REVIEWS OF TOPICAL PROBLEMS

Thermally stimulated electromagnetic fields of solids

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Abstract. Different ways to calculate the spectral properties of fluctuating electromagnetic fields produced by solids are reviewed, all of which essentially reduce to solving the Maxwell equations for a specified geometry and boundary conditions and then using the fluctuation–dissipation theorem. It is shown that in the practical case of plane-layered solids, all correlation characteristics of thermal fields can be expressed in terms of the Fresnel coefficients. The experimental results on thermally stimulated electromagnetic fields from solids are in qualitative and quantitative agreement with model calculations and theoretical expectations. The dispersion interaction between solid bodies in different thermodynamic states, the fluctuating fields as a means of body-to-body energy transfer, and the shift, broadening, and deexcitation of energy levels in a particle near a solid surface are discussed using the theory of thermally stimulated electromagnetic fields.

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1. Introduction

Electromagnetic fluctuations are part of one of the most fundamental phenomena in nature, Brownian motion. In the theoretical works of Einstein, Smoluchowski, and Langevin [1–3], the fluctuational nature of this motion has been revealed and its basic statistical features have been investigated.

The history of studies of electromagnetic fluctuations, or electrical noise, is well known [4–7]. In the early 20th century, the relevant regularities were reliably established in two cases; in the case where the characteristic wavelength of the problem is much longer than its characteristic size, and in the opposite case of geometric optics, where the wavelength is much less than the characteristic scale of the problem. In the first case, which corresponds to the quasistationary region of the spectrum, the Nyquist formula was obtained, which describes the spectral intensity of noise in an arbitrary passive one-port with a given impedance; in the second case, Planck's and Kirchhoff's formulas were obtained, which describe the energy spectral density and the equilibrium radiation intensity. One of Kirchhoff's laws in fact allows determining the spectral intensity of radiation of a body into a less heated space, i.e., is associated with a thermodynamically nonequilibrium situation. This is the classical Kirchhoff law, according to which the intensity of emission of a flat element of a body surface in a certain direction at a fixed frequency is given by

$$I(\omega, \theta) = I_0(\omega)\left[1 - R(\omega, \theta)\right], \qquad (1)$$

where θ is the angle between the normal to the surface and the direction to the radiation detector, $R(\omega, \theta)$ is the reflection

coefficient, and $I_0(\omega)$ is the equilibrium intensity of emission, which is independent of the incidence angle and of the material of the blackbody cavity.

Relation (1) allows finding the power transferred through a unit area into a fixed solid angle, for example, determined by the specific geometric features of the radiation detector in the experiment,

$$P(\omega, \Delta\Omega) = \int_{\phi_1}^{\phi_2} \int_{\theta_1}^{\theta_2} \mathrm{d}\theta \, \mathrm{d}\phi \, I_0(\omega) \left[1 - R(\omega, \theta) \right] \cos\theta \sin\theta \,, \quad (2)$$

where $R(\omega, \theta)$ is the reflection coefficient of the body at a given frequency of the field in the direction specified by the angle θ. The solid angle cut out by the radiation detector, whose axis makes an angle θ with the normal to the surface, is written as

$$\Delta\Omega = \int_{\phi_1}^{\phi_2} \int_{\theta_1}^{\theta_2} \mathrm{d}\theta \, \mathrm{d}\phi \, \sin\theta \,, \quad (3)$$

and the intensity of the blackbody radiation in the vacuum, as

$$I_0(\omega) = \frac{c}{4\pi} \, U_0(\omega) = \frac{\hbar\omega^3}{4\pi^3 c^2} \left[\frac{1}{2} + \frac{1}{\exp\left(\hbar\omega/k_{\mathrm{B}}T\right) - 1} \right] \,, \quad (4)$$

where $U_0(\omega)$ is the spectral density of the blackbody radiation, c is the speed of light in the vacuum, k_{B} is the Boltzmann constant, and T is the temperature of the equilibrium system.

