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Nonlinear dynamics of the brain: emotion and cognition

M I Rabinovich, M K Muezzinoglu

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Abstract. Experimental investigations of neural system functioning and brain activity are standardly based on the assumption that perceptions, emotions, and cognitive functions can be understood by analyzing steady-state neural processes and static tomographic snapshots. The new approaches discussed in this review are based on the analysis of transient processes and metastable states. Transient dynamics is characterized by two basic properties, structural stability and information sensitivity. The ideas and methods that we discuss provide an explanation for the occurrence of and successive transitions between metastable states observed in experiments, and offer new approaches to behavior analysis. Models of the emotional and cognitive functions of the brain are suggested. The mathematical object that represents the observed transient brain processes in the phase space of the model is a structurally stable heteroclinic channel. The possibility of using the suggested models to construct a quantitative theory of some emotional and cognitive functions is illustrated.

M I Rabinovich, M K Muezzinoglu Institute for Nonlinear Science, University of California, San Diego, 9500 Gilman Drive, La Jolla, CA 92093-0402, USA Tel. +1-858-534-6753. Fax +1-858-534-7664 E-mail: mrabinovich@ucsd.edu, mrabinovich@gmail.com

Received 26 June 2009, revised 4 August 2009 Uspekhi Fizicheskikh Nauk **180** (4) 371–387 (2010) DOI: 10.3367/UFNr.0180.201004b.0371 Translated by M I Rabinovich; edited by M S Aksent'eva "It is a great pleasure for us to think about how we think." Anonymus

1. Introduction: what we discuss and why now

1.1 Dynamical modeling of cognition

The tradition of understanding thought based on dynamical systems theory has its roots in the cybernetics era of the 1940s [1]. However, with the dominance of symbolic artificial intellect and 'information-processing psychology' and the absence of a good experimental technology in the 1960s and 1970s, dynamical-systems-based approaches were not extensively pursued. More recently, the idea that dynamics is a relevant framework for understanding cognition has become popular again. For example, [2] describes the development of kicking and reaching in infants in terms of dynamical notions such as the stability of attractors in a phase space that is defined by the body and environmental parameters. Movements to new stages in development are explained in terms of bifurcations to new attractors as a result of the change in order parameters, e.g., infant weight or body length, as the infant grows. Thelen and Smith believe that 'higher cognition' is ultimately rooted in these types of spatial skills learned in infancy, and thus that higher cognition will itself be best understood dynamically. They contrast their account with traditional 'information processing' theories of development, in which new developmental stages are caused by brain maturation and the increasing ability of maturing infants to reason logically.

The study [3] formulates the general idea of continuity, namely that cognition should be characterized as a continual coupling among brain, body, and environment that unfolds in real time, as opposed to the discrete time steps of the artificial intellect. This is said to contrast with computations that focus on 'internal structure,' i.e., its concern with the static organization of information processing and representational structure in a cognitive system. A dynamical approach means that the organization of brain structures is also dynamical and functional, i.e., not only anatomical. Thus, the dynamical approach to cognition is a confederation of research efforts bound together by the idea that natural cognition is a dynamical phenomenon and best understood in dynamical terms. This contrasts with the 'law of qualitative structure' [4] governing orthodox or 'classical' cognitive science, which holds that cognition is a form of digital computation.

The temporal characteristics of information flow in the brain depend on both the neural network architecture and the type of connections. In particular, inhibitory connections supported by interneurons are responsible for the spatio-temporal transient activity. On the other hand, excitatory cell assemblies, and associated connections, ensure that information 'goes to the right place at the right time' [5]. One can say that any coarse-grain cognitive pattern (mode or representation) observed in experiments can inhibit alternative activity patterns among workspace neurons [6]. At the same time, these modes *sequentially* inhibit each other, because only one representation is possible at any given time. As a result, mental activities, i.e., cognition, emotion, and consciousness, arise in the form of *sequences of representations*.

A dynamical system for the modeling of emotion and cognition and their interactions is a set of quantitative variables changing continually, concurrently, and interdependently over quantitative time in accordance with dynamical principles, which are embodied in a set of differential equations. Dynamics in this sense includes a very successful experience of dynamical modeling that scientists use to understand natural phenomena via nonlinear dynamical models. This experience includes a set of concepts, proofs, and tools for understanding the behavior of systems in general. An important insight of dynamical systems theory is that behavior can be understood geometrically in some projection of the state (phase) space. The behavior can then be described in terms of attractors, transients, stability, coupling, bifurcations, chaos, etc.

Although classical cognitive science has interpreted cognition in principle as something that happens over time, the dynamical approach sees cognition as being in time, i.e., as an essentially temporal phenomenon. For example, when a dynamical model of the information (sensory) coding is created, time is included in the coding space [7]. Details of timing (durations, rates, synchronies, etc.) matter [8].

1.2 Brain imaging

Recent work in brain imaging has revealed many fundamental properties, and, in particular, the functional organization of brain systems. Most of these results have been expressed in the form of averaged-in-time spatial patterns indicating the brain areas that are simultaneously activated in various emotional and cognitive states. These findings create an impression that we can get a clear fingerprint, i.e., a portrait of the specific emotion or the execution of the specific cognitive function. However, neither emotions nor cognition are frozen functional patterns. The underlying neuro-dynamics, i.e., the temporal evolution of emotion and cognition and their reciprocal link, can be extracted only by detection of the sequential brain activity in representing and translating a sequence [9].

Some brain imaging methods of today provide very high temporal resolution. For example, Positron Emission Tomo-

graphy (PET) is characterized by low time resolution (about 10 s [10]). High temporal resolution is characterized by electro-encephalography (EEG), whose resolution is about 2 ms [11]. Unfortunately, the EEG method has a very blurry spatial resolution.

Blood-oxygen-level dependent (BOLD) imaging, a Magnetic Resonance Imaging (MRI) technique, can provide whole brain coverage approximating a one-second temporal and one-millimeter spatial resolution, hence making it a strong candidate for the application of nonlinear dynamical systems theory to emotion, cognition, and their interaction in the brain [12].

1.3 Dynamics of emotions

In 1872 Charles Darwin published the book Expression of the Emotion in Man and Animals. He formulated there the idea that the evolution principle is applicable to the emotionalbehavioral development of animals, but not only to their biophysiological development. There is a clear evolutionary connection between the behavior of humans and animals. The important theoretical point here is that, for Darwin, similar bodily expressions indicate the same state of mind in both animals and humans. Clinical reports of congenitally blind children also served as apparent corroboration of this position: "The inheritance of most of our expressive actions explains the fact that those born blind display them both with man and animals, express the same state of mind by the same movements" [13]. Darwin's idea became a very powerful one. Today the point of view that emotions are the archaic form of a consciousness is still popular.

It is well known in psychology that both emotion and cognition are multistep dynamical processes of the perception, processing, and creation of new information. Each metastable state that is represented by such process is characterized by specific cause-response connections between external and internal informational flows [14].

As we already said, the dynamics of emotion and cognition are strongly connected. Thus, positive emotions appear, as a rule, when the outcome of our activity corresponds to the expected one. If not, negative emotions appear. This is cognitive dissonance. Everybody knows how to avoid such an uncomfortable state: we have to persuade ourselves that our expectation exactly corresponds to the outcome, or to get one's way in the next attempt. Nowadays even an 'informational theory of emotion' exists [15]. According to this theory, the intensity and sign of our emotion is determined by the strength of the desire and the probability of getting a satisfactory result in existing environment [16-18].

