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Sixty years of broken symmetries in quantum physics

(from the Bogoliubov theory of superfluidity to the Standard Model)

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Abstract. **This is a retrospective historical review of the ideas that led to the concept of spontaneous symmetry breaking (SSB), the issue that has been implemented in quantum field theory in the form of the Higgs mechanism. The key stages covered include: the Bogoliubov microscopic theory of superfluidity (1946); the Bardeen – Cooper – Schrieffer – Bogoliubov microscopic theory of superconductivity (1957); superconductivity as the superfluidity of Cooper pairs (Bogoliubov, 1958); the extension of the SSB concept to simple quantum field models (early 1960s); and the triumph of the Higgs model in the electroweak theory (early 1980s). The role and status of the Higgs mechanism in the current Standard Model are discussed.**

"Phase transition in a quantum system is typically accompanied by spontaneous symmetry breaking"
Folklore of the middle of the 20th century

1. Introduction

Spontaneous symmetry breaking (SSB) is a well-established term in quantum theory; its essence is simple. We mean a physical system that can be described by expressions (Lagrangian, Hamiltonian, equations of motion) obeying some symmetry, while a real physical state of the system corresponding to some particular solution of the equations of motion does not obey this symmetry. We meet such a case when the lowest of possible symmetric states does not provide the system with the absolute energy minimum and turns out to be unstable. A particular lowest state is not unique; a full collection of them forms a symmetric set. The real cause of symmetry breaking and transition of the system to some of the lowest nonsymmetric states is usually an arbitrary small asymmetric perturbation.

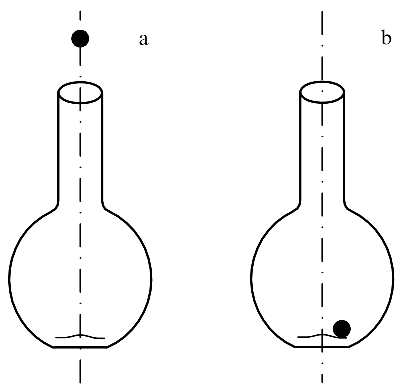

Figure 1. A simple mechanical system illustrating spontaneous symmetry breaking: (a) initial state; (b) final state.

As a simple illustration, we take a system of a tiny ball and an empty vessel with a convex bottom. Let the vessel, which is a rotating body, stand vertically and the ball be located above it, just on the axis (Fig. 1a). The system is symmetric with respect to rotation around the vertical axis. Let the ball fall down due to the force of gravity. Upon reaching the bottom, the ball does not stand at the center of the convex surface and rolls down to some point at the periphery of the bottom (Fig. 1b). Thus, the initial conditions are symmetric, but the final state is not.

A more pithy example is a magnetized ferromagnet. The compass was known to the ancient Chinese, but only at the beginning of the 18th century did an Oxford professor of astronomy, John Keill [1], notice that heating destroys the magnetic property:[1] "...if a Loadstone be put into the Fire, insomuch that the internal Structure of its Parts be changed or

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[1] The quotation is given with the orthography of the original.

wholly destroyed, then it will lose all its former Virtue, and will scarce differ from other Stones." A systematic study of the thermal properties of magnetic substances was undertaken by Pierre Curie, who discovered a sharp decrease in magnetization as the temperature approached the critical value, now called the Curie point. Above the critical temperature, ferromagnetism disappears. With decreasing the temperature from the critical point, the magnetization direction may be reversed if the ferromagnet is placed in an external field opposite to the reference direction of the magnetization and then this field is removed. Thus, ferromagnetic magnetization is related to two important notions. First, it is spontaneous symmetry breaking, because the external field may be chosen as weak as one wishes. Second, the value of magnetization is just the quantity that was called the order parameter in the Landau theory of phase transitions [2] (see also pp. 234–252 in [3]). This parameter is nonzero in the ferromagnetic region and continuously decreases in approaching the critical point, where it vanishes.

The main subject of this contribution is set out on the material of quantum statistics (superfluidity and superconductivity) with a smooth transition to quantum field theory, as far as the recent increase of interest in SSB stems from the quantum-field context. The previous contribution by Dremin, which plunged the audience into the bulk of technical details of future experiments at the Large Hadron Collider (LHC), reminded us of the 'Higgs expectations,' which are closely related to SSB.

Incidentally, in our exposition, we mention two diverse and partially opposing ways of conceiving the main ideas of the structure of the physical world, that is, ways of constructing the physical theory.

The initial nurturing material of our science, data from observations, are to be systematized and understood. To put things in order, a phenomenological model is usually constructed based on some physical idea; the model invested in a mathematical form, the form of a physical law. An important criterion of success of the scheme and its grounds is not only a reasonable correlation of the initial data but also the possibility of predicting new effects with a clear-cut implementation. This is the usual path of the phenomenologist, the way 'from a phenomenon to a theoretical scheme' and back.

Along with this, many important steps in the building of the physical theory are performed in another, more speculative way. We recall the unification of celestial and terrestrial gravity, electricity and magnetism, as well as the recently discovered principle of *dynamics from symmetry* that formed the foundation of the electroweak theory and quantum chromodynamics.

Adherents to this way of thinking, people who try to start from deep and profound ideas, from primary *ab initio* principles, are known as 'reductionists'.[2] In statistical physics, they are typically adherents of the microscopic approach.[3]

At the same time, the reductionists comprise an overwhelming majority of the founders of basic fundamentals of modern physics, like relativity, quantum mechanics, and the theory of quantum field theory.

Meanwhile, in our opinion, we should not be unduly carried away with the opposing points of view to these two modes of reflection. An important detail is that between equations [e.g., equations of classical mechanics or Maxwell equations in a medium (plasma)] and laws that describe a sequel of observed events (e.g., laws of a planet's motion or the Meissner law in a superconductor), there is a space, a logical gap. Only here does the phenomenology work. Therefore, the efforts by reductionists and phenomenologists, in the very end, supplement each other. We now turn to examples.

In the early 1930s, by heuristic reasonings, Fermi devised a four-fermion Lagrangian for the weak nuclear force, initially with a single coupling constant G_{F}. The Fermi Lagrangian, with subsequent modifications, played an important role in understanding and regulating numerous data on lepton dynamics. A modification of the Fermi model of the mid-50s included up to 10 coupling parameters.

