order for $(D, F)_{\pm 1}^{\mathrm{E,H}}$ to two sets of the fourth order. In this case, as a result of the hybridization of electric and magnetic components, there appear individual resonances determined by the vanishing the factor $\varepsilon + 1$ in the electric components. The renormalization factors $G$, which are responsible for the collective effects, also change; however, because of the retention of the polarization degeneracy, the new $G_y^{\mathrm{E,H}}$ components are then determined by only quasistatic fields, as before.

If we take the gyrotropy into account, then in the dipole approximation, the system analogous to (6) remains a general system of the eighth order, and hybridization must also occur in the vector $x$ and $y$ components of the electric and magnetic dipole moments, causing a mixing of the equations for the $x$ and $y$ projections of the fields and dipole moments. In the dipole components $D_{\pm 1}$, all the resonances are then present, both individual and collective. Because these resonances are shifted in the different components due to the gyrotropy, resonance effects of the polarization plane rotation must be observed in both the reflected and the transmitted radiation. Especially promising for the enhancement of the magnetooptical effects seems to be the diffraction resonance; because of the presence of singularities in frequency in the derivatives of the excitation coefficient of the open channel, the difference in the excitation coefficients of the left-handed and right-handed components are anomalously large, which should lead to anomalously strong Kerr and Faraday effects. It is well possible that a significant enhancement of magnetooptical effects observed recently in experiments [17] is connected precisely with this mechanism.

## 5. Conclusions

We have considered the influence of nanoinhomogeneities on the magnetooptical effects in ferromagnetic films. It has been demonstrated that as a result of the retardation of waves in nanowaveguides and the presence of individual internal resonances in the waveguides and collective effects of multiple scattering, the magnetooptical effects can be considerably enhanced. We expect that the effects of a resonant enhancement of magnetooptical effects can be used for creating new devices for the recording and processing of information and for the diagnostics of magnetic states in composite ferromagnetic films.

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# Superresolution and enhancement in metamaterials

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## 1. Introduction

We discuss the optical and microwave properties of artificial materials that can have negative dielectric and magnetic constants simultaneously. Backward electromagnetic waves can propagate in such metamaterials, which leads to a negative refraction. We discuss some unusual properties of metamaterials, in particular, the effect of superresolution. The large losses predicted in such materials in optics can be compensated by using an amplifying laser medium. We also consider the possibility of designing a nanolaser with a size several dozen times less than the wavelength of light. This article is intended as a general introduction to this thriving field.

More than 100 years have passed since Lamb's work appeared [1], where he first noted the possibility of the existence of backward waves, i.e., unusual wave processes with oppositely directed phase and group velocities. The properties of backward electromagnetic waves were also discussed by Schuster [2]. Almost simultaneously, in the article "Growth of a wave-group when the group velocity is negative," Pocklington [3] showed that in a medium that supports backward waves, a point source excites *convergent* waves, and the group velocity of waves is directed from the source. These works did not attract much attention for almost 40 years, until the well-known work of Mandel'shtam [4] was published, in which he predicted a new physical phenomenon, the negative refraction. This phenomenon can exist only in the case where the refracted waves propagate in a medium that supports backward waves. A discussion of article [4] can be found, for example, in recent work [5].

The next important step was made by Sivukhin in Ref. [6], where it was first shown that in a medium with simultaneously negative dielectric ($\varepsilon$) and magnetic ($\mu$) permeabilities, the group and phase velocities of the wave are oppositely directed. Until the appearance of Ref. [6], this sufficiently fine circumstance remained unnoticed, possibly because the wave equation preserves its form in the case of a simultaneous change of the signs of $\varepsilon$ and $\mu$.

The work by Veselago [7] became a revolutionary step in the study of negative refraction; the idea of a completely original lens was suggested there based on a surprising property of the plane-parallel layer of a material with $\varepsilon = \mu = -1$ (which is now called a metamaterial) to focus the image of an object placed in front of it. Veselago also noted that the optical properties of a metamaterial with negative $\varepsilon$ and $\mu$ can be described by introducing a negative refractive index. Moreover, in the electromagnetic wave propagating in such a metamaterial, the electric field $\mathbf{E}$, the magnetic field $\mathbf{H}$, and the wave vector $\boldsymbol{\kappa}$ form a *left-handed* triple. In all materials known at that time, these vectors formed a right-handed system. Among other predictions made by Veselago, we mention the change in sign of light pressure in a metamaterial with a negative refractive index. Veselago's work was much ahead of its time. More than 30 years passed after the publication of Ref. [7] until a 'big bang' — the appearance of numerous works on metamaterials — occurred, initiated by Pendri [8], who showed that Veselago's lens has an even more remarkable property: it can create an image of a source without the usual distortions on the wavelength scale. This means that such a lens gives an image whose quality is not confined by the diffraction limit. It is therefore frequently called a superlens. Pendri explained this phenomenon by the amplification in the medium with negative $\varepsilon$ and $\mu$ of waves that exponentially decay in the usual optical materials and media. The first experiment that demonstrated such a superresolution was performed in 2003 [9, 10].

We note that materials with negative $\varepsilon$ and $\mu$ were generally developed and used long before the appearance of Pendri's work. It suffices to recall that the majority of well-conducting metals (gold, silver, aluminum, etc.) have a negative dielectric constant in the visible and infrared spectral ranges. On the other hand, the phenomenon of ferromagnetic resonance, which has been known already for many decades, is very frequently accompanied by the appearance of a negative magnetic permeability. But it is only after the appearance of [8] in 2000 that the creation and study of metamaterials with simultaneously negative $\varepsilon$ and $\mu$ became a new scientific avenue, in which dozens of researchers in many countries worldwide now work.