The relative power of emission at a given frequency into a given solid angle is given by

$$\frac{P(\omega, \Delta\Omega)}{P_0(\omega, \Delta\Omega)} = \frac{\displaystyle\int_{\phi_1}^{\phi_2} \int_{\theta_1}^{\theta_2} \mathrm{d}\theta \, \mathrm{d}\phi \, \left[1 - R(\omega, \theta) \right] \cos\theta \sin\theta}{\displaystyle\int_{\phi_1}^{\phi_2} \int_{\theta_1}^{\theta_2} \mathrm{d}\theta \, \mathrm{d}\phi \, \cos\theta \sin\theta} \,, \quad (5)$$

where $P_0(\omega, \Delta\Omega)$ is the spectral power of blackbody emission at the frequency ω into the solid angle $\Delta\Omega$.

We note that for the spectral composition of emission into a given solid angle, the modern phenomenological theories of thermal radiation give the same result as the classical Kirchhoff model of thermal radiation. In this sense, these theories contain the Kirchhoff law, which describes the properties of the field of propagating, or traveling, waves as the limit case, supplementing the general theory of a fluctuating electromagnetic field by the description of the properties of its quasistationary part. Moreover, theoretical methods have been developed for calculating the space–time correlation characteristics of thermal fields at any distances from bodies with arbitrary geometric shapes.

The study of electrodynamic fluctuations is an important part of modern fundamental and applied science, because it is precisely the fluctuations of dynamic variables of the system that determine a large class of the most important physical phenomena. In particular, fluctuations ensure the van der Waals interaction of bodies; the Casimir force, which can be considered a special case of the van der Waals interaction; the heat transfer between the bodies separated by a vacuum gap; the capture of atoms, molecules, and coherent material states by electromagnetic traps; and a number of major physicochemical phenomena near the surface of condensed media, such as the adsorption and desorption of atoms and molecules. Electromagnetic fluctua-

tions lead to a change in the conditions and characteristics of the spontaneous emission of atoms and molecules near the surface, the shift of their levels, and the complete or partial removal of degeneracy, which can substantially change the dynamics of the phenomena. We emphasize that a study of resonance states in the spectra of thermostimulated fields allows finding the eigenmodes of the system, i.e., its volume and surface polaritons, whose properties are determined by the totality of the electrodynamic and geometric characteristics of the system.

2. Statistical description of the thermal electromagnetic field generated by macroscopic bodies. Specific features of the statistical model of random electromagnetic fields

To date, the theory that has been developed in sufficient detail is the correlation theory of the properties of thermal electromagnetic fields induced by neutral macroscopic bodies. The electroneutrality of a body implies that the mean fluctuating local charge density vanishes, $\left\langle \rho^{\mathrm{fluct}}(\mathbf{r}, t) \right\rangle = 0$, and, as a consequence of charge conservation, the mean fluctuating current density vanishes as well, $\left\langle \mathbf{j}^{\mathrm{fluct}}(\mathbf{r}, t) \right\rangle = 0$; here, $\mathbf{r}$ is the coordinate of a point inside the body. The retarding vector and scalar potentials at a certain point $\mathbf{R}$ of space outside the body are also equal to zero on average: $\left\langle \mathbf{A}^{\mathrm{fluct}}(\mathbf{R}, t) \right\rangle = 0$, $\left\langle \phi^{\mathrm{fluct}}(\mathbf{R}, t) \right\rangle = 0$. In turn, the vanishing of the mean retarded potentials indicates the vanishing of the mean value of the electromagnetic field. We note that other averaged characteristics, for example, those that are quadratic in the field, can differ from zero. The physical interpretation here is that the known square-law characteristics of the thermal electromagnetic field determine its energy, which has a nonzero finite value if the source of the field — the macroscopic body — is heated to a certain temperature and is a reservoir of energy that exists in different forms, for example, in the form of the thermal kinetic energy of the atomic nuclei in the body lattice and of corresponding electrons, or in the form of the electromagnetic energy of fields inside and outside the body, which are generated by charges and currents and have a random, fluctuation nature.

In experiments, averaged characteristics of fields are typically recorded in the vacuum at a certain distance from a heated body. In the theoretical model, it is assumed that the source of a thermostimulated fluctuation electromagnetic field is a set of independent emitters, which emit waves of random amplitude, phase, and polarization, and that each elementary radiating volume emits nonmonochromatic waves. The resulting field changes in space and time in a complex, random way.

