

METHODOLOGICAL NOTES

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Energy, linear momentum, and mass transfer by an electromagnetic wave in a negative-refraction medium

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Contents

Abstract. The problem of photon linear momentum in a refracting medium is discussed. It is shown that the relations $P = \hbar k$ and $\Delta M = E/c^2$ cannot hold simultaneously in a refracting medium and that in the particular case of a negative-refraction medium, light pressure is replaced by light attraction. It is also shown that the Abraham energy–momentum tensor is actually not a tensor because of its lack of relativistic invariance.

1. Introduction

The fact that the refractive index n can be negative raises the question of whether and how physics formulas valid for positive n can be applied to negative-n materials. It is easy to show that many commonly known formulas of electrodynamics and optics often produce gross errors when used straightforwardly for negative values of n [1, 2]. If $n < 0$ and hence the phase and group velocities are antiparallel, some other formulas (those that do not involve n directly) should also be used with extreme caution. One example is the well-known formula $P = \hbar k$ relating the photon momentum P (linear momentum in the present context) and the photon wave vector k. Clearly, for oppositely directed phase and group velocities, when the wave vector k is negative, this formula gives a negative value of the photon linear momentum, meaning that light absorbed or reflected by a negative-refraction medium should produce attraction instead of pressure [2 – 4]. This statement is of course strong enough to require solid foundation and a through analysis of its consequences, especially because, strange though this may seem, the value of the photon linear momentum is still sometimes the subject of discussion even for usual, positive-n materials. Studies on this topic abound, in Russia in particular [5 – 9]. A current bibliography on this subject is given, in particular, in Ref. [10] and in review paper [11].

The question of the direction and magnitude of the field linear momentum is closely related to the more general question of how energy, linear momentum, and mass are transferred as the field propagates in the material. In particular, it is necessary to determine the mass transfer from the emitter to the receiver in the situation where the space between the two is filled not with the vacuum but with the medium in which the phase (v_{ph}) and group (v_{gr}) velocities of radiation differ from the speed of light in the vacuum c. We then assume that, on the one hand, the medium has a certain amount of frequency dispersion (and hence $v_{\mathrm{ph}} \neq v_{\mathrm{gr}}$ in the general case) and, on the other hand, we are far from the absorption lines of the medium, meaning that the absorption is not too strong and the very concept of group velocity still has its usual meaning.

2. Light pulse propagation in the vacuum

It is known that if the space between the emitter and the receiver is filled with the vacuum, then the transfer between them of electromagnetic radiation with the energy E and the linear momentum

$$P = \frac{E}{c} \tag{1}$$

is accompanied by the transfer of the mass

$$\Delta M = \frac{E}{c^2}\,. \tag{2}$$

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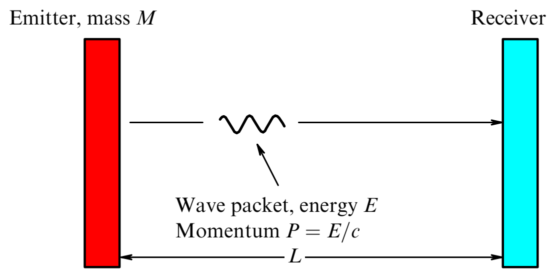

Figure 1. Transfer of a wave packet from an emitter to a receiver in the vacuum.

This relation readily follows by considering Fig. 1 along the lines of Ref. [12].

The emitter of mass M that emits a wave packet (or a photon or a light pulse) of energy E and linear momentum P receives a leftward recoil

$$v = \frac{P}{M} = \frac{E}{Mc} . \tag{3}$$

The wave packet reaches the receiver at the instant

$$t = \frac{L}{c} , \tag{4}$$

whereas the emitter travels in this time leftward over the distance

$$\Delta x = tv = \frac{LE}{Mc^2} . \tag{5}$$

The requirement that the center of inertia of the system as a whole not move leads to the relation

$$\Delta x M = \frac{LE}{c^2} . \tag{6}$$

This relation can be interpreted as meaning that as the energy E is transferred from the emitter to the receiver, the former loses and the latter acquires the mass ΔM equal to E/c^2 in accordance with Eqn (2). We note that the photon itself remains massless in this geometry [13].

