the special issue of the journal *Electromagnetic Waves and Electronic Systems* devoted to the 100th anniversary of the birth of this brilliant scientist.

PACS numbers: 42.25.Bs, 42.25.Dd, 72.15.Rn
DOI: 10.3367/UFNe.0179.200905h.0534

Asymptotic limit of the radiative transfer theory in problems of multiple wave scattering in randomly inhomogeneous media

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1. Introduction

Wave propagation in disordered systems is considered one of the most difficult subjects of theoretical physics. The traditional approach involves the phenomenological radiative transfer theory [1, 2], which originated more than a century ago in the studies by Khvol'son (1890), Schuster (1905), and Schwarzschield (1906) devoted to light scattering in milk glasses and solar and foggy earth atmospheres; it is based on the notions of linear kinetic theory involving an elementary scattering act and radiation free path. In the 1950s, the development of the theory of partial coherence for wave fields prompted active studies of the applicability limits for the radiative transfer theory from the standpoint of the statistical theory of multiple wave scattering in randomly inhomogeneous media. Their results were regularly reported and critically discussed at the All-Moscow Radiophysics Seminar headed by S M Rytov from 1965 to 1985. Although this research was purely theoretical, it resulted in the prediction of weak light localization in randomly inhomogeneous media in 1973. Because this phenomenon lay at the applicability boundary of the radiative transfer theory, it could not be ignored.

As early as 1967, a derivation was proposed [3] (see also [4]) of the phenomenological radiative transfer equation in a discrete randomly inhomogeneous medium, with due regard for the correlation of scatterers in all orders and mutual irradiation of scatterers within the same effective inhomogeneity, i.e., a cluster of scatterers. A single scatterer was characterized by its shape, dielectric permeability, and conductivity; the method of Dyson and Bethe–Salpeter equations, the Feynman diagram technique, and the concept of the quantum mechanical scattering operator were used [5]. The transfer equation [3], with pair correlation of weak scatterers satisfying the applicability conditions of the Born approximation taken account, was used to investigate an increase in the free path length of microwaves in snow layers and conduction electrons in liquid metals [6], as well as the opposite effect of a decrease in the free path of light [7] and neutrons in a liquid [6] in the vicinity of a phase transition critical point. The contribution from higher correlations of weak scatterers to the light free path at critical opalescence was considered in Ref. [8], while Ref. [9] dealt with the influence of higher correlations due to large, optically dense scatterers on the transmittance of a layer composed of such particles.

Simultaneously with the derivation of the transfer equation [3], the applicability condition for the phenomenological transfer theory was formulated as the single-group approximation, i.e., the possibility to discard all repeated scattering of radiation on the same inhomogeneity. Additionally, it is required that inhomogeneities be in the far field of Fraunhofer diffraction with respect to each other. The first part of this condition, later called the approximation of independent scattering on effective inhomogeneities [10], disregards all loops in multiple wave scattering with a diameter of the order of or larger than the free path length of radiation.

The decisive role of the single-group approximation for the transfer theory was convincingly demonstrated with the model of multiple scattering of a nonstationary wave field in a randomly inhomogeneous and randomly variable medium, for which the concept of a finite 'lifetime' of an inhomogeneity can be introduced. From this model, thoroughly studied in Refs [6, 11], the transfer theory follows as an asymptotically exact Van Hove limit [12] under the condition that the ratio of the inhomogeneity lifetime to the radiation free path time tends to zero, but the ratio of the observation time to the free path time remains fixed. The second condition of the Van Hove limit prevents repeated scattering due to a selected inhomogeneity from occurring on a large time or space scale of radiation propagation. In repeated scattering, the effects of multi-group or dependent scattering, i.e., the loops, would have to be taken into account, and the phenomenological radiative transfer theory would need modifications.

The most thoroughly studied effect of multi-group or dependent scattering by effective inhomogeneities is manifested through the coherent enhancement of backscattering. It was first predicted as a wave correction to the solution of the transfer equation for scattering directed exactly 'backward' [13]. Reference [14] shows, based on the technique of cyclic (maximally crossed) Feynman diagrams, that the wave correction attains a relative value close to unity in a narrow backscattering cone whose angular width is of the order of the ratio of the wave length to the free path length of radiation.