A more profound understanding of the weak interaction was achieved a quarter of a century later, in the Glashow–Salam–Weinberg (GSW) gauge theory of electroweak interaction with its massive vector W and Z bosons that appeared to be the 'missing link' transmitters of forces between lepton currents. The origin of the heavy masses ($\sim$ 90 GeV) of these particles is related to SSB. The GSW theory is elegant and rather simple, being based on the new general principle 'dynamics from symmetry.' The transition to a deeper level greatly reduced the number of parameters.

In 1941, quite soon after the experimental discovery of superfluidity, Lev Davidovich Landau, "just in time" as Kapitza said, devised a phenomenological model [6] (see also pp. 352–385 in [3] and [7]) that quite well described some essential properties of HeII — thermodynamics, kinetics, and so on.

The pith of Landau's reasoning was the assumption of the dominating role of the collective quantum effect. An analysis at the microscopic level appeared five years later as a model of a weakly imperfect Bose gas, when Nikolai Nikolaevich Bogoliubov proposed treating atoms of HeII as weakly repulsing particles interacting with a condensate. Here, the key element consisted in the assumption that the condensate contained a macroscopically large number of helium atoms. That was the hypothesis that led to the elucidation of the nature of the Landau collective effect. In his paper [8] (see also pp. 108–112 in [5] and [9]), the famous (u, v) transformation was introduced, which is closely related to the spontaneous breaking of phase symmetry responsible for the conservation of the number of particles.

The third example, finally, is the remarkable 1950 paper by Ginzburg and Landau [10] (see also pp. 126–152 in [11]) — the phenomenological description of superconductivity by a specially devised, rather abstract, wave-like function $\Psi(\mathbf{r})$ (the two-component order parameter) of the collective of superconducting electrons. However, the understanding of the physical idea of the function $\Psi(\mathbf{r})$ appeared 8–9 years later, after elaborating the Bardeen–Cooper–Schrieffer and,

[2] Because they tend to *reduce* the observed variety of phenomena to a small number of simple notions and general principles.

[3] We quote the definition formulated by Bogoliubov in the 1958 paper "Basic principles of the theory of superfluidity and superconductivity" [4] (see also pp. 297–309 in Ref. [5]): "The goal of macroscopical theory can be said as obtaining equations, similar to classical equations of mathematical physics, that describe a majority of data related to macroscopical objects under study..." and then "In microscopical theory, a more profound problem is posed: to understand an intrinsic mechanism of the phenomena, in terms of quantum mechanics notions and equations... Here, in particular, one should also obtain relations between dynamical variables; relations that yield equations of macroscopical theory."

particularly, Bogoliubov microscopic constructions, explicitly taking the interaction of electrons with ion lattice vibrations into account.

2. Spontaneous symmetry breaking in quantum statistics

2.1 Superfluidity

The theory of superfluidity is a good example of the interconnection between phenomenological ideas and mathematical constructions. The original explanation of the phenomenon of superfluidity offered by Landau [6, 7] was based on the idea that at low temperatures, the properties of liquid ^{4}He are determined by collective excitations (phonons) rather than a quadratic spectrum of individual particle excitations. It follows from this assumption that in moving with a velocity not exceeding a certain critical value, it is impossible to slow down the liquid by transferring energy and momentum from the wall to individual atoms because the linear form of the phonon spectrum does not allow the energy and momentum conservation to be satisfied simultaneously. The need for agreement between the form of the spectrum and the thermodynamic properties of liquid helium motivated Landau to introduce particular excitations, in addition to phonons, with a quadratic spectrum beginning with a certain energy gap, which he called rotons.[4]

Bogoliubov's theory is based on the physical assumption that in a weakly nonideal Bose gas, there is a condensate akin to an ideal Bose gas. The existence of the Bose condensate leads to a unique wave function of the whole system, i.e., a collective effect. Therefore, the presence of even a weak interaction transforms single-particle excitations into the spectrum of collective excitations. To calculate this spectrum, Bogoliubov inferred that at low temperatures, the Bose condensate contains a macroscopically large number of particles N_0,[5] of the order of the Avogadro constant N_{A}, and hence matrix elements of the creation and annihilation operators of particles in the condensate are proportional to the 'large' number $\sim \sqrt{N_0}$ and the main contribution to the system dynamics comes from the processes of particle transition from the condensate to the continuous spectrum and back to the condensate.

Following [8, 9], we start with the secondary-quantized description of the system of Bose particles in the coordinate representation. The Hamiltonian of the system with a pair interaction is given by

$$H = -\frac{\hbar^2}{2m}\int \mathrm{d}x\, \Psi^*(x)\Delta\Psi(x) + \int \mathrm{d}x \int \mathrm{d}y\, \Psi^*(x)\Psi(x)\, V(x-y)\, \Psi^*(y)\Psi(y)\,. \qquad (1)$$

Extraction of the condensate corresponds to passing from the Ψ function to the sum

$$\Psi(x) = C + \phi(x)\,, \qquad \Psi^*(x) = C + \phi^*(x) \qquad (2)$$

of a 'large constant' C (containing the identity operator) and a 'small operator' $\phi(x)$. Because the Fourier transform of a constant is the Dirac delta function, in the discrete momentum representation

$$\Psi(x) = \frac{1}{\sqrt{V}}\sum_k a_k \exp\frac{\mathrm{i}kx}{\hbar}\,, \quad \phi(x) = \frac{1}{\sqrt{V}}\sum_{p\neq 0} b_p \exp\frac{\mathrm{i}px}{\hbar} \qquad (3)$$

we can write

$$a_k = \delta_{k,0}\, c + (1-\delta_{k,0})\,\delta_{k,p}\, b_p\,, \qquad c = \frac{C}{\sqrt{V}} = \sqrt{\frac{N_0}{V}}, \qquad (4)$$

where a_k, a_k^* and b_p, b_p^* are operators with the Bose commutation relations

$$a_k\, a_q^* - a_q^*\, a_k = \delta_{k,q}\,, \qquad b_p\, b_l^* - b_l^*\, b_p = \delta_{p,l}\,.$$