The quest for the dynamical origin of emotions goes back many decades. Franz [19] devoted attention to the dynamics of emotions by describing emotional sequences together with their content. He emphasized that only the balance between incorporation, elimination, and retention represents the fundamental dynamics of the biological process called life. Recently, Zautra [20] has developed a two-dimensional approach in which both positive and negative emotions are conceptualized and measured as co-occurring simultaneous dynamical processes.

Whereas behaviorism dominated the psychological and psychiatric sciences during the first half of the 20th century, cognitive science became the central paradigm of the latter half. This new line of interest was fostered by the promise of the *dynamics* of cognition as an integrated and fertile approach to understanding the mind [21]. 'The cognitive revolution' has started to inform us about the dynamics of emotion, as well.

Due to the great progress in measurement and imaging technologies within the last two decades, we now have a deeper understanding of the neural substrates of emotion and cognition. Although further progress, especially in temporal resolution, is still needed to delineate many important details, it is clear from the current evidence that human mental life is governed by a complex nonlinear dynamical system in a nonstationary (i.e., transient) regime.

The experimental insight into the matter corresponds to a period when the mature theory of dynamical systems was itself in flux. In particular, the recent shift in interest towards complex systems has been establishing a valuable array of tools and an important motivation for physicists and engineers to tackle mental phenomena — an ancient problem that still lacks a rigorous formulation.

The extension of dynamical systems research towards complex systems analysis [22] promises resolutions of these qualitative aspects, which are required to apply the rich theory of dynamical systems to brain-like complex systems. It is reasonable to emphasize here three approaches that are closely connected with each other: (i) a structural approach, which focuses on the specific architecture of connections (e.g., 'small-world' type [23]); (ii) an informational approach that describes the dynamics of principal information flows [24, 25]; and (iii) an approach considering functional modes [26, 27]. In this review we mostly follow the last one.

2. Mental modes

2.1 State space

As the two main players in human mental life, emotion and cognition have been under the spotlight for researchers for a long time, both individually and jointly. Given the necessity that any useful analysis must be based on an accurate quantification of the investigated phenomenon, numerous attempts have been made to assess cognition and emotion. Being directly related to the processing of auxiliary information, cognition has attracted relatively more attention in these efforts, particularly in the form of task development assessing decision making tasks [28, 29]. Although tests aimed solely at emotional quantities also exist (see, for example [30]), the assessment of emotions or their effects have often been attempted in a cognitive theme involving, for example, appraisal of an emotional stimulus [31, 32] on decisionmaking with the participation of memory [33, 34].

The critical constructive question to tackle the problem is: What is the best state space to describe the evolution of mental, i.e., emotional and cognitive, modes while capturing the functional complexity? To answer this question, we can use the experience of the investigation of complex systems in nature, in particular, turbulent flows [35]. It tells us that we need to know just the equations for coarse grain liquid particles, i.e., the micro details related to molecular dynamics are not directly effective for a macroscopic description. However, such details are consequential for the parameters of a macroscopic model. Although the situation with cortex dynamics is much more complex, the analogy suggests some insight and direction. A similar approach, a neural mass model, has been suggested in [36, 37]. We also use the coursegrain description, but in a different form (see Section 4).

2.2 Functional networks

The dynamical system perspective and the models of both cognitive functions and emotions are based on the assumption that the brain is a complex neural network of many dynamical sub-networks (neural clusters) working in coherence within a sequential time structure (see Fig. 1). Understanding the dynamics of this structure underlying certain emotional and cognitive functions and constructing a reasonable model based on this can be helpful in evaluating and predicting specific features in a psychiatric disorder.

New results in brain imaging, particularly functional MRI (fMRI) data, have revealed some fundamental properties and the functional organization of the brain systems that correlate with emotion and cognitive functions [38, 39].

Each of the brain centers that form the functional emotional subcircuit or mode is itself a very complex dynamical system with several characteristic time scales (see Fig. 2). These systems are open to an enormous range of neural stimulations from a wide range of brain areas. The spatiotemporal pattern of brain activity underlying an emotion is typically very sensitive to external or internal stimuli. For example, the amygdala, which plays a key role in emotional modes, receives information from both cortical and subcortical structures. These include highly processed information from the visual system, the auditory cortex, the olfactory and gustatory neocortex, and the somatosensory cortex. In short, it is directly informed about each of the five senses. The amygdala also receives projections from the association cortex, from the thalamus (relaying basic, unprocessed sensory signals), the hippocampus (high level information about the relationship between objects and events in the external world), and from a range of structures that represent internal bodily states, such as hunger and thirst.

It is important to take into account that *emotions and cognition are active processes* that result in a specific changing of the organization of the brain in time and the brain's dynamical response to environmental information and representation of the self. These processes are determined by the functional (not necessarily anatomical) connections



Figure 1. Schematic representation of three different functional modes in a complex neural network. (Different colors in the electronic version.) The nodes are different brain centers. The connections between them can be activated or inhibited depending on informational inputs. Figure modified from [40].



Figure 2. (Left) Possible dynamical regimes of specific brain centers: (a) a heteroclinic chain, (b) a heteroclinic cycle, (c) a strange attractor. (Right) Global connections between brain centers.



Figure 3. An illustration of the emotion-cognition tandem including the self. All three mental activities develop in time in parallel and continuously exchange information.

between brain areas or neural circuits that participate in the execution of cognitive functions and the generation of emotions. At different segments or steps of temporal emotional or cognitive processes, these networks form different temporal sequences that execute and represent different emotions and cognitive functions in the brain.

Dynamical variables describing emotion, cognition, and their mutual interaction form a joint workspace (or state space). To understand how to choose such variables, we have to know some details of the organization of emotional and cognitive modes. Let us consider emotion. As we already pointed put, an emotion is a result of the coordinated dynamical activity of many brain areas (sub-networks). Examples of such areas are the posterior orbitofrontal cortex, the anterior temporal sensory association areas, and the amygdala. These areas are not all inclusive, but they have a key role in temporal emotional processing. They *sequentially interact in evaluating the sensory and emotional aspects of the environment* for decision and action of the complex behavioral coping response (see Fig. 3).

2.3 Emotion-cognition tandem

Most theories view the emotion-cognition interaction crosssectionally, as disease-specific or based on a particular level of knowledge (i.e. neuroanatomical, pharmacological, etc.). To illustrate our point, let us refer to the current theories of Panic Disorder. The neuroanatomical theory of panic, as proposed in [41], describes a functional relationship between different anatomical parts of the brain. However, the theory of anxiety and panic disorders must be multi-dimensional. In particular, the theory must handle 'alarm' responses (amygdala and central gray nuclei), abnormal cognitions (striatal circuits), and controlling pathological behaviors (involving executive cortices). Under stress, people lose the ability to maintain positive feelings because they have become inversely linked, causally, with negative states. This state of affect simplification has the effect of reducing information-processing capacity and therefore reducing emotional clarity, one of the key components of emotional cognition.