The statistical model of fluctuating electromagnetic fields is based on the fundamental theorem in probability theory, the central limit theorem (see, e.g., Ref. [5]). This name unites several theorems with different degrees of generality and applicability, which allow solving the problem of the distribution of the sum of independent random quantities. In the most general form, this problem was analyzed by A M Lyapunov, who proved that under some conditions and an unlimited increase in the number of terms of a normalized sum of unequally distributed independent random quantities, the distribution of such a sum approaches the normal distribution.

Thus, it is reasonable to assume that the source of a thermal field — a macroscopic body — can be conditionally divided into independently radiating volume elements and that all conditions of the central limit theorem are satisfied in this case. Then the random electromagnetic fields belong to the class of normal objects, for which all n-dimensional probability distributions are Gaussian; for their comprehensive statistical description, it suffices to know only the two-dimensional density of the probability distribution. Therefore, a special role in the description of statistical characteristics of thermal fields is played by the correlation theory of random stationary processes, which deals with only one-dimensional and two-dimensional distributions.

The stationarity of a process implies that the mean value of the random field is constant and, according to the above-described physical model, must be assumed to be zero. The correlation function of the components of the random stationary field depends only on the difference in the time moments and uniquely determines the spectral composition of the field according to the Wiener–Khinchin theorem [5]. In the literature, there exist different definitions of averaged products of fields taken at different space–time points. In particular, the concept of an analytic signal is frequently used, and normal and antinormal correlation functions are introduced, for example, in quantum statistical optics [8, 9]. A Fourier decomposition is frequently used, assuming an auxiliary nature of this way of writing and considering that all integrals and derivatives of the fields are understood in the sense of convergence with respect to probability or as root-mean square values:

$$\mathbf{A}(t) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi}\, \mathbf{A}(\omega) \exp\left(-\mathrm{i}\omega t\right), \quad \mathbf{A} = \mathbf{E}, \mathbf{H}, \mathbf{D}, \mathbf{B}, \quad (6)$$

where $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic field strengths and $\mathbf{B}$ and $\mathbf{D}$ are the magnetic and electric inductions.

The correlation functions can be conveniently written in a symmetrized form; for example, for the electric field components, as

$$\gamma_{ij}(\mathbf{r}, \mathbf{r}'; t - t') = \frac{1}{2}\left\langle E_i(\mathbf{r}, t)\, E_j(\mathbf{r}', t') + E_j(\mathbf{r}', t')\, E_i(\mathbf{r}, t)\right\rangle, \quad (7)$$

where the angular brackets denote ensemble averaging.

The Fourier transform of the correlation function, according to the Wiener–Khinchin theorem, represents the spectral density of fluctuations

$$\gamma_{ij}(\mathbf{r}, \mathbf{r}'; \omega) = \int_{-\infty}^{+\infty} \gamma_{ij}(\mathbf{r}, \mathbf{r}'; \tau) \exp\left(\mathrm{i}\omega\tau\right) \mathrm{d}\tau. \quad (8)$$

The main task of the correlation theory of fluctuating electromagnetic fields is the development of methods of calculating correlation functions of fields or their spectral characteristics for a given geometry of bodies with realistic electrodynamic properties.

It is obvious that the dynamics of any electromagnetic process, including one having a fluctuation nature, must be described by the set of Maxwell equations. Therefore, as the starting equations here, we take equations for the microscopic fluctuating field strengths $\mathbf{E}^{\mathrm{M}}$ and $\mathbf{H}^{\mathrm{M}}$ in the vacuum:

$$\operatorname{rot} \mathbf{E}^{\mathrm{M}}(\mathbf{r}, t) = -\frac{1}{c}\frac{\partial \mathbf{H}^{\mathrm{M}}(\mathbf{r}, t)}{\partial t}, \quad \operatorname{div} \mathbf{H}^{\mathrm{M}}(\mathbf{r}, t) = 0,$$

$$\operatorname{rot} \mathbf{H}^{\mathrm{M}}(\mathbf{r}, t) = \frac{1}{c}\frac{\partial \mathbf{E}^{\mathrm{M}}(\mathbf{r}, t)}{\partial t}, \quad \operatorname{div} \mathbf{E}^{\mathrm{M}}(\mathbf{r}, t) = 0. \quad (9)$$

