at any fixed point $\mathbf{R}$ in space. This is indicative of the emergence of a cluster (caustic) structure in the intensity field. The formation of statistics (for instance, of moment functions $\langle I^n(x,\mathbf{R})\rangle$) proceeds through large overshoots of the process $I(x)$ with respect to this curve.

The description of intensity fluctuations, obtained in the first order of the MSP, is valid for $\beta_0(x) \leqslant 1$. As the parameter $\beta_0(x)$ increases further, this approximation becomes violated and the nonlinear character of the equation for the complex phase of the wave field has to be taken into account. This range of fluctuations, called *the region of strong focusing*, is very difficult for analytical research. For even larger values of parameter $\beta_0(x)$, the statistical characteristics of intensity reach saturation, in which case $\beta(x) \to 1$ as $\beta_0(x) \to \infty$. This region of the parameter $\beta_0(x)$ variations is called *the region of strong intensity fluctuations*.

In this region, the statistical characteristics of the wave field cease to depend on the distance and one has

$$\left\langle I^n(x,\mathbf{R})\right\rangle = n!\,, \qquad P(x,I) = \exp(-I)\,.$$

In this case, the mean specific area of regions within which $I(x,\mathbf{R}) > I$ and the mean specific power concentrated in these regions are constant and do not describe the behavior of the wave field intensity in separate realizations. Likewise, passage to a statistically equivalent random process is not informative in this case since its curve of typical realization assumes a constant value. An understanding of the wave field structure in specific realizations can only be gained in this case from the analysis of such quantities as the specific mean length of contours and mean specific number of wave field intensity contours. These quantities continue to grow with the parameter $\beta_0(x)$, implying that the splitting of contours takes place (see Fig. 4).

3. Conclusions

In closing, I would like to reiterate once more the main point of this talk. The approach to analysis of stochastic dynamical problems rooted in the ideas of stochastic topography, which enables, given the one-point statistical characteristics of processes and fields, determining quantitative and qualitative characteristics of behavior of their particular realizations for all times (in the entire space), has emerged as a result of discussions with experimenters who largely deal with separate realizations. For a comprehensive description of stochastic dynamical systems, it is insufficient to formulate a basic equations with respective boundary and initial conditions. It is necessary first and foremost to understand which coherent phenomena (occurring with the probability of unity, i.e., in almost all realizations of their solutions) are contained in these systems, and proceed with a statistical analysis in a related way.

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Development of the radiative transfer theory as applied to instrumental imaging in turbid media

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1. Introduction

This talk presents the basic elements of instrumental imaging theory in media with strongly anisotropic scattering and a technique devised to compute images of diffusively reflecting objects, accounting for the effects of light absorption and multiple scattering. It discusses peculiarities of different variants of the radiative transfer equation in the small-angle approximation, used in the imaging theory and optical coherence 'tomography' (OCT) of turbid media. A new method of computing the temporal moments of a pulsed light beam transmitted through a layer of a turbid medium is described. The results of theoretical and experimental studies of shadow noises in OCT images of turbid media with fluctuating optical parameters are outlined.

By scattering light a turbid medium limits the visibility range of objects located within it and becomes visible itself. Therefore, the development of the methods and theory of instrumental imaging in turbid media was directed toward solving two interconnected tasks — the removal of the adverse influence of the medium on the visibility of objects, and the remote sensing of inherent optical properties of the medium itself.

The Koshmider equation [1] expresses the fundamental result of the imaging theory by relating the image contrast of a black object (observed in the sky background near the horizon) to the light attenuation coefficient in the atmosphere. The relationships for estimating the contrast of the image and visibility range of underwater objects under natural illumination were obtained in a now classical work by Duntley [2]. In this case, it was assumed that the angular size of the observed object is small, so that its apparent radiance is attenuated by the medium according to Buger's law. The need in a more universal imaging theory emerged in connection with the development of laser methods of underwater vision.

Pioneering works in this area were performed under the supervision of A V Gaponov-Grekhov in the Radiophysical Research Institute (NIRFI in *Russ. abbr.*) (Gor'ky) in the 1960s. They have led to the design of the first prototype of a laser-pulse system of underwater imaging with the help of which the feasibility of an essential increase in the visibility range of underwater objects was demonstrated in sea conditions. It relied on laser target illumination and pulse gating of a useful signal. It was at that time that the main results laying the foundation for the modern theory of laser location and instrumental imaging in turbid media were obtained: based on the radiative transfer equation (RTE) in the small-angle approximation, an analytical model was proposed for blurring and attenuation of a laser beam in its passing through a medium with strongly anisotropic scattering [3, 4]; formulas were derived for gauging the characteristics of underwater object images taking into account the effects of light absorption and multiple scattering in water [5–10], and a universal technique was suggested for estimating the potential range of underwater imaging systems of different types, including laser-based ones [11–14].

The plausibility of applying the phenomenological radiative transfer theory to the analysis of coherent light beam propagation in a turbid medium was a subject of certain concern. It was partly removed owing to Refs [15–17], which derived the equation for the coherence function of a wave beam in a medium with strongly anisotropic scattering and showed that the Fourier transform of the coherence function satisfies the RTE in the small-angle approximation and, consequently, that it is a wave analogue to the light field radiance (these results were reported at S M Rytov's seminar in 1966, through the initiative of M A Miller who was in charge of theoretical research in hydrooptics at NIRFI and offered everyday help with his advice and criticism to researchers involved in this work). The justification of RTE in a more general formulation was a subject of intensive research, with results reported in a set of reviews and monographs [18–20].

Later on, different variants of RTE solutions in the small-angle approximation were used for developing the theory of laser location and imaging of underwater objects through a rough sea surface and lidar methods of determining the optical characteristics of natural scattering media, and also in problems concerning the optical tomography of biological tissues.