Under the assumption of the decisive role of the condensate, we can neglect any terms responsible for the interaction of above-condensate atoms with each other. Then the total Hamiltonian of a Bose gas in the momentum representation

$$H_{\mathrm{B}_0} = \sum_k T(k)\, a_k^+ a_k + \sum_{k,q} v(k_1-k_2)\, a_{k_1}^+ a_{k_2}\, a_{q_1}^+ a_{q_2}\, \delta_{k_1-k_2,\, q_1-q_2}\,, \qquad T(k) = \frac{k^2}{2m}\,, \qquad (5)$$

with the Fourier transform $v(k) > 0$ of the potential energy of the weak pair repulsion of helium atoms[6] results in the Bogoliubov Hamiltonian [8, 9] of the weakly nonideal Bose gas model:[7]

$$H_{\mathrm{B}_0} \to H_0 + H_{\mathrm{B}_1}; \qquad H_0 = \frac{v(0)\,\mathrm{N}_0^2}{2V}\,, \qquad \mathrm{N}_0 = a_0^+ a_0\,,$$

where $\mathrm{N}_0 = a_0^+ a_0$ is the particle number (i.e., occupation number) operator in the condensate and

$$H_{\mathrm{B}_1} = \sum_{p\neq 0}\left\{T(p) + \frac{\mathrm{N}_0 v(p)}{V}\right\} b_p^+ b_p + \frac{1}{2V}\sum_{p\neq 0} v(p)\{b_p^+ b_{-p}^+ a_0 a_0 + a_0^+ a_0^+\, b_p b_{-p}\}\,. \qquad (6)$$

The second sum describes particle transitions from the condensate and back, i.e., the production of pairs with zero total momentum from the condensate and their annihilation.

Bogoliubov's next step rested on the fact that the operators a_0 and a_0^+ of condensate atoms enter the Hamiltonian in the combinations $a_0/\sqrt{V}$ and $a_0^+/\sqrt{V}$ and approximately commute with each other in the large-volume limit. At the same time, their matrix elements contain $\sqrt{N_0}$. Therefore, the *operators* a_0 *and* a_0^+ *can be treated as numbers* $\sqrt{N_0/V}$, and the operator N_0, divided by V, can be replaced by the finite density of Bose condensate $\rho_0 = N_0/V$. As a result, the Hamiltonian H_{B_1} becomes a homogeneous bilinear form in

[4] See Fig. 2a below, in which formulas (2.2) and (2.3) from [6] are used.

[5] Bogoliubov's intuitive guess later received direct data support (see [12–14] and the note added at the end of the paper).

[6] Summation is over the 3-dimensional discrete momentum space corresponding to the system final volume V in the coordinate space. The three-dimensional Kronecker symbol is related to the three-dimensional delta function δ by $V\delta_{k,q} \to (2\pi)^3\delta(k-q)$ as $V\to\infty$.

[7] Here and below in Section 2.1, the momentum p, in contrast to k and q, does not take a zero value, being referred only to above-condensate particles.

operators with nonzero momentum:

$$H_{B_2} = \sum_{p\neq 0}\Big\{\big[T(p)+\rho_0 v(p)\big]\, b_p^+ b_p + \frac{\rho_0 v(p)}{2}\big[b_p^+ b_{-p}^+ + b_p b_{-p}\big]\Big\}. \quad (7)$$

We note that the initial expression (5), like (6), is invariant under the phase transformation[8] of the operators

$$a_k \to \exp(\mathrm{i}\phi)\, a_k\,, \qquad a_k^+ \to \exp(-\mathrm{i}\phi)\, a_k^+\,, \quad (8)$$

which corresponds to the particle number conservation. Indeed, the Hamiltonian H_{B_1}, like H_{B_0}, commutes with the total particle number operator $\mathrm{N} = \sum_k a_k^+ a_k$. However, this property is not inherent in the approximation H_{B_2}, which does not contain condensate operators. Precisely this step, i.e., a transition to bilinear (exactly solvable) approximate Hamiltonian (7), leads to spontaneous symmetry breaking.

The diagonalization of the bilinear Hamiltonian H_{B_2} is not a particular problem and can be accomplished by the famous Bogoliubov canonical (u, v) transformation

$$\begin{aligned} &b_p \to \xi_p = u_p b_p + v_p b_{-p}^+\,, \\ &b^+ \to \xi_p^+ = u_p b_p^+ + v_p b_{-p}\,, \qquad u_p^2 - v_p^2 = 1\,, \end{aligned} \quad (9)$$

with real coefficients 'braiding' the creation and annihilation operators. Thus, the new operators ξ_p and ξ_p^+ are a superposition of the old ones. A 'hyperbolic rotation' of operators (9) corresponds to the unitary transformation[9]

$$\begin{aligned} &\xi_p = U_\alpha^{-1} b_p U_\alpha = u_p b_p + v_p b_{-p}^+\,, \\ &U_\alpha = \exp\Big[\sum_p \alpha(p)(b_p^+ b_{-p}^+ - b_p b_{-p})\Big]\,, \end{aligned} \quad (10)$$

where the coefficient $\alpha(p)$ depends on the parameters of the initial Hamiltonian. The transformed Hamiltonian

$$H = \sum_{p\neq 0} E(p)\,\xi_p^+ \xi_p \quad (11)$$

has the spectrum

$$E(p) = \sqrt{T^2(p) + T(p)\,\rho_0\, v(p)}\,. \quad (12)$$

The new ground state

$$\Psi_0(\alpha) = U_\alpha^{-1}\,\Phi_0 = \exp\Big(-\sum_p \alpha(p)\, b_p^+ b_{-p}^+\Big)\Phi_0 \quad (13)$$

includes superpositions of correlated pairs with a total zero momentum.[10]

Transformation (9), (10) leads to the spectrum of collective excitations in (12). The dependence of energy on momentum has an initial linear part, which is necessary for explaining superfluidity, and a nonlinear part with flexure that places Landau's rotons[11] into the required position (see Fig. 2b). The absence of single-particle excitations, as in the phenomenological approach, underlies the formulation of the model, although the operator form of the canonical transformation gives information about the nature of collective excitations and the structure of the new ground state (13).

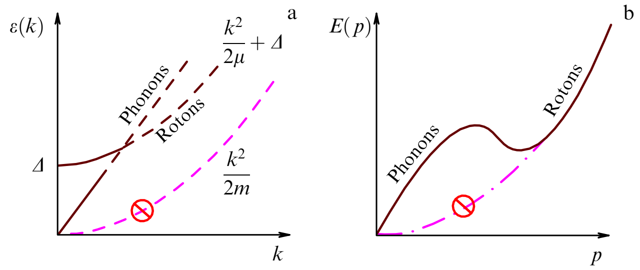

Figure 2. (a) Spectrum of phonons and rotons in the Landau phenomenological theory; (b) The Bogoliubov–Landau spectrum of collective excitations following from expression (12) of the Bogoliubov microscopic theory [8, 9].

