Physics – Uspekhi **53** (12) 1281 – 1286 (2010)

METHODOLOGICAL NOTES

PACS numbers: 62.25.Fg, 62.25.Jk, 63.20.Ry, 82.20.Rp

IN MEMORY OF VITALY LAZAREVICH GINZBURG

Vibrational analogue of nonadiabatic Landau – Zener tunneling and a possibility for the creation of a new type of energy traps

Yu A Kosevich, L I Manevitch, E L Manevitch

DOI: 10.3367/UFNe.0180.201012e.1331

Contents

Abstract. The problem of irreversible targeted energy transfer is approached in a new way using the analogy between a system of two weakly coupled parametric pendulums or oscillators and nonadiabatic Landau – Zener tunneling in a two-state quantum system. This analogy predicts that efficient irreversible transfer of vibrational energy is possible between two subsystems if the frequency of at least one of them changes adiabatically slowly with time, thus allowing an internal resonance to occur between them. We also show that evolution equations for the transition of the Landau – Zener tunneling type give a quantitative prediction for the part of the initially imparted energy that is retained asymptotically in the protected classical system. The findings made can be used for designing new types of energy traps for the dynamical protection of various mechanical systems.

1. Introduction

Tunneling is one of the most striking manifestations of quantum behavior and has been the subject of extensive research in both fundamental and applied physics [1]. A well-known generic example of a tunneling phenomenon is Landau–Zener tunneling (LZT) in which a quantum system subject to an external force tunnels across an energy gap between two anticrossing energy levels [2–5] (see also recent review [6]). Quantum LZT was observed in semiconductor superlattices for electrons [7, 8], as well as in optical lattices for ultracold atoms [9, 10] and Bose–Einstein condensates [11, 12]. In the case of electrons in a semiconductor superlattice, the external force responsible for nonadiabatic energy-level crossing and LZT is exerted by an external electric field. Gravitation or acceleration fields play a similar role for ultracold atoms and Bose–Einstein condensates. Landau–Zener tunneling of optical waves has been observed in optical lattices [13] and optical waveguide arrays [14]. Recently, the Landau–Zener tunneling of bulk and surface acoustic waves in ultrasonic superlattices was predicted and observed [15, 16]. In addition, these predictions and observations were also extended to macroscopic two-dimensional phononic crystals made of rigid cylinders immersed in water [17, 18]. Effective external forces in optical or acoustic LZT are produced by the perturbation of the corresponding optical or ultrasonic superlattice.

The common feature of all the aforementioned examples of nonadiabatic LZT is the irreversible (and almost unidirectional) exchange of energy between two energy states caused by an external force or perturbation. The possibility of this type of exchange would also be desirable in vibrating mechanical systems, e.g., in bridges, in towers, in airplane wings, and in other structures. (The macroscopic dynamical instability of an automotive bridge, which was observed on 20 May 2010 in the city of Volgograd, gives evidence of the desirability of the construction of such systems.) Here, the impact excitation, which is capable of causing dynamic instability or destruction of the mechanical system, must be irreversibly transferred to another system which plays the role of an energy trap. It turns out that a classical system governed by equations similar to those of a quantum system can in fact be designed. We noticed earlier a profound analogy between *adiabatic* quantum tunneling and energy exchange between weakly coupled linear or nonlinear classical oscillators [19–21]. In this study, we present for the first time a vibrational analogue of *nonadiabatic* quantum Landau–Zener tunneling, which reveals a possibility for the creation of a new type of energy traps. We demonstrate analytically and numerically that a transition of the LZT type can be realized in a system of

Yu A Kosevich, L I Manevitch Semenov Institute of Chemical Physics, Russian Academy of Sciences,
ul. Kosygina 4, 119991 Moscow, Russian Federation
Tel. (7-495) 939 71 39. Fax (7-495) 137 82 84
E-mail: yukosevich@gmail.com
E L Manevitch Department of Mechanical Science and Engineering, University of Illinois at Urbana-Champaign, Urbana, IL 61801, USA

Received 31 May 2010, revised 3 August 2010
Uspekhi Fizicheskikh Nauk **180** (12) 1331 – 1336 (2010)
DOI: 10.3367/UFNr.0180.201012e.1331
Translated by Yu A Kosevich, L I Manevitch; edited by A Radzig

two weakly coupled pendulums or oscillators. For this to occur, the length of at least one of the pendulums (or the mass or spring stiffness of at least one of the oscillators) should be changed adiabatically slow during the vibrations. As a result, an efficient *irreversible* transfer of vibrational energy from one pendulum or oscillator to the other occurs under the condition that the weakly coupled subsystems pass through the internal resonance. Precisely such resonant vibrational mechanical systems represent the simplest energy traps of a new type, which can be utilized in more complex systems for their dynamical protection from vibro-impact actions, with possible applications in macro-, micro-, and nanomechanics.