2. How to build a model image on the basis of the radiative transfer equation

When solving the problems of imaging theory in turbid media one can assume, without loss of generality, that the observing system (Fig. 1) comprises the illuminating source S and optical receiver R, whereas the image is formed by detecting the power of the incoming signal $P_{\rm R}$ as a function of coordinates $\mathbf{r}_0$ of the point at which the axes of the directivity patterns of the source $\mathbf{n}_{\rm S}$ and the receiver $\mathbf{n}_{\rm R}$ intersect the surface of the object $S_{\rm ob}$. To determine the signal $P_{\rm R}$, the radiative transfer equation [21] is applied:

$$\left(\frac{1}{c}\,\frac{\partial}{\partial t}+\mathbf{n}\nabla_{\mathbf{r}}+\alpha\right) I(\mathbf{r},\mathbf{n},t)=\sigma\int_{4\pi} I(\mathbf{r},\mathbf{n}',t)\,x(\gamma)\,\mathrm{d}\mathbf{n}'+Q\,, \tag{1}$$

where $I(\mathbf{r},\mathbf{n},t)$ is the intensity of radiation (radiance) at a point in space $\mathbf{r}$ in the direction of unit vector $\mathbf{n}$ at the instant of time t, c and $\alpha=\sigma+\kappa$ are the speed of light and attenuation coefficient in the medium, respectively, σ and κ are the scattering and absorption coefficients, $x(\gamma)$ is the scattering phase function normalized as $2\pi\int_0^{\pi} x(\gamma)\sin\gamma\,\mathrm{d}\gamma=1$,

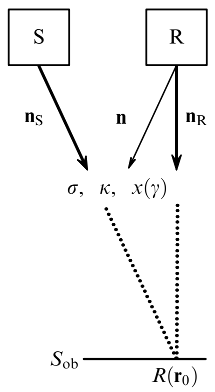

Figure 1. Schematics of observations.

$\gamma=\arccos(\mathbf{n}\mathbf{n}')$ is the scattering angle, $\mathrm{d}\mathbf{n}'$ is the solid angle element around the direction $\mathbf{n}'$, and Q are the volume sources of radiation.

Scattering phase functions of natural turbid media exhibit well-expressed anisotropy. They therefore can be described with sufficient accuracy by the expression

$$x(\gamma)=(1-2p_{\rm b})\,x_1(\gamma)+\frac{p_{\rm b}}{2\pi}\,, \tag{2}$$

where $p_{\rm b}=2\pi\int_{\pi/2}^{\pi} x(\gamma)\sin\gamma\,\mathrm{d}\gamma$ is the backscattering probability, $p_{\rm b}\ll 1$, $x_1(\gamma)$ is the narrow part of the scattering phase function satisfying the conditions $2\pi\int_0^{\pi} x_1(\gamma)\sin\gamma\,\mathrm{d}\gamma=1$, and $x_1(\gamma)\ll p_{\rm b}/(2\pi)$ for $\gamma>\pi/2$. The power of a received signal $P_{\rm R}(\mathbf{r}_0,t)$ is expressed through the radiance of light $I_{\rm R}(-\mathbf{n},t)$ incident on the receiver aperture as

$$P_{\rm R}(\mathbf{r}_0,t)=\Sigma_{\rm R}\int I_{\rm R}(-\mathbf{n},t)\,D_{\rm R}(\vartheta)\,\mathrm{d}\mathbf{n}\,,$$

where $\Sigma_{\rm R}$ is the area of the detector entrance pupil, and $D_{\rm R}(\vartheta)$ is its directivity pattern: $D_{\rm R}(0)=1$, $\vartheta=\arccos(\mathbf{n}\,\mathbf{n}_{\rm R})$. The light field from source S is expanded as a sum of 'directed' I_1 and diffusive I_2 components. The first of them plays the major role in illuminating the object, while the second one corresponds to a signal reflected by the medium (backscattering noise, 'haze'). The field I_1 is identified with that emitted by the source S in an 'auxiliary' medium with a narrow scattering phase function $x_1(\gamma)$, scattering coefficient $\sigma_1=\sigma-2\sigma_{\rm b}$, and absorption coefficient $\kappa_1=\alpha-\sigma_1=\kappa+2\sigma_{\rm b}$, where $\sigma_{\rm b}=p_{\rm b}\sigma$ is the backscattering coefficient of the real medium. Computation of the diffusive field component in the approximation of single light scattering over large angles reduces to finding the field of distributed radiation sources

$$Q=\frac{\sigma_{\rm b}}{2\pi}\int_{4\pi} I_1\,\mathrm{d}\mathbf{n}$$

in the auxiliary medium. In this case, the total power of the received signal is presented in the form

$$P_{\rm R}=P_{\rm ob}+P_{\rm b}\,, \tag{3}$$

$$P_{\rm ob}=\frac{\Sigma_{\rm R}\Omega_{\rm R}}{\pi}\iint_{S_{\rm ob}} R(\mathbf{r}') \times\left[\int_{-\infty}^{\infty} E^{(\rm s)}(\mathbf{r}',t')\,E^{(\rm r)}(\mathbf{r}',t-t')\,\mathrm{d}t'\right]\mathrm{d}\mathbf{r}'\,, \tag{4}$$

$$P_{\mathrm{b}} = \frac{\Sigma_{\mathrm{R}} \Omega_{\mathrm{R}}}{2\pi} \iiint_V \sigma_{\mathrm{b}}(\mathbf{r}) \times \left[\int_{-\infty}^{\infty} E_0^{(\mathrm{s})}(\mathbf{r}, t')\, E_0^{(\mathrm{r})}(\mathbf{r}, t - t')\, \mathrm{d}t' \right] \mathrm{d}\mathbf{r}\,, \tag{5}$$

where P_{ob} is the signal power from the object; P_{b} is the power of the volume backscattering signal; $\Omega_{\mathrm{R}} = 2\pi \int_0^{\pi/2} D_{\mathrm{R}}(\vartheta) \sin\vartheta\, \mathrm{d}\vartheta$ is the effective solid receiving angle; $E^{(\mathrm{s})}(\mathbf{r}', t) = \int_{\mathbf{nN}>0} (\mathbf{nN})\, I_1\, \mathrm{d}\mathbf{n}$ and $E_0^{(\mathrm{s})}(\mathbf{r}, t) = \int_{4\pi} I_1\, \mathrm{d}\mathbf{n}$ are the irradiance of the object surface at point $\mathbf{r}'$ and spatial irradiation of the medium at point $\mathbf{r}$ at the instant of time t, and $E^{(\mathrm{r})}(\mathbf{r}', t)$ and $E_0^{(\mathrm{r})}(\mathbf{r}, t)$ are, respectively, the irradiance of the object and medium due to an auxiliary δ-pulse source of light with a unit energy and the directivity pattern of the detector. The integration in formula (4) is performed over the object's surface S_{ob}, whereas that in formula (5) is carried out over the illuminated volume V of the medium. For a stationary illumination (when functions with superscript (s) are time independent), the time convolution drops out from formulas (4) and (5), and the functions with superscript (r) in the integrands become irradiances of the object and medium from the auxiliary continuous light source with unit power. Formulas (3)–(5) allow the received signal to be expressed through the reflection coefficient of the observed object, backscattering coefficient of the turbid medium, and irradiation fields created by the real and auxiliary radiation sources in a turbid medium that scatters light only 'forward'.

