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Commemoration of the 80th anniversary of the birth of Academician B B Kadomtsev
(Scientific session of the Physical Sciences Division of the Russian Academy of Sciences, 10 December 2008)

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The scientific session of the Physical Sciences Division of the Russian Academy of Sciences (RAS) was held on December 10, 2008 at the conference hall of the P N Lebedev Physical Institute, RAS. It was devoted to the commemoration of the 80th anniversary of the birth of Academician B B Kadomtsev. The following reports were presented at the session:

(1) **Smirnov V P** (Nuclear Fusion Institute of the Russian Research Centre 'Kurchatov Institute', Moscow) "Commemorating the 80th anniversary of the birth of Boris Borisovich Kadomtsev (opening address)";

(2) **Mirnov S V** (State Scientific Center of the Russian Federation 'Troitsk Institute of Innovative and Thermonuclear Research', Troitsk, Moscow region) "Academician B B Kadomtsev and the International Thermonuclear Experimental Reactor (ITER)";

(3) **Kruglyakov E P** (G I Budker Institute of Nuclear Physics of the Siberian Branch of the RAS, Novosibirsk) "Open magnetic systems for plasma confinement";

(4) **Kovrizhnyh L M** (A M Prokhorov Institute of General Physics, RAS, Moscow) "The current status of the stellarator program";

(5) **Gurevich A V** (P N Lebedev Physical Institute, RAS, Moscow) "Nonlinear phenomena in the ionospheric plasma";

(6) **Ilgisonis V I** (Nuclear Fusion Institute of the Russian Research Centre 'Kurchatov Institute', Moscow) "Classical results of B B Kadomtsev and the plasma rotation in modern tokamaks".

An abridge version of the opening address and reports 2, 4, and 6, as well as a paper written on the basis of report 5, is given below.

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Boris Borisovich Kadomtsev
(09.11.1928 – 19.08.1998)

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Commemorating the 80th anniversary of the birth of Boris Borisovich Kadomtsev
(opening address)

V P Smirnov

Today's session is unusual. Recently we have conducted at the Physical Sciences Division of the RAS a relatively large number of sessions commemorating centenaries of the births of outstanding Russian scientists. Today, however, Boris

Borisovich Kadomtsev would be only 80 years old; all of us still remember him very well and it feels as if he may walk in right now, right into this hall.

The first thing I wish to tell you, on behalf of Evgenii Pavlovich Velikhov, is that he very much wanted to be present at this session and deliver the opening address, but his many responsibilities in the Civic Chamber of the Russian Federation — I talked to him today — truly precluded him from doing it. Therefore, E P Velikhov has begged us to excuse him for his inability to be present in person at this gathering and requested that I say a few words on his behalf.

Boris Borisovich is no more... . This is a colossal loss to the entire Kurchatov Institute pursuing plasma research, and, I believe, a huge loss for the entire Russian physics community, not to mention the world physics community. The talks that we are to hear today are a reflection of the large scale of Boris Borisovich's activities in the physics of hot plasma and thermonuclear fusion that he conducted during his entire span of work at the Kurchatov Institute. B B Kadomtsev laid the scientific foundation for the tokamak reactor, and together with L A Artsimovich they for the first time formulated a very important statement: tokamak-based thermonuclear fusion is feasible — a thermonuclear reactor can be built, and it can be built in spite of those numerous instabilities and physical problems we keep coming across. S V Mirnov will speak today about the analysis of these instabilities, including those of magnetohydrodynamic (MHD) processes.

Boris Borisovich made an enormous contribution to finding a solution to the problem of thermonuclear fusion, and when we meet at our traditional conference — the biannual IAEA conference on thermonuclear fusion — the ideas advanced by Boris Borisovich surface again and again, and people do not forget it.

We need to say too that it was not only hot plasma physics and controlled thermonuclear fusion that lost their great scientist, one who had this ability to find order in enormously complicated processes with incomparable clarity. Boris Borisovich's scientific interests were never limited to plasma physics and nuclear fusion; all of us who had any sufficiently close interaction with Boris Borisovich were amazed by the ease with which he would perceive the most diverse fields of physics. When, for instance, I or my colleagues talked to him about papers that seemed to lie far from the boundaries of his everyday scientific activities, he would catch on very fast to the subject and pose key questions. This was Boris Borisovich's wonderful ability.

And I wish to add that he was a great teacher. He headed for a long time Chair of Plasma Physics and Chemistry at the Moscow Institute of Physics and Technology (MFTI in *Russ. abbr*.), and many of those who are now at the frontline of research in plasma theory and plasma experiment, and perhaps not only in Russia, passed through this chair, heard his lectures, absorbed the system of perceiving the physical processes that he succeeded in somehow instilling into his students. In fact, the work being done now by his students still bears the very imprint that he left on all of us.

We feel especially happy of course that when we commemorate B B Kadomtsev's 80th birthday today, we witness very positive steps in the progress of thermonuclear fusion that at the same time place very high responsibility on our shoulders. We all know well that Russia is a member of the international cooperation that is building the International Thermonuclear Experimental Reactor (ITER). The creation of this reactor extends the activity linked to the research at the I V Kurchatov Institute of Atomic Energy that started as early as the 1970s; B B Kadomtsev played a very important and significant role in this program. When working on the ITER project, B B Kadomtsev was a permanent member of the ITER International Research and Consultative Committee opened under the auspices of the IAEA, and he introduced into the committee a number of outstanding representatives of our technology and our nuclear science. I need to mention among these the names of Academician V A Glukhikh and E O Adamov who, by his criticism, contributed very positively to designing the ITER, and M I Solonin (now Corresponding Member of the RAS). On the whole, this was a delightful team that accomplished its work so well that Russia's contribution during the times of the funding crisis and minimum support of science from the Russian State became commensurate to the contributions of other participants of the project.

B B Kadomtsev thought much about thermonuclear fusion as an energy source and came up with sometimes positive, but on other occasions rather negative, assessments of specific solutions concerning thermonuclear fusion. He understood at the same time that the road to thermonuclear fusion is extremely hard and will require enormous efforts, including the efforts of physicists performing fundamental research.

I should also mention another area of research which began on B B Kadomtsev's initiative. A department was set up in the I V Kurchatov Institute of Atomic Energy, headed by Viktor Vladimirovich Orlov. This department worked on technological and physical aspects of thermonuclear reactors and, in particular, on the analysis of the feasibility of creating a hybrid thermonuclear reactor. At this moment, after complete rejection following the activities of the 'green opposition', this idea is reborn, not only in this country (in fact, in Russia it mostly has 'acoustic', not real, power; nevertheless, we are discussing it). However, in other countries — those participating in thermonuclear fusion activities — the idea of the hybrid reactor is attracting more and more attention. We have a proposal from the United States to collaborate on it, and China is manifesting a very active and perfectly focused desire to solve the problems of producing fuel for the atomic power industry at hybrid power stations. In a word, the activities of Boris Borisovich, both as a scientist and as the actual leader of the thermonuclear program at the Kurchatov Institute, have created inroads into, and opened access to, very many research avenues. I see that Aleksandr Grigor'evich Litvak is looking at me. I am obliged to say that through Kadomtsev's support, work on gyrotrons was commissioned and extended, and this work was carried out at the highest physical and technological level with excellent results by the Institute of Applied Physics of the RAS in Nizhny Novgorod, now headed by Academician A G Litvak. This wide scope of studies that were accomplished at the Kurchatov Institute when Kadomtsev was Head of the Plasma Studies Division is very important and promises great gains in the future.

The talks prepared for this session will describe in detail the progress of a number of Kadomtsev's ideas at the current stage of our science.

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Academician B B Kadomtsev and the International Thermonuclear Experimental Reactor (ITER)

S V Mirnov

Kadomtsev Boris Borisovich (1928–1998), physicist, Full Member of the Russian Academy of Sciences (1970). Main works on plasma physics (stability, transport processes, thermal insulation, etc.) and controlled thermonuclear fusion. USSR State Prize (1970), and Lenin Prize (1984).
Great Russian Encyclopedia, 2007

Academician Boris Borisovich Kadomtsev belonged to a group of physicists of whom their colleagues can speak only in superlatives. The spectrum of his scientific interests is astonishing. The posthumous collection of his works in two volumes contain not only the widely known publications on high-temperature plasma physics that became classical but also those on quantum mechanics (his first and lifelong love as a disciple of D I Blokhintsev), the behavior of matter in superstrong magnetic fields, the theory of nonlinear and stochastic processes, magnetic reconnection, the theories of solitons and ball lightning, and above all controlled thermonuclear fusion. Boris Borisovich treated all these issues with plenty of original thinking. The seeds sown by him will bring forth fruit at some time or another. Suffice it to mention his interpretation of the known *Sokolov effect* (appearance of coherent addition to an excited hydrogen atom flying near a metallic surface). Boris Borisovich expounded his vision of quantum mechanics and the Sokolov effect in the book *Dynamics and Information* [2][1] that he finished not long before his death. In a way, he had cast a glance 'behind the horizon'. It is as likely as not that in some 20 years an encyclopedia will mention B B Kadomtsev not only as a founder of the theory of thermonuclear plasma transport and stability, but also as an author of a new interpretation of quantum mechanics.

I happened to collaborate with Boris Borisovich only in a relatively narrow area of his scientific interests, namely plasma stability and transport in tokamaks, hence the choice of the subject of this report. Despite the seeming narrowness of this subject, it was B B Kadomtsev's focus of attention for a long period of his scientific and administrative work as Director of the Nuclear Fusion Institute (NFI) in the Russian Research Centre 'Kurchatov Institute'. That period was a time of rapid progress in research on controlled thermonuclear fusion carried out abroad after the powerful incentive given by the tokamak concept suggested by Andrey D Sakharov and I E Tamm in the early 1950s and universally accepted in the mid-1960s through efforts of L A Artsimovich, B B Kadomtsev's predecessor in the capacity of Director of the Kurchatov Institute's NFI.

Figure 1, borrowed from display materials of the Princeton Plasma Physics Laboratory (PPPL), USA, illustrates the growth dynamics of deuterium–deuterium (DD) and deuterium–tritium (DT) thermonuclear fusion power achieved in tokamaks of different countries from 1970 to 1996. The lower portion of the curve shows the results

[1] See also Refs [3–8].

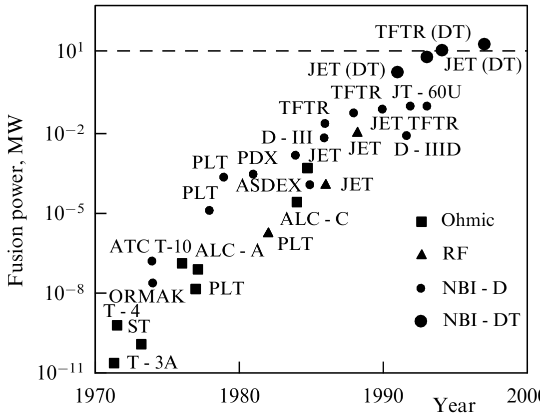

Figure 1. Progress of tokamaks: Ohmic — ohmic heating alone; RF — high-frequency heating; NBI-DT and NBI-D — heating by neutral deuterium beams with and without tritium, respectively.

obtained with Soviet tokamaks (T-3A, T-4, and T-10) in the late 1960s and early 1970s.

The overall success of the tokamak program, i.e., the construction of fusion facilities as the first demonstration nuclear power plants, provided a physics basis for the world's first industry-scale International Thermonuclear Experimental Reactor (ITER) currently under construction in France through the joint efforts of Europe, India, China, Russia, USA, South Korea, and Japan. It is appropriate to recall that diplomatic and organizational activities concerning this project were initiated by our country and translated into meaningful proposals at the well-known meeting of Mikhail Gorbachev and Ronald Reagan in 1985. Further coordination of these activities from the Russian side was entrusted to E P Velikhov, while B B Kadomtsev acted in the capacity of key native expert in tokamak research as a recognized founder of modern high-temperature plasma physics.

It is believed that Kadomtsev first addressed the physical aspects of tokamaks as potential candidates for fusion reactors in the paper published by *Sov. Physics – Uspekhi* in 1967 under the title of "Plasma instability and controlled thermonuclear reactions" [9]. There is little doubt that the paper was inspired by L A Artsimovich, who prefaced it by his own review in the same issue concerning the state of the art of thermonuclear research and reporting the absence of so-called Bohm diffusion in Soviet T-3 and TM-3 tokamaks where the plasma energy lifetime was 10 times the calculated 'Bohm' lifetime. Artsimovich summarized the review [10] with the following conclusion: "This result affords ground for cautious optimism with which we are now looking at prospects for the development of future research in this area [i.e., tokamak physics]."

Despite the delicate form of this assertion, it was a rather radical statement for that time: the most promising foreign thermonuclear facilities (stellarators) almost universally suffered substantial anomalous plasma leakage across the strong magnetic field, i.e., turbulent diffusion named after the American scientist D Bohm, who observed it in gas discharges in a strong magnetic field in the 1940s. Had this diffusion really been universal, the idea of thermonuclear fusion in closed magnetic traps (tokamaks and stellarators) would have had to have been abandoned. Many serious theorists of that time tried to prove the universal character of Bohm diffusion. Boris Borisovich was among them.

In this context, the downfall of Bohm universality in relation to tokamaks looked like sheer nonsense. The point is that many Russian and foreign physicists of that period regarded the tokamak concept as physically ungrounded, in contrast to the stellarator one. Electron flux forming current that traverses plasma along the direction of the strong tokamak magnetic field and giving it a helical shape was thought to be an additional energy reservoir for a battery of magnetohydrodynamic and kinetic instabilities. Moreover, if tokamak plasma were macroscopically stable, the longitudinal electric field applied to a plasma turn would gradually convert all current-carrying electrons into a relativistic electron beam (plasma betatron). Tokamaks would be a step down 'from the Bohm' rather than an improvement.

However, the real situation proved more complicated than *a priori* considerations. In 1962, E P Gorbunov and K A Razumova showed for the first time that current-carrying electrons did not enter betatron acceleration in the usual macroscopically stable regime of the TM-2 tokamak. This is a fundamental physical phenomenon underlying the current tokamak concept. Later studies demonstrated that the passage of electrons to acceleration was hampered by certain electron beam instability scattering electrons being accelerated over a range of angles. Spontaneous suppression of this instability in certain exotic regimes immediately transformed the tokamak into a superpowerful betatron.

The first explanation of the nature of this tokamak-saving kinetic instability of electrons was offered by B B Kadomtsev and O P Pogutse [11] in 1967 by invoking the mechanism of the anomalous Doppler effect. It removed a major restriction from tokamaks. The absence of universal Bohm diffusion opened up prospects for using these magnetic traps as thermonuclear reactors. It appears that Artsimovich wanted to emerge together with Kadomtsev as a leading expert in plasma turbulence that many regarded as limitless at that time. Now, what conclusions did Kadomtsev arrive at in his keynote paper?

"Some time ago, a pour of publications reporting more and more new instabilities gave reason to think that we shall never have a complete description of rarified plasma instability in a strong magnetic field. Fortunately, the situation is beginning to improve. Despite continuing extensive studies of plasma instabilities, it is becoming clear that only few of them pose a real threat to hot plasma confinement in the magnetic field" [9]. In other words, the number of dangerous instabilities was argued to be limited. This conclusion was not universally accepted, since pessimism about the stability of high-temperature plasma was a given at that time. Kadomtsev considered drift-temperature instability to be the most dangerous among remaining instabilities; it was expected to manifest itself as enhanced plasma leakage across the magnetic field, functionally resembling Bohm instability but multiplied by the ratio of the Larmor ion radius to the minor radius of the plasma column. Today, transfer of this type is referred to as gyroBohm transport and accepted as the most probable one to occur in the ITER based on experimental data obtained with operating tokamaks for the last 40 years, rather than on theoretical predictions (see Fig. 1).

In 1966, the available experimental database was too small for such generalizations. However, it allowed for reasoning 'in Bohm fractions'. Kadomtsev calculated the minimal size of a DT-fuelled reactor-tokamak proceeding from the maximally possible (from the theorist's standpoint) magnetic field induction ($B_{\rm T} = 10$ T) created by superconductors and unbelievably high (at that time) plasma temperature ($T = 10$ keV). He found that in the case of Bohm leakage, the minor torus radius a should be unreasonably large (14 m). In the case of a two-order-of-magnitude-lower leakage (lifetime of '100 Bohms'), the minor radius had an acceptable size of 1.4 m. Thus, the values of $a = 1.4$ m and $B = 10$ T were chosen as the tentative parameters of the DT-fusion reactor-tokamak.

In 1967, the possibility of reducing plasma losses to 1/100 of the Bohm ones seemed practically unattainable. In conclusion, Kadomtsev cautioned enthusiasts of thermonuclear fusion [9]: "In order to achieve controlled thermonuclear fusion by this approach [i.e., in tokamaks], it will be necessary to overcome huge technical difficulties of creating a magnetic field on the order of 10 T in a cubic meter-scale volume. These are preliminary conclusions. Extensive physical surveys are needed to verify them... ."

Indeed, later research clarified the situation. Within a year after Kadomtsev's publication, L A Artsimovich reported at a conference held under the auspices of IAEA (Novosibirsk, 1968) that the plasma energy lifetime obtained in domestic T-3 and TM-3 tokamaks was 40 times the Bohm lifetime. Certain suspicious foreign researchers wished to check our measurements of the electron temperature, a key parameter of the T-3 experiment, by the direct laser scattering method then developed in the UK. In a year (in 1969), the joint British-Soviet team successfully repeated these measurements and gave reporters cause to announce with much ado that "the results proved even higher than those reported by the Russians." (The Russians measured average temperature by plasma diamagnetism, while the laser-assisted method offered a local temperature. The results obtained by the two methods were in good agreement within the scatter of measurements.)

Direct existence proof of an electron temperature around 1 keV in tokamaks caused sensation and led to restructuring several major research programs abroad. New specialists were recruited and newly built large tokamaks (see Fig. 1) were equipped with additional sources of plasma heating, such as beams of neutral atoms (NBI, Fig. 1), high-frequency heating systems (RF, Fig. 1), and present-day diagnostic facilities. As a result, the ion temperature of about 10 keV was recorded in the Princeton Large Torus (PLT) tokamak (USA, 1979) and the so-called breakeven condition was achieved in 1997 in experiments with a DT-mixture (NBI-DT, see Fig. 1) in the Joint European Torus (JET) tokamak (rough equality of thermonuclear fusion power (ca. 17 MW) and heat flux from the hot zone of the plasma column, i.e., thermal plasma losses). These experimental results constitute the basis for ITER. The gap between the Bohm lifetime and real energy lifetime of the plasma (ca. 1 s) amounted to hundreds of times, which allowed the ITER toroidal field to be reduced from 10 to 6 T at a minor plasma column radius of 2 m. As a result, the concept of the reactor-tokamak acquired concrete features remaining in the same energy limits as declared by B B Kadomtsev in 1967. (To recall, the first toroidal magnetic reactor proposed by Andrey D Sakharov and I E Tamm in the early 1950s had a similar minor radius (2 m) and magnetic field (5 T) but was designed to be used for DD-fusion instead of DT-fusion!)

After the death of L A Artsimovich in 1973, B B Kadomtsev took up his post of Head of the Plasma Studies Division at I V Kurchatov Institute of Atomic Energy (later the

Nuclear Fusion Institute at the Kurchatov Centre). Kadomtsev was in a way forced to take this position by his colleagues, who did not wish to see it occupied by an 'outsider'.

As Boris Borisovich Kadomtsev himself said in an interview to *Literaturnaya gazeta* (*Literary Gazette* magazine) soon before these dramatic events, he was a loner type of researcher, choosing to work with no more than one or two postgraduate students. In his new capacity, he had to alternate social, administrative, and other nonscientific activities, but in fact remained inwardly a skeptical researcher always concerned about the possibility of developing new spontaneous plasma instability able to reduce to nought his efforts as director. It seemed to hang heavy on him. He did not have inborn leadership qualities, and this fact proved highly beneficial for the tokamak program. Intense thinking about scientific problems being developed in his Institute brought forth fresh fruitful ideas that gave new impetus to experimental and theoretical research. In my opinion as an experimentalist in the field of plasma stability, two most popular ideas of B B Kadomtsev had the greatest influence on the understanding by plasma physicists of processes responsible for the macroscopic stability of tokamaks ('tokamak limits'). These ideas are indispensable components of the ITER physical concept:

(1) The idea of magnetic reconnection during development of large-scale instabilities in tokamaks (*Kadomtsev's internal disruption model*, 1975);

(2) The idea of ideal kink instability of the *vacuum bubble* type in the presence of low magnetic shear (in co-authorship with O P Pogutse, 1973).

The idea of magnetic reconnection was derived by Kadomtsev from astrophysics to account for transition of an ideal (without a change in the total magnetic flux) kink instabilities to resistive ones developing with a substantial loss of magnetic fluxes. The first example of successful application of this idea was interpretation of the so-called internal disruption or 'sawtooth' oscillations, i.e., characteristic instability at the center of a tokamak plasma column, manifesting itself as strictly periodic relaxation oscillations of its electron temperature.

It was shown in experiment that relaxations were preceded by the development of helical $m = 1$ perturbation resonant (i.e., having the same geometry) with the closed helical magnetic structure created by plasma current near the center.

In order to clearly demonstrate the mechanism of this phenomenon, Kadomtsev made use of the so-called additional magnetic field $\mathbf{B}^*$ (introduced earlier jointly with O P Pogutse) defined in the vicinity of a certain chosen closed helical magnetic structure as the vector difference between the real magnetic field $\mathbf{B}$ and calculated field, i.e., the magnetic field of the said magnetic structure extended over the entire region of interest. In other words, field $\mathbf{B}^*$ is identically equal to zero in the zone occupied by this structure. All magnetic lines in this zone have a similar helical geometry and therefore do not cross as they move relative to one another. Such a magnetic configuration was called configuration with *zero magnetic shear*.

The departure of the field $\mathbf{B}^*$ from zero means that the magnetic lines of force have a pitch angle different from that given by a closed magnetic structure; therefore, they ought to intersect during their motion relative to one another, i.e., to reconnect (*nonzero shear* case). The electroconducting plasma frozen into a magnetic field must hamper such an intersection

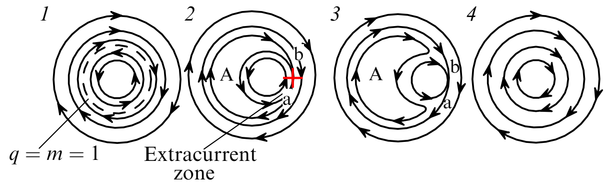

Figure 2. Schematic of internal disruption in a tokamak as postulated by Kadomtsev.

and reconnection. In other words, electric currents (*extracurrents*) must be induced in the putative intersection zone to hinder intersection of magnetic lines of force. By this means, magnetic fluxes and the plasma-confining magnetic configuration are retained in tokamaks. Abnormally fast dissipation of induced currents implies equally rapid reconnection of magnetic fields $\mathbf{B}^*$, loss of magnetic fluxes, and (accordingly) magnetic and plasma energies.