The mental process in the joint emotion-cognition workspace is indeed a competition for resources that are needed to carry out each process. Two basic types of such finite resources are the energy (e.g., oxygen and glucose) and the information (attention and memory). Emotional appraisal and cognitive-emotional dynamics interplay continuously in time. This interplay could have an extremely important (perhaps central) role in diagnosis. Nevertheless, little attention has been given to the temporal dimension of emotion and emotion-cognition interaction processes [42-44]. By focusing on nonlinear dynamical interaction between cognition and emotions, [45] provides a valuable platform for integrating psychological and neural perspectives on the emotion-cognition interface. The author discussed a wideranging and timely theoretical formulation of emotioncognition relations and, in particular, emphasized: (a) bidirectional interactions between appraisal and emotion; (b) sub-cortical psychological and neural constituents underlying the emergence of emotion-appraisal processes; and (c) large-scale functional coupling through oscillatory neurophysiological mechanisms.

The joint emotio-cognitive behavior has its basis in the dynamical coordination of many brain centers, which often participate in both emotional and cognition activity [45, 46]. Due to this overlap, emotion and cognition are integrated in the sense of being partly separable [47, 48].

Emotional and cognitive modes in the brain interfere and exchange information reciprocally. As we have already emphasized, this relation is driven by a competition for energy and informational resources [48–51]. On the neurobiological level, the cognitive control of emotion follows from the direct inhibition of negative emotional modes by the centers that underlie the correct behavior [51].

On this basis, much is known about the interaction of emotion and cognition. However, to the best of our knowledge, the developing theory of emotion still lacks a mathematical model that accounts for emotion-cognition interaction in time based on observable principles. Such models should describe and predict the mental processes and their bifurcations with control parameters (i.e., the physiological state, drug concentration, etc.).

2.4 Dynamical model of consciousness

Consciousness underlines the main aspects of cognitive human behavior. Understanding the neurobiological and dynamical mechanisms of consciousness has proved to be one of the most mysterious problems for neuroscientists. Progress in recent years, however, allows for the development of theories of consciousness through integration of evidence from physiological, behavioral, and modeling studies. Any kind of cognition (perception, image recognition, self, awareness, etc.) can be conscious. For example, a conscious awareness is the process in which external or internal stimuli are perceived, are recognized, and can be intentionally acted on. Experiments using simple sensory stimuli suggest that even the primary sensory areas in the brain may be involved in the process of conscious awareness [52]. Once conscious awareness is established, it is fed back in time to process the primary input [53].

It has been accepted since the time of William James that consciousness is a continuous and transient process [54]. After half a century, Gurwitsch [55] has put this pioneering idea on a time axis: transients link the current mental state to what came before and what is to come after. According to him, every mental state has a (finite) duration and complies with a temporal order as imposed by the continuously flowing stream of consciousness. In the context of our work, mental states correspond to conscious metastable states, and temporal order to a sequence of metastable states.

Due to the provocative nature of this topic for physicists, we would like to clarify the dynamical modeling of consciousness.

What are the main differences between conscious and unconscious cognitive functions? It is the ability of a human being to predict the future based on past experience and knowledge, and to use it in the present in order to create the future. A dynamical model of any conscious cognition has to include two sub-systems, namely, the part that represents the cognitive process in the present [it can be a basic model: see Eqns (6)–(9) below], and the part that reconstructs a possible future based on the analyses of the past and using the present cognitive metastable state as initial conditions. In contrast to the first sub-system, the second one resides in an imaginary universe where time is compressed. The fast prediction together with the best possible version of the future (as a result of decision-making) is used for the correction of the present. Because conscious activity is able hold only a limited number of items at once, it only gives us a fragmentary picture of the future. This is one of the reasons why we are mistaken so often.

From the neurophysiological point of view, consciousness is a temporal process associated with low-amplitude irregular high-frequency EEG activity (20–70 Hz). In contrast, unconscious states like deep sleep, a coma, general anesthesia, and epileptic states show a predominance of slow rhythms, highamplitude, and a more regular spectrum at less than 4 Hz [56]. As fMRI investigations indicate, the consciousness involves widespread, low-amplitude interactions of many brain centers.

The fact that time is a critical attribute of the consciousness is clearly illustrated by recent experiments [57], which show that the human conscious experience is the ability to not only experience the present moment but also recall the past and predict the future, a facility that is called 'mental time travel.'

To build the dynamical model of the consciousness, it is necessary to formalize the main properties of consciousness (see, for example, [58]). These are:

(i) Consciousness is a simulator (or game generator) that enables the brain to process experiences that are not actually occurring. Such a simulator of experience can attach to or detach from perception and physical action as necessary the decision to attach is under the control of the simulator, but may be overruled by emotion and perceptions (for example, pain). During the representation, details of perception are substituted by simulation, causing various illusions and false memories.

(ii) The simulator detaches during dreaming, remembering, and imagining—detached simulation of an event causes the same cortical neurons to fire that the actual event does.

(iii) Simulation allows complex situations to be explored many times for more efficient learning (e.g., thinking about future moves in a chess game, rather than actually making each move).

(iv) Consciousness is a sequential dynamical process: The constantly evolving sequence of conscious content simulates cause and effect in a sequence of world events. Conscious content may evolve due to an interruption of attention to simulate a different sequence of events. Its availability depends on the quality of representation, where quality is defined by stability in time, strength, reproducibility, sensitivity, and metastability.

The quality of simulation depends on the stability against diverse perturbations, on the reproducibility that is necessary for the repeated analysis of the same events, and on the lifetime of metastable states — too short a lifetime may not be sufficient for storing the results in memory.

One of the most popular approaches to consciousness modeling is based on the global workspace theory [6, 59]. The main idea of this theory is the following. Informational inputs from inside or outside (external stimuli) the brain activate excitatory neurons with long-range axons, leading to the emergence of a global pattern among workspace neurons. According to the global workspace theory, transient links among specialized brain processing modules form a dynamic network that generates new information depending on incoming signals [60]. However, this theory cannot explain the underlying dynamical mechanism. To model the main dynamical properties of the consciousness, we suggest here merging the global workspace theory with the theory of the transient dynamics of complex networks based on the winnerless competition principle (see below, and also [27, 61, 62]). The most difficult problem here is to find time dependent variables convenient for measurements and sufficient for the dynamical description. We suggest using as dynamical variables the strengths of cognitive modes, which is discussed below. Nonlinear differential equations that describe the interaction of these variables is a dynamical model that we are looking for.

We would like to mention here one more problem that is directly related to consciousness, namely the self. Despite being treated continuously as a fundamental question since the earliest era of psychiatry [54], the concept of 'self' is a relatively new and very active field of study in neuropsychiatry. Among a few establishments in this line, there is growing evidence that self-reference is a stand-alone process in the human brain [64–67]. It is also believed to be closely related to a distinct brain system, called the default network.

The search for a baseline condition in brain imaging studies has revealed the default network (see Fig. 4), whose activity was not initially interpreted beyond assuming it as a 'resting state.' The pioneering studies [68–70] enlightened the neural substrates of this activity, its distinct connectivity among other brain systems, and its crucial role in self-referencing in both active and passive cognitive states.