For the given positions of the source, the detector, and the object of observation, the functions $E^{(\mathrm{s,r})}$ and $E_0^{(\mathrm{s,r})}$ in Eqns (4) and (5) depend not only on variables $\mathbf{r}$, $\mathbf{r}'$ and t, but also on the coordinates of object point $\mathbf{r}_0$, toward which point the directivity patterns of the source and detector. If the field of view of the observing system is sufficiently small and the distance z_{ob} to the object is large compared to the 'source–detector' base, then, for stationary object illumination, the dependence of the signal P_{ob} on $\mathbf{r}_0$ is given by the formulas

$$P_{\mathrm{ob}}(\mathbf{r}_0, z_{\mathrm{ob}}) = \frac{P_{\mathrm{S}}}{\pi} \iint_{S_{\mathrm{ob}}} R(\mathbf{r}')\, A_{\mathrm{ob}}(\mathbf{r}_0 - \mathbf{r}', z_{\mathrm{ob}})\, \mathrm{d}\mathbf{r}', \tag{6}$$

$$A_{\mathrm{ob}}(\mathbf{r}_\perp, z) = \Sigma_{\mathrm{R}} \Omega_{\mathrm{R}} \bar{E}^{(\mathrm{s})}(\mathbf{r}_\perp, z)\, \bar{E}^{(\mathrm{r})}(\mathbf{r}_\perp, z)\,, \tag{7}$$

where P_{S} is the power of the illuminating source, and $\bar{E}^{(\mathrm{s,r})}$ is the distribution of irradiance in the plane $z = \mathrm{const}$ due to real and auxiliary illuminating beams with unit power, when the beam axes are oriented toward a point $\mathbf{r}_\perp = 0$ in this plane.

The functions $\bar{E}^{(\mathrm{s,r})}$ could be termed the effective directivity patterns of the source and receiver. In order to retrieve the image, at least one of them should be narrow. In the standard television system, an image is formed owing to the directivity pattern $\bar{E}^{(\mathrm{r})}$, and in a system with a running light beam, owing to $\bar{E}^{(\mathrm{s})}$. According to Eqn (6), a turbid medium transforms the image as a linear filter of two-dimensional signals. The function $A_{\mathrm{ob}}(\mathbf{r}_\perp, z)$, dubbed the point spread function (PSF), characterizes the structure of the image of a pointwise object and serves as an analogue of the pulse characteristic for filters of electric signals. To find this function, it suffices to know the directivity patterns of the source and detector and the distribution of irradiance $e(r_\perp, z)$ in a cross section of an infinitely narrow light beam transmitted through a medium layer of thickness z. The normalized spatial spectrum of this distribution,

$$T(k, z) = \frac{\int_0^\infty e(r_\perp, z)\, J_0(kr_\perp)\, r_\perp\, \mathrm{d}r_\perp}{\int_0^\infty e(r_\perp, z)\, r_\perp\, \mathrm{d}r_\perp}\,,$$

is called the modulation transfer function of the turbid medium layer.

For a pulse illumination of the object, the energy of a useful signal at the image element can be estimated from Eqns (6) and (7): $W_{\mathrm{ob}}(\mathbf{r}_0) = \int P_{\mathrm{ob}}(\mathbf{r}_0, t)\, \mathrm{d}t$. This knowledge is what is needed to estimate the image quality. When computing W_{ob} it is necessary to make the substitutions in Eqn (6): $P_{\mathrm{ob}} \to W_{\mathrm{ob}}$ and $P_{\mathrm{S}} \to W_{\mathrm{S}}$, where W_{S} is the energy of the sounding pulse.

If a turbid medium is sounded with light pulses of duration Δt, a signal backscattered at an instant t returns from the depth $z_t = ct/2$ from a layer with a thickness of $c\Delta t/2$. As follows from Eqn (5), if the optical axes of the source and detector are directed toward the point $\mathbf{r}_0$ in the plane $z = z_t$, the power of the detected signal is expressed in the form

$$P_{\mathrm{b}}(\mathbf{r}_0, z_t) = \frac{cW_{\mathrm{S}}}{4\pi} \iint_\infty \sigma_{\mathrm{b}}(\mathbf{r}_\perp, z_t)\, A_{\mathrm{b}}(\mathbf{r}_0 - \mathbf{r}_\perp, z_t)\, \mathrm{d}\mathbf{r}_\perp\,, \tag{8}$$

$$A_{\mathrm{b}}(\mathbf{r}_\perp, z) = \Sigma_{\mathrm{R}} \Omega_{\mathrm{R}} \bar{E}_0^{(\mathrm{s})}(\mathbf{r}_\perp, z)\, \bar{E}_0^{(\mathrm{r})}(\mathbf{r}_\perp, z) \tag{9}$$

through the medium backscattering coefficient σ_{b} and distributions $\bar{E}_0^{(\mathrm{s,r})}(\mathbf{r}_\perp, z)$ of spatial irradiance in the cross section of real and auxiliary light beams at a distance z from the source. Formulas (8) and (9) are applicable provided the pulse length $c\Delta t$ is small relative to the photon mean free path $1/\alpha$ and the scale Δz of longitudinal inhomogeneity σ_{b}. They indicate that the pulse imaging system is equally applicable to observing objects in a turbid medium and spatial variations of the backscattering coefficient of the medium proper. The distributions $\bar{E}_0^{(\mathrm{s,r})}$ and $\bar{E}^{(\mathrm{s,r})}$ differ only slightly in an auxiliary medium with a narrow scattering indicatrix. Thus, computing images of the object or medium, one can set $A_{\mathrm{b}}(\mathbf{r}_\perp, z) \approx A_{\mathrm{ob}}(\mathbf{r}_\perp, z)$ and use one and the same PSF.