Evidently, really observable anomalously fast dissipation of extracurrents has a collisionless nature; in all likelihood, it is underlain by a certain variety of electron beam kinetic instability in the plasma. In astrophysics, anomalously fast magnetic reconnection is described in terms of the known Sweet – Parker model. Unfortunately, the same processes in tokamaks proceed more abruptly. Their physics is the subject of ongoing research.

Figure 2 presents a schematic of internal disruption in a tokamak as postulated *by Kadomtsev*. According to this model, the physical cause of internal disruption is an excessive concentration of plasma current close to the axis of the column due to a preferred heating of the plasma at the center and the resulting enhancement of its electric conduction. Concentration of plasma current in the central part of the column gives rise to a closed helical magnetic structure (shown by the dashed line in Fig. 2) formed by magnetic field lines that close up on themselves every other turn around the torus ($m = 1$). This is the first necessary condition for internal disruption to occur. Figure 2 illustrates the evolution of field $\mathbf{B}^*$ distributions at the center of the plasma column as inferred by Kadomtsev. There are four characteristic positions. Position *1* is the starting stationary position with an annular closed magnetic structure (dashed line), the maximum magnetic flux and energy accumulated at the center of the column. Current density dies away from column center to periphery; accordingly, field $\mathbf{B}^*$ changes sign as it crosses the dashed line. Position *2* corresponds to the evolution of the central ideal (i.e., retaining total magnetic flux) kink ($m = 1$) instability provoking internal disruption. The appearance of this instability is the second necessary condition for disruption; it is due to the accumulation of excessively high plasma energy near the axis and manifests itself as a helical ($m = 1$) displacement of the current axis with respect to the column center. Region a – b is the extracurrent generation zone (shown by the cross); the extracurrent prevents further outward shift of helical current perturbation as yet on retention of the total magnetic flux enveloped by perturbation helical axis. Position *3* displays the internal disruption proper or dissipation of extracurrent, implying reconnection of oppositely directed magnetic fields $\mathbf{B}^*$ on either side of the region of a closed helical magnetic structure ($m = 1$), *pressing-out* of the hot plasma of the former column center to the outside, loss of magnetic flux, and the formation of a central relatively cold *magnetic island* A with uniformly

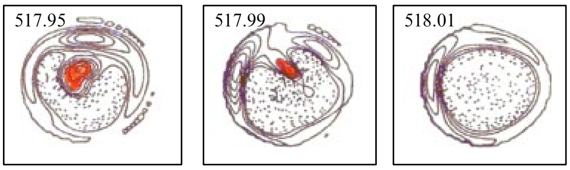

Figure 3. Time (in microseconds) sequence of SX-profiles during development of internal disruption in the Tokamak de Varenna [12].

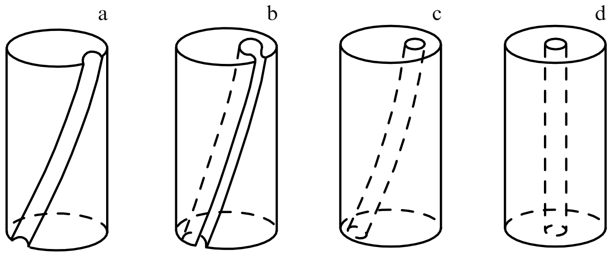

Figure 4. Successive steps of the penetration of a vacuum bubble into the center of the column [13].

distributed field $\mathbf{B}^*$, i.e., elimination of the closed structure ($m = 1$) within the entire central region (position *4*), signifying the appearance of the flattened current distribution at the column center. Thus, the process that started from ideal instability passes to dissipative instability accompanied by the loss of plasma magnetic and thermal energies. A distinctive feature of internal disruption is its local nature. It does not cause radical deterioration of the state of the plasma column as a whole. Flattening of current distribution at the center is followed by a smooth phase during which the flattened current profile gradually sharpens, and the process repeats itself in the form of quasistationary relaxations.

The *Kadomtsev model* has received wide recognition. Its validity was confirmed in numerous experiments, and the magnetic reconnection concept is extensively exploited by tokamak physicists.

Figure 3 presents X-ray tomographic images of the column central region obtained during the course of internal disruption in the Canadian Tokamak de Varenna. Equal-intensity contours of soft X-ray radiation roughly correspond to equal electron temperature. Evidently, the hot plasma core is pressed out from the center within tens of microseconds and substituted by a cold zone, closely following the scheme displayed in Fig. 2.

Another constructive idea of Kadomtsev suggested jointly with Pogutse permits us to explain the nature of major disruption in tokamaks. The authors hypothesized the development of ideal kink instability of the vacuum-bubble type under small magnetic shear conditions.

Unlike internal disruption, major disruption (or simply disruption) in a tokamak affects the entire zone through which the current flows and cools the whole column from the edge to the center for a few hundred microseconds. This is the most serious instability threatening the tokamak as a reactor. The zone where disruption instability develops bounds the region of permissible operating conditions and determines limiting parameters of the device ('tokamak limits'). The first question that arises is: what causes the rapid development of instability and fast transport of the cold plasma from the periphery to the center of the column? The vacuum-bubble capture model gives an answer to this question. It is schematically represented in Fig. 4.

Given perturbation at the edge of the plasma column in the form of a vacuum bubble or a vacuum tube (Fig. 4a) oriented along the tokamak helical field created by superposition of toroidal magnetic field $\mathbf{B}_\mathrm{T}$ and poloidal current field $\mathbf{B}_\mathrm{p}$, the magnetic energy stored in such a bubble must equal $(B_\mathrm{T}^2 + B_\mathrm{p}^2)/8\pi$ multiplied by the tube volume. The energy of a similar tube at the center of the column (Fig. 4d) will be proportional only to $B_\mathrm{T}^2/8\pi$ (the poloidal magnetic field at the center being negligibly small). Therefore, it is energetically advantageous for the bubble to pass from the periphery to the center of the plasma column; in other words, the vacuum bubble created at the edge of the column appears to be potentially unstable. Such motion of the bubble is illustrated in Figs 4b, c. What can hamper it? It can easily be shown that the answer is: the magnetic shear can. Such a passage of the vacuum tube is feasible only under zero shear conditions over the entire cross section of the column, when perturbation propagating toward the center draws the magnetic lines of force apart but does not intersect them. In this case, magnetic fluxes remain virtually unperturbed. In other words, ideal kink instability takes place with an inherently high increment. But zero shear in a tokamak must correspond to the ideally flat current distribution over the cross section, unattainable in experiment, where current density changes in parallel with electron temperature (i.e., electrical conduction) of the plasma that takes dissimilar values in different parts of the column. Under real conditions, it is the magnetic shear that maintains the existence of plasma column in a tokamak by preventing penetration of peripheral cold plasma into the core. As exemplified above by the development of internal disruption, internal instability may lead to the flattening of current distribution at the center of the column, i.e., cause local lowering of magnetic shear. Experiments (S V Mirnov, I B Semenov, T-4 tokamak, 1976) demonstrated the development of a powerful single analog of internal disruption at the column center a few dozen microseconds before the onset of major disruption; it spreads not only over an $m = 1$ perturbation region but also over $m = 2$. As a result, magnetic shear decreases practically across the entire hot zone of the plasma column; this signifies fulfillment of the condition of 'cold bubble' penetration into its center set by the Kadomtsev – Pogutse model. Elucidation of the mechanism of disruption, identification of its precursors and the safe operation zone of the reactor-tokamak were priority issues for B B Kadomtsev at the time of developing the physical basis of the ITER.

What gives us reason to expect that the planned power (500 MW) of DT-fusion will be achieved in the ITER? The necessary plasma temperature (above 10 keV) has already been obtained and the necessary plasma density n on the order of 10^{20} m^{-3} is not too high for tokamaks (the highest reported $n \approx 10^{21}$ m^{-3}). What remains to be reached is the record energy lifetime τ_E, which must be as large as 3 – 5 s against the present 1 s.

Parametric analysis of databases obtained with different tokamaks of ITER-like magnetic geometry allowed deriving the ITER's scaling law for τ_E:

$$\tau_{E,98} = 0.0365\, I_\mathrm{p}^{0.97} B_\mathrm{T}^{0.08} P_\mathrm{H}^{-0.63} n^{0.41} M^{0.20} R^{1.93} \left(\frac{a}{R}\right)^{0.23} k^{0.67}\ \mathrm{s}\,,$$

where $2a$ is the lateral dimension in meters, R is the major radius of the torus [m], I_p is the current [MA] flowing through

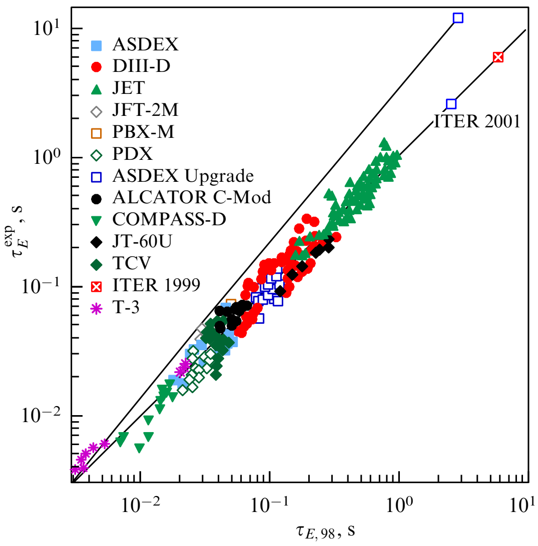

Figure 5. Experimental τ_E values and extrapolations of $\tau_{E,98}$ and $\tau_{E,68}$.

the plasma, n is the number density [10^{19} m^{-3}], B_T is the toroidal magnetic field [T], P_H is the heating power [MW], M is the mass of ions measured in the proton mass, and k is the vertical elongation of the plasma column. ITER parameters allow τ_E to be around 4 s. Figure 5 gives an idea of the departure of real τ_E from this law in different tokamaks. Rounding off and neglecting power terms with exponents smaller than 0.2, this law can be written out in a simpler form:

$$\tau_{E,98} \sim I_p n^{0.4} R^{1.7} a^{0.2} P_H^{-0.6} k^{0.7} .$$

In this form, it closely resembles the similarity rule offered by L A Artsimovich at the IAEA Conference in Novosibirsk (1968): $\tau_{E,68} \sim B_p a^2 n^{1/3}$ or $\tau_{E,68} \sim I_p n^{0.33} a$, since $B_p \sim I_p/a$.

Substituting ITER parameters into this expression, a physicist of 1968 (when absolute τ_E values were 1–6 ms) would have had 12 s, i.e., only 3 times what the same physicist would obtain 30 years later. In Figure 5, extrapolation from 1968 is shown by the upper straight line. Thus, it can be concluded that even at a 1000-fold rise in τ_E, the tokamak as a physical object exhibits surprisingly stable similarity. Evidently, it ought to persist, i.e., τ_E will increase only 4-fold upon passage from current tokamaks to the ITER.

However, an important distinctive feature of the expression for $\tau_{E,98}$ (decreasing dependence on heating power $\sim P_H^{-0.6}$) causes some concern. It was absent in the similarity law for $\tau_{E,68}$ because the latter was derived only for conditions of ohmic plasma heating, during which P_H varies but insignificantly. This feature allows a simple physical interpretation. Bearing in mind that $\tau_E = W/P_H$, where W is the plasma thermal energy, P_H can be eliminated from the right-hand side of the expression for $\tau_{E,98}$, which then takes the form

$$\tau_{E,98} \sim \left(\frac{1}{\beta_T}\right)^{1.7} V^{1/2} B_T^{0.6} ,$$

where V is the plasma volume, β_T is the ratio of plasma pressure to toroidal magnetic field pressure $(B_T^2/8\pi)$ — the second most important parameter characterizing hot plasma confinement in tokamaks. In all likelihood, it is a rise in this parameter, i.e., the energy of the plasma being heated, that is responsible for $\tau_{E,98}$ degradation with increasing P_H. Evidently, its optimization will be the main problem facing the ITER experimental program, the more so as it directly determines the commercial attractiveness of the tokamak as a reactor.

With β_T remained at the planned level (2.5%), it would be commercially justified to use the tokamak as a breeder reactor for completely burning radioactive wastes of conventional nuclear power plants. It was estimated that the introduction of a ^{232}Th blanket into a fusion reactor analogous to the ITER would yield an electric power of up to 1 GW and annually produce up to 2 tons of ^{233}U as a fuel for nuclear reactors. This amount of ^{233}U would be sufficient to generate an additional 2 GW of electric power. To recall, a power of 3 GW roughly corresponds to half the total power of the Krasnoyarsk hydroelectric plant. Such are the stakes on the line. Evidently, the road from the ITER to commercially efficient energy-producing systems will be at least as long and hard as that from the first atomic reactors of Fermi and Kurchatov to modern industrial-scale nuclear power plants. However, the first psychologically most difficult step has been made: it was shown that such a conversion *is physically possible*.

We are proud that the first physical thermonuclear reaction was accomplished in this country and in our tokamaks in the late 1960s and early 1970s, that the world's scientific community took up and further developed this idea (see Figs 1 and 5), and that the theory of thermonuclear plasma behavior was for the major part developed by our scientists and was enthusiastically accepted by their foreign colleagues. Boris Borisovich Kadomtsev was a key actor in this global process.

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The current status of the stellarator program

L M Kovrizhnykh

1. Introduction

Briefly reviewed in this report is the world stellarator program, i.e., a research program having the aim of developing a controlled fusion reactor based on a stellarator type magnetic trap. The main stages of program advancement, its current status, and plans for further research are discussed. The author took the liberty to omit, as far as possible, mathematical formulas and experimental data plots to make the subject of the report comprehensible to a wider audience of listeners (readers). I apologize to specialists for the unavoidable inaccuracy of certain my assertions and formulations.

2. The origin of the stellarator program and the main stages of its development

Studies to explore the possibility of using controlled thermonuclear fusion (CTF) as a source of energy were initiated in the Soviet Union and USA almost simultaneously circa 1951. However, up to 1958 this work was assigned a security label, and the first results were made available for the world's scientific community only at the 2nd International Congress on Peaceful Uses of Atomic Energy (September 1958, Geneva). In the USSR, these studies were based on Tokamak machines invented by I E Tamm and Andrey D Sakharov, and in the USA on facilities of the stellarator type developed by Professor L Spitzer, an astrophysicist affiliated with Princeton University.

As the story goes, Spitzer, while on a ski holiday, happened to read a report saying that certain R Richter from the Argentine had successfully demonstrated the possibility of CTF. Understandably, this news agitated the scientific community. Upon his return to Princeton, Spitzer, assisted by coworkers, developed a research program to assess the possibility of CTF and advanced the scheme of an experimental reactor for its realization. This facility, called the stellarator (from Latin stella — 'star'), was actually a toroidal magnetic trap of tokamak type. Spitzer appears to have proposed this system on the following theoretical grounds: (1) magnetic field lines should not extend beyond a closed volume (topological torus); (2) they should lie at the surfaces of toroids inserted into each other; (3) the turn angle of a line of force should not be too small (the turn angle or rotational transform angle is the angle by which a line of force turns with respect to the torus small azimuth upon its complete revolution about the principal axis of the torus), and (4) the system should be suitable for operating in a steady-state regime.

Both tokamak and stellarator facilities meet all but the last of these conditions. However, tokamaks are distinct from stellarators in that their magnetic surfaces, as surfaces of constant plasma pressure, appear due to ohmic current running through a plasma; in stellarators, magnetic surfaces are created by a system of external current-carrying conductors wrapped around a vacuum chamber. These currents are responsible for the rotational transform necessary to compensate for toroidal drift of the particles and confine them within the closed volume (toroid). It should be recalled that the last requirement is very important because an industrial power plant must operate in the steady-state regime, which is difficult to maintain in a tokamak machine, where ohmic current creating the magnetic field to ensure the plasma equilibrium is generated by an eddy electric field. There is no such problem with stellarators, where the magnetic field is created by external conductors that may be superconducting and fed from a regular current generator.

The Princeton Plasma Physics Laboratory (PPPL) designed and built a family of stellarators (A, B, C models) and proposed model D (the project has yet to be implemented) with a view to demonstrating CTF.

Unfortunately, model C experiments have failed. Despite expectations and theoretical predictions, the plasma lifetime in this system proved too short, whereas experiments on the Soviet tokamak T3 demonstrated rather efficient plasma heating and confinement. As a result, the US stellarator research program was abandoned in 1969, stellarator C was transformed into a tokamak, and tokamaks gained triumphant worldwide approval. To cite H Furth, the then PPPL Director, "each housewife in America wished to have her own small tokamak."

Such tremendous growth of tokamak developments had a negative impact on the stellarator program: its funding was curtailed and further progress slowed down. Meanwhile, despite skepticism about stellarator systems and some loss of interest in them amongst the world fusion community, stellarator research continued in the USSR [P N Lebedev Physical Institute (FIAN), A M Prokhorov Institute of General Physics (IOFAN), Khar'kov Physico-Technical Institute (KhFTI)][1], Germany, and Japan, soon yielding interesting and promising results. This provided an incentive to revitalize stellarator projects in the USA and to build the Advanced Toroidal Facility (ATF) at the Holifield National Laboratory[2] (Oak Ridge, 1988–1994). At that time, it was the largest stellarator having the powerful heating sources and continuously maintaining the discharge as long as 1 hour, i.e., operating practically in the steady-state regime. However, ATF experiments inexplicably ceased 6 years after their onset, and the facility was decommissioned despite a number of interesting results obtained with this machine. Figures 1 and 2 give an idea of the ATF stellarator, showing its helical conductors (inducing a magnetic field) and the shape of confined plasma, and illustrating its general view, respectively.

The results of research conducted in the USSR, Germany, and Japan proved highly impressive, which allowed, in my opinion, regarding the stellarator program as a plausible alternative to the tokamak project. These results, both theoretical and experimental, are briefly discussed below.

3. Main results of the theoretical studies

I. Fundamentals of the neoclassical transport theory have been developed (*Galeev, Sagdeev*; *Kovrizhnykh*, 1967–1969 [1, 2]). The theory predicts that the plasma lifetime at high temperatures characteristic of thermonuclear reactors is longer in tokamaks than in stellarators. However, lifetimes examined experimentally proved very close in these

[1] Present names of research institutions.
[2] Oak Ridge National Laboratory before 1975.

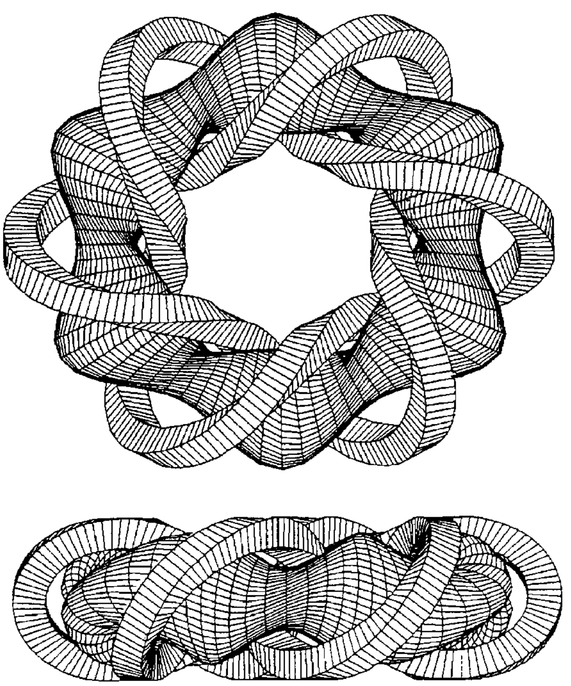

Figure 1. Helical conductors creating a magnetic field and shapes of magnetic surfaces in the ATF stellarator.

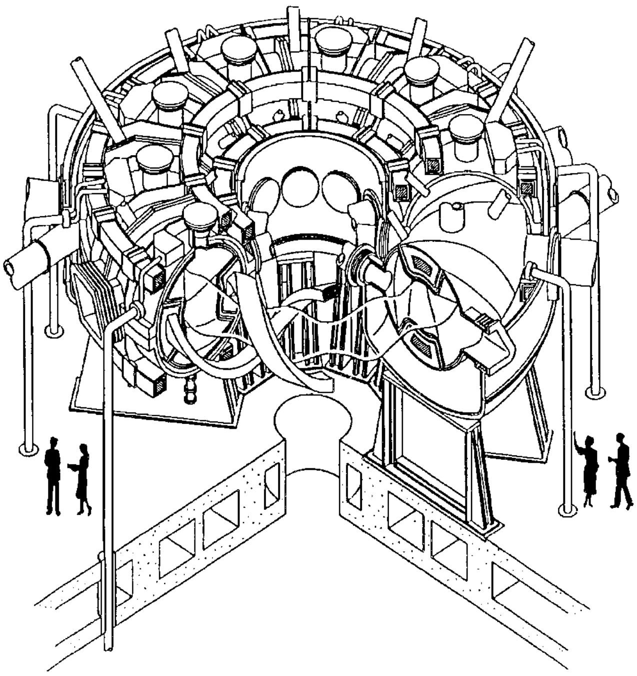

Figure 2. General view of ATF machine.

two facilities, which points to the occurrence of some additional mechanism of losses common to both magnetic systems.

II. It is theoretically demonstrated that neoclassical transport coefficients in stellarators can be reduced (optimization) by the special choice of external conductors creating a helical magnetic field (*Kovrizhnykh*, 1982 [3]). The idea of optimization has recently acquired wide recognition; special mathematical codes are being developed and powerful modern computers used for its realization (*Nuhrenberg et al.*).

III. Studies on the influence of magnetic field perturbations on the structure of magnetic surfaces suggest the existence of 'magnetic islands' that may, under certain conditions, be responsible for strong degradation of magnetic surfaces (*Kovrizhnykh, Morozov, Solov'ev*, 1961 – 1963 [4, 5]). These studies imply that current conductors should be manufactured with a high degree of accuracy if good magnetic surfaces are to be obtained.

IV. A simplified system (averaged over helical harmonics) of magnetohydrodynamic equations for stellarators, characterized by axial symmetry (as in the case of tokamaks), has been derived; this made possible the application of the methods previously developed for tokamaks to stellarators (*Kovrizhnykh, Shchepetov*; *Strauss*, 1980 [6, 7]).