We pause briefly at the history of metamaterials. In 1952, a monograph was published [11] that contained a chapter devoted to composite materials used for the optimization of the work of radio antennas. To create artificial magnetic permeability, it was proposed to use conducting inclusions in the form of a horseshoe or in the form of a ring resonator with a cut. The equations given in [11] demonstrate the typical resonant behavior of $\mu$ with a negative value at high frequencies. In 1990, monograph [12] was published in Russia, summarizing the results of some studies on the electrodynamics of such composite materials performed at the Institute of Theoretical and Applied Electrodynamics, Russian Academy of Sciences. The results of the further development of this work were published in [13–15]. In [14], experimental studies were described of the dielectric constant of metamaterials containing pieces of metallic microwires (microdipoles) that resonate in the microwave range. Two different values of the length of the microwire pieces were chosen that ensured resonance at two frequencies, and a composite material was demonstrated that had two minima in the frequency dependence of the dielectric constant, both having negative values. The position of the minima is determined by the different length of the conducting inclusions used in the mixture. In [16], it was shown that the inclusions in the form of a pair of conducting cylinders allow obtaining a nonzero magnetic permeability at optical frequencies, which later served as the basis for creating artificial magnetism in the infrared and visible ranges. In 1997 [17], as an outgrowth of this work, experimental data were obtained for a mixture with inclusions in the form of bifilar spirals with negative $\varepsilon$ and $\mu$, and equations were also proposed that satisfactorily reproduced experimental data. These studies were not aimed at obtaining a negative refraction but were part of a systematic work on obtaining metamaterials with an assigned frequency dispersion of the dielectric and magnetic constants. In spite of the large freedom in the selection of shape and concentration of the conducting inclusions, it turned out that the Kramers–Kronig relations impose very stringent constraints on the frequency dependence of the effective parameters. One of the possible applications of these studies is the creation of highly efficient materials for the absorption of radio waves.

## 2. Superresolution in flat focusing systems

The ideas presented in [8] stimulated a detailed study of superresolution mechanisms. In Refs [18–24], which appeared almost simultaneously, it was shown that achieving superresolution requires metamaterials with extremely low losses. In [22, 23], it was noted that the negative influence of Ohmic losses can be substantially reduced in a very thin Veselago lens; therefore, a superresolution can also be achieved under realistic conditions, with the use of accessible metamaterials [9, 10]. In Ref. [23], it was also shown that the focusing and superresolution in a Veselago lens, in contrast to conventional lenses, can be achieved even with a small size of the plate (aperture), which can even be shorter than the wavelength.

In a typical Veselago lens made of a modern metamaterial, the size of the conducting inclusions is comparable to the lens thickness. Therefore, the concept of effective parameters (for example, dielectric and magnetic constants) must be used with care. It has been shown in [16, 25, 26] that for planar metamaterials containing strongly elongated conducting inclusions, the concept of a dielectric constant can be introduced only if the thickness of the material layer exceeds a certain critical value. The distribution of an electromagnetic field in the lens also differs significantly from that obtained in calculations with the use of effective parameters.

Taking the above considerations into account, we selected a flat lens consisting of a single layer of resonators for experiment [10] (Fig. 1). This structure can hardly be considered a plate of a uniform material, not least because it is not possible to clearly define the boundaries of the material in the direction perpendicular to the plate. In a computer model that we developed to describe the operation of the planar superlens, we used the direct solution of the Maxwell equations rather than the effective parameters. In the case under consideration, i.e., for a metamaterial consisting of wire inclusions, solving the Maxwell equations was reduced to solving Pocklington-type equations, which are based on the thin-wire approximation with capacitive load. In particular, the double-coil short spiral used as the inclusion in the metamaterial can be approximated by a metallic ring with a capacitor inserted into the ring break.

In calculations, we also took the finite conductivity of the metal and the corresponding skin effect into account. Our

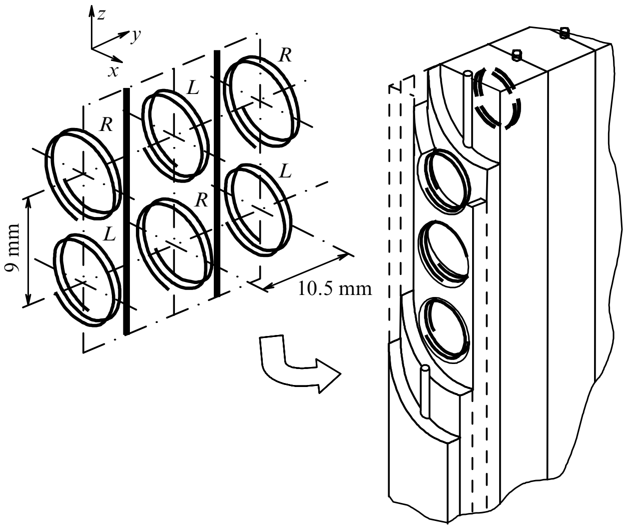


**Figure 1.** Distribution of inclusions in a metamaterial plate.

computational programs allowed calculating the electromagnetic fields generated by different sources both in a finite set of resonators and in an infinite two-dimensional periodic system.

The calculations not only reproduce the effects of focusing and superresolution but also allow comparing the electrodynamic properties of a real metamaterial with those of an ideal completely uniform metasubstance. In particular, it was shown that a plate consisting of only one layer of resonators partly demonstrates the properties of a plate made of an ideal metasubstance. For example, there is a frequency range (positioned somewhat higher than the resonance frequency of inclusions) in which superresolution is observed. This phenomenon can be seen well in Fig. 2a, where the upper part shows a 3D plot of the electric field strength and the lower part shows contour lines calculated both inside and outside the plate. For comparison, Fig. 2b shows the distribution of the field of the same sources in free space. All distances in the figures are given in dimensionless units, i.e., they are multiplied by $k = 2\pi/\lambda$.

On the whole, the plate of our metamaterial can be described as a device that supports backward waves, because a computer simulation indicates the presence of a zone near the plate where the phase and group velocities have opposite directions. However, there is an important difference in the distribution of the local field in a layer of resonators and in an ideal metamaterial. For example, in an ideal uniform metamaterial with $\varepsilon = \mu = -1$, the phase and group velocities of the propagation of electromagnetic waves are opposite to each other only inside the metamaterial layer. The excitation of currents in the layer of resonators forming a real metamaterial leads to the appearance of a spatial zone of backward waves that extends beyond the geometric boundaries of the real metamaterial (the details of the calculation are described in [27, 28]). It is also known [10, 29] that when a plane-parallel layer of an ideal metamaterial is excited, the field energy is concentrated near the farther (relative to the radiation source) face of the layer. This effect is precisely the physical basis of the superresolution phenomenon. In the plate of resonators, an accumulation of energy also occurs, but this energy is concentrated only near specific elements. In what follows, we consider the physical causes for superresolution in a real metamaterial consisting of a planar layer of ringlike resonators and elongated conducting inclusions.