Formulas (8) and (9) underlie the theory of laser sounding of the ocean and atmosphere and algorithms for the remote assessment of their optical characteristics. These formulas can also be adapted to describe images of a scattering medium, obtained using the method of optical coherence tomography [22]. In OCT setups, a continuous light is used with femtosecond coherence times and the interferometric method is exploited to determine the depth from which the backscattered signal comes. Optical signals are emitted and received by the end of a single-mode optical fiber. This creates conditions for the backscattering amplification effect to manifest itself [23], which is missing from the transfer equation. In order to address it, the development of a wave model of OCT-imaging [24] was needed, relying on the hybrid method of evaluating field fluctuations in a medium with coarse and fine inhomogeneities of permittivity [25]. It was shown [24] that the OCT system with a heterodyne detector can be brought into correspondence with an equivalent system of pulse location with direct signal detecting, while the specifics of the single-position sounding method can be taken into account by setting

$$A_{\mathrm{b}}(\mathbf{r}_\perp, z) = \Sigma_{\mathrm{R}} \Omega_{\mathrm{R}} \left\lfloor 2E^2(\mathbf{r}_\perp, z) - E_{\mathrm{ns}}^2(\mathbf{r}_\perp, z) \right\rfloor$$

in Eqn (8), where E is the total irradiance of the medium at point $\mathbf{r}_\perp, z$, and E_{ns} is the medium irradiance through nonscattered (direct) light from the source.

Thus, the theory of stationary light beam propagation in a medium with a narrow scattering indicatrix can serve as a basis for computing images formed by active vision systems, both continuous and pulsed. In order to analyze the performance of passive observing facilities, one also needs a model of a natural light field.

3. Analytical models of light fields for problems of imaging theory

The theory describing the propagation of a narrow light beam in media with strongly anisotropic scattering relies on the radiative transfer equation in the small-angle approximation [12]. Assuming that the beam propagates in the direction of the z-axis, this equation can be cast in the form

$$\left[c^{-1}\frac{\partial}{\partial t}+n_z\frac{\partial}{\partial z}+\mathbf{n}_\perp\nabla_\perp+\alpha\right]I(\mathbf{r}_\perp,z,\mathbf{n}_\perp,t)$$
$$=\sigma_1\iint_\infty I(\mathbf{r}_\perp,z,\mathbf{n}'_\perp,t)\,x_1\big(|\mathbf{n}_\perp-\mathbf{n}'_\perp|\big)\,\mathrm{d}\mathbf{n}'_\perp\,,\qquad(10)$$

where $\mathbf{r}_\perp$ and $\mathbf{n}_\perp$ are the components of $\mathbf{r}$ and $\mathbf{n}$ in the plane $z=\text{const}$, and $n_z=(1-n_\perp^2)^{1/2}$.

A rigorous analytical solution of Eqn (10) can only be obtained in the approximation $n_z\approx 1$, which ignores the effects of photon multipath propagation (distortion of the light signal as it travels through the medium, the formation of stationary angular radiance distribution in a continuous beam at large optical distances from its source). Originally, this equation was exploited in the theory of multiple scattering of fast charged particles in matter [26–29]; with its assistance, an expression for the angular distribution of particles in an infinitely broad beam was found [26, 27] and the functions of the type $\iint_\infty I(x,y,z,n_x,n_y)\,\mathrm{d}y\,\mathrm{d}n_y$, which characterize the structure of a thin beam, were analyzed [28]. The solution to Eqn (10) at $n_z=1$ and an arbitrary boundary condition for the radiance at the source aperture, $I(\mathbf{r}_\perp,0,\mathbf{n}_\perp)=I_0(\mathbf{r}_\perp,\mathbf{n}_\perp)$, was obtained in Ref. [3] and later generalized for stratified turbid media in Ref. [30]. According to this solution, the distribution of irradiance $E(\mathbf{r}_\perp,z)=\iint_\infty I\,\mathrm{d}\mathbf{n}_\perp$ over the cross section of a light beam exiting the layer of a turbid medium with a narrow scattering phase function $x_1(\gamma)$ and optical parameters $\alpha(z)$, $\sigma_1(z)$, and $\kappa_1(z)=\alpha-\sigma_1$ is expressed in a spectral form as

$$E(\mathbf{r}_\perp,z)=\iint_\infty F(\mathbf{k},z)\,T(k,z)\exp\left(-\tau_\kappa+\mathrm{i}\mathbf{k}\mathbf{r}_\perp\right)\mathrm{d}\mathbf{k}\qquad(11)$$

through the functions

$$F=\frac{1}{(2\pi)^2}\iint_\infty\iint_\infty I_0(\mathbf{r}_\perp,\mathbf{n}_\perp)\exp\left[-\mathrm{i}\mathbf{k}(\mathbf{r}_\perp+z\mathbf{n}_\perp)\right]\mathrm{d}\mathbf{r}_\perp\,\mathrm{d}\mathbf{n}_\perp\,,$$
$$T=\exp\left[-\int_0^z\sigma_1(z-z')\left[1-x_{\mathrm{S}}(kz')\right]\mathrm{d}z'\right],\qquad(12)$$
$$\tau_\kappa=\int_0^z\kappa_1(z')\,\mathrm{d}z'\,,\qquad x_{\mathrm{S}}(p)=2\pi\int_0^\infty x_1(\gamma)\,J_0(p\gamma)\,\gamma\,\mathrm{d}\gamma\,,$$

the first of which (F) determines the beam structure at a distance z from the source in an absolutely transparent medium, while the second (T) represents the modulation transfer function of the turbid medium layer through which the beam was traveling.