The predicted cone of enhanced backscattering was experimentally observed by several groups [15–17]. It strongly stimulated research on the weak localization phenomenon in optics [18]. Coherent enhanced backscattering stems from coherent loops in which the field and its complex conjugate go around a given set of inhomogeneities in opposite ways. There also exist incoherent loops, in which case a set of inhomogeneities is passed by the field and its complex conjugate in the same order. The incoherent loops lead to the backscattering effect for nonstationary radiation in a modified transfer theory with time delay [19], where the delay time is of the order of the radiation free path time. Another variant of the transfer theory with time delay emerges on considering the effect of trapping [20] under multiple scattering of wave radiation, for example, a short femtosecond laser pulse [21], in a randomly inhomogeneous medium composed of resonant scatterers.

Taking account of loops in multiple wave scattering implies lifting the part of the applicability condition of the phenomenological transfer equation [3] that requires omitting repeated scattering of radiation by the same inhomogeneity. No principal obstacles are seen in using this equation to explore multiple light scattering by new artificial systems such as statistical ensembles of nanosclusters [22], in which each cluster represents a combination of a possibly large but finite number of atoms (or molecules) and requires describing light scattering with methods of quantum mechanics or electrodynamics. Admittedly, it may call for considering effects from near fields in the form of evanescent waves in some vicinity of effective inhomogeneities (nanoclusters), i.e., abandoning the condition that the inhomogeneities are located in the far wave zone of Fraunhofer diffraction with respect to each other.

Recently, considerable progress has been achieved in the theoretical understanding of the role played by near fields in multiple scattering of electromagnetic waves by inhomogeneous dielectric media. The progress was facilitated by using the Sommerfeld–Weyl representation for angular spectral amplitudes of local electromagnetic waves propagating along and against the axis of the parameter of embedding into a layer of a three-dimensional medium, and by exploring the phenomenon of energy emission from an evanescent wave scattered by a dielectric structure [23], with checks for the extended unitarity of the 2×2 block S-matrix [24] and the interpretation of the mentioned energy emission through the interference of two evanescent waves decaying toward each other [25]. As a result, an approach was formulated [26] to establishing a modified theory of electromagnetic radiation transfer in a randomly inhomogeneous medium with the effects of near fields and interference of oppositely directed wave beams taken into account.

Finally, perspectives of exploring the electrodynamic properties of artificial materials exemplified by statistical ensembles of particles with a given shape, dielectric permeability and conductivity (see, e.g., Ref. [27]), and, possibly, high spatial packing parameter, put forward the task of radically modifying the phenomenological radiative transfer theory while preserving some of its properties. Such a modification hinges on the Ambartsumyan method of layer summation [28], according to which the scattering medium is split in virtual layers (slices) separated by small gaps, with a subsequent derivation of transfer relations in terms of the intensity between radiation fluxes in gaps and the fluxes reflected from and transmitted through the entire medium.

The idea about the virtual layering of the medium into slices and gaps is evidently applicable to any medium, in particular, that consisting of the particles mentioned previously, for further derivation of transfer relations between the radiation fluxes, and not only for wave intensities but also for wave amplitudes, as in Refs [29, 30]. The transfer relations obtained there imply a system of four differential Riccati equations for the blocks of the S-matrix, written for a layer of a medium in the form of Reid [31] and Redheffer [32], with differentiation over the parameter of embedding into the layer. Using the Reid method, a relation is established between the solution of the system of nonlinear matrix Riccati equations and that of the linear differential equation for the transfer matrix. Paper [30] elaborates on the extended relations of unitarity and invariance under time reversal for the solution of the system of Riccati equations and of the transfer matrix equation. The extended relations augment, by near-field effects, the more habitual quantum mechanical relations used in the widely known theory for the transmission coefficient and electron localization length in N connected disordered chains developed by Dorokhov [33] and the authors of Ref. [34] based on the Fokker–Planck equation for the probability distribution function for transfer matrix elements.