Figure 4. The default network, as explored so far, comprises six brain regions: Ventral medial prefrontal cortex (vMPFC), posterior cingulate/ retosplenial cortex (PCC/Rsp), inferior parietal lobule (IPL), lateral temporal cortex (LTC), dorsal medial prefrontal cortex (dMPFC), and hippocampal formation (HF+). The approximate locations of these core areas are marked in the figure (adapted from [71]).

In recent decades, because of some success in quantum computing, we can observe an increase in interest in the very contradictory problem of "Quantum Consciousness", which has already been discussed for many years [72–74, 76]. Here is not the right place to consider the details of this subject. We just wish to say that the basic idea supporting the approach is Everett's Many-Worlds Interpretation of quantum measurement, which was formulated more than half a century ago (see for details [77]). The main assumption in the corresponding theory is this: consciousness selects a specific solution within the possible multi-variance of the informational world. As we already emphasized, one of the important functions of consciousness is the generation of new information. It is a clear contradiction of a purely 'selective' approach to "Quantum Consciousness."

3. Competition — reliability and sensitivity

3.1 Transients versus attractors in the brain

Traditional efforts in modeling dynamical phenomena are predominantly based on the fact that dynamical systems tend to converge to stable fixed points or dynamical states (limit cycles or strange attractors) where the density of all flows (matter, energy, or information) are balanced. A dissipative complex network of agents (i.e., neurons, brain centers, etc.) with symmetric interactions gives rise to a convergent behavior involving multiple attractors [78, 79]. The basins of attraction can be arranged to partition the state space to satisfy certain needs. When applicable, a Lyapunov function can be a handy tool in the analysis and design of such systems, since it translates the fixed points as extrema on an energy landscape while providing a clear image of the basins (see Fig. 5). There may be certain neural phenomena, such as an associative memory [80], that would benefit from a dynamical model operated in this mode, i.e., as a map from initial conditions to the attractors. However, computing with attractors makes very limited use of complex dynamical networks in general: once the attractor (or its vicinity) is reached, the 'dynamical' nature of the system becomes irrelevant. Since attractors mark the terminal states of the process, this perspective assigns a mere quantizer role to such a network that could be formulated equally effectively by an algebraic transform. Furthermore, it overlooks the qualities of the path from the initial condition to the attractor, a phase where nonlinear systems could exploit their remarkable repertoire of behaviors. Therefore, confining dynamical models to the symmetry assumption not only is unrealistic, but also rules out a continuum of opportunities within the modeling capacity of dynamical systems. Let us discuss here a contrary paradigm.

A fundamental responsibility of the nervous system that shapes the dynamical processes in the brain is the internal regulation of the organism so that the available resources are distributed properly among emotions, thoughts, and actions. This constitutes the origin of the time dependence in the functional brain organization underlying the emotional or the cognitive processes: at different time steps of these processes, the participating brain centers (or even the networks of centers) can be different and the activity in any one of them alone might not be sufficient to identify the ongoing emotion or cognitive function. A spatiotemporal competition of multiple networks of brain centers, on the other hand, can



Figure 5. Energy landscapes are illustrative tools in attractor-oriented analysis and design. The top row shows three landscapes accommodating different types of attractors: (a) a global attractor, (b) distinct local attractors, and (c) a continuous attractor. The surfaces on the bottom row are associated with two multi-stable systems where the basins of attraction are color-coded. In simple systems, one can obtain clear-cut borders as in panel (d). As system complexity increases, the borders can become fractal [see panel (e)], which makes the computation with attractors questionable. (Figure courtesy of T Nowotny.)

achieve this. In fact, the dynamic interaction of modes forms a reproducible temporal pattern that is specific to the ongoing emotion or cognitive function. Technically, such a temporal aspect of encoding is the only possible scheme for the brain to be capable of executing a continuum of emotions and virtually infinitely many cognitive functions while possessing a finite number of elements (i.e., centers). Thus, *emotions and cognition are sequential dynamic processes resulting from the interactions of different brain subsystems (modes) and their coordination and synchronization in time.* (About the synchronization in neuronal systems, see [5, 81, 82].)

3.2 Cognitive variables

The critical constructive question for tackling the problem is the choice of the best medium to describe the evolution of such emotional and cognitive modes while capturing the functional complexity.

We suppose that the specific cognitive activities can be described by the interaction of a finite number of cognitive modes. We can describe in this way the fMRI series of snapshots taken at consecutive instants (i.e., an fMRI movie), when a subject is busy with the execution of some cognitive job, for example, decision making. As experiments showed, the distinct networks of coherently working brain centers underlie this activity and they constitute an image of a spatial pattern in each frame. In spite of the vast diversity of these patterns across the frames, each of them can be considered as a sequential activity of the finite number N of functional modes. The overall movie, then, is a reproducible spatio-temporal sequence illustrating the time-varying ordered contribution of each (observable) mode in the process. There are several ways to extract these modes from the experimental data; for example, it can be the main principal components that describe brain activity in time [83, 84]. Thus, a snapshot of brain activity at time t captures a combination of cognitive variables in the form $\sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{i=1}^{N} \sum_{i=1}^{N} \sum_{i=1}^{N} \sum_{i=1}^{N} \sum_{i=1}^{N} \sum_{i=1}^{N} \sum_{i=1}^{N} \sum_{i=1}^{N} \sum_{$ $\sum_{i=1}^{N} A_i(t) U_i$, where U_i is the fixed spatial form (e.g., coordinates) of the *i*th cognitive mode, and $A_i(t)$ is its level of activity, e.g., the average intensity of voxels covering U_i in the BOLD image at time [85]. The number N of these modes depends on the level of detail that we wish to describe.

These modes in the modeling may include appraisal or the execution of a strategy (policy), or may even be as broad as decision making, a default network, or self-reflection activities. Despite being a coarse-grain partitioning of the cognitive universe, these functions can still be distinguished by the active brain subnetworks underlying them. We denote these cognitive modes by the nonnegative time-varying variables A_i , i = 1, ..., N, indicating the average activity of the corresponding sub-networks.

As a mental activity, a cognitive process requires both energy and informational resources (e.g., attention and working memory) to proliferate. In the ecological model that we suggest below, we encapsulate all these resources in a real variable R_A within the interval [0, 1], which denotes the ratio of the resources supplied to the total demand from the cognitive process.

3.3 Emotional variables

It is known that distinct emotional modes have certain mental fingerprints characterized by the specific connections of brain centers involved in it (and this is an active field of research). A well-explored example is the sadness network in the human brain [86].

Following the same reasoning presented for the cognitive variables above, the emotion-related functional can be a readout in the form $\sum_{i}^{M} B_{i}(t)V_{i}$, where B_{i} are the emotion modes and V_{i} are their temporal and spatial activities. Since the spatial functions are assumed to be known for all modes and fixed for all times, the temporal evolution of emotion modes are captured by $B_{i}(t)$.

Another critical observation is that the activity patterns of emotions are unlikely to coexist for a long time, yet overlap during smooth transitions. This observation is consistent with the general ecological principle that we expand on below and adopt throughout the modeling effort.

Analogously to R_A , which represents the resources allocated to cognition, we introduce the positive quantity R_B denoting the supplied rate of resource demand from the emotional process.