As is known, the electromagnetic field radiated by a pointlike source can be represented in the form of a three-dimensional spectrum of plane waves. The coefficients of

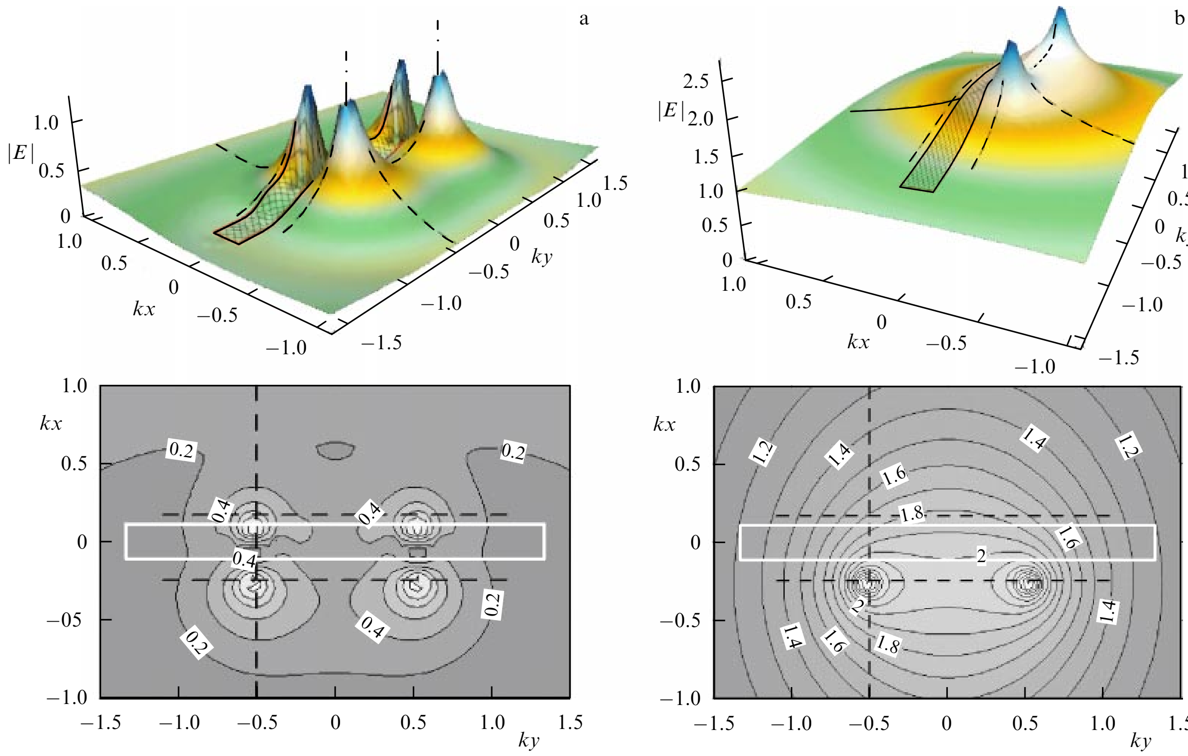


**Figure 2.** Field of linear sources: (a) in the presence of a metamaterial plate and (b) without such a plate.

wave propagation in this case take real and imaginary values. The harmonics with real propagation coefficients are the usual propagating waves. The harmonics with imaginary propagation coefficients describe a wave process that is exponentially damped with distance. To describe propagating waves, a classical beam approach is frequently used. Light beams are focused by a usual optical lens and give an image of an object with a spatial resolution of the order of the light wavelength. For obtaining superresolution, it is necessary to supplement this image with that part of the electromagnetic field that is lost in the damped harmonics.

The problem lies in the fact that exponentially decaying oscillations do not interact with a usual lens so as to be focused into an image, and their amplitude unavoidably decays both in free space and in the usual transparent material. But for obtaining a superresolution, the relation between the amplitudes of the propagating and evanescent waves at the focus must be the same as near the source. According to Pendri's original result, the damped harmonics in the plate of a metamaterial with a negative refractive index begin to increase exponentially when approaching the far (unilluminated) face. In the particular case where $\varepsilon = \mu = -1$, the relation between the amplitudes of the propagating and damped waves is restored at the focus, where an 'exact' image of the object is obtained, which is unrestricted by the diffraction limit. Therefore, we can regard a Veselago lens as an optical device that transmits propagating waves without distortions, but amplifies harmonics with imaginary propagation coefficients, preserving the necessary phase relation.

In Pendri's work and in several dozen subsequent works, objects made of an ideal metasubstance uniquely characterized by their $\varepsilon$ and $\mu$ were examined. We here consider a 'microscopic' theory of superresolution in a metamaterial consisting of electric and magnetic resonators.

For simplicity, we consider a single layer consisting of metallic needles, which play the role of electric resonators, and of split rings, which play the role of magnetic resonators. It is important that the propagating and damped harmonics excite the resonators differently. The difference appears because the electric **E** and magnetic **H** fields are in phase in the propagating waves, and are shifted by 90° in the damped harmonics. The electric and magnetic resonators are excited differently by the propagating and decaying oscillations and, correspondingly, differently emit the secondary electromagnetic field.

Therefore, the electromagnetic response of an electric-resonator–magnetic-resonator pair depends on the nature of the exciting wave, as in the Veselago lens. For example, the current in the resonators (Fig. 3a) determines the magnitudes of the equivalent electric and magnetic moments (Fig. 3b) and, eventually, the magnitudes of the effective $\varepsilon$ and $\mu$. Further studies [27, 28] showed that with the correctly chosen phase and amplitude characteristics of the dipoles (equivalents of the resonators), the system of electric and magnetic dipoles gives clear separate images of point sources in the region behind the plane of a plate made of such a metamaterial; the spacing between the sources is in this case much less than the wavelength. The frequency at which the superresolution effect appears is 3–5% higher than the frequency of the electric and magnetic resonances. If it were possible to introduce effective dielectric and magnetic constants, then this frequency range would correspond to negative values of $\varepsilon$ and $\mu$.

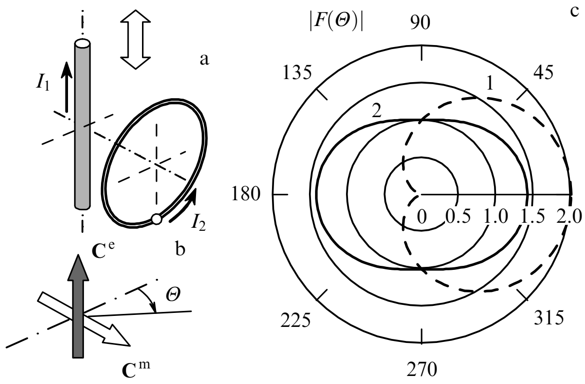


**Figure 3.** (a) Electromagnetic excitation of a pair of interacting resonators (electric and magnetic). (b) Equivalent electric and magnetic dipoles. (c) Directivity diagram for the emission of this pair.