Noteworthy in the just mentioned system of matrix Riccati equations is the fact that the governing role belongs to the equation for the coefficient of wave reflection from a layer of a three-dimensional randomly inhomogeneous dielectric medium written for angular spectral amplitudes, which is independent of the other equations. It was studied in [35] and served as a starting point in Ref. [36], where the functional Fokker–Planck equation for the functional of the probability distribution of the reflection coefficient was written.

The treatment of the Fokker–Planck equation in variational derivatives is exceptionally difficult. However, one can change to ordinary derivatives by discretizing the space of the wave vector transverse to the axis of the embedding parameter, in analogy with the method of the transfer matrix with a finite number of propagation channels in the quantum mechanical theory dealing with interference effects in metal conductors [37].

In exploring multiple wave scattering by statistical ensembles of particles characterized by a large packing parameter, it seems reasonable to assume that the distribution of particles is periodic in the zeroth approximation, as in the structure of the photon crystal in [38]. In this case, for example, the matrix Riccati equation for the reflection coefficient turns out to be naturally discretized over diffraction orders of the radiation reflected from and transmitted through the structure layer. This equation was successfully used in Ref. [39] for numerical computation via the Runge–Kutta method of formation of gaps in the spectrum of radiation transmission through an ordered system of dielectric cylinders with arbitrary cross sections.

Some of the aforementioned questions of the asymptotic limit of the radiative transfer theory are considered in detail in what follows.

2. Single-group approximation and the Van Hove limit

In the scalar case, the phenomenological transfer equation written for the ray intensity $I(\mathbf{R}, \mathbf{s})$ at a point $\mathbf{R}$ in the direction of a unit vector $\mathbf{s}$ takes the form

$$\mathbf{s}\nabla I(\mathbf{R}, \mathbf{s}) = -\frac{1}{l}\, I(\mathbf{R}, \mathbf{s}) + \int_{4\pi} \mathrm{d}\mathbf{s}'\, W(\mathbf{s}, \mathbf{s}')\, I(\mathbf{R}, \mathbf{s}')\,. \qquad (1)$$

The coefficients of extinction $1/l$ and scattering $W(\mathbf{s}, \mathbf{s}')$ due to an elementary volume are expressed by via Fourier transformation through the mass operator M_1 and intensity operator K_1 of the Dyson and Bethe–Salpeter equations in the single-group approximation. They are symbolically represented by the series

$$\begin{matrix}\mathrm{M}_1 \\ \mathrm{K}_1\end{matrix} = \sum_{n=1}^{\infty} \frac{1}{n!} \int \mathrm{d}1 \ldots \int \mathrm{d}n g_n(1, \ldots, n) \begin{matrix} T^{\mathrm{gr}}_{1\ldots n} \\ (T_{1\ldots n} \otimes T^*_{1\ldots n})^{\mathrm{gr}} \end{matrix}\,. \qquad (2)$$

Here, natural numbers $1, \ldots, n$ label spatial coordinates of points, $T_{1\ldots n}$ is the operator of scattering by a system of n scatterers centered at these points, and $g_n(1, \ldots, n)$ is the correlation function of the scatterers. The group subtraction operation gr removes small orders of scattering in the group of scatterers and acts in the simplest case as $T^{\mathrm{gr}}_{12} = T_{12} - T_1 - T_2$. The symbol $\otimes$ denotes the tensor product.

Originally, single-group approximation (2) was derived using the Feynman diagram technique. However, it was recognized later that it could be obtained on a purely analytic level, based on the exact Bethe–Salpeter relation [20, 40] of the form

$$(\mathrm{L}\otimes I - I\otimes \mathrm{L})\langle G\otimes G^*\rangle = \Big\langle \big[\mathbf{M}\otimes I - I\otimes \mathbf{M}^* - (G\otimes I - I\otimes G^*)\,\mathbf{K}\big]\, G\otimes G^*\Big\rangle + I\otimes\langle G^*\rangle - \langle G\rangle\otimes I\,, \qquad (3)$$

where L is the differential operator of the wave equation in free space, G is the Green's function of the wave equation in a randomly inhomogeneous medium, and the angular brackets denote averaging over the statistical ensemble of scatterers. The operators $\mathbf{M}$ and $\mathbf{K}$ formally have the same analytic structure as the mass and intensity operators in single-group approximation (2), except for the replacement of the T operators of free space scattering with self-consistent random operators of scattering in a randomly inhomogeneous medium.