The manner in which individuals receive, process, and interpret information in time is the key to understanding



emotional representation, the execution of cognitive functions, and biobehavioral organization [87, 88]. For healthy people, these processes are sequentially organized in time following a stable (i.e., robust) one-directional information flow. Although the execution of multiple mental processes seems feasible in a healthy human brain, the critical resource of attention is available to just one of them at any given time. Therefore, progress (in the form of a metastable state sequence) can actually occur in only one process at an arbitrary instant.

3.4 Metastability and dynamical principles

We suppose that the reader has got some knowledge about emotion and cognition interaction; thus, it is the right time to introduce the principles that form the basis of their dynamical modeling. These principles are:

(i) the existence of metastable states representing modes in the unified emotion-cognition working space,

(ii) the structural stability of the transients that are formed by the switching of brain modes among these states,

(iii) ecological, i.e., competition, principles governing these switchings.

The first item, metastability, is a general nonlinear dynamics concept, which describes states of delicate equilibrium. Metastability in the brain is a phenomenon which is being studied in neuroscience to elucidate how the human mind processes information and recognizes patterns. There are semi-transient equilibria in the brain which persist for a while and are different from the usual equilibrium state [89]. The metastable activity of the cortex can also be inferred from the behavior [90]. Metastability is a principle that describes the brain's ability to make sense out of seemingly random environmental cues [91, 92].

Metastability is supported by the flexibility of coupling among diverse brain centers or neuron groups [83, 85, 93–95]. The temporal order of the metastable states is determined by the functional connectivity of the underlying networks and their causality structure [96]. The mathematical image of a metastable state is a saddle in the state space of the dynamical model. The image of the transition between these saddles is the unstable separatrix connecting them (see Fig. 6). Such a construction is named a heteroclinic chain.

3.5 Winnerless competition — structural stability of transients

Competition without a winner (or continuously changing winners) is a widely-known phenomenon in systems involving more than two interacting agents that satisfy a relationship similar to the popular game rock-paper-scissors or the voting paradox [97, 98]. The participants in such a process can become winners periodically or, especially when the number of participants is more than three, the process can be noncyclic and can even be terminated following a stable sequence of transients, and one participant becomes the ultimate winner. Busse and Heiskes [99] modeled the thermo-convection in a horizontal liquid layer rotating around a vertical axis. Different convective patterns with rolls orientations of 0° , 120° , and 240° periodically switch among each other [100].

As a generic dynamical phenomenon, which is rare in simple systems yet common in complex ones, sequential switching among saddles can provide concise and constructive formulations in a variety of real-world problems [101]. Prototype dynamical models that are widely accepted in computational neuroscience [102] and ecology [103–106] have been shown to exhibit a transitive winnerless competition for a fairly broad range of parameters [101, 107, 108].

Since the time spent within a saddle vicinity is inversely proportional to the (logarithm of the) noise level (variance) [109, 110], the characteristic time of such a transient varies in a wide range. In a stable heteroclinic sequence, the order of temporal winners is fixed and the noise is able to accelerate the process. *Thus, the noise must be large enough to maintain the switching behavior at the desired rate (on average) and small enough to keep the heteroclinic nature of the system on track, i.e. to maintain stability.*

Embedding a structurally stable heteroclinic skeleton in the phase space (see Fig. 6b) results in a channel which routes the volume around it along the imposed sequence. Within this volume the system behavior is reproducible with finite accuracy. Since the location of the saddles conveys inputspecific information, which is activated the corresponding metastable states and their sequential order by the strength and the topology of the connections, *the system becomes both noise-rejecting and input-sensitive (due to following the stimulus-specific channel) simultaneously*. The key mechanism underlying the winnerless competition in the brain is inhibition, which is known to exist in neural systems at microand macroscopic levels [111–114].

There is substantial experimental support [62, 89, 115] (also outlined below) that metastability and transient dynamics are the key phenomena that can contribute to the modeling of cortical processes and thus yield a better understanding of a dynamical brain.

There is a fundamental contradiction between the stability of transient brain dynamics and their sensitivity to information signals. The resolution of this contradiction, as we discussed above, can be found by informational reconstruc-





Figure 7. Neurons in the rat's gustatory cortex generate a taste-specific sequential pattern. The top row shows the sequential WLC activity among 10 cortex neurons in response to four taste stimuli. A Hidden Markov Model (HMM) of joint temporal activity (the ticks denoting the action potentials) reveals that the network behavior is best represented by four discrete states in a winnerless competition setting. The dashed horizontal line denotes the threshold, above which (with probability 0.8) the network is considered to be occupying the corresponding state (i.e., the state becomes the winner). The second row lists the outcome of the four replicates of the previous experiment on the same network and confirms the reproducibility of the sequential activity: the order of the observed states is the same in each trial. Note, however, that the switching times are irregular. The translation of the four HMM states into firing rates for each stimulus are given in the last row. (Figure adapted from [115].)

tion of a heteroclinic channel, i.e., by choosing new metastable states. It is an example of the interaction of dynamical and informational processes in nonequilibrium dissipative systems. The general problems of such interactions are discussed in Kadomtsev's interesting book [116] and reviews [117–119].

3.6 Examples: competitive dynamics in sensory systems

The way sensory signals are processed in animals is through the activation of specific groups of neurons, which are determined by both the quality and the quantity of the stimulus. The intrinsic dynamics of neural networks produces firing patterns that encode the informational input and relay them to further processing centers upstream. In general, this code is spatio-temporal and sequential. Transitive winnerless competition is a sound mathematical method to explain these reproducible transient phenomena. The competitive nature of neuronal processing in general provides another clue that winnerless competition may underlay a sequential activity. When characterizing the dynamics of biological sensory systems, the variability observed in recordings from the neural activity appears as noise, but is not actually a noise component. It should be viewed as fragments of the competitive sequential activity. Such a phenomenon has been observed recently in experiments with olfactory and gustatory sensory systems [115, 120].

An analysis of the response in a rat's gustatory cortex to prototype tastes reveals that a reproducible taste-specific switching pattern is triggered shortly after the stimulus is presented [115] (see Fig. 7).

Experimental observations in the olfactory systems of locusts [121] and zebrafish [122] reveal odor- and concentration-specific, reproducible, and transient patterns of activity in principal neurons. Here, the odor representations are spatio-temporal successions of states, or trajectories, each corresponding to one odor identity and one concentration [121] (see also Fig. 8).

The rules governing the WLC resemble the competition among different species for resources [101]. This analogy links the WLC concept with brain functions via a popular standpoint in brain research. This ecological perspective on the brain's organization suggests that it is the strict competition for finite resources among brain networks that maintains the collective dynamics. For example, a competitive activity of multiple brain areas [123, 124] is fundamentally important for thinking, in particular, for sentence comprehension [125]. These results provide indirect support for the argument that the WLC is widespread in the nervous system. The WLC opens a new avenue in modeling and analysis of complex cortical processes.

3.7 Formalism for structural stability of transients

The mathematical method that represents a reproducible transient activity is the Stable Heteroclinic Channel (SHC), consisting of saddle sets, their vicinities, and the separatrices connecting them. An SHC is characterized by two properties: (i) a sufficiently strong convergence of the phase volume in the vicinity of each saddle with respect to the stretching along unstable separatrices, and (ii) the relatively long (but finite) passage (or exit) time that the system spends in the vicinity of a saddle in the presence of moderate noise.

