PACS numbers: **01.65.+g**, **52.35.–g**, **52.55.–s**
DOI: 10.3367/UFNe.0179.200907i.0790

B B Kadomtsev's classical results and the plasma rotation in modern tokamaks

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1. Introduction. Stages of Kadomtsev's work at the Kurchatov Institute

This report is focused on three theoretical works by B B Kadomtsev concerning controlled thermonuclear fusion, which in my opinion give a good idea of several periods of his work affiliated with the Kurchatov Institute. Kadomtsev was a highly productive theorist with a very wide spectrum of interests. Therefore, in choosing which of his works to present in a report like the present one, I could only rely on my subjective impressions. The first period is distinctive for a series of research undertakings on the so-called energy stability principle [1]. Two other works to be considered deal with trapped particle instability [2] and reduced equations describing plasma dynamics [3]. All these works became classical and opened up a new area of further investigations. Moreover, each of them gives a striking example of the productivity of the theory that was later brilliantly confirmed in experiment. What follows is intended to illustrate the importance of these works from a present-day perspective with special reference to plasma rotation, a new phenomenon that was not discussed in the periods in question. In modern tokamaks, plasma can rotate at a high speed, and this rotation is presently regarded as a key factor promoting plasma confinement. Rotation requires modification of most results obtained for motionless equilibrium plasma.

The first period of Kadomtsev's work at the Kurchatov Institute roughly covers the years of 1956 – 1962. In 1956, Boris Borisovich joined the Theory Division headed by Mikhail Aleksandrovich Leontovich, who immediately appreciated the young researcher as a man of great intelligence. Within a few years, Leontovich managed to bring together a small but highly efficient team of gifted theorists with high scientific potential. In this context, Vitaly Dmitrievich Shafranov should be given credit for having introduced his fellow-student Boris Kadomtsev to M A Leontovich.

Boris Borisovich started with kinetic research, partly continuing his previous investigations in Obninsk, but very soon his exceptional ability to see the essence of an issue made him switch over to the problem of macroscopic plasma confinement. This ability to see all sides of a problem and focus on its key aspects distinguished Kadomtsev throughout all periods of his scientific work. It is this ability that put him in the forefront of a new science, the theory of high-temperature plasma, and made him one of its leading actors. He began to develop the theory of plasma stability and continued this work one way or another for the rest of his life. In this period, Kadomtsev completed several important studies on the magnetohydrodynamic (MHD) stability of the plasma in magnetic traps and formulated the energy principle of stability of MHD equilibrium, which will be discussed below. His efforts culminated in experimental verification of the 'minimum B' principle in the laboratory of M S Ioffe [4]. This period logically ended in the defense of his doctoral thesis (1961) and election as a Corresponding Member of the USSR Academy of Sciences (1962).

In the second period (1962 – 1970), Boris Borisovich elaborated the theory of magnetized plasma turbulence and related transport processes. This greatly contributed to the understanding that Bohm diffusion (long considered to be an insurmountable obstacle for thermonuclear fusion) is not inevitable and can be obviated. Further development of this theory brought Kadomtsev to the concept of a tokamak-based thermonuclear reactor. In parallel, Boris Borisovich continued the search for instability, a key prerequisite for a fusion reaction in tokamaks, and discovered trapped particle instability. In 1970, he was elected a Full Member of the Academy and awarded the USSR State Prize.

During the next period (1971 – 1990), large tokamaks were built at the Kurchatov Institute and in leading research centers abroad. Powerful gyrotron heating was used to obtain record-breaking plasma parameters in the tokamak T-10. The subsequent tokamak T-15 was a unique machine with superconducting windings. At that time, Boris Borisovich showed special interest in tokamak physics as a whole. He formulated principles of plasma self-organization in tokamaks and continued to develop the theory of stability with reference to disruption instability, of primary importance for tokamak operation. Simultaneously, he made an important contribution to a physics of nonlinear phenomena (the well-known Kadomtsev – Petviashvili and Kadomtsev – Pogutse equations, the latter being considered in Section 4 below). In 1984, Boris Borisovich Kadomtsev was awarded the Lenin Prize. The period under consideration naturally ends with participating in ambitious international projects initiated by Evgenii Pavlovich Velikhov. After 1990, Kadomtsev published a few reviews of tokamak plasma physics, serving as a basis for the International Thermonuclear Experimental Reactor (ITER) project, in which Boris Borisovich was an active participant. Kadomtsev described the history of tokamak concept from the very first idea to ITER in review [5]. The world's thermonuclear community recognized the outstanding scientific contributions of Boris Borisovich Kadomtsev by awarding him the J C Maxwell Prize (1998) of the American Physical Society.

2. Energy stability principle

The macroscopic stability of the plasma in fusion devices, first and foremost MHD stability, became a priority issue in the second half of the 1950s. Researchers very soon recognized the importance of flute instability in magnetic traps, at which plasma tongues stretch parallel to the magnetic field and penetrate through the lines of force without perturbing the field. This instability is an MHD analog of Rayleigh – Taylor instability, which it is natural to analyze from the energy standpoint.

The following 'energy principle of stability' of static equilibria is quite obvious for simple Hamiltonian systems: owing to the positive definitiveness of the kinetic energy, the positive definitiveness of the potential energy guarantees the stability of the initial equilibrium (in accordance with the Lyapunov theorem). S Lundquist was the first to suggest this approach in relation to MHD problems in 1951 [6]. It was further developed by Kruskal, Kalsrud, Schlüter, Rosenbluth, Longmire et al. (see, for instance, Refs [7, 8]) during the next 6 years. Some important results were obtained, and comprehensive mathematical formulation of the energy principle for arbitrary MHD systems was proposed by Bernstein, Frieman, Kruskal, and Kalsrud (BFKK principle) in 1958 [9]. By that time, the works of Kadomtsev and the above authors had already contained main elements of the BFKK energy principle [10]. However, due to the comprehensiveness and generality of the BFKK formulation, the energy principle of MHD stability is presently associated with Ref. [9], most frequently cited in plasma physics publications. This paper was followed by a large number of others reporting application of the energy principle to the description of stability of concrete magnetic systems, modes, etc.