We assume that the random Green's function $G(\mathbf{r},\mathbf{r}')$ satisfies the locality property and hence differs only slightly from its ensemble mean and the Green's function in free space if the distance between the observation and source points $\mathbf{r}$ and $\mathbf{r}'$ is of the order of the scatterer radii and their correlation radii. In that case, the random operators $\mathbf{M}$ and $\mathbf{K}$ practically coincide, respectively, with the mass and intensity operators in single-group approximation (2), while relation (3) transforms into the Bethe–Salpeter equation in the form of a kinetic equation [20].

The locality property of the random Green's function formulated above, in particular, implies omission of all loops in multiple wave scattering with a diameter of the order of or greater than the free path length of radiation. Can this property be justified in an asymptotically exact way? Such justification is known, for example, for the problem of the propagation of a quantum mechanical stream of particles in the field of a randomly varying potential $V(\mathbf{r},t)$. Based on the stochastic Liouville–von Neumann equation for the density matrix $\rho(t)$ of a particle stream under the assumption that the probability distribution of random potential realizations is Gaussian, the kinetic equation for the ensemble mean density matrix $\bar\rho(\mathbf{R},\mathbf{k},t)$ in the Wigner variables was derived in Refs [6, 20] in the form

$$\left(\frac{\partial}{\partial t} + \frac{\hbar\mathbf{k}}{m}\,\nabla_{\mathbf{R}}\right)\bar\rho(\mathbf{R},\mathbf{k},t) = \int \mathrm{d}\mathbf{k}'\,W(\mathbf{k},\mathbf{k}')\big[\bar\rho(\mathbf{R},\mathbf{k}',t) - \bar\rho(\mathbf{R},\mathbf{k},t)\big]\,. \qquad (4)$$

Equation (4) is asymptotically exact in the Van Hove limit $\tau_0/t_M \to 0$, $t/t_M = \mathrm{const}$, where t_M estimates the relaxation time t_r of the kinetic equation from below and the lifetime τ_0 of the effective inhomogeneity is expressed in terms of the cumulant of random potential fluctuations. Paper [11] considers the passage from kinetic equation (4) to the limit of stationary transfer equation (1) through the replacement of the observation time with the absorption time in the second condition of the Van Hove limit.

3. Coherent loops and weak localization

Coherent loops are linked to cyclic Feynman diagrams of the intensity operator [14]. Cyclic diagrams are omitted when transfer equation (1) is derived in the framework of single-group approximation (2). Nevertheless, they are, in some sense, equivalent to 'ladder' diagrams, which constitute the main element of the transfer theory. This equivalence property follows upon inversion of the upper or lower rows of a cyclic diagram and using the reciprocity property of the Green's function and of the intensity operator in the single-group approximation. The equivalence property of cyclic diagrams enabled the author of Ref. [14] to conclude that there exists a cone of enhanced backscattering with the width $\delta\theta \approx \lambda/l$ measured by the ratio of the wavelength λ to the free path length l.

It is worth noting that Ref. [14] was published in relation to the discussion of the work by Gazaryan [41] which, based on the exact solution to the Anderson strong localization problem [42] in a one-dimensional randomly inhomogeneous medium, demonstrated, following Ref. [43], an exponentially fast decay of the layer transmittance for the mean intensity as a function of the layer thickness, instead of a power-law decay predicted by the transfer equation [1]. In other words, the transfer equation provides a reduced value for the reflective capability of the one-dimensional scattering layer. The goal of Ref. [14] was to demonstrate the reduction of a similar type in the framework of transfer theory for the reflective capability of a three-dimensional scattering layer, although a reduction in a weak sense.