As mentioned above, the initial Hamiltonian of a weakly nonideal Bose gas (5) is invariant under gauge transformation (8) providing conservation of the total particle number N. However, Bogoliubov's bilinear Hamiltonian (7) does not have this property, which corresponds to symmetry breaking. This Hamiltonian appeared as a result of the substitution of operator 'condensate' contributions (at $k = 0$) by c-numbers. This substitution assumes nonzero values of vacuum averages $\langle a_0^+\rangle$ and $\langle a_0\rangle$, which are connected with a transition to the new vacuum by the unitary operator[12]

$$U_c = \exp\big[c(a_0^* - a_0)\big]\,, \qquad a_q \to U_c^{-1} a_q U_c = b_q + c a_0\,. \quad (14)$$

2.2 Superconductivity

Another example of spontaneous symmetry breaking is the phenomenon of superconductivity, where phase invariance is violated, as in the case of phase transition to a superfluid state. Although superconductivity was discovered in 1911, significantly earlier than ^{4}He superfluidity, theoretical insight into the phenomenon of superconductivity was gained much later than the explanation of superfluidity. A breakthrough along this line was the phenomenological theory suggested by Ginzburg and Landau (GL). In the GL theory [10], a superconducting state is described by an effective 'wave function' of superconducting electrons playing the role of a two-component order parameter

$$\Psi(\mathbf{r}) = \big|\Psi(\mathbf{r})\big|\,\exp\big[\mathrm{i}\Phi(\mathbf{r})\big]\,. \quad (15)$$

[8] For historical reasons, transformation (8) is often called a gauge transformation, which might inevitably lead to association with 'electromagnetic gauge transformation' (as, e.g., in [15]), i.e., with the law of electric charge conservation. The last Nobel press release [16] contains this error.

[9] For technical details, see, e.g., § 12 and Appendix IV in textbook [17].

[10] It is interesting to note that a procedure similar to the Bogoliubov (u, v) transformation is used (see, e.g., Ref. [18]) in quantum optics in determining 'squeezed' states $\Psi_0(q) \sim \exp\{\sum_k a(k)[b_k^+ b_{q-k}^+]\}\,\Phi_0$, where an important role is played by correlated pairs of photons with a nonzero total momentum q.

[11] The curve with a flexure was published by Landau in paper [19] (also see pp. 32–34 in [11]) written soon after the discussion with Bogoliubov of his presentation of paper [8] given on 21 October 1946. In his paper, Landau used Bogoliubov's idea of a unique spectrum of collective excitations in a quantum liquid. In a more detailed paper [20] (see also pp. 42–46 in [11] and [21]), he emphasized Bogoliubov's priority: "It is worthwhile to point out that N.N. Bogoliubov has recently succeeded in determining in a general form an energy spectrum of Bose–Einstein gas with a weak interaction between particles with the help of ingenious application of the second quantization." Therefore, we consider it appropriate to call the curve in Fig. 2b the Bogoliubov–Landau spectrum.

[12] See footnote 8 above.

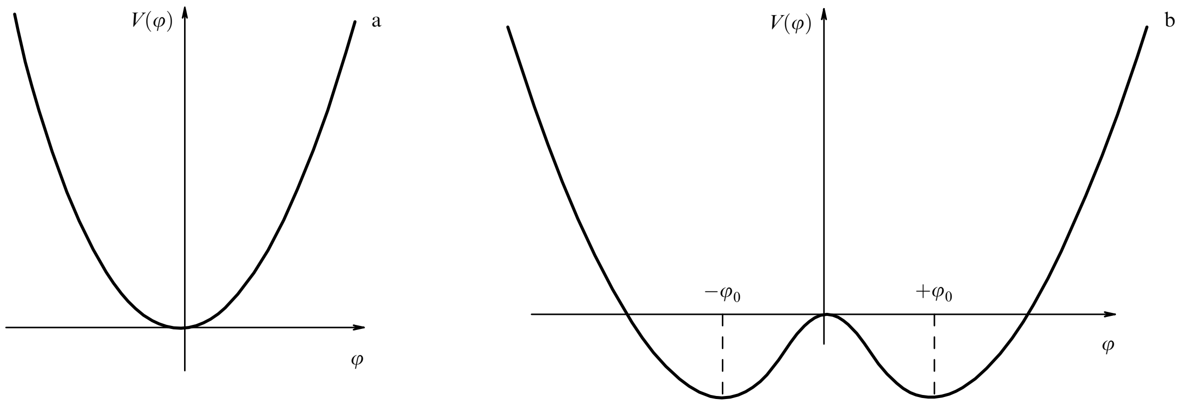

Figure 3. (a) Potential energy of the free scalar field with mass $m^2 > 0$. (b) Potential function of the scalar field with self-interaction and an unstable symmetric state.

The equilibrium properties of a superconductor are defined there by the free-energy functional depending on $\Psi(\mathbf{r})$ and an external magnetic field $\mathbf{B}(\mathbf{r})$:

$$F(\Psi) = F_{n0} + \int d\mathbf{r} \left\{ \frac{|\mathbf{B}|^2}{8\pi} + a|\Psi|^2 + \frac{1}{2} b|\Psi|^4 + \sum_\alpha \frac{1}{2m^*} \left| \left(-i\hbar\nabla_\alpha - \frac{q}{c} A_\alpha \right) \Psi(\mathbf{r}) \right|^2 \right\}, \quad (16)$$

where F_{n0} is free energy in the normal state, $\mathbf{B} = \operatorname{rot} \mathbf{A}$, and q and m^* are the effective charge and mass of superconducting electrons. In the original paper, those were arbitrary parameters, put equal to the electron charge and mass based on general physical grounds. The modulus of order parameter (15) is proportional to the density of superconducting electrons n_s, and its phase $\Phi(\mathbf{r})$ defines the superconducting current

$$j_\alpha = \frac{q\hbar}{m^*} |\Psi|^2 \nabla_\alpha \Phi(\mathbf{r}) . \quad (17)$$

The essential feature of the GL theory is that at the temperature T_c of a superconducting transition, the coefficient $a \sim (T - T_c)$ changes sign, while the positive coefficient b, the effective mass m^*, and the charge q are independent of the temperature. In such a case, GL functional (16) describes a transition from the normal state with $\Psi = 0$ to a superconducting state at $T = T_c$, at which a nonzero order parameter $\Psi \neq 0$ arises. In the absence of a magnetic field, a second-order phase transition with the mean-field critical indices occurs. In the framework of the GL theory, the behavior of a superconductor in an external magnetic field, including the Abrikosov vortex lattice in second-type superconductors, was successfully described [22]. At the same time, the nature of the superconducting transition remained unclear.

