

LETTERS TO THE EDITORS

PACS number: **71.36.+c**

On the 70th anniversary of the Leontovich boundary condition

(comments on "Generalization of the Leontovich approximation for electromagnetic fields on a dielectric – metal interface" by V I Alshits and V N Lyubimov)

A V Guglielmi

DOI: 10.3367/UFNe.0180.201001g.0105

Abstract. The question of Leontovich's 70-year old boundary condition at the surface of a well conducting body is discussed. Doubt is cast on the assertion by Alshits and Lyubimov (*Usp. Fiz. Nauk* 179 865 (2009) [*Phys. Usp.* 52 815 (2009)]) that this boundary condition is more accurate than Leontovich himself believed. The 1940 paper by Rytov written on the suggestion of Leontovich is indicated as evidence for the latter's full awareness of exactly how accurate his proposed boundary condition was.

The Leontovich impedance boundary condition

$$\mathbf{E}_t = \zeta \mathbf{H}_t \times \mathbf{n} \tag{1}$$

holds true approximately on surfaces of well conducting bodies [1]. Here, $\mathbf{E}_t$ and $\mathbf{H}_t$ are the tangent components of the electric and magnetic fields, respectively, $\mathbf{n}$ is the inward normal to the body surface, and ζ is the surface impedance. The Leontovich condition has been used for solving electrodynamic problems in many fields covering the extremely wide spectral range of vacuum wavelengths from 4×10^{-5} cm in optics [2] to 10^{14} cm in geophysics [3].

M A Leontovich formulated boundary condition (1) in the late 1930s. It is difficult to specify the time more exactly. By a strange quirk of fate, the result was only published after World War II; however, radiophysicists were aware of it earlier. For example, we may recall the article by S M Rytov [4] presented to *ZhETF* on December 19, 1939. The article was written at the suggestion of Leontovich who, judging from the context of paper [4], stated the problem of finding corrections to formula (1) in the framework of an asymptotical theory of the skin-effect.

In the present comments I question the validity of the following sentences by V I Alshits and V N Lyubimov: "It is interesting that Leontovich himself paradoxically underestimated the accuracy of the approximation... The accuracy of an impedance approximation turns out to be higher than its developer himself believed" [2]. From formulas presented in Ref. [4] it is quite obvious that Leontovich and Rytov correctly estimated the accuracy of formula (1). Namely, according to Ref. [4], the corrections to formula (1) in a certain class of models of electromagnetic wave propagation start with the cubic term rather than with the quadratic term with respect to small parameter ζ. Alshits and Lyubimov [2] independently come to the same conclusion; however, in contrast to their opinion, we do not assume Leontovich was wrong in estimating the accuracy of formula (1).

Corrections cubic in ζ break the local character of the boundary condition. Consider, for example, a homogeneous conducting body that occupies half-space $z \geqslant 0$ and borders a vacuum, and assume that the electromagnetic field depends on x and z being, however, independent of y, and that it has a TM-wave structure. Then at $z = 0$ we have

$$E_x = \zeta H_y + \frac{c^2 \zeta^3}{2\omega^2} \frac{\partial^2 H_y}{\partial x^2}\,, \tag{2}$$

where ω is the wave frequency, and c is the speed of light. Equation (2) was easily derived in Ref. [5] by means of the Leontovich parabolic equation and it was directly stressed that the more general, albeit more complicated, formula similar to Eqn (2) had been earlier obtained in another way in Ref. [4]. One can see that the quadratic in ζ correction to condition (1) is indeed absent. But this property of the boundary condition is immediately broken if the body surface is not a plane. In other words, the boundary condition acquires the terms on the order of ζ^2 if the surface curvature is taken into account. Leontovich was aware of this fact, as well.

It should be noted that these remarks only concern the historical aspect of the problem. I believe work [2] is interesting and important as a whole.

References

1. Landau L D, Lifshitz E M *Elektrodinamika Sploshnykh Sred* (Electrodynamics of Continuous Media) (Moscow: Nauka, 1982) [Translated into English (Oxford: Pergamon Press, 1984)]
2. Alshits V I, Lyubimov V N *Usp. Fiz. Nauk* **179** 865 (2009) [*Phys. Usp.* **52** 815 (2009)]
3. Guglielmi A V *Usp. Fiz. Nauk* **158** 605 (1989) [*Phys. Usp.* **32** 678 (1989)]
4. Rytov S M *Zh. Eksp. Teor. Fiz.* **10** 180 (1940)
5. Guglielmi A V *Pis'ma Zh. Eksp. Teor. Fiz.* **89** 439 (2009) [*JETP Lett.* **89** 377 (2009)]

A V Guglielmi O Yu Schmidt Institute of Physics of the Earth, Russian Academy of Sciences,
ul. B. Gruzinskaya 10, 123995 Moscow, Russian Federation
Tel. (7-495) 582 99 71
E-mail: guglielmi@mail.ru

Received 24 August 2009
Uspekhi Fizicheskikh Nauk **180** (1) 105 – 106 (2010)
DOI: 10.3367/UFNr.0180.201001g.0105
Translated by N A Raspopov; edited by A Radzig