It is worth mentioning that we do not claim a strict analogy with the quantum LZT in an infinite time interval, which was considered both in the papers by L Landau and C Zener and in numerous subsequent papers. The papers by Dykhne [22], Demkov and Osherov [23], and Demkov and Kunike [24] allow one to describe the nonadiabatic transition which occurs in a finite time interval (see also more recent paper [25]). Naturally, accounting for the finiteness of the time interval for the energy exchange between the subsystems is very important for mechanical systems. The analogy we are discussing is due to the fact that the two weakly coupled classical subsystems in the vicinity of the internal resonance show the behavior similar to that in the case of quantum LZT. The essence of the described effect lies in the irreversible energy transfer from one subsystem to another, which is determined by the system behavior within a finite time interval. It turns out that this time interval is relatively narrow in comparison with the characteristic time of the transition of the Landau–Zener tunneling type [6, 26, 27], but this time interval remains wide enough in comparison with the period of classical vibrations. As we will show, the deviation from a behavior similar to that in the case of quantum LZT turns out to be inessential for the irreversible energy transfer in mechanical systems far from the point of internal resonance.

From the physical point of view, the irreversible energy exchange in question can be considered as the targeted energy transfer which is under active discussion in recent literature (see Refs [19–21, 28–30]). However, the mechanisms of irreversible targeted energy transfer considered in these papers are based on nonlinear resonance at which the nonlinearity in the system of interest is assumed to be rather strong. In contrast to that, the mechanisms of irreversible energy exchange considered in our paper are revealed in linear systems, although, as we will show in Section 4, these mechanisms also remain in nonlinear systems. The possibility of irreversible energy exchange in a linear mechanical system was recently predicted and verified experimentally [31].

2. Statement of the problem

We consider a system of two plane pendulums with lengths l_1 and l_2, and masses m_1 and m_2, weakly coupled by a spring (with a length comparable to l_1). The Lagrange function of the system is written as follows:

$$L = \frac{1}{2}\left[m_1 l_1^2\left(\frac{\mathrm{d}\varphi_1}{\mathrm{d}t}\right)^2 + m_2 l_2^2\left(\frac{\mathrm{d}\varphi_2}{\mathrm{d}t}\right)^2\right] - g\left[m_1 l_1(1-\cos\varphi_1) + m_2 l_2(1-\cos\varphi_2)\right] - \frac{1}{2}k_{12}(l_1\sin\varphi_1 - l_2\sin\varphi_2)^2\,, \tag{1}$$

where φ_1 and φ_2 are the deflection angles, and k_{12} is the spring constant. Let l_1 be a constant, and l_2 be a specified function of time. Then the corresponding equations of motion are written out as

$$\frac{\mathrm{d}^2\varphi_1}{\mathrm{d}t^2} + \frac{g}{l_1}\sin\varphi_1 + \frac{k_{12}}{m_1}\cos\varphi_1\left(\sin\varphi_1 - \frac{l_2}{l_1}\sin\varphi_2\right) = 0\,,$$
$$\frac{\mathrm{d}^2\varphi_2}{\mathrm{d}t^2} + \frac{2}{l_2}\frac{\mathrm{d}l_2}{\mathrm{d}t}\frac{\mathrm{d}\varphi_2}{\mathrm{d}t} + \frac{g}{l_2}\sin\varphi_2 + \frac{k_{12}}{m_2}\cos\varphi_2\left(\sin\varphi_2 - \frac{l_1}{l_2}\sin\varphi_1\right) = 0\,. \tag{2}$$

We assume further that

$$l_2(t) = l_1\big(1 + \varDelta_2(t)\big)\,, \tag{3}$$

where $\varDelta_2(t)$ describes a relatively small change of l_2 with time. In the following we consider three functions $\varDelta_2(t)$ — one smooth and two piecewise smooth functions:

$$\varDelta_2(t) = \delta_2 - f_2 T_2 \tanh\frac{t}{T_2}\,, \tag{4}$$

$$\varDelta_2(t) = \begin{cases} \delta_2 - f_2 t\,, & 0 < t \leqslant T_2\,, \\ \delta_2 - f_2 T_2\,, & t > T_2\,, \end{cases} \tag{5}$$

$$\varDelta_2(t) = \begin{cases} \delta_2 - g_2 t^2\,, & 0 < t \leqslant T_2\,, \\ \delta_2 - g_2 T_2^2\,, & t > T_2\,, \end{cases} \tag{6}$$

where $\delta_2, f_2/\omega_1$, and g_2/ω_1^2 are independent small parameters of the same sign, and $\omega_1 = \sqrt{g/l_1}$. Saturation time $T_2 > \delta_2/f_2$, $T_2 > \sqrt{\delta_2/g_2}$, $T_2 \gg 1/\omega_1$ makes l_2 positive at any instant for positive δ_2, f_2, and g_2. As we will show in Sections 3 and 4, all the functions (4), (5), and (6), which describe both linear and quadratic in time changes in l_2 for $t < T_2$, lead to the irreversible energy exchange between the pendulums with qualitatively similar features. This confirms that the effect revealed by us indeed has the generic origin.