Equation (10) served as a 'bridge' connecting for the first time the theory of radiative transfer with that of wave propagation in randomly inhomogeneous media. This link was made explicit when considering the propagation of a wave beam $u=V(\mathbf{r}_\perp,z)\exp(\mathrm{i}\omega t-\mathrm{i}kz)$ through a medium with the large-scale fluctuations of permittivity $\varepsilon=\langle\varepsilon\rangle[1+\delta\varepsilon(\mathbf{r}_\perp,z)]$. Based on the equation

$$\left[\Delta_\perp-2\mathrm{i}k\frac{\partial}{\partial z}+k^2\delta\varepsilon\right]V=0$$

for the field correlation function

$$\Gamma(\mathbf{r}_\perp,z,\boldsymbol{\rho}_\perp)=\left\langle V\left(\mathbf{r}_\perp+\frac{\boldsymbol{\rho}_\perp}{2},z\right)V^*\left(\mathbf{r}_\perp-\frac{\boldsymbol{\rho}_\perp}{2},z\right)\right\rangle,\qquad(13)$$

an equation of the form [16, 17]

$$\left\{\nabla_{\mathbf{r}_\perp}\nabla_{\boldsymbol{\rho}_\perp}-\mathrm{i}k\frac{\partial}{\partial z}-\mathrm{i}k^2\left[b(0)-b(\rho_\perp)\right]\right\}\Gamma=0\,,$$
$$b(\rho_\perp)=\frac{k}{4}\int_{-\infty}^{\infty}\left\langle\delta\varepsilon(\mathbf{r}_\perp+\boldsymbol{\rho}_\perp,z+\xi)\,\delta\varepsilon(\mathbf{r}_\perp,z)\right\rangle\mathrm{d}\xi,$$
$$k=\omega\,\frac{\sqrt{\langle\varepsilon\rangle}}{c}$$

was derived. It was also shown that the Fourier transform of the correlation function:

$$I(\mathbf{r}_\perp,z,\mathbf{n}_\perp)=\frac{1}{\lambda^2}\iint_\infty\Gamma(\mathbf{r}_\perp,z,\rho_\perp)\exp(\mathrm{i}k\mathbf{n}_\perp\boldsymbol{\rho}_\perp)\,\mathrm{d}\boldsymbol{\rho}_\perp\,,$$
$$\lambda=\frac{2\pi}{k}\,,\qquad(14)$$

satisfies Eqn (10) with the coefficient $n_z=1$, describing the radiation field in a medium with optical characteristics

$$x_1(n_\perp)=\frac{2\pi}{\lambda^2 b(0)}\int_0^\infty b(\rho_\perp)\,J_0(kn_\perp\rho_\perp)\,\rho_\perp\,\mathrm{d}\rho_\perp\,,$$
$$\sigma_1=kb(0)\,,\quad\kappa_1=0\,.$$

The implication was that the mathematical apparatus of the radiative transfer theory can be employed to analyze the influence of a randomly inhomogeneous medium on the correlation and energy characteristics of a wave beam with due regard for its diffractive broadening, provided that its radiance is defined not in the energy terms (as the radiation flux per unit area and per unit solid angle), but through relationship (14).

The influence of a photon spread in ranges on the characteristics of nonstationary light fields was explored with the help of an equation of the Fokker–Planck type [12]:

$$\left[\frac{1}{c}\frac{\partial}{\partial t}+n_z\frac{\partial}{\partial z}+\mathbf{n}_\perp\nabla_\perp+\kappa_1-\frac{d_2}{4}\sigma_1\Delta_{\mathbf{n}_\perp}\right]I(\mathbf{r}_\perp,z,\mathbf{n}_\perp,t)=0\qquad(15)$$

with the coefficient $n_z=1-n_\perp^2/2$. Equation (15) follows from Eqn (10) under the condition that the scattering phase function be narrow compared to the width of angular radiance distribution, and contain only the integral indicatrix parameter $d_2=2\pi\int_0^\pi\gamma^2x_1(\gamma)\sin\gamma\,\mathrm{d}\gamma$. On the basis of

Eqn (15), a theory of multiple scattering for sinusoidally modulated light beams was developed [31]. It was shown that the sinusoidal component of the radiation field behaves similarly to a wave with specific dispersive properties. This equation was exploited as well for the analysis of the spatio–temporal structure of a pulsed light beam [32] and the construction of simple analytical models of stationary radiation fields formed by sources of various types in strongly absorbing media with narrow scattering phase functions [33].

The radiation field of a δ-pulsed source $Q(\mathbf{r},\mathbf{n},t) = \bar{Q}(\mathbf{r},\mathbf{n})\,\delta(t)$ in a turbid medium with an absorption coefficient κ_1 is expressed in the form

$$I = \frac{1}{2\pi}\int \bar{I}\left(\mathbf{r},\mathbf{n},\kappa_1 + \mathrm{i}\,\frac{\omega}{c}\right)\exp(\mathrm{i}\omega t)\,\mathrm{d}\omega$$

through the solution $\bar{I}(\mathbf{r},\mathbf{n},\kappa_1)$ of the transfer equation with a stationary source $\bar{Q}(\mathbf{r},\mathbf{n})$. Hence, the integral parameters of the pulsed signal — its mean propagation time $\bar{t}$ and typical duration Δt — can be found by differentiating $\bar{I}(\mathbf{r},\mathbf{n},\kappa_1)$ with respect to the parameter κ_1:

$$\bar{t} = \frac{\int tI\,\mathrm{d}t}{\int I\,\mathrm{d}t} = -\frac{1}{c}\,\frac{\mathrm{d}\ln\bar{I}}{\mathrm{d}\kappa_1}\,, \tag{16}$$

$$(\Delta t)^2 = \frac{\int (t-\bar{t})^2 I\,\mathrm{d}t}{\int I\,\mathrm{d}t} = \frac{1}{c^2}\,\frac{\mathrm{d}^2\ln\bar{I}}{\mathrm{d}\kappa_1^2}\,. \tag{17}$$

Owing to relationships (16) and (17), the models of stationary radiation fields presented above turn out to be very useful also for the theory of pulsed signal propagation in turbid media.

