

ORAL ISSUE OF THE JOURNAL "USPEKHI FIZICHESKIKH NAUK"

PACS numbers: **34.80.–i**, **51.50.+v**, **52.20.–j**, **52.80.–s**

Modeling of gas discharge plasma

B M Smirnov

DOI: 10.3367/UFNe.0179.200906e.0591

Contents

Abstract. The condition for the self-maintenance of a gas discharge plasma (GDP) is derived from its ionization balance expressed in the Townsend form and may be used as a definition of a gas discharge plasma in its simplest form. The simple example of a gas discharge plasma in the positive column of a cylindrical discharge tube allows demonstrating a wide variety of possible GDP regimes, revealing a contradiction between simple models used to explain gas discharge regimes and the large number of real processes responsible for the self-maintenance of GDP. The variety of GDP processes also results in a stepwise change of plasma parameters and developing some instabilities as the voltage or discharge current is varied. As a consequence, new forms and new applications of gas discharge arise as technology progresses.

1. Introduction

Gas discharge plasma is a widespread type of a plasma with the largest number of its applications. The practice of gas discharge and its basic forms were worked out in the 19th century, whereas the understanding of gas discharge as a self-maintaining phenomenon providing passage of an electric current through a gas under the action of an electric field was reached in the beginning of the 20th century [1, 2] and was presented in the simplest form in books by Townsend [3–5]. The understanding of the nature of gas discharge gave the impetus for the development of a new atomic physics and the creation of nuclear physics, where the gas discharge techniques became the basis of experimental methods for these new areas of physics (see, for example, Ref. [6]). The understanding of the nature of gas discharge brought about more detailed research on a gas discharge plasma, which in turn supported new applications of gas discharge. The experience acquired in developing gas discharge techniques showed that the number of gas discharge constructions and regimes of gas discharge is inexhaustible. Therefore, even now new types and new regimes of known forms of gas discharge are being created for new applications.

Leaving aside a variety of constructions of gas discharge as a self-maintaining phenomenon, one can state that a variety of regimes of gas discharge is determined by a large number of processes which govern the state of a gas discharge plasma. We will consider below from this standpoint the character of the processes that provide the ionization balance of a simplest gas discharge plasma. The variety of processes leads to the absence of a universal model that allows one within the framework of a general scheme to determine the numerical parameters of a gas discharge plasma under given conditions and construction of gas discharge. This variety also results in a stepwise change of plasma parameters and developing instabilities as the discharge current or voltage varies. In addition, this creates nonstationarities and nonuniformities of the gas discharge plasma.

B M Smirnov Institute for High Temperatures, Russian Academy of Sciences,
ul. Izhorskaya 13/19, 127412 Moscow, Russian Federation
Tel./Fax (7-499) 190 42 44
E-mail: bmsmirnov@gmail.com

Received 4 December 2008
Uspekhi Fizicheskikh Nauk **179** (6) 591 – 604 (2009)
DOI: 10.3367/UFNr.0179.200906e.0591
Translated by B M Smirnov; edited by A Radzig

2. Condition of self-maintenance of gas discharge

Gas discharge is a self-sustaining system, so that electrons leaving a gas discharge plasma must be reproduced in it due to some ionization processes. The ionization balance provides a

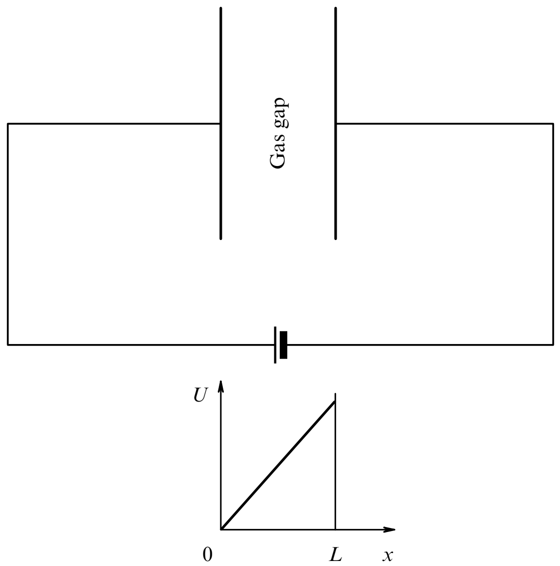

Figure 1. The simplest scheme for maintenance of the gas ionization state in a gas gap between two infinite parallel electrodes, and the spatial distribution of the electric potential in this gap in the limit of a low number density of charged particles.

plasma self-maintenance for gas discharge of any type. Below, following Townsend, we consider the simplest example of gas discharge self-maintenance, which is given in Fig. 1. In this case, secondary electrons are formed at the cathode as a result of its bombardment by ions, and then secondary electrons are multiplied in ionization collisions with atoms. This character of self-maintenance of gas discharge is governed by two parameters: the first Townsend coefficient α, so that $1/\alpha$ is the electron mean free path in a gas with respect to the atom ionization taking place in the course of electron drift, and the second Townsend coefficient γ, the probability of formation of a secondary electron as a result of cathode bombardment by an ion.

Let us derive the condition of self-maintenance of gas discharge in the simplest case given in Fig. 1, when gas discharge plasma is kept up in a gas gap between two parallel infinite electrodes, and the particle's number density of the gas discharge plasma is small, so that it does not disturb the voltage distribution inside the gas gap (see Fig. 1). Under this assumption about the voltage uniformity, the number of electrons formed from a test secondary electron after moving a distance L is equal to $\exp(\alpha L) - 1$. From this we obtain the Townsend condition for self-maintenance of gas discharge on the basis of the balance of ionization processes and processes involving the loss of electrons and ions:

$$\gamma\left[\exp(\alpha L) - 1\right] = 1 \,. \tag{2.1}$$

This case gives the simplest example of the ionization balance in a plasma. As a matter of fact, the Townsend condition (2.1) within the framework of the processes used sets the minimum voltage between electrodes at a given distance between them, which corresponds to the minimum of the Paschen curve that determines the dependence of the breakdown voltage on the product of the gas pressure p inside the gas gap and the distance L between electrodes. An increase or decrease in the parameter pL with respect to its optimal value leads to increasing breakdown voltage that, in accordance with Fig. 1, occurs in the absence of a gas discharge plasma inside a gas gap.

The ionization balance is of fundamental importance for gas discharge, as was analyzed in Townsend's books [3–5]. Townsend's contribution to the physics of gas discharge is not restricted to the analysis of conditions of gas discharge self-maintenance. Townsend elaborated methods of measurements and carried out measurements of gas discharge plasma parameters, such as the drift velocity of electrons in an external electric field and the diffusion coefficient of electrons in gases (see, for example, Refs [7–11]), and his measurements are still relevant. Incidentally, it was Townsend who first established the relation between zero-field mobility and the diffusion coefficient of electrons in gases [12–15]. Subsequently, it was used by Einstein in the analysis of Brownian motion [16, 17] and received the name the Einstein relation.

The self-maintenance condition of a gas discharge plasma is fundamental to the physics of gas discharge, and equation (2.1) provides the simplest example of this condition. But in reality there are a large number of types and regimes of gas discharge, and the condition of the discharge self-maintenance may have a different form depending on the processes which are responsible for the ionization equilibrium. The simplest mechanisms offering the self-maintenance of gas discharge are presented in contemporary gas-discharge books, and this may mislead those who are encountering gas discharge for the first time. This problem is absent for professionals of gas discharge because they deal with certain regimes of self-maintenance of gas discharge and are restricted by certain processes related to their particular case. The peculiarities of various schemes of self-maintenance of a gas discharge plasma and the description of various regimes of gas discharge are contained in gas discharge books, in particular, in books [18–37], but the number of gas discharge regimes under consideration is less than what exists in reality. Therefore, in analyzing the gas discharge physics, we encounter a contradiction when the explanation of certain regimes of a gas discharge plasma is based on the simplest models and schemes of processes, while additional processes may influence the self-maintenance condition of a gas discharge plasma. This contradiction has existed for decades, and below we demonstrate it on the basis of simple examples.

Condition (2.1) for the balance of processes of atom ionization and loss of electrons and ions relates to a uniform plasma. But even at a low number density of electrons and ions, the spatial distribution of the electric field potential inside a gas gap differs from that presented in Fig. 1. Figure 2 exhibits the redistribution of this electric voltage due to screening of an external electric field by electrons and ions located in a gas gap. Say, if the Debye–Hückel radius r_D [38], which is responsible for the screening of an external field in the statical regime, is equal to $r_D = 0.8$ cm at room temperature of electrons and ions, this corresponds to the number density of electrons and ions $N_e = 10^4\ \mathrm{cm}^{-3}$, i.e., even this number density of charged particles provides the screening of an external electric field at a gap width $L \sim 1$ cm. Note that the number density of electrons and ions in glow discharge ranges from $N_e \sim 10^{10}$ to $10^{12}\ \mathrm{cm}^{-3}$, and the Debye–Hückel radius r_D is less than the above value by three to four orders of magnitude — that is, a gas discharge plasma creates a nonuniform electric field in gas discharge which is maintained by an external electric field.

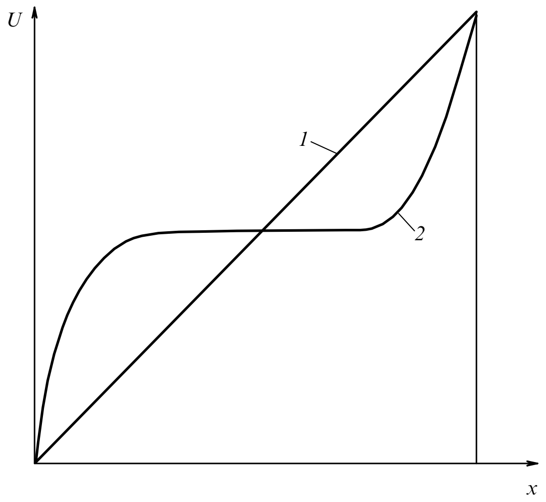

Figure 2. The spatial distribution of the electric potential in a gas gap, if the charge of electrons and ions is relatively small and does not influence the voltage distribution (*1*), and the redistribution of electrons and ions under the action of an external electric field leads to screening of this field (*2*).