3. Irreversible energy exchange in the linear regime

Since the LZT is basically a linear phenomenon, we start the analysis of irreversible energy exchange process with considering linearized equations (2) in the limit of $\varphi_1 \ll 1$ and $\varphi_2 \ll 1$. There are several ways to proceed from two real differential equations of the second order to four complex equations of the first order. Following the approach used in Ref. [32], we introduce two complex envelopes a_1 and a_2 of the real deflection angles φ_1 and φ_2:

$$\varphi_{1,2} = \frac{1}{2}\left[a_{1,2}\exp(-\mathrm{i}\omega_1 t) + a_{1,2}^*\exp(\mathrm{i}\omega_1 t)\right]\,, \tag{7}$$

where it is assumed that $\mathrm{d}a_{1,2}/\mathrm{d}t \ll \omega_1 a_{1,2}$.

As follows from equation (7), the real part of the variable a_i determines the envelope of φ_i, while its imaginary part determines the envelope of the dimensionless time derivative $\mathrm{d}\varphi_i/\mathrm{d}t/\omega_1$, $i = 1, 2$. These properties of complex envelopes allow us to relate easily the envelope modulus $|a_i|$ with the vibrational energy of the linearized i-th pendulum: $E_i = 0.5 g l_1 m_i |a_i|^2$.

Substituting equations (3) and (7) into linearized equations (2), we get the following two evolution equations for the complex envelopes a_1 and a_2 in the main approximation with

respect to small parameters δ_2, f_2/ω_1, g_2/ω_1^2, and $k_{12}/\mu\omega_1^2$ [$\mu = 1/(1/m_1 + 1/m_2)$ is a reduced mass of m_1 and m_2]:

$$\mathrm{i}\,\frac{\mathrm{d}a_1}{\mathrm{d}t} = \frac{k_{12}}{2m_1\omega_1}\,(a_1 - a_2)\,,$$
$$\mathrm{i}\,\frac{\mathrm{d}a_2}{\mathrm{d}t} = \frac{k_{12}}{2m_2\omega_1}\,(a_2 - a_1) - \frac{1}{2}\,\omega_1\Delta_2(t)\,a_2\,. \tag{8}$$

We also obtain two corresponding equations for the complex-conjugate envelopes a_1^* and a_2^*. It follows from equations (8) that the total vibrational energy of the coupled linearized pendulums, $0.5gl_1\left(m_1|a_1|^2 + m_2|a_2|^2\right)$, is the integral of motion. For $t < T_2$ and $\Delta_2(t)$ given by equation (4) or equation (5), equations (8) coincide with the modified description of the quantum Landau–Zener transition [3]. The multiple scale expansion procedure presented, for instance, in Ref. [33], leads to similar complex evolution equations.

The same equations (8) describe the dynamics of the complex envelopes of the real displacements u_1 and u_2 of two oscillators with masses m_1 and m_2 and springs with similar modulus of rigidity κ_1, or two oscillators with equal masses m_1 and springs with moduli of rigidity κ_1 and κ_2, weakly coupled by a spring with the modulus of rigidity $k_{12} \ll \kappa_1$, when either m_2 or $1/\kappa_2$ changes in time according to equation (3) (where $l_{1,2}$ should be replaced by $m_{1,2}$ or $1/\kappa_{1,2}$, respectively). Introducing two complex envelopes a_1 and a_2 of the real displacements u_1 and u_2 according to representation (7) in which $\varphi_{1,2}$ should be replaced by displacements $u_{1,2}$, under the same assumption $\mathrm{d}a_{1,2}/\mathrm{d}t \ll \omega_1 a_{1,2}$ we arrive at evolution equations (8) for the complex envelopes a_1 and a_2, in which $\omega_1 = \sqrt{\kappa_1/m_1}$ now. In this case, one should put $m_1 = m_2$ into the evolution equation for a_2.

The tunneling dynamics of two weakly coupled ideal Bose–Einstein condensates in a macroscopic double-well potential are also described by equations similar to equations (8) (see Refs [19–21, 34–37]). In this case, the beatings between the coupled states of the condensates constitute quantum Rabi oscillations, and the parameter $\Delta_2(t)$ in equations (8) describes the time dependence of the depth of the trapping potential in one of the coupled quantum wells. But in contrast to practically linear mechanical systems, ideal (noninteracting and nonsuperfluid) Bose–Einstein condensates do not exist in nature and this limit can be realized only at the magnetically tunable Feshbach resonance (see, e.g., Ref. [38]). [1]

[1] Equations describing the dynamics of the macroscopic tunneling of two weakly linked nonideal Bose–Einstein condensates in a double-well potential [19–21, 34–37] in the mean-field approximation can be derived with the use of the Gross–Pitaevskii equation [39–41]. It is worth mentioning in this connection that an equation similar to the Gross–Pitaevskii equation was derived earlier by V L Ginzburg and L P Pitaevskii [42, 43] in the framework of the theory of superfluid helium near the λ-point (see also the review paper [44].) The dynamical (time-dependent) Ginzburg–Pitaevskii equation for the macroscopic Ψ function of superfluid helium near the λ-point in the single-velocity regime, for $v_n = 0$, and neglecting specifically quantum effects and relaxation phenomena is equivalent to the Gross–Pitaevskii equation for the classical macroscopic Ψ function of an inhomogeneous nonideal Bose–Einstein condensate, although nonlinear terms in these two equations have different origins. The single-velocity regime for the superfluid helium can be realized in narrow capillaries in which the motion of the normal component is blocked by wall friction and the wave of the fourth sound can propagate in the superfluid component [45, 46]. The dispersion and absorption of the fourth sound near the λ-point are affected by the slow relaxation of the small density of the superfluid component, which leads to the appearance of the effective second viscosity in the system [45, 47, 48].