We comment on the structure of a 'potential' term in expression (16),

$$V(\varphi) = a\varphi^2 + \frac{b}{2} \varphi^4 , \qquad \varphi = |\Psi| , \quad (18)$$

in terms of a nonlinear (classical or quantum) oscillator. At $T > T_c$, the coefficient a is positive and can be expressed in terms of mass, $a \to m^2/2$. The first term dominates at small values of φ and corresponds to an ordinary oscillator, as in Fig. 3a. Below the critical temperature, this term is negative (Fig. 3b) and the value $\varphi = 0$ becomes unstable, which results in a spontaneous breaking of the discrete symmetry related to the reflection $\varphi \to -\varphi$. Equation (18) and illustrations in Fig. 3 correspond to a one-component order parameter. The two-component case corresponds to illustrations in Fig. 1 describing violation of the continuous symmetry of rotation.

The microscopic theory of superconductivity was developed only in 1957 by Bardeen, Cooper, and Schrieffer (BCS) [23, 24] and Bogoliubov [25, 26] (see also § 2 in [27], [28], and pp. 200–208 in [5]). BCS considered a simplified model in which the interaction of electrons due to an exchange of phonons was substituted for an effective attraction of electrons near the Fermi surface

$$H_{BCS} = \sum_{\mathbf{k},\sigma} \varepsilon(\mathbf{k})\, c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} - V_{BCS} \sum_{\mathbf{k},\mathbf{k}'} v(\mathbf{k},\mathbf{k}')\, c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} c_{-\mathbf{k}'\downarrow} c_{\mathbf{k}'\uparrow} ,$$
$$v_{\mathbf{k},\mathbf{k}'} = \begin{cases} 1, & |\varepsilon(\mathbf{k}) - \varepsilon(\mathbf{k}')| < \omega_{ph} , \\ 0, & |\varepsilon(\mathbf{k}) - \varepsilon(\mathbf{k}')| > \omega_{ph} , \end{cases} \quad (19)$$

where $c^\dagger_{\mathbf{k}\sigma}(c_{\mathbf{k}\sigma})$ are the electron creation (annihilation) operators with momentum $\mathbf{k}$ and spin $\sigma = (\uparrow, \downarrow) = (+1/2, -1/2)$, obeying the Fermi anticommutation relations $[c_{\mathbf{k}\sigma}, c^\dagger_{\mathbf{k}'\sigma'}]_+ = \delta_{\mathbf{k},\mathbf{k}'}\delta_{\sigma,\sigma'}$. The Bloch electron energy in the normal phase $\varepsilon(\mathbf{k})$ is referenced to the Fermi energy E_F, and hence $\varepsilon(\mathbf{k}) \approx \mathbf{v}_F(\mathbf{k} - \mathbf{k}_F)$ near the Fermi surface, where $\mathbf{v}_F = \partial\varepsilon(\mathbf{k})/\partial\mathbf{k}$ and $\mathbf{k}_F$ are the Fermi velocity and momentum. The coupling constant V_{BCS} defines the attraction of electrons near the Fermi surface in a narrow energy layer $\pm\omega_{ph}$, where ω_{ph} is a specific phonon energy. A variational wave function was used for calculation of the ground-state energy and the spectrum of electron excitations

$$|\Psi_{BCS}\rangle = \prod_{\mathbf{k}} \left[\sqrt{1 - h_{\mathbf{k}}} + \sqrt{h_{\mathbf{k}}}\, c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} \right] |\Phi_0\rangle , \quad c_{\mathbf{k}\sigma} |\Phi_0\rangle = 0 , \quad (20)$$

where the variational parameter $h_{\mathbf{k}}$ was determined from the minimum of the ground-state energy $W_0 = \langle \Psi_{BCS} | H_{BCS} | \Psi_{BCS} \rangle$. It was established that an energy gap appears in the superconducting phase in the spectrum of one-electron excitations

$$\Delta \sim \exp\left(-\frac{1}{\lambda}\right), \qquad E(\mathbf{k}) = \sqrt{\varepsilon^2(\mathbf{k}) + |\Delta|^2} ,$$

where the coupling constant $\lambda = V_{\rm BCS}\, N(0)$ is determined by the effective interaction from Hamiltonian (19) and the density of electron states on the Fermi surface $N(0)$. The thermodynamics and electrodynamics of a superconductor were considered, the superconducting transition temperature $T_{\rm c} = 1.14\,\omega_{\rm ph} \exp(-1/\lambda)$ was calculated, and a universal relation between the gap in the spectrum at zero temperature and the superconducting transition temperature $2\Delta_0 = 3.52\, T_{\rm c}$ was obtained. The gap in the spectrum arises due to the formation of bound states of electron pairs with opposite momenta and spins, the 'Cooper pairs.' The corresponding vacuum expectation value

$$\langle c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow} \rangle = \Psi(\mathbf{k}) = \left|\Psi(\mathbf{k})\right| \exp\left[\mathrm{i}\Phi(\mathbf{k})\right] \qquad (21)$$

represents an order parameter written in form (15). This expression is explicitly related to the violation of phase (gauge) invariance

$$\begin{aligned} c^{\dagger}_{\mathbf{k}\uparrow} &\to c^{\dagger}_{\mathbf{k}\uparrow} \exp(\mathrm{i}\varphi)\,, \\ \langle c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow} \rangle &\to \langle c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow} \rangle \exp(\mathrm{i}2\varphi)\,, \end{aligned} \qquad (22)$$

as in the theory of superfluidity [8]. In this case, a long-range order in the superconducting phase is specified not only by the appearance of Cooper pairs with $|\langle c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow} \rangle| \neq 0$ but also by fixing the order parameter phase in the bulk of the superconductor.

Based on the BCS semiphenomenological theory, Gor'kov [29] gave a consistent derivation of GL functional (16) and showed that the effective charge corresponds to a Cooper pair, i.e., $q = 2e$, and the effective mass should be taken equal to the mass of a Cooper pair $m^* = 2m$. In so doing, it is convenient to normalize the modulus of the order parameter to the density of superconducting electron pairs $|\Psi(\mathbf{r})|^2 = n_{\rm s}/2$.