4. Near fields and the tunnel component of radiative transfer

Near fields arise during multiple wave scattering in a randomly inhomogeneous medium as evanescent waves in the vicinity of effective inhomogeneities and participate in a specific way in radiation transfer together with propagating waves. Only the latter are taken into account by the phenomenological transfer theory. The principal role is here played by a formula derived in Ref. [25] for the radiation energy flux in an inhomogeneous dielectric medium. In the scalar version, this formula has the form

$$P_z(z) = \int_{\mathbf{k}_\perp} H^{\mathrm{pr}}(k_\perp)\big[\rho_{11}(\mathbf{k}_\perp,\mathbf{k}_\perp;z) - \rho_{22}(\mathbf{k}_\perp,\mathbf{k}_\perp;z)\big] + \int_{\mathbf{k}_\perp} \mathrm{i}H^{\mathrm{ev}}(k_\perp)\big[\rho_{12}(\mathbf{k}_\perp,\mathbf{k}_\perp;z) - \rho_{21}(\mathbf{k}_\perp,\mathbf{k}_\perp;z)\big], \qquad (5)$$

where $P_z(z)$ is the component of the Poynting vector along the z axis of the parameter of embedding into the medium layer $0 < z < L$, integrated in the plane perpendicular to this axis, $\rho_{mm'}(\mathbf{k}_\perp,\mathbf{k}'_\perp;z)$ is the density matrix [25] of (slowly varying) angular spectral amplitudes of local waves propagating in forward (index 1) and backward (index 2) directions relative to the z axis ($m, m' = 1, 2$), $\mathbf{k}_\perp$ and $\mathbf{k}'_\perp$ are the components of $\mathbf{k}$ and $\mathbf{k}'$ perpendicular to the z axis, $H^{\mathrm{pr}}(k_\perp)$ and $H^{\mathrm{ev}}(k_\perp)$ are the respective projectors on the propagating ($k_\perp < k_0$) and evanescent ($k_\perp > k_0$) waves, for which the longitudinal wave number $\gamma_k = [k_0^2 - k_\perp^2]^{1/2}$ takes a real or a purely imaginary value, and k_0 is the wave number of free space; the compact notation for integrals, $\int_{\mathbf{k}_\perp} = (2\pi)^{-2}\int \mathrm{d}\mathbf{k}_\perp$, is used.

According to formula (5), the contribution of propagating waves to the total energy flux along the z axis is determined by intensities of angular spectral amplitudes of waves propagating in forward and backward directions, whereas the contribution of evanescent waves to this energy flux is expressed through the term corresponding to the interference of two evanescent waves decaying toward each other. Turning to a randomly inhomogeneous medium, we need to consider the coherence matrix for angular spectral amplitudes of local waves for two values of the parameter of embedding into the medium, $\overline{\rho}_{mm'}(\mathbf{p}, \mathbf{p}'; z, z')$, where the bar denotes ensemble averaging and the short notation $\mathbf{p}$ and $\mathbf{p}'$ is used instead of $\mathbf{k}_\perp$ and $\mathbf{k}'_\perp$. The Bethe–Salpeter equation for this coherence matrix with the mass and intensity operators is derived in single-group approximation (2).

In the simplest case of weak uncorrelated scatterers, we arrive at the model of a medium with Gaussian fluctuations of the potential, used in Ref. [26] to analyze effects of near fields in the transfer theory. After some simplification, the Bethe–Salpeter equation for the coherence matrix of angular spectral amplitudes is reduced to four equations, two of which allow computing ensemble-averaged intensities of angular spectral amplitudes for the forward and backward directions, and the other two, for complex-conjugate mutual coherences of evanescent waves decaying toward each other. One of the first two equations has the form

$$\begin{aligned}\hat{\rho}_{11}(\mathbf{p}, \mathbf{p}, z, z) &= \exp\left(-2\Im\gamma_{1p} z\right)\left|f(\mathbf{p})\right|^2 \\ &+ \int_0^z \mathrm{d}z_1 \exp\left[-2\Im\gamma_{1p}(z - z_1)\right] \int_{\mathbf{p}'} \frac{1}{4|\gamma_p||\gamma_{p'}|} \\ &\times \left[B(\mathbf{p} - \mathbf{p}', \gamma_p - \gamma_{p'})\,\hat{\rho}_{11}(\mathbf{p}', \mathbf{p}'; z_1, z_1)\right. \\ &+ B(\mathbf{p} - \mathbf{p}', \gamma_p + \gamma_{p'})\,\hat{\rho}_{22}(\mathbf{p}', \mathbf{p}'; z_1, z_1) \\ &+ B(\mathbf{p} - \mathbf{p}', \gamma_p - \gamma_{p'})\,\hat{\rho}_{12}(\mathbf{p}', \mathbf{p}'; z_1, z_1) \\ &\left.+ B(\mathbf{p} - \mathbf{p}', \gamma_p + \gamma_{p'})\,\hat{\rho}_{21}(\mathbf{p}', \mathbf{p}'; z_1, z_1)\right]. \qquad (6)\end{aligned}$$