For large positive t, the asymptotic analytical solution of equations (8) with $\Delta_2(t)$ given by equation (4) or equation (5) and the initial conditions $\left|a_1(-\infty)\right|^2 = 1$, $a_2(-\infty) = 0$ has the following form

$$\left|a_1(\infty)\right|^2 = \exp\left(-R\right), \quad R = \frac{\pi k_{12}^2}{m_1 m_2 |f_2| \omega_1^3}\,. \tag{9}$$

This quantity describes the part of the initial vibrational energy of pendulum 1 that is retained asymptotically in this subsystem (at a large delay time).

Remarkably, the 'tunneling' exponential function (9) for the system of two classical particles corresponds to the nonperturbative approximation, as in the case of a quantum Landau–Zener transition, but it does not contain the Planck constant — the cornerstone parameter in quantum mechanics. Function (9) takes into account, inter alia, that only one of the coupled pendulums is driven parametrically and that the inequality of pendulum masses takes place in general. In the case of positive δ_2 and f_2, the applicability of equation (9) to classical systems, which are described by equations (2)–(5), is limited by the finite width of the 'time window' of the parametric drive: $t \in (0, T_2)$, and $T_2 \leqslant (1 + \delta_2)/f_2$. Exponentially small probability $|a_1(\infty)|^2$ of the 'survival amplitude' of the final state of subsystem 1 implies the existence of a wide enough time window for the parametric drive (as in the case of quantum LZT). Therefore, equation (9) cannot be applied to classical oscillator systems with high values of the parameter R and mass ratio m_1/m_2. Parameter δ_2 entering into Eqns (4)–(6) effectively stretches out this time window.

To check the efficiency of the system of pendulums in the capacity of an energy trap, we simulated the time evolution of vibrational energies of the coupled pendulums from the solution of linearized input equations (2) for real deflection angles φ_1 and φ_2, and compared it with the numerical solution of equations (8) for the complex envelopes a_1 and a_2, which are similar to the equations describing quantum transition like Landau–Zener tunneling. Since the damping of low-frequency vibrations of pendulums is very small, we neglected the damping effect on the energy exchange between the pendulums.

Time-dependent vibrational energies E_1 and E_2 of pendulums 1 and 2 with equal masses and their total energy E_T, obtained as solutions of linearized equations (2) (solid lines *1–3*), together with the solution of equations (8) describing the LZT type transition (dashed lines *4* and *5*), and with the energy given by Eqn (9) that is retained asymptotically in pendulum 1 (solid line *6*) are shown in Figs 1a–c, respectively, for $\Delta_2(t)$ given by equations (4), (5), or (6). The initial conditions correspond to the impact excitation of pendulum 1. The following realistic parameters and initial conditions were taken in simulations: $l_1 = 0.305$ m, $m_1 = 0.244$ kg, $k_{12} = 0.785$ N m^{-1}, $\delta_2 = 0.22$, $f_2 = 0.0625$ s^{-1}, and $T_2 = 15.6$ s in Figs 1a, b; $g_2 = 0.0177$ s^{-2} and $T_2 = 8$ s in Fig. 1c, and

$$\varphi_1(0) = 0,\ \varphi_2(0) = 0,\ \dot\varphi_2(0) = 0,\ \dot\varphi_1(0) = 0.61 \text{ rad s}^{-1},$$
$$\mathrm{i}a_1(0) = \frac{\dot\varphi_1(0)}{\omega_1},\ a_2(0) = 0\,. \tag{10}$$

As evident from Fig. 1, an irreversible and intensive energy flow from pendulum 1 to pendulum 2 occurs. One can also conclude from this figure that equations (8) describing an LZT type transition correctly reflect the