The spatial structure of a narrow light beam is described by Eqn (15) with a large error. For this reason, Eqn (10) with the coefficient $n_z = 1 - n_\perp^2/2$, which is devoid of this drawback, holds some interest for a certain set of applications. Based on this equation, a theory was developed for plane pulsed wave propagation in a turbulent medium [34]. An approximate solution to this equation was also obtained for a monodirected δ-pulsed source [35]. The analysis of this solution has, in particular, shown that the time moment (17) is determined from Eqn (15) with a noticeable error and depends not only on the scattering phase function dispersion d_2, but also on the parameter

$$g = \frac{2\pi}{d_2^2}\int_0^\pi \gamma^4 x_1(\gamma)\,\sin\gamma\,\mathrm{d}\gamma,$$

which characterizes its form.

The absence of an exact analytical solution to Eqn (10) with the coefficient $n_z = 1 - n_\perp^2/2$ does not exclude the possibility of finding the integral characteristics of its exact solution. If one passes on in Eqn (10) to dimensionless variables [34] $s = ct$ and $\zeta = ct - z$, then for the moments of longitudinal radiance distribution in the pulse volume, viz.

$$M_m(\mathbf{r}_\perp,\mathbf{n}_\perp,s) = \int_0^\infty \zeta^m I(\mathbf{r}_\perp,\zeta,\mathbf{n}_\perp,s)\,\mathrm{d}\zeta, \quad m = 0,1,\ldots\,, \tag{18}$$

follow the equations [36]

$$\left(\frac{\partial}{\partial s} + \mathbf{n}_\perp\nabla_\perp + \alpha\right)M_m(\mathbf{r}_\perp,\mathbf{n}_\perp,s)$$
$$= \sigma_1\iint_\infty M_m(\mathbf{r}_\perp,\mathbf{n}'_\perp,s)\,x_1\left(|\mathbf{n}_\perp - \mathbf{n}'_\perp|\right)\,\mathrm{d}\mathbf{n}'_\perp + \frac{m}{2}\,n_\perp^2 M_{m-1}\,, \tag{19}$$

which yield the exact analytical solution because they are identical to the stationary equation (10) with the coefficient $n_z = 1$. The moment M_0 is found directly by replacing $z \to s$, $I \to M_0$, and $I_0 \to M_0(\mathbf{r}_\perp,\mathbf{n}_\perp,0)$, while computation of the higher moments requires solving the RTE with volume sources. Notice that temporal moments of the radiation field can easily be expressed through the spatial ones (M_m), provided the shape of the light pulse varies only slightly as it displaces over the proper length.

4. Fluctuating light fields and images

Random distortions of images occur when objects are observed through a turbulent atmosphere, rough water surface, or turbid medium, the optical parameters of which vary randomly in space (when the Earth is observed from space, the role of such a medium can be played by fragmented cloudiness). In order to analyze the influence of turbulence on images, the wave theory is routinely used; in the other cases mentioned above, the methods and apparatus of the radiative transfer theory are employed.

The theory of instrumental imaging through the rough water surface [37, 38] is similar to that of underwater imaging in the sense that in both cases the signals from the object and the medium are determined from relationships (4) and (5). The influence of the surface on the image is taken into account by substituting fields $E^{(\mathrm{s,r})}$ found with regard for light refraction at the air–water interface into these relationships. The fields $E^{(\mathrm{s,r})}$ are expressed through the radiance of light incident on the water surface and Green's function of the transfer equation in the small-angle approximation. This yields the general expression for a random realization of an image, which is further used to analyze its statistical characteristics. One need not solve the transfer equation all over again.

The theory of imaging in turbid media with fluctuating optical characteristics calls for qualitatively new solutions to the RTE. Such solutions are also necessary for problems of the optical diagnostics of similar media. The research on light propagation in turbid media with randomly inhomogeneous optical characteristics was initially prompted by problems of Earth's radiative balance [39]. In that case, relatively simple models of scattering objects were employed (a homogeneous layer of turbid medium with a fluctuating optical thickness or a smoothly inhomogeneous layer). Later on, the focus shifted toward statistical models of radiative transfer in media with three-dimensional concentration inhomogeneities of absorbing and scattering substances [40–43]. Along with models of the statistically mean radiation field, the models of its fluctuations were developed, too. In particular, equations have been obtained for computing the mean radiance and the function of the spatial radiance correlation for a light beam leaving a layer of a turbid medium with a narrow scattering phase function and randomly inhomogeneous coefficients of absorption and scattering [41]. These equations have allowed one to quantitatively describe the bleaching effect of the medium, caused by fluctuations in its parameters, and also the processes of random modulation of radiance distribution in a narrow light beam upon its multiple passing through absorbing inhomogeneities and the 'smoothing' of occurring radiance fluctuations because of multiple 'forward' scattering. It was shown that relative radiance fluctuations can grow without limits as the thickness of the scattering layer increases and that a stationary regime of fluctuations (their saturation) can settle down depending on the medium parameters [41, 44, 45].

These equations served as a basis for developing a statistical model of OCT — imaging of a layered turbid medium with three-dimensional inhomogeneities of absorption and backscattering coefficients $\kappa = \bar{\kappa}(z) + \tilde{\kappa}(\mathbf{r}_\perp, z)$ and $\sigma_\mathrm{b} = \bar{\sigma}(z) + \tilde{\sigma}(\mathbf{r}_\perp, z)$, $(\bar{\kappa} = \langle \kappa \rangle$, $\bar{\sigma} = \langle \sigma_\mathrm{b} \rangle)$, respectively. Expressions for spatial correlation function $B_P\,(\boldsymbol{\rho}, z_1, z_2)$ of relative fluctuations of tomographic signal power $P(\mathbf{r}_\perp, z)$, coming from two separate locations within a medium with coordinates $\mathbf{r}_\perp + \boldsymbol{\rho}/2$, z_1 and $\mathbf{r}_\perp + \boldsymbol{\rho}/2$, z_2, were found and it was shown that fluctuations of medium parameters lead to the appearance of spatial noise with specific properties ('shadow' noise) [46, 47] in the medium image. This noise arises as a result of inhomogeneous shading of each medium layer by clusters of the absorbing or scattering substance located in the upper layers. Notice that fluctuations of κ are manifested only through the shadow noise, while those of σ_b show up through the shadow noise and images of inhomogeneities σ_b proper. Figures 2 and 3 give examples of function $B_P(\boldsymbol{\rho}, z_1, z_2)$ computed for those cases when only one of these parameters fluctuates. The computations used the formulas