The energy stability principle for an MHD system with the Hamiltonian

$$H = K + W = \int_{\Gamma} \frac{\rho \mathbf{V}^2}{2}\, \mathrm{d}^3 r + \int_{\Gamma} \left(\frac{p}{\gamma - 1} + \frac{\mathbf{B}^2}{8\pi} \right) \mathrm{d}^3 r \qquad (1)$$

is formulated as a requirement for positive semi-definitiveness of the second potential energy variation:

$$\delta^2 W \geqslant 0 \qquad (2)$$

in the vicinity of static equilibrium position $\mathbf{V} = \mathbf{0}$, $\nabla p = \operatorname{rot} \mathbf{B} \times \mathbf{B}/4\pi$ (standard notations are used: $\mathbf{V}$ and p are the macroscopic velocity and pressure, respectively, of the plasma with adiabatic exponent γ, confined by magnetic field $\mathbf{B}$ in volume Γ assumed to be fixed for simplicity). It is convenient to express quantity $\delta^2 W$ in terms of displacement ξ of a plasma element:

$$\begin{aligned} \delta^2 W &= -\frac{1}{2} \int \xi\, \mathbf{F}(\xi)\, \mathrm{d}^3 r \\ &= \frac{1}{2} \int \mathrm{d}^3 r \left\{ \frac{1}{4\pi} \Big(\operatorname{rot}^2 [\xi \times \mathbf{B}] + [\xi \times \operatorname{rot} [\xi \times \mathbf{B}]] \operatorname{rot} \mathbf{B} \Big) \right. \\ &\quad \left. + \xi\, \nabla p \operatorname{div} \xi + \gamma p \operatorname{div}^2 \xi \right\}. \end{aligned} \qquad (3)$$

This expression clearly demonstrates the physical nature of possible instability: the second and the third terms on its right-hand side are responsible for two feasible instability mechanisms, one associated with the electric current flowing in the plasma, the other with its pressure, whereas perturbation of the magnetic field and plasma compressibility serve as stabilizing factors. Productivity of the energy principle is closely related to self-conjugacy (hermiticity) of linearized force operator $\mathbf{F}$, understood in the usual sense:

$$\int_{\Gamma} \boldsymbol{\eta}\, \mathbf{F}(\xi)\, \mathrm{d}^3 r = \int_{\Gamma} \xi\, \mathbf{F}(\boldsymbol{\eta})\, \mathrm{d}^3 r$$

(arbitrary vectors ξ and $\boldsymbol{\eta}$ vanish at the boundary of the integration domain Γ).

Self-conjugacy of the force operator $\mathbf{F}$ guarantees the necessity of stability condition (2) and its completeness for systems with magnetic surfaces, such as the majority of the known magnetic traps. In other words, the following assertion can be proved: if the potential energy of a certain displacement ξ is negative, there is an eigenmode of the small-oscillation equation

$$\rho \ddot{\xi} = \mathbf{F}(\xi)\,,$$

which exponentially grows with time [9]. The set of eigenmodes forms a complete system.

Kadomtsev applied this principle to the analysis of flute modes and found the stability condition

$$\nabla p\, \nabla U + \frac{\gamma p (\nabla U)^2}{U} > 0\,, \qquad (4)$$

where $U = \int \mathrm{d}l/B$ is the integral taken along a magnetic field line (along the flute length). The first term in condition (4) describes the 'mean magnetic well' effect contributing to stability and showing itself as the magnetic field grows from the plasma confinement region. The second term takes account of the stabilizing effect of plasma compressibility. Kadomtsev summarized this and some other practical applications of the energy principle in a comprehensive and easily understandable review [1].

The energy principle has an important nuance. The Lyapunov theorem demands sign-definiteness of the functional [strict inequality in formula (2)], hence there is the problem with *neutral* displacements that do not change potential energy W, i.e., those corresponding to zero frequency in terms of eigenmodes. It is these displacements that cause concern as regards nonlinear instability. Moreover, it can be shown that such neutral displacements always exist in MHD systems and that they are nontrivial, i.e., nonreducible to global displacements and turns of the plasma as a whole, which are of no interest for the problem under consideration. In the systems with nested magnetic surfaces, whose state is beyond boundary stability, neutral perturbations reduce to relabeling transformations of fluid elements that do not perturb physical quantities characterizing plasma state, viz. pressure, density, and magnetic field [11]. Thus, the energy principle in the form of expressions (2), (3) is exhaustive for plasma static equilibria in the systems with magnetic surfaces. However, the existence of relabeling symmetries suggests the possibility of a shift in equilibrium along such transformations, i.e., the flows. Therefore, attempts to extend this approach to the case of plasma with flows seem natural.

Such an attempt was made by Frieman and Rotenberg as early as 1960 [12]; they derived the energy condition from the general linearized equation of motion

$$\rho \ddot{\xi} + 2\rho (\mathbf{V} \nabla)\, \dot{\xi} - \mathbf{F}(\xi) = 0\,, \qquad (5)$$

where ρ and $\mathbf{V}$ are the stationary values of mass density and plasma flow velocity, and the operator $\mathbf{F}$ is modified compared with the operator $\mathbf{F}$ in Eqn (3) but still retains the property of hermiticity. Conservatism of the system [antisymmetric operator with $\dot{\xi}$ in Eqn (5) drops out of the energy balance equation in integration] again permits obtaining (in analogy with the static energy principle) a sufficient condition for stability in the form

$$\begin{aligned} \delta^2 W \approx \frac{1}{2} \int_{\Gamma} \mathrm{d}^3 r \Bigg\{ & -\frac{1}{\rho} \operatorname{rot}^2 [\xi \times \rho \mathbf{V}] - [\xi \times \operatorname{rot} [\xi \times \rho \mathbf{V}]] \operatorname{rot} \mathbf{V} \\ & + \frac{\mathbf{V}^2}{\rho} \operatorname{div}^2 (\rho \xi) + \left(\xi \nabla \frac{\mathbf{V}^2}{2} - 2 \mathbf{V} (\mathbf{V} \nabla)\, \xi \right) \operatorname{div} (\rho \xi) \\ & + \frac{1}{4\pi} \Big(\operatorname{rot}^2 [\xi \times \mathbf{B}] + [\xi \times \operatorname{rot} [\xi \times \mathbf{B}]] \operatorname{rot} \mathbf{B} \Big) \\ & + \xi\, \nabla p \operatorname{div} \xi + \gamma p \operatorname{div}^2 \xi \Bigg\} \geqslant 0\,, \end{aligned} \qquad (6)$$

which is too (unnecessarily) 'rigorous', unlike the condition in the static case, and is not satisfied for systems of any practical interest barring a few rather special cases (e.g., plasma flow strictly along magnetic lines of force, $\mathbf{V} \parallel \mathbf{B}$). Interest in this

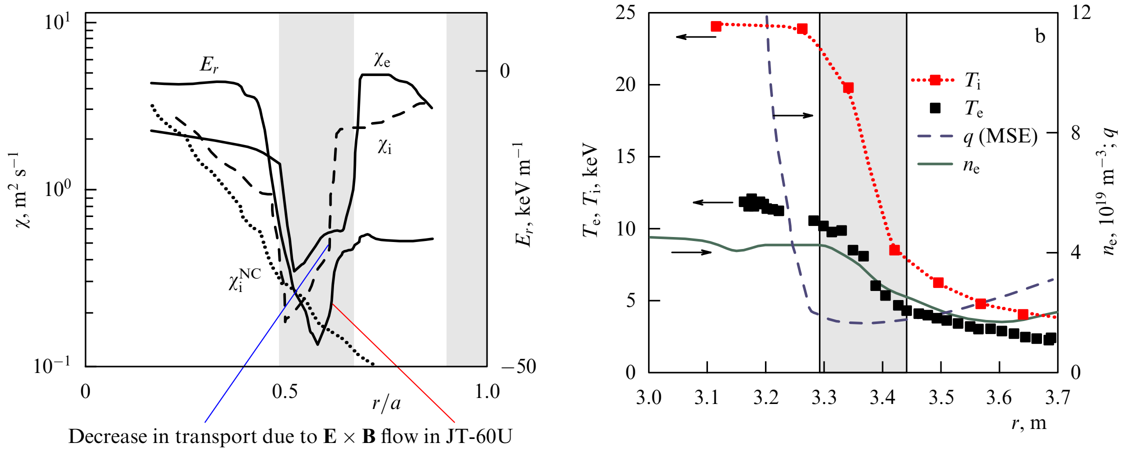

Figure 1. Transport barrier in the JT-60U tokamak, Japan (see Ref. [13]). (a) Jumps in electric field E_r and effective coefficients of ion (χ_i) and electron (χ_e) heat conductivity in the barrier zone. Ion heat conductivity decreases to χ_i^{NC} calculated from a neoclassical (NC) theory (dotted curve). (b) Large temperature (T_e, T_i) and density (n_e) gradients in this zone illustrate the notion of 'transport barrier'; q is the safety factor measured by the MSE (Motional Stark Effect) method.