3. On the concept of field linear momentum in matter

One more point to note in the above discussion is the quantity c that occurs twice in the denominator of Eqn (2). While this quantity is numerically equal to the speed of light, it is not entirely understood what role it plays in either of these two quantities (2). In one possibility, c^2 is simply a numerical factor introduced to make both sides of Eqn (2) dimensionally equal; in another, it may have a more definite physical meaning. The following analysis of the origin of these two c's in Eqn (2) readily shows that the latter is the case. It is clear that one of these quantities came to the denominator in the right-hand side of Eqn (2) from relation (1), whereas the other came from Eqn (4). On the other hand, it is also clear that the factor c in Eqn (1) has the meaning of the phase velocity of light c_{ph}, whereas the same factor in Eqn (4) has the meaning of the group velocity c_{gr}. That both factors are numerically equal to the speed of light in the vacuum does not change their physical meaning, nor does it change the fact that each of them has a meaning of its own. Equation (2) can then be rewritten in the somewhat different form

$$\Delta M = \frac{E}{c_{\text{ph}} c_{\text{gr}}} . \tag{7}$$

A quite reasonable question that arises from this equation is: How much mass is transferred from the emitter to the receiver if (in the configuration of Fig. 1) all the space between them is filled with a material whose phase velocity v_{ph} and group velocity v_{gr} are different from c? Does the modification of Eqn (7) to

$$\Delta M = \frac{E}{v_{\text{ph}} v_{\text{gr}}} \tag{8}$$

hold in this case (see Refs [14, 15])?

Perhaps the even more interesting question to ask is: Does Eqn (8) hold if the space between the emitter and the receiver is filled with a negative-refraction material? In that case, the phase and group velocity vectors are antiparallel, making mass transfer negative. This is equivalent to saying, paradoxically, that an emitter in a negatively refracting medium does not lose, but rather gains, mass and that the radiation receiver is subject to attraction rather than pressure from light [14–16].

To clarify these statements, we must first define the concept of the linear momentum of an electromagnetic field in a medium. It is commonly pointed out when discussing this question [7, 8] that the linear momentum of a field propagating in a medium cannot be distinguished (or, at least is difficult to distinguish) from that of the medium itself. Indeed, if an electromagnetic field is present in a certain volume of the medium (and is defined by the field strength and induction vectors), particles in the medium undergo some kind of motion that can be described in terms of the displacements of and tension between the particles. We are primarily interested in the field-period-averaged quadratic functions of these quantities, because these functions determine the permanent mechanical (ponderomotive) forces acting on the medium. Clearly, in this setting, we actually treat the electromagnetic field as an ensemble of some kind of quasiparticles, which can be called quasiphotons. Nevertheless, asking how much linear momentum the flux of such particles transfers — or, equivalently, what the field linear momentum in the medium is — is quite legitimate, as is asking about the size of the emitter-to-receiver mass transfer in the presence of the flow of such quasiparticles. (We emphasize once again that the mass transferred from the emitter to the receiver need not necessarily be equal to the mass of the quasiphoton [13].)

Given the ambiguity in splitting the field linear momentum in a medium into contributions from the field proper and the medium, we rely on Fig. 2 to define the field linear momentum in the medium. In this figure, light enters from the vacuum on the left, travels through a transparent nonabsorbing body, and is absorbed in an absorber that is located on the right and matched with the transparent body, such that the light is not reflected as it enters the absorber. Nor does the light entering the transparent body at the matched left face undergo reflection. The left face is fixed, as the two vertical arrows indicate. The energy flux density of the light entering the transparent body is equal to the Poynting vector S and remains the same within it. The linear momentum flux density of light is $P_0 = S/c$ in the vacuum and

$$nP_0 = \frac{nS}{c} = \frac{S}{v_{\text{ph}}}$$

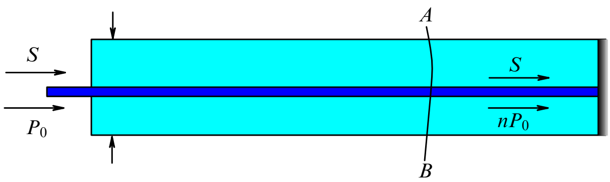

Figure 2. Schematic of light propagating in a transparent body and then absorbed in a matched absorber.

inside the body. Here, n is the refractive index of the medium. The fact that the linear momentum flux density in the vacuum differs from that in the medium results in the mechanical force density

$$F_1 = (1 - n)\,P_0 \tag{9}$$

on the input face of the transparent body. But in the geometry that we use (the left face is fixed!), this force leaves the transparent body unmoved as a whole.