## 3. Magnetic plasmonic resonance in optics. Active metamaterials

In the microwave range, as was shown in Section 2, metamaterials with a negative refractive index are prepared using split ring resonators or spirals, which ensure negative values of the effective magnetic permeability, $\mathrm{Re}\,\mu < 0$. In the microwave range, the metals can be considered almost ideal conductors, because the skin depth ($\sim 1-10$ μm) in them is much less than the characteristic size of metallic inclusions in metamaterials. The magnetic response is reached in the vicinity of the *LC* resonance in spirals or in split rings [17, 30, 31]. Consequently, the frequencies of the *LC* resonances are completely determined by the shape and sizes of inclusions. The resonance appears under specific relations between the size of the split ring and the wavelength of the exciting field. Subsequently, we call the LC resonances in the ideally conducting structures the geometric (G*LC*) resonances.

The situation changes dramatically in the visible and infrared ranges, where the nanosize metallic inclusions behave quite specifically when their thickness becomes less than the skin depth. For example, a plasmonic resonance appears as a result of collective oscillations of electrons. Because of these oscillations, the dielectric constant of metals $\varepsilon_{\mathrm{m}}$ is negative in the visible and infrared ranges. The plasmonic resonances cause many interesting optical phenomena, e.g., the propagation of surface plasmons, anomalous absorption, giant Raman scattering, and light supertransmission (see, e.g., [31, 32]).

The near-field superresolution also appears as a result of the excitation of plasmons in metamaterials with $\varepsilon = -1$ [6]. The near-field superresolution can be explained on the basis of the elementary solution of a problem in electrostatics (see, e.g., book of problems [41], problem no. 209). The plasmonic response of metals is the basic reason why the G*LC* resonance method is not directly applicable in optics.

Optical metamaterials with a negative refractive index were first demonstrated in [34–36]. In [34, 35], a plasmonic resonance that appears in a system of parallel nanowires was used. Such resonances were first examined in our previous works [16, 31, 37]. In [36], a negative real part of the refractive index was observed at the wavelength 2.0 μm in a system consisting of two parallel gold nanofilms with the openings of a size much smaller than the wavelength. The metallic connections between the openings play the role of nanoantennas analogous to pairs of nanowires.

The first work on obtaining and studying optical metamaterials was continued by other successful experiments [38–43]. For example, the creation of a prism from an optical metamaterial and the demonstration of a negative deviation of a light beam were described in [43]. The negative optical magnetic permeability was first announced in [44]. But we believe that the geometry used in that experiment (vertical metallic columns perpendicular to the film plane) does not allow exciting magnetic resonances in the case of normal incidence of light on the film. Indeed, irrespective of the polarization of the incident wave, the electric field is perpendicular to the axis of the metallic columns and cannot excite closed electric currents that flow in opposite directions along the metal inclusions. Some other problems related to the experiment in [44] were discussed in [45].

As was noted above, the losses are most important in the microwave range. With decreasing the wavelength (shifting it toward the visible range), the Ohmic losses become the decisive factor limiting the application of metamaterials [46, 47]. In particular, these losses radically decrease the chances of obtaining superresolution and make the creation of a flat optical Veselago lens with a superresolution virtually impossible. In other optical instruments based on the use of metamaterials, such as a hyperlens [48–52] or an invisibility device (a 'cloak') [53–56], the losses do not lead to the disappearance of efficiency, but sharply reduce the optical power of promising instruments. The problem of losses can be solved by using amplifying laser materials.

A plasmonic resonance in a metallic nanoantenna placed in an amplifying medium can be used for the excitation of magnetic and electric dipoles. The amplifying medium increases the amplitude of the excited dipoles and can in principle lead to the complete compensation of losses in the metamaterial. Because the enhancement of the electromagnetic field in a laser material implies the presence of an external energy source, this means that a metamaterial including an active medium is a dissipative system. Consequently, the substantial limitations imposed by the Kronig–Kramers relations on the behavior of the effective parameters become unobvious.

As an example of the use of an amplifying medium, we consider the phenomenon of the magnetic plasmonic resonance (MPR) in an optical nanoantenna placed into such a medium [69]. An MPR has a very important property: its frequency depends on the structure of the nanoantenna but not on its overall size. An MPR can be excited in a metallic 'nanohorseshoe' (Fig. 4). Structures of this form act as optical antennas, concentrating electric and magnetic fields on a scale that is much smaller than the wavelength of light. The magnetic response of nanohorseshoes is characterized by the magnetic polarizability $\alpha_M$, which exhibits a Lorentz resonance: the real part reverses sign near the resonance frequency and becomes negative, as is necessary for creating optical metamaterials with a negative magnetic permeability.

The concept of a magnetic plasmonic resonance, which leads to optical magnetism, is relatively new and, of course, contradicts the known concept [57] of the impossibility of magnetism in optics. However, this only seem to be a contradiction: the authors of [57] mean the microscopic magnetism, while the negative magnetism we discuss here arises at a mesoscopic level, as a result of collective electron motion.

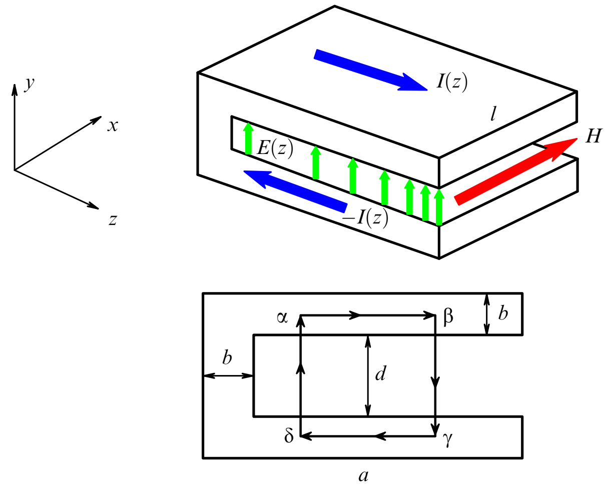


**Figure 4.** Nanoantenna in the form of a horseshoe (nanohorseshoe). The parameters used in computer simulation: $a = 300$ nm, $d = 70$ nm, and $b = 34$ nm.