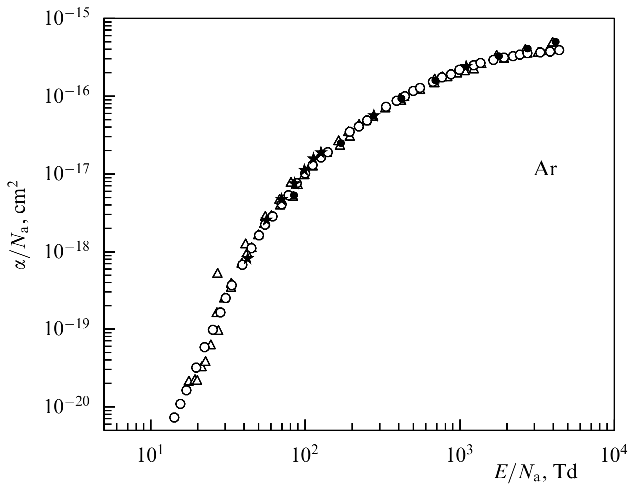

Figure 3. The reduced first Townsend coefficient as a function of the reduced electric field strength [70].

Along with the statical redistribution of electrons and ions in a gas discharge plasma as a result of the screening of an external electric field, the dynamic charge redistribution is of importance due to the passage of electrons and ions to the electrodes and walls of the discharge chamber. Let us analyze the self-maintenance condition of gas discharge for the case of Fig. 1, assuming the electric field inside the gas gap to be nonuniform. Figure 3 gives the dependence of the reduced first Townsend coefficient α/N_a in argon on the reduced electric field strength E/N_a, where N_a is the number density of argon atoms (the unit of the reduced electric field strength is 1 Td (Townsend) $= 10^{-17}$ V cm^2). As is seen, the first Townsend coefficient decreases sharply with decreasing electric field strength. This leads to the conclusion that it is more effective to provide self-maintenance of a gas discharge plasma by maintaining a higher electric field strength in a small region of gas discharge and an almost zero field in other regions of a gas gap (Fig. 4).

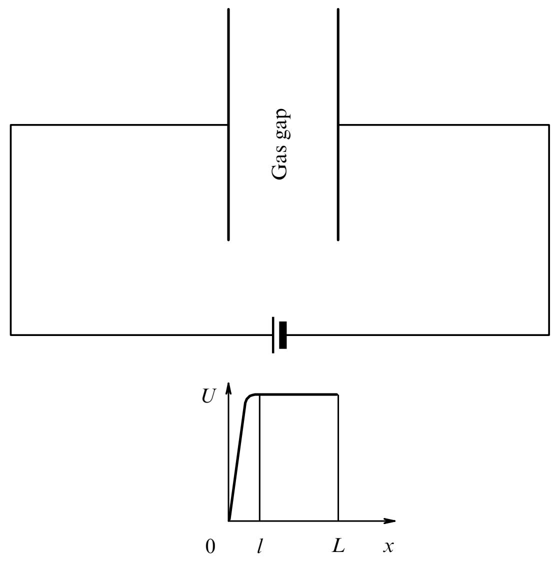

Figure 4. The spatial distribution of the electric field strength in a gas gap between two infinite parallel electrodes, when a voltage is applied to them.

Let us rewrite the self-maintenance condition of a nonuniform GDP (2.1) in the form

$$\int_0^l \alpha \, dx = \ln\left(1 + \frac{1}{\gamma}\right). \tag{2.2}$$

We denoted by l a dimension of the region that provides self-maintenance of a gas discharge plasma (evidently, $l < L$, where L is a width of a gas gap). Let us construct the region of formation of charged particles, the process that compensates the departure of ions to the cathode, such that the voltage applied to electrodes is minimal. As a matter of fact, we construct in this manner the cathode region of glow discharge, where reproduction of charged particles in a gas discharge is connected with the production of the secondary electrons as a result of cathode bombardment by ions. Then we find that the optimal voltage between electrodes depends weakly on the charge distribution in the cathode region and is equal to approximately 130 V for argon. The maximum electric field strength is then attained near the cathode, and the maximum reduced electric field strength in the case of argon is $E_c/N_a = 1300$ Td, while the reduced size of the cathode region is equal to $lN_a = 1.4 \times 10^{16}$ cm^{-2}. In particular, at the argon pressure $p = 1$ Torr and temperature $T = 300$ K, we have $E_c = 400$ V cm^{-1} for the electric field strength near the cathode and $l = 0.45$ cm for the cathode region size.

Thus, gas discharge in a gas gap between two parallel infinite electrodes is divided into two regions (see Fig. 4), so that the cathode region is responsible for a self-maintenance of GDP, and in the positive column, the other region of gas discharge, the electric field strength is practically zero — that is, a weak electric field maintains a discharge electric current. If a gas is located in a cylindrical tube, the electric field strength in this region is nonzero, because charged particles go to the walls, and new charged particles are formed under the action of an electric field in order to compensate for losses due to the attachment of charged particles to the walls. A schematic spatial distribution of the electric field is depicted in Fig. 5 for a cylindrical discharge tube, and Fig. 6 displays the

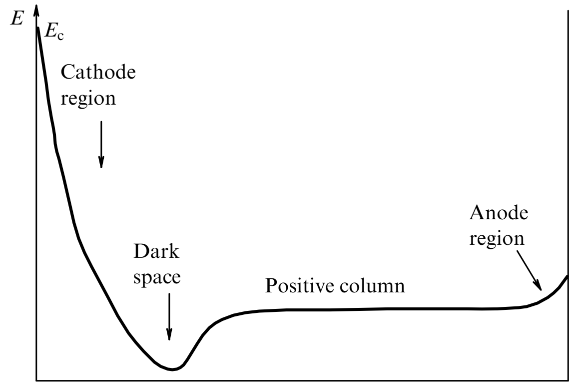

Figure 5. The distribution of the electric field strength along a cylindrical discharge tube for glow discharge; E_c is the electric field strength near the cathode.

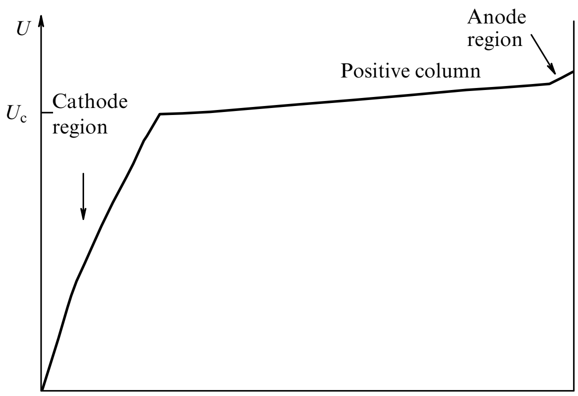

Figure 6. The spatial distribution of the electric field potential along a cylindrical discharge tube for glow discharge; U_c is the cathode drop.

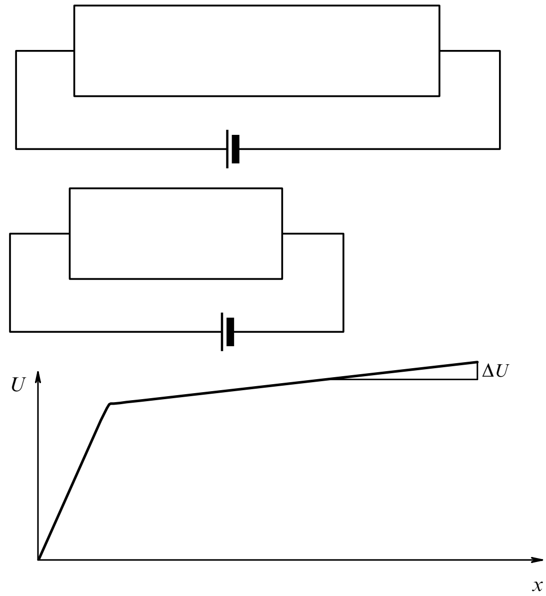

Figure 7. Gas discharges in cylindrical discharge tubes of different lengths, but of the same radius and filled by the same gas under identical pressures. The small voltage difference because of different lengths of the positive column is $\Delta U = E(L_1 - L_2)$, where E is the electric field strength in the positive columns, L_1 and L_2 are the lengths of the discharge tubes.

spatial distribution of the electric potential for glow discharge. As in the case of parallel infinite electrodes, there is a narrow cathode region that is responsible for the reproduction of charged particles, and the positive column, where the ionization balance requires the equality between the rate of atomic ionization in collisions with electrons of a heightened energy due to the electric field and the rate of attachment of charged particles to the walls. The peculiarity of the positive column of a cylindrical tube is such that the ionization balance is identical for each cross section of the positive column — that is, GDP is uniform along the discharge tube. This relates to a high-pressure gas discharge, which satisfies the criterion

$$\alpha L \gg 1 \,, \tag{2.3}$$

where L is the discharge tube length. Since the electric field strength in the positive column is less significantly than that in the cathode region, the electric field strengths are close in magnitude for discharge tubes of different lengths, but with the same tube radii, gas pressures, types of gas, and discharge currents, as shown in Fig. 7.

The simplest geometry under consideration is convenient for ascertaining the nature of gas discharge as the number density of electrons (or the discharge current) grows, and the passage takes place from the Townsend discharge, where a space charge weakly influences the discharge parameters, to a glow discharge with a separation of the cathode region, which is responsible for the reproduction of electrons, and the positive column of gas discharge with a relatively low electric field strength. But a detailed study of passage between these discharge types [39–45] also includes the character of current distribution near the cathode, so that the current of Townsend discharge occupies all the cathode area, while in the normal regime of glow discharge the current occupies a part of the cathode area. Thus, the electric current (or the number density of electrons) increases as a result of charge separation, both along the axis and in the perpendicular direction.

3. Model of gas discharge plasma of a positive column

We consider below a gas discharge plasma of a positive column for a high-pressure gas discharge, where the mean free path λ of atoms or molecules in the gas is small compared to the dimensions of the gas discharge tube, in particular, to its radius. For example, taking the radius of the discharge tube to be $r_0 \sim 1$ cm and the gas-kinetic cross section of atom collisions to be $\sigma_g \sim 3 \times 10^{-15}$ cm^2, we find $N_a \gg 3 \times 10^{14}$ cm^{-3} for the number density of atoms N_a in a high-pressure gas discharge or $p \gg 1$ Pa for the gas pressure.