The boundary between the body and the absorber is also acted upon by the mechanical (ponderomotive) force whose magnitude is equal to the linear momentum flux density in the transparent body, i.e.,

$$F_2 = nP_0\,. \tag{10}$$

Due to this force, a corresponding stress develops in an arbitrary cross section $A-B$ of the body; very importantly, it changes sign with the sign of n. If n is positive, the boundary between the transparent body and the absorber is subject to light pressure and the cross section $A-B$, correspondingly, to tension. For a negative n, light pressure is replaced by light attraction [3, 14–16], and the cross section $A-B$ turns out to be under pressure.

The important point is that for any value of n, the sum F_0 of the forces acting on both faces of the transparent body is determined only by the linear momentum P_0 transferred by the incident beam, and is equal to

$$F_0 = F_1 + F_2 = P_0\,. \tag{11}$$

The force F_0 is balanced by the reaction force of the fixing device on the left face of the transparent body.

4. Ponderomotive force at the emitter–medium interface

We return to relation (8), which is thus far a certain assumption that extends the results in Ref. [12] to the case where all the space between the emitter and the receiver is filled by a material whose phase velocity $v_{\rm ph}$ and group velocity $v_{\rm gr}$ are both different from c (see Fig. 3). In discussing the forces F_1 and F_2, the uncertainty arises as to how to determine the points (or more precisely, planes) of their application. It is not clear, for example, whether F_1 is applied to the emitter or the transparent body. This uncertainty makes it difficult to determine the displacements that occur in the system during the passage of light. Changing the system geometry to that in Fig. 4 helps elucidate this question. A new feature in Fig. 4 is that it shows the vacuum gaps separating the transparent body from the emitter and from the receiver. The width of the gaps is much less than the length of the transparent body but much larger than the light

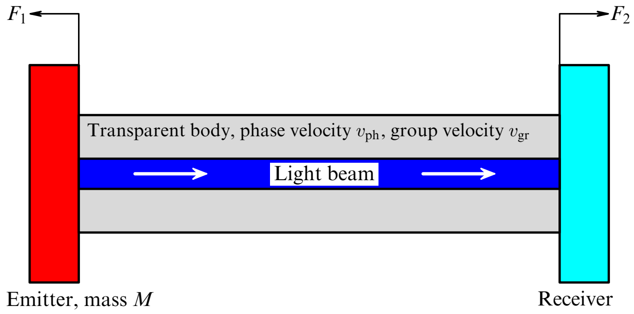

Figure 3. Linear momentum and mass transfer from emitter to receiver in the case where the space between them is filled with material with phase and group velocities different from c.

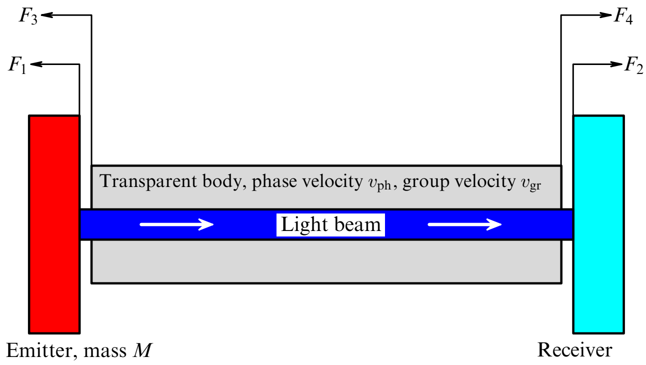

Figure 4. Light passage in the presence of gaps at the boundary of the transparent body. Forces F_1-F_4 correspond to the case of continuous radiation from the emitter to the receiver.

wavelength. In this situation, for example, the force between the emitter and the transparent body can be decomposed into two components, F_1 between the emitter and the vacuum, and F_3 between the vacuum and the transparent body. Clearly, the total force between the emitter and the transparent body is now the sum of F_1 and F_3 and is equal to the corresponding total force in Fig. 3 (also denoted by F_1).