Our discussion in what follows is based on the consideration of the collective effects in a metallic nanohorseshoe. The results obtained can easily be extended to other antennas.

We consider the interaction of a nanohorseshoe with an amplifying medium simulated by a two-level amplifying system (TLS) represented, for example, by quantum dots or molecules of a dye. The metallic horseshoe that interacts with the TLS is arguably the simplest plasmon system; based on this system, we can study the basic properties of active metamaterials, including processes of nanolasing. The nonradiative energy transfer from the active medium to quasistatic plasmonic oscillations has been discussed in [58]. The processes of propagation of a surface plasmon–polariton at the boundary between a metal and an active medium have been studied since the 1960s [59–63]. The superresolution in the near-field lens due to the compensation of losses in the presence of an amplifying medium was discussed in [64]. Work on active metamaterials performed before 2006 was discussed in review [41]. The first experimental and theoretical work on plasmonic resonance in metallic nanoparticles placed into an active medium was performed in [65–67]. The work that is nearest to our approach is [68], where a dipole laser was considered.

We have already mentioned that the simple compensation of losses in metamaterials does not necessarily lead directly to an increase in superresolution. Nevertheless, active metamaterials offer new possibilities for the optimization of the operation of superresolution optical systems. The active metamaterials are also important for practical applications different from those related to superresolution. For example, the plasmonic nanolaser discussed in Section 4 is a source of coherent emission, whose size can be several dozen times less than the wavelength of light. Such a nanolaser can be regarded as a nanogenerator for the power supply of future plasmonic devices, e.g., those intended for information processing.

We consider a metallic nanohorseshoe with a TLS introduced into it. The population inversion in the TLS is ensured by external pumping. The pumping can be optical or electrical, when the carriers are injected into the TLS, for example, into a quantum dot, from the surrounding material. The TLS interacts with the electromagnetic field that is excited inside the nanohorseshoe. In the equations of motion, we use a phenomenological description of pumping, characterizing the TLS by the value of the stationary inversion of the population. In other words, we characterize the TLS by the level of inversion that would exist if the TLS did not interact with the nanohorseshoe. An external AC magnetic field $\mathbf{H} = (H_0(t), 0, 0)$ is applied in the plane of the nanohorseshoe, as is shown in Fig. 4. The displacement currents in the gap of the horseshoe close the circuit.

The closed electric current $I(z)$ flowing in the nanohorseshoe generates the magnetic field $H(z) = 4\pi I(z)/c$ in the gap, where $I(z)$ is the density of the surface current in the upper plate of the capacitor (i.e., in the plate $\alpha\beta$ in Fig. 4) and $c$ is the speed of light. To obtain a closed equation for the current, we integrate the Maxwell equation $\operatorname{rot}\mathbf{E} = -\dot{\mathbf{H}}/c$, which expresses the Faraday induction law, along the contour $\alpha\beta\gamma\delta$ and obtain the equation

$$\left[2I(z)\,Z - \frac{\partial U}{\partial z}\right]\Delta z = -\frac{d}{c}\left(\frac{4\pi}{c}\,\dot{I}(z) + \dot{H}_0\right)\Delta z\,, \qquad (1)$$

where $\Delta z$ is the distance between the points $\alpha$ and $\beta$ along the integration contour shown in Fig. 4, dots denote time derivatives, $Z = 1/(\sigma b) = 4\mathrm{i}\pi/(\varepsilon_{\mathrm{m}}\omega b)$ is the surface impedance, and $\varepsilon_{\mathrm{m}}$ is the complex dielectric constant of the metal.

We substitute the potential difference $U(z) = E_y(z)\,d = -4\pi(Q(z) + P(z))\,d$ in (1), where $Q(z)$ is the charge per unit area and $P(z)$ is the polarization of the medium inside the nanohorseshoe. We then differentiate both parts of Eqn (1) with respect to time and use the charge conservation law $\partial I/\partial z = -\partial I_1/\partial z = -\partial Q/\partial t$, where $I_1$ is the current in the lower plate. Thus, we obtain the basic equation for the current in the nanoantenna:

$$\frac{\partial^2 I(z,t)}{\partial z^2} - \frac{\partial \dot{P}(z,t)}{\partial z} - \frac{Z}{2\pi d}\,\dot{I}(z,t) = \frac{1}{4\pi c}\left[\frac{4\pi}{c}\,\ddot{I}(z) + \ddot{H}_0\right]. \qquad (2)$$

This equation is analogous to the well-known telegrapher equation [57, p. 91]. For determining the polarization $P$, a matter equation must be added to Eqn (2). The polarization of the medium inside the nanohorseshoe is the sum of two polarizations: $P = P_1 + P_2$, where $P_1 = \chi_1 E_y$ is the usual polarization of a dielectric and $P_2$ is the 'anomalous' polarization due to pumping of the active medium; $\chi_1$ denotes the usual (nonresonant) polarizability of the medium. We substitute $P = \chi_1 E_y + P_2$ in (1) and obtain

$$\frac{\partial^2 I(z,t)}{\partial z^2} - \frac{\partial \dot{P}_2(z,t)}{\partial z} - \frac{Z\varepsilon_{\mathrm{d}}}{2\pi d}\,\dot{I}(z,t) = \frac{\varepsilon_{\mathrm{d}}}{4\pi c}\left[\frac{4\pi}{c}\,\ddot{I}(z) + \ddot{H}_0\right], \qquad (3)$$

where the polarizability $\chi_1$ now enters the 'regular' part of the dielectric constant $\varepsilon_{\mathrm{d}} = 1 + 4\pi\chi_1$.