Before the appearance of a detailed BCS paper [24], Bogoliubov succeeded in constructing a microscopic theory of superconductivity for the original Fröhlich electron–phonon model

$$\begin{aligned} H_{\rm Fr} = &\sum_{\mathbf{k},\sigma} \varepsilon(\mathbf{k})\, c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \sum_{\mathbf{q}} \omega(\mathbf{q})\, b^{\dagger}_{\mathbf{q}} b_{\mathbf{q}} \\ &+ g_{\rm Fr} \sum_{\mathbf{k},\mathbf{q},\sigma} \sqrt{\frac{\omega(\mathbf{q})}{2V}}\, c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}+\mathbf{q}\sigma} (b^{\dagger}_{\mathbf{q}} + b_{-\mathbf{q}})\,, \end{aligned} \qquad (23)$$

where $\omega(\mathbf{q}) = sq$, s is the speed of sound, and the interaction of electrons with acoustic phonons is described by the Fröhlich coupling constant $g_{\rm Fr}$. Generalizing the method of a canonical (u, v) transformation from the theory of superfluidity [8, 9], Bogoliubov introduced new Fermi amplitudes $\alpha_{\mathbf{k},\sigma}$, superpositions of electron creation and annihilation operators [25, 26] (see also § 2 in [27]):

$$\alpha_{\mathbf{k}\uparrow} = u_k c_{\mathbf{k}\uparrow} - v_k c^{\dagger}_{-\mathbf{k}\downarrow}\,, \quad \alpha_{\mathbf{k}\downarrow} = u_k c_{-\mathbf{k}\downarrow} + v_k c^{\dagger}_{\mathbf{k}\uparrow}\,, \quad u_k^2 + v_k^2 = 1\,, \qquad (24)$$

where u_k and v_k are real functions.

The new Fermi amplitudes $\alpha_{\mathbf{k}\sigma}$ and $\alpha^{\dagger}_{\mathbf{k}\sigma}$ were used to compensate for the so-called 'dangerous diagrams' responsible for the production of electron pairs with opposite momenta and spins. In the Fermi amplitude representation in (24), the Hamiltonian of electrons in the superconducting state takes the form of the Hamiltonian of a quasiparticle ideal gas:

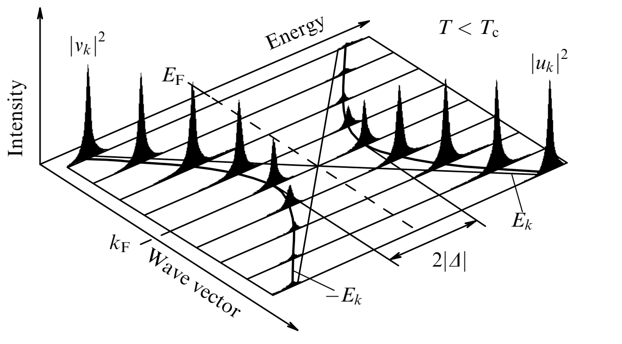

Figure 4. Spectral function of one-electron quasiparticle exitations (26) of the Bogoliubov theory [25–27] in the superconducting phase (taken from paper [30]).

$$\begin{aligned} &H_{\rm Fr} \to H_{\rm B} = \sum_{\mathbf{k},\sigma} E(\mathbf{k})\, \alpha^{\dagger}_{\mathbf{k}\sigma} \alpha_{\mathbf{k}\sigma} + U_0\,, \\ &E(\mathbf{k}) = \sqrt{\varepsilon^2(\mathbf{k}) + \left|\Delta(\mathbf{k})\right|^2}\,, \end{aligned} \qquad (25)$$

where the spectrum of excitations of quasiparticles $E(\mathbf{k})$ is defined by the spectrum of electrons in the normal phase $\varepsilon(\mathbf{k})$ and the gap $\Delta(\mathbf{k})$ in superconducting state, depending on momentum $\mathbf{k}$ in general. The equations derived by Bogoliubov for the gap and the superconducting temperature coincide with those in the BCS theory with the intensity directly determined by the Fröhlich coupling constant in Hamiltonian (23): $\lambda = g^2_{\rm Fr}\, N(0)$.

Bogoliubov's quasiparticles (24) (sometimes called 'bogolons') provide us with a clear physical picture of the spectrum of quasiparticle excitations as a superposition of a particle and a hole with a gap in the spectrum on the Fermi surface. We give the spectral function of quasiparticle excitations in the superconducting phase,

$$A_{\rm sc}(\mathbf{k}, \omega) = u^2_{\mathbf{k}}\, \delta(\omega - E_{\mathbf{k}}) + v^2_{\mathbf{k}}\, \delta(\omega + E_{\mathbf{k}})\,, \qquad (26)$$

with due account for the expressions derived by Bogoliubov for the coefficients in transformation (24):

$$u_k^2 = \frac{1}{2}\left(1 + \frac{\varepsilon(k)}{E(\mathbf{k})}\right), \qquad v_k^2 = \frac{1}{2}\left(1 - \frac{\varepsilon(\mathbf{k})}{E(\mathbf{k})}\right).$$

Away from the Fermi surface, $|\varepsilon(\mathbf{k})| \gg |\Delta(\mathbf{k})|$, $E(\mathbf{k}) \approx |\varepsilon(\mathbf{k})|$, quasiparticle excitations are either electrons outside the Fermi sphere for $\varepsilon(k) > 0$, $u_k^2 \approx 1$, $v_k^2 \approx 0$, or holes inside the Fermi sphere for $\varepsilon(k) < 0$, $u_k^2 \approx 0$, $v_k^2 \approx 1$. In the vicinity of the Fermi surface, $|\varepsilon(\mathbf{k})| \ll |\Delta(\mathbf{k})|$, $E(\mathbf{k}) \approx |\Delta(\mathbf{k})|$, excitations are a coherent superposition of an electron and a hole, and hence spectral function (26) has two peaks with equal weights: $u_k^2 \approx v_k^2 \approx 1/2$. In this case, the energy gap for electron excitations equals $2|\Delta(\mathbf{k})|$. In passing to the normal phase, $|\Delta(\mathbf{k})| = 0$, the spectral function takes a standard form $A_{\rm n}(\mathbf{k}, \omega) = \delta(\omega - \varepsilon(k))$ with a linear spectrum of excitations near the Fermi surface: $\varepsilon(\mathbf{k}) \approx \mathbf{v}_{\rm F}(\mathbf{k} - \mathbf{k}_{\rm F})$. Figure 4 shows the spectral phase of quasiparticle excitations (26), two of whose branches correspond to the spectrum $\omega = \pm E(\mathbf{k})$ with the gap $|\Delta(\mathbf{k})|$. A similar quasiparticle spectrum with two peaks has been observed, for instance, in photoemission experiments in high-temperature superconductors [30]), which proves the coherent nature of quasiparticles in the superconducting phase.