V. The newly discovered self-stabilization effect was used to confine plasma in a stellarator at rather high β values (up to 10%, with $\beta = 8pNT/B^2$, where N is the plasma density, T is its temperature, and B is the magnetic field induction (*Kovrizhnykh, Shchepetov*, 1981 [8]). This finding is very important in that one of the main drawbacks of stellarators, for which they could not be regarded as prototype thermonuclear reactors, was too low (as was believed in those days) a critical value of β; the plasma became unstable as this value was surpassed.

VI. Mathematical codes have been developed for calculating magnetic surfaces, plasma equilibrium, and plasma stability in different, very complicated magnetic configurations (mostly by foreign researchers).

In my opinion, the period from the late 1970s to the early 1980s was especially productive for the development of stellarator theory. Those years were a time of fierce disputes and heated debates, sometimes with strong language, so strikingly different from the politically correct atmosphere prevailing at today's forums. Nevertheless, they were purely scientific discussions and never upset personal relations between the participants.

4. Main experimental findings

I. The methods developed for the measurement of magnetic surfaces allowed the cause of the failure of stellarator C to be elucidated. It appears to have been due to the bad quality (destruction) of magnetic surfaces (FIAN).

II. It was shown that plasma can be produced, heated, and confined in a stellarator without ohmic current.

III. The effect of plasma self-stabilization was confirmed and transition to the second stability zone was accomplished (ATF).

IV. Different methods of plasma heating were developed and verified. Steady-state plasma confinement was achieved [for over 1 hour; ATF, Large Helical Device (LHD)].

V. The subthermonuclear parameters of the plasma were obtained in the LHD stellarator (*although not all of them at the same time!*): $\beta = 4.5\%$, $N(0) = 10^{15}$ cm^{-3}, $T_i(0) = 13.5$ keV, $T_e = 10$ keV, $NT\tau = 4.4 \times 10^{19}$ m^{-3} s keV, $\tau > 0.02$ s (τ is the hot plasma lifetime), and discharge duration 1 h. It should be remembered that the triple product $NT\tau$ should be 1.5 – 2 orders of magnitude greater for a thermonuclear fusion reaction to be ignited.

5. Merits and demerits of stellarators

Let us formulate the advantages and disadvantages of stellarators in comparison with tokamaks.

Advantages of stellarators:

— ability to operate in the steady-state regime;

— avoidance of rather hazardous disruption instabilities characteristic of tokamaks due to ohmic current;

— presence of a natural diverter (a special system to remove impurities from the plasma and reduce the thermal load on the chamber);

— potential for optimization of the magnetic configuration in order to weaken neoclassical transport processes (diffusion and heat conduction);

— absence of plasma density cut-off limit. In all likelihood, such a limit does exist but as found experimentally it is higher than 10^{15} cm^{-3} (an acceptable value for a future thermonuclear reactor);

Drawbacks of stellarators:

— higher neoclassical transport coefficients in the region of low collision frequencies. Nevertheless, as noted above, there are optimization methods to reduce these coefficients. True, efficacious optimization is hardly possible if anomalous losses are commensurate with or exceed neoclassical ones;

— more complicated system of magnetic field windings and a high degree of accuracy needed to manufacture them;

— smaller experimental database attributable to the successful development of tokamak programs and failure of stellarator experiments in the late 1970s and the early 1980s.

The current project of a prototype thermonuclear reactor [International Thermonuclear Experimental Reactor (ITER)] is based on the tokamak concept. However, future research must show whether the above advantages of stellarators outweigh their drawbacks to enable the choice of one system or the other as the basis on which to build a commercial fusion reactor. Thus far, both programs have been developing successfully, providing plausible and significant mutual rewards because, for all the difference between stellarators and tokamaks, many processes in them are similar and likely to have a common nature.

Today, nine stellarators are operating throughout the world (see Table 1).

In what follows, only the main characteristics of these stellarators and the most interesting photos of them are presented without reference to the results obtained with each of them.

1. *Large Helical Device* (LHD) is currently the largest operating stellarator in the word, comparable in terms of size to such large tokamaks as Tore-Supra (France), JT-60 (Japan), DIII-D (USA), and TFTR (USA; dismantled). In terms of total plasma volume, the LHD is a disadvantage in relation to the JET tokamak (UK) only. The National Institute of Fusion Studies (NIFS) was established in Japan to accommodate the LHD and the device itself was built in a relatively short time.

Table 1. Stellarators operated in different countries.

Facility	Country	Startup	$V_p B$, m^3 T *
LHD	Japan	1998	80
Heliotron J	Japan	1999	1.4
TJ-II	Spain	1997	1.4
HSX	USA	1999	0.66
U-3M	Ukraine	1981	0.6
H-1	Australia	1998	0.7
L-2M	Russia	1975 (1993)	0.4
WEGA	Germany	1975 (2001)	0.16
STH	USA	2005	0.2

* Product of plasma volume V_p and magnetic field induction modulus determining the lifetime of the hot plasma: the higher this value the better its confinement and the higher its temperature.

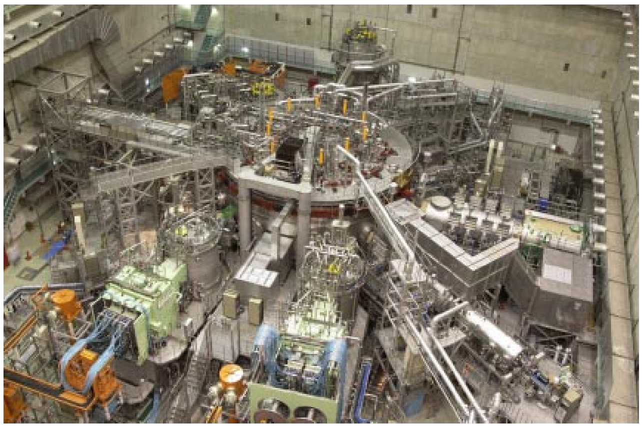

Figure 3. Overall view of the hall accommodating the LHD and accessory equipment.

In principle, a LHD is a classical stellarator designed similarly to the aforementioned ATF machine (see Fig. 1). However, it employs superconducting external winding, and our Japanese colleagues call it a heliotron or torsotron. The following characteristics give some idea of the size of the LHD.

Major plasma radius $R = 3.6$ m, minor radius $r = 0.6$ m, magnetic field induction $B = 3$ T, plasma volume ~ 30 m^3, magnetic field energy on the order of 1 GJ. The device is equipped with various plasma heating sources, such as powerful microwave and high-frequency generators for heating at electron cyclotron resonance (ECR) and ion cyclotron resonance. Also, it has injectors for high-energy beams of neutral atoms and a large set of modern diagnostic tools. Further enhancement of heating power is necessary to improve the plasma parameters. Temperatures close to magnitudes needed to trigger a fusion reaction have been reached, however plasma density remains 1.5 – 2 orders of magnitude lower than that sought for the purpose. Clearly, the probability of igniting the reaction in the LHD is a negligibly small value, but it is quite possible to come close to this goal but it will require substantially increasing the heating power. A general view of the LHD and accessory equipment is shown in Fig. 3.

2. *TJ-II stellarator*. This facility is substantially different from the so-called classical stellarator in terms of design and methods for the creation of the magnetic field. Nevertheless, it is ranked as a stellarator because its magnetic field is generated by a system of external conductors rather than plasma current. The size of this stellarator is relatively small: major plasma radius $R = 1.5$ m, average minor radius $r = 0.22$ m, and magnetic field induction $B = 1.2$ T. The heating sources comprise a microwave generator using electron cyclotron resonance and beams of neutral atoms. The plasma electron temperature reached thus far is 1 keV, and a plasma density of several units of [10^{13} cm^{-3}] has been attained. Although plasma parameters obtained with such small and relatively low-cost facilities are much lower than those in the LHD, they may be used in experiments to gather novel and very useful information about various processes proceeding in hot plasma and the influence of magnetic field structure on transport processes. A schematic of field-generating conductors and a general view of TJ-II are shown in Figs 4 and 5.

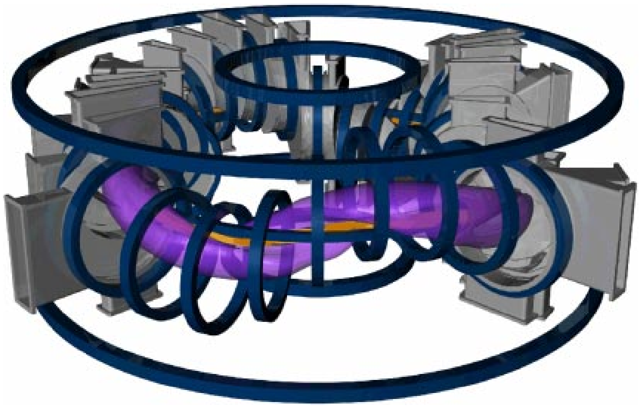

Figure 4. Schematic arrangement of conductors creating a magnetic field in the vacuum chamber of the TJ-II stellarator.

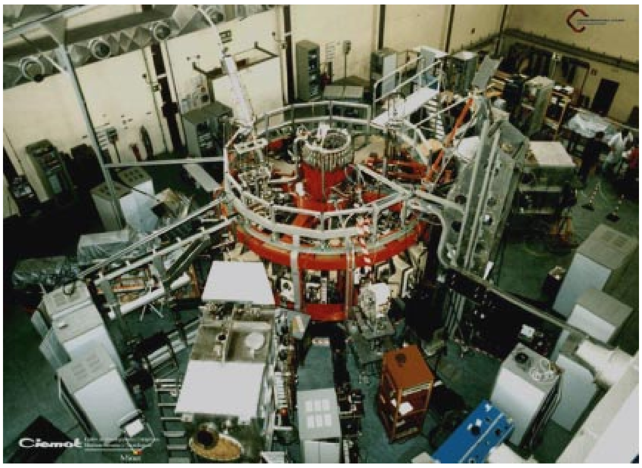

Figure 5. General view of TJ-II and accessory equipment.

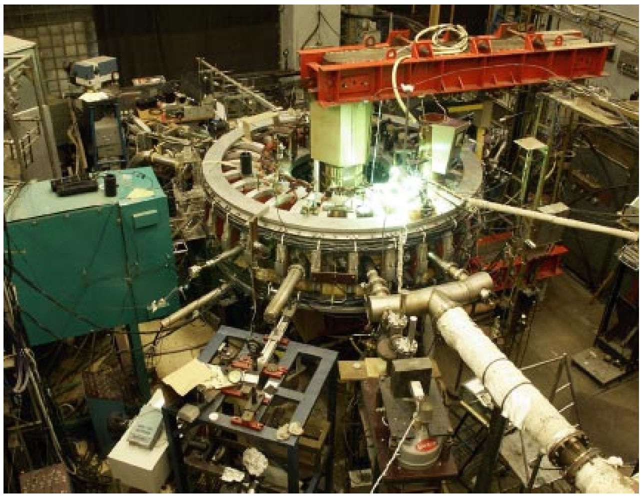

Figure 6. Russian L-2M stellarator.

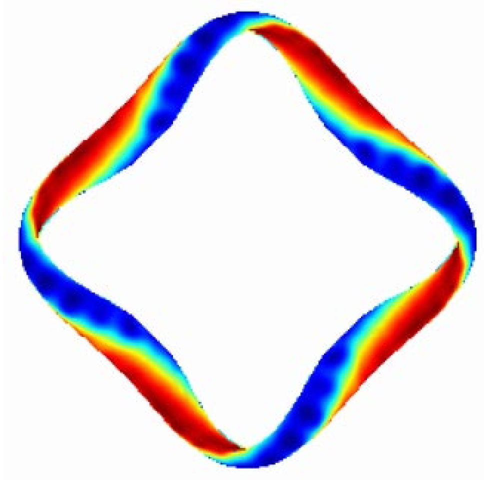

Figure 7. Shape of the plasma cord in the HSX (numerical simulation).

3. *L-2M stellarator*. I cannot help saying a few words about this (also small) facility for two reasons: first, it is accommodated at my Institute (IOFAN), and second, it is now the only stellarator operating in Russia. L-2M has the following characteristics: major plasma radius is 1 m, minor radius 11.5 cm, magnetic field induction 1.3 T, maximum electron temperature reached thus far 1.5 keV, and ion temperature is 150 eV at plasma density of 3×10^{13} cm^{-3}. Presently, installation and adjustment of a new powerful microwave complex are underway, consisting of two highly economical gyrotrons of 0.8 MW each expected to produce a record-breaking heating power density in excess of 5 MW m^{-3}. It should be mentioned that the L-2M team twice developed and proposed projects of even larger facilities that, however, could not be implemented for lack of funding. Project materials were used in other countries where similar reactors were built. The general view of L-2M is presented in Fig. 6.

4. *Helically Symmetric eXperiment* (*HSX*) *stellarator* is distinctive for the complexity of its system of conductors arranged so as to meet the quasihelical symmetry condition and thereby minimize (optimize) transport processes. In other words, its magnetic field structure was chosen in such a way that it reduces to a minimum the effect of toroidicity responsible for greater diffusion and heat conduction in stellarators compared with tokamaks. Note that prefix 'quasi' points to the impossibility of reaching total symmetry in stellarators. In fact, what can be done is to make the system close to a symmetric one, where the toroidicity effect is partly, albeit not totally, eliminated. Since the available photo of this facility conveys little useful information, I present here only a simulated profile of its plasma cord (Fig. 7) and main parameters, viz. major plasma 'radius' of 1.2 m, minor radius 0.12 m, magnetic field induction 0.5 T, and ECR heating power not more than 100 kW. Despite the rather long period of stellarator operation, the HSX collaboration still fails to answer the principal question of how, if at all, efficient the optimization procedure is.

6. Stellarators being designed and constructed

Several institutions are combining ongoing theoretical and experimental studies with designing and building new stellarators and developing a future thermonuclear reactor based on the stellarator concept. Although the reactor issues are beyond the scope of the present report, a few words about the stellarators under construction are certainly in order.

The project of a new superconducting, partly optimized stellarator with modular coils has been underway at the Max Planck Institute of Physics in Greifswald, Germany since the early 1980s. Let us recall that the stellarator magnetic field

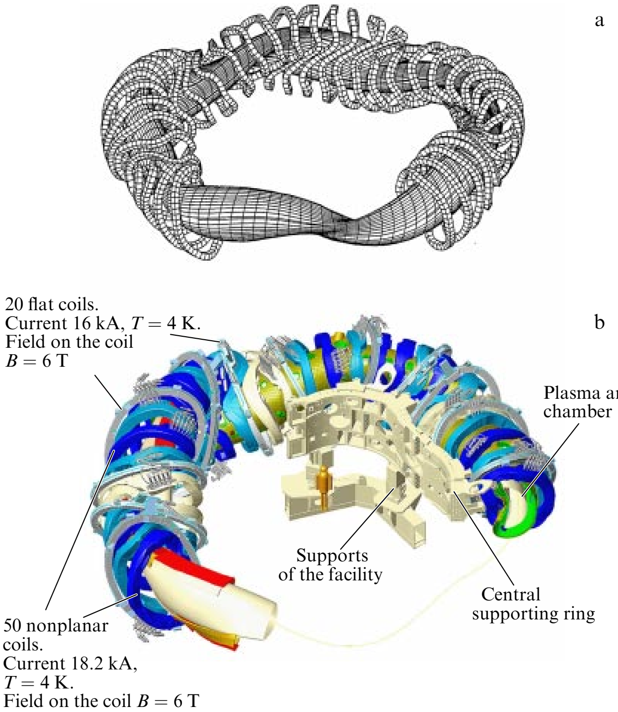

Figure 8. (a) Modular coils and shape of the plasma cord in the W-7X stellarator. (b) Mock-up of the W-7X stellarator as an assembly.

Figure 9. A fragment of the W-7X vacuum chamber (half the module).

can be generated not only by continuous helical conductors but also by properly shaped modular coils, as proposed for the first time by *Popov and Popryadukhin* [9] in 1966.

After thorough preparatory work that was, in my opinion, too long, the building of the W-7X stellarator was started in 2000. The reactor was planned for 2005 with preliminary experiments to start in the same year. However, technical difficulties encountered in manufacturing modular coils of intricate shape postponed completion of the project first till 2009 and thereafter till 2012. It is hoped that the work will be finished by this time.

W-7X has the following basic characteristics: major plasma radius $R = 5.5$ m, minor radius $r = 0.5$ m, magnetic field induction in the plasma core $B = 3$ T, and magnetic field energy $W = 620$ MJ. Figures 8 and 9 give a better idea of this grandiose machine.

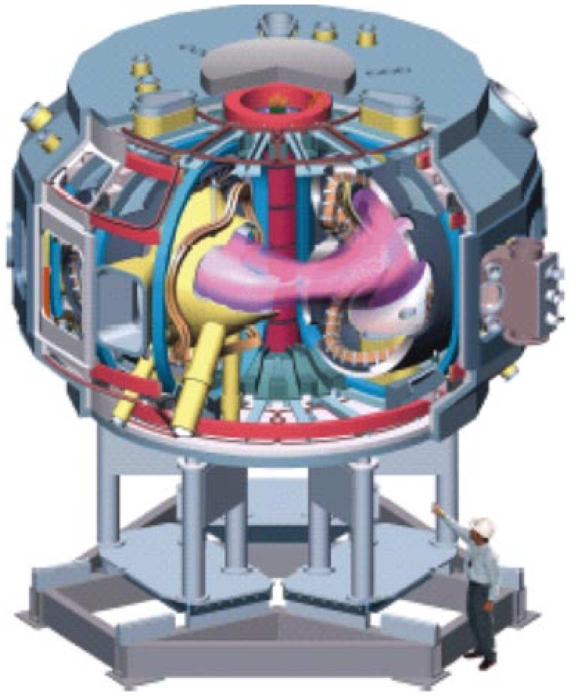

Figure 10. A full-size mock-up of the NCSX stellarator.

One more stellarator was designed for a much shorter time at PPPL (USA). It is called the National Compact Stellarator eXperiment or NCSX. The aim of the program is to address the following issues: the possibility of operations at high β with bootstrap current but avoiding the development of disruption instability, the reduction of neoclassical losses through establishing quasisymmetry conditions, and the possibility of imposing limits on beta. Although the NCSX is smaller than the LHD and W-7X, it is large enough to produce new interesting results owing to novel ideas employed in its construction. A mock-up of the NCSX is presented in Fig. 10. The average major radius of the plasma is 1.42 m, minor radius 0.33 m, magnetic field induction 2 T, and heating source power 3 – 12 MW. The completion of building the facility and the onset of experiments were planned as of 2009, but the project was suspended in 2008 (seemingly due to significantly overrunning an outlay) and the facility operation was stopped.

7. Pressing tasks and plans

I would formulate the goals of further studies as follows.

I. Studies of the nature of anomalous losses and modes of their suppression. Both stellarator and tokamak experiments have shown that the observed coefficients of heat conduction and diffusion are much higher than those predicted by neoclassical theory. The cause of this discrepancy is believed to be the presence of as yet unknown additional mechanism called anomalous losses. The physical nature of this mechanism awaits elucidation: can we influence it to totally or partially reduce anomalous transport? Given such a possibility, the development of transport-optimized systems should be continued. Otherwise, optimization is of little promise.

II. Simulation of transport processes and the development of increasingly more adequate numerical codes for investigations into plasma transport, equilibrium, and stability. There are two options for achieving this goal. One implies more or less the exact numerical solution of starting equations and then the transport equations. In principle, this option looks promising, but it requires powerful computers, sophisticated software programs, and a large amount of computing time. The other approach consists in elaborating a relatively simple analytical model that would phenomenologically take account of anomalous losses and allow a simple and rapid solution. In all likelihood, theorists will work along both lines.

III. Search for limiting density in stellarators. The fact that limiting density has not thus far been found in stellarators does not mean that it is altogether nonexistent. The solution to this problem is of primary importance because it would permit achieving the ignition conditions of a fusion reaction at lower temperatures.

IV. Diverter performance testing. A reactor cannot work without a diverter. However, an optimal design of a diverter remains to be developed and its efficiency needs to be assessed more thoroughly.

V. Extension of international collaboration, creation of comprehensive experimental database, and specification of scaling properties. Cooperation between different research groups in the framework of the stellarator program has been successfully developing in the past years in the form of exchange of scientists, the organization of joint working groups for the collection and analysis of experimental data obtained at different facilities, regular meetings to discuss newly obtained results, joint exploitation of powerful computers, and extensive use of Internet resources.

VI. Improvement of plasma parameters to be achieved by using more powerful facilities and enhancing power of plasma heating sources.

VII. Elaboration of a stellarator-based reactor project. Although building a stellarator-based fusion reactor is not a matter for tomorrow, the work toward this goal needs to be carried out today because many problems apart from purely physical ones (engineering, technological, economic, etc.) will have to be resolved. The development and construction of ITER would be of great help in this context.

I cannot help observing here that unfortunately the presence of Russian scientists in international efforts within the framework of the world stellarator program is rapidly decreasing. The same is true of the tokamak program and collaboration between Russian and foreign researchers. The causes are well known: from the lack of adequate financial support (and, as a consequence, the absence of new experimental devices and modern diagnostic equipment) to the aging of research groups, loss of interest, and unwillingness of young specialists to associate themselves with science, plus the waning prestige of the scientist at large. With this in mind, it appears safe to predict that the participation of Russia in the ITER program may end in complete failure for the simple reason that there will be nobody to recruit into it.

To conclude, I would like to say a few words about the man whose memory has brought us all together here.

I was not an intimate friend of Boris Borisovich Kadomtsev nor even his close associate, but I frequently met with him at numerous conferences, seminars, and workshops. What always impressed me was his remarkable physical intuition. Within a few days after a new experimental effect or phenomenon was reported, Boris Borisovich explained its physical nature in hand-waving terms speaking at a seminar or in a narrow circle of colleagues. Surprisingly, this first explanation of his was exactly in line with what was postulated after a time by a well-considered theory or in his own work. Surely, B B Kadomtsev was an outstanding scientist, a heaven-sent theorist. His death is an irreparable loss, not only for the plasma community but also for physics in general. He will be sorely missed by all who knew him. The sole consolation is the splendid reviews and those (alas!) few monographs [10–13] that he left to be studied, as I hope, by seasoned researchers and fledgling students alike.

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Nonlinear phenomena in the ionospheric plasma. Effects of cosmic rays and runaway breakdown on thunderstorm discharges

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1. Introduction

This article reviews some recent progress in the theory of cosmic rays and runaway breakdown (RB) along with new observations of their effects on thunderstorm processes in the atmosphere. An asymptotic solution of the linear kinetic equation is considered and similarity relations for runaway breakdown are proposed. The paper describes the Groza experimental installation measuring different forms of radiation during thunderstorms that is operated at the high-altitude cosmic ray station of the P N Lebedev Physical

Institute, RAS, located in the mountains of Tien Shan. The main concern is with observations of intense gamma-ray bursts in the active phase of thunderstorms. Specific atmospheric discharges caused by the joint action of runaway breakdown and extensive air showers on a thundercloud are recorded for the first time. The discovery of intense long-lasting gamma bursts and their correlation with radio emission gives the first experimental evidence of the key role of cosmic ray-initiated runaway breakdown in charge accumulation and transfer from clouds to a lightning leader during an active thunderstorm period.