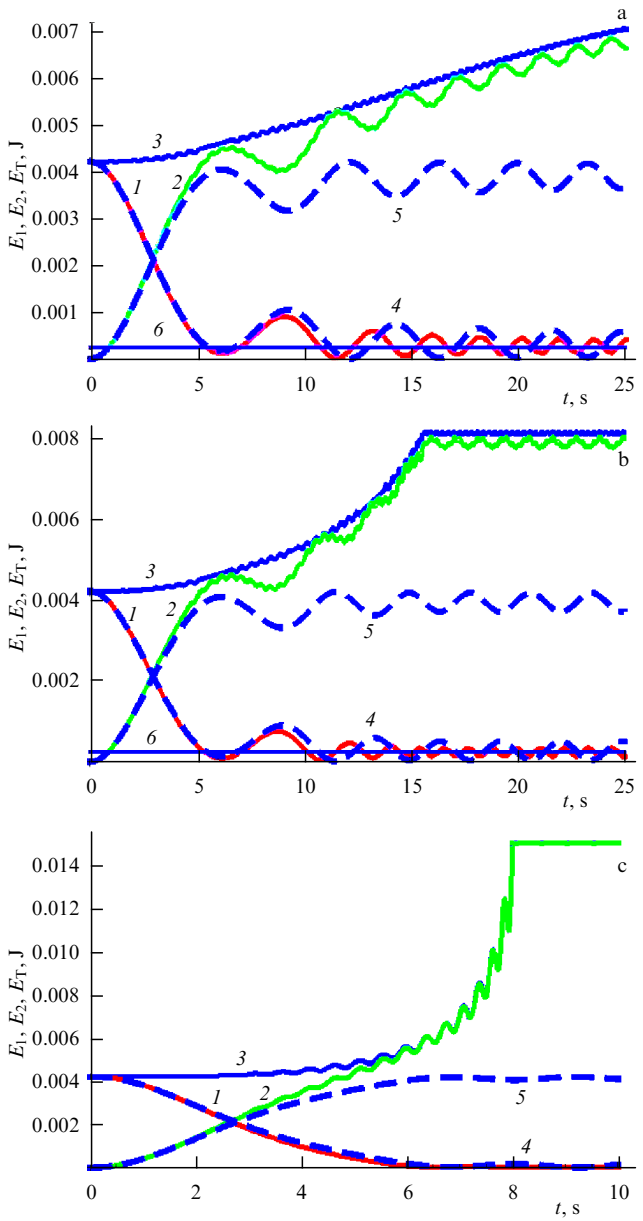

Figure 1. The vibrational energies E_1 and E_2 of pendulums 1 and 2 and their total energy E_T vs. time as the solutions of linearized equations (2) (solid lines *1–3*), and the vibrational energies of pendulums 1 and 2 as the solutions of evolution equations (8) describing the LZT type transition (dashed lines *4* and *5*) for the values of $\Delta_2(t)$ given by (a) equation (4), (b) equation (5), and (c) equation (6). The part of the imparted initial vibrational energy that is retained asymptotically in pendulum 1, given by equation (9), is shown by solid line 6 in plots (a) and (b). Parameters used in the calculations are given by equations (3)–(6) and (10) in the case of equal pendulum masses.

regularities of the process during its initial stage, when the most intensive resonance energy exchange proceeds. The prediction for the part of the initial vibrational energy that is retained asymptotically in pendulum 1 is also impressively confirmed in our simulations, although the factor R in equation (9) is not small for the parameters utilized in Fig. 1: $R = 2.85$. According to our simulations, this value of R gives an approximate upper limit of the applicability of equation (9) for the classical systems considered. A large enough saturation time T_2 only influences the transient dynamics of energy exchange without affecting the asymptotic value (9) of the energy of pendulum 1 (as in the case of LZT in the two-level quantum system which is described by equations similar to equations (4) and (8); see Refs [24, 25]).

The exact internal resonance is reached in the system under consideration at the instant when $l_2 = l_1$ (or $m_2 = m_1$, $\kappa_2 = \kappa_1$) and the eigenfrequencies of the coupled pendulums (or oscillators) become equal. This occurs at $t = \delta_2/f_2$ or $t = \sqrt{\delta_2/g_2}$. As the system moves out of the resonance (for $t > \delta_2/f_2$ or $t > \sqrt{\delta_2/g_2}$), there is no considerable reverse energy flow from pendulum 2 to pendulum 1. It is precisely this phenomenon that allows considering pendulum 2 as an energy trap. It is also worth mentioning that the irreversibility of the energy transfer is achieved in the linear systems considered only due to the parametric drive and the LZT-like dynamic behavior, but not due to nonlinear resonance as, for example, in the paper [29], where nonlinear resonance has played, in contrast, an important role in the irreversible energy transfer in the system of interest.

Importantly, for the times $t > 2\delta_2/f_2$ or $t > \sqrt{2\delta_2/g_2}$ we are dealing with a phenomenon that is beyond the considered analogy with the LZT. Nevertheless, the irreversible energy transfer goes on even though the energy of the first pendulum has already substantially decreased (see Fig. 1). The quantity $m_1|a_1|^2 + m_2|a_2|^2$ is not conserved anymore, even approximately. Essentially, in all the cases presented in Fig. 1 those characteristics of the process considered that are important for possible applications, such as the time evolution and