problem was lost for the next 20 years because the role of macroscopic plasma motion (flow) was deemed unessential at a flow rate much lower than the speed of sound. It should be noted that this argument is not quite correct since the characteristic size of spatial inhomogeneity of the flow may be significantly different from that of pressure, density, and magnetic field inhomogeneities, hence the taking into account plasma motion at much lower flow velocities can be important. However, such a possibility was disregarded in early thermonuclear experiments.

Interest in plasma flows was renewed with the advent of new powerful plasma-heating sources in modern tokamaks. Uncompensated injection of fast atomic beams into a tokamak sets the plasma in rotational motion with a rate that may reach the same order of magnitude as the speed of sound. In this case, *improved confinement* regimes associated with the appearance of relatively narrow layers of nonuniform rotation develop. Figure 1 demonstrates the so-called transport barrier phenomenon typical of such regimes. A narrow layer undergoes a jump in the electric field and, accordingly, in the rate of plasma rotation. A temperature jump in this layer corresponds to a sharp fall in effective heat conductivity. The presence of such a layer makes it possible to significantly increase permissible parameters of the plasma confined within the barrier. Taken together, these facts dictated the necessity of studying plasma rotation effects in both the stability problems and closely related problems of transport theory.

One of the probable causes of the excessively large discrepancy between the sufficient Frieman – Rotenberg stability condition (6) and the necessary MHD stability condition is underestimation of the relationship between the displacement and the speed inherent in the real dynamics of the system. This assertion is illustrated by a simple example sometimes referred to as the Prendergast problem. Let us consider the motion of a charge over a symmetric hill in a gravitational field and in a vertical magnetic field. The magnetic field does not change the charge energy and conclusions based on analysis of the sign of the second variation of potential energy point to possible instability at any hill slope. Positive definiteness of the potential energy guarantees stability only in a gravitational well, even though the magnetic field clearly affects the

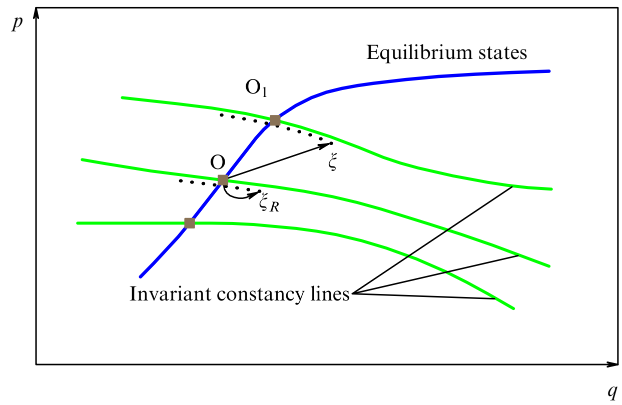

Figure 2. A case of degenerate equilibrium [dark curve (blue in on-line version)]. Oscillations (dots) occur along invariant constancy lines.

charge dynamics. The magnetic field being strong enough, equilibrium at the top of the hill or rotation around it may prove stable. This is easy to see since the problem has an exact solution. The redundant freedom in variable functions can be eliminated by taking account of conservation laws inherent in the system, differing from the law of conservation of energy. Thus, the generalized angular momentum must be conserved in this problem. In the general case, in the presence of additional motion invariants shown by level lines on the conditional phase plane (Fig. 2), it is enough to study perturbations ξ_R retaining the meaning of such invariants instead of arbitrary displacements ξ. Interestingly, using this procedure and taking into account the law of conservation of the generalized angular momentum *in variations* allow, in our example, obtaining an exact (necessary and sufficient) stability criterion.

In 1965, V I Arnold suggested this idea in application to hydrodynamics [14, 15] and proposed taking into account the conservation of vorticity in the analysis of flow stability. In MHD conditions, vorticity is not conserved, whereas systems with magnetic surfaces retain (under certain conditions) cross helicity I_1 and its 'counterpart' I_2:

$$I_1 = \int \mathbf{V}\,\mathbf{B}\,\mathrm{d}^3 r\,, \qquad I_2 = \int \mathbf{V}\,\mathbf{D}\,\mathrm{d}^3 r\,. \tag{7}$$

Here, $\mathbf{D}$ and $\mathbf{B}$ are linearly independent, the former being divergenceless vector frozen into the plasma and also tangential to the magnetic surfaces; integration in formulas (7) is taken over the volume between any adjacent magnetic surfaces. The use of Arnold's scheme to take account of limitations on the variable functions, which are imposed by the condition of conservation of quantities (7) in variations, permitted obtaining common equilibria with flows and simultaneously a milder stability condition [16, 17] as against the Frieman – Rotenberg condition (6). In the general case, elimination of excess freedom in variations of independent variables (coordinates and momenta of 'fluid elements' in the medium) is achieved by splitting perturbations in accordance with invariance of quantities of the form

$$\int_{\Gamma} \lambda \mathbf{P}\,\mathbf{V}\,\mathrm{d}^3 r\,,$$

where $\mathbf{P}$ is the canonical momentum (bearing in mind perturbations), $\mathbf{V}(\mathbf{r})$ is the equilibrium velocity field in volume Γ, and λ is the weight factor related to system topology. It is essential that such splitting should be taken into account in both the first and the second functional variations. Although consideration of the first variation yields an equilibrium condition of the most general functional form, the stability condition may still be far from the necessary one.

It is methodically relevant to draw attention to a misapprehension widespread in the literature that the formal addition of conserved quantities [e.g., integrals (7)] with undetermined Lagrange multipliers to a variable functional and variation of the new functional automatically lead to an improved (milder) stability condition. This procedure described, for instance, in the well-known review [18] leads only to a more general class of equilibria but does not restrict perturbations in variations and therefore results in a loss of information about the derived integrals of motion in studies of convexity of the functional, i.e., again in a more stringent stability condition than in the Arnold method. The same drawback is inherent in most studies of nonlinear stability and flow stability performed later as recommended in Ref. [18].