The ensemble averaging of microscopic fields in fact implies averaging over fluctuations, which results in zero mean values of the fields: $\langle \mathbf{E}^{\mathrm{M}} \rangle = \langle \mathbf{H}^{\mathrm{M}} \rangle = 0$. Nevertheless, the statistical information about the thermal field is not lost, because it can be extracted from an analysis of the dynamics of correlation functions. As was shown in [10, 11], the correlation functions satisfy the usual wave equation in the vacuum, and their Fourier transforms satisfy the Helmholtz equation. Using Green's theorems, it can easily be shown (see, e.g., Ref. [8]) that the spectral density then satisfies the integral equation

$$\gamma(\mathbf{r}, \mathbf{r}'; \omega) = \iint_S \mathrm{d}s\, \mathrm{d}s'\, \frac{\partial G}{\partial n}(\mathbf{r}, \mathbf{s})\, \frac{\partial G^*}{\partial n'}(\mathbf{r}', \mathbf{s}')\, \gamma(\mathbf{s}, \mathbf{s}'; \omega), \quad (10)$$

where G is the Green's function of a certain boundary problem (in this case, it is chosen such that it vanishes on the body surface S), $\mathbf{s}$ is the coordinate of a point on the body surface, and $\mathbf{n}$ is the normal to the surface of the body. Relation (10) shows that the spectral density of the field outside the body can be found if its value on the surface of a heated body is known. As frequently happens in physics, this relation can also be read from the right to left. Namely, given the spectral density of radiation in the space outside the body, we can, in principle, find the correlation properties of the thermal field on the body surface by solving integral equation (10).

In the bulk of the body, i.e., in a medium, the equations for microscopic fields are more complex than in the vacuum, because it is necessary to consider the motion of charges of the medium:

$$\operatorname{rot} \mathbf{E}^{\mathrm{M}}(\mathbf{r}, t) = -\frac{1}{c}\frac{\partial \mathbf{H}^{\mathrm{M}}(\mathbf{r}, t)}{\partial t}, \quad \operatorname{div} \mathbf{H}^{\mathrm{M}}(\mathbf{r}, t) = 0,$$

$$\operatorname{rot} \mathbf{H}^{\mathrm{M}}(\mathbf{r}, t) = \frac{1}{c}\frac{\partial \mathbf{E}^{\mathrm{M}}(\mathbf{r}, t)}{\partial t} + \frac{4\pi}{c}\, \mathbf{j}^{\mathrm{M}}(\mathbf{r}, t),$$

$$\operatorname{div} \mathbf{E}^{\mathrm{M}}(\mathbf{r}, t) = 4\pi\rho^{\mathrm{M}}(\mathbf{r}, t). \quad (11)$$

The microscopic charge density and the current density, under the assumption that the charges are pointlike, can be written as

$$\rho^{\mathrm{M}}(\mathbf{r}, t) = \sum_i e_i\, \delta\!\left(\mathbf{r} - \mathbf{r}_i(t)\right),$$

$$\mathbf{j}^{\mathrm{M}}(\mathbf{r}, t) = \sum_i e_i \mathbf{v}_i\, \delta\!\left(\mathbf{r} - \mathbf{r}_i(t)\right), \quad (12)$$

where e_i, $\mathbf{v}_i$, and $\mathbf{r}_i$ are the charge, velocity, and coordinate of the ith charge.

Ensemble averaging also leads to zero mean currents $(\langle \mathbf{j}^{\mathrm{M}} \rangle = 0)$, zero charges $(\langle \rho^{\mathrm{M}} \rangle = 0)$, and zero fields $(\langle \mathbf{E}^{\mathrm{M}} \rangle = \langle \mathbf{H}^{\mathrm{M}} \rangle = 0)$. In practice, however, in fluctuation electrodynamics, as in many other branches of the physics of Brownian motion, it is frequently expedient to deal with equations for dynamic variables in order to obtain the correlation characteristics of the fields via standard methods of solving electrodynamic problems. For this, Eqn (11) can be averaged over elementary volumes, allowing the fields to change in time as rapidly as possible:

$$\mathbf{E}(\mathbf{r}, t) = \frac{1}{\Delta V} \int_{\Delta V} \mathrm{d}^3\rho\, \mathbf{E}^{\mathrm{M}}(\mathbf{r} + \boldsymbol{\rho}, t), \quad (13)$$

(here, $\boldsymbol{\rho}$ is the position vector of a certain point in the chosen elementary volume ΔV, whose center is determined by the

coordinate **r**). It is seen from Eqn (13) that averaging over elementary volumes in fact implies a spatial smoothing of the fields with the preservation of their coordinate dependence [12, 13].