The thunderstorm discharge, i.e., the lightning, is a result of three major processes:

(1) Accumulation of electric charges from a large mass of clouds (in fact, the initiation of lightning) in which they are resided on water droplets or small pieces of ice.

(2) Charge transfer from the cloud to the ground or between clouds.

(3) 'Burning' of the charge in the thunderstorm discharge (lightning proper).

The third process — return stroke of lightning — is fairly well known and has been studied rather extensively. The second one (formation of the lightning leader) is studied equally well, although many its aspects await further clarification. The first one has not been discussed in the literature at any length.

The fact is that earlier investigators observed radio emission from multiple discharges proceeding in the cloud. These discharges were believed to result from conventional air breakdown. However, numerous measurements (very accurate of late) have shown that the electric field in thunderclouds never reaches the strength necessary to initiate a normal atmospheric discharge (as definitively confirmed by recent explorations [1, 2]). For this reason, charge accumulation from a large mass of clouds remains an open issue, virtually untouched in the literature (see, e.g., monographs [3, 4]).

The present report demonstrates for the first time that the active phase of a thunderstorm discharge is actually accompanied by powerful gamma-ray fluxes in the clouds and radio emission correlated with gamma radiation. It suggests runaway breakdown. The electric field is stronger than the critical field characteristic of runaway breakdown. The breakdown is triggered by secondary electrons of cosmic rays (CRs). Therefore, the runaway breakdown in the clouds may account for charge accumulation from clouds, necessary for a thunderstorm discharge to be produced. This opens up a new line of inquiry for the study of the first and main phase — the basic process of thunderstorm discharge. The whole problem is closely related to the ideas and concepts of plasma physics developed in the principal works of B B Kadomtsev. To recall, runaway breakdown is a process in which any matter behaves like a plasma.

Atmospheric discharges have been extensively investigated in recent years. New forms of discharges between atmosphere and ionosphere (sprites, elves, and blue jets) have been discovered [5, 6]. Strong terrestrial gamma flashes (TGFs) have been recorded [7]. High-altitude discharges generating superpowerful radio pulses, i.e., narrow bipolar events (NBEs), have been thoroughly studied [8].

The results of new measurements have posed new questions. By way of example, the latest work has given indisputable experimental evidence that the electric field strength E in the clouds is much lower than the threshold electric breakdown E_{th} [1, 2, 9]. How then is lightning initiated? How do numerous discharges generating radio emission bursts develop in the active period of a thunderstorm [10]? The new physical process, *runaway breakdown*, helps to answer these questions [11, 12]. RB has a low excitation threshold corresponding to electric fields present in the thunderstorm atmosphere.

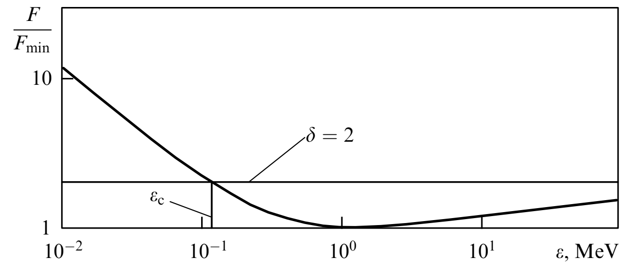

Figure 1. Dependence of braking force F on electron energy ε. Force F is normalized to $F_{\min}$, and the parameter $\delta = E/E_c$.

Conventional breakdown results from the heating of electrons in an electric field. In this process, fast electrons that belong to the tail of the distribution function become able to ionize matter and, therefore, to generate new free electrons, while slow electrons disappear either owing to recombination in the bulk or on the walls of the discharge chamber. As soon as the electric field becomes sufficiently strong, the generation of new electrons via ionization exceeds their disappearance due to recombination, and their number begins to increase exponentially. This phenomenon is termed electric breakdown of matter. Characteristic energies of electrons responsible for ionization are 10–20 eV, while recombination mostly takes place at low energies. For this reason, mean electron energy $\bar{\varepsilon}$ does not normally exceed several electron-volts. For instance, this energy in the air is $\bar{\varepsilon} \sim 2$ eV.

Runaway breakdown has an essentially different nature [13], resulting from the interaction between fast particles and matter. The braking force F acting on an energetic particle in matter is determined by ionization losses [14]. Figure 1 shows that force F decreases with increasing electron energy $\bar{\varepsilon}$ because fast electrons interact with the electrons and nuclei of neutral matter as if they were free particles, i.e., according to the Coulomb law. The Coulomb scattering cross section is the Rutherford cross section $\sigma \sim 1/\varepsilon^2$. That is why the braking force $F \sim \varepsilon\sigma N_{\text{m}} \sim 1/\varepsilon$ in the nonrelativistic region where it is proportional to molecular number density N_{m} and inversely proportional to the electron energy ε. The decrease in the ionization braking force becomes weaker due to relativistic effects. For $\varepsilon \geqslant 1$ MeV, it falls to a minimum and thereafter begins to slowly (logarithmically) increase (see Fig. 1).

The decrease in the friction force is related to the possibility of the appearance of runaway electrons in a substance placed in an electric field. Indeed, if a constant field E is present in a medium, such that $E > E_c = F_{\min}/e$, an electron with a sufficiently high energy $\varepsilon > \varepsilon_c$ will be continuously accelerated by the field (see Fig. 1). Such electrons are called runaway electrons [15].

Runaway breakdown is associated with the generation of secondary electrons due to the ionization of neutral molecules by fast runaway particles. Although the majority of secondary electrons have low energies, electrons with rather high energy $\varepsilon > \varepsilon_c$ can be produced as well. These will also become runaway electrons, i.e., they will be accelerated by the field (see Fig. 1), and may in turn generate particles with $\varepsilon > \varepsilon_c$ in the ionization process. As a result, an exponentially growing avalanche of runaway electrons develops. In parallel, an immensely large number of slow electrons are generated, which ultimately leads to an electric breakdown of the matter. It is of importance that runaway breakdown occurs in a relatively weak field $E \geqslant 1.3E_c$ which is an order of magnitude smaller than the field strength E_{th} of conventional electric breakdown. For instance, $E_{th} \approx 23$ kV cm^{-1} and $E_c \approx 2.16$ kV cm^{-1} at atmospheric pressure in the air.

However, condition $E \geqslant 1.3E_c$ alone is insufficient for runaway breakdown to develop. The presence of fast bare electrons with energies in excess of critical runaway energy $\varepsilon > \varepsilon_c$ is necessary. Such electrons effectively generated by cosmic rays are always present in the atmosphere. They are responsible for RB during thunderstorms and its action on the development of atmosphere electric discharges in the active thunderstorm period.

This period comprises several stages, namely, preliminary breakdown, the formation and motion of leaders, the main return stroke, repetitive return strokes, etc. It may be supposed that in the active period the in-cloud electric field exceeds E_c in a broad zone [1] and can be even higher, $E > E_{th}$, in local areas (e.g., in the leader core) [3]. In these areas, RBs may locally develop, accompanied by gamma-ray and radio pulses. The existence proof of RB and revealing of its participation in atmosphere discharges would be direct observation of high-energy electrons and gamma-ray pulses inside thunderstorm clouds, since neither travels a long distance. It should be noted that an important role is played not only by secondary electrons with energies ranging $10^4 - 10^6$ eV but also by primary CR particles with energies of $10^{14} - 10^{16}$ eV initiating extensive air showers (EASs). 'Cloud-to-ground' lightning is typically generated at an altitude of 4–6 km, corresponding to the characteristic altitude of the maximum number of particles in the showers produced by CR with an energy in excess of 10^{15} eV. Hence the extreme importance of the facility operating at such altitudes for comprehensively surveying high-energy CRs, gamma-ray, X-ray, and radio emissions from lightning discharges.

The Tien Shan high-altitude cosmic ray station of P N Lebedev Physical Institute (TSCRS) is a unique site for investigations into the physics of thunderstorm discharges. Its Groza installation is designed to systematically study atmospheric discharges and simultaneously record different types of radiation (electron, gamma, X-ray, and radiofrequency) in the range from 0.1 to 30 MHz and at a frequency of 250 MHz. It has been collecting statistics for several years now. All its detectors continuously operate in the automatic mode during the thunderstorm season (May–September). An important advantage of this installation is its location at an altitude between 3340 and 4000 m above sea level, where thunderstorm clouds are formed in the mountains of northern Tien Shan; in other words, its detectors are sort of embedded in the clouds.

The very first data suggesting markedly enhanced radiation in a range of 100–500 keV during thunderstorms were obtained as early as 2002 [16]. Thereafter, correlation between short radio pulses and the arrival of EASs was documented [17]. Bipolar pulses of short-wave (SW) radio emission coincident (within 50 μs) with trigger signals of the EAS recording system were observed. Such pulses never occurred in the absence of a thunderstorm. Later investigations showed that SW radio emission from each stroke of lightning starts as a short bipolar pulse with a rise time of order 100 ns [20], similar to what was observed in lowland terrains [18, 19]. The shape, width, and amplitude of the initial pulse were consistent with the respective values predicted by the theory of joint RB/EAS action induced by a primary particle with an energy of $10^{15} - 10^{16}$ eV.

This paper presents results of new observations of the influence of CRs and RB on thunderstorm processes in the atmosphere. Section 2 contains brief remarks on the theory of RB. In Section 3, we describe the Groza experimental installation measuring different types of radiation during thunderstorms. Section 4 deals with a method for selection of events recorded by different Groza detectors that may be of interest for the study of processes in the thunderstorm atmosphere. The main results of experiments carried out in the season of 2007 are described at some length in Section 5. Worthy of special note are observations of short but intense gamma-ray bursts in the active phase of thunderstorms. Specific RB – EAS discharges caused by the combined action of RB and an EAS in the thunderstorm atmosphere were recorded for the first time. The discovery of long (10–100 ms) intense gamma-ray bursts and their correlation with radio emission gives the first experimental evidence of the key role of RB in charge accumulation and transfer from clouds to the lightning leader during the active thunderstorm period.

2. Notes on the theory of runaway breakdown

2.1 Asymptotic solution

The breakdown problem in a kinetic theory is formulated as follows. The kinetic equation for electrons in the coordinate space **r** and momentum space **p** is considered. In the simplest representation, electric field E, number density N of neutral molecules, their charge Z, and other structural parameters of matter determining electron–electron collisions are assumed to be constant. The collision integral for electrons is taken in the linear form. Then, the solution of the kinetic equation for the electron distribution function $f(\mathbf{p}, t)$ — homogeneous, independent of spatial coordinates, and asymptotic in time t — has an exponential character:

$$f(\mathbf{p}, t) \to f_{01}(p) \exp(\nu_1 t) + f_{02}(\mathbf{p}) \exp(\nu_2 t) .$$

Here, ν_i is the eigenvalue of the linear kinetic equation, $i = 1, 2$, and $f_{0i}(\mathbf{p})$ is the corresponding eigensolution.

The exponential growth of the distribution function, hence the number of electrons, is what is known as electric breakdown of matter. Parameter ν_i defines the ionization frequency, and $\nu_i^{-1} \approx \tau_i$ is the characteristic time of breakdown. The co-existence of two independent solutions of the linear kinetic equation suggests the presence of two types of breakdown, conventional (ν_1) and RB (ν_2), in any dielectric.

2.2 Similarity relations

Runaway breakdown occurs in all kinds of matter. Because fast electrons interact with other electrons according to the Coulomb law, the character of interaction is always the same; therefore, RB possesses an identical structure in all substances and remarkable similarity properties, meaning that the critical electric field E_c in any matter is proportional to its density ρ. If ρ is expressed in grams per cm^3, then one finds

$$E_c = 1.8\rho \ [\text{MeV cm}^{-1}] .$$

Accordingly, the critical runaway electron energy ε is related to the electric field strength E by the expression

$$\varepsilon \cong \frac{m_e c^2}{2\delta},$$

where $\delta = E/E_c$.

The characteristic breakdown length l determines the growth scale of the RB exponent:

$$l = \frac{6.1}{\rho\delta^2}\ [\text{cm}]\,.$$

It can be seen that l rapidly falls with increasing electric field strength ($\sim \delta^{-2}$).

Characteristic breakdown time τ_2 also decreases ($\sim \delta^{-3/2}$), while ionization frequency grows:

$$\tau_2 \cong 10^{-10}\rho^{-1}\delta^{-3/2}\ [\text{s}]\,,$$

$$\nu_2 \cong 10^{10}\rho\delta^{3/2}\ [\text{s}^{-1}]\,.$$

In dense matter with $\rho \approx 10-100$ g cm^{-3}, characteristic breakdown times are extremely small and rapidly decrease as parameter δ grows.

Notice that the above similarity relations hold in a limited region of parameter δ variations:

$$1.5 \leqslant \delta \leqslant 100-150\,.$$

2.3 Runaway breakdown in the atmosphere

Figure 2 depicts RB ionization frequency ν_2 in the air depending on the electric field strength E. Evidently, the frequency monotonically increases with E ($\sim E^{3/2}$), whereas the ionization frequency ν_1 of ordinary breakdown first grows very rapidly with E (roughly as $\sim E^{5.5}$) and then becomes saturated [21–23].

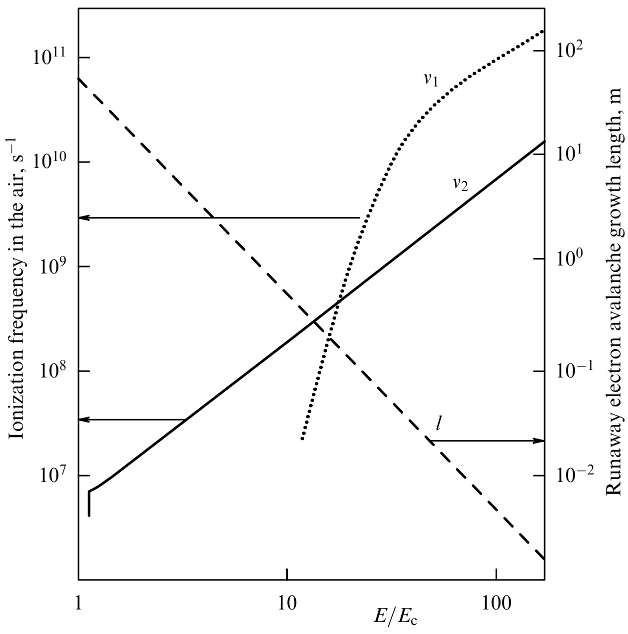

Figure 2. Dependences of ionization frequency in the case of conventional (ν_1) and runaway (ν_2) breakdown in the air on the electric field (left scale). Also shown is field dependence of runaway electron avalanche growth length (right scale).

Importantly, RB is possible even at low values of the electric field, i.e., in a *weak field*, provided the condition $E_{th} > E > E_c$ is met. In this case, the ionization frequency is rather high:

$$\nu_2 > 10^7-10^8\ \text{s}^{-1}\,.$$

Conventional electric breakdown in a weak field is impossible and occurs only if $E > E_{th}$. Despite rapid elevation of the ionization frequency of an ordinary breakdown with E, it remains lower than that of RB as the field grows up to $E \simeq 2E_{th}$. This fact is of great importance since it may lead to the appearance of a large number of fast electrons in a normal discharge for $2E_{th} \geqslant E \geqslant E_{th}$. It may be a cause of observed gamma-ray bursts both in a lightning leader [24–26] and in laboratory discharges [27–29].

Thus, RB proves to be the predominant breakdown in the air in terms of ionization rate, not only for $E < E_{th}$ but also at higher field strengths up to $E = 2E_{th}$.

The second important parameter plotted in Fig. 2 is characteristic length l that determines the minimal spatial scale of the breakdown region. It is seen that the characteristic length in weak fields $E \sim E_c$ is rather large ($l \approx 30-50$ m). However, it rapidly decreases ($\sim E^{-2}$) with increasing electric field strength to become 10–30 cm at $E \approx (1-2)E_{th}$. In high fields, $E \gg E_{th}$, the characteristic length l is quite small. This means that RB studies at low electric fields are feasible only in thunderstorm clouds, whereas at higher fields RB effects can just as well be observed in laboratory experiments.

2.4 The effect of cosmic rays

As noted above, fast bare electrons with an energy in excess of runaway energy ($\varepsilon > \varepsilon_c$) are needed for RB to occur, besides fulfillment of the condition $E > E_c$. The two conditions are satisfied in thunderstorm clouds. It has been found experimentally that maximally strong electric fields in the atmosphere thunderstorm clouds are close to the critical RB field E_c [1, 2, 9]. Secondary CR electrons having the relatively high mean flux density $\Phi_e \approx 10^3$ m^{-2} s^{-1} at altitudes of 4–8 km behave like fast bare particles in thunderstorm clouds.

An essential difference of RB from conventional electric breakdown consists in efficacious generation of X-ray and gamma radiation. It is the observation of gamma emission with an energy of $\varepsilon \sim 50-100$ keV that suggests the possibility of RB. Simultaneous excitation of strong electric currents promotes intense generation of radio emission.

Naturally, the number of runaway electrons is proportional to the number of bare ones resulting from the interaction between primary CRs in the atmosphere. The total number N_e of bare electrons in an EAS increases in proportion to the energy of a primary CR particle. By way of example, the shower formed by a primary particle with energy $E_{CR} \approx 10^{15}$ eV contains 10^6 bare electrons; at $E_{CR} \approx 10^{18}$ eV, $N_e \approx 10^{10}$.

The passage of an EAS through a thunderstorm cloud with $E > E_c$ gives rise to an avalanche of runaway electrons that cause an exponential rise in the number of high-energy electrons in the EASs. Simultaneously, the number of thermal electrons increases millions of times. Collectively, they produce an RB – EAS discharge [30]. Owing to the huge number of high-energy electrons, this discharge should be naturally accompanied by a strong pulse of gamma-ray radiation.

3. Groza experimental installation

The Groza installation comprises the following facilities: an EAS registering system, the system of NaI scintillation detectors, two independent radiosystems, a detector of jumps in the static electric field and its high-frequency component (Fig. 3). The maximum spacing between the detectors in the horizontal plane runs to 2–2.5 km, which enables observation of both temporal and spatial distributions of different forms of radiation in the clouds by intensity (and even monitoring movements of radiation sources together with the clouds). The relief of the Tien Shan station is very convenient for carrying out such surveys: the two closely spaced slopes near the mountain pass harboring the station allow the radiation detection sites to be located at different altitudes (from 3.4 to 4 km) above sea level. This makes it possible to obtain radiation distribution profiles inside the clouds, not only in the horizontal but also in the vertical plane.

3.1 Registering system of extensive air showers

The EAS registering system consists of a few dozen SI5G Geiger counter-based detectors spread over the area of the station to record the passage of an EAS, and to measure its size and primary particle energy. Each EAS detector contains 20 SI5G gas-discharge counters with parallel anodes; its sensitive area is around 0.6 m^2. Each detector occupies a specific location. Their mutual alignment is shown in Fig. 3. All EAS detectors are screened with 10-cm-thick iron filters to suppress the background of low-energy electrons. The adjacent detectors are spaced roughly 65 m apart. The sensitive area of the entire system is around 0.1 km^2.

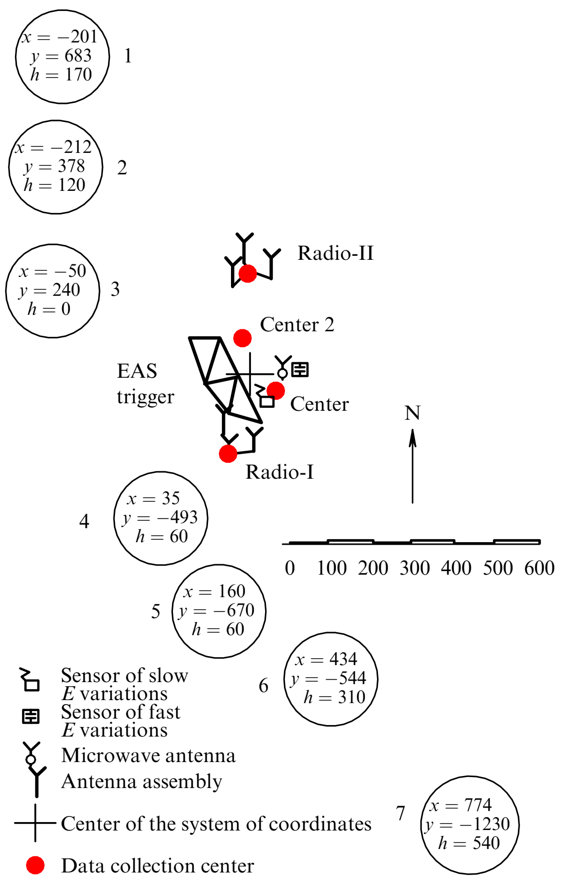

Figure 3. Groza experimental installation: polygon — EAS recording system (EAS trigger); circles (1–7) — array of NaI scintillation detectors, with numerals in the circles specifying coordinates (in meters) with respect to the Center (TSCRS) altitude; two independent radiosystems (Radio-I, Radio-II); detectors of 'fast' and 'slow' electric fields. Relative positions of all TSCRS facilities are to a scale.

3.2 System of gamma-ray scintillation detectors

Soft gamma radiation and hard X-ray radiation from electrons accelerated in the electric fields of a thunderstorm cloud are registered by 14 NaI crystal-based scintillation detectors. Seven registering sites are arrayed at the slopes of the surrounding mountains in a chain running across the usual direction of thunderstorm cloud movements (see Fig. 3). The distance between the ends of the chain reaches ≈ 2 km, and maximum vertical spacing between the detectors equals ≈ 600 m. Such a structure of the scintillation system makes it possible to study the spatial distribution of radiation inside thunderstorm clouds in both vertical and horizontal directions.

Two scintillation detectors are placed at each site. In one of them, an NaI crystal connected with an FEU-49 photoelectron multiplier is enclosed in an aluminium jacket with 1-mm-thick walls (Sc-II); in the other, the crystal is placed in a polyethylene tube with 10-mm-thick walls (Sc-I). As a result, the two detectors have different registering thresholds for gamma-ray radiation. In addition, one 'sham' FEU-49 detector without an NaI crystal is installed at several of the most remote sites from the system center (sites 1, 2, 6, and 7 in Fig. 3). The signals of these detectors serve to monitor electromagnetic pickup by the cables connecting the detectors with the observatory campus.