Another purely physical cause of the difficulties encountered in the energy approach is concerned with negative energy waves. Indeed, energy analysis of perturbations for the study of stable oscillations in the system of interest may prove unproductive if these oscillations possess not only positive but also negative energy. It should be emphasized that Kadomtsev paid his attention to negative energy waves, but his well-known work [19] concerned only interaction between electromagnetic waves in media with different dispersions. It is important for our purposes that MHD oscillations may have negative energy, too. Indeed, the following dispersion equation for eigenfrequency ω formally follows from the Frieman – Rotenberg equation (5):

$$A\,\omega^2 - 2B\,\omega - C = 0\,, \tag{8}$$

where for $\boldsymbol{\xi}$ in the form of normal modes, viz.

$$\boldsymbol{\xi}(\mathbf{r}, t) = \hat{\boldsymbol{\xi}}(\mathbf{r}) \exp(-\mathrm{i}\omega t)\,, \tag{9}$$

the coefficients $A = \int \rho |\hat{\boldsymbol{\xi}}|^2\,\mathrm{d}^3\mathbf{r}$, $B = -\mathrm{i}\int \rho \hat{\boldsymbol{\xi}}^*\,(\mathbf{V}\nabla)\,\hat{\boldsymbol{\xi}}\,\mathrm{d}^3\mathbf{r}$, and $C = -\int \hat{\boldsymbol{\xi}}^*\,\mathbf{F}[\hat{\boldsymbol{\xi}}]\,\mathrm{d}^3\mathbf{r}$ are real by definition. The solution of equation (8) has the form

$$\omega = \frac{B + s\sqrt{B^2 + AC}}{A}\,, \tag{10}$$

where $s = 1$ or $s = -1$ for a given eigenwave. Therefore, the eigenwave is unstable only if $B^2 + AC < 0$. The eigenmode energy can be written out as

$$E = \frac{1}{2}\left(A|\omega|^2 + C\right) \exp(2\gamma t)\,, \tag{11}$$

where the increment $\gamma = \operatorname{Im}\omega$. Because the energy is conserved, E in expression (11) cannot depend on time and must be zero for any unstable eigenmode with $\gamma \neq 0$.

The energy of a stable eigenmode with $\gamma = 0$ is given by the expression

$$E = s\omega\,\sqrt{B^2 + AC} \tag{12}$$

and can be either positive (positive energy waves, PEWs) or negative (negative energy waves, NEWs). NEWs exist as eigenmodes with $-B^2/A < C < 0$ and $\operatorname{sign}(B) = -s$. It follows from Eqns (8), (12) that all NEWs are asymmetric, i.e., show spatial dependence in the direction of the stationary flow, so that $B \neq 0$. As shown in Ref. [20], there is an interval of equilibrium parameters within which PEWs and NEWs coexist. When their frequencies coincide (resonance conditions), the energy may be transferred from a NEW to a PEW, which leads to instability. In point of fact, such NEW/PEW pairs constitute a universal mechanism of any asymmetric instability in an ideal MHD system with flows.

Eigenmodes with purely real or purely imaginary eigenvalues producing a spectrum symmetric with respect to the origin of coordinates on the plane $\operatorname{Re}\omega - \operatorname{Im}\omega$ are referred to below as symmetric. They correspond, in particular, to static equilibria or modes homogeneous along the flow direction ($B = 0$). The standard energy principle holds true for symmetric modes because their energy (12) is always nonnegative and passes through zero during the transmission from stability zone to instability zone. Certainly, this principle is violated in the case of excitation of NEWs in the system since zero energy is attainable in a wholly stable zone, too.

This NEW-related inconveniency can also be avoided by taking into account the necessary number of additional integrals of motion, at least in the case of a discrete spectrum. The linear equation of motion (5) has an infinite set of energy type integrals [21] but not-reducible-to-energy integrals:

$$E_n = \frac{1}{2}\int \left(\rho|\boldsymbol{\xi}^{(n+1)}|^2 - \boldsymbol{\xi}^{*(n)}\,\mathbf{F}[\boldsymbol{\xi}^{(n)}]\right) \mathrm{d}^3\mathbf{r}\,, \tag{13}$$

where $\boldsymbol{\xi}^{(n)}$ is the nth derivative in time. In the main, these integrals are independent. E_0 corresponds to energy, and integral E_1 to type (7) invariants. Higher order invariants (13) have no explicit nonlinear analogs. Using a recurrent relation directly following from equation (5), namely

$$\boldsymbol{\xi}^{(n+2)} = -2(\mathbf{V}\nabla)\,\boldsymbol{\xi}^{(n+1)} + \frac{\mathbf{F}[\boldsymbol{\xi}^{(n)}]}{\rho}\,, \tag{14}$$

it is possible to express all integrals (13) through initial perturbations $\dot{\boldsymbol{\xi}}_0 = \dot{\boldsymbol{\xi}}|_{t=0}$ and $\boldsymbol{\xi}_0 = \boldsymbol{\xi}|_{t=0}$. Specifically, one finds

$$E_1(\dot{\xi}_0, \xi_0) = \frac{1}{2}\int\left(\frac{1}{\rho}\left|\mathbf{F}[\xi_0] - 2\rho(\mathbf{V}\nabla)\,\dot{\xi}_0\right|^2 - \dot{\xi}_0^*\,\mathbf{F}[\dot{\xi}_0]\right)\mathrm{d}^3\mathbf{r}\,. \tag{15}$$

Integrals of motion (13) can be introduced into the Lyapunov functional by the method of Arnold [15] using the Lagrange multipliers λ_n:

$$U(\dot{\xi}_0, \xi_0) = \sum_{n=0}^{N} \lambda_n E_n(\dot{\xi}_0, \xi_0)\,. \tag{16}$$

The following theorem provides a sufficient condition for formal stability of the system described by equation (5).

Theorem. *If there exist real numbers λ_n and integer $N \in [0, \infty]$ such that the form* (16) *is positively definite for all $\dot{\xi}_0$ and ξ_0, then the form* (16) *is the Lyapunov functional and the equilibrium state is formally* (*spectrally*) *stable*.

The proof of this theorem and more detailed description of this approach can be found in Ref. [22]. Under certain assumptions, the theorem also provides necessary conditions for spectral stability because, given that the system is stable, there exist such λ_n whereat the functional U is nonnegative at any perturbation.

