PACS numbers: 11.10.Gh, **11.15.–q**, **12.38.–t**
DOI: 10.3367/UFNe.0180.201003j.0328

Screening and antiscreening of charge in gauge theories

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We discuss charge renormalization in vector theories with an Abelian and non-Abelian gauge group on the qualitative level.

1. The discussed studies began with the remarkable paper by Landau, Abrikosov, and Khalatnikov [1], published more than half a century ago. In particular, it was demonstrated therein that the observable electron charge e in quantum electrodynamics is related to the bare charge e_0 as

$$e^2 = e_0^2 \left(1 + \frac{e_0^2}{12\pi^2} \ln \frac{\Lambda^2}{m^2}\right)^{-1} < e_0^2 , \tag{1}$$

where m is the electron mass and Λ is the cut-off parameter for divergent integrals (of course, $\Lambda \gg m$). This and other relations discussed below are presented in the leading logarithmic approximation, i.e., we keep only the leading power of the large logarithm in the coefficient at a given power of the coupling constant (which is assumed to be small).

The fact that the observable charge is less than the bare one is a quite natural and obvious result of vacuum polarization: the bare charge attracts virtual particles with a charge of the opposite sign, and repulses virtual particles with a charge of the same sign (Fig. 1).

On the other hand, it can be easily demonstrated that inequality (1) naturally follows from the unitarity relation, according to which the imaginary part of the photon polarization operator is positive definite, $\mathrm{Im}\,\Pi > 0$. It should be combined, of course, with the dispersion relation for the polarization operator.

This is a result for all time in quantum electrodynamics.

2. However, 11 years later, Vanyashin and Terentjev [2], investigating the contribution of a charged vector particle to the nonlinear Lagrangian of a constant electromagnetic field, discovered that the contribution of this particle (with the gyromagnetic ratio $g = 2$) to the charge renormalization is quite different:

$$e^2 = e_0^2 \left(1 - \frac{7e_0^2}{12\pi^2} \ln \frac{\Lambda^2}{m^2}\right)^{-1} > e_0^2 . \tag{2}$$

In other words, the antiscreening of a charge occurs in the electrodynamics of a vector particle. But how can this be reconciled with the simple qualitative arguments presented above? What is the difference between an electron with spin $s = 1/2$ and a W boson with $s = 1$?

The difference is first of all that the electrodynamics of a vector particle is a nonrenormalizable theory, [1] in which the photon polarization operator diverges, generally speaking, not logarithmically, as is the case of the electrodynamics of spin-1/2 particles [see (1)], but as Λ. Of course, the leading, quadratically divergent contribution to the charge renormalization, proportional to Λ^2/m^2, would have the same sign as the logarithmic contribution in formula (1), and would therefore result in screening. But the technique used in [2] for calculation of a nonlinear Lagrangian of the electromagnetic field was such that power-like divergences in Λ^2/m^2 were eliminated from the result. As regards the sign of the logarithmically divergent contribution to the charge renormalization, it is not then fixed by simple qualitative arguments.

Result (2) is certainly quite meaningful and interesting. Relations of this type arise in modern models of the electroweak interaction where power-like divergencies are absent.

3. Four years later, the structure of the polarization operator was found for a massless vector field with self-coupling described by the non-Abelian gauge group SU(2); the Coulomb gauge was used in the calculation [3].

The charge renormalization is described in this gauge by two diagrams (see Figs 2 and 3). The dashed line refers, as previously, to the Coulomb field; the wavy line refers to actually propagating three-dimensionally transverse vector quanta. Because these quanta are massless, the divergence at small momenta are cut off at $\bar{q}^2$.

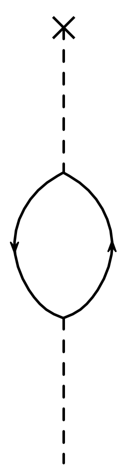

Figure 1. Vacuum polarization in quantum electrodynamics.