Equation (6) is written in terms of the coherence matrix of rapidly varying angular spectral amplitudes $\hat{\rho}_{mm'}(\mathbf{p}, \mathbf{p}'; z, z') = \exp\left(\mathrm{i}\xi_m\gamma_p z - \mathrm{i}\xi_{m'}\gamma^*_{p'} z'\right)\overline{\rho}_{mm'}(\mathbf{p}, \mathbf{p}'; z, z')$ with $\xi_1 = 1$ and $\xi_2 = -1$. The quantity $\Im\gamma_{1p}$ represents the imaginary part of the longitudinal wave number in a deterministic medium with the effective complex wave number k_1; it is assumed that a propagating wave with the angular spectral amplitude $f(\mathbf{p})$ is incident on the left layer boundary. The function $B(\mathbf{k}_\perp, k_z)$ is the three-dimensional Fourier transform of the cumulant of random potential fluctuations. Equation (6) describes multiple scattering of ensemble-averaged intensities of angular spectral amplitudes by effective inhomogeneities with a free path between them that is typical of the transfer theory. In addition, this equation, evidently, contains the contribution to mean intensity of waves propagating forward brought about by mutual coherences of evanescent waves as they are scattered on effective inhomogeneities.

The equation for computing the ensemble-averaged intensity of angular spectral amplitudes for waves propagating backward is similar to Eqn (6) and is not reproduced here.

We turn to the equation for mutual coherence of pairs of evanescent waves decaying toward each other, of the form

$$\begin{aligned}\hat{\rho}_{12}(\mathbf{p}, \mathbf{p}; z, z) &= \int_0^z \mathrm{d}z_1 \int_z^L \mathrm{d}\zeta_1 \exp\left[\mathrm{i}\gamma_{1p}(z - z_1)\right] \\ &\times \exp\left[\mathrm{i}\gamma^*_{1p}(z - \zeta_1)\right] \int_{\mathbf{p}'} \frac{1}{4|\gamma_p||\gamma_{p'}|} \\ &\times B(\mathbf{p} - \mathbf{p}', z_1 - \zeta_1) \sum_{m,n} \exp\left[-\mathrm{i}\xi_m\gamma_{p'}(z - z_1)\right] \\ &\times \exp\left[\mathrm{i}\xi_n\gamma^*_{p'}(z - \zeta_1)\right]\hat{\rho}_{mn}(\mathbf{p}', \mathbf{p}'; z, z), \qquad (7)\end{aligned}$$

where $B(\mathbf{k}_\perp, z)$ is the two-dimensional Fourier transform of the cumulant of random potential fluctuations in the plane perpendicular to the z axis.

Equation (7) differs principally from Eqn (6) by its structure. Indeed, the inequality $z_1 < z < \zeta_1$ holds in the double integral in the right-hand side of Eqn (7), with $\zeta_1 - z_1 \approx r_0$, where r_0 is the spatial scale of effective inhomogeneities. This implies that the observation point z and two scattering points z_1 and ζ_1 are confined to the same inhomogeneity, whereas two elementary evanescent waves appearing during scattering decay in the directions toward each other with interference at the observation point. This interference leads, as follows from a more detailed analysis of Eqn (7), to the following tunnel component of the radiation energy flux:

$$\begin{aligned}-2\int_{\mathbf{p}(k_0<p<p_0)} \Im\hat{\rho}_{12}(\mathbf{p}, \mathbf{p}; z, z) \\ = \frac{g}{1-g}\int_{\mathbf{p}(p<k_0)} \left[\hat{\rho}_{11}(\mathbf{p}, \mathbf{p}; z, z) - \hat{\rho}_{22}(\mathbf{p}, \mathbf{p}; z, z)\right]. \qquad (8)\end{aligned}$$