Let us analyze the character of the ionization balance in the positive column of gas discharge for a cylindrical discharge tube. This balance is similar to that described by equations (2.1) and (2.2) for self-maintenance of GDP located in a gas gap between two parallel infinite electrodes. Assume that this balance is maintained by the production of charged atomic particles as a result of atom ionization by electron impact in an external electric field and by attachment of electrons and ions to walls of the discharge tube, so that the ionization equilibrium is described by the balance equation

$$\alpha(E)\, w_e(E) = \frac{1}{\tau_D} \,, \tag{3.1}$$

where w_e is the electron drift velocity, and τ_D is a typical time for diffusion of electrons and ions to the walls. The solution of this equation gives the strength E of the electric field that maintains the positive column plasma. The balance equation in this case has the form

$$\frac{D_a}{\rho}\frac{d}{d\rho}\left(\rho\frac{dN_e}{d\rho}\right)+k_{ion}N_eN_a=0\,, \tag{3.2}$$

where N_e and N_a are the number densities of electrons and atoms, respectively, ρ is the distance from the tube axis, r_0 is the radius of the discharge tube, D_a is the ambipolar diffusion coefficient for the gas discharge plasma as a whole, and k_{ion} is the rate constant of atom ionization by electron impact. The solution of equation (3.2) has the form [46]

$$N_e(\rho)=N_0J_0\left(\frac{2.4\rho}{r_0}\right), \tag{3.3}$$

and the boundary condition $N_e(r_0)=0$ gives [46]

$$N_a\frac{k_{ion}r_0^2}{D_a}=5.78\,. \tag{3.4}$$

As a matter of fact, equations (3.1) and (3.4) are identical, if a typical time τ_{ion} of ionization and a typical time τ_D of plasma diffusion to the walls are equal:

$$\frac{1}{\tau_D}=\frac{5.78D_a}{r_0^2}\,,\qquad \frac{1}{\tau_{ion}}=\alpha w_e=N_ak_{ion}\,, \tag{3.5}$$

and

$$\tau_D=\tau_{ion}\,. \tag{3.6}$$

It should be noted that the GDP regime under consideration relates to a low number density of electrons N_e, so that electrons do not influence the plasma behavior. In addition, the case of high-pressure discharge is considered in accordance with criterion (2.3).

4. Nonequilibrium gas discharge plasma

The variety of processes in a gas discharge plasma and the variety of its regimes create nonequilibrium conditions that may be regarded as one of the principal properties of a gas discharge plasma [47, 48]. Some plasma components, such as electrons, ions, atoms, or molecules in the ground or excited states, partake in certain processes which lead to an appropriate plasma state. Therefore, usually a stationary gas discharge plasma is not in thermodynamic equilibrium, and its equilibrium state is maintained by certain processes, including those related to the above balance equations (3.1), (3.2).

This means that thermodynamic equilibrium is not suitable for a gas discharge plasma, and other parameters rather than thermodynamic ones must be used for the analysis of a GDP. In particular, the description of a GDP relies on the distribution functions of particles and includes the kinetics of evolution for each plasma component on the basis of the Boltzmann kinetic equation [49]. For the evolution of electrons located in a gas in an external electric field and scattered by atoms, it is of importance that electron scattering on atoms through large angles is accompanied by a strong change in electron momentum and by a small change in its energy due to the small electron mass compared with the atom mass. Therefore, the basic part of the electron velocity distribution function is angle-symmetrical, and a small antisymmetrical part of the distribution function is responsible for energy transfer from the field to atoms through electrons. This form of the distribution function [50] allows one to represent the kinetic equation for the energy distribution function of electrons in the form of the Fokker–Planck equation [51, 52] in which the electron diffusion coefficient due to electron–atom scattering in the energy space has the form [53, 54]

$$B(\varepsilon)=\frac{m_e}{M}\,Tm_ev^2N_av\sigma^*(v)\,. \tag{4.1}$$

Here, v is the electron velocity, N_a is the atom number density, $\sigma^*(v)=\int(1-\cos\vartheta)\,d\sigma$ is the diffusion electron–atom cross section, so that ϑ is the scattering angle, m_e, M are the electron and atom masses, respectively, and T is the gas temperature.

In the case of motion of a test electron in an electron gas, the change in the electron energy proceeds by small portions because small scattering angles give the main contribution to the diffusion cross section of two electrons in a plasma. Then, the electron distribution function in a space of electron momenta obeys the Landau equation [55] which, rewritten in an energy space for a fast electron, gives the following expression for the electron diffusion coefficient in the energy space [56]:

$$B_{ee}(\varepsilon)=\frac{4\pi}{3}\,e^4vN_e\ln\Lambda\,, \tag{4.2}$$

where N_e is the electron number density, and $\ln\Lambda$ is the Coulomb logarithm that accounts for electron scattering in an ideal plasma when the distance between colliding electrons in large-angle scattering, at which the energy of Coulomb interaction is comparable to the kinetic energy of the electrons, is small compared to the Debye–Hükkel radius [38] that characterizes screening of the Coulomb interaction of electrons.

Depending on the relation between the electron diffusion coefficients in the energy space, we have two forms of the energy distribution functions of electrons, and the boundary between them is estimated from the equality of the diffusion coefficients (4.1) and (4.2) in the electron energy space:

$$\left(\frac{N_e}{N_a}\right)_b=\frac{3}{8\pi}\,\frac{m_e}{M}\,\frac{T\varepsilon}{e^4\ln\Lambda}\,\sigma^*(v)\,. \tag{4.3}$$

As is seen, the boundary degree of GDP ionization is small for a transition between the above regimes of establishment of the electron distribution function, which is formed by two factors in formula (4.3), where the first one is the ratio of the electron and atom masses, and the second one is the ratio between the diffusion cross section of electron–atom scattering and the cross section of Coulomb scattering of two electrons in a plasma.

Let us find the boundary degree of ionization (4.3) for helium, taking a room temperature gas and a typical value of the Coulomb logarithm $\ln\Lambda=7$, and accounting for the diffusion cross section of electron scattering on the helium atom to be independent of the electron energy in the wide energy range $0-10$ eV, where it is equal to $\sigma^*=(6\pm1)$ Å^2

[57]. This allows us to represent formula (4.3) for helium in the form

$$\left(\frac{N_{\mathrm{e}}}{N_{\mathrm{a}}}\right)_{\mathrm{b}} = 1.6 \times 10^{-9}\,\varepsilon\,, \tag{4.4}$$

where the electron energy ε is measured in eV.

If electron–electron collisions are of importance for establishing the electron distribution function, i.e., the degree of ionization of a gas discharge plasma exceeds the value given by formula (4.3), the distribution function has the Maxwell form, and the electron temperature T_{e} follows from the energy balance for electrons which acquire their energy from an external electric field and transfer it to gas atoms in collisions with them. At a low degree of ionization of a gas discharge plasma, which is below than that given by formula (4.3), the distribution function of electrons is connected with the velocity dependence $\sigma^*(v)$ of the diffusion cross section of electron–atom collisions. In particular, if this cross section is independent of the collision velocity, it is convenient to introduce the mean free path of electrons in a gas as $\lambda = 1/(N_{\mathrm{a}}\sigma^*(v))$. Then, the dependence of the electron distribution function at its tail, where the electron energy significantly exceeds the thermal atomic energy, takes the form [58, 59]

$$f_0(\varepsilon) \sim \exp\left[-\frac{m_{\mathrm{e}}}{M}\left(\frac{\varepsilon}{eE\lambda}\right)^2\right], \tag{4.5}$$

where E is the electric field strength.

Thus, a gas discharge plasma as a physical object is a nonequilibrium system in thermodynamic terms, and the stationary state of GDP depends on processes that establish this equilibrium. Hence, different regimes are possible for the establishment of an equilibrium in GDP, and an analysis of GDP must be fulfilled in terms of the distribution functions of electrons and ions.

5. Electron transport in a gas discharge plasma

Ionization balance does not exhaust all the information about a gas discharge plasma. Additional information may be gained from the analysis of the current–voltage characteristic of gas discharge. In the limit of low number densities of electrons and ions, the electric field strength does not depend on the discharge electric current. But this dependence occurs in the cathode region at high currents, where the number density of ions exceeds the number density of electrons, and one can assume that the number densities of electrons and ions in the positive column are close to those at the boundary of the cathode region if the discharge current covers all the cathode area. At low discharge currents, when the discharge current covers a small part of the cathode area, a transient region arises between the cathode region and the positive column, where the discharge current occupies a certain part of the cross section of the discharge tube. This region may be significant and play a self-dependent role. The electron current I in the positive column (ions give a small contribution to the discharge current) is determined by the following expression

$$I = e\int_0^{r_0} 2\pi\rho\,\mathrm{d}\rho\,N_{\mathrm{e}}(\rho)w_{\mathrm{e}}\,, \tag{5.1}$$

where e is the electron charge, and we assume that, as in the case of weak electric field strengths E, the electron drift velocity is proportional to the electric field strength: $w_{\mathrm{e}} = K_{\mathrm{e}}E$, i.e., the electron mobility K_{e} is independent of the electric field strength. This corresponds to the tau-approximation, where a typical time τ_{e} of electron–atom collisions is independent of the electron energy. In this case, the electron drift velocity follows from the equation of motion for electrons, which has the following form

$$m_{\mathrm{e}}\,\frac{\mathrm{d}w_{\mathrm{e}}}{\mathrm{d}t} = eE - m_{\mathrm{e}}\,\frac{w_{\mathrm{e}}}{\tau_{\mathrm{e}}}\,.$$

Since $1/\tau_{\mathrm{e}} = N_{\mathrm{a}}k_{\mathrm{e}}$, where the rate constant k_{e} of elastic electron–atom collisions is assumed to be independent of the electron velocity, we have for the electron drift velocity the following expression:

$$w_{\mathrm{e}} = \frac{eE\tau_{\mathrm{e}}}{m_{\mathrm{e}}} = \frac{eE}{m_{\mathrm{e}}N_{\mathrm{a}}k_{\mathrm{e}}}\,. \tag{5.2}$$

The above-indicated assumption about the constancy of the rate constant of electron–atom scattering simplifies the computations and is in common use in books on gas discharge.