The averaged Maxwell equations in a medium with given statistics of external or total currents along with the boundary conditions on the body surface and linear matter equations allow determining all statistical characteristics of thermo-stimulated random fields.

We note that the macroscopic Maxwell equations in a medium can be written in various forms. This is caused, first and foremost, by the method of defining induced currents. For example [13, 14], if we introduce the total induction $\mathbf{D}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}, t) + 4\pi\mathbf{P}(\mathbf{r}, t)$, which takes all effects of motion of charges and the magnetization currents into account, and if we assume that the polarization

$$\mathbf{P}(\mathbf{r}, t) = \int_{-\infty}^{t} \mathbf{j}(\mathbf{r}, t')\, \mathrm{d}t'$$

includes the entire induced current without its division into separate contributions, then the set of Maxwell equations in a medium is written as

$$\operatorname{rot}\mathbf{E}(\mathbf{r}, t) = -\frac{1}{c}\frac{\partial\mathbf{B}(\mathbf{r}, t)}{\partial t}\,, \quad \operatorname{div}\mathbf{B}(\mathbf{r}, t) = 0\,,$$
$$\operatorname{rot}\mathbf{B}(\mathbf{r}, t) = \frac{1}{c}\frac{\partial\mathbf{D}(\mathbf{r}, t)}{\partial t} + \frac{4\pi}{c}\mathbf{j}_{\mathrm{ext}}(\mathbf{r}, t)\,, \quad (14)$$
$$\operatorname{div}\mathbf{D}(\mathbf{r}, t) = 4\pi\rho_{\mathrm{ext}}(\mathbf{r}, t)\,.$$

In this case, the set of equations involves the external currents $\mathbf{j}_{\mathrm{ext}}$ and charges ρ_{ext} related by the continuity equation. These can be random external currents or auxiliary fictitious currents, i.e., those currents that induce the fluctuation currents **j** in the system. The total current, equal to the sum of currents, is the source of the fluctuating field. Frequently, the external (fixed) random currents are called Langevin currents, by analogy with the description of mechanical Brownian motion. Naturally, the mean value of external currents vanishes, $\langle\mathbf{j}_{\mathrm{ext}}\rangle = 0$. The tensor of the spectral density of external currents is determined by the fluctuation–dissipation theorem.

If for some reasons it is expedient to divide the induced currents into the current of free charge carriers, the current of the polarization of bound charges, and the eddy current of magnetizing, then different electric induction vector $\mathbf{D}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}, t) + 4\pi\mathbf{P}(\mathbf{r}, t)$ and magnetic field strength vector $\mathbf{H}(\mathbf{r}, t) = \mathbf{B}(\mathbf{r}, t) - 4\pi\mathbf{M}(\mathbf{r}, t)$ are introduced. The polarization of the medium **P** may involve the contribution of the currents of free and bound carriers, or only the contribution of the current of bound carriers, with the current of free charge carriers isolated as a special contribution. Then the vector of magnetization of the medium **M** determines eddy currents. In this case, the set of Maxwell equations in the medium is written as

$$\operatorname{rot}\mathbf{E}(\mathbf{r}, t) = -\frac{1}{c}\frac{\partial\mathbf{B}(\mathbf{r}, t)}{\partial t}\,, \quad \operatorname{div}\mathbf{B}(\mathbf{r}, t) = 0\,,$$
$$\operatorname{rot}\mathbf{H}(\mathbf{r}, t) = \frac{1}{c}\frac{\partial\mathbf{D}(\mathbf{r}, t)}{\partial t} + \frac{4\pi}{c}\mathbf{j}_{\mathrm{ext}}(\mathbf{r}, t)\,, \quad (15)$$
$$\operatorname{div}\mathbf{D}(\mathbf{r}, t) = 4\pi\rho_{\mathrm{ext}}(\mathbf{r}, t)\,.$$

The set of equations (15) also contains the external currents $\mathbf{j}_{\mathrm{ext}}$. The name 'external' currents means that the dynamics of processes in the problem in question does not affect any characteristics of these currents, which can be caused by nonelectromagnetic phenomena; in the general case, the induced currents and charges are functionals of the field strengths, which are in turn determined by the sum of the induced and external currents and charges. The external and induced currents and charges separately and also their sum are related by the appropriate continuity equations.

