinvarianz was coined by H Weyl [2] in 1919 in the framework of his unsuccessful attempt to construct a unified theory of gravitational and electromagnetic interactions. In the context of Ref. [2], Eichinvarianz was equivalent to Masstabinvarianz (scale invariance). The construction in Ref. [2] was purely classical, without any elements of quantum mechanics, which did not exist at that time.

A few years later, Th Kaluza made an attempt to construct a unified classical theory of electromagnetic and gravitational interactions by invoking a fifth dimension [3].

In 1925, L de Broglie introduced the notion of wavelength for particles of matter [4]. In 1926, E Schrödinger proposed his famous nonrelativistic quantum mechanical wave equation [5]. A few months later, O Klein [6], trying to establish a relation between quantum theory and the five-dimensional relativity theory of Kaluza, suggested the relativistically invariant generalization of the Schrödinger equation.

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Independently, the same generalized Schrödinger equation was proposed by V A Fock [7]. (In a note added in the proof, Fock wrote that while his paper was in press, 'a beautiful work' by O Klein appeared.) It is interesting that Fock also used five dimensions to write his equation. Later in 1926, the same equation was published by W Gordon [8]. Thus the Klein–Fock–Gordon equation appeared, which described scalar particles and their interaction with an electromagnetic field in a relativistic invariant form.

The new transformation introduced by Fock and later named 'gradient transformation' by him is given by Eqn (5) of his paper [7]:

$$A = A_1 + \operatorname{grad} f,$$

$$\phi = \phi_1 - \frac{1}{c}\frac{\partial f}{\partial t},$$

$$p = p_1 - \frac{e}{c} f,$$

where f is an arbitrary function of space–time coordinates, while the role of the additional coordinate parameter p "is precisely to insure the invariance of the equations under the addition of an arbitrary gradient to the four-potential", $\exp(\mathrm{i}p/\hbar)$ being the phase factor of the charged scalar field. It is the third equation which is characteristic of quantum theory. For classical electrodynamics, it is sufficient to have the first two equations, which transform the vector and scalar parts of the electromagnetic potential [the invariance with respect to them being guaranteed by the electromagnetic current conservation (see, e.g., Refs [9, 10])]. We attach the adjective 'classical' to gauge transformations and invariance involving only the first two equations.

† Paper [1] prepared for celebration of the 100th anniversary of V A Fock's birthday was published in English in a dedicated volume *Quantum Theory, in honour of Vladimir A. Fock*' (Proceedings of the VIII UNESCO International School of Physics, St. Petersburg, 25 May – 6 June 1998) Part 2 (Eds Yu Novozhilov, V Novozhilov) (St. Petersburg: Univ. Euro-Asian Phys. Soc., 1998) pp. 13 – 17. The text of the paper has been partly revised by the author for this publication and is published here by the kind permission of UNESCO.

2. From V A Fock to H Weyl

The phase factor $\exp(\mathrm{i}ef/c\hbar)$, introduced by V Fock, was rediscovered in the same journal, three volumes later, by F London [11] (without any reference to V Fock; there is a reference to O Klein [6] and V Fock [7] in a short note [12]). F London noticed that without i in the exponent, the phase factor becomes a scale factor, similar to the Eichfactor of Weyl's 1919 theory [2].

The same statement was repeated by Weyl in 1928 in his book [13] and in 1929 in article [14] (without any reference to Fock or London). Weyl went even further and gave the old name *Eichinvarianz* to the new invariance discovered by Fock.

In Ref. [7], Fock did not use any name for the new invariance he discovered. The earliest use of the term gradient invariance that I found, was in the first Russian edition (1941) of the book *Classical Field Theory* by L Landau and E Lifshitz [15], where they attributed this term to Fock. The title of § 16 was "Gradient Invariance." In the text of this section, the German and English terms were also given: *Eichinvarianz* and *gauge invariance*. Fock himself used the term gradient invariance in 1950 in Ref. [16]. (According to Yu V Novozhilov, that was the earliest reference he could find.) Meanwhile, in 1929, Fock [17] used the term *Eichinvarianz* with reference to [13] and [7].

In his book [13] and in the introduction to his paper [14], Weyl proclaimed Eichinvarianz as one of the basic principles of his new unified theory of gravity, electricity, and matter (an aim which is still in front of us seventy years later). It is interesting that in his beautiful popular book *Symmetry* [18], where he describes various types of symmetries, Weyl did not mention gauge invariance.

The physical meaning of gauge invariance in quantum electrodynamics (QED) does not seem to be extremely profound in and of itself. The tiny mass of the photon would destroy the gauge invariance of QED without changing its excellent agreement with experimental facts and, in particular, its renormalizability. (According to [19], the nonvanishing photon mass does not violate gauge invariance, but in this case the function f has to be very special.) On the other hand, the renormalizability would be destroyed by an anomalous magnetic moment term in the Lagrangian, $\mu\bar{\psi}\sigma_{\mu\nu}\psi F_{\mu\nu}$, in spite of its manifest gauge invariance. What is really fundamental in electrodynamics is the conservation of electromagnetic current (see [20, 21]) or, in other words, conservation of charge. Due to the charge conservation, the effects caused by a nonvanishing mass of the photon m_γ are proportional to m_γ^2 and therefore negligibly small for small enough values of m_γ. On the other hand, small or vanishing m_γ guarantees the electric charge conservation [22–24].