Let us consider a channel that consists of saddles, each having one-dimensional unstable manifolds, i.e., a separatrix leading to the next saddle. To obtain the condition of the channel's stability, we must consider elementary phase volume in the neighborhood of each saddle that is compressed along the stable separatrices and stretched along the unstable separatrix. Let us order the eigenvalues of saddle *i* as

$$\lambda_1^l > 0 > \operatorname{Re}\left\{\lambda_2^l\right\} \ge \operatorname{Re}\left\{\lambda_3^l\right\} \ge \dots \ge \operatorname{Re}\left\{\lambda_n^l\right\}. \tag{1}$$

The number

$$v_i = \frac{-\operatorname{Re}\left\{\lambda_2^i\right\}}{\lambda_1^i} \tag{2}$$

is called the saddle value.

If

$$v_i > 1 \,, \tag{3}$$

the compression along the stable manifolds dominates the stretching along the unstable manifold, and the saddle is called a *dissipative* saddle. If all saddles in the heteroclinic chain are dissipative, the trajectories in their vicinity cannot escape from the chain, providing stability.

In the absence of noise, a state vector approaching a saddle along a stable manifold is confined to the neighborhood of the saddle indefinitely. The exit from a saddle's neighborhood is possible only due to a strong enough perturbation. The dependence of the exit time on the noise level was studied in [109] and [110]. A local stability analysis around a saddle fixed point results in the relation

$$\tau^{i} = \frac{1}{\lambda_{1}^{i}} \ln \frac{1}{|\eta|}, \qquad (4)$$

where τ^i is the mean time spent in the neighborhood of saddle *i* (provided that initial points belong to the stable manifold), i.e., the lifetime of a metastable state, and $|\eta|$ is the level of noise. In the framework of a specific model, one can derive the inequalities on the model parameters using conditions (2) and (3), which guarantee the stability of the heteroclinic channel.

3.8 Hierarchy of brain informational flows: heteroclinic tree

The brain's hierarchical organization [126], i.e., from the perception levels to more complex sub-core and cortex structures, supports the idea of the hierarchical organization of information flows in the brain. In fact, the idea of onedimensional flows of visual, auditory, and other information about environment, which transfer through centers of the preliminary processing to the cortex decision-making centers and behavior generation, looks very attractive. However, the reality is more challenging. The brain is characterized by a huge number of informational feedbacks. This is a way to control sensory systems by the cortex. In particular, by concentrating attention the brain is choosing which sensory information is critical for executing cognitive functions and surviving in extreme conditions.

At the same time, the idea of the hierarchical organization of brain informational flows looks very promising [127]. In contrast to the traditional view, we have to consider the informational flows in the phase space of the corresponding dynamical model, but not in the physical space of the brain.

Figure 8. Spatio-temporal representation of sensory information in the olfactory system of a locust (antennal lobe): (a) Schematic of an insect antennal lobe sectioned through its equatorial plane; (b) response of 110 neurons of the antennal lobe to an odorant that lasted 1.5 s; (c) projection of neural activity on 3D principal component space (black trajectory is the average of 10 different experiments); (d) diversity of heteroclinic sequences in an ensemble of seven neuronal groups.





Figure 9. A heteroclinic tree in the multi-dimensional phase space of the emotion-cognition interaction dynamical model. The terminal state corresponding to the final decision (or strategy) of behavior is indicated by the white circle.

Figure 9 illustrates divergent informational flows along chains of metastable states. Different flows converge into the main channel and end in a state corresponding to the final decision or event. Such a representation naturally satisfies the causal answer relationship and suggests the basis for solving new problems. It can be analyses of the working memory capacity [128] or the stability dependence of cognitive information flow on emotion.

The 'information theory of living systems' is only in its infancy; however, some interesting results have already been published [127, 129–132].

4. Basic ecological model

4.1 The Lotka–Volterra system

The competition within and between cognitive and emotional modes can be described by the Generalized Lotka–Volterra (GLV) model, given by

$$\tau \frac{\mathrm{d}}{\mathrm{d}t} x_i(t) = x_i \left[\mu_i(E) - \sum_{i=1}^n \varphi_{ij}(E) x_j(t) \right] + x_i(t) \eta(t) ,$$

$$i = 1, \dots, n .$$
(5)

Here, $x_i \ge 0$ is the *i*th competing agent, *E* is the input that captures all (known) external effects on the competition, τ is the time-constant, μ_i 's are the increments that represent the resources available to the competitor *i* to prosper, φ_{ij} is the competition matrix, and $\eta(t)$ is a multiplicative noise perturbing the system.

Depending on the ratio of the control parameters, this model can describe a vast diversity of behaviors. When connections are nearly symmetric, i.e., $\varphi_{ij} \approx \varphi_{ji}$, two or more stable states can co-exist, yielding a multi-stable behavior—the initial condition determines the final state. When the connections are strongly nonsymmetric, heteroclinic contours or limit cycles in their vicinities can emerge.

Dynamical chaos can be observed in this case [133]. A specific kind of dynamical chaos, where the order of the switching is deterministic, but the life-time of the metastable states is random, can also be observed [134]. We think that such reproducibility of the order of metastable states, despite the irregularity in timing, can be interesting for the processing of observed data.

For a given model, the values of the control parameters that ensure the stability of the transients can be obtained from inequalities derived from (1) and (2). In [135], such conditions have been generalized in the case of weakly-interacting subsystems like (5).

As we already discussed, cognition and emotion are strongly connected. Nevertheless, it is reasonable to suppose that the modes within one family are more strongly connected than the modes between these two families. One can thus consider that one family does not 'destroy' the dynamics of the other family, but modulates it. In particular, cognition supports emotional equilibrium, whereas emotion excites or inhibits cognition. Therefore, it is natural to describe their interaction based on coupled subsystems of type (5). Also taking into account the dynamics of resources, we should write a third set of equations describing the resource modes (i.e., attention, memory, and energy). It is important to emphasize the special role of attention in this interaction: it selects the sensory cues that are critical for the current decision-making process. Based on experimental evidence, the dynamics of attention can also be described by a competition among informational entities. For the sake of simplicity, let us consider that these entities are total emotional $\tilde{B} = \sum_{i=1}^{M} B_i(t)$ and total cognitive $\tilde{A} = \sum_{i=1}^{N} A_i(t)$ activities. We then write the model in the following form:

$$\tau_{A} \frac{\mathrm{d}}{\mathrm{d}t} A_{i}(t) = A_{i}(t) \left[\sigma_{i}(I, \mathbf{B}, D) R_{A} - \sum_{j=1}^{N} \rho_{ij}(D) A_{j}(t) \right] + A_{i}(t) \eta(t), \quad i = 1, \dots, N,$$
(6)

$$\tau_{B} \frac{\mathrm{d}}{\mathrm{d}t} B_{i}(t) = B_{i}(t) \left[\zeta_{i}(S, \mathbf{A}, D) R_{B} - \sum_{j=1}^{M} \zeta_{ij}(D) B_{j}(t) \right] + B_{i}(t) \eta(t), \quad i = 1, \dots, M,$$
(7)

$$\tau_{R_A} \frac{\mathrm{d}}{\mathrm{d}t} R_A(t) = R_A(t) \Big[\tilde{A} - \left(R_A(t) + \phi_A(I, D) R_B(t) \right) \Big], \quad (8)$$

$$\tau_{R_A} \frac{\mathrm{d}}{\mathrm{d}t} R_B(t) = R_B(t) \Big[\tilde{B} - \big(R_B(t) + \phi_B(S, D) R_A(t) \big) \Big].$$
(9)