Solution (8) of the imaginary part of Eqn (7) is obtained in the limit of small-scale fluctuations of the random potential, when two evanescent waves, one incident on the inhomogeneity and the other scattered by it, decay only slightly on the inhomogeneity scale, $|\gamma_p|r_0 \ll 1$ and $|\gamma_{p'}|r_0 \ll 1$, with $k_0 < p < p_0$ and $k_0 < p' < p_0$, where $p_0 \approx 1/r_0$ is some cutoff parameter. The parameter of the tunnel component of the energy flux is of the order of $g \approx r_0/l$, which represents the ratio of the inhomogeneity scale to the free path length, defined as $1/l = \langle V^2 \rangle r_0^3$. The observation point z is assumed to be located far enough from the layer boundaries with respect to the scale of the effective inhomogeneity.

Equality (8) demonstrates the existence of a homogeneous tunnel energy flux in a randomly inhomogeneous medium. It is proportional to the energy flux of propagating waves, as is common in the phenomenological transfer theory. In a weakly scattering medium, the proportionality coefficient is small compared to unity. However, in a medium with strong scattering, for example, the one consisting of resonant scatterers, the proportionality coefficient may increase in absolute value.

5. Conclusions

Elucidation of the applicability bounds for the phenomenological radiative transfer theory from the standpoint of the statistical theory of multiple wave scattering has led to the prediction and discovery of the phenomenon of weak light localization in a randomly inhomogeneous medium. The proposed modifications to the transfer theory have enabled the treatment of phenomena such as the trapping of radiation in a resonant medium and the tunnel transfer of radiation energy by near fields of effective scattering inhomogeneities. The ideas and methods of the transfer theory were fruitful in studies dealing with the formation of forbidden zones in spectra of radiation transmission through photon crystals and can be used to explore multiple light scattering by statistical ensembles of nanoclusters and conducting particles with a large packing parameter.

This work was supported in part by the Russian Foundation for Basic Research grant 06-02-17451.

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PACS numbers: **41.20.–q**, 42.65.Pc, **85.50.–n**
DOI: 10.3367/UFNe.0179.200905i.0539

Local fields in nanolattices of strongly interacting atoms: nanostrata, giant resonances, 'magic numbers,' and optical bistability

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S M Rytov took an interest in many things, including the theory of layered media with a period much smaller than the wavelength [*Zh. Eksp. Teor. Fiz.* **29** 605 (1955)]. One of us, A E K, who participated in Rytov's seminars for 20 years, until 1979, was also involved with multiple and various things, and was sometimes surprised to realize that his work touches upon old areas of interests of Rytov. Of course, there is little surprise here, because Rytov had an intuition for unusual and fundamental things, and he often looked far ahead. Our new results presented here echo, to an extent, those old interests of Rytov.

In this report, following our recent brief publication [1], we consider a number of new effects emerging in one- and two-dimensional ordered systems of two-level atoms with a sufficiently strong dipole interaction. We have shown that in systems smaller than the wavelength of light, an excitation of the atomic dipole moments may become substantially inhomogeneous, forming strata and two-dimensional structures of a nanometer scale. Such behavior of the local field in a dielectric system is significantly different from the results of the Lorentz–Lorenz theory for local fields; it gives rise to resonances defined by the size and geometry of the system and is capable of inducing a giant local-field enhancement. We demonstrated that the saturation nonlinearity in two-level atoms may cause optical bistability, in particular, in the simplest case where the system is comprised of two atoms only. We also predicted 'magic' system sizes and geometries that, unlike the Lorentz model, do not result in a suppression of the local field in the system when the laser frequency is tuned to the resonance of the two-level atom.

A known fact of the electrodynamics of continuous media is that the microscopic field acting on atoms or molecules (known as the 'local field') is generally different from the macroscopic (average) field because of the dipole interaction between the particles composing the medium. This difference is a central point of the classical theory of local fields in dielectrics advanced by Lorentz and Lorenz [2]. An important, albeit implicit, assumption of that theory is that the local field remains virtually unchanged from atom to atom over distances much shorter than the wavelength of light λ. The theory is therefore essentially based on the so-called 'mean-field approximation.'