In a realistic problem concerning the thermostimulated emission of heated bodies, several spatial regions with different electrodynamic properties are considered. Therefore, it is necessary to solve the set of equations (14) or (15) with suitable boundary conditions and linear matter equations. The boundary conditions and the matter equations must be formulated in accordance with the specific features of the problem and with the model of the medium (see, e.g., Refs [13, 15]).

To find correlation functions of the components of fields, the correlation functions of currents in the system or Green's functions, specified by the fluctuation–dissipation theorem (FDT), must be known. As is known [14, 16, 17], the FDT relates spontaneous fluctuations of the parameters of a system to its dissipative properties. In particular, the FDT relates the spectral density of fluctuations of external currents to the anti-Hermitian part of the dielectric constant tensor of matter: $\varepsilon_{ij}(\mathbf{r}, \mathbf{r}'; \omega) = \varepsilon'_{ij}(\mathbf{r}, \mathbf{r}'; \omega) + \mathrm{i}\varepsilon''_{ij}(\mathbf{r}, \mathbf{r}'; \omega)$. The spectral density of fluctuations of external currents is defined as the Fourier transform of the correlation function of the external currents:

$$\Phi_{ij}^{\mathrm{ext}}(\mathbf{r}, \mathbf{r}'; \omega) = \int_{-\infty}^{+\infty}\mathrm{d}\tau\, \Phi_{ij}^{\mathrm{ext}}(\mathbf{r}, \mathbf{r}'; \tau)\exp{(\mathrm{i}\omega\tau)}\,, \quad (16)$$

and the correlation function can be written in the symmetrized form

$$\Phi_{ij}^{\mathrm{ext}}(\mathbf{r}, \mathbf{r}'; t - t')$$
$$= \frac{1}{2}\operatorname{Sp}\left\{\rho_0\left[j_i^{\mathrm{ext}}(\mathbf{r}, t)\, j_j^{\mathrm{ext}}(\mathbf{r}', t') + j_j^{\mathrm{ext}}(\mathbf{r}', t')\, j_i^{\mathrm{ext}}(\mathbf{r}, t)\right]\right\}\,, \quad (17)$$

where $\rho_0 = \exp{\left[(F - H_0)/k_{\mathrm{B}}T\right]}$ is the equilibrium density matrix, F is the free energy, and H_0 is the unperturbed Hamiltonian of the system. The calculations lead to the well-known formula

$$\Phi_{ij}^{\mathrm{ext}}(\mathbf{r}, \mathbf{r}'; \omega) = -\frac{\mathrm{i}\omega\Theta(\omega, T)}{4\pi}\left(\varepsilon_{ij}(\mathbf{r}, \mathbf{r}'; \omega) - \varepsilon_{ji}^*(\mathbf{r}, \mathbf{r}'; \omega)\right)\,, \quad (18)$$

where $\Theta(\omega, T) = (\hbar\omega/2)\coth{(\hbar\omega/2k_{\mathrm{B}}T)}$ is the mean energy of an oscillator at a temperature T. In the literature, this quantity is frequently written in the form that explicitly separates the mean occupation number per mode, or the degeneracy factor $\delta = \langle n(\omega)\rangle = [\exp{(\hbar\omega/k_{\mathrm{B}}T)} - 1]^{-1}$; in this case, $\Theta(\omega, T) = \hbar\omega(1/2 + \langle n(\omega)\rangle)$.

We note that the FDT can be expressed in terms of the susceptibility of the system, which is unambiguously related to the dielectric constant: $\chi_{ij}(\omega) = (\varepsilon_{ij}(\omega) - \delta_{ij})/(4\pi)$. In this case, a dual treatment of the polarization of a spatially bounded system in a field is possible, namely, $P_i(\omega) = \chi_{ij}(\omega)E_j(\omega) = \chi_{ij}^{(\mathrm{e})}(\omega)E_j^{(\mathrm{e})}(\omega)$; that is, the polarization can be defined as a response to the external field $\mathbf{E}^{(\mathrm{e})}$ in which the

body is placed, or as a response to the true field $\mathbf{E}$ in the medium. In the general case, the connection between the tensors χ and $\chi^{(e)}$ is determined by an integral equation. One classical case, however, is well known, which is connected with the polarization of a uniform and isotropic body of an ellipsoidal form in a quasistatic uniform field [12]. In this simplest case, $\chi^{(e)} = 4\pi\chi/(1 + 4\pi\chi n^{(\alpha)})$, where $n^{(\alpha)}$ is the so-called depolarizing factor ($\alpha = x, y, z$).