But transferring to real conditions, one can turn down this assumption, according to which the electron drift velocity w_{e} is proportional to the electric field strength E. In the general case, the electron drift velocity is given by [60]

$$w_{\mathrm{e}} = -\frac{eE}{3m_{\mathrm{e}}N_{\mathrm{e}}}\int_0^{\infty}\frac{4\pi v_{\mathrm{e}}^3}{\nu_{\mathrm{ea}}}\,\frac{\mathrm{d}f_0}{\mathrm{d}v_{\mathrm{e}}}\,\mathrm{d}v_{\mathrm{e}}\,, \tag{5.3}$$

where the rate of elastic electron–atom collisions is defined as

$$\nu_{\mathrm{ea}} = N_{\mathrm{a}}v_{\mathrm{e}}\sigma^*_{\mathrm{ea}}\,. \tag{5.4}$$

Here, $\sigma^*_{\mathrm{ea}} = \int(1-\cos\vartheta)\,\mathrm{d}\sigma_{\mathrm{ea}}$ is the diffusion electron–atom cross section, ϑ is the scattering angle, and $\mathrm{d}\sigma_{\mathrm{ea}}$ is the differential cross section of electron–atom scattering. The spherically symmetric part $f_0(v_{\mathrm{e}})$ of the distribution function is normalized by the condition

$$\int_0^{\infty} f_0(v_{\mathrm{e}})\,4\pi v_{\mathrm{e}}^2\,\mathrm{d}v_{\mathrm{e}} = N_{\mathrm{e}}\,. \tag{5.5}$$

From this it follows that formula (5.2) is valid if the diffusion cross section σ^*_{ea} of electron–atom scattering is inversely proportional to the electron velocity v_{e}, i.e., $\sigma^*_{\mathrm{ea}} \sim 1/v_{\mathrm{e}}$.

This simple dependence of the cross section on the collision velocity is not fulfilled under the real conditions. In particular, in the case of electron scattering on argon, krypton, and xenon atoms, the diffusion cross section has a deep minimum at electron energies ranging $0.3-0.5$ eV because of the Ramsauer effect [61–63], and the cross section of electron–atom scattering at its minimum is more than two orders of magnitude lower than the appropriate value at zero energy. This fact leads to the dependence of the electron mobility in a gas on the electric field strength, and in some cases this causes instabilities of the gas discharge plasma. As an example of this dependence, Fig. 8 depicts the electron drift velocity in argon as a function of the reduced electric field strength (according to measurements [9, 64]). Note that the electron velocity distribution function in formula (5.3) includes the velocity dependence of the diffusion cross section $\sigma^*_{\mathrm{ea}}(v)$ of electron–atom scattering.

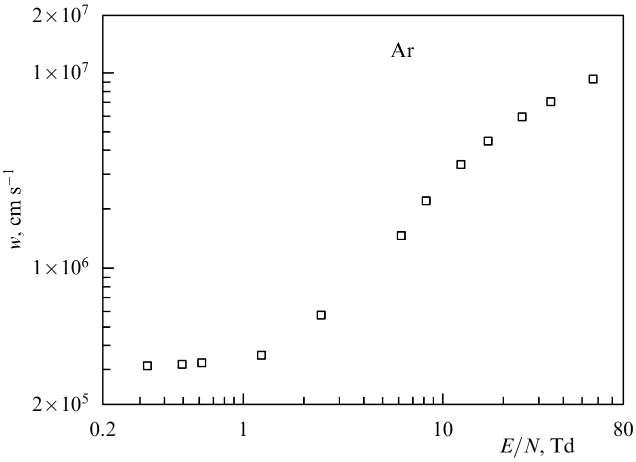

Figure 8. The electron drift velocity in argon in the limit of a low number density of electrons [9, 64].

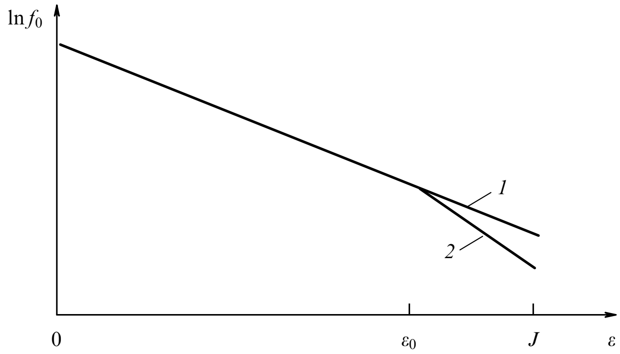

Figure 9. The energy distribution function of electrons when the tail of the distribution function is restored (*1*) and not restored (*2*) as a result of elastic electron–atom collisions.

6. Ionization in single electron–atom collisions in a gas discharge plasma

Introducing the first Townsend coefficient α as a characteristic of the ionization rate in gas discharge, we assumed that atomic ionization by electron impact proceeds in single electron–atom collisions according to the scheme

$$\mathrm{e} + A \to 2\mathrm{e} + A^{+} . \tag{6.1}$$

Then, according to the definition, the rate constant of ionization in equation (3.4) of the ionization balance is expressed through the electron distribution function by the formula

$$k_{\mathrm{ion}} = \frac{1}{N_{\mathrm{e}}} \int_{v_{\mathrm{i}}}^{\infty} 4\pi v_{\mathrm{e}}^{3} \sigma_{\mathrm{ion}}(v_{\mathrm{e}})\, f_0(v_{\mathrm{e}})\, \mathrm{d}v_{\mathrm{e}} , \tag{6.2}$$

where $\sigma_{\mathrm{ion}}(v_{\mathrm{e}})$ is the cross section of atom ionization by electron impact, and v_{i} is the threshold electron velocity for atomic ionization. Since the average electron energy in a gas discharge is small compared to the atomic ionization potential J, the rate constant of ionization k_{ion} is determined by the tail of the electron energy distribution function.

Being guided by atomic gases, we have that the symmetric velocity distribution function $f_0(v_{\mathrm{e}})$ below the excitation threshold is determined by elastic electron–atom collisions. Then, a remarkable change in the electron energy results from many collisions with gas atoms, and there are various regimes for the establishment of a tail of the electron distribution function above the atomic excitation threshold within the framework of scheme (6.1) for the ionization process in electron–atom collisions. Indeed, after atomic excitation, the electron slows down and acquires its energy in an electric field up to the excitation threshold for a long time after many elastic collisions with atoms. Therefore, there are two regimes for the establishment of the electron energy distribution function $f_0(\varepsilon)$ at small electron number densities [65, 66], as represented in Fig. 9. In the first case, a typical time of atom excitation is small in comparison with a typical time of establishment of the distribution function tail, and then the energy derivative of the electron distribution function does not vary near the excitation threshold. In the second case, with the inverse relation between indicated times, the electron energy distribution function decreases sharply behind the excitation threshold because of the processes of atomic excitation. This leads to different regimes for excitation and ionization of atoms by electron impact in a gas [67].

7. Ionization in a gas discharge plasma involving excited atoms

Ionization balance described by equations (3.1) and (3.4) is based on scheme (6.1) for the ionization process when it results from single electron–atom collisions. Other channels of atom ionization are also possible in a dense gas through the formation of excited atoms. The stepwise atomic ionization is one of such channels, where the first stage of the ionization process is the formation of excited atomic states, and the ionization process proceeds according to the scheme

$$\mathrm{e} + \mathrm{A} \to \mathrm{e} + \mathrm{A}^{*} , \qquad \mathrm{e} + \mathrm{A}^{*} \to 2\mathrm{e} + \mathrm{A}^{+} . \tag{7.1}$$

The second stage of the ionization process is realized if the quenching rate of an excited atom formed by electron impact exceeds the rates of atom quenching in other processes, such as atomic radiation emission or quenching on the walls. Therefore, the stepwise character of atom ionization in a gas discharge plasma is realized at relatively high electron number densities, so that an increase in the discharge current leads to a change in the ionization balance regime, from single ionization up to stepwise ionization of atoms.

If the equilibrium between atoms in the ground and excited states results from collisions with electrons according to scheme (7.1), and the electron energy distribution function is determined by electron drift in a gas in an external electric field, the stepwise ionization is more effective than the single one, until the average electron energy is relatively small. Let us demonstrate this for the example of atomic ionization in a helium GDP, where metastable helium atoms determine the stepwise ionization of atoms in a plasma. The following scheme of processes takes place in this case in accordance with formula (7.1):

$$\begin{gathered} \mathrm{e} + \mathrm{He}(1^1\mathrm{S}) \leftrightarrow \mathrm{e} + \mathrm{He}(2^3\mathrm{S}) , \\ \mathrm{e} + \mathrm{He}(2^3\mathrm{S}) \to 2\mathrm{e} + \mathrm{He}^{+} . \end{gathered} \tag{7.2}$$

For example, at the reduced electric field strength of $E/N_{\mathrm{a}} = 10$ Td, which is a typical value of the reduced electric field strength for the positive column of glow discharge, the rate constant of single atomic ionization equals $k_{\rm ion} = 1.1 \times 10^{-13}$ cm^3 s^{-1} under the assumption that atom excitation by electron impact does not influence the tail of the distribution function. The rate constant of ionization of a metastable atom is $k_{\rm ion}^{\rm m} = 4.2 \times 10^{-7}$ cm^3 s^{-1} under these conditions (for comparison, the quenching rate constant of the metastable helium atom by a slow electron is $k_{\rm q} = 3.1 \times 10^{-9}$ cm^3 s^{-1} [68, 69]), and the concentration of metastable atoms reaches $N_{\rm m}^{\rm eq}/N_{\rm a} = 1.2 \times 10^{-3}$ — that is, the rate of atom ionization through formation of the metastable state exceeds the rate of single atomic ionization by electron impact by more than three orders of magnitude at the indicated reduced electric field strength. Note that the ionization potentials of a helium atom in the ground and metastable states are 24.59 and 4.77 eV, respectively, and the excitation energy for the helium metastable state is 19.82 eV.