The use of the geometry in Fig. 4 avoids the need to consider the forces acting in the near-interface region at the junction between the emitter (receiver) and the transparent body. We consider a short-duration wave packet of energy E traveling from the emitter to the receiver in this geometry. When leaving the emitter, the wave packet imparts the linear momentum $P = E/c$ to the emitter, causing it to move to the left at the speed $v = E/Mc$. From Eqn (2), this means that the emitter loses the mass E/c^2. When the wave packet reaches the front face of the transparent material, the linear momentum $P = E/c$ and the mass E/c^2 are transferred to the material. After crossing the front face of the transparent body, the energy of the wave packet remains equal to E, and the linear momentum is given by

$$P_1 = \frac{E}{v_{\rm ph}} = \frac{En}{c}\,, \tag{12}$$

where n is the refractive index of the transparent material. We thus see that the wave packet that entered the transparent material with the linear momentum $P = E/c$ has the linear momentum $P_1 = En/c$ in that material, in accordance with Eqn (12). Also, by the linear momentum conservation law, the transparent material gains the linear momentum

$$P_2 = P_1 - P = (n-1)\,P \tag{13}$$

and starts moving to the left (if $n > 1$) at the speed

$$v = \frac{P_2}{M_1} = \frac{(n-1)\,P}{M_1}\,, \tag{14}$$

where M_1 is the mass of the transparent material.

When the wave packet crosses the back face of the transparent material, the process occurs in reverse: the transparent material stops, the receiver starts moving to the right after gaining the linear momentum $P = E/c$ and mass E/c^2, and the center of inertia of the system as a whole comes to a halt.

How far did the center of inertia move during the packet's travel time from the emitter to the receiver? This travel time is readily shown to be

$$t = \frac{L}{v_{\mathrm{gr}}}\,, \tag{15}$$

where L is the distance from the emitter to the receiver and v_{gr} is the group velocity. The length of the transparent body is assumed to be equal to L.

The distance the emitter travels in time t is

$$\Delta x_1 = \frac{tP}{M} = \frac{EL}{Mcv_{\mathrm{gr}}}\,. \tag{16}$$

In the same time, the transparent body travels the distance

$$\Delta x_2 = \frac{EL(n-1)}{M_1 c v_{\mathrm{gr}}}\,. \tag{17}$$

The requirement that the center of inertia of the entire system be at rest is given by

$$M\Delta x_1 + M_1 \Delta x_2 = L\Delta m\,, \tag{18}$$

where Δm is the mass the emitter loses and the receiver gains. From Eqns (16)–(18) (and in full agreement with Eqn (8)), this mass can be expressed as $\Delta m = E/v_{\mathrm{ph}} v_{\mathrm{gr}}$. The term 'emitter' refers in this case to the emitter as such, which loses the energy E, linear momentum P, and mass E/c^2 when emitting a wave packet; and the front face wall of the transparent material is a face to which these energy, linear momentum, and mass are transferred by the wave packet. We note, however, that the packet then carries the energy E, the linear momentum $(n-1)\,P$, and the mass $E(1/v_{\mathrm{ph}} v_{\mathrm{gr}} - 1/c^2)$ from the wall, and hence the loss of mass is $\Delta m = E/v_{\mathrm{ph}} v_{\mathrm{gr}}$, in full agreement with Eqn (8). In the neighborhood of the absorber, things go in reverse.

5. Energy–momentum relation for a field in a medium

5.1 The Minkowski and Abraham energy–momentum tensors

Equation (8) sharply contradicts the familiar relation (2). The reason for this is the energy–momentum relation

$$P = \hbar k = \frac{E}{v_{\mathrm{ph}}} = \frac{En}{c} \tag{19}$$

which we used for our quasiparticle and which, importantly, involves the phase velocity v_{ph} rather than simply the speed of light c.