The productivity of this approach can be illustrated by a simple example of Rayleigh – Taylor instability of a rotating cold gravitating gas. All equilibrium quantities can depend only on radius r in a cylindrical system of coordinates (r, φ, z). The equilibrium velocity is expressed as

$$\mathbf{V} = r\Omega(r)\mathbf{e}_\varphi\,, \qquad r\Omega^2(r) = \frac{\partial\Phi}{\partial r}\,, \tag{17}$$

where $\Omega(r)$ is the angular frequency of rotation in a gravitational field with the potential $\Phi(r)$, and $\mathbf{e}_\varphi = r\nabla\varphi$. The stability condition for such rotation is fairly well known. It is the Rayleigh criterion (a necessary and sufficient condition of spectral stability) reducible in the present case to the requirement for the so-called epicyclic frequency κ to be real:

$$\kappa^2 = 4\Omega^2 + r\,\frac{\partial\Omega^2}{\partial r} \geqslant 0\,. \tag{18}$$

Let us apply the above-described variational method to this problem. In this case, all invariants (13) are local, and the first two, E_0 and E_1, have the following form for the modes rotating with frequency $\Omega(r)$:

$$\begin{aligned} E_0 &= \frac{1}{2}\left(|\dot{\xi}|^2 - \xi^{*\mathrm{T}}\hat{\mathbf{B}}\xi\right) \\ &= \frac{1}{2}\left(|\dot{\xi}_r|^2 + |\dot{\xi}_\varphi|^2 + |\dot{\xi}_z|^2 + r\,\frac{\partial\Omega^2}{\partial r}\,|\xi_r|^2\right), \qquad (19) \\ E_1 &= \frac{1}{2}\left(|\hat{\mathbf{B}}\xi - 2\Omega\hat{\mathbf{A}}\dot{\xi}|^2 - \dot{\xi}^{*\mathrm{T}}\hat{\mathbf{B}}\dot{\xi}\right) \\ &= \frac{1}{2}\left[\left|r\,\frac{\partial\Omega^2}{\partial r}\,\xi_r - 2\Omega\dot{\xi}_\varphi\right|^2 + \left(4\Omega^2 + r\,\frac{\partial\Omega^2}{\partial r}\right)|\dot{\xi}_r|^2\right] \end{aligned}$$

where $\hat{\mathbf{B}}$ is the matrix: $B_{ij} = 2r\Omega\Omega_r'\delta_{i1}\delta_{j1}$, and δ is the Kronecker symbol. Choosing E_1 for U and putting $\lambda_{i\neq 1} = 0$ in formula (16) leads to the spectral stability condition that is exactly the Rayleigh criterion (18). As follows from Eqn (19), the energy principle ($U = E_0$) gives a more rigorous sufficient stability condition: $r\,\partial\Omega^2/\partial r \geqslant 0$, confirming the efficiency of the proposed method.

Another example is E P Velikhov's magnetorotational instability (MRI) [23] supposed to be responsible for turbulent processes in accretion disks. Let us calculate energies and eigenmode frequencies in an experiment simulating magnetorotational instability. Consider an incompressible conducting fluid rotating across a uniform magnetic field $\mathbf{B} = B_0\mathbf{e}_z$ with angular velocity

$$\mathbf{V} = r\Omega(r)\mathbf{e}_\varphi\,, \qquad \Omega(r) = \frac{\Omega_1 r_1^2}{r^2}\,, \tag{20}$$

given in the cylindrical system of coordinates (r, φ, z). Let us choose, for definiteness, $r_2/r_1 = 5$, where r_1 and r_2 are the inner and outer radii of the fluid-containing channel, respectively, and Ω_1 is the angular velocity at radius r_1. A detailed study of the stability of such a flow was reported in Refs [20, 24] for normal modes represented in the form $\xi(\mathbf{r}, t) = \xi(\mathbf{r})\exp(-\mathrm{i}\omega t + \mathrm{i}m\phi + \mathrm{i}k_z z)$.

Figure 3 depicts the frequency and the energy of axisymmetric ($m = 0$) and nonaxisymmetric ($m = 1$) eigenwaves depending on the equilibrium parameter $\Omega_1/\omega_\mathrm{A}$ that characterizes rotational velocity (ω_A is the Alfvén frequency). The instability zone is shaded. In the axially symmetric case (Fig. 3a), only positive energy waves can be excited in the system. The value of $\Omega_1/\omega_\mathrm{A} \approx 2.0$ (MRI threshold for $m = 0$) corresponds to the point of merging of two branches in Fig. 3a. The nature of axially symmetric MRI is unrelated to negative energy waves and can be associated with a mechanism resembling Rayleigh – Taylor instability [23].

Positive and negative energy waves with $m = 1$ (Fig. 3b) can coexist in the system when $\Omega_1/\omega_\mathrm{A} > 1$. In this case, the instability threshold is $\Omega_1/\omega_\mathrm{A} \approx 1.7$ (which corresponds to a radial mode with $n_r = 0$), when NEW and PEW frequencies coincide as mentioned earlier. The discreteness of the spectrum also permits utilizing the above combined functional (16). This example is considered at greater length in Ref. [22].

To sum up, generalization of the classical energy principle for the case of dynamic equilibria, i.e., flows, is feasible, albeit not universal.

3. Trapped particle instability

During the second period of work at the Kurchatov Institute, Boris Borisovich Kadomtsev devoted much attention to the nature of plasma turbulence in tokamaks and the closely related problem of anomalous particle and energy transport across a magnetic field. According to Oleg Pavlovich Pogutse, a disciple and the then closest associate of Boris Borisovich, Kadomtsev thought of something as simple as flute instability but unique to tokamaks. In the long run, Kadomtsev arrived at the notion of trapped particle instability [2, 25], the nature of which can be described as follows.

In tokamaks and some other toroidal systems with nested magnetic surfaces created by lines of force with ergodic winding, flute instability of a low-pressure (compared with magnetic field pressure) plasma is stabilized by magnetic shear, i.e., the intersection of magnetic lines of force at adjacent magnetic surfaces. Physically, such a stabilization is achieved by efficient redistribution of local perturbation of electrostatic potential over the entire magnetic surface under the effect of rapid (with thermal