Evidently, stepwise ionization is not the dominate channel at low electron number densities, when destruction of excited atoms is determined by other channels rather than by electron impact, and the number density of excited atoms is significantly lower than the equilibrium one. In the positive column of gas discharge, such processes of destruction of excited atoms are the radiation emission of excited atoms and the departure of excited atoms to the walls as a result of their diffusion in a gas. The latter process is of importance for metastable atoms and determines the lifetime of metastable atoms in a plasma at low electron concentrations, being expressed through the diffusion coefficient of metastable atoms in a gas. Table 1 contains the reduced diffusion coefficients of metastable atoms of inert gases in parent gases [70]. Then, the helium metastable state is He(2^3S), and for other inert gases consisting of atoms A, this state is $A(^3{\rm P}_2)$. As is seen, the diffusion coefficient of metastable inert-gas atoms in parent gases is close to the diffusion coefficient of molecular ions in this gas. If destruction of excited atoms is determined by their diffusion to the walls, the lifetime of excited atoms is expressed through the coefficient of their diffusion in a gas by formula (3.5).

Table 1. Diffusion coefficients for metastable atoms, D^*, atomic ions, $D(A^+)$, and molecular ions, $D(A_2^+)$, of inert gases in a parent gas at the gas temperature $T = 300$ K. The diffusion coefficients are reduced to the normal number density of atoms $N = 2.69 \times 10^{19}$ cm^{-3} [70].

Gas (A)	He	Ne	Ar	Kr	Xe
D^*, cm^2 s^{-1}	0.59	0.20	0.067	0.039	0.024
$D(A^+)$, cm^2 s^{-1}	0.28	0.10	0.041	0.022	0.013
$D(A_2^+)$, cm^2 s^{-1}	0.49	0.16	0.049	0.031	0.018

Let us rewrite equation (3.5) of ionization balance, including in it the single and stepwise ionization processes and assuming that the stepwise ionization (7.1) contributes to the total ionization balance of the GDP. Then, the equation of ionization balance takes the following form

$$\frac{1}{\tau_{\rm D}} = N_{\rm a} k_{\rm ion} + N_{\rm m} k_{\rm ion}^{\rm m} \,, \tag{7.3}$$

where $N_{\rm a}$ and $N_{\rm m}$ are the number densities of atoms in the ground and excited states, and $k_{\rm ion}$, $k_{\rm ion}^{\rm m}$ are the rate constants of single ionization of a helium atom in the ground and excited states by electron impact. Let us determine the boundary of change between the single and stepwise ionization characters, when both terms on the right-hand side of equation (7.3) are equal. Evidently, this takes place for the regime of destruction of excited atoms as a result of their departure to the walls rather than their quenching by electron impact. Then, the number density of metastable atoms in equation (7.3) is equal to

$$N_{\rm m} = N_{\rm m}^{\rm eq} \frac{\tau_{\rm D}}{\tau_{\rm e}} = N_{\rm m}^{\rm eq} \frac{5.78 D^*}{k_{\rm q} r_0^2} \,, \tag{7.4}$$

where $N_{\rm m}^{\rm eq}$ is the equilibrium number density of metastable atoms, when electron quenching of metastable atoms dominates, and $\tau_{\rm D}$, $\tau_{\rm e}$ are the times of departure of metastable atoms to the walls and quenching of metastable atoms by electron impact, respectively.

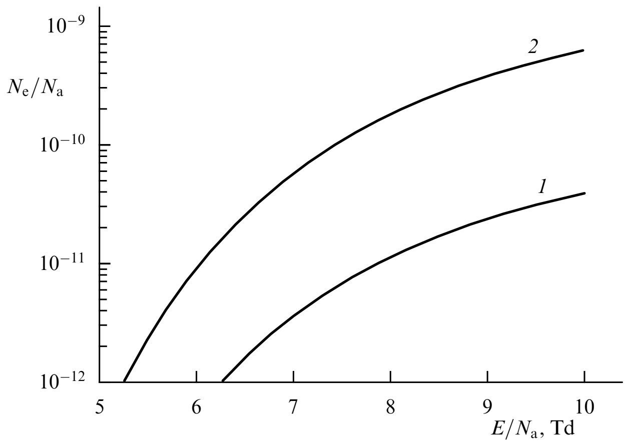

Figure 10. The boundary degree of ionization $N_{\rm e}/N_{\rm a}$ for helium atoms in GDP ($N_{\rm e}$ is the number density of electrons, and $N_{\rm a}$ is the number density of helium atoms) for the transition between single and stepwise mechanisms of ionization of helium atoms by electron impact. The boundary degree of ionization is determined by the equality of the two terms on the right-hand side of equation (7.3) at a given electric field strength. The reduced number density of helium atoms is $N_{\rm a} r_0 = 1 \times 10^{17}$ cm^{-2} (*1*), and $N_{\rm a} r_0 = 4 \times 10^{17}$ cm^{-2} (*2*).

Figure 10 [71] gives the boundary of the ionization regimes under consideration at some values of the reduced radius of the discharge tube that are typical for the positive column of glow discharge. Note that we consider the regime of low electron number densities for the establishment of the electron energy distribution function below the excitation threshold, when this equilibrium results from elastic collisions of electrons with atoms of a gas located in an external electric field. As follows from the data of Fig. 10, the stepwise ionization of atoms can dominate at a low degree of atomic ionization and is possible in glow discharge. Then, the equilibrium between atoms in the ground and metastable states according to the scheme of stepwise ionization (7.2) has a threshold that may be the threshold of stepwise ionization.

The boundary number density of electrons, when the rates of single and stepwise ionization of atoms by electron impact are equal, relates to the regime of destruction of metastable atoms as a result of their departure to the walls. We determine now a higher boundary electron number density $N_{\rm e}$ at which the rates of metastable atom destruction on the walls and by electron impact are equal. This boundary electron number density, by analogy with equations (3.5) and (3.6), is given by the equation $\tau_{\rm D} = \tau_{\rm e}$, namely

$$N_{\rm e} k_{\rm q} = \frac{5.78 D^*}{r_0^2} \,, \tag{7.5}$$

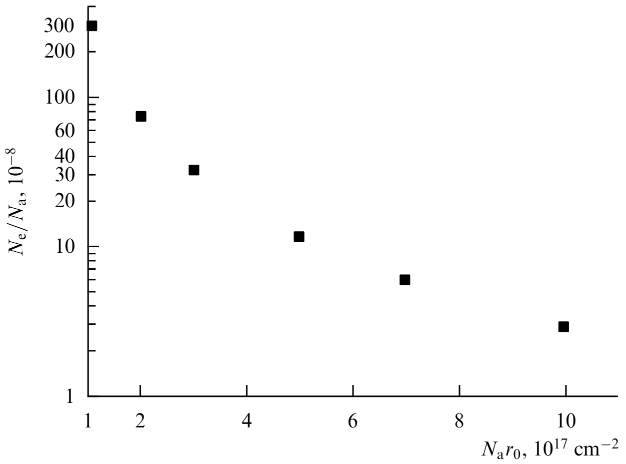

Figure 11. The boundary degree of helium atom ionization in GDP, at which the quenching rates of metastable atoms by electron impact and on the walls are equal, in accordance with equation (7.6).

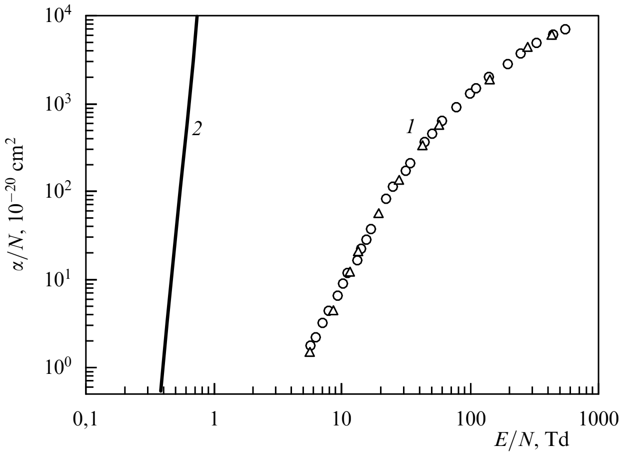

Figure 12. The reduced first Townsend coefficient for neon as a function of the reduced electric field strength: *1* — ionization proceeds in single electron–atom collisions and the values of the reduced first Townsend coefficient results from measurements at low number densities of electrons [64], and *2* — equilibrium between electrons and excited atoms is maintained at each electric field strength, and new electrons and ions result from ionization of excited atoms in accordance with scheme (7.1) [71].

where k_q is the rate constant of quenching of an excited atom by electron impact, D^* is the diffusion coefficient of excited atoms in a gas, and r_0 is a discharge tube radius. Equation (7.5) leads to the following relation for the boundary degree of ionization N_e/N_a of the gas discharge plasma depending on the reduced atom number density $N_a r_0$:

$$\frac{N_e}{N_a} = \frac{C}{(N_a r_0)^2}\,, \qquad C = \frac{5.78 D^* N_a}{k_q}\,, \tag{7.6}$$

and because the diffusion coefficient D^* of an excited atom in a gas is inversely proportional to the number density of atoms N_a, the proportionality coefficient C is independent of the atom number density. Figure 11 depicts dependence (7.6) for the stepwise ionization in helium under consideration, when the ionization process proceeds according to scheme (7.2).

As follows from the data of Figs 10 and 11, stepwise ionization in helium starts at low degrees of plasma ionization and takes place in gas discharge of moderate powers. Note that an energy dependence for the diffusion cross section of elastic electron scattering on the helium atom in the ground state, which in a wide range of electron energies can be set $\sigma^* = 6$ Å^2 [57], leads to a simple analytical expression (4.5) for the energy distribution function of electrons at energies below the excitation threshold, and this simplifies the analysis (see, for example, Ref. [71]). In particular, at a typical value of the reduced electric field strength $E/N_a = 10$ Td in helium, an electron acquires the energy $eE\lambda = 0.17$ eV between neighboring collisions with atoms, while the average electron energy is $\bar{\varepsilon} = 6.1$ eV at this electric field strength. This relation shows that an electron experiences many elastic collisions with atoms until it reaches the average energy or the excitation threshold.