Because the susceptibility of a system can be introduced in two ways, the FDT can also be written in two forms. It turns out that apart from formula (18), which, together with the relation $\varepsilon_{ij} = \delta_{ij} + 4\pi\chi_{ij}$, relates the true susceptibility of the system χ to the spectral density of fluctuations of external currents, it is possible to find a similar relation of the susceptibility $\chi^{(e)}$ to the spectral density of fluctuations $\Phi_{ij}(\mathbf{r}, \mathbf{r}'; \omega)$ of the true current in the system, which is the sum $\mathbf{j} = \mathbf{j}_{\text{ext}} + \mathbf{j}_{\text{ind}}$ of the external and induced currents [14]. These relations are identical in form, but differ essentially in meaning. It is obvious that the true susceptibility χ characterizes the properties of the medium itself, while $\chi^{(e)}$ also takes the geometrical features of the sample into account. Therefore, as was noted in Ref. [14], a more 'fundamental' approach consists in the use of correlation functions of external currents in the sample.

3. Theoretical methods for calculating the properties of thermally stimulated electrodynamic fields

3.1 Langevin approach. Rytov theory

As is known [1–3], there exist alternative approaches to the description of Brownian motion. Langevin [3] proposed using mechanical equations of motion for a Brownian particle in a liquid. For this, an extra force is introduced into the equations of motion, given by a random function of time $\mathbf{f}^{\text{ext}}(t)$, which reflects the molecular structure of the liquid:

$$m \frac{d\mathbf{u}(t)}{dt} = -m\gamma\mathbf{u}(t) + \mathbf{f}^{\text{ext}}(t) , \qquad (19)$$

where m is the mass of the particle, γ is the friction coefficient, and $\mathbf{u}(t)$ is the instantaneous velocity of the particle.

The friction force $-m\gamma\mathbf{u}(t)$ in Eqn (19) corresponds to the continuum approximation, and the random force $\mathbf{f}^{\text{ext}}(t)$ is responsible for the fluctuational nature of the motion of a particle. Because the mass of the particle is much greater than the mass of a molecule in the liquid, it is reasonable to assume that the particle motion is the result of an enormous number of impacts by the molecules; therefore, the conditions of the central limit theorem are satisfied. In addition, it is assumed that there are no preferred directions of the random force in the medium; therefore, its mean value is equal to zero and its components do not correlate between themselves. Hence, the process described by the $\mathbf{f}^{\text{ext}}(t)$ function can be considered to be Gaussian, with zero mean:

$$\langle f_i^{\text{ext}}(t) \rangle = 0 , \qquad \langle f_i^{\text{ext}}(t) f_k^{\text{ext}}(t') \rangle = 2D\delta_{ik}\delta(t - t') , \quad (20)$$

where the random force strength $D = \gamma m k_B T$ is found from the condition of the statistical equilibrium between the Brownian particles and the surrounding medium [18]. The delta-correlated singularity in (20) in fact implies a zero approximation in the small parameter $\tau_{\text{cor}}/\tau_{\text{rel}}$, where τ_{cor} is the correlation time of molecular impacts and $\tau_{\text{rel}} = \gamma^{-1}$ is the relaxation time, or the characteristic time scale of Brownian motion. The Einstein formula $D = \gamma m k_B T$, which relates the intensity of the Langevin source to the dissipative characteristic of the medium γ and the temperature T, is the first known fluctuation–dissipation relation. We note that the mathematical features of the Langevin description of Brownian motion and its connection with alternative approaches have been investigated, e.g., in Refs [5, 19, 20].

By analogy with the Langevin approach to the description of a mechanical problem, Rytov has developed a theory of equilibrium electromagnetic fluctuations [4–7] based on the set of macroscopic Maxwell equations (15). For the spectral