The gauge symmetry of electrodynamics allows considering A_μ with various gauge conditions:

$$\partial_i A_i = 0 \quad (i = 1, 2, 3) \text{ — the Coulomb gauge},$$

$$\partial_\mu A_\mu = 0 \quad (\mu = 0, 1, 2, 3) \text{ — the Lorenz gauge},$$

$$A_0 = 0 \text{ — the Hamilton gauge},$$

$$n_\mu A_\mu = 0 \quad (n^2 = 0) \text{ — the light-cone gauge, etc.}$$

An adequate choice of the gauge simplifies the calculations.

In quantum electrodynamics, the gauge degree of freedom manifests itself in the expression for the propagator of a virtual photon with a 4-momentum k:

$$D_{\mu\nu}(k) = -\frac{1}{k^2}\left[g_{\mu\nu} + (\alpha - 1)\,\frac{k_\mu k_\nu}{k^2}\right].$$

The most frequently used cases are:
$\alpha = 1$ — the Feynman gauge,
$\alpha = 0$ — the Landau gauge.

3. From L V Lorenz to H A Lorentz

Looking at the three gauge equations, it is natural to ask yourself when the first two of them were written? As I did not find the answer to this question in various textbooks and monographs, I sent an e-mail to John David Jackson an expert in classical electrodynamics. After a few days of research in the Berkeley libraries, he sent me a reference to H A Lorentz [25]. He also added that the first equation was already known to J C Maxwell, who also used the Coulomb gauge [26], while the Lorenz gauge was introduced in 1867 by L V Lorenz. Hence, various gauges were used long before the full gauge invariance was discovered, although it is clear that both Lorenz and Maxwell understood the physical equivalence of different forms of the vector potential.

4. From the 1920s to the 1990s

The Abelian gauge invariance of QED served as a testing ground for the non-Abelian gauge symmetries, which became the dynamical basis for the present-day Standard Model of the electroweak and strong interactions. The first attempt at such a model was published by O Klein in 1938 [27]. It contained isotopic doublets of a proton and neutron and of a neutrino and electron. It had four gauge fields, or in other words, four gauge bosons: a photon, W^+, W^-, and Z^0 (in modern notation). It contained nonlinear interactions of these fields. Unfortunately, it had only one coupling constant, α, which was too small for strong interactions. O Klein never returned to this model. It was fully forgotten, when in 1954 C N Yang and R L Mills wrote their seminal paper [29], in which they coupled the gauge and isotopic symmetries. [1]

In the 1960s the broken non-Abelian gauge symmetry was used to unify the electromagnetic and weak interactions, and in the 1970s QCD appeared, as did various models of grand unification. In the 1980s, W-bosons, Z-bosons, and gluons were discovered. They were thoroughly studied in the 1990s. But that development has been described in many textbooks and lies beyond the scope of this short note.

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Appendix. August 2010

I would like to make some additions to the translation of my article [1], and to comment on some of the publications concerning the early history of gauge invariance and Fock's role in its creation.

[1] Paper [27] has been reprinted in Refs [21] and [28].

(1) Appendices to the lectures "Introduction to gauge theories" at the JINR–CERN school [30] show photocopies of and selected excerpts from papers by V Fock, F London, O Klein, and H Weil published between 1919 and 1938. In 1986, these lectures and the appendices to them were reproduced in a separate publication [21].

(2) The background of gauge ideas from Ampère and Faraday to Fock and Klein was discussed in the review "Historical roots of gauge theory," in 2001 [31].

(3) In order to understand Fock's paper [7], it is very important to have knowledge of his preceding paper [32] — something that I unfortunately failed to mention in the main body of my paper [1]. Paper [32] proposed the method used in Ref. [7].

(4) In a footnote to the title of his paper [7], V A Fock wrote: "...the idea of this work originated in a conversation with Prof. V. Frederiks, and I am also much obliged to him for a number of valuable suggestions." Perhaps not every reader knows about Vsevolod Konstantinovich Frederiks, so I would like to recommend a book about him [33], as well as his articles published in *Physics–Uspekhi* [34, 35].

(5) Fock's paper [7] was tranlated into English and published in book [36]. It is dated 1927 both in the table of contents of the book and in the subtitle of the article, while in fact Fock's paper was in reality published in 1926. This misprint is possibly the explanation of the mistaken evaluation of the role played by Fock in the discovery of gauge symmetry that the reader finds both in [36] and in the publications by other authors based on this book.

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