Competition between the single and stepwise mechanisms of atom ionization by electron impact has a general character. As a demonstration, the rates of single and stepwise ionization in neon are compared in Fig. 12 in accordance with the process schemes (6.1) and (7.2), where the values of the first Townsend coefficient for the first ionization process are taken from the review [64] for measurements at low electron number densities, when excited atoms do not give a contribution to the total atom ionization. As is seen, the transition from the single to stepwise mechanisms of atom ionization leads to a remarkable change in the electric field strength that maintains a GDP. This may cause a stepwise change of discharge parameters as a result of variation of the discharge current and can lead to the creation of a nonuniform distribution of the charge in the GDP, as it takes place in a striated gas discharge [72, 73].

The presence of excited atoms in a GDP opens up new channels of atom ionization, such as the associative ionization of atoms:

$$A^* + B \rightarrow e + AB^+\,, \tag{7.7}$$

where the energy required for electron release is taken from the dissociation energy of a molecular ion being formed. In this case, atoms A and B may be of the same sort, as it takes place in the photoresonant plasma. Another ionization process involving excited atoms is the Penning process [74, 75] proceeding in accordance with the scheme

$$A^* + B \rightarrow e + A + B^+\,, \tag{7.8}$$

where the ionization potential of an atom B is lower than the excitation energy of an atom A. An example of this process is given by

$$\mathrm{He}(2^3\mathrm{S}) + \mathrm{Ar} \rightarrow \mathrm{He}(1^1\mathrm{S}) + \mathrm{Ar}^+ + e\,, \tag{7.9}$$

so that a small admixture (of the order of $\sim 10^{-4}$) of heavy inert gases added to helium decreases the breakdown voltage of this mixture compared to pure helium. Thus, excited atoms in a GDP influence its ionization balance and lead to appearance of additional channels of ionization in the plasma.

8. Character of destruction of gas discharge plasma of a positive column

We assume in equations (3.1) or (3.4) regarding ionization equilibrium that the loss of electrons and ions in GDP is determined by their departure to the walls as a result of ambipolar diffusion. The coefficient of ambipolar diffusion in GDP is determined to a greater extent by the ion diffusion coefficient, which in turn depends on the ion sort in a plasma. For example, in inert gases at low temperatures (in particular, below 700 K under typical conditions of glow discharge in argon) the basic ion sort is Ar_2^+ and then the ambipolar diffusion coefficient is expressed through the diffusion coefficient of these ions. At high gas temperatures, atomic ions are present in a gas, and the diffusion coefficients of atomic and molecular ions are different in this case (see Table 1), because the diffusion coefficient of molecular Ar_2^+ ions is determined by elastic ion–atom scattering, while in the case of atomic Ar^+ ions the diffusion coefficient results from the resonant charge exchange process. The difference in the diffusion coefficients of molecular and atomic ions in GDP increases with an increase in temperature. Since a change in the gas temperature leads to a change in the ion sort, it is accompanied by a change in the ionization balance of GDP.

Transition from atomic ions to molecular ones may lead to a principal change in the character of the ionization balance in a gas discharge plasma. Indeed, in the case of molecular ions, destruction of charged particles in a plasma may be determined by dissociative recombination that for a gas discharge plasma of inert gases corresponds to the scheme

$$\mathrm{e} + \mathrm{Ar}_2^+ \to \mathrm{Ar}^* + \mathrm{Ar}\,. \tag{8.1}$$

The diffusion mechanism of destruction of charged particles in GDP dominates until the number density of electrons is low, but at relatively high number densities of electrons the dissociative recombination of electrons and molecular ions (8.1) may be responsible for loss of charged particles in GDP. Hence, there are different regimes of ionization balance in GDP depending on the character of destruction of charged atomic particles.

Thus, we are faced with a large variety of ionization processes that determine the ionization balance in GDP. The formation and destruction of excited atoms in gas discharge of moderate pressure, including processes with the participation of electrons, influence the character of the ionization balance. In considering the ionization balance in terms of the electron distribution function over energies or velocities, when electrons are moving in a gas in an external electric field, we note that this problem is self-consistent since the rate of atomic excitation and ionization is determined by the tail of the electron distribution function, and it in turn depends both on the rate of elastic electron–atom collisions and on the number density of excited atoms in a gas discharge plasma. This also creates a variety of regimes for the electron distribution function (see, for example, Refs [67, 76, 77]).

9. Heat balance for a positive column of gas discharge

In analyzing the properties of a gas discharge plasma of the positive column in a cylindrical discharge tube, we assumed implicitly that the presence of electrons and ions does not influence the properties of a gas located in a discharge tube. In reality, this influence may be different. For example, the formation of ions in inert gases with a lightly ionized admixture (in particular, this mixture is formed by the addition of a metal vapor to an inert gas) results from ionization of this admixture. Since ions are moving towards the cathode under the action of the discharge electric field, this transport may lead to a nonuniform distribution of the admixture along a discharge tube. If a discharge glow is created by radiation emission of excited admixture atoms, we observe a nonuniform discharge glowing along the discharge tube, and the maximum radiation intensity is concentrated near the cathode. This phenomenon, known as electrophoresis, is possible under typical conditions of glow discharge.

Passage of an electric current through a gas discharge tube under the action of an external electric field causes heat release and leads to gas heating. We considered above the case where heating of a GDP changes the ion sort and leads to gas heating and correspondingly to a change of the regime of the ionization balance in a plasma. Then, the change from atomic ions to molecular ions as a basic ion sort proceeds at a temperature of hundreds of degrees Kelvin, which is typical of a glow discharge.

Another example relates to molecular gas lasers that work on transitions between vibrational states of molecules in a restricted range of temperatures of a gas discharge plasma. The mechanism of this laser operation is based on the selective population of molecular levels in the upper state of the radiative transition, and the lower molecular state of the laser transition is destroyed in collisions with gas atoms or molecules. Under these conditions, an increase in the specific power of the positive column of gas discharge leads to growth in the number density of excited molecules, i.e., to an increase in the laser radiation power. But this simultaneously leads to an increase in the temperature of the GDP, which causes a nonlinear growth in the molecular population at the low laser level. At some temperature (for a CO_2-laser, this temperature equals approximately 600 – 700 K [78, 79]), the inverse population of these vibrational states disappears, and laser generation ceases.

At large discharge currents, gas heating is of importance for the parameters of GDP. To describe the properties of GDP of the positive column, it is necessary to add the heat balance equation to the equation of the ionization balance and Ohm's law. If electron transport gives a small contribution to heat transport, the heat balance equation for a plasma located inside a cylindrical discharge tube has the form of the Elenbaas–Heller equation [23, 80], namely

$$\frac{1}{\rho}\frac{\mathrm{d}}{\mathrm{d}\rho}\left[\rho\kappa(T)\,\frac{\mathrm{d}T}{\mathrm{d}\rho}\right] + p(\rho) = 0\,. \tag{9.1}$$

Here, κ is the gas thermal conductivity coefficient, and the second term of this equation, the specific power of heat release due to a discharge electric current, is equal to $p(\rho) = iE$, where i is the electric current density, and E is the electric field strength. From Ohm's law it follows that $i = \Sigma E$, where Σ is the conductivity of GDP. Therefore, heat balance equation (9.1) characterizes heat release as a result of the passage of electric current through the GDP and corresponds to heat transport due to the thermal conductivity of a gas.

Heat processes are of particular importance for an arc plasma, where the influence of thermal processes on parameters of a gas discharge plasma is strong. Below we consider the contraction of gas discharge in a dense gas [81–83] as the clearest example of this type, when the electric current occupies a small part of the cross section of the positive column near the tube axis. Indeed, let us assume that thermodynamic equilibrium is established in a dense gas discharge plasma, so that the number density of electrons and ions is determined by an equilibrium similar to the Saha one, and the number density of charged particles increases sharply with increasing temperature. But the heat release does not increase so sharply with increasing temperature if the heat release is determined by thermal conductivity of GDP in accordance with the heat balance equation (9.1). As a result, an instability develops that causes contraction of the discharge current. When the GDP occupies a small part of the tube near its axis, the temperature gradient becomes large, which provides heat balance in accordance with the heat balance equation (9.1). It should be noted that large temperature gradients can cause convective instability, leading to convective gas mixing and meaning a change of the heat release process in a gas discharge plasma.

10. Nonstationarities and nonuniformities of a gas discharge plasma

The variety of GDP regimes and their competition may lead to a stepwise change in plasma parameters as a result of variations in external parameters and can lead to developing instabilities due to nonuniformities of GDP. Let us demonstrate this by the simple example of a stationary GDP located in a constant electric field. Electrons travel in a plasma with an average drift velocity. Let us initiate a drift wave (see, for example, Ref. [67]) by creating a perturbation δN_e in the electron number density near the cathode. This perturbation propagates with the electron drift velocity and is smeared as a result of both electron diffusion and Coulomb repulsion of electrons.

We now consider the regime of drift wave propagation, when the rate of atom ionization by electron impact in GDP increases with an increase in the number density of electrons because of a change in the regime of GDP evolution or a change in the ionization regime. Then self-organization arises for a nonuniform distribution of electrons, and the ionization waves being formed are striations. There are various regimes of striations, both in the form of traveling waves and standing waves, and they have been studied for a long time [84–91]. If we consider striations as ionization waves from the standpoint of gas discharge, we find that a given discharge current of the positive column is characterized by a lower voltage for the striated structure than that for a uniform gas discharge plasma [29].

A general criterion for the propagation of ionization waves in GDP is connected with the marked dependence of the ionization rate on the electron number density, which, for example, takes place for a change from single to stepwise atom ionization by electron impact in GDP. Next, a nonuniform distribution for GDP parameters, including the GDP voltage distribution along the axis, must maintain nonuniform distribution. These criteria may be fulfilled for various GDP regimes, but this phenomenon requires a more detailed analysis [92–94].