$$B_P(\boldsymbol{\rho}, z_1, z_2) = \left\{ \left[1 - \frac{4A_\sigma(\boldsymbol{\rho}, z_m)}{\bar{\sigma}(z_m) - 2A_\sigma(0, z_m)} + \frac{B_\sigma(\boldsymbol{\rho}, z, \xi)}{\left[\bar{\sigma}(z_1) - 2A_\sigma(0, z_1)\right]\left[\bar{\sigma}(z_2) - 2A_\sigma(0, z_2)\right]} \right] \times \exp\left[\int_0^{z_m} \left[4A_\kappa(\boldsymbol{\rho}, z') + 16A_\sigma(\boldsymbol{\rho}, z') \right] \mathrm{d}z' \right] \right\} - 1\,, \quad (20)$$

$$A_\kappa(\boldsymbol{\rho}, z) = \int_{-\infty}^{\infty} B_\kappa(\boldsymbol{\rho}, z, \xi)\, \mathrm{d}\xi\,,$$

$$A_\sigma(\boldsymbol{\rho}, z) = \int_{-\infty}^{\infty} B_\sigma(\boldsymbol{\rho}, z, \xi)\, \mathrm{d}\xi\,, \quad (21)$$

$$B_\kappa(\boldsymbol{\rho}, z, \xi) = \left\langle \tilde{\kappa}\left(\mathbf{r} + \frac{\boldsymbol{\rho}}{2}, z + \frac{\xi}{2} \right) \tilde{\kappa}\left(\mathbf{r} - \frac{\boldsymbol{\rho}}{2}, z - \frac{\xi}{2} \right) \right\rangle, \quad (22)$$

$$B_\sigma(\boldsymbol{\rho}, z, \xi) = \left\langle \tilde{\sigma}\left(\mathbf{r} + \frac{\boldsymbol{\rho}}{2}, z + \frac{\xi}{2} \right) \tilde{\sigma}\left(\mathbf{r} - \frac{\boldsymbol{\rho}}{2}, z - \frac{\xi}{2} \right) \right\rangle, \quad (23)$$

$$z = \frac{z_1 + z_2}{2}\,, \quad \xi = z_1 - z_2\,, \quad z_m = z - \frac{1}{2}\,|\xi|\,, \quad (24)$$

which are applicable under the condition that the width of the PSF be small compared to the typical horizontal size of inhomogeneities. As follows from the figures, fluctuations of the absorption coefficient lead to the appearance of noise, the correlation function of which is everywhere positive and has the shape of a wave crest. When the backscattering coefficient is fluctuating, the correlation function can change its sign since the image of each inhomogeneity and its shadow form the combined signal with sign-changing intensity variations.

These theoretical conclusions got qualitative support from experiments with a model turbid medium containing absorbing inhomogeneities and the results of correlation processing of biotissue tomograms. Figure 4 presents a comparison between the theoretical and experimental dependences of dispersion in relative fluctuations of tomographic signal power $d_P(z) = B_P(0, z, z)$ on the depth z it originated from. In processing the tomograms, the following functions

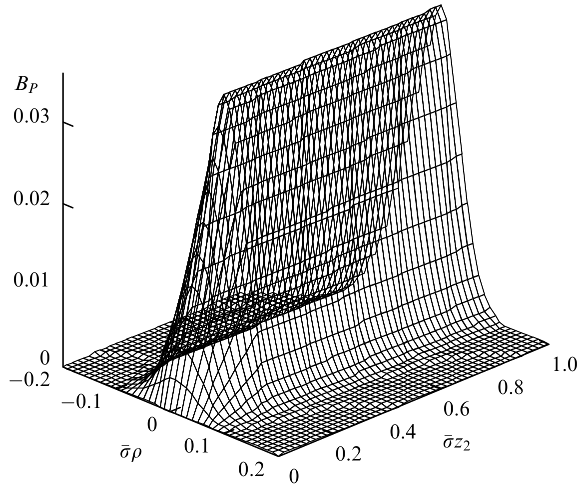

Figure 2. The function of spatial correlation B_P of relative fluctuations of tomographic signal power in a medium with a fluctuating absorption coefficient as a function of variables $\bar{\delta}\rho$ and $\bar{\sigma}z_2$ for $\bar{\sigma}z_1 = 0.2$. The dispersion of fluctuations of absorption index is $d_\kappa = 0.5\bar{\sigma}^2$, and the correlation radius $\rho_\kappa = 0.05/\bar{\sigma}$.

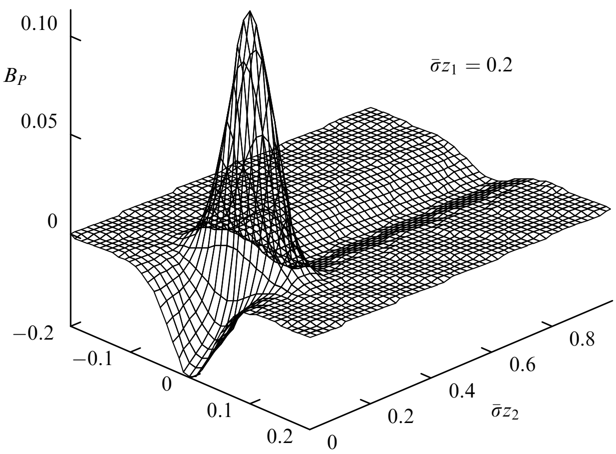

Figure 3. The same as in Fig. 2 but for a medium with a fluctuating backscattering coefficient for $d_\sigma = 0.125\bar{\sigma}^2$ and $\rho_\sigma = 0.05/\bar{\sigma}$.

were determined in addition to the dependence $d_P(z)$:

$$R(z_1, z_2) = \frac{B_P(0, z_1, z_2)}{\sqrt{d_P(z_1)}\sqrt{d_P(z_2)}}\,,$$

$$\bar{B}_P(\rho, \xi) = \frac{1}{z_0} \int_0^{z_0} B_P\left(\rho, z + \frac{\xi}{2}, z - \frac{\xi}{2} \right) \mathrm{d}z\,.$$

They correspond to the coefficient of longitudinal correlation of relative signal fluctuations and their correlation function averaged over the thickness z_0 of the medium layer under study. The data presented in Fig. 5 illustrate the possibility of fitting the theoretical predictions to experimental results by exhausting medium parameters appearing in formulas used and, in this way, the feasibility of determining them by the OCT method.