The nonnegative variables A_i and B_i , as described above, correspond to the cognitive and emotional modes, the union of which is denoted by **A** and **B**, respectively. The general intensities of these variables are denoted by A and B, respectively. The proposed model is merely a formulation of the competition within and among these two sets of modes. Both of these modes are open to the external world through the quantities I and S, denoting the cognitive load and the stressor, respectively, and D is the control parameter characterizing the medication. The coupled processes evolve on time scales determined by the parameters τ_A and τ_B . Both processes are open to brain noise, which appears as the multiplicative perturbation $\eta(t)$ in the equations. The variables R_A and R_B characterize the resource dynamics, where ϕ_A and ϕ_B control the level of competition for resources.



Figure 10. Simulation of the stressor-induced emotion-cognition interaction generated by the proposed ecological model. The bottom curve is the temporal profile of the stressor, which triggers the emotional activity depicted in the second row. Arousal of these emotional modes affects the ongoing cognitive activity negatively, as seen in the first row. This effect is due to two couplings between the cognitive and emotional processes: (i) the direct interaction encoded in the cognitive increments σ (see text), and (ii) resource competition, which is shown in the third row.

Cognitive and emotional brain processes have different qualities: the former is usually characterized as a sequentially ordered brain activity advancing at a regular pace, whereas the latter is a highly variable, fast, and sometimes unpredictable activity. Based on these observations, the suitable operating regime for a (healthy) cognitive process is the stable heteroclinic chain. There is no particular constraint posed at this point on the quality of emotional dynamics; they can also follow a heteroclinic sequence with a short switching period, a recurrence behavior, or a strange attractor.

The competition within cognitive and emotional modes is regulated by the self-excitations σ and ζ , by the competition matrices ρ and ξ , and by the time constants τ_A and τ_B , respectively. The dependence of these increments on **A** and **B**, respectively, describes the direct influence of cognition on emotion, and vice versa. For example, these could be set to describe the excitatory or inhibitory action of the emotion on cognition. They could also reflect the suppression of emotion through a correct behavioral strategy under stress.

4.2 Stress and hysteresis

It is well known that different emotions can switch between each other. Therefore, the cognitive-emotion dynamics can bifurcate from one dynamical image to another. It would be very interesting to find the relationship between the control parameters that correspond to the bifurcation surface based on the models (6)–(9). Finding such relationship may allow quantitative prediction of psychiatric disorders. It is interesting that in some areas of the parameter space two different stable objects. Hence, bistability and hysteresis are observed [136].

In Fig. 10, we present an example where an auxiliary stressor *S* triggers these emotions, which in turn disrupts an ongoing cognitive sequence.

Let us consider N = 5 cognitive modes and M = 5emotional components. The multiplicative perturbation $\eta(t)$ is white noise with variance 10^{-8} and 10^{-3} for the cognitive and the emotional dynamics, respectively, and the time constants are $\tau_A = \tau_B = 20$.

Without loss of generality, we prescribed the finite heteroclinic sequence of saddles $e_1 \rightarrow 2 \rightarrow \cdots \rightarrow e_5$ for emotional modes. The mode e_5 is a stable attractor (i.e., without any unstable manifold so that the system is confined to the vicinity of e_5 once it enters its domain of attraction). This state marks the terminal cognitive mode, such as the execution of a certain coping strategy, whereas the preceding modes denote the cognitive tasks that lead to this resulting activity. They could be called, for instance, perception, appraisal, evaluation, and selection, in their order of appearance in the sequence.

The feasible values of ρ_{ij} that can establish the desired heteroclinic skeleton in the *A* network constitute a broad continuum in the parameter space. A set of sufficient conditions that determine a part of this region in the form of simple inequalities on σ_i and ρ_{ij} can be found in [61]. Following these conditions, we set $\rho_{ii} = 1.0$ for $i \in \{1, \ldots, 5\}$, $\rho_{i-1,i} = 1.5$ for $i \in \{2, \ldots, 5\}$, $\rho_{i,i+1} = 0.5$ for $i \in \{1, \ldots, 4\}$, and $\rho_{ij} = \rho_{j-1,j} + 2$ for $j \in \{2, 3, 4\}$ and $i \notin j - 1, j, j + 1$.

In this illustration, the five emotional modes were organized as a heteroclinic sequence, albeit a cyclic one, by introducing the $e_5 \rightarrow e_1$ transition. We note that we do not necessarily name the emotional components individually, but interpret their mean activity as the degree of anxiety, a negative emotional state. In this respect, the precise dynamical quality of the emotional network is not of primary consideration in our design; for the sake of our illustrations, the emotional behavior could have been realized simply as a limit cycle, or as a strange attractor. The ξ_{ij} was evaluated as done above for ρ_{ij} , yet taking into account the $e_5 \rightarrow e_1$ transition, which results in $\xi_{5,5} = 0.5$.

We disregard any transient drug effect in the simulations, thus assume that D, thus both matrices ρ and ξ are fixed. These matrices configure the competition within the cognitive and the emotional modes. The interaction between them is regulated by the choice of the increment functions σ and ζ , as well as through resource competitions (8) and (9).

All five increments σ_i in the cognitive process were modelled as $1 - \sum_{i=1} B_i(t)$, i.e., inversely proportional to the total (negative) emotional activity. The increments ζ_i for the five emotional modes were considered independent of the cognitive activity in this example; they were all equal to the externally applied stressor quantity *S*, which was assumed to be nonnegative.

The resource competition R_A vs. R_B is regulated by the equations (8) and (9) with parameters $\phi_A = \phi_B = 0.3$ and random initial conditions.

With the selected parameters, the integration of ordinary differential equations were performed by the Milstein approximation. Figure 10 illustrates the suppression of and delay in the cognition due to emotional activity, which is induced by an external stressor.

An interesting prediction that can be derived from the model is the contrast in the switching regimes of the total activity in the cognitive and the emotional network during the rising and decay periods of *S*. This can be better observed in Fig. 11.



Figure 11. Bifurcation origin of hysteresis: (top row) The sequence of phase portraits that corresponds to increasing stressor intensity, with the basins of attractors *A*' and *B*' indicated. (bottom left) Witney fold or Cusp catastrophe [137]; (bottom right) the hysteresis obtained from the model.



Figure 12. The change in the cognitive activity $\sum_{i=1}^{5} A_5$ with respect to emotional activity, which is separated into two axes. The color change from dark to light shows the time arrow.

4.3 Mood-cognition interaction in the resting state

Such interaction in the absence of stress depends on the psychiatric profile of the individuals. Individuality is fixed by the value of parameters in the framework of our model. In fact, all we need to know is just the ratio of characteristic timescales, the level of excitation, and the degree of the nonsymmetry of the inhibitory connections. Let us first consider an average (i.e., healthy) person.