We first consider the simplest case where the laser polarizability $P_2$ is linear in the applied field, $P_2 = \chi_2 E_y$. This is possible if we are far from the generation threshold and therefore the interaction with the plasmons does not lead to the depletion of the upper level of the TLS. We also assume that the external field oscillates with a frequency $\omega$, $H_0(t) = H_0 \exp(-\mathrm{i}\omega t)$. Under these assumptions, Eqn (3) takes the form

$$\frac{\partial^2 I(z)}{\partial z^2} = -g^2 I(z) - \frac{\varepsilon_{\mathrm{d}}\omega k}{4\pi}\,H_0\,, \qquad (4)$$

where the coordinate $z$ varies in the range $0 < z < a$, and the coordinates $z = 0$ and $z = a$ correspond to the beginning and end of the nanohorseshoe, such that $\mathrm{d}I(0)/\mathrm{d}z = I(a) = 0$; $k = \omega/c$; and the wave vector of the plasmon $g$ is determined from the equation

$$g^2 = \varepsilon_{\mathrm{d}}\,k^2 - \frac{2\varepsilon_{\mathrm{d}}}{bd\varepsilon_{\mathrm{m}}}\,, \qquad (5)$$

where the dielectric constant includes both the ordinary part and the contribution of the TLS. The second term in the right-hand side of Eqn (5) can be represented in the form $\sim k^2(\delta/b)^2$, where $b$ is the characteristic size of the system (for example, the thickness of the capacitor plate), and $\delta$ is the skin depth. If $\delta \ll b$, which is typical of the microwave range, we obtain the usual G$LC$-antenna resonance. In the opposite case $k^2 bd|\varepsilon_{\mathrm{m}}| \ll 1$, the parameter $g = \sqrt{-2\varepsilon_{\mathrm{d}}/(\varepsilon_{\mathrm{m}} bd)}$ is independent of the absolute length of the nanohorseshoe and does not depend explicitly on the frequency. This is a situation characteristic of the MPR, which occurs for the nanohorseshoes in the visible range [69]. It is interesting that the electric field is nonpotential under the conditions of MPR; the $E_y$ component depends on the coordinate $z$, while the component of the electric field $E_z$ depends on the coordinate $y$. The presence of a solenoidal optical field at scales much smaller than the wavelength of light is a characteristic feature of the MPR.

The electric current $I(x)$ found from Eqn (4) allows calculating the magnetic moment of the nanohorseshoe. The magnetic moment $m$ has a resonance if the condition $ga = \pi/2$ is satisfied as the magnitude of $m$ becomes large. We note that the resonance condition is satisfied not for the absolute size of the nanohorseshoe but for the ratio of its length to its width. For a typical metal, the frequency behavior of the dielectric constant is qualitatively described by the Drude formula $\varepsilon_{\mathrm{m}} = -(\omega_{\mathrm{p}}/\omega)^2(1 + \omega_\tau/\omega)^{-1}$, where $\omega_{\mathrm{p}}$ is the plasmonic frequency and $\omega_\tau$ is the relaxation frequency, which are estimated, for example, as $\hbar\omega_{\mathrm{p}} = 9.6$ eV and $\hbar\omega_\tau = 0.02$ eV for silver. In this notation, the magnetic moment of the horseshoe is written as

$$\alpha_{\mathrm{M}} = V\,\frac{bd\omega_{\mathrm{p}}^2}{\pi\lambda^2\omega_{\mathrm{r}}^2}\,\frac{1}{1 - \omega/\omega_{\mathrm{r}} - \mathrm{i}(\varkappa_{\mathrm{m}} + \varkappa_{\mathrm{d}})/2}\,, \qquad (6)$$

where the resonance frequency is $\omega_{\mathrm{r}} = \omega_{\mathrm{p}}\pi\sqrt{bd/[8\operatorname{Re}(\varepsilon_{\mathrm{d}})a^2]}$, $V$ is the volume of the horseshoe, $\varkappa_{\mathrm{m}}$ is the dimensionless loss in the metal ($\varkappa_{\mathrm{m}} = \operatorname{Im}\varepsilon_{\mathrm{m}}/\operatorname{Re}\varepsilon_{\mathrm{m}} \approx \omega_\tau/\omega \ll 1$), and $\varkappa_{\mathrm{d}}$ is the dimensionless loss in the dielectric, also assumed to be small: $\varkappa_{\mathrm{d}} = \operatorname{Im}\varepsilon_{\mathrm{d}}/\operatorname{Re}\varepsilon_{\mathrm{d}} \ll 1$.

Expression (6) for $\alpha_{\mathrm{M}}$ contains the factor $bd/\lambda^2 \ll 1$, which is small for the nanohorseshoes; however, near the resonance, the condition $|\alpha_{\mathrm{M}}| \gg 1$ can be satisfied in the visible and infrared ranges as a result of the high quality of the MPR. The presence of a frequency range where the magnetic polarizability $\alpha_{\mathrm{M}}$ is negative and large in magnitude allows creating optical metamaterials with a negative magnetic permeability.

The distribution of the magnetic field in the nanohorseshoe for a frequency close to the resonance is shown in Fig. 5. The behavior of the optical magnetic permeability for a metamaterial consisting of nanohorseshoes is shown in Fig. 6. If the dielectric is an active medium, then the dimensionless losses $\varkappa_{\mathrm{d}}$ become negative under pumping. This leads to a compensation of losses in the metal. As the

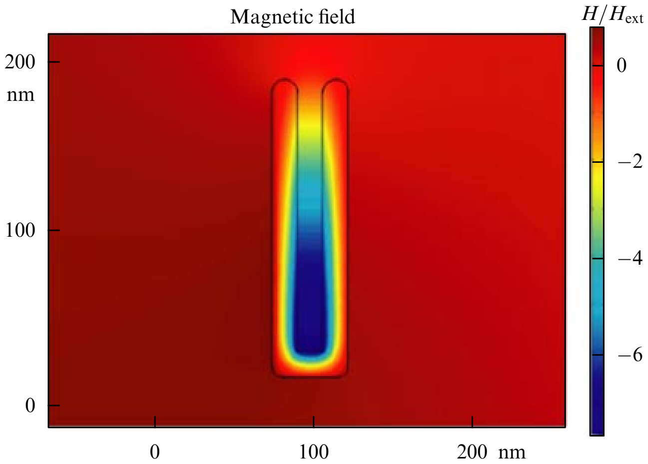


**Figure 5.** Magnetic plasmonic resonance in a silver nanohoof excited by an external magnetic field $H_{ext}$ perpendicular to the figure plane. The external field wavelength is $\lambda = 1.5$ μm; $\varepsilon_d = 2$. The magnetic field $H$ inside the hoof is directed against the external field, which corresponds to a negative polarizability.

losses are compensated due to the active medium and the total losses $\varkappa = \varkappa_m + \varkappa_d$ decrease, the absorption line (dashed curve in Fig. 6) becomes narrower. At some moment, the losses become negative, which indicates the loss of stability. The metamaterial begins lasing.