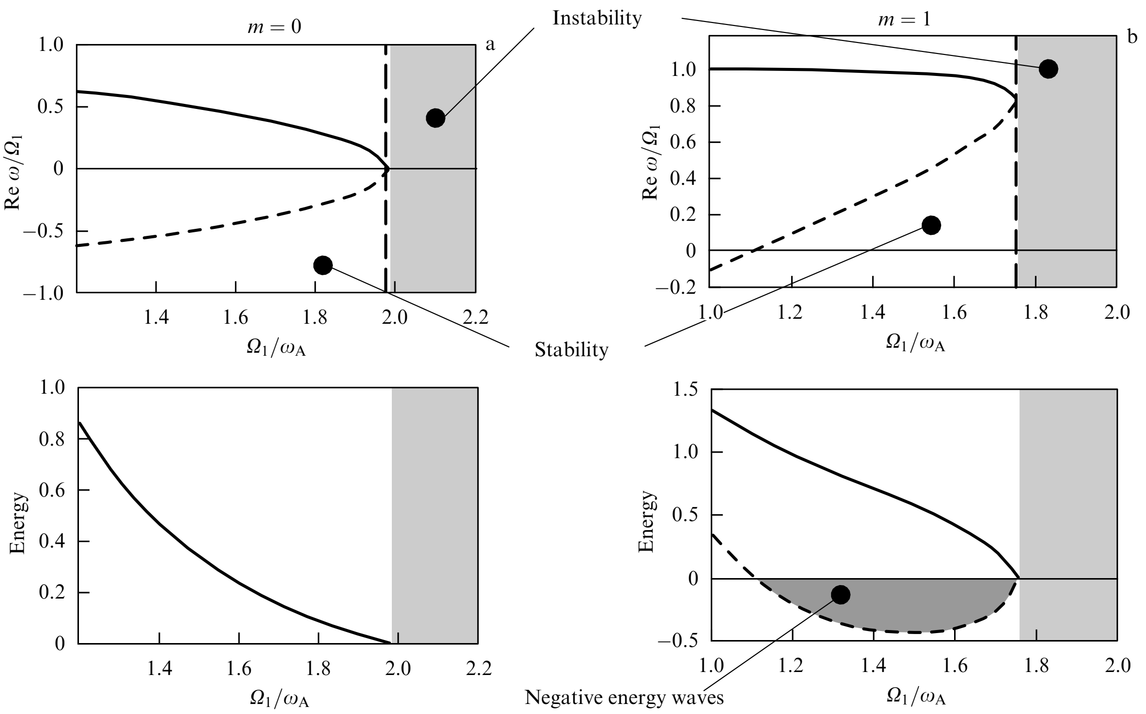

Figure 3. Energy (arbitrary units) and frequency of the most unstable eigenmodes: (a) axially symmetric with $m = 0$, and (b) asymmetric with $m = 1$. The instability region is shaded [20].

speed) charge flow along magnetic lines of force. As a result, a magnetic surface becomes an equipotential that hinders percolation of plasma flutes arrayed in the poloidal direction along the radius, as in open traps. However, the concept of free flow of charges over magnetic surfaces during their motion along magnetic lines of force is not quite correct. Figure 4 depicts projections of typical trajectories of charged particles in the tokamak magnetic field onto its poloidal (left) and toroidal (right) cross sections (for definiteness, Fig. 4 displays a situation in which directions of toroidal current and toroidal magnetic field coincide; the trajectories of positively charged particles are only presented). Figure 4a shows the so-called transit particles whose trajectories enclose both magnetic and geometric axes of the tokamak and only slightly deflect from the respective magnetic surface. The trajectory thickness in the figure is given by the diameter of the particle's Larmor orbit. Figure 4b presents particle trajectories having a small cosine of the pitch angle, i.e., angle α between the directions of particle velocity and magnetic field. Such particles are highly sensitive to magnetic field nonuniformities along the trajectory and may be trapped between magnetic mirrors formed at the magnetic surface due to nonuniformity of the toroidal magnetic field ($B_T \sim 1/r$, where r is the distance to the tokamak axis). Poloidal projections of trapped particle trajectories are sometimes called 'banana' orbits for their shape. In other words, the trajectory of a trapped particle does not enclose the entire magnetic surface but spreads over a part of its area only. Therefore, it can be imagined for sufficiently low-frequency processes that such particle movements fail to ensure exact compensation for perturbation of the electric potential by longitudinal motion along the lines of force. The trapped particle simply cannot move under the action of perturbation, being confined between the magnetic mirrors. Certainly, the fraction of trapped particles is

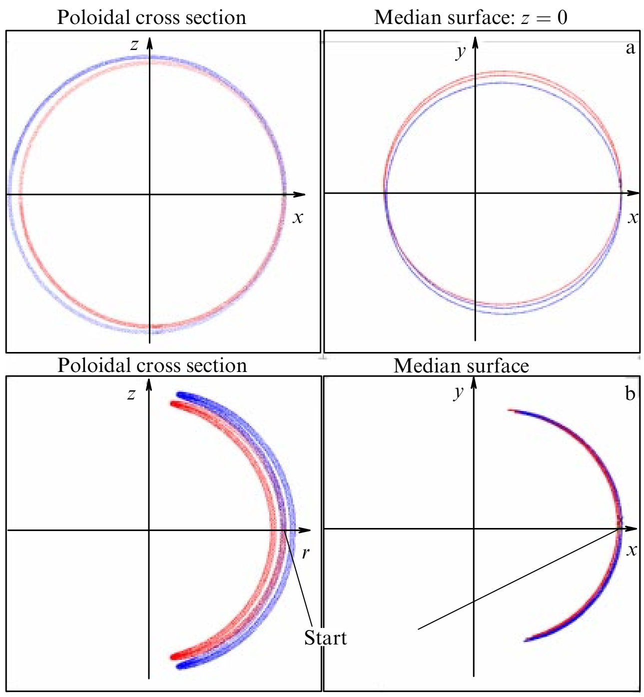

Figure 4. Typical trajectories of transit (a) and trapped (b) particles in a tokamak starting from the same point with opposite velocities. The dark color (blue in the on-line version) corresponds to $v > 0$, the light one to $v < 0$.

relatively small. The maximum mirror ratio on a magnetic surface of radius ρ is given by

$$\Pi = \frac{1+\varepsilon}{1-\varepsilon}, \qquad (21)$$

and those particles whose pitch angle α satisfies the relation

$$\left|\cos\alpha\right| = \left|\frac{v_{\parallel}}{v}\right| \leqslant \sqrt{\frac{\Pi - 1}{\Pi}} \approx \sqrt{2\varepsilon}\,,$$

where $\varepsilon = \rho/R$, and R is the major radius of the tokamak (magnetic axis radius), prove to get trapped. Thus, the fraction of trapped particles (in the case of isotropic distribution in the phase space) $\sim \varepsilon \ll 1$, and charges produced due to them are to a large extent compensated by redistribution of transit particles. Owing to this effect, the increment of trapped particle instability is relatively small [2].

How then can plasma rotation affect this instability? Seemingly, toroidal rotation at the tokamak periphery may not appreciably influence the instability because it simply leads to cooperative displacement of particles (both trapped and transit) along the torus; the effect of poloidal rotation is not so obvious. Indeed, rotation of a magnetized plasma (i.e., collective motion of ions and electrons) is normally associated with the presence of a radial electric field, the electric drift being the sole type of drift motion whose velocity does not depend on the charge sign (the central region of the plasma column in a tokamak is usually negatively charged). As shown in Fig. 4b, a positively charged trapped particle starting parallel to the field direction deflects *inwardly* due to toroidal drift and acquires kinetic energy in the presence of a radial electric field. This excess energy may be sufficient for the particle to pass through a magnetic mirror and become a transit particle. For a particle with energy E, the mirror ratio in formula (21) should be effectively decreased by $1 + e\phi'\Delta_{\mathrm{b}}/E$ times, where $\phi(\rho)$ is the electric potential, and Δ_{b} is the half-width of the banana orbit. A particle starting from the same point in the opposite direction drifts outward from the original magnetic surface and, consequently, turns out to be trapped even more strongly. Electrons drift in opposite directions, but the charge sign in the above correction for the mirror ratio also changes. The situation at the center of the plasma column is more interesting due to the known asymmetry of the velocity space at certain $|\cos\alpha|$ values; namely, one of the two particles starting in opposite directions may prove to be transit, while the other trapped (Fig. 5). The former remains transit even if it loses speed when moving away from the center, while the latter is still trapped; only radii of their orbits decrease in the poloidal cross section. It should be borne in mind that particle trajectories in the core region barely follow the magnetic surfaces (see Fig. 5), whereas rotation diminishes this difference. Naturally, the effect will be opposite when $\phi'(\rho)$ has the opposite sign.