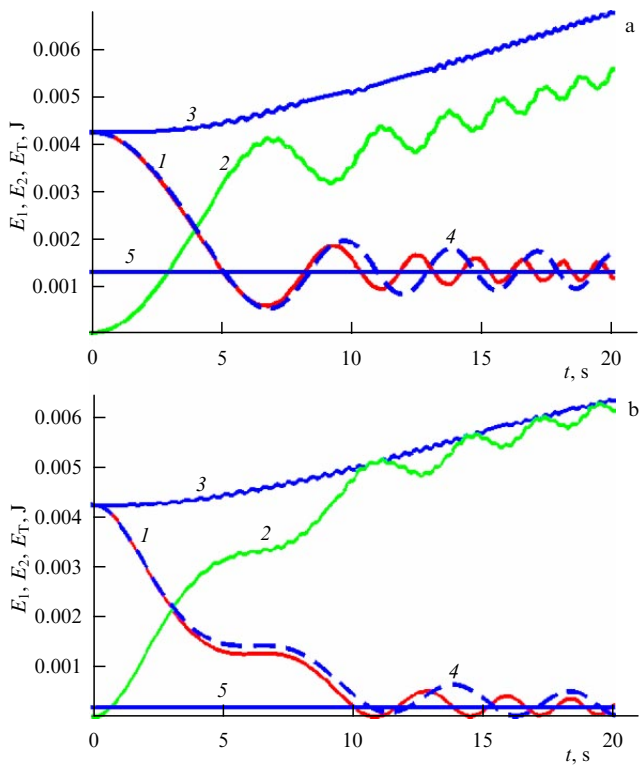

Figure 2. The vibrational energies E_1 and E_2 of pendulums 1 and 2 with nonequal masses and their total energy E_T vs. time as the solutions of linearized equations (2) (solid lines *1–3*), and the vibrational energy of pendulum 1 as the solution of evolution equations (8) describing the LZT type transition (dashed line *4*) for (a) $m_2 = 0.5m_1$, $k_{12} = 0.3925$ N m^{-1}, $\delta_2 = 0.26325$, $f_2 = 0.07476$ s^{-1}, and (b) $m_2 = 2m_1$, $k_{12} = 1.1775$ N m^{-1}. The rest of the parameters are given by equations (3), (4), and (10). The part of the imparted initial vibrational energy that is retained asymptotically in pendulum 1 [see equation (9)] is shown by solid line *5*.

averaged asymptotic value of the vibrational energy of pendulum 1, are correctly described by equations (8) and (9), which are related to conservative LZT type transitions (dashed line *4* in all plots in Fig. 1). At the same time, the classical systems under consideration are inherently nonconservative.

Our calculations also demonstrate that the employment of a larger or smaller mass for pendulum 2 does not suppress the irreversible energy transfer from pendulum 1 (see Fig. 2). By the proper choice of the parameters δ_2 and k_{12} together with the mass ratio m_1/m_2, we can reach a good agreement with the predictions given by equations (8) and (9) describing the LZT type transition, both for $m_2 < m_1$ (see Fig. 2a at $m_2 = 0.5m_1$) and for $m_2 > m_1$ (see Fig. 2b at $m_2 = 2m_1$). These calculations, together with the results shown in Fig. 1, confirm the robustness of the revealed effect of irreversible energy exchange in the linear regime.

4. Irreversible energy exchange in the nonlinear regime

In this section we will describe the effect of the nonlinear properties of the coupled pendulums or oscillators on the irreversible energy exchange between them. This effect, being described in the general case by equations (2), strengthens with an increase in the initial momentum imparted to pendulum 1, which is proportional to $\dot{\varphi}_1(0)$. In general, the nonlinearity enlarges that part of the initial vibrational energy that is retained asymptotically in pendulum 1 (cf. Fig. 1b for $\dot{\varphi}_1(0) = 0.61$ rad s^{-1} with Fig. 3a for $\dot{\varphi}_1(0) = 7.42$ rad s^{-1}, when the rest of the parameters in Fig. 3a are the same as in Fig. 1b). But the parametric system under consideration possesses the effective separatrix which detaches two modes with the intensive and weakened energy exchanges. As one can see in Fig. 3, the relatively small change in the initial momentum imparted to pendulum 1 (from $\dot{\varphi}_1(0) = 7.42$ rad s^{-1} in Fig. 3a to $\dot{\varphi}_1(0) = 8.19$ rad s^{-1} in Fig. 3c) results in an apparent change from an almost complete to a relatively weak exchange of vibrational energy. The separatrix mode (shown in Fig. 3b for $\dot{\varphi}_1(0) = 7.93$ rad s^{-1}) detaches the two modes with different degrees of completeness of the energy exchange. In the energy transfer mode shown in Fig. 3c, pendulum 2 at $t \approx 10$ s finds itself in the whirling mode. In this mode, the reverse energy flow to pendulum 1 is suppressed and the irreversible character of energy transfer is correspondingly enhanced. A similar transition between the two modes of targeted energy transfer, determined by the existence of the separatrix, is known in passive nonlinear systems in which the nonlinearity substantially affects both the rate and completeness of the energy transfer (see Refs [19–21, 28–30]). Our results demonstrate that nonlinearities of the coupled elements can substantially influence the targeted energy transfer in active (parametric) systems as well.