## 4. Interaction of plasmons with an amplifying medium. Plasmonic nanolaser

To explain the nature of plasmonic lasing, we consider the microscopic model suggested in [70–72]. In this model, the equations of motion are derived from quantum mechanics, but they are solved without taking the fluctuations into account and with quantum mechanical operators regarded as complex quantities. This approximation allows obtaining an analytic solution and carrying out a qualitative analysis of the system shown in Figs 4 and 7.

The Hamiltonian of a nanoantenna interacting with a TLS is given by the sum of Hamiltonians $H = H_0 + H_{TLS} + V_{int} + \Gamma$, where $H_0$ and $H_{TLS}$ respectively describe the nanohorseshoes and the TLSs, $V_{int} = -P_2\langle E_y\rangle Sd = -p\langle E_y\rangle NSd$ is the operator of the averaged interaction between a TLS and a nanohorseshoe, $p$ is the dipole moment operator, $N$ is the density of TLSs in the nanohorseshoe, $S$ is the area of the nanohorseshoe, $d$ is the distance between the plates of the capacitor, and $\Gamma$ describes the effects of dissipation and pumping.

The electrons and the related electric field oscillate with a frequency $\omega$ close to the MPR frequency. These oscillations are plasmons in the nanoantenna. We regard the electric charge and field as classical quantities.

We introduce operators $b$ and $b^+$ corresponding to the transition between the excited and ground states of the TLS. Then the Hamiltonian of the TLS takes the form $H_{TLS} = \hbar\omega_2 b^+ b$. The operator of the dipole moment can be

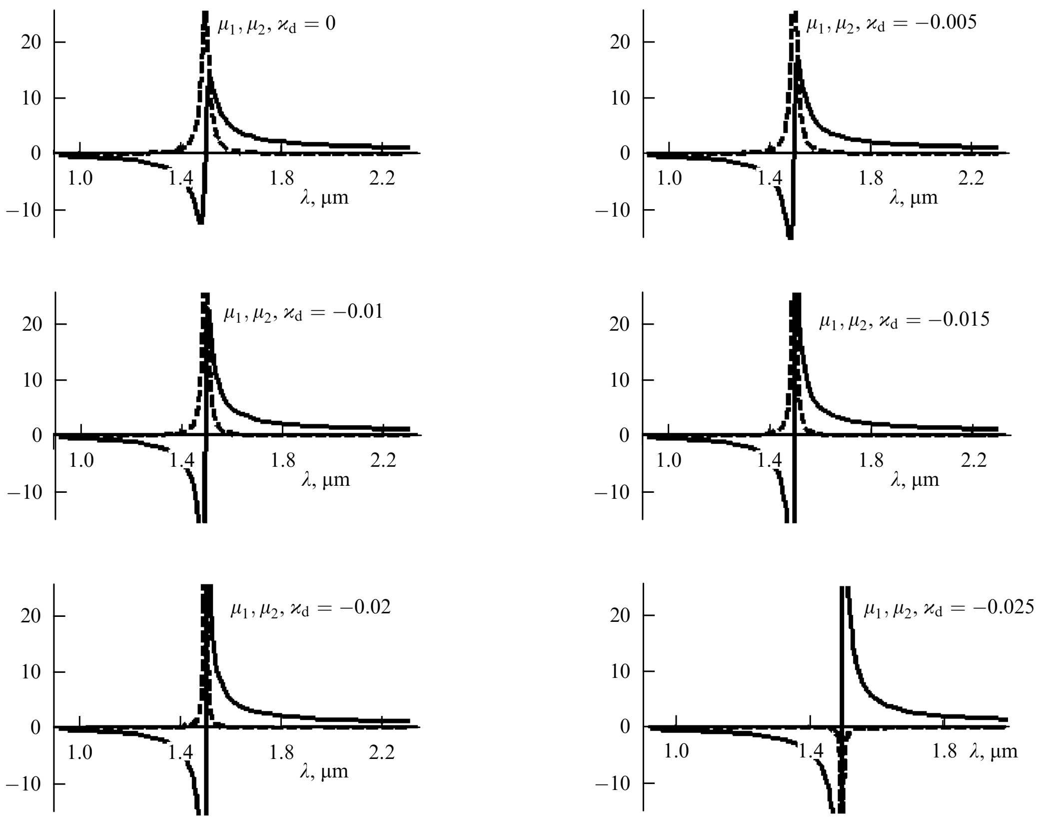


**Figure 6.** Effective magnetic permeability $\mu = \mu_1 + i\mu_2$ of a metamaterial made of silver nanohorseshoes placed in an active medium with the dielectric constant $\varepsilon_d = 4(1 + i\varkappa_d)$, where the loss factor is negative ($\varkappa < 0$) because of the pumping of the medium. The nanohorseshoe parameters used in computer simulation are $a = 300$ nm, $d = 70$ nm, and $b = 34$ nm; the bulk concentration of the nanohorseshoes is $p = 0.3$. The real part of the magnetic permeability $\mu_1$ is shown by the solid line, and the imaginary part $\mu_2$ is shown by the dashed line. Upon passing through the value $\varkappa_d = -0.025$, the metamaterial loses stability and starts lasing.

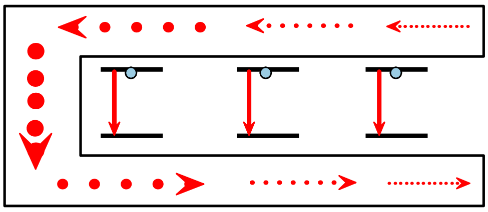

**Figure 7.** At plasmon oscillating in a nanohorseshoe (dotted lines); its amplitude increases due to the interaction with excited two-level systems, which give their energy to the plasmon.

written as

$$P_2 = \Pi b \exp(-\mathrm{i}\omega t) + \Pi^* b^+ \exp(\mathrm{i}\omega t) \,, \qquad (7)$$

where $\Pi \approx \langle g|\, r \,|e\rangle$ is the matrix element of the TLS dipole operator. We also introduce the population inversion operator $D(t) = n_{\mathrm{g}}(t) - n_{\mathrm{e}}(t)$, where $n_{\mathrm{e}}(t) = b^+ b$ and $n_{\mathrm{g}}(t) = bb^+$ are the respective operators of the population of the upper and lower levels. We assume that the TLS oscillates between the excited and ground levels with a frequency $\omega$ that is close to the frequency $\omega_2$ ($\hbar\omega_2$ is the difference between the energy levels of the TLS).