We can now formulate the problem in a somewhat different way by asking the following: What form should the energy–momentum relation for a quasiphoton take in a medium in order that, as the quasiphoton passes the medium, the mass transferred from the emitter to the receiver be exactly E/c^2? It can be shown following Ref. [6] that the mass transfer is given by E/c^2 if the energy and linear momentum of the quasiphoton are related by

$$P = \frac{E}{cn}\,. \tag{20}$$

With the frequency dispersion neglected, this relation is identical to the well-known relativistic formula [Ref. [17], Eqn (9.8)]

$$P = \frac{Ev}{c^2}\,. \tag{21}$$

The inconsistency between Eqns (19) and (20) fundamentally stems from the following inconsistency inherent to the very concept of the particle–wave dualism. If a photon (or a quasiphoton) is considered a particle, then its linear momentum can to a certain approximation be written as $P = Mv$ and is therefore proportional to the velocity. If it is considered a wave, its linear momentum is $P = \hbar k = \hbar\omega/v$, that is, inversely proportional to the velocity. Because of this difference, a wave and a particle have different energy–momentum relations [see Eqns (19) and (20)]. This is of little or no importance for photons propagating in the vacuum but leads to problems when considering a photon (quasiphoton) in a medium.

We note that the 'nonstandard' expression $\Delta M = E/v_{\mathrm{ph}} v_{\mathrm{gr}}$ is found in some papers published even before our papers [14, 15], where some bibliography is provided. Later, this expression was also obtained in Ref. [16], neglecting frequency dispersion. Choosing between Eqns (19) and (20) for the photon energy–momentum relation is a century-old problem, dating back to the works of Minkowski [18] and Abraham [19], each of whom suggested a form of his own for the energy–momentum tensor of the electromagnetic field.

The energy–momentum tensor T_{ik} is important in that it allows expressing the four-dimensional ponderomotive force f_i acting on an electromagnetic medium. The tensor T_{ik} and the force f_i are related by

$$f_i = \frac{\partial T_{ik}}{\partial x_k}\,. \tag{22}$$

We note that Minkowski's and Abraham's forms of the tensor can be written in combined form as

$$T_{ik} = \begin{bmatrix} \theta_{\alpha\beta} & \mathbf{g}c \\ \mathbf{S}/c & W \end{bmatrix}, \tag{23}$$

where the quantities $\theta_{\alpha\beta}$ are the spatial components of the tensor, with $\alpha, \beta = x, y, z$; $\mathbf{g}$ is the field linear momentum density; $\mathbf{S}$ is the Poynting vector (energy flux density); and W is the field energy density. In the Appendix, the concrete forms of all of these quantities (taken from Ref. [8]) are provided for either tensor form.

As can be seen from the Appendix, the only difference between the tensors — and exactly the one that matches the difference between Eqns (19) and (20) — is in the magnitude of the linear momentum density **g**.

Obviously, calculating the ponderomotive forces from Eqn (22) yields different results depending on which tensor is taken to be T_{ik}. Whereas the Minkowski tensor causes no problems when using Eqn (22), determining the forces f_i by means of the Abraham tensor turns out to require [8] that Eqn (22) be modified by introducing the so-called 'Abraham force'

$$f_i^{\mathrm{A}} = \frac{n^2 - 1}{4\pi c} \frac{\partial}{\partial t} [\mathbf{EH}]_i ,$$

such that Eqn (22) becomes

$$f_i = \frac{\partial T_{ik}}{\partial x_k} + f_i^{\mathrm{A}} = \frac{\partial T_{ik}}{\partial x_k} + \frac{n^2 - 1}{4\pi c} \frac{\partial}{\partial t} [\mathbf{EH}]_i . \tag{24}$$

This expression, with T_{ik} taken to be the Abraham from tensor, yields the same f_i as follows from Eqn (2) with the Minkowski tensor. Another reason why the Abraham tensor cannot be directly used in Eqn (22) is that it is not relativistically invariant, as direct calculation shows.