Based on the Bogoliubov representation of quasiparticles, it is easy to calculate the thermodynamic and electrodynamic properties of a superconductor. The Bogoliubov canonical (u, v) transformation in (24) is widely used in solving present-day problems in the theory of superconductivity. Noteworthy also is the generalization of the (u, v) transformation to the case of inhomogeneous systems, the *Bogoliubov–De Gennes transformation* (see, e.g., [31]) that can be written in terms of coordinate-dependent $(u(\mathbf{r}), v(\mathbf{r}))$ wave functions of electrons in the superconducting phase.

Although the results obtained on the basis of the BCS simplified model Hamiltonian (19) were impressive, the problem of the accuracy of the obtained solutions remained unsolved. Omitting the details, we note that further analysis [32] (see also pp. 168–176 in [5]) showed that the superconducting phase represents a condensate of Cooper pairs (i.e., bosons) consisting of 'attracted' electrons. The spectrum of excitations of a pair condensate satisfies the Landau superfluidity criterion. Thus, Bogoliubov came to the conclusion of the unity of these two phenomena: it is superfluidity of Cooper pairs that creates a superconducting current.[13]

We note that the identity of both these phenomena has recently been confirmed directly in experiments with ultracold fermion gases (see recent reviews [34, 35]).

Summarizing the discussions of phase transitions in quantum statistics, we emphasize that in passing to the superfluid and superconducting states, the system undergoes spontaneous symmetry breaking, specifically, the breaking of the phase (otherwise, gauge) invariance.

3. Spontaneous symmetry breaking in quantum field theory

3.1 The 1960 events

The first attempts to use the SSB mechanism in quantum field theory (QFT) arose in 1960. At that time, this idea was just floating about. Almost concurrently there appeared several investigations within two-dimensional $(1+1)$ QFT models. The very first ones submitted for publication were papers by Vaks and Larkin [36] (see also p. 873 in [37]). Almost simultaneously, the first results by Tavkhelidze and Nambu [38] were obtained. That summer Bogoliubov and Nambu met in Utrecht,[14] and three months later in Rochester (USA) at the High Enegry Physics Conference. There, Nambu delivered a draft of his first paper[15] with Jona-Lasinio [40]. In a comment on Nambu's talk, Bogoliubov said (see p. 865 in [37]): "...one of my collaborators (Tavkelidze) has considered a Thirring-type one-dimensional model in which massless fermions interact with massive bosons. His calculations are not based on the self-consistent principle but on the ordinary Feynman diagram approach. The result is that there is a degeneracy in such a simple case."

All the QFT models in the early papers [36–40] of 1960, including the most well-known, second paper by Nambu and Jona-Lasinio [41], were nonrenormalizable, the results being dependent on the cutoff. This drawback was avoided only by Arbuzov, Tavkhelidze, and Faustov [42] (see also pp. 527–530 in [43]).

The first realistic use of SSB took place several years later in the realistic model of the electroweak interaction by Glashow, Salam, and Weinberg (GSW), where heavy gauge vector W and Z bosons acquire masses due to the Higgs mechanism.

3.2 The Higgs mechanism in the Standard Model

The Lagrangian of the complex (pseudo)scalar field with a quartic self-coupling

$$L(\varphi, g) = \frac{1}{2}(\partial_\mu \varphi)^2 - V(\varphi)\,,$$
$$V(\varphi) = \frac{m^2}{2}\,\varphi^2 + g\,\varphi^4\,, \qquad g > 0\,,$$

and a stable lower state at $\varphi = 0$ differs from the Lagrangian of a (two-component) Higgs field,

$$V_{\mathrm{Higgs}}(\Phi^2) = \lambda\big(\mathbf{\Phi}^2(x) - \Phi_0^2\big)^2\,, \qquad \mathbf{\Phi}^2 = \Phi_1^2 + \Phi_2^2\,, \quad \Phi_0^2 = \mathrm{const}\,, \tag{27}$$

by the sign of the quadratic term.[16] This corresponds to a pure imaginary initial mass $\mu_{\mathrm{H}}^2 = -4\lambda\Phi_0^2$. After the shift $\Phi_1(x) \to \varphi_1(x) = \Phi_1(x) - \Phi_0$ by a constant Φ_0, the Higgs field acquires the physical mass

$$M_{\mathrm{Higgs}} = 2\sqrt{2\lambda}\,\Phi_0\,, \tag{28}$$

proportional to the vacuum expectation value Φ_0.

The main reason for this formal trick is to provide nonzero masses to quanta of the above-mentioned gauge vector fields and to leptons and quarks. The first ones are expressed in terms of the coupling constants of the electroweak interaction and Φ_0, like, for example, $M_Z = e\sqrt{2}\,\Phi_0/\sin 2\theta_{\mathrm{W}}$, and the last ones are expressed via Φ_0 and some Yukawa couplings. The relevant Yukawa interactions are *specially added* to the Lagrangian of the Standard Model for this and only this purpose! After the shift by Φ_0,

$$g_i \bar{\psi}_i\,\Phi(x)\,\psi_i \to g_i \bar{\psi}_i\,\varphi(x)\,\psi_i + m_i\,\bar{\psi}_i\,\psi_i\,; \qquad m_i = g_i\,\Phi_0\,,$$

these Yukawa interactions provide masses for fermions.

This recipe, devoid of elegance, gives masses to leptons and quarks through the barter rule — *one mass for one coupling constant*. As a result, of about 25 parameters (not counting neutrino masses) of the current Standard Model, just 12 are Yukawa constants, added 'by hand.'

[13] We give a quotation from Bogoliubov's review paper [33] (see also pp. 289–296 in [5]) of that time: "...the property of superconductivity may be treated as a property of superfluidity of a system of electrons in metal."

[14] See the last sentence in paper [39].