The possibility of arising ionization waves in a GDP is connected with the character of change of the electron energy in collisions with atoms. The mean free path of electrons λ_ε with respect to variation of their energy, if the electron energy does not exceed the threshold for atomic excitation, is more than the mean free path of electrons λ with respect to the momentum variation in elastic collisions by roughly $\sqrt{M/m_e}$ times (m_e is the electron mass, M is the atom mass), i.e., approximately by two orders of magnitude. For example, in helium at a pressure of $p = 1$ Torr the electron mean free path with respect to the momentum variation is equal to $\lambda \approx 0.05$ cm, whereas the electron mean free path with respect to the energy change exceeds a typical radius of a laboratory discharge tube. This relation between the mean free paths leads to the following expression for the electron energy $\varepsilon(\mathbf{r}_2)$ at point $\mathbf{r}_2$, if at a point $\mathbf{r}_1$ it has the energy $\varepsilon(\mathbf{r}_1)$ [95]:

$$\varepsilon(\mathbf{r}_2) = \varepsilon(\mathbf{r}_1) + eU(\mathbf{r}_2) - eU(\mathbf{r}_1)\,, \tag{10.1}$$

where $U(\mathbf{r}_2)$, $U(\mathbf{r}_1)$ are the electric potentials of the GDP at the indicated points. This is essential for nonlocal phenomena in the kinetics of GDP and allows one to understand the character of electron transport and creation of nonuniformities in a gas discharge plasma, including stratification of gas discharge [96–98].

Note that the stratified character of the spatial distribution of GDP is possible not only for various gas sorts, but also for various discharge geometries. In particular, an inverse direction of the electric field in a striation with respect to the direction of the electric field strength in gas discharge is observed in the spherical stratified discharge [99–101].

Electric domains form another structure of GDP and have as their basis drift waves, so that plasma bunches propagate in GDP with the electron drift velocity. Though this phenomenon was discovered for a semiconductor plasma [102, 103], it is typical for GDP also [67, 104]. A nonmonotonic dependence of the electron drift velocity on the electric field strength, which provides the electric domain stability, is necessary for the existence of the electric domain.

11. Models of a gas discharge plasma

The above analysis demonstrated the variety of GDP regimes and allowed us to extract the basic factors which determine GDP properties under certain conditions. On the basis of this analysis one can formulate a general approach to the analysis of a uniform GDP, being guided by the following equations which determine the properties of a certain plasma and the regime of its evolution:

(1) equation of the ionization balance;

(2) equation for the gas discharge current or the current–voltage characteristic;

(3) equation of heat balance for a gas discharge plasma.

Since these equations in the general case include a large number of processes which provide the balance of charged particles and the heat balance of GDP, it is not convenient to use a universal approach in a general form. But this approach is productive in the analysis of a certain gas discharge plasma with a restricted number of processes that are responsible for its properties. One can extract in this case basic elements which influence the properties of plasma, taking into consideration the restricted number of regimes of its development. These general principles are also conserved in the analysis of a nonuniform and nonstationary gas discharge plasma and form the basis of plasma modeling.

12. Conclusions

As follows from the above analysis, even in the simplest case of GDP located in the positive column of a cylindrical discharge tube there are many regimes maintaining a gas discharge plasma and many ways of its evolution, depending on the electric field strength, the electric current, the gas pressure, and the number density of electrons and atoms. Above we restricted ourselves to a simple geometric construction and a gas sort, whereas many additional regimes of GDP arise beyond this scope. Indeed, transitions between vibrational and rotational states significant for molecular systems, the processes of formation and destruction of negative ions essential for electronegative gases, and chemical processes may be of principle importance for GDP in chemically active gases. Next, we were concerned with a uniform or almost uniform plasma, while nonuniformities may be of importance for some forms of GDP.

We define a gas discharge plasma as a matter resulting from the action of an electric field and providing the passage of an electric current through a gas. Being a system formed as a result of an external action, a gas discharge plasma is nonuniform in principle, and contains regions of a certain destination, in particular, the cathode region for the reproduction of charged particles to compensate for ion losses at the cathode, and the positive column that comprises a uniform plasma with a small electric field gradient along the tube. We were guided by a DC gas discharge located in a cylindrical discharge tube. Cases of alternative or pulse fields and combinations of electric and magnetic fields also enlarge possible regimes of gas discharge and GDP. From this we conclude that the number of regimes of a gas discharge plasma seems to be boundless.

We thus move to the contradiction between the principles of GDP and regimes of its evolution. The basic principle of GDP consists in the condition of its self-maintenance that has the form of equation (2.1) in the Townsend scheme with the appropriate processes that are responsible for the loss and origin of charged particles. Therefore, self-maintenance equations (2.1), (3.1) are often used in textbooks for the explanation of GDP principles, while a certain real GDP is based on other processes and equations. Therefore, we have a contradiction between the principles of GDP and the methods of its description. This contradiction is known by professionals, but escapes the attention of somebody who is just getting acquainted with GDP. This fact should be noted here in the first place.

But the role of a large number of GDP regimes extends the indicated contradiction further. Transition between regimes of gas discharge in the course of variation of the discharge voltage or current may be accompanied by a stepwise change in gas discharge parameters or can lead to developing instabilities. In such phenomena of gas discharge as striations or contraction of the discharge electric current, we encounter stepwise changes in the electric current or voltage of gas discharge.

Another consequence of the large number of GDP regimes consists in a large number of applications which are renewed as the technology develops. One can use as an example gas discharge lamps which, being filled by inert gases, about hundred years ago were used for commercial purposes. Luminous lamps as light sources found a wide distribution with the creation of a fluorescent material that covers the surface of a cylindrical discharge tube and transforms ultraviolet radiation of gas discharge into light; as a result, this increases the efficiency of such light sources. One-half century later sodium lamps of a yellow color become prevalent for street lighting; haloid lamps with a high efficiency of transformation of electric energy into light also appeared at that time. The application of new devices becomes possible first and foremost due to the development of a new technology that allows the use of new light sources to be economically profitable. Hence, one can expect new gas discharge sources of light in the future.

Let us draw attention to one more aspect of the problem under consideration. Though the study of GDP was begun in the 19th century, research on gas discharge plasma as a physical object has moved in new directions, as some of its aspects and problems are solved. The reason for this is the variety of conditions and processes that organize a gas discharge plasma. As an example, one can cite a laboratory dusty plasma [105, 106] or a cluster plasma [107] — new types of gas discharge plasma.

References

1. Thomson J J *Conduction of Electricity Through Gases* (Cambridge: Univ. Press, 1903)
2. Townsend J S, Hurst H E *Philos. Mag.* **8** 738 (1904)
3. Townsend J S *Electricity in Gases* (Oxford: Clarendon Press, 1915)
4. Townsend J S *Motion of Electrons in Gases* (Oxford: Clarendon Press, 1925)
5. Townsend J *Electrons in Gases* (London: Hutchinson's Sci. Tech. Publ., 1947)
6. Smirnov B M *Physics of Atoms and Ions* (New York: Springer, 2003)
7. Townsend J S *Proc. R. Soc. London A* **80** 207 (1908)
8. Townsend J S *Proc. R. Soc. London A* **81** 464 (1908)
9. Townsend J S, Bailey V A *Philos. Mag.* **43** 593 (1922)
10. Townsend J S, Bailey V A *Philos. Mag.* **44** 1033 (1922)
11. Townsend J S, Bailey V A *Philos. Mag.* **46** 657 (1923)
12. Townsend J S, Bailey V A *Philos. Trans. A* **193** 129 (1899)
13. Townsend J S, Bailey V A *Philos. Trans. A* **195** 259 (1900)
14. Huxley L G, Crompton R W, Elford M T *Brit. J. Appl. Phys.* **17** 1237 (1966)
15. Huxley L G, Crompton R W *The Diffusion and Drift of Electrons in Gases* (New York: Wiley, 1974) [Translated into Russian (Moscow: Mir, 1977)]
16. Einstein A *Ann. Physik* **17** 549 (1905)
17. Einstein A *Ann. Physik* **19** 371 (1906)
18. Engel A, Steenbeck M *Elektrische Gasenladungen, ihre Physik und Technick* (Berlin: J. Springer, 1932 – 1934) [Translated into Russian (Moscow–Leningrad: ONTI, 1935)]
19. Loeb L B *Fundamental Processes of Electrical Discharge in Gases* (New York: J. Wiley & Sons, 1939) [Translated into Russian (Moscow–Leningrad: GITTL, 1950)]
20. Penning F M *Electrical Discharges in Gases* (New York: Macmillan, 1957) [Translated into Russian (Moscow: IL, 1960)]
21. Brown S C *Basic Data of Plasma Physics* (Cambridge: Technol. Press of the M.I.T., 1959) [Translated into Russian (Moscow: Gosatomizdat, 1961)]
22. Llewellyn-Jones F *The Glow Discharge, an Introduction to Plasma Physics* (London: Methuen, 1966)
23. Hoyaux M F *Arc Physics* (Berlin: Springer, 1968)
24. Engel A *Ionized Gases* (Oxford: Clarendon Press, 1955) [Translated into Russian (Moscow: Fizmatgiz, 1959)]
25. Granovskii V L *Elektricheskii Tok v Gaze. Ustanovivshiisya Tok* (Electric Current in Gas. Steady Current) (Eds L A Sena, V E Golant) (Moscow: Nauka, 1971)
26. Nasser E *Fundamentals of Gaseous Ionization and Plasma Electronics* (New York: Wiley-Intersci., 1971)
27. Aleksandrov A F, Rukhadze A A *Fizika Sil'notochnykh Elektrorazryadnykh Istochnikov Sveta* (Physics of High-Current Electric-Discharge Light Sources) (Moscow: Atomizdat, 1976)
28. Chapman B *Glow Discharge Processes* (New York: Wiley, 1980)
29. Raizer Yu P *Fizika Gazovogo Razryada* (Gas Discharge Physics) (Moscow: Nauka, 1987) [Translated into English (Berlin: Springer-Verlag, 1991)]
30. Neumann W *The Mechanism of the Thermoemitting Arc Cathode* (Berlin: Academie-Verlag, 1987)