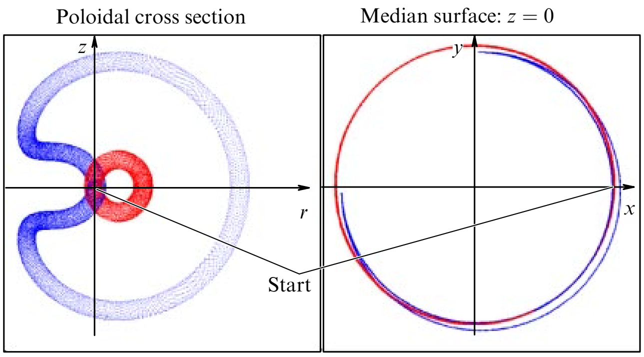

Figure 5. Asymmetry of the trajectories of particles starting from the same point with opposite velocities in the center of a tokamak.

Nevertheless, it can be concluded that trapped particle instability does not suffer variation to any great extent in a rotating plasma.

4. Reduced magnetohydrodynamic equations

Large tokamaks were extensively designed and built in different countries in the 1970s. The striking success of the tokamak T-10 at the Kurchatov Institute and the Princeton Large Torus (PLT) opened the gate to bigger tokamaks of the next generation, such as the T-15 in the USSR, the Tokamak Fusion Test Reactor (TFTR) and Doublet III in the USA, the Tore-Supra in France, the Joint European Torus (JET) in the UK, and the JT-60 in Japan. In those years, Kadomtsev formulated the concept of switching from physical research to thermonuclear engineering. He became interested in plasma self-organization, which needed nonlinear equations to be described. As is known, consideration of nonlinearity is equally important to address disruption instability, which is especially dangerous for tokamak plasma that first develops as a helical mode and thereafter leads to ejection of plasma and current channel onto the chamber wall. The physics of such instability was highlighted in the report by S V Mirnov at the present session (p. 725 of this issue). We shall focus here on the formalism invoked for the description of this instability.

A simplified (but adequate for the phenomenon under consideration) nonlinear model is needed because both MHD equations and drift equations are too complicated for comprehensive three-dimensional simulation, mainly by virtue of their multiscale nature. For example, MHD phenomena involve physical processes having totally different (by several orders of magnitude) spatial and temporal scales, including Alfvén, thermal, inertial, resistive and so forth. Direct numerical simulation of such complex phenomena is impracticable since small-scale errors accumulate into uncontrollable errors on large scales. Moreover, the power of even the best supercomputers is thus far insufficient for such calculations with the necessary accuracy within observable time. Therefore, Boris Borisovich decided to derive simplified (reduced) equations suitable for practical numerical simulation based on kink mode dynamics, including a nonlinear one.

The main objective of such a work was to derive equations describing the low-frequency nonlinear dynamics of tokamak plasma by canceling out higher-frequency stable magnetoacoustic oscillations from original MHD equations. In practical terms, this objective could be achieved by performing expansion in a small parameter characteristic of tokamaks (poloidal-to-toroidal magnetic field ratio $\epsilon = B_{\perp}/B_0 \ll 1$) and thereby moving from a three- to a two-dimensional problem. Somewhat later, the idea of reduced equations for tokamaks and stellarators was further developed in the works by such reputed researchers as M Rosenbluth, R Haseltine, and R White, and in many studies by H Strauss (see, for instance, Ref. [26]); this explains why the equations first derived by Kadomtsev and Pogutse [3] are not infrequently associated with the name of Strauss.

Of utmost importance was simplifying the description of the nonlinear dynamics of Alfvén perturbations by utilizing the freezing-in equation for an effective magnetic field defined by a single scalar flow function ψ:

$$\mathbf{B}_* = \mathbf{B}_\perp - \mu \frac{\rho}{R} B_0 \mathbf{e}_\theta = \nabla\zeta \times \nabla\psi \,, \tag{22}$$

where μ is the rotational transform angle in a tokamak with major radius R, and $\mathbf{e}_\theta = \rho\nabla\theta$ and $R\nabla\zeta$ are the unit vectors in the poloidal and toroidal directions, respectively. For $\epsilon \ll 1$, this freezing-in equation for the magnetic field reduces to the freezing-in equation for field B_*, automatically fulfilled for incompressible flows with $\mathbf{v} \approx \mathbf{v}_\perp$, $\operatorname{div}\mathbf{v}_\perp = 0$, with the frozen-in flux ψ: $\partial\psi/\partial t + \mathbf{v}\nabla\psi = 0$. Then, the Euler equation reduces to

$$\rho \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} + \nabla P = \frac{1}{4\pi} (\mathbf{B}_* \nabla)\mathbf{B}_* \,,$$

where

$$P = \frac{1}{8\pi} \left(2B_0 B_\zeta' + B_\perp^2 + 4\mu^2 \frac{\rho^2}{R^2} B_0^2\right) + \frac{\mu B_0 \psi}{2\pi R^2}$$

plays the part of pressure. Thereby, the plasma motion problem is reduced to the problem of two-dimensional flow of an incompressible ideally conducting fluid with the frozen-in magnetic field $\mathbf{B}_*$. The reduced equations under discussion made it possible to simply and demonstrably simulate the evolution of so-called bubbles, disruptions, and other nonlinear phenomena in tokamaks. This reduction procedure proposed by Kadomtsev and Pogutse for an ideal single-fluid MHD model provided a basis for a new field of research on nonlinear dynamics of magnetized plasma. Its principles were later applied to simplify more complicated models, such as Braginskii's two-fluid dissipative equations employed for the description of peripheral plasma.

For all the advantages of this reduction procedure, it is not free from some drawbacks. It is easy to see that perturbations of $B_\parallel$ and $\operatorname{div}\mathbf{v}_\perp$ can be neglected only in the principal order of expansion in parameter ϵ. Therefore, the procedure lacks self-consistency, and the dynamics of the system violate the assumptions on which they were derived. Moreover, the reduced equations do not admit stationary states with flows due to broken relabeling symmetry intrinsic in original MHD equations (as mentioned in Section 2, it is relabeling symmetry that signifies the admissibility of stationary flows in the hydrodynamic system of interest). In order to overcome this drawback and generalize the Kadomtsev–Pogutse approach, the research group headed by V P Pastukhov in the Plasma Theory Division of Kurchatov Institute NFI undertook the development of the method for adiabatic separation of fast and slow motions, allowing ideal and weakly dissipative dynamic systems to be reduced using different small parameters [27]. A given method is essentially the generalization of the classical Van der Pol method to the case of continual Lagrangian systems.