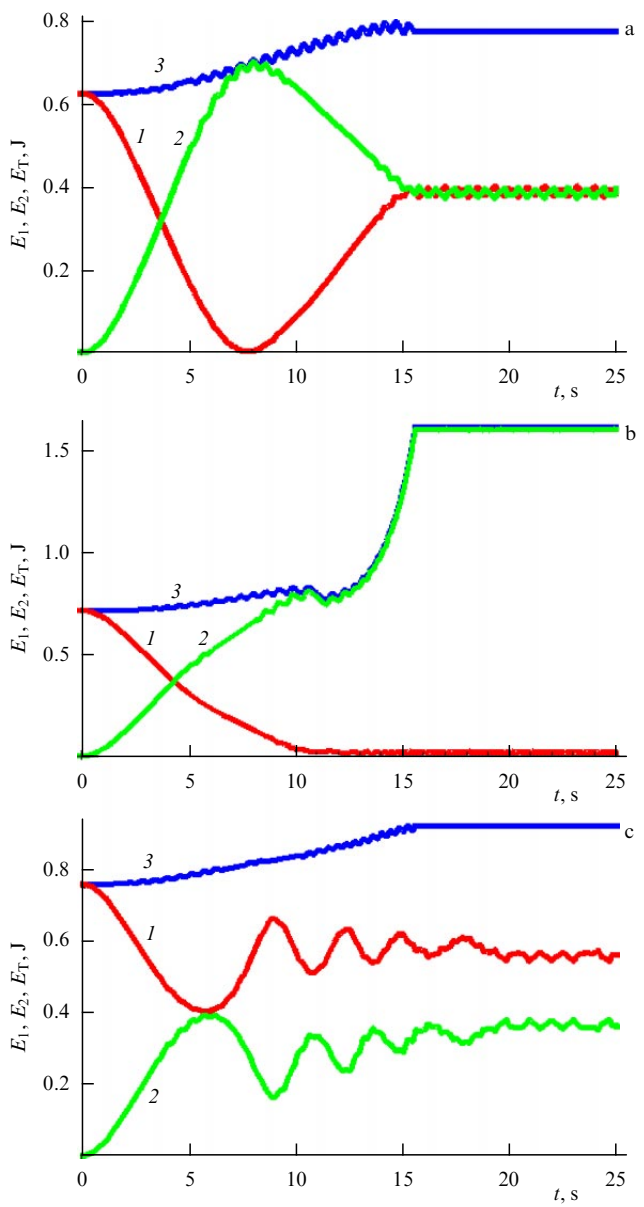

Figure 3. The vibrational energies E_1 and E_2 of pendulums 1 and 2 and their total energy E_T vs. time as the solutions of nonlinear Eqns (2), (3), and (5) (lines *1*, *2*, and *3*, respectively) for (a) $\dot{\varphi}_1(0) = 7.42$ rad s^{-1}, (b) $\dot{\varphi}_1(0) = 7.93$ rad s^{-1}, and (c) $\dot{\varphi}_1(0) = 8.19$ rad s^{-1}. Parameters utilized in the calculations are given by equations (3), (5), and (10) in the case of equal pendulum masses. Plot (b) corresponds to the separatrix mode of the targeted energy transfer.

5. Conclusion

Thus, we presented a novel principle of 'trapping' of the vibrational energy. This principle is based on the profound analogy between the irreversible transfer of the vibrational energy in a classical parametric system and nonadiabatic quantum Landau–Zener tunneling. We demonstrated analytically and numerically that in a system of two weakly coupled pendulums or oscillators an efficient irreversible transfer of vibrational energy from one subsystem to another occurs under the condition that the coupled subsystems pass adiabatically slow through the internal resonance. Nonlinear effects can enhance the irreversible character of vibrational energy transfer between the subsystems. The revealed phenomena open up the possibility of designing fundamentally new types of energy traps for the dynamic protection of various mechanical systems. Our simulations also show that the complex evolution equations describing the Landau–Zener quantum transition can give a quantitative prediction for the part of the imparted initial vibrational energy that is retained asymptotically in the 'protected' mechanical subsystem. This means that the regularities of quantum Landau–Zener tunneling can also be experimentally studied with the employment of classical systems.

We dedicate this paper to the memory of V L Ginzburg. Vitaly Lazarevich showed a deep interest in the problems of nonstationary dynamics in different areas of physics, first and foremost in the problem of the transition radiation of moving charged particles.

The authors are grateful to G M Sigalov for the constructive remarks.

The work was performed with a financial support from RFBR (through grant No. 10-01-00698).