A particle passing through the NaI crystal of a scintillation detector produces electric pulses with a varying amplitude proportional to the energy dissipated by the charged particle or gamma quantum inside the crystal (for complete absorption in the crystal, the amplitude is proportional to the primary particle or quantum energy). Signals of the scintillators are transmitted to fast-acting amplitude analyzers (a set of parallelly-operating amplitude discriminators tuned to six startup thresholds). Output signals of the discriminators are pulses with the standard amplitude and duration, the intensity of which is estimated separately for each of the six amplitude ranges using a count-conversion algorithm. Absolute energy calibration of the detectors was performed with the use of ^{241}Am and ^{137}Cs gamma sources.

The intensity of gamma radiation emitted during the passage of thunderstorm clouds was measured in the high time-resolution mode. The intensity of signals from each scintillator was deduced from a time scan consisting of 4000 successive 200-μs intervals; this allows the time scan of short radiation bursts to be thoroughly analyzed. At each instant of continuous operation, the time scanning system stores the scintillation pulse intensity in memory for the past 0.8 s. If it receives a trigger signal during this time, the data collection system goes on working for another 0.4 s, after which the information is written to the disk of the master computer. In this way, the time sweep of signal intensity with a resolution of 200 μs 0.4 s before and after the arrival of each trigger signal is preserved for each recorded event.

3.3 Radiosystems

The electromagnetic radiation of thunderstorm discharges was studied with the employment of two radiosystems

(Radio-I and Radio-II in Fig. 3) operating in a frequency range of 0.1–30 MHz and recording the waveform of radiation pulses with a time resolution of around 16 ns. Also, the systems determined the direction to radiation sources from the relative time delays of radio signals [18, 19]. Each radiosystem has three antennas: two frame ones crossed at 90° to measure the horizontal magnetic component, and a whip antenna measuring vertical electric component of the electromagnetic field.

The radiosystems operate in the external trigger mode with a total recording duration of 200 ms and a prehistory (i.e., the record length before the arrival of the trigger pulse) of 160 ms.

The third Radio-E system is used to register ultrashort (metric) radiowaves at ~ 250 MHz. This system also contains detectors measuring electric field variations in the thunderstorm atmosphere: a quasistatic 'slow' field is measured with a 'field mill' electrostatic fluxometer, and electric field variations in the frequency range of 0.5–25 kHz ('fast' field) with a capacitor type sensor.

4. Selection of events for analysis

Indispensable for the study of CR and RB effects on the development of discharges in a thunderstorm atmosphere are continuous EAS monitoring; recording of short bursts of gamma-ray, X-ray, and radio emission, and elucidation of time correlations between them in time. The runaway breakdown theory implies that short gamma-ray and X-ray bursts from thunderstorm clouds have to result from an avalanche of high-energy electrons accelerated in the electric field of a thunderstorm cloud. Secondary CR electrons serve as bare electrons having the energy necessary for acceleration. Specifically, discharges initiated by the passage of EASs through the atmosphere may be ignited in thunderstorm clouds due to CR particles with energy $E_{\mathrm{CR}} \geqslant 10^{15}$ eV. These physical mechanisms of burst generation were taken into account in the choice of conditions for generating trigger signals to be used in the registering system.

4.1 The trigger of extensive air showers

The shower trigger was implemented to study time correlations between the instant of EAS passage and gamma-ray, X-ray and radio emission. Its signal developed when pulses from four neighboring detectors of the shower-recording system coincided within the interval of 5 μs. Configuration of the shower trigger system illustrated in Fig. 3 permits one to effectively select an EAS with primary particle energy $E_{\mathrm{CR}} \geqslant 10^{15}$ eV.

4.2 The trigger of an electric field jump

The trigger of an electric field jump was utilized to investigate intensity variations of gamma-ray, X-ray, and radio emission associated with the initiation and development of lightnings in a thunderstorm atmosphere. Because a cloud–earth discharge is accompanied by a rapid fall (for a few microseconds) in the electric field strength, the trigger signal was formed by a sensor of electric field variations at the instant of a sharp change ('jump') in the field strength. We chose a high startup threshold for the trigger-generating device, which corresponded to very closely spaced lightning discharges. This accounted for the small number of trigger signals of this type.

4.3 The electromagnetic trigger

Because the length of the cables connecting remote detectors with the data collection center amounted to 2 km, electromagnetic pulses produced by atmosphere discharges in the vicinity of the detector array were induced on long cables. Such a pulse coinciding with the in-cloud discharge was also used as the trigger signal.

4.4 Statistics of recorded events

The measurement data yielded by the Groza experiments were analyzed by comparing two samples of events: one obtained in clear cloudless weather, the other during the traversing of dense electrically charged clouds with lightning discharges over the TSCRS mountain pass. In the latter case, the uppermost detection sites (6 and 7 in Fig. 3) proved to be located deeper in the thunderstorm cloud than those at lower altitudes.

The total number of events recorded during both measurement periods is shown in Table 1.

Table 1. Statistics of events recorded in clear weather and during thunderstorms.

Type of event	Number of events in clear cloudless weather for 11 hours (13 August 2007)	Number of events during a thunderstorm for 11 hours (4, 7, 8, 15 August 2007)
EAS triggers	611	600
Number of events per hour	55 ± 2	54 ± 2
Events with EAS triggers containing short gamma-ray bursts	0	14
Triggers of an electric field jump	0	13
Electromagnetic triggers	0	503

It should be noted that an EAS trigger and a trigger of an electric field jump initiated both scintillation detectors and radiosystems. Electromagnetic triggers failed to develop under normal conditions but were formed by a discharge close to the cable. Therefore, statistics of relevant events fully reflect only those registered by scintillation detectors. Time synchronization of the readings of the scintillation detectors and radiosystems actuated by the electromagnetic trigger unambiguously suggest their correlation.

5. Results of the Groza experiments

5.1 Long-lasting gamma bursts and their correlation with radio emission

In most cases, strong electromagnetic pulses giving rise to an electromagnetic trigger result in powerful gamma-ray bursts. A typical example of gamma bursts registered upon initiation of scintillation detectors by the electromagnetic trigger is given in Fig. 4 showing the time-base sweep of signal intensity measured by one of the two scintillators (with a lower energy threshold for registering gamma quanta) located at sites 4, 6, and 7. The time scans for each detector are presented in three energy regions differing in lower amplitude thresholds corresponding to the minimal energy of recorded

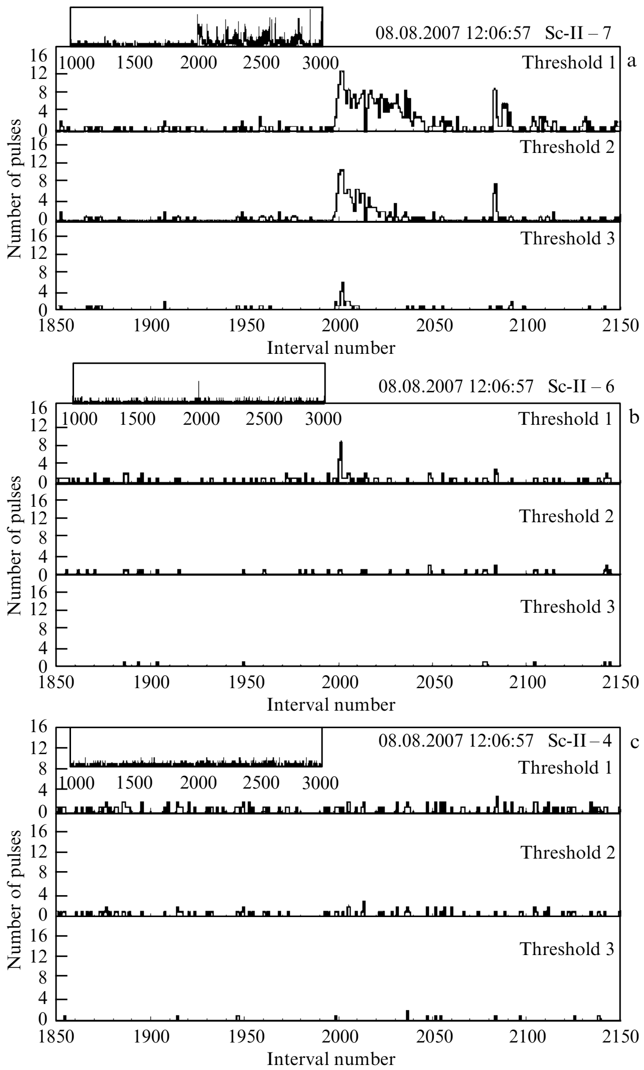

Figure 4. Gamma-ray bursts registered upon initiation of scintillation detectors by the electromagnetic trigger. The intensities are shown for three detection sites: (a) 7 (540 m above TSCRS), (b) 6 (310 m), and (c) 4 (TSCRS level). For each site, intensities of one counter are shown at three registering thresholds corresponding to the minimal energy of the recorded gamma quanta (30, 60, and 120 keV).

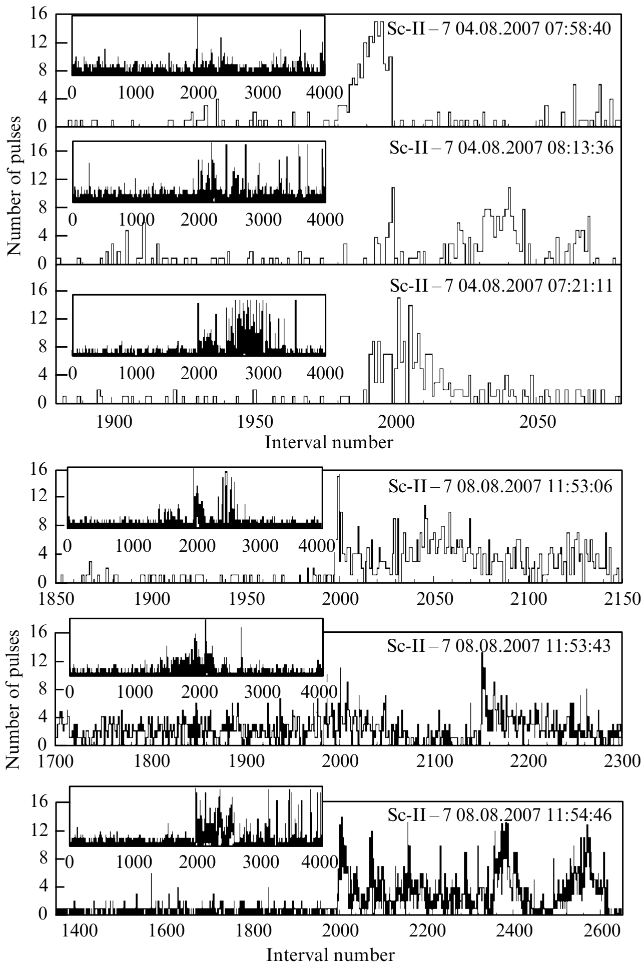

Figure 5. Time scans of gamma radiation intensity recorded with the electromagnetic trigger at 540 m above TSCRS (site 7) by the Sc-II detector with a threshold of 30 keV.

gamma quanta (30, 60, and 120 keV). The axes of abscissas in Fig. 4 are graduated in sequential numbers (200-microsecond time scan intervals); interval No. 2000 corresponds to the arrival of the trigger.

It appears from the top graph (4a) that the uppermost scintillation detector (site 7) begins to record enhanced gamma radiation 200–400 μs before the electromagnetic trigger. This radiation grows very rapidly, reaches a maximum when the trigger arrives, and persists for the next 10 ms. The inset to the top graph displays numerous similar flashes generated up to the 3000th time interval (200 ms after trigger actuation).

The background level of the signals throughout the first 1600 intervals of the time scan being considered amounts to 0.35 ± 0.01 pulses per interval. This value increases to 10–12 in the flash maximum, which corresponds to a rise in gamma radiation intensity by a factor of 30–35. Mean signal intensity during the first flash (from the 1998th to the 2046th interval) is 5.9 ± 0.3 pulses per interval (17 times above the background). Signal intensity throughout the flash duration rapidly decreases with gamma quanta energy, even if it remains an order of magnitude higher than the background value. Mean flash intensity in the second energy region (with a threshold of 60 keV) reaches 3.0 ± 0.3 pulses per interval (15 times the background value), and in the third one (120 keV) only 0.6 ± 0.1 pulses per interval (10 times the background value). In light of these statistics, the observed effect shows itself beyond any doubt.

Figure 4b illustrates the intensity of gamma quanta recorded (at three different energy thresholds) by a detector located 230 m below the previous one. The same flash is much weaker and manifests itself only in signals with a minimal energy threshold. Finally, the detector placed even lower (another 130 m down) (Fig. 4c) does not record any noticeable peculiarities at the instant of discharge.

The above character of radiation flashes is very typical and well apparent in most time scans of events registered by scintillation detectors inside thunderstorm clouds during electric discharges. Such flashes accompany at least 80% of the events recorded during a thunderstorm with a field jump trigger or an electromagnetic trigger; however, they do not occur in clear weather. This means that flashes are due to the presence of electrically charged clouds over detection sites. Examples of other events with long-lasting gamma bursts are presented in Fig. 5. Specifically, the uppermost graph in Fig. 5

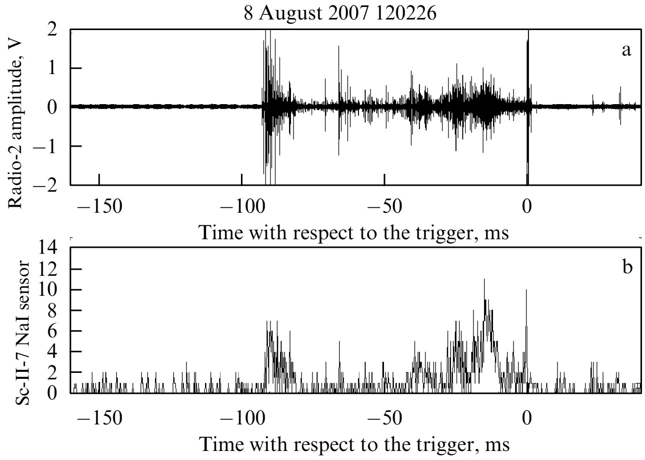

Figure 6. Radiofrequency radiation (a) and gamma-ray bursts (b) recorded with the use of an electromagnetic trigger.

shows a sharp cutoff of the gamma burst upon actuation of the electromagnetic trigger. Probably, it was the first return stroke of a lightning because radio emission died out for some time after it. Here (see the time scan), we observe similar attenuation of gamma activity.

A characteristic of long gamma-ray bursts is the distribution of their radiation in the vertical plane; all such bursts are especially well apparent at the uppermost detection site located deep in the thunderstorm cloud. The main signal component is built up by pulses with the lowest energy threshold. Thus, it can be concluded that the region where the bursts are generated lies in the depth of the thunderstorm cloud, while the bulk of radiation thus produced consists of low-energy gamma quanta.

In the above examples, the role of the trigger was played by a pulse of electromagnetic radiation created by an electric discharge. At the same instant of time, the scan displays a strong gamma pulse, suggesting close correlation between an electric discharge in the thunderstorm atmosphere and gamma-ray bursts. This is confirmed by the simultaneous observation of gamma and radiofrequency radiation. An example in Fig. 6 demonstrates close synchronization between gamma and radiofrequency radiation during a span of 100 ms.

5.2 Intercloud discharge induced by an extensive air shower (RB – EAS discharge)

An analysis of the time scans of the events presented in Table 1 permits distinguishing four short intense gamma-ray bursts in a thunderstorm atmosphere, which exactly coincide in time with the EAS trigger. One such an event occurred at the beginning of a thunderstorm on 15 August 2007, and the remaining three in the active phase of another thunderstorm (8 August 2007). Let us consider two of these events, one in the initial phase, the other in the active phase.

The first event (August 15, 2007) occurred during a thunderstorm that lasted 4 hours (from 05:18 to 09:40). The event occurred at 05:18:40 at the outset of the thunderstorm. EAS was recorded by all 6 detectors of the EAS trigger system. This means that the EAS was generated by a CR particle with an energy of 10^{16} eV or higher. The gamma pulse was registered at three sites (7, 6, and 4). The time scan of gamma-quantum intensity is presented in Fig. 7. The following characteristic features of this flash can be distinguished: (1) a strong gamma pulse was simultaneously detected at all detection sites; (2) the flash duration felt within a single time scan interval, i.e., was less than 200 μs; (3) the instant of gamma flash at all detection sites exactly coincided with the arrival of EAS, and (4) a sharp drop in background gamma radiation was recorded at the same instant of time.

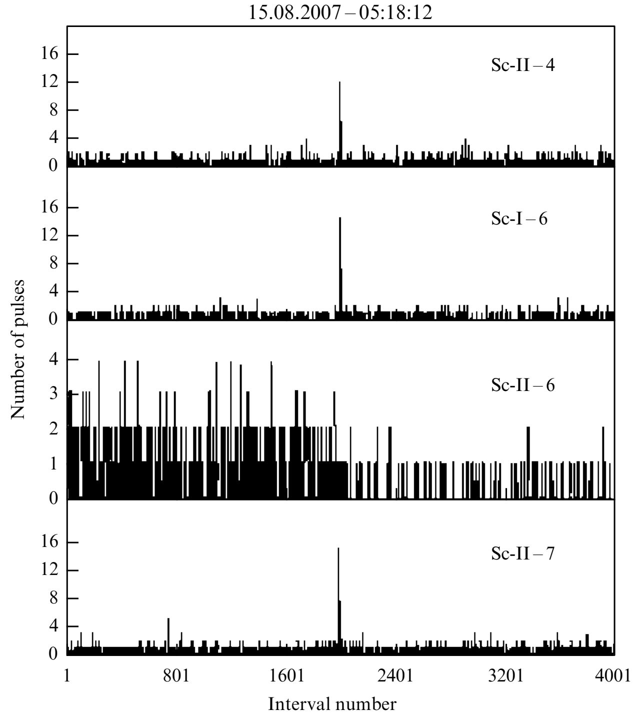

Figure 7. Time scan of gamma radiation intensity in an event exactly coincident in time with the EAS trigger. The event occurred at the beginning of a thunderstorm.

Notice that the distance between sites 4 and 7 is roughly 1100 m in the horizontal and 600 m in the vertical plane (see Fig. 3). In other words, strong gamma radiation extended over a large area. Also, Fig. 7 shows that the observed energy distribution of gamma quanta has the form of classical RB (see Section 5.3), i.e., a small difference between the cases of 30 keV (first threshold) and 60 keV (second threshold) and a sharp fall in the case of 120 keV (third threshold).

An abrupt change in the gamma-ray background at sites 6 and 7 (see Fig. 7) deserves special attention. The mean background value at site 6 decreases by a factor of 3 exactly at the moment of the trigger. At site 7, the background grows for 50 ms before the moment of the trigger and sharply decreases by approximately 3 times upon its arrival. The elevated gamma background created by CR is a consequence of enhancement of the fast electron flux in the strong thunderstorm field under the effect of RB[1]. The sharp decrease in the background reflects a sudden reduction in the electric field strength of the cloud crossed by EAS. In other words, an electric discharge in the cloud results from the

[1] Note a recent paper [33] where a long-term elevation of gamma background caused by fast electrons was observed during a thunderstorm at a height of 2770 m. The elevation lasted for 90 s, and the data were one-second averaged. The authors attributed the elevation to the enhancement of CR gamma background to RB effect. (*Authors' note to English translation.*)

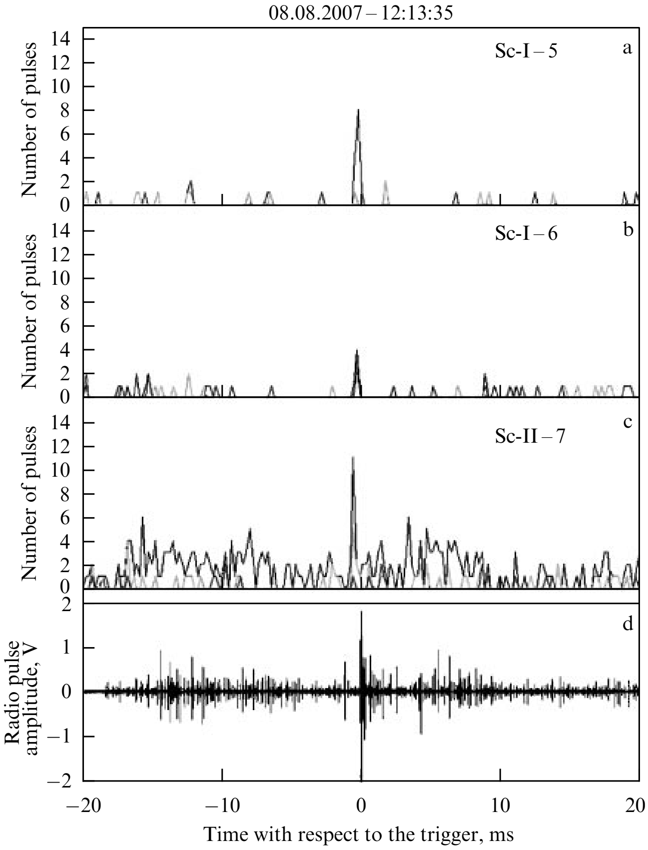

Figure 8. An event that occurred in the active phase of a thunderstorm, in which the instant of a gamma flash exactly coincided in time with that of the EAS trigger and radio emission: scans (a–c) show the time of the flash registered by scintillation detectors at sites 4, 6, and 7 with respect to the trigger pulse; scan (d) reveals the coincidence between radiofrequency and trigger pulses.

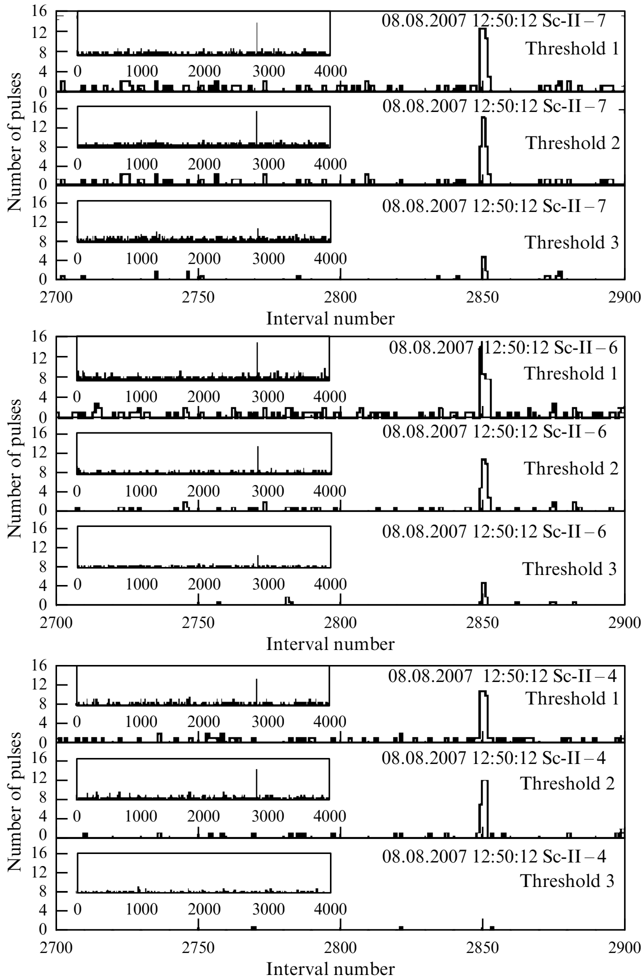

Figure 9. Time scans of gamma radiation intensity recorded by scintillation detectors using the EAS trigger. The scan sequence is similar to that in Fig. 4.

macroscopic effect of the joint RB/EAS action. Thus, we directly observe an RB – EAS discharge.