[15] Unfortunately, Nambu's publications contain only a slipshod reference to the preprint of the first Bogoliubov paper on superconductivity already published in *JETP* [25] two years before. Partially due to this, the appearance of the SSB phenomenon as early as 1946 in Bogoluibov's theory of superfluidity [8, 9] (see pp. 108–112 in [5]) (as well as later on in the theory of superconductivity) relating to the (u, v) transformation and being physically responsible for the nonconservation of the number of noncondensed particles (of Cooper pairs) remained unnoticed by succeeding authors.

[16] Cf. Fig. 3 and expression (18).

Nevertheless, in the gauge sector of the SM, the SSB phenomenon implemented in the form of the Higgs model led, about 40 years ago, to one of the greatest triumphs of QFT — the prediction of the existence of neutral currents and numerical values of intermediate boson $W_{\pm}$ and Z_0 masses. The 1979 Nobel Prize was awarded to theoreticians Glashow, Salam, and Weinberg a few years before the experimental observation of the $W_{\pm}$ and Z_0 particles, which, in turn, was marked by another Nobel Prize in 1984.

Along with quantum electrodynamics and quantum chromodynamics, the GSW theory of electroweak interactions represents a splendid achievement of the human intellect. Being based upon an elegant and powerful principle — *dynamics from symmetry* — it constitutes a foundation of the Standard Model.

3.3 Search for the Higgs boson

Meanwhile, the v.a.v. $\Phi_0 \sim 250$ MeV determined from the electroweak theory is not sufficient for the estimation of the Higgs mass itself. Expression (28) for the mass value also contains the self-coupling constant λ, which remains free. The current combination of theoretical and experimental restrictions results in a small window for possible mass values

$$114 \text{ GeV} < M_{\text{Higgs}} < 154 \text{ GeV},$$

that, hopefully, will quite soon be studied at the Large Hadron Collider.

In the context of these 'great LHC expectations,' it is worth remembering that a rather artificial Higgs construction in (27), with its pure imaginary initial mass, looks like a simple-minded relativistic replica of the Ginzburg–Landau classic functional (16), (18) with all its pragmatic advantages and physical shortcomings. The real underlying physical reason for SSB remains unknown, despite the success of the electroweak theory. In such a situation, any aspirations to directly observe Higgs particles, in our opinion, look unjustifiably straightforward.

4. Conclusion

4.1 Regarding the practice of the Nobel Committee on Physics

Now a few words about the awarding of the Nobel Prize. The Committee on Physics is the Class for Physics of the Royal Swedish Academy consisting of six members. Only these Swedish academicians make the decision according to the Alfred Nobel testament, taking into account the opinions of leading specialists, mainly the community of Nobel laureates, with its well-known specific features.

We recall a few well-known incidents.

Piotr Kapitza discovered superfluidity in 1937. All his outstanding results in low-temperature physics were also obtained in the late 1930s. He happened to be lucky enough to survive until his early nineties, when they bethought him, more than 40 years later. We all remember in what a desperate state, following an accident, Landau received his prize.

Items of another kind.

The 1999 Prize to 't Hooft and Veltman. The renormalization of a non-Abelian vector field, which acquires mass due to symmetry breaking, was a physically important and mathematically intricate problem. Its masterly solution by the rather complicated combination of formal tricks formed a base of the electroweak theory in the late 1960s and, subsequently, in quantum chromodynamics. However, the contribution of three Russian theorists to this solution is, at least, of no less importance than that of the laureates. Everyone calculates matrix elements (in electroweak theory and QCD) by the Feynman rules formulated by Faddeev and Popov and performs renormalization with due account of Slavnov's identities.

Now the last case. It combines two important elements of the Standard Model that are rather distinct from each other. Their junction seems rather deliberate. The first one, spontaneous symmetry breaking in the theory of quantum gauge fields, in the current context of the 21st century could be connected (after the discovery of the Higgs particle) to the names of Nambu, Goldstone and Higgs. The second one — the formal mixing of three lepton generations (via the Cabbibo–Kobayashi–Maskawa matrix) in the current version of the Standard Model — lies completely outside our scope.

4.2 Summary

Above, we attempted, in fairytale form, to trace the development of a topical issue, spontaneous symmetry breaking, in the field of quantum physics during the 20th century.

It is evident that the 'Nobel race' is won by pragmatic theorists of the phenomenological kind, in terms of the discussion in the Introduction. And this is natural, in a sense. It is precisely in this mood that the inventor of dynamite set his priorities to benefit humanity. A very clear-cut implementation of this spirit of the Nobel testament was the 2007 Prize in physics.

Meanwhile, the reductionists have no reasons to be dejected or envious. Their efforts' reward lies in other fields. Thanks to their achievements, a more complete picture of the physical universe appears; ties of affinity are established between what are at first sight unrelated phenomena such as between electromagnetic and nuclear forces and, quite hopefully, between the dynamics of the Universe's evolution and some hypothetic generalization of the Standard Model (with additional spatial dimensions).

The author is indebted to Prof. Oleg Rudenko for the impetus of this talk and paper and for continuous moral support. In the course of implementation of the initial plan, the two mighty figures of Landau and Bogoliubov and the complementary interference of their creative methods came to the fore. By a lucky chance, this paper is being published just between their centennial jubilees.

The role of Drs. N M Plakida and V B Priezzhev in composing Section 2 is indispensable. They are practically coauthors of it. Besides, they provided the author with many subtle comments on the whole text. It was also a pleasure to follow the essential advice of Dr. V A Zagrebnov. This investigation was supported in part by the presidential grant Scientific School–1027.2008.2.

Notes added in English proofreading

In 1966, Hohenberg and Platzman [12] proposed using deep inelastic neutron scattering for experimental observation of the Bose condensate in ^{4}He. Different groups performed experiments in this direction. For example, the "Dubna–Obninsk group" was the first not only to reexamine the fraction of the Bose–Einstein condensate, $(3.6 \pm 1.4)\%$ at $T = 1.2$ K [13], but also to measure its temperature dependence. Moreover, they found the rather accurate value

$T_c = (2.24 \pm 0.04)$ K of the critical temperature, with the λ-point for superfluidity being equal to $T_\lambda = 2.18$ K!

Much later, Bose–Einstein condensation was observed for alkali metals in traps [14] (see [34, 35] for recent reviews).

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[17] Square brackets stand for the date of submission, parentheses stand for the date of publication.