References

1. Razavy M *Quantum Theory of Tunneling* (Singapore: World Scientific, 2003)
2. Landau L *Phys. Z. Sowjetunion* **2** 46 (1932)
3. Zener C *Proc. R. Soc. London A* **137** 696 (1932)
4. Stückelberg E C G *Helv. Phys. Acta* **5** 369 (1932)
5. Majorana E *Nuovo Cimento* **9** 43 (1932)
6. Pokrovsky V L *Usp. Fiz. Nauk* **179** 1237 (2009) [*Phys. Usp.* **52** 1169 (2009)]
7. Schneider H et al. *Phys. Rev. Lett.* **65** 2720 (1990)
8. Rosam B et al. *Phys. Rev. B* **68** 125301 (2003)
9. Bharucha C F et al. *Phys. Rev. A* **55** R857 (1997)
10. Anderson B P, Kasevich M A *Science* **282** 1686 (1998)
11. Cristiani M et al. *Phys. Rev. A* **65** 063612 (2002)
12. Sias C et al. *Phys. Rev. Lett.* **98** 120403 (2007)
13. Ghulinyan M et al. *Phys. Rev. Lett.* **94** 127401 (2005)
14. Trompeter H et al. *Phys. Rev. Lett.* **96** 023901 (2006)
15. Sanchis-Alepuz H, Kosevich Yu A, Sánchez-Dehesa J *Phys. Rev. Lett.* **98** 134301 (2007)
16. de Lima M M (Jr.), Kosevich Yu A, Santos P V, Cantarero A *Phys. Rev. Lett.* **104** 165502 (2010)
17. Sánchez-Dehesa J, Sanchis-Alepuz H, Kosevich Yu A, Torrent D *J. Acoust. Soc. Am.* **120** 3283 (2006)
18. He Z et al. *Phys. Rev. E* **76** 056605 (2007)
19. Kosevich Y A, Manevitch L I, Savin A V *J. Phys. Conf. Ser.* **92** 012093 (2007)
20. Kosevich Yu A, Manevitch L I, Savin A V *Phys. Rev. E* **77** 046603 (2008)
21. Kosevich Yu A, Manevitch L I, Savin A V *J. Sound Vibr.* **322** 524 (2009)
22. Dykhne A M *Zh. Eksp. Teor. Fiz.* **41** 1324 (1961) [*Sov. Phys. JETP* **14** 941 (1962)]
23. Demkov Yu N, Osherov V I *Zh. Eksp. Teor. Fiz.* **53** 1589 (1967) [*Sov. Phys. JETP* **26** 916 (1968)]
24. Demkov Yu N, Kunike M *Vestn. Leningr. Univ. Ser. Fiz. Khim.* **16** 39 (1969)
25. Suominen K-A, Garraway B M *Phys. Rev. A* **45** 374 (1992)
26. Niu Q, Raizen M G *Phys. Rev. Lett.* **80** 3491 (1998)
27. Vitanov V N *Phys. Rev. A* **59** 988 (1999)
28. Kopidakis G, Aubry S, Tsironis G P *Phys. Rev. Lett.* **87** 165501 (2001)
29. Maniadis P, Aubry S *Physica D* **202** 200 (2005)
30. Vakakis A F et al. *Nonlinear Targeted Energy Transfer in Mechanical and Structural Systems* (Berlin: Springer, 2008)
31. Manevitch L I et al. *Int. J. Non-Linear Mech.* **46** 247 (2011)
32. Kosevich Yu A, Lepri S *Phys. Rev. B* **61** 299 (2000)
33. Manevich A I, Manevitch L I *The Mechanics of Nonlinear Systems with Internal Resonances* (London: Imperial College Press, 2005)
34. Milburn G J et al. *Phys. Rev. A* **55** 4318 (1997)
35. Smerzi A et al. *Phys. Rev. Lett.* **79** 4950 (1997)
36. Raghavan S et al. *Phys. Rev. A* **59** 620 (1999)
37. Nistazakis H E et al. *Phys. Rev. A* **78** 023635 (2008)
38. Inouye S et al. *Nature* **392** 151 (1998)
39. Pitaevskii L P *Zh. Eksp. Teor. Fiz.* **40** 646 (1961) [*Sov. Phys. JETP* **13** 451 (1961)]
40. Gross E P *Nuovo Cimento* **20** 454 (1961)
41. Lifshitz E M, Pitaevskii L P *Statisticheskaya Fizika* (Statistical Physics) Pt. 2 (Moscow: Fizmatlit, 2001) [Translated into English (Oxford: Butterworth-Heinemann, 2002)]
42. Ginzburg V L, Pitaevskii L P *Zh. Eksp. Teor. Fiz.* **34** 1240 (1958) [*Sov. Phys. JETP* **7** 858 (1958)]
43. Pitaevskii L P *Zh. Eksp. Teor. Fiz.* **35** 408 (1958) [*Sov. Phys. JETP* **8** 282 (1959)]
44. Ginzburg V L, Sobyanin A A *Usp. Fiz. Nauk* **120** 153 (1976) [*Sov. Phys. Usp.* **19** 773 (1976)]
45. Landau L D, Lifshitz E M *Gidrodinamika* (Fluid Mechanics) (Moscow: Nauka, 1986) [Translated into English (Oxford: Pergamon Press, 1987)]
46. Khalatnikov I M *Teoriya Sverkhtekuchesti* (Theory of Superfluidity) (Moscow: Nauka, 1971)
47. Mandelstam L I, Leontovich M A *Zh. Eksp. Teor. Fiz.* **7** 438 (1937)
48. Landau L D, Khalatnikov I M *Dokl. Akad. Nauk SSSR* **96** 469 (1954)