An event that took place in the active phase of the thunderstorm on 8 August 2007 is presented in Fig. 8. The thunderstorm lasted 1 hour (from 11:50 to 12:50). The event occurred at 12:35 during the active phase of the thunderstorm. An EAS was recorded at four detection sites. This means that the energy of the CR particle was at least 10^{15} eV. A strong gamma pulse was registered at sites 5, 6, and 7. The time scans of gamma radiation in Fig. 8 show that the strong gamma-ray pulse coincided with the EAS trigger at all detection sites. Figure 8d depicts in addition the measured radio signal. The strong pulse of radio emission is exactly coincident in time with the moment of EAS trigger, and gives evidence that the passage of the EAS through the thunderstorm cloud was accompanied by a strong electric discharge.

The above events give evidence of direct observation of RB – EAS discharges. This phenomenon was first examined in experiment. The probability of observation depends on two main factors. First, the path of a primary CR particle inducing the RB – EAS discharge must traverse a thunderstorm cloud at a distance of no more than 400–500 m from the gamma-ray scintillation detectors. Otherwise, the gamma emission will be absorbed in the atmosphere and remain undetected. On the other hand, the same path must be registered by the EAS-recording system. These conditions impose strong constraints on the number of permissible trajectories. Second, the strength E of the electric field in the cloud crossed by the EAS must be higher than the critical RB field: $E > E_c$. The latter condition is more frequently fulfilled in the active phase of a thunderstorm. The necessity to meet both conditions mentioned above reduces the probability of directly observing RB – EAS discharges to one in 100 'simple' EASs. We have observed only 4 RB – EAS discharges of the 600 recorded with the EAS trigger.

As noted above, the theory predicted the possibility of RB – EAS discharge, regarding it as one of the most important events in the thunderstorm atmosphere. Specifically, it associated the RB – EAS discharge with the high-altitude intercloud NBE discharge [8]. Characteristic features of NBE, namely, huge power of the radio pulse, weak optical radiation, absence of a step leader, and complete development within a few microseconds, agree with predictions of the RB – EAS discharge theory [31]. However, the RB – EAS discharge has never been directly observed.

5.3 Gamma spectrum of the lightning step leader

Analysis of all 600 events recorded with the EAS trigger showed that 14 included short intense gamma-ray flashes 400–800 μs in duration. Typical short gamma flashes registered by scintillation counters are presented in Figs 9 and 10 showing time scans of signal intensity from SC-II scintillators

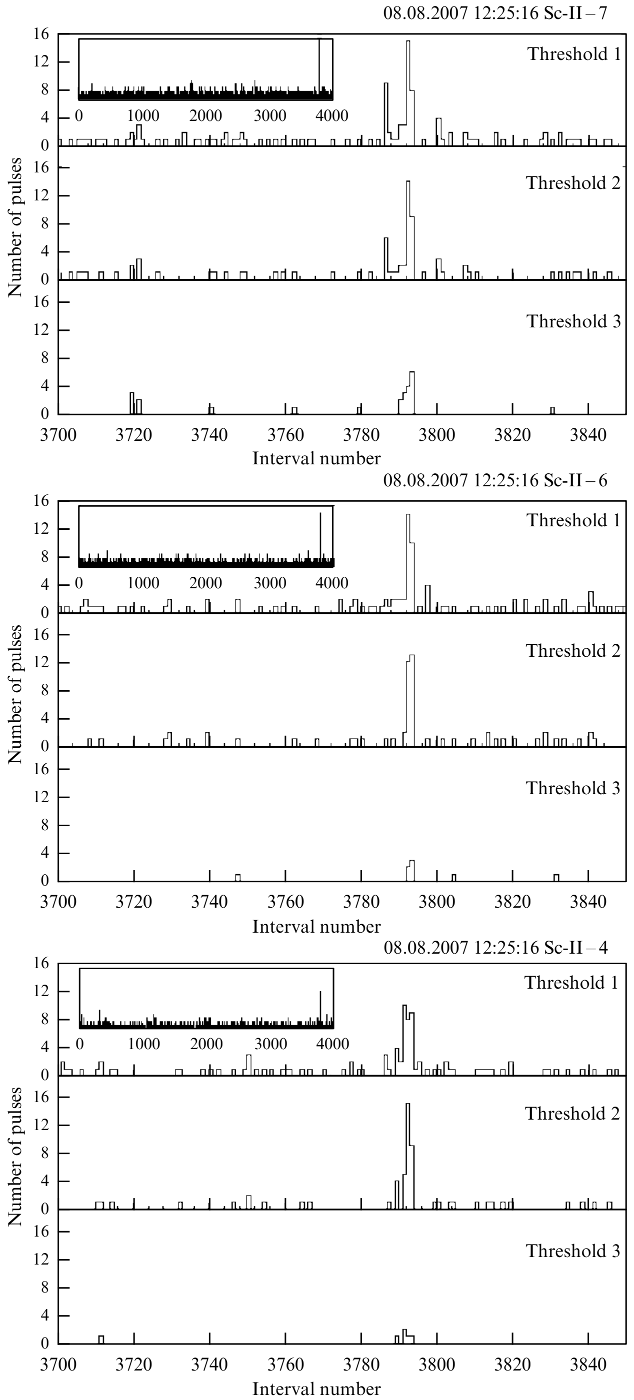

Figure 10. An event with a short gamma-ray flash similar to that in Fig. 9.

(with a lower energy threshold for recording gamma-quanta energy) located at sites 4, 6, and 7. Figure 9 illustrates the event in which the intense gamma-ray burst occurred 170 ms after trigger actuation. The burst was 400–600 μs long and spread over a distance of around 1 km, since it was simultaneously seen at four detection sites (7, 6, 4, and 3) and at all energy thresholds. The time scan of another event is displayed in Fig. 10.

We analyzed 14 events to reconstruct the energy spectrum of gamma quanta. We counted the number of gamma quanta with energies above 30, 40, 60, 70, 120, and 320 keV in these events for each scintillation detector that recorded them, taking into account detector calibration results. The comparison of observations with different energy thresholds at all

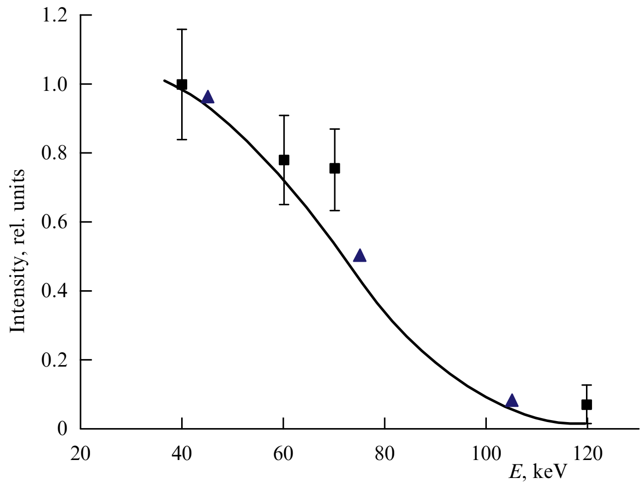

Figure 11. Integral spectrum of short gamma flashes recorded in thunderstorm events in the atmosphere: squares (with vertical error bars) — data of the Groza experiment; triangles — data of a balloon experiment, and solid line — theoretical curve.

detection sites showed that their integral spectra look alike. The characteristic spectrum thus obtained is represented in Fig. 11. Mean gamma-quantum energy in a short flash was estimated at 80 keV.

Short gamma-ray bursts associated with the lightning step leader were reported in Ref. [24]. It was shown that they significantly correlate with step leader jumps [25]. Recent studies of this phenomenon have demonstrated that gamma radiation is generated in the step leader prior to the return stroke and lasts 200–400 μs [26], during which the leader covers a distance of 200–400 m. This distance is a measure of gamma-quanta absorption in the atmosphere at sea level. Absorption at the TSCRS altitudes is 1.5 times lower, being proportional to the air density. It is therefore natural to suggest that the short (400–800 μs) intense gamma-ray bursts observed in our experiments actually emanate from the step leader before the return stroke of a lightning. It may be assumed from the theory that gamma radiation in the case of RB is generated in the leader's typical electric field $E \simeq 2E_{\mathrm{th}}$ (see Section 2.3). Then, the RB ionization frequency should strongly depend on the macroscopic parameters of the medium, such as electric field strength E and molecular number density N_{m}. At the same time, the energy spectra of both runaway electrons and the gamma radiation they generate have the same characteristic shape.

In Fig. 11, the normalized integral spectrum predicted by the theory is compared with our experimental data. Evidently, the two spectra agree fairly well. The figure also shows the results of a balloon experiment during a thunderstorm at an altitude of 4 km [32]. Numerous balloon experiments demonstrated that the electric field in the atmosphere is rather weak (max. 3–9 kV cm^{-1} [6, 7]), and RB is initiated by secondary EAS electrons passing through the cloud. As follows from Fig. 11, the gamma spectrum obtained in the balloon experiment is in excellent agreement with both the theory and our observations. We interpret the results of our experiments on the assumption that RB is caused by fast electrons produced during electric breakdown in the air in the strong electric field (50–60 kV cm^{-1}) of the step leader's head. Thus, the integral RB spectrum is very similar in both experiments and agrees with the theory (see Section 2.3) despite the significant difference between the electric field strengths in the cloud (3–9 kV cm^{-1}) and in the step leader (50–60 kV cm^{-1}).

6. Conclusion

This report considers selected data from the RB theory and the results of experimental studies concerned with thunderstorm events in the atmosphere. They are discussed with special reference to the active role of RB in the development of thunderstorm discharges.

Runaway breakdown is a novel and exceptionally common phenomenon underlain by Coulomb interactions between fast electrons and matter. We considered two new important issues of the RB theory. One is related to the proposed similarity relation that implies a similar RB structure in all substances. The other is the identification of a formerly unexplored strong region in the RB electric field: $2E_{\text{th}} \geqslant E \geqslant E_{\text{th}}$. Both conventional and runaway electric breakdowns are feasible at such field strengths. However, the ionization frequency for RB, i.e., its growth rate, is higher than for usual breakdown. This means that RB may play an important role in this region. These features are characteristic of the leader in a spark discharge and, perhaps, in a lightning leader.

The main achievement of the reviewed studies is the discovery of intense gamma-ray bursts in a thunderstorm cloud during the active phase of the discharge. The difficulty is posed by the fact that thunderstorm discharges are random and rapidly developing large-scale phenomena, hence the importance of drawing an analogy to lengthy spark discharges investigated under laboratory conditions. One element of a spark discharge is the leader transporting the electric charge. There is a similar leader in a thunderstorm discharge. The fast and extremely powerful return stroke of the lightning has been thoroughly investigated. However, a distinctive feature of spark discharges is the presence of an electrode that shed the electric charge accumulated in the capacitor; in a thunderstorm cloud, the charge is carried by water droplets and small pieces of ice over a large area (km-scale). How is such a charge collected from these structures and delivered to the leader within a few milliseconds? This problem has not thus far been discussed at any length in the literature. Electrical conduction of the air is not high enough for charge transfer and needs to be increased by at least 5–6 orders of magnitude.[2] It is still lower in the clouds. The cloud electric field measured many times in experiment is insufficient for direct breakdown of the air. Therefore, mechanisms underlying accumulation and transport of the charge to the lightning leader remained enigmatic (see Refs [3, 4, 34]). The discovery of intense gamma radiation and its correlation with radio emission suggests that RB may play a key role in these processes.

Thunderstorm clouds contain electric fields stronger than the RB critical field. The number of secondary EAS electrons entering the area of 1 km^2 for 1 ms amounts to $\sim 10^6$. They serve as a 'seed' for RB. Further studies are needed to clarify whether they can ensure fulfillment of necessary conditions for the charge transfer from a cloud to the lightning leader.

The results of Groza experiment underway at FIAN's TSCRS gave an idea of the mechanism behind electric charge accumulation from thunderstorm clouds for initiation of the lightning. Another new fact discovered in this experiment is an RB – EAS-induced discharge. The theory predicts that this discharge may be a source of narrow bipolar events (NBEs).

To sum up, the development of the active phase of a thunderstorm discharge (from preliminary breakdown to initiation of the leader and its propagation until the occurrence of return stroke) is totally governed by the accumulation and transfer of the electric charge from the clouds. Our study demonstrated that this process is accompanied by correlated powerful gamma-ray and radio emission fluxes created by runaway breakdown.

It is safe to conclude that runaway breakdown initiated by cosmic rays is the main driving mechanism of thunderstorm discharge.

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[2] More correctly, "...3–4 orders of magnitude." (*Authors' note to English translation.*)

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B B Kadomtsev's classical results and the plasma rotation in modern tokamaks

V I Ilgisonis

1. Introduction. Stages of Kadomtsev's work at the Kurchatov Institute

This report is focused on three theoretical works by B B Kadomtsev concerning controlled thermonuclear fusion, which in my opinion give a good idea of several periods of his work affiliated with the Kurchatov Institute. Kadomtsev was a highly productive theorist with a very wide spectrum of interests. Therefore, in choosing which of his works to present in a report like the present one, I could only rely on my subjective impressions. The first period is distinctive for a series of research undertakings on the so-called energy stability principle [1]. Two other works to be considered deal with trapped particle instability [2] and reduced equations describing plasma dynamics [3]. All these works became classical and opened up a new area of further investigations. Moreover, each of them gives a striking example of the productivity of the theory that was later brilliantly confirmed in experiment. What follows is intended to illustrate the importance of these works from a present-day perspective with special reference to plasma rotation, a new phenomenon that was not discussed in the periods in question. In modern tokamaks, plasma can rotate at a high speed, and this rotation is presently regarded as a key factor promoting plasma confinement. Rotation requires modification of most results obtained for motionless equilibrium plasma.

The first period of Kadomtsev's work at the Kurchatov Institute roughly covers the years of 1956 – 1962. In 1956, Boris Borisovich joined the Theory Division headed by Mikhail Aleksandrovich Leontovich, who immediately appreciated the young researcher as a man of great intelligence. Within a few years, Leontovich managed to bring together a small but highly efficient team of gifted theorists with high scientific potential. In this context, Vitaly Dmitrievich Shafranov should be given credit for having introduced his fellow-student Boris Kadomtsev to M A Leontovich.

Boris Borisovich started with kinetic research, partly continuing his previous investigations in Obninsk, but very soon his exceptional ability to see the essence of an issue made him switch over to the problem of macroscopic plasma confinement. This ability to see all sides of a problem and focus on its key aspects distinguished Kadomtsev throughout all periods of his scientific work. It is this ability that put him in the forefront of a new science, the theory of high-temperature plasma, and made him one of its leading actors. He began to develop the theory of plasma stability and continued this work one way or another for the rest of his life. In this period, Kadomtsev completed several important studies on the magnetohydrodynamic (MHD) stability of the plasma in magnetic traps and formulated the energy principle of stability of MHD equilibrium, which will be discussed below. His efforts culminated in experimental verification of the 'minimum B' principle in the laboratory of M S Ioffe [4]. This period logically ended in the defense of his doctoral thesis (1961) and election as a Corresponding Member of the USSR Academy of Sciences (1962).

In the second period (1962 – 1970), Boris Borisovich elaborated the theory of magnetized plasma turbulence and related transport processes. This greatly contributed to the understanding that Bohm diffusion (long considered to be an insurmountable obstacle for thermonuclear fusion) is not inevitable and can be obviated. Further development of this theory brought Kadomtsev to the concept of a tokamak-based thermonuclear reactor. In parallel, Boris Borisovich continued the search for instability, a key prerequisite for a fusion reaction in tokamaks, and discovered trapped particle instability. In 1970, he was elected a Full Member of the Academy and awarded the USSR State Prize.

During the next period (1971 – 1990), large tokamaks were built at the Kurchatov Institute and in leading research centers abroad. Powerful gyrotron heating was used to obtain record-breaking plasma parameters in the tokamak T-10. The subsequent tokamak T-15 was a unique machine with superconducting windings. At that time, Boris Borisovich showed special interest in tokamak physics as a whole. He formulated principles of plasma self-organization in tokamaks and continued to develop the theory of stability with reference to disruption instability, of primary importance for tokamak operation. Simultaneously, he made an important contribution to a physics of nonlinear phenomena (the well-known Kadomtsev – Petviashvili and Kadomtsev – Pogutse equations, the latter being considered in Section 4 below). In 1984, Boris Borisovich Kadomtsev was awarded the Lenin Prize. The period under consideration naturally ends with participating in ambitious international projects initiated by Evgenii Pavlovich Velikhov. After 1990, Kadomtsev published a few reviews of tokamak plasma physics, serving as a basis for the International Thermonuclear Experimental Reactor (ITER) project, in which Boris Borisovich was an active participant. Kadomtsev described the history of tokamak concept from the very first idea to ITER in review [5]. The world's thermonuclear community recognized the outstanding scientific contributions of Boris Borisovich Kadomtsev by awarding him the J C Maxwell Prize (1998) of the American Physical Society.

2. Energy stability principle

The macroscopic stability of the plasma in fusion devices, first and foremost MHD stability, became a priority issue in the second half of the 1950s. Researchers very soon recognized the importance of flute instability in magnetic traps, at which plasma tongues stretch parallel to the magnetic field and penetrate through the lines of force without perturbing the field. This instability is an MHD analog of Rayleigh – Taylor instability, which it is natural to analyze from the energy standpoint.

The following 'energy principle of stability' of static equilibria is quite obvious for simple Hamiltonian systems: owing to the positive definitiveness of the kinetic energy, the positive definitiveness of the potential energy guarantees the stability of the initial equilibrium (in accordance with the Lyapunov theorem). S Lundquist was the first to suggest this approach in relation to MHD problems in 1951 [6]. It was further developed by Kruskal, Kalsrud, Schlüter, Rosenbluth, Longmire et al. (see, for instance, Refs [7, 8]) during the next 6 years. Some important results were obtained, and comprehensive mathematical formulation of the energy principle for arbitrary MHD systems was proposed by Bernstein, Frieman, Kruskal, and Kalsrud (BFKK principle) in 1958 [9]. By that time, the works of Kadomtsev and the above authors had already contained main elements of the BFKK energy principle [10]. However, due to the comprehensiveness and generality of the BFKK formulation, the energy principle of MHD stability is presently associated with Ref. [9], most frequently cited in plasma physics publications. This paper was followed by a large number of others reporting application of the energy principle to the description of stability of concrete magnetic systems, modes, etc.

The energy stability principle for an MHD system with the Hamiltonian

$$H = K + W = \int_{\Gamma} \frac{\rho \mathbf{V}^2}{2}\, \mathrm{d}^3 r + \int_{\Gamma} \left(\frac{p}{\gamma - 1} + \frac{\mathbf{B}^2}{8\pi} \right) \mathrm{d}^3 r \qquad (1)$$

is formulated as a requirement for positive semi-definitiveness of the second potential energy variation:

$$\delta^2 W \geqslant 0 \qquad (2)$$

in the vicinity of static equilibrium position $\mathbf{V} = \mathbf{0}$, $\nabla p = \operatorname{rot} \mathbf{B} \times \mathbf{B}/4\pi$ (standard notations are used: $\mathbf{V}$ and p are the macroscopic velocity and pressure, respectively, of the plasma with adiabatic exponent γ, confined by magnetic field $\mathbf{B}$ in volume Γ assumed to be fixed for simplicity). It is convenient to express quantity $\delta^2 W$ in terms of displacement ξ of a plasma element:

$$\begin{aligned} \delta^2 W &= -\frac{1}{2} \int \xi\, \mathbf{F}(\xi)\, \mathrm{d}^3 r \\ &= \frac{1}{2} \int \mathrm{d}^3 r \left\{ \frac{1}{4\pi} \Big(\operatorname{rot}^2 [\xi \times \mathbf{B}] + [\xi \times \operatorname{rot} [\xi \times \mathbf{B}]] \operatorname{rot} \mathbf{B} \Big) \right. \\ &\quad \left. + \xi\, \nabla p \operatorname{div} \xi + \gamma p \operatorname{div}^2 \xi \right\}. \end{aligned} \qquad (3)$$

This expression clearly demonstrates the physical nature of possible instability: the second and the third terms on its right-hand side are responsible for two feasible instability mechanisms, one associated with the electric current flowing in the plasma, the other with its pressure, whereas perturbation of the magnetic field and plasma compressibility serve as stabilizing factors. Productivity of the energy principle is closely related to self-conjugacy (hermiticity) of linearized force operator $\mathbf{F}$, understood in the usual sense:

$$\int_{\Gamma} \boldsymbol{\eta}\, \mathbf{F}(\xi)\, \mathrm{d}^3 r = \int_{\Gamma} \xi\, \mathbf{F}(\boldsymbol{\eta})\, \mathrm{d}^3 r$$

(arbitrary vectors ξ and $\boldsymbol{\eta}$ vanish at the boundary of the integration domain Γ).

Self-conjugacy of the force operator $\mathbf{F}$ guarantees the necessity of stability condition (2) and its completeness for systems with magnetic surfaces, such as the majority of the known magnetic traps. In other words, the following assertion can be proved: if the potential energy of a certain displacement ξ is negative, there is an eigenmode of the small-oscillation equation

$$\rho \ddot{\xi} = \mathbf{F}(\xi)\,,$$

which exponentially grows with time [9]. The set of eigenmodes forms a complete system.

Kadomtsev applied this principle to the analysis of flute modes and found the stability condition

$$\nabla p\, \nabla U + \frac{\gamma p (\nabla U)^2}{U} > 0\,, \qquad (4)$$

where $U = \int \mathrm{d}l/B$ is the integral taken along a magnetic field line (along the flute length). The first term in condition (4) describes the 'mean magnetic well' effect contributing to stability and showing itself as the magnetic field grows from the plasma confinement region. The second term takes account of the stabilizing effect of plasma compressibility. Kadomtsev summarized this and some other practical applications of the energy principle in a comprehensive and easily understandable review [1].