There is one more problem that arises when using Eqn (22) to calculate the forces the field exerts on a frequency-dispersive medium, in particular, if the medium has both its dielectric constant ε and magnetic permittivity μ negative. In this case, the energy density should be written as

$$W = \frac{\partial(\omega\varepsilon)}{\partial\omega} E^2 + \frac{\partial(\omega\mu)}{\partial\omega} H^2 , \tag{25}$$

instead of the simpler expression $W = \varepsilon E^2 + \mu H^2$. Equation (25) corresponds to the component T_{44} of the energy–momentum tensor (in any form). Usually, however, no question arises as to whether all the other components of the energy–momentum tensor should also be modified in some way in the presence of dispersion.

5.2 Energy–momentum tensor in Rytov and Polevoi forms

In this connection, we note the Rytov–Polevoi modification of the energy–momentum tensor [20]. Rytov and Polevoi showed that the energy–momentum tensor can be greatly simplified by introducing the four-dimensional group velocity

$$U_k = \left(\frac{\mathbf{u}}{\sqrt{1 - u^2/c^2}}, \frac{c}{\sqrt{1 - u^2/c^2}} \right) \tag{26}$$

and the four-dimensional wave vector

$$K_i = \left(\mathbf{k}, \frac{\omega}{c} \right) . \tag{27}$$

Expressed in terms of these quantities, the components of the energy–momentum tensor take the very compact form

$$T_{ik} = \frac{W}{\omega} \sqrt{1 - \frac{u^2}{c^2}}\, K_i U_k . \tag{28}$$

We note that the energy density is $W = T_{44}$, the Poynting vector is $S_\alpha = T_{4\alpha}$, the linear momentum density is $g_\alpha = 1/cT_{\alpha 4}$, $\theta_{\alpha\beta} = T_{\alpha\beta}$ and u is the three-dimensional group velocity. Equation (28) is valid in the presence of frequency dispersion, whatever the sign of n. Substituting the energy density from Eqn (25) in Eqn (28) automatically introduces necessary changes into all the components of the energy–momentum tensor.

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6. Appendix

The components of the energy–momentum tensors according to Ref. [8]

The Minkowski and the Abraham tensors have the common form given in (23).

The components of the Minkowski tensor are

$$\theta_{\alpha\beta} = \frac{1}{4\pi} (E_\alpha D_\beta + H_\alpha B_\beta) - \frac{1}{8\pi} \delta_{\alpha\beta}(\mathbf{ED} + \mathbf{HB}) , \tag{A.1}$$

$$\mathbf{S} = \frac{c}{4\pi} [\mathbf{EH}], \; \mathbf{g} = \frac{1}{4\pi c} [\mathbf{DB}], \; W = \frac{1}{8\pi} (\mathbf{ED} + \mathbf{HB}). \tag{A.2}$$

The components of the Abraham tensor are

$$\theta_{\alpha\beta} = \frac{1}{4\pi} (E_\alpha D_\beta + H_\alpha B_\beta) - \frac{1}{8\pi} \delta_{\alpha\beta}(\mathbf{ED} + \mathbf{HB}) , \tag{A.3}$$

$$\mathbf{S} = \frac{c}{4\pi} [\mathbf{EH}], \; \mathbf{g} = \frac{1}{4\pi c} [\mathbf{EH}], \; W = \frac{1}{8\pi} (\mathbf{ED} + \mathbf{HB}). \tag{A.4}$$

Here, **E** and **H** are the electric and magnetic field strengths, and **D** and **B** are their respective inductions.

Note added to the English proofs

After the publication date of the Russian original of this paper, at the PIERS-2009 Conference (Moscow, Russia, August 18–21), the talk ''Negative radiation-pressure response of a left-handed plasmonic metamaterial'' was presented by H Lezec and K Chau (NIST, USA). The authors interpreted their results as an experimental evidence of the validity of the Minkowski tensor. At the same time, other scientists have argued against the Minkowski tensor and in favor of the Abraham tensor; these arguments were not considered in our paper. Given the evident scientific interest in this problem, the author hopes to discuss it in the future publications.

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