Suppose that a cognitive activity, in the form of a transient process, is to be finished in solving an internally-formulated problem. We represent this cognitive process by five modes, mood (in the absence of stress) by three modes, and emotional memory by two modes. The model parameters are indicated in the caption of Fig. 9. The initial conditions are as follows:



Figure 13. A self-induced emotion-cognition interaction as generated by the proposed ecological model. The interpretations are as in the previous figure. Here, a certain cognitive mode, the A_4 denoted by the green curve, triggers emotional activity, which suppresses cognitive activity in return. The emotional activity is time-limited as encoded in ξ (see text); the cognitive process returns to back its track after this period.

at t = 0 all but one cognitive mode is equal to zero. One emotional mode, namely the one representing a negative mood, is not equal to zero, whereas the other is set to zero. The memory mode, reminiscent of a positive image, is slightly larger than the intensity of the memory mode, which we assume to be representing a negative image. The phase portrait of the considered dynamical process is given in Fig. 12, and the corresponding time series in Fig. 13. One can see that, at the first stage of the process, the positive reminiscences suppress the negative emotions and support cognitive activity. Then, the negative reminiscences push out M I Rabinovich, M K Muezzinoglu

the positive ones and negative emotions become prevalent (see also [138]). As a result, cognitive activity goes down. However, cognition still controls emotion partially, and eventually negative reminiscences vanish, yielding the success of the cognitive job. Of course, for another psychiatric profile, the interaction between mood and cognition could be different. For example, reminiscences could be exchanging with each other chaotically, as could emotion. In some region of the control parameters, negative reminiscences can generate a depression lasting for a very long time.

The discussed dynamical model is a viable mathematical description of the mental brain dynamics delineating the crucial elements of emotion, cognition, and attention memory.

The analysis and simulation of this self-perturbating system can reveal different characteristics and interaction schemes of the two processes. This should be a scenario-based approach, where the exact emotional and cognitive modes, as well as all known (or investigated) interactions, are encoded in the model.

What aspects of the proposed dynamical model of the emotion-cognition interaction are specific to an individual or to any given psychiatric disorder? We can think of two answers to this important question: (i) the coupling schemes connecting emotion \leftrightarrow cognition \leftrightarrow memory processes; and (ii) the time constants setting the pace for each process. The former determines the *scheduling* of the components in the course of mental activity. For instance, the appraisal is likely to trigger a coping strategy, which is shaped by the current emotional state. The co-occurrence or a particular sequential order of these processes in time forms a non-coincident pattern that may be unique for an individual and/or may be indicative of a disorder.

The timing of these episodes is another mental characteristic. For instance, prematurely terminated appraisal may result in an improper coping strategy, or coping may not arise at all if the appraisal *gets stuck*, occupying the stage indefinitely.

5. Conclusion

Coming back to the epigraph of the review, we have to say that we do not know the origin of thought. However, we hope that we have delivered to the reader our vision of how to build a theory of cognition and emotion and how to develop corresponding dynamical models. Our focus here was on the specific mental phenomenon: the emotion-cognition interaction.

Principles: Because of the brain's exclusive complexity and the necessity to get a successful description of a continuum of different cognitive and behavioral brain functions, it is possible to build such a theory only if we can find a reasonable level of abstraction. We do not think that this abstraction should be at the level of neuronal groups, or even of brain centers, but of transient cognitive modes and intermediate entities, particularly metastable states. The main principles that have to be the basis of such a theory are: (i) the robustness of brain dynamics against noise, (ii) the reproducibility of mental activities in a similar environment and personal shape, and (iii) the ecological principle—competition of mental modes for energy and information resources.

In the framework of such a theory, normal mental activities can be viewed as emergent properties of the dynamics of complex functional networks, whereas mental disorders can be viewed as distortions of these dynamical networks. Quantification of mental processes based on types of dynamical distortion is a new discipline in diagnostics, modeling, and tackling mental disorders.

Analyses of transient patterns of mood have emerged as a new field in psychiatry. The creation of this new field can be called the "Project for a Dynamic Psychiatry", as a paraphrase of Freud's "Project for a Scientific Psychology" [139].

Dynamical Images: Under basal resting conditions, most healthy neural sub-systems demonstrate irregular complex dynamics that represent weakly interacting multiple mental processes that operate over multiple time scales. These processes prime the brain for an adaptive response, making it ready and able to react to new cognitive information or internal and external psychological perturbations. This reaction in a normal situation leads to a robust and reliable condition.

The dynamical principles that we have discussed above provide us the understanding of the origin of the robustness and the reliability of mental behavior. This behavior is a result of temporal brain activity that is an open complex nonequilibrium system with finite energy and informational resources. We have shown that the competition between different modes, each functionally depending on incoming information, solve the fundamental contradiction between robustness and sensitivity to a weak informational signal. The dynamical image of such activity in the state space of the corresponding dynamical model is a Stable Heteroclinic Channel (SHC) that is a sequence of metastable states, whose vicinities are connected by unstable trajectories, i.e., separatrices.

We have analyzed here the simplest variant of the heteroclinic channel, i.e. with one-dimensional unstable separatrices. As our preliminary computer experiments indicate, a heteroclinic channel that consist of saddles with many dimensional unstable separatrices can nevertheless be stable. Everything depends on the values of the positive Lyapunov exponents: if one of them is clearly dominant, it decisively determines the departing direction, consolidating the robustness of transient behavior. When there are several positive Lyapunov exponents of one order, functionally oriented cognitive behavior can be distorted. From this point of view, certain psychiatric disorders involving emotion and cognition can be distinguished by the dynamical parameters quantifying these exponents.

Using Dynamical Characteristics in a Clinic: Recent clinical observations have shown that a mental disorder (just like mental health) cannot be described by analysis of the mood over a short period of time. Such mental disorders are dynamical. For example, in a recent paper, Katerndahl et al. [140] have asked the basic questions about the levels of mood variability between healthy and unhealthy people. They analyze and compare the dynamic patterns of hourly mood variation among newly diagnosed primary care patients to determine whether the major depressive disorder or panic disorder compared with patterns in patients without either disorder. Their premise is that in 'normal' people, mood states might vary over time in a dynamical pattern similar to that seen for heart rate. Heart rate variability in normal people has been shown to have some level of chaoticity. Normal controls displayed a circadian mood pattern with chaotic dynamics. Depressed subjects did not show a circadian pattern of mood variation. Panic disorder subjects had variable patterns of mood dynamics but generally did not match the combination of circadian pattern and dynamical chaos seen in controls. Taken together, these results suggested that healthy individuals (i.e. without a disorder) might experience a normal circadian rhythm in mood with superimposed mood changes as the chaotic response to multiple social or biological stressors during a day, while either the circadian rhythm or the responsiveness to stressors is impaired in those with mood or anxiety disorders.

Our efforts that focus on new dynamical models of emotion and cognition in fact suggest to clinicians new approaches for recording and analyzing data, and, furthermore, for diagnosis. Now is the time to use functional brain imaging to identify patterns of brain activity in response to selected stimuli, and gene mapping to identify genetic features associated with specific mental disorders. If we can add transient brain dynamics to the psychiatry toolbox, the ability to identify and classify mental and behavioral disorders will be greatly enhanced.

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