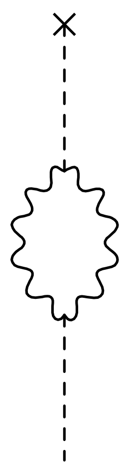

Figure 2. Contribution of three-dimensionally transverse quanta to vacuum polarization.

[1] The importance of this fact was emphasized by I Ya Pomeranchuk as soon as paper [2] appeared.

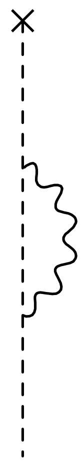

Figure 3. Nondispersion contribution to vacuum polarization.

The contribution of Fig. 2 to the observable charge g^2 is given by

$$g_0^2\left(1+\frac{g_0^2}{12\pi^2}\ln\frac{\Lambda^2}{\bar{q}^2}\right)^{-1}. \qquad (3)$$

There is nothing surprising here: everything agrees quite naturally with result (1) for quantum electrodynamics.

But in the theory with a non-Abelian gauge group, a diagram arises that is absent in electrodynamics; this diagram has no imaginary part because the Coulomb field (dotted line) does not propagate in time. The nature of this contribution is the interaction of the Coulomb field with the fluctuations of the three-dimensionally transverse physical degrees of freedom in the second order of the perturbation theory.

The sign of this contribution is opposite to that of (3), and numerically this contribution is much larger. With both contributions taken into account, the total result for the coupling constant is

$$g^2 = g_0^2\left[1+\left(\frac{1}{12}-1\right)\frac{g_0^2}{\pi^2}\ln\frac{\Lambda^2}{\bar{q}^2}\right]^{-1} = g_0^2\left(1-\frac{11g_0^2}{12\pi^2}\ln\frac{\Lambda^2}{\bar{q}^2}\right)^{-1}. \qquad (4)$$

Instead of the Abelian screening of the charge, its non-Abelian antiscreening arises!

4. For the further physical interpretation, it is convenient to pass to the running coupling constant $g(\bar{q}^2)$ in result (4), which in the same logarithmic approximation is

$$g^2(\bar{q}^2) = g^2\left(1+\frac{11g^2}{12\pi^2}\ln\frac{\bar{q}^2}{\bar{q}_0^2}\right)^{-1}, \qquad (5)$$

where g is the renormalized coupling constant and $\bar{q}_0$ is the normalization point in the momentum transfer.

It is clear from expression (5) that in the limit as $\bar{q}^2 \to \infty$, i.e., at short distances, the effective coupling constant tends to zero, $g^2(\bar{q}^2) \to 0$. This allows using the perturbation theory in this limit, as $\bar{q}^2 \to \infty$. The remarkable fact is that the interaction of quarks at small distances is well described by such a vector theory (which has the SU(3) gauge group, however). This is the asymptotic (in the sense of large momenta), or ultraviolet freedom [4 – 6].

On the other hand, at small $\bar{q}^2$, i.e., at large distances, the effective coupling constant

$$g^2(\bar{q}^2) = g^2\left(1+\frac{11g^2}{12\pi^2}\ln\frac{\bar{q}^2}{\bar{q}_0^2}\right)^{-1},$$

increases and the perturbation theory becomes inapplicable. The interaction between quarks at large distances becomes so strong that quarks do not exist in a free state at all. This is the region of quark confinement, or infrared slavery. A closed quantitative theory that describes quark confinement does not exist at present.

5. I have to add that paper [1] is in no way the only study Isaak Markovich Khalatnikov made for all time.

And last but not least, I have heard that to his centenary

For his Merit and his classic works,
Order they will add to his awards!
He'll get certainly this honor,
And to celebrate the Order
We will get together here of course!

* * *

I am grateful to V V Sokolov and M I Vysotsky for the discussions. The work was supported by the Russian Foundation for Basic Research Grant No. 08-02-00960-a.

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