The energy principle has an important nuance. The Lyapunov theorem demands sign-definiteness of the functional [strict inequality in formula (2)], hence there is the problem with *neutral* displacements that do not change potential energy W, i.e., those corresponding to zero frequency in terms of eigenmodes. It is these displacements that cause concern as regards nonlinear instability. Moreover, it can be shown that such neutral displacements always exist in MHD systems and that they are nontrivial, i.e., nonreducible to global displacements and turns of the plasma as a whole, which are of no interest for the problem under consideration. In the systems with nested magnetic surfaces, whose state is beyond boundary stability, neutral perturbations reduce to relabeling transformations of fluid elements that do not perturb physical quantities characterizing plasma state, viz. pressure, density, and magnetic field [11]. Thus, the energy principle in the form of expressions (2), (3) is exhaustive for plasma static equilibria in the systems with magnetic surfaces. However, the existence of relabeling symmetries suggests the possibility of a shift in equilibrium along such transformations, i.e., the flows. Therefore, attempts to extend this approach to the case of plasma with flows seem natural.

Such an attempt was made by Frieman and Rotenberg as early as 1960 [12]; they derived the energy condition from the general linearized equation of motion

$$\rho \ddot{\xi} + 2\rho (\mathbf{V}\, \nabla)\, \dot{\xi} - \mathbf{F}(\xi) = 0\,, \qquad (5)$$

where ρ and $\mathbf{V}$ are the stationary values of mass density and plasma flow velocity, and the operator $\mathbf{F}$ is modified compared with the operator $\mathbf{F}$ in Eqn (3) but still retains the property of hermiticity. Conservatism of the system [antisymmetric operator with $\dot{\xi}$ in Eqn (5) drops out of the energy balance equation in integration] again permits obtaining (in analogy with the static energy principle) a sufficient condition for stability in the form

$$\begin{aligned} \delta^2 W \approx \frac{1}{2} \int_{\Gamma} \mathrm{d}^3 r & \left\{ -\frac{1}{\rho} \operatorname{rot}^2 [\xi \times \rho \mathbf{V}] - \big[\xi \times \operatorname{rot} [\xi \times \rho \mathbf{V}]\big] \operatorname{rot} \mathbf{V} \right. \\ & + \frac{\mathbf{V}^2}{\rho} \operatorname{div}^2 (\rho \xi) + \left(\xi\, \nabla \frac{\mathbf{V}^2}{2} - 2 \mathbf{V}\, (\mathbf{V}\, \nabla)\, \xi \right) \operatorname{div} (\rho \xi) \\ & + \frac{1}{4\pi} \Big(\operatorname{rot}^2 [\xi \times \mathbf{B}] + \big[\xi \times \operatorname{rot} [\xi \times \mathbf{B}]\big] \operatorname{rot} \mathbf{B} \Big) \\ & \left. + \xi\, \nabla p \operatorname{div} \xi + \gamma p \operatorname{div}^2 \xi \right\} \geqslant 0\,, \end{aligned} \qquad (6)$$

which is too (unnecessarily) 'rigorous', unlike the condition in the static case, and is not satisfied for systems of any practical interest barring a few rather special cases (e.g., plasma flow strictly along magnetic lines of force, $\mathbf{V} \parallel \mathbf{B}$). Interest in this

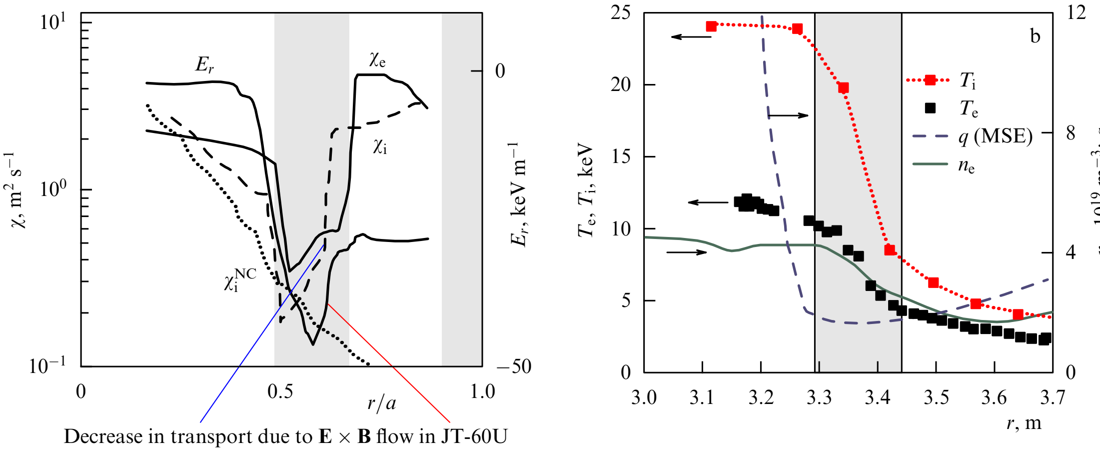

Figure 1. Transport barrier in the JT-60U tokamak, Japan (see Ref. [13]). (a) Jumps in electric field E_r and effective coefficients of ion (χ_i) and electron (χ_e) heat conductivity in the barrier zone. Ion heat conductivity decreases to χ_i^{NC} calculated from a neoclassical (NC) theory (dotted curve). (b) Large temperature (T_e, T_i) and density (n_e) gradients in this zone illustrate the notion of 'transport barrier'; q is the safety factor measured by the MSE (Motional Stark Effect) method.

problem was lost for the next 20 years because the role of macroscopic plasma motion (flow) was deemed unessential at a flow rate much lower than the speed of sound. It should be noted that this argument is not quite correct since the characteristic size of spatial inhomogeneity of the flow may be significantly different from that of pressure, density, and magnetic field inhomogeneities, hence the taking into account plasma motion at much lower flow velocities can be important. However, such a possibility was disregarded in early thermonuclear experiments.

Interest in plasma flows was renewed with the advent of new powerful plasma-heating sources in modern tokamaks. Uncompensated injection of fast atomic beams into a tokamak sets the plasma in rotational motion with a rate that may reach the same order of magnitude as the speed of sound. In this case, *improved confinement* regimes associated with the appearance of relatively narrow layers of nonuniform rotation develop. Figure 1 demonstrates the so-called transport barrier phenomenon typical of such regimes. A narrow layer undergoes a jump in the electric field and, accordingly, in the rate of plasma rotation. A temperature jump in this layer corresponds to a sharp fall in effective heat conductivity. The presence of such a layer makes it possible to significantly increase permissible parameters of the plasma confined within the barrier. Taken together, these facts dictated the necessity of studying plasma rotation effects in both the stability problems and closely related problems of transport theory.

One of the probable causes of the excessively large discrepancy between the sufficient Frieman – Rotenberg stability condition (6) and the necessary MHD stability condition is underestimation of the relationship between the displacement and the speed inherent in the real dynamics of the system. This assertion is illustrated by a simple example sometimes referred to as the Prendergast problem. Let us consider the motion of a charge over a symmetric hill in a gravitational field and in a vertical magnetic field. The magnetic field does not change the charge energy and conclusions based on analysis of the sign of the second variation of potential energy point to possible instability at any hill slope. Positive definiteness of the potential energy guarantees stability only in a gravitational well, even though the magnetic field clearly affects the

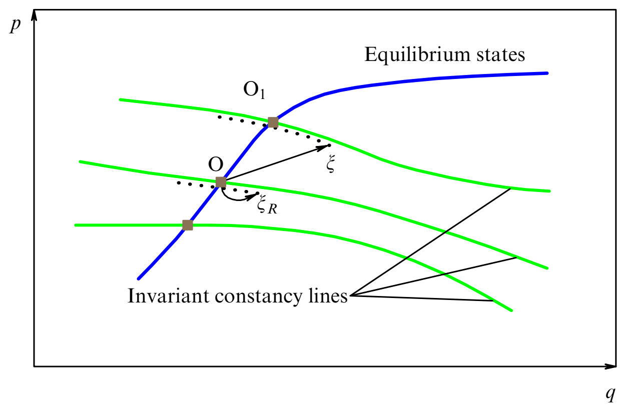

Figure 2. A case of degenerate equilibrium [dark curve (blue in on-line version)]. Oscillations (dots) occur along invariant constancy lines.

charge dynamics. The magnetic field being strong enough, equilibrium at the top of the hill or rotation around it may prove stable. This is easy to see since the problem has an exact solution. The redundant freedom in variable functions can be eliminated by taking account of conservation laws inherent in the system, differing from the law of conservation of energy. Thus, the generalized angular momentum must be conserved in this problem. In the general case, in the presence of additional motion invariants shown by level lines on the conditional phase plane (Fig. 2), it is enough to study perturbations ξ_R retaining the meaning of such invariants instead of arbitrary displacements ξ. Interestingly, using this procedure and taking into account the law of conservation of the generalized angular momentum *in variations* allow, in our example, obtaining an exact (necessary and sufficient) stability criterion.

In 1965, V I Arnold suggested this idea in application to hydrodynamics [14, 15] and proposed taking into account the conservation of vorticity in the analysis of flow stability. In MHD conditions, vorticity is not conserved, whereas systems with magnetic surfaces retain (under certain conditions) cross helicity I_1 and its 'counterpart' I_2:

$$I_1 = \int \mathbf{V}\,\mathbf{B}\,\mathrm{d}^3 r\,, \qquad I_2 = \int \mathbf{V}\,\mathbf{D}\,\mathrm{d}^3 r\,. \tag{7}$$

Here, $\mathbf{D}$ and $\mathbf{B}$ are linearly independent, the former being divergenceless vector frozen into the plasma and also tangential to the magnetic surfaces; integration in formulas (7) is taken over the volume between any adjacent magnetic surfaces. The use of Arnold's scheme to take account of limitations on the variable functions, which are imposed by the condition of conservation of quantities (7) in variations, permitted obtaining common equilibria with flows and simultaneously a milder stability condition [16, 17] as against the Frieman–Rotenberg condition (6). In the general case, elimination of excess freedom in variations of independent variables (coordinates and momenta of 'fluid elements' in the medium) is achieved by splitting perturbations in accordance with invariance of quantities of the form

$$\int_{\Gamma} \lambda \mathbf{P}\,\mathbf{V}\,\mathrm{d}^3 r\,,$$

where $\mathbf{P}$ is the canonical momentum (bearing in mind perturbations), $\mathbf{V}(\mathbf{r})$ is the equilibrium velocity field in volume Γ, and λ is the weight factor related to system topology. It is essential that such splitting should be taken into account in both the first and the second functional variations. Although consideration of the first variation yields an equilibrium condition of the most general functional form, the stability condition may still be far from the necessary one.

It is methodically relevant to draw attention to a misapprehension widespread in the literature that the formal addition of conserved quantities [e.g., integrals (7)] with undetermined Lagrange multipliers to a variable functional and variation of the new functional automatically lead to an improved (milder) stability condition. This procedure described, for instance, in the well-known review [18] leads only to a more general class of equilibria but does not restrict perturbations in variations and therefore results in a loss of information about the derived integrals of motion in studies of convexity of the functional, i.e., again in a more stringent stability condition than in the Arnold method. The same drawback is inherent in most studies of nonlinear stability and flow stability performed later as recommended in Ref. [18].

Another purely physical cause of the difficulties encountered in the energy approach is concerned with negative energy waves. Indeed, energy analysis of perturbations for the study of stable oscillations in the system of interest may prove unproductive if these oscillations possess not only positive but also negative energy. It should be emphasized that Kadomtsev paid his attention to negative energy waves, but his well-known work [19] concerned only interaction between electromagnetic waves in media with different dispersions. It is important for our purposes that MHD oscillations may have negative energy, too. Indeed, the following dispersion equation for eigenfrequency ω formally follows from the Frieman–Rotenberg equation (5):

$$A\,\omega^2 - 2B\,\omega - C = 0\,, \tag{8}$$

where for $\boldsymbol{\xi}$ in the form of normal modes, viz.

$$\boldsymbol{\xi}(\mathbf{r}, t) = \hat{\boldsymbol{\xi}}(\mathbf{r}) \exp(-\mathrm{i}\omega t)\,, \tag{9}$$

the coefficients $A = \int \rho |\hat{\boldsymbol{\xi}}|^2\,\mathrm{d}^3\mathbf{r}$, $B = -\mathrm{i}\int \rho \hat{\boldsymbol{\xi}}^*\,(\mathbf{V}\nabla)\,\hat{\boldsymbol{\xi}}\,\mathrm{d}^3\mathbf{r}$, and $C = -\int \hat{\boldsymbol{\xi}}^*\,\mathbf{F}[\hat{\boldsymbol{\xi}}]\,\mathrm{d}^3\mathbf{r}$ are real by definition. The solution of equation (8) has the form

$$\omega = \frac{B + s\sqrt{B^2 + AC}}{A}\,, \tag{10}$$

where $s = 1$ or $s = -1$ for a given eigenwave. Therefore, the eigenwave is unstable only if $B^2 + AC < 0$. The eigenmode energy can be written out as

$$E = \frac{1}{2}\left(A|\omega|^2 + C\right) \exp(2\gamma t)\,, \tag{11}$$

where the increment $\gamma = \operatorname{Im}\omega$. Because the energy is conserved, E in expression (11) cannot depend on time and must be zero for any unstable eigenmode with $\gamma \neq 0$.

The energy of a stable eigenmode with $\gamma = 0$ is given by the expression

$$E = s\omega\,\sqrt{B^2 + AC} \tag{12}$$

and can be either positive (positive energy waves, PEWs) or negative (negative energy waves, NEWs). NEWs exist as eigenmodes with $-B^2/A < C < 0$ and $\operatorname{sign}(B) = -s$. It follows from Eqns (8), (12) that all NEWs are asymmetric, i.e., show spatial dependence in the direction of the stationary flow, so that $B \neq 0$. As shown in Ref. [20], there is an interval of equilibrium parameters within which PEWs and NEWs coexist. When their frequencies coincide (resonance conditions), the energy may be transferred from a NEW to a PEW, which leads to instability. In point of fact, such NEW/PEW pairs constitute a universal mechanism of any asymmetric instability in an ideal MHD system with flows.

Eigenmodes with purely real or purely imaginary eigenvalues producing a spectrum symmetric with respect to the origin of coordinates on the plane $\operatorname{Re}\omega - \operatorname{Im}\omega$ are referred to below as symmetric. They correspond, in particular, to static equilibria or modes homogeneous along the flow direction ($B = 0$). The standard energy principle holds true for symmetric modes because their energy (12) is always nonnegative and passes through zero during the transmission from stability zone to instability zone. Certainly, this principle is violated in the case of excitation of NEWs in the system since zero energy is attainable in a wholly stable zone, too.

This NEW-related inconveniency can also be avoided by taking into account the necessary number of additional integrals of motion, at least in the case of a discrete spectrum. The linear equation of motion (5) has an infinite set of energy type integrals [21] but not-reducible-to-energy integrals:

$$E_n = \frac{1}{2}\int \left(\rho |\boldsymbol{\xi}^{(n+1)}|^2 - \boldsymbol{\xi}^{*(n)}\,\mathbf{F}[\boldsymbol{\xi}^{(n)}]\right) \mathrm{d}^3\mathbf{r}\,, \tag{13}$$

where $\boldsymbol{\xi}^{(n)}$ is the nth derivative in time. In the main, these integrals are independent. E_0 corresponds to energy, and integral E_1 to type (7) invariants. Higher order invariants (13) have no explicit nonlinear analogs. Using a recurrent relation directly following from equation (5), namely

$$\boldsymbol{\xi}^{(n+2)} = -2(\mathbf{V}\,\nabla)\,\boldsymbol{\xi}^{(n+1)} + \frac{\mathbf{F}[\boldsymbol{\xi}^{(n)}]}{\rho}\,, \tag{14}$$

it is possible to express all integrals (13) through initial perturbations $\dot{\boldsymbol{\xi}}_0 = \dot{\boldsymbol{\xi}}|_{t=0}$ and $\boldsymbol{\xi}_0 = \boldsymbol{\xi}|_{t=0}$. Specifically, one finds

$$E_1(\dot{\xi}_0, \xi_0) = \frac{1}{2}\int\left(\frac{1}{\rho}\left|\mathbf{F}[\xi_0] - 2\rho(\mathbf{V}\nabla)\,\dot{\xi}_0\right|^2 - \dot{\xi}_0^*\,\mathbf{F}[\dot{\xi}_0]\right)\mathrm{d}^3\mathbf{r}\,. \tag{15}$$

Integrals of motion (13) can be introduced into the Lyapunov functional by the method of Arnold [15] using the Lagrange multipliers λ_n:

$$U(\dot{\xi}_0, \xi_0) = \sum_{n=0}^{N} \lambda_n E_n(\dot{\xi}_0, \xi_0)\,. \tag{16}$$

The following theorem provides a sufficient condition for formal stability of the system described by equation (5).

Theorem. *If there exist real numbers λ_n and integer $N \in [0, \infty]$ such that the form* (16) *is positively definite for all $\dot{\xi}_0$ and ξ_0, then the form* (16) *is the Lyapunov functional and the equilibrium state is formally* (*spectrally*) *stable*.

The proof of this theorem and more detailed description of this approach can be found in Ref. [22]. Under certain assumptions, the theorem also provides necessary conditions for spectral stability because, given that the system is stable, there exist such λ_n whereat the functional U is nonnegative at any perturbation.

The productivity of this approach can be illustrated by a simple example of Rayleigh – Taylor instability of a rotating cold gravitating gas. All equilibrium quantities can depend only on radius r in a cylindrical system of coordinates (r, φ, z). The equilibrium velocity is expressed as

$$\mathbf{V} = r\Omega(r)\mathbf{e}_\varphi\,, \qquad r\Omega^2(r) = \frac{\partial\Phi}{\partial r}\,, \tag{17}$$

where $\Omega(r)$ is the angular frequency of rotation in a gravitational field with the potential $\Phi(r)$, and $\mathbf{e}_\varphi = r\nabla\varphi$. The stability condition for such rotation is fairly well known. It is the Rayleigh criterion (a necessary and sufficient condition of spectral stability) reducible in the present case to the requirement for the so-called epicyclic frequency κ to be real:

$$\kappa^2 = 4\Omega^2 + r\,\frac{\partial\Omega^2}{\partial r} \geqslant 0\,. \tag{18}$$

Let us apply the above-described variational method to this problem. In this case, all invariants (13) are local, and the first two, E_0 and E_1, have the following form for the modes rotating with frequency $\Omega(r)$:

$$\begin{aligned} E_0 &= \frac{1}{2}\left(|\dot{\xi}|^2 - \xi^{*\mathrm{T}}\hat{\mathbf{B}}\xi\right) \\ &= \frac{1}{2}\left(|\dot{\xi}_r|^2 + |\dot{\xi}_\varphi|^2 + |\dot{\xi}_z|^2 + r\,\frac{\partial\Omega^2}{\partial r}\,|\xi_r|^2\right), \\ E_1 &= \frac{1}{2}\left(|\hat{\mathbf{B}}\xi - 2\Omega\hat{\mathbf{A}}\dot{\xi}|^2 - \dot{\xi}^{*\mathrm{T}}\hat{\mathbf{B}}\dot{\xi}\right) \\ &= \frac{1}{2}\left[\left|r\,\frac{\partial\Omega^2}{\partial r}\,\xi_r - 2\Omega\dot{\xi}_\varphi\right|^2 + \left(4\Omega^2 + r\,\frac{\partial\Omega^2}{\partial r}\right)|\dot{\xi}_r|^2\right] \end{aligned} \tag{19}$$

where $\hat{\mathbf{B}}$ is the matrix: $B_{ij} = 2r\Omega\Omega_r'\delta_{i1}\delta_{j1}$, and δ is the Kronecker symbol. Choosing E_1 for U and putting $\lambda_{i\neq 1} = 0$ in formula (16) leads to the spectral stability condition that is exactly the Rayleigh criterion (18). As follows from Eqn (19), the energy principle ($U = E_0$) gives a more rigorous sufficient stability condition: $r\,\partial\Omega^2/\partial r \geqslant 0$, confirming the efficiency of the proposed method.

Another example is E P Velikhov's magnetorotational instability (MRI) [23] supposed to be responsible for turbulent processes in accretion disks. Let us calculate energies and eigenmode frequencies in an experiment simulating magnetorotational instability. Consider an incompressible conducting fluid rotating across a uniform magnetic field $\mathbf{B} = B_0\mathbf{e}_z$ with angular velocity

$$\mathbf{V} = r\Omega(r)\mathbf{e}_\varphi\,, \qquad \Omega(r) = \frac{\Omega_1 r_1^2}{r^2}\,, \tag{20}$$

given in the cylindrical system of coordinates (r, φ, z). Let us choose, for definiteness, $r_2/r_1 = 5$, where r_1 and r_2 are the inner and outer radii of the fluid-containing channel, respectively, and Ω_1 is the angular velocity at radius r_1. A detailed study of the stability of such a flow was reported in Refs [20, 24] for normal modes represented in the form $\xi(\mathbf{r}, t) = \xi(\mathbf{r})\exp(-\mathrm{i}\omega t + \mathrm{i}m\phi + \mathrm{i}k_z z)$.

Figure 3 depicts the frequency and the energy of axisymmetric ($m = 0$) and nonaxisymmetric ($m = 1$) eigenwaves depending on the equilibrium parameter $\Omega_1/\omega_\mathrm{A}$ that characterizes rotational velocity (ω_A is the Alfvén frequency). The instability zone is shaded. In the axially symmetric case (Fig. 3a), only positive energy waves can be excited in the system. The value of $\Omega_1/\omega_\mathrm{A} \approx 2.0$ (MRI threshold for $m = 0$) corresponds to the point of merging of two branches in Fig. 3a. The nature of axially symmetric MRI is unrelated to negative energy waves and can be associated with a mechanism resembling Rayleigh – Taylor instability [23].

Positive and negative energy waves with $m = 1$ (Fig. 3b) can coexist in the system when $\Omega_1/\omega_\mathrm{A} > 1$. In this case, the instability threshold is $\Omega_1/\omega_\mathrm{A} \approx 1.7$ (which corresponds to a radial mode with $n_r = 0$), when NEW and PEW frequencies coincide as mentioned earlier. The discreteness of the spectrum also permits utilizing the above combined functional (16). This example is considered at greater length in Ref. [22].

To sum up, generalization of the classical energy principle for the case of dynamic equilibria, i.e., flows, is feasible, albeit not universal.

3. Trapped particle instability

During the second period of work at the Kurchatov Institute, Boris Borisovich Kadomtsev devoted much attention to the nature of plasma turbulence in tokamaks and the closely related problem of anomalous particle and energy transport across a magnetic field. According to Oleg Pavlovich Pogutse, a disciple and the then closest associate of Boris Borisovich, Kadomtsev thought of something as simple as flute instability but unique to tokamaks. In the long run, Kadomtsev arrived at the notion of trapped particle instability [2, 25], the nature of which can be described as follows.