31. Velikhov E P, Kovalev A S, Rakhimov A T *Fizicheskie Yavleniya v Gazorazryadnoi Plazme* (Physical Phenomena in Gas-Discharge Plasma) (Moscow: Nauka, 1987)
32. Mesyats G A, Proskurovsky D I *Impul'snyi Elektricheskii Razryad v Vakuume* (Pulsed Electrical Discharge in Vacuum) (Novosibirsk: Nauka, 1984) [Translated into English (Berlin: Springer-Verlag, 1989)]
33. Smirnov B M *Gazorazryadnaya Plazma* (Gas Discharge Plasma) (Moscow: Izd. MFTI, 1992)
34. Lagarkov A N, Rutkevich I M *Ionization Waves in Electrical Breakdown in Gases* (New York: Springer-Verlag, 1994)
35. Lieberman M A, Lichtenberg A J *Principles of Plasma Discharges and Materials Processing* (New York: Wiley, 1994)
36. Raizer Y P, Shneider M N, Yatsenko N A *Radio-Frequency Capacitive Discharges* (Boca Raton: CRC Press, 1995)
37. Korolev Yu D, Mesyats G A *Physics of Pulsed Breakdown in Gases* (Ekaterinburg: URO Press, 1998)
38. Debye P, Hückel E *Phys. Z.* **24** 185 (1923)
39. Klyarfel'd B N, Guseva L G, Pokrovskaya-Soboleva A S *Zh. Tekh. Fiz.* **36** 704 (1966) [*Sov. Phys. Tech. Phys.* **11** 520 (1966)]
40. Emeleus K G *Z. Phys.* **268** 175 (1974)
41. Sena L A, Ryazantseva O L *Zh. Tekh. Fiz.* **48** 1643 (1978) [*Sov. Phys. Tech. Phys.* **23** 930 (1978)]
42. Melekhin V N, Naumov N Yu *Zh. Tekh. Fiz.* **54** 1521 (1984) [*Sov. Phys. Tech. Phys.* **29** 888 (1984)]
43. Petrović Z Lj, Phelps A V *Phys. Rev. E* **47** 2806 (1993)
44. Kaganovich I D, Fedotov M A, Tsendin L D *Zh. Tekh. Fiz.* **64** (3) 22 (1994) [*Tech. Phys.* **39** 241 (1994)]
45. Kolobov V I, Fiala A *Phys. Rev. E* **50** 3018 (1994)
46. Schottky W *Phys. Z.* **25** 342 (1924)
47. Biberman L M, Vorob'ev V S, Yakubov I T *Usp. Fiz. Nauk* **128** 233 (1979) [*Sov. Phys. Usp.* **22** 411 (1979)]
48. Biberman L M, Vorob'ev V S, Yakubov I T *Kinetika Neravnovesnoi Nizkotemperaturnoi Plazmy* (Kinetics of Nonequilibrium Low-Temperature Plasmas) (Moscow: Nauka, 1982) [Translated into English (New York: Consultants Bureau, 1987)]
49. Boltzmann L *Wien. Ber.* **66** 275 (1872)
50. Lorentz H A *The Theory of Electrons* (New York: Stechert, 1923)
51. Fokker A D *Ann. Physik* **43** 810 (1914)
52. Planck M *Preuß. Akad. Wiss. Phys. Mat. Kl.* 324 (1917)
53. Davydov B *Phys. Z. Sowjetunion* **9** 433 (1936)
54. Gurevich L E *Osnovy Fizicheskoi Kinetiki* (Fundamentals of Physical Kinetics) (Leningrad–Moscow: GITTL, 1940)
55. Landau L D *Zh. Eksp. Teor. Fiz.* **7** 203 (1937)
56. Smirnov B M *Principles of Statistical Physics* (Weinheim: Wiley-VCH, 2007)
57. Pack J L et al. *J. Appl. Phys.* **71** 5363 (1992)
58. Druyvesteyn M J *Physica* **10** 61 (1930)
59. Druyvesteyn M J, Penning F M *Rev. Mod. Phys.* **12** 87 (1940)
60. Allis W P, in *Handbuch der Physik* Vol. 21 (Ed. S Flügge) (Berlin: Springer, 1956) p. 383
61. Ramsauer C *Ann. Physik* **66** 545 (1921)
62. Ramsauer C *Ann. Physik* **72** 345 (1923)
63. Ramsauer C, Kollath R *Ann. Physik* **3** 536 (1929)
64. Dutton J *J. Phys. Chem. Ref. Data* **4** 577 (1975)
65. von Smit J A *Physica* **3** 543 (1936)
66. Loeb L B *Fundamental Processes of Electrical Discharge in Gases* (New York: J. Wiley & Sons, 1939)
67. Smirnov B M *Physics of Ionized Gases* (New York: John Wiley, 2001)
68. Smirnov B M *Vozbuzhdennye Atomy* (Excited Atoms) (Moscow: Energoizdat, 1982)
69. Kolokolov N B, in *Khimiya Plazmy* (Plasma Chemistry) Vol. 12 (Ed. B M Smirnov) (Moscow: Energoatomizdat, 1985) p. 56
70. Smirnov B M *Reference Data on Atomic Physics and Atomic Processes* (New York: Springer, 2008)
71. Smirnov B M *Plasma Processes and Plasma Kinetics* (Berlin: Wiley-VCH, 2007)
72. Platonov A A et al. *Zh. Tekh. Fiz.* **76** (7) 22 (2006) [*Tech. Phys.* **51** 841 (2006)]
73. Kudryavtsev A A , Morin A V, Tsendin L D *Zh. Tekh. Fiz.* **78** (8) 71 (2008) [*Tech. Phys.* **53** 1029 (2008)]
74. Kruithof A A, Penning F M *Physica* **4** 430 (1937)
75. Kruithof A A, Druyvesteyn M J *Physica* **4** 450 (1937)
76. Smirnov B M *Fizika Slaboionizovannogo Gaza v Zadachakh s Resheniyami* (Physics of Weakly Ionized Gases: Problems and Solutions) (Moscow: Nauka, 1978) [Translated into English (Moscow: Mir Publ., 1981)]
77. Eletskii A V, Kulagin V D *Fiz. Plazmy* **5** 98 (1979) [*Sov. J. Plasma Phys.* **5** 55 (1979)]
78. Eletskii A V, Smirnov B M *Gazovye Lazery* (Gas Lasers) (Moscow: Atomizdat, 1971)
79. Eletskii A V, Smirnov B M *J. Sov. Laser Res.* **7** 207 (1986)
80. Elenbaas W *The High Pressure Mercury Vapour Discharge* (Amsterdam: North-Holland, 1951)
81. Baranov V Yu, Ul'yanov K N *Zh. Tekh. Fiz.* **39** 249, 259 (1969) [*Sov. Phys. Tech. Phys.* **14** 176, 183 (1969)]
82. Rakhimov A T, Ulinich F R *Dokl. Akad. Nauk SSSR* **187** 72 (1969) [*Sov. Phys. Dokl.* **14** 655 (1969)]
83. Eletskii A V, Rakhimov A T, in *Khimiya Plazmy* (Plasma Chemistry) Vol. 4 (Ed. B M Smirnov) (Moscow: Atomizdat, 1977) p. 123
84. Pupp W *Phys. Z.* **33** 844 (1932)
85. Wojaczek K *Beitr. Plasmaphys.* **1** 30 (1960)
86. Wojaczek K *Beitr. Plasmaphys.* **6** 319 (1966)
87. Nedospasov A V *Usp. Fiz. Nauk* **94** 439 (1968) [*Sov. Phys. Usp.* **11** 174 (1968)]
88. Pekarek L *Usp. Fiz. Nauk* **94** 463 (1968) [*Sov. Phys. Usp.* **11** 188 (1968)]
89. Tsendin L D *Zh. Tekh. Fiz.* **40** 1600 (1970) [*Sov. Phys. Tech. Phys.* **15** 1245 (1971)]
90. Nedospasov A V, Khait V D *Kolebaniya i Neustoichivosti Nizkotemperaturnoi Plazmy* (Oscillations and Instabilities of Low-Temperature Plasma) (Moscow: Nauka, 1979)
91. Tsendin L D *Zh. Tekh. Fiz.* **52** 635, 643 (1982)
92. Golubovskii Yu B et al. *Phys. Rev. E* **63** 036409 (2001)
93. Golubovskii Yu B et al. *Phys. Rev. E* **72** 026414 (2005)
94. Kolobov V I *J. Phys. D* **39** R487 (2006)
95. Bernstein I B, Holstein T *Phys. Rev.* **94** 1475 (1954)
96. Tsendin L D *Plasma Sources Sci. Technol.* **4** 200 (1995)
97. Kolobov V I, Godyak V A *IEEE Trans. Plasma Sci.* **23** 503 (1995)
98. Golubovskii Yu B et al. *J. Phys. D* **32** 1391 (1999)
99. Nerushev O A et al. *Pis'ma Zh. Tekh. Fiz.* **30** (3) 47 (2004) [*Tech. Phys. Lett.* **30** 106 (2004)]
100. Novopashin S A et al. *Pis'ma Zh. Tekh. Fiz.* **33** (5) 25 (2007) [*Tech. Phys. Lett.* **33** 196 (2007)]
101. Novopashin S A, Radchenko V V, Sakharov S Z *IEEE Trans. Plasma Sci.* **36** 998 (2008)
102. Gunn J B *Solid State Commun.* **1** 88 (1963)
103. Gunn J B *Usp. Fiz. Nauk* **89** 147 (1966); "The Gunn effect" *Int. Sci. Technol.* (46) 43 (1965)
104. Kadomtsev B B *Kollektivnye Yavleniya v Plazme* (Moscow: Nauka, 1988) [Translated into English: "Cooperative effects in plasmas", in *Reviews of Plasma Physics* Vol. 22 (Ed. V D Shafranov) (New York: Kluwer Acad./Consultants Bureau, 2001) p. 1]
105. Nefedov A P, Petrov O F, Fortov V E *Usp. Fiz. Nauk* **167** 1215 (1997) [*Phys. Usp.* **40** 1163 (1997)]
106. Fortov V E, Khrapak A G, Yakubov I T *Fizika Neideal'noi Plazmy* (Physics of Nonideal Plasma) (Moscow: Fizmatlit, 2004)
107. Smirnov B M *Usp. Fiz. Nauk* **170** 495 (2000) [*Phys. Usp.* **43** 453 (2000)]