Using known commutation relations between the operators $b$, $b^+$, and $n_{\mathrm{e,\,g}}$, we can derive the Heisenberg equation of motion for operators $\mathrm{i}\hbar\dot{b} = [b, H]$ and $\mathrm{i}\hbar\dot{D} = [D, H]$. We consider lasing as the process of oscillations of the electric charge in the nanohoof even in the absence of an external magnetic field. We assume that this is a stationary process, i.e., the oscillation amplitude does not vary with time. Then the equation for the charge and the equation for $b$ and $D$ can be written as

$$(\mathrm{i}\delta + \gamma)\, q_2 - \mathrm{i}b = 0\,, \quad (\mathrm{i}\Delta + \Gamma)\, b - \mathrm{i}ADq_2 = 0\,, \qquad (8)$$

$$\frac{D - D_0}{\tau} - 2\mathrm{i}A(q_2^* b - q_2 b^+) = 0\,,$$

where $q_2 = q/(SN\Pi)$ is the dimensionless electric charge, $\delta = 1 - (\omega/\omega_{\mathrm{r}})^2$, $\gamma = (\varepsilon''_{\mathrm{m}}/|\varepsilon'_{\mathrm{m}}|)(\omega/\omega_{\mathrm{r}})^2 \approx \varepsilon''_{\mathrm{m}}/|\varepsilon'_{\mathrm{m}}|$, $\Delta = (\omega_2 - \omega)/\omega_{\mathrm{r}}$ [70–72], and the terms with $\Gamma$ and $\tau$ respectively take the processes of relaxation of the dipole moment and population into account. In the 'laser' terminology, these are the processes of transverse and longitudinal relaxation; $D_0$ is the value of the population that would be achieved by pumping if the TLS did not interact with the nanohorseshoe. We assume that we are dealing with inversion, i.e., $D_0 < 0$. Disregarding quantum fluctuations and correlations, $D$ and $b$ can be considered complex quantities with the replacement $b^+ \to b^*$. The dimensionless constant is written as

$$A = \frac{4\pi N|\Pi|^2}{\omega_{\mathrm{r}}\hbar n^2} > 0\,,$$

where $N$ is the bulk density of TLSs and $n$ is the refractive index of the medium in which the TLSs are located, for example, quantum dots. Equation (8) has a nontrivial solution only if the following conditions, which are simultaneously the conditions of lasing, are satisfied:

$$\frac{\Delta}{\Gamma} = -\frac{\delta}{\gamma}\,, \quad \left(\frac{\delta}{\gamma}\right)^2 + 1 + \frac{AD_0}{\Gamma\gamma} = 0\,. \qquad (9)$$

The first condition gives the frequency of lasing, which always lies between the MPR frequency $\omega_{\mathrm{r}}$ and the TLS resonance frequency $\omega_2$. All terms in (9) are positive, except the population in the second lasing condition. Consequently, this condition is satisfied only in the case of inversion $n_{\mathrm{e}} > n_{\mathrm{g}}$, when $D_0 < 0$. According to the definition, $D_0$ cannot be less than $-1$, which corresponds to the case where all the TLSs are in an excited state. Thus, we obtain the condition necessary for lasing: $A/(\Gamma\gamma) > 1$. As soon as the second condition in (9) is satisfied, the interaction between the TLS and the nanohorseshoe leads to coherent oscillations of the electric charge, current, and magnetic moment even in the absence of an external electromagnetic field.

The lasing condition can be expressed in terms of the amplification coefficient $G$ in the active medium located in the nanohorseshoe. The amplification in the medium must be large, and hence the inequality

$$\frac{G\lambda}{2\pi n\gamma} > 1 \qquad (10)$$

is satisfied, where $\gamma = \varepsilon'_{\mathrm{m}}/|\varepsilon''_{\mathrm{m}}| \ll 1$ is the dimensionless factor of losses in the metal and $n \sim 1$ is the refractive index.

We note that the lasing condition depends on the amplification in the active medium and on the losses in the metal. We assume that this is a universal condition for the operation of a plasmonic nanolaser with any configuration of the metallic nanoantenna. For example, a silver nanoantenna lases at the wavelength 1.5 μm if the active medium that fills it has an amplification factor larger than $G_{\mathrm{c}} \approx 5 \times 10^3$ cm$^{-1}$ at this frequency.

We now consider the effect of an external magnetic field on the operation of a nanolaser. A high-frequency magnetic field excites currents in the nanohorseshoe and acts as a driving force. In the absence of this force, the plasmonic nanolaser, which should be regarded as a nonlinear oscillator, self-oscillates and moves along its limit cycle with the lasing frequency given by Eqn (9). When we apply an external force, the plasmonic laser continues moving along the same limit cycle but already with the frequency of the external force. In other words, an external electromagnetic wave can retune the nanolaser. This fantastic possibility requires further study.

## 5. Conclusions

We see that metamaterials offer new possibilities for developing different devices in the microwave and visible ranges, such as focusing systems, nanolasers, absorbers, resonators, and many other devices. The development of new electromagnetic materials, which starts from the construction of unit cells with predetermined properties that may or may not exist in nature, is a new technique that opens unique prospects. The spectrum of the potential applications of metamaterials that is discussed in the contemporary literature extends from unique sensors of Raman scattering to the creation of cloaking devices ('magic caps' and 'magic carpets'). Moreover, work on the creation and analysis of mechanical (e.g., acoustic) metamaterials has actively been developed recently. Nevertheless, we emphasize that in spite of all the progress achieved in experimental and theoretical studies, no commercially successful metamaterials or devices based on them have been developed so far. This is partly connected with the problem of losses, which was discussed in Sections 2 and 3. We attempted to show that the physics of metamaterials is very interesting not only because of its attractive potential applications but also in and of itself, and that many fundamental problems remain unsolved to date.

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# Application of the scattering matrix method for calculating the optical properties of metamaterials

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We consider the application of the scattering matrix formalism for calculating the eigenfrequencies, radiation widths, and field distributions of quasiwaveguide modes in photonic crystal layers (PCLs) of finite thickness.

At present, investigations are being performed of one-dimensional (1D) or two-dimensional (2D) periodic layers of photonic crystals whose vertical geometry can be arbitrarily complex [1–3]. Such PCLs have proved to be very interesting and promising structures; they can be prepared by the modern methods of layer-by-layer lithography; their optical properties are of practical interest in connection with their potential compatibility with microelectronic devices.