5. Conclusions

In this report we presented the problems of imaging theory, which effectively use the small-angle approximation of radiative transfer theory. The list could be further continued. However, it should be borne in mind that the accuracy of

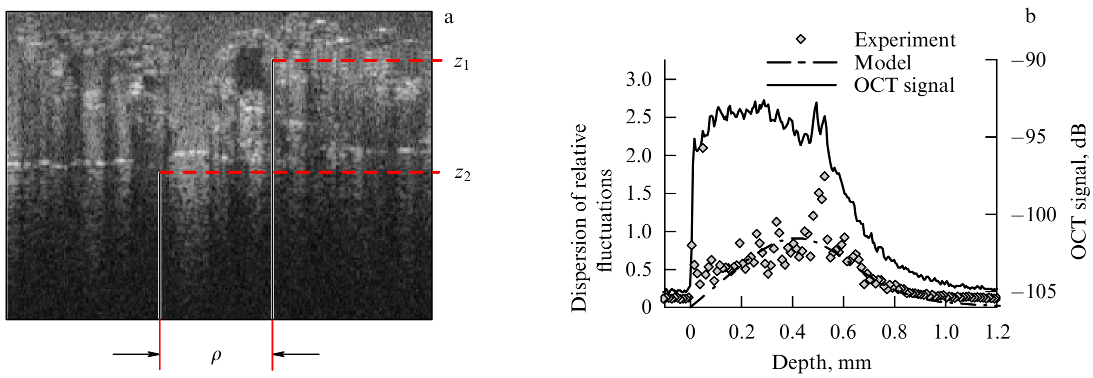

Figure 4. Tomogram of a model turbid medium (a) and results of its analysis (b). The solid line plots the mean power of the OCT signal as a function of depth, squares correspond to the dispersion of relative fluctuations of the measured signal, and the dashed curve presents the theoretical dependence of the dispersion of fluctuations on the depth.

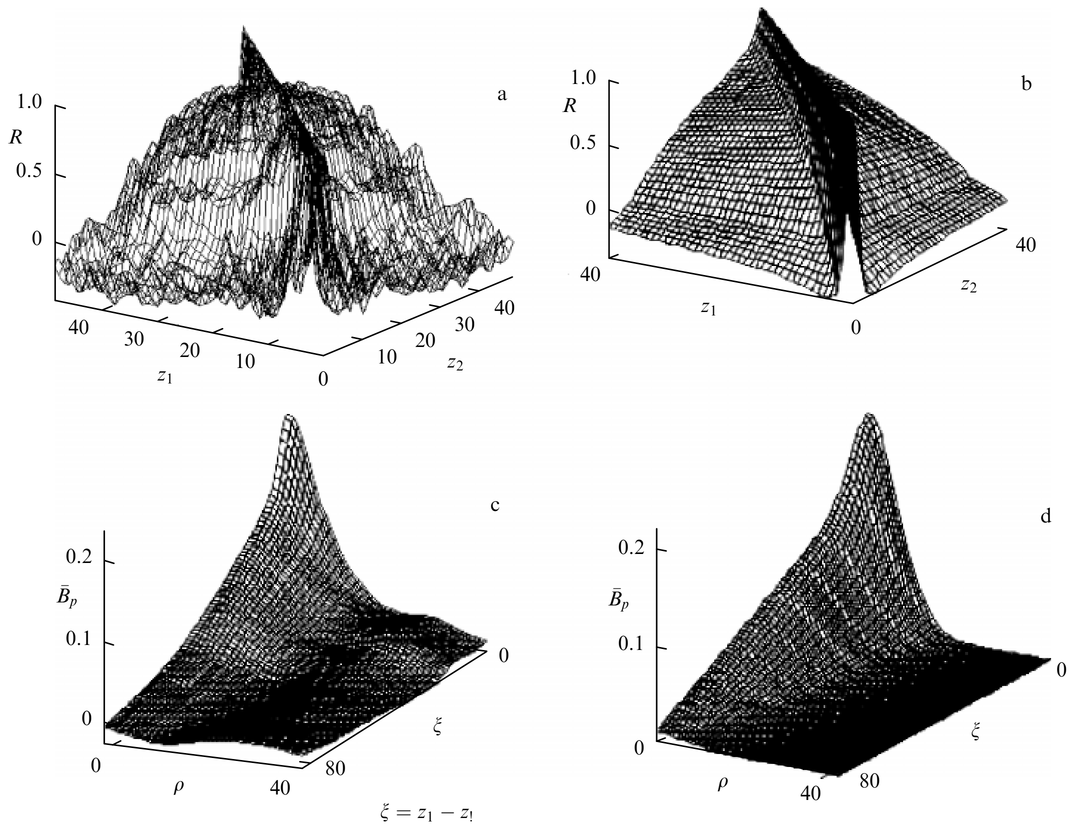

Figure 5. Correlation characteristics of the tomogram presented in Fig. 4: (a, b) the coefficient of longitudinal correlation $R(0, z_1, z_2)$ of image noise, and (c, d) the depth-averaged function of spatial noise correlation $\bar{B}_P(\rho, \xi)$. Panels (a) and (c) present the results of tomogram processing, and (b) and (d) display the theoretical results.

this approximation is strongly sensitive to the angular width of the scattering indicatrix and absorption ability of the medium. In seawater, for which the albedo of single scattering (Λ) is commonly below 0.9, this approximation performs reliably for optical thicknesses reaching up to $\tau \sim 15$. For biological tissues, where $\Lambda \sim 0.99$, the diffusive and directional components of irradiance in a narrow light beam become equal already at $\tau \sim 5$. This calls for hybrid models of the light field [12, 48], which allow for the effects of multiple light scattering over large angles.

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