The principle of the method is as follows. Let a weakly dissipative system have fast and stable collective degrees of freedom with characteristic frequencies $\sim \omega_{\mathrm{F}}$ and slow collective degrees of freedom with the frequencies $\sim \omega_{\mathrm{S}} \sim \epsilon\omega_{\mathrm{F}}$, where $\epsilon \ll 1$, as before (the putative smallness of system deviation from ideality is also related to the value of ϵ). Adiabatic transformation of generalized (flow) coordinates α^i in the form $\delta_{\mathrm{a}}\alpha^i = -\xi_{\mathrm{a}} \nabla\alpha^i$ is sought by analogy with relabeling symmetry transformation. This transformation does not change a Lagrangian with an accuracy up to terms of order ϵ^2:

$$\delta_{\mathrm{a}} \int_\Gamma L\big(\{\alpha^i\},\, \{\partial_t \alpha^i\},\, \{\nabla\alpha^i\},\, \epsilon\big)\, \mathrm{d}^3 r = O(\epsilon^2)\,.$$

The velocity field of slow (adiabatic) motion has the same functional structure and does not perturb fast degrees of freedom. Then, the reduced equation of motion is derived from Hamilton's principle of least action using ξ_{a} as a variable.

The simplest model of turbulent convection and transport is based on single-fluid magnetohydrodynamics with the adiabaticity parameter $\epsilon^3 \sim \chi/c_{\mathrm{s}} a \ll 1$ and adiabatic velocity field

$$\mathbf{v}_{\mathrm{a}} = \frac{\mathbf{B}_{\mathrm{p}} \times \nabla\Phi}{B_{\mathrm{p}}^2} \sim \epsilon c_{\mathrm{s}}\,.$$

Here, χ is the classical heat conductivity coefficient serving as a 'priming' dissipative process, c_{s} is the speed of sound in a plasma with transverse size a and poloidal magnetic field $\mathbf{B}_{\mathrm{p}}$, and Φ is the toroidal magnetic flux frozen-in to the plasma (for certain reasons, the discussion of which is beyond the scope of this report, the use of quantity Φ instead of poloidal flux ψ contained in formula (22) may be more favorable). The characteristic frequencies of the low-frequency convection under discussion, $\omega \sim \epsilon k_\perp c_{\mathrm{s}}$, are significantly lower than those of the following stable oscillation branches: magnetoacoustic with the frequency $\omega \sim k_\perp c_{\mathrm{A}}$, Alfvénean with $\omega \sim k_\parallel c_{\mathrm{A}}$, and longitudinal acoustic with $\omega \sim k_\parallel c_{\mathrm{s}}$. The reduction procedure formalized as expansion in the parameter ϵ of the action integral permits cutting off the above stable degrees of freedom and obtaining self-consistent equations for low-frequency convection of the plasma. In this scheme, the simplest expression for P present in Kadomtsev–Pogutse equations is replaced by the heat transfer equation written for the plasma entropy function, and the heat energy fluctuation equation taking into account all sources and sinks of energy in the system of interest (high-frequency heating, ohmic heating, viscous heat release, radiation losses, etc.) [27].

The reduced equations thus obtained make it possible to use an affordable personal computer for unique numerical calculations of the self-consistent nonlinear dynamics of a plasma system for time periods on the order of its lifetime. Notice that the most advanced gyrokinetic codes currently available, in which reduction has been performed to date for a single fast time (Larmor gyration period of charged particles), allow only a few dozen characteristic times of turbulence development to be computed. The results of these calculations demonstrate universal properties of fully developed plasma turbulence, which manifest themselves in experiments on tokamaks and other plasma confinement devices. These properties are as follows:

— wide frequency spectrum of observed oscillations with one or several dominant frequencies;

— intermittency and non-Gaussian statistics;

— nondiffusive character of transverse (with respect to magnetic field direction) transport of particles and energy;

— formation and presence of long-lived nonlinear structures ('filaments', 'blobs', 'streamers', etc.) in plasma;

— well-apparent trend toward self-organization of dynamic and transport processes (self-consistency of plasma

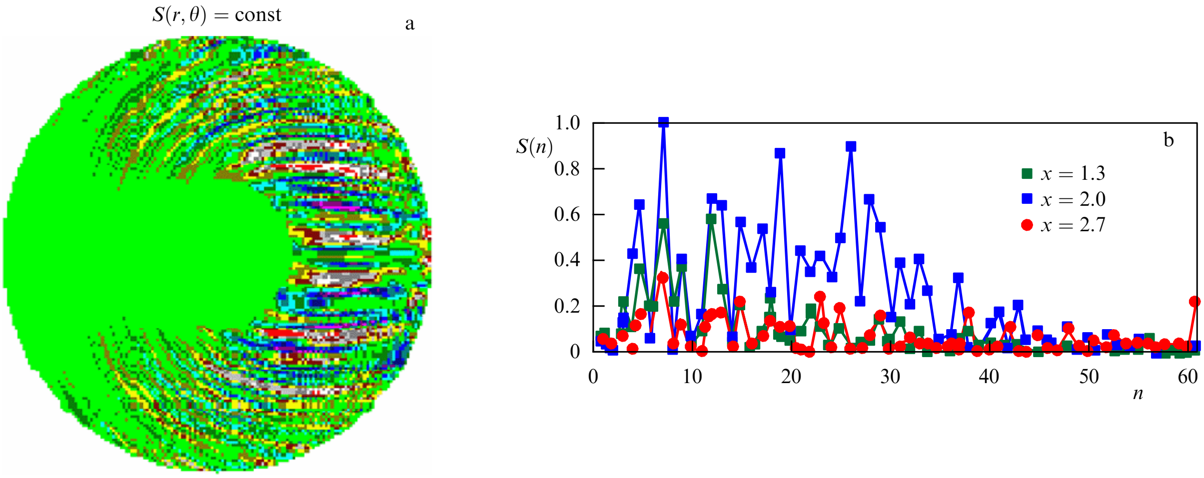

Figure 6. Cross section of isoentropic surfaces (a) and entropy fluctuation spectrum (n is the wave number) for the regime of fully developed MHD turbulence (b) [28].

parameter profiles, L–H transitions, 'transport barriers', etc.).

By way of illustration, Fig. 6 shows typical cross sections of isoentropic surfaces and the spectrum of entropy function fluctuations. The wide fluctuation spectrum does not lead, however, to oscillations of an averaged entropy spatial profile or other plasma parameters that remain quasistationary. The essence of turbulent self-organization is that deviation from the established profile immediately leads to the enhancement of oscillations and transfers to compensate for such a deviation. A practical consequence of the above physical picture is the possibility of controling turbulent transport by means of spatial redistribution of the sources of particles and power introduced into the system [28].

In conclusion, I would like to emphasize once again that many problems, the importance of which B B Kadomtsev understood fairly well at the early stages of the development of the hot plasma theory (in particular, plasma turbulence and self-organization, mechanisms and methods of suppression of large-scale instabilities, physics of transport processes, and nonlinear dynamics), remain of utmost significance in the modern period of translating fusion research into practical reactor-scale thermonuclear facilities. Just as much credit is due to Kadomtsev for his remarkable physical intuition, foresight, and ability to see exactly what is needed at the moment and act accordingly. The principles and approaches to the solution of the aforementioned problems, formulated and developed by Kadomtsev, continue to be relevant and are being successfully developed by the present generation of theorists, his followers, as I tried to briefly illustrate in this report.

My sincere gratitude is due to M S Aksent'eva, to whose perseverance and enthusiasm I owe publication of the print version of this report.

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