In tokamaks and some other toroidal systems with nested magnetic surfaces created by lines of force with ergodic winding, flute instability of a low-pressure (compared with magnetic field pressure) plasma is stabilized by magnetic shear, i.e., the intersection of magnetic lines of force at adjacent magnetic surfaces. Physically, such a stabilization is achieved by efficient redistribution of local perturbation of electrostatic potential over the entire magnetic surface under the effect of rapid (with thermal

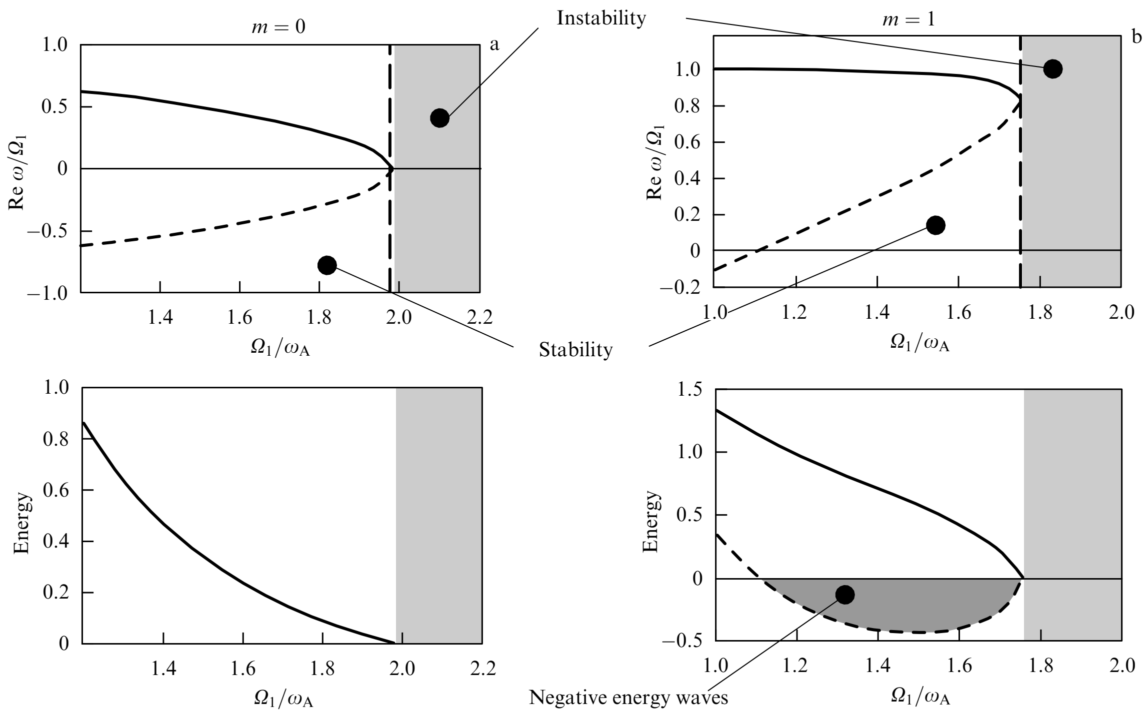

Figure 3. Energy (arbitrary units) and frequency of the most unstable eigenmodes: (a) axially symmetric with $m = 0$, and (b) asymmetric with $m = 1$. The instability region is shaded [20].

speed) charge flow along magnetic lines of force. As a result, a magnetic surface becomes an equipotential that hinders percolation of plasma flutes arrayed in the poloidal direction along the radius, as in open traps. However, the concept of free flow of charges over magnetic surfaces during their motion along magnetic lines of force is not quite correct. Figure 4 depicts projections of typical trajectories of charged particles in the tokamak magnetic field onto its poloidal (left) and toroidal (right) cross sections (for definiteness, Fig. 4 displays a situation in which directions of toroidal current and toroidal magnetic field coincide; the trajectories of positively charged particles are only presented). Figure 4a shows the so-called transit particles whose trajectories enclose both magnetic and geometric axes of the tokamak and only slightly deflect from the respective magnetic surface. The trajectory thickness in the figure is given by the diameter of the particle's Larmor orbit. Figure 4b presents particle trajectories having a small cosine of the pitch angle, i.e., angle α between the directions of particle velocity and magnetic field. Such particles are highly sensitive to magnetic field nonuniformities along the trajectory and may be trapped between magnetic mirrors formed at the magnetic surface due to nonuniformity of the toroidal magnetic field ($B_T \sim 1/r$, where r is the distance to the tokamak axis). Poloidal projections of trapped particle trajectories are sometimes called 'banana' orbits for their shape. In other words, the trajectory of a trapped particle does not enclose the entire magnetic surface but spreads over a part of its area only. Therefore, it can be imagined for sufficiently low-frequency processes that such particle movements fail to ensure exact compensation for perturbation of the electric potential by longitudinal motion along the lines of force. The trapped particle simply cannot move under the action of perturbation, being confined between the magnetic mirrors. Certainly, the fraction of trapped particles is

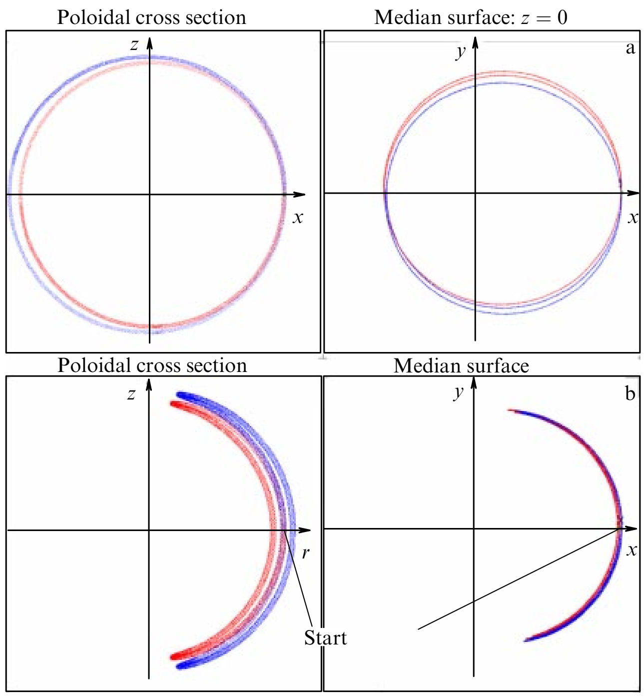

Figure 4. Typical trajectories of transit (a) and trapped (b) particles in a tokamak starting from the same point with opposite velocities. The dark color (blue in the on-line version) corresponds to $v > 0$, the light one to $v < 0$.

relatively small. The maximum mirror ratio on a magnetic surface of radius ρ is given by

$$\Pi = \frac{1 + \varepsilon}{1 - \varepsilon}, \tag{21}$$

and those particles whose pitch angle α satisfies the relation

$$\left|\cos\alpha\right| = \left|\frac{v_\parallel}{v}\right| \leqslant \sqrt{\frac{\Pi - 1}{\Pi}} \approx \sqrt{2\varepsilon}\,,$$

where $\varepsilon = \rho/R$, and R is the major radius of the tokamak (magnetic axis radius), prove to get trapped. Thus, the fraction of trapped particles (in the case of isotropic distribution in the phase space) $\sim \varepsilon \ll 1$, and charges produced due to them are to a large extent compensated by redistribution of transit particles. Owing to this effect, the increment of trapped particle instability is relatively small [2].

How then can plasma rotation affect this instability? Seemingly, toroidal rotation at the tokamak periphery may not appreciably influence the instability because it simply leads to cooperative displacement of particles (both trapped and transit) along the torus; the effect of poloidal rotation is not so obvious. Indeed, rotation of a magnetized plasma (i.e., collective motion of ions and electrons) is normally associated with the presence of a radial electric field, the electric drift being the sole type of drift motion whose velocity does not depend on the charge sign (the central region of the plasma column in a tokamak is usually negatively charged). As shown in Fig. 4b, a positively charged trapped particle starting parallel to the field direction deflects *inwardly* due to toroidal drift and acquires kinetic energy in the presence of a radial electric field. This excess energy may be sufficient for the particle to pass through a magnetic mirror and become a transit particle. For a particle with energy E, the mirror ratio in formula (21) should be effectively decreased by $1 + e\phi'\Delta_{\mathrm{b}}/E$ times, where $\phi(\rho)$ is the electric potential, and Δ_{b} is the half-width of the banana orbit. A particle starting from the same point in the opposite direction drifts outward from the original magnetic surface and, consequently, turns out to be trapped even more strongly. Electrons drift in opposite directions, but the charge sign in the above correction for the mirror ratio also changes. The situation at the center of the plasma column is more interesting due to the known asymmetry of the velocity space at certain $|\cos\alpha|$ values; namely, one of the two particles starting in opposite directions may prove to be transit, while the other trapped (Fig. 5). The former remains transit even if it loses speed when moving away from the center, while the latter is still trapped; only radii of their orbits decrease in the poloidal cross section. It should be borne in mind that particle trajectories in the core region barely follow the magnetic surfaces (see Fig. 5), whereas rotation diminishes this difference. Naturally, the effect will be opposite when $\phi'(\rho)$ has the opposite sign.

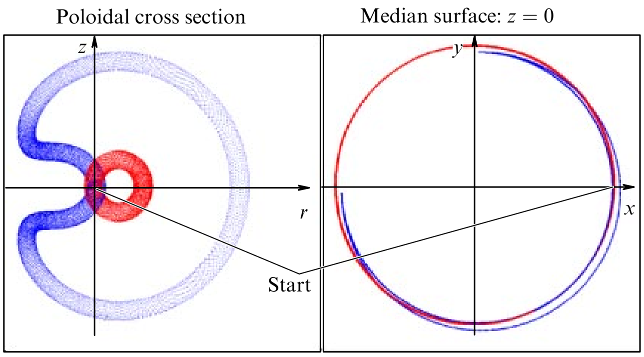

Figure 5. Asymmetry of the trajectories of particles starting from the same point with opposite velocities in the center of a tokamak.

Nevertheless, it can be concluded that trapped particle instability does not suffer variation to any great extent in a rotating plasma.

4. Reduced magnetohydrodynamic equations

Large tokamaks were extensively designed and built in different countries in the 1970s. The striking success of the tokamak T-10 at the Kurchatov Institute and the Princeton Large Torus (PLT) opened the gate to bigger tokamaks of the next generation, such as the T-15 in the USSR, the Tokamak Fusion Test Reactor (TFTR) and Doublet III in the USA, the Tore-Supra in France, the Joint European Torus (JET) in the UK, and the JT-60 in Japan. In those years, Kadomtsev formulated the concept of switching from physical research to thermonuclear engineering. He became interested in plasma self-organization, which needed nonlinear equations to be described. As is known, consideration of nonlinearity is equally important to address disruption instability, which is especially dangerous for tokamak plasma that first develops as a helical mode and thereafter leads to ejection of plasma and current channel onto the chamber wall. The physics of such instability was highlighted in the report by S V Mirnov at the present session (p. 725 of this issue). We shall focus here on the formalism invoked for the description of this instability.

A simplified (but adequate for the phenomenon under consideration) nonlinear model is needed because both MHD equations and drift equations are too complicated for comprehensive three-dimensional simulation, mainly by virtue of their multiscale nature. For example, MHD phenomena involve physical processes having totally different (by several orders of magnitude) spatial and temporal scales, including Alfvén, thermal, inertial, resistive and so forth. Direct numerical simulation of such complex phenomena is impracticable since small-scale errors accumulate into uncontrollable errors on large scales. Moreover, the power of even the best supercomputers is thus far insufficient for such calculations with the necessary accuracy within observable time. Therefore, Boris Borisovich decided to derive simplified (reduced) equations suitable for practical numerical simulation based on kink mode dynamics, including a nonlinear one.

The main objective of such a work was to derive equations describing the low-frequency nonlinear dynamics of tokamak plasma by canceling out higher-frequency stable magnetoacoustic oscillations from original MHD equations. In practical terms, this objective could be achieved by performing expansion in a small parameter characteristic of tokamaks (poloidal-to-toroidal magnetic field ratio $\epsilon = B_\perp/B_0 \ll 1$) and thereby moving from a three- to a two-dimensional problem. Somewhat later, the idea of reduced equations for tokamaks and stellarators was further developed in the works by such reputed researchers as M Rosenbluth, R Haseltine, and R White, and in many studies by H Strauss (see, for instance, Ref. [26]); this explains why the equations first derived by Kadomtsev and Pogutse [3] are not infrequently associated with the name of Strauss.

Of utmost importance was simplifying the description of the nonlinear dynamics of Alfvén perturbations by utilizing the freezing-in equation for an effective magnetic field defined by a single scalar flow function ψ:

$$\mathbf{B}_* = \mathbf{B}_\perp - \mu \frac{\rho}{R} B_0 \mathbf{e}_\theta = \nabla\zeta \times \nabla\psi \,, \tag{22}$$

where μ is the rotational transform angle in a tokamak with major radius R, and $\mathbf{e}_\theta = \rho\nabla\theta$ and $R\nabla\zeta$ are the unit vectors in the poloidal and toroidal directions, respectively. For $\epsilon \ll 1$, this freezing-in equation for the magnetic field reduces to the freezing-in equation for field B_*, automatically fulfilled for incompressible flows with $\mathbf{v} \approx \mathbf{v}_\perp$, $\operatorname{div}\mathbf{v}_\perp = 0$, with the frozen-in flux ψ: $\partial\psi/\partial t + \mathbf{v}\nabla\psi = 0$. Then, the Euler equation reduces to

$$\rho \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} + \nabla P = \frac{1}{4\pi} (\mathbf{B}_* \nabla)\mathbf{B}_* \,,$$

where

$$P = \frac{1}{8\pi} \left(2B_0 B'_\zeta + B_\perp^2 + 4\mu^2 \frac{\rho^2}{R^2} B_0^2 \right) + \frac{\mu B_0 \psi}{2\pi R^2}$$

plays the part of pressure. Thereby, the plasma motion problem is reduced to the problem of two-dimensional flow of an incompressible ideally conducting fluid with the frozen-in magnetic field $\mathbf{B}_*$. The reduced equations under discussion made it possible to simply and demonstrably simulate the evolution of so-called bubbles, disruptions, and other nonlinear phenomena in tokamaks. This reduction procedure proposed by Kadomtsev and Pogutse for an ideal single-fluid MHD model provided a basis for a new field of research on nonlinear dynamics of magnetized plasma. Its principles were later applied to simplify more complicated models, such as Braginskii's two-fluid dissipative equations employed for the description of peripheral plasma.

For all the advantages of this reduction procedure, it is not free from some drawbacks. It is easy to see that perturbations of $B_\parallel$ and $\operatorname{div}\mathbf{v}_\perp$ can be neglected only in the principal order of expansion in parameter ϵ. Therefore, the procedure lacks self-consistency, and the dynamics of the system violate the assumptions on which they were derived. Moreover, the reduced equations do not admit stationary states with flows due to broken relabeling symmetry intrinsic in original MHD equations (as mentioned in Section 2, it is relabeling symmetry that signifies the admissibility of stationary flows in the hydrodynamic system of interest). In order to overcome this drawback and generalize the Kadomtsev – Pogutse approach, the research group headed by V P Pastukhov in the Plasma Theory Division of Kurchatov Institute NFI undertook the development of the method for adiabatic separation of fast and slow motions, allowing ideal and weakly dissipative dynamic systems to be reduced using different small parameters [27]. A given method is essentially the generalization of the classical Van der Pol method to the case of continual Lagrangian systems.

The principle of the method is as follows. Let a weakly dissipative system have fast and stable collective degrees of freedom with characteristic frequencies $\sim \omega_{\mathrm{F}}$ and slow collective degrees of freedom with the frequencies $\sim \omega_{\mathrm{S}} \sim \epsilon\omega_{\mathrm{F}}$, where $\epsilon \ll 1$, as before (the putative smallness of system deviation from ideality is also related to the value of ϵ). Adiabatic transformation of generalized (flow) coordinates α^i in the form $\delta_{\mathrm{a}}\alpha^i = -\xi_{\mathrm{a}} \nabla\alpha^i$ is sought by analogy with relabeling symmetry transformation. This transformation does not change a Lagrangian with an accuracy up to terms of order ϵ^2:

$$\delta_{\mathrm{a}} \int_\Gamma L\big(\{\alpha^i\},\, \{\partial_t \alpha^i\},\, \{\nabla\alpha^i\},\, \epsilon\big)\, \mathrm{d}^3 r = O(\epsilon^2) \,.$$

The velocity field of slow (adiabatic) motion has the same functional structure and does not perturb fast degrees of freedom. Then, the reduced equation of motion is derived from Hamilton's principle of least action using ξ_{a} as a variable.

The simplest model of turbulent convection and transport is based on single-fluid magnetohydrodynamics with the adiabaticity parameter $\epsilon^3 \sim \chi/c_{\mathrm{s}} a \ll 1$ and adiabatic velocity field

$$\mathbf{v}_{\mathrm{a}} = \frac{\mathbf{B}_{\mathrm{p}} \times \nabla\Phi}{B_{\mathrm{p}}^2} \sim \epsilon c_{\mathrm{s}} \,.$$

Here, χ is the classical heat conductivity coefficient serving as a 'priming' dissipative process, c_{s} is the speed of sound in a plasma with transverse size a and poloidal magnetic field $\mathbf{B}_{\mathrm{p}}$, and Φ is the toroidal magnetic flux frozen-in to the plasma (for certain reasons, the discussion of which is beyond the scope of this report, the use of quantity Φ instead of poloidal flux ψ contained in formula (22) may be more favorable). The characteristic frequencies of the low-frequency convection under discussion, $\omega \sim \epsilon k_\perp c_{\mathrm{s}}$, are significantly lower than those of the following stable oscillation branches: magnetoacoustic with the frequency $\omega \sim k_\perp c_{\mathrm{A}}$, Alfvénean with $\omega \sim k_\parallel c_{\mathrm{A}}$, and longitudinal acoustic with $\omega \sim k_\parallel c_{\mathrm{s}}$. The reduction procedure formalized as expansion in the parameter ϵ of the action integral permits cutting off the above stable degrees of freedom and obtaining self-consistent equations for low-frequency convection of the plasma. In this scheme, the simplest expression for P present in Kadomtsev – Pogutse equations is replaced by the heat transfer equation written for the plasma entropy function, and the heat energy fluctuation equation taking into account all sources and sinks of energy in the system of interest (high-frequency heating, ohmic heating, viscous heat release, radiation losses, etc.) [27].

The reduced equations thus obtained make it possible to use an affordable personal computer for unique numerical calculations of the self-consistent nonlinear dynamics of a plasma system for time periods on the order of its lifetime. Notice that the most advanced gyrokinetic codes currently available, in which reduction has been performed to date for a single fast time (Larmor gyration period of charged particles), allow only a few dozen characteristic times of turbulence development to be computed. The results of these calculations demonstrate universal properties of fully developed plasma turbulence, which manifest themselves in experiments on tokamaks and other plasma confinement devices. These properties are as follows:

— wide frequency spectrum of observed oscillations with one or several dominant frequencies;

— intermittency and non-Gaussian statistics;

— nondiffusive character of transverse (with respect to magnetic field direction) transport of particles and energy;

— formation and presence of long-lived nonlinear structures ('filaments', 'blobs', 'streamers', etc.) in plasma;

— well-apparent trend toward self-organization of dynamic and transport processes (self-consistency of plasma

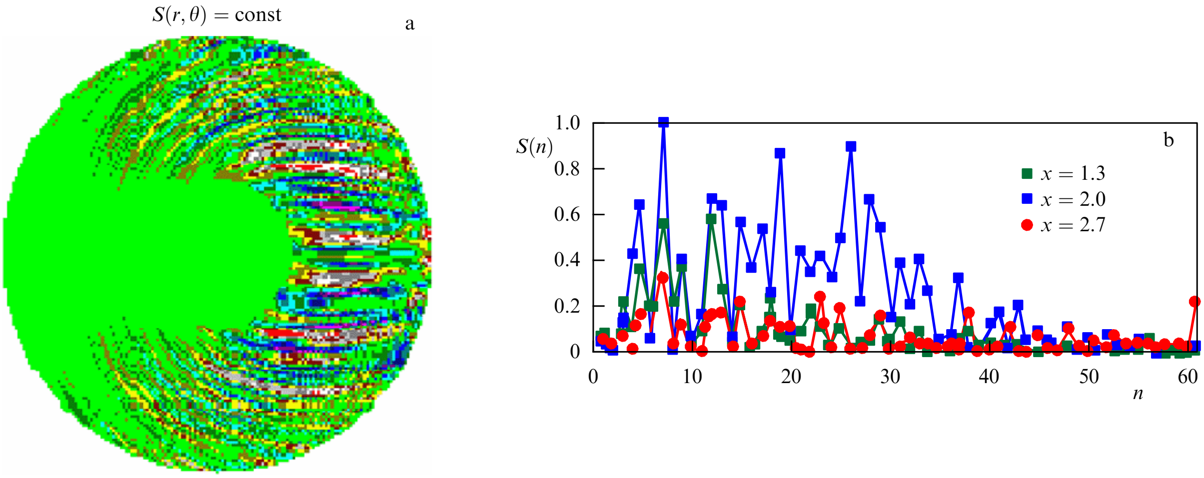

Figure 6. Cross section of isoentropic surfaces (a) and entropy fluctuation spectrum (n is the wave number) for the regime of fully developed MHD turbulence (b) [28].

parameter profiles, L–H transitions, 'transport barriers', etc.).

By way of illustration, Fig. 6 shows typical cross sections of isentropic surfaces and the spectrum of entropy function fluctuations. The wide fluctuation spectrum does not lead, however, to oscillations of an averaged entropy spatial profile or other plasma parameters that remain quasistationary. The essence of turbulent self-organization is that deviation from the established profile immediately leads to the enhancement of oscillations and transfers to compensate for such a deviation. A practical consequence of the above physical picture is the possibility of controling turbulent transport by means of spatial redistribution of the sources of particles and power introduced into the system [28].

In conclusion, I would like to emphasize once again that many problems, the importance of which B B Kadomtsev understood fairly well at the early stages of the development of the hot plasma theory (in particular, plasma turbulence and self-organization, mechanisms and methods of suppression of large-scale instabilities, physics of transport processes, and nonlinear dynamics), remain of utmost significance in the modern period of translating fusion research into practical reactor-scale thermonuclear facilities. Just as much credit is due to Kadomtsev for his remarkable physical intuition, foresight, and ability to see exactly what is needed at the moment and act accordingly. The principles and approaches to the solution of the aforementioned problems, formulated and developed by Kadomtsev, continue to be relevant and are being successfully developed by the present generation of theorists, his followers, as I tried to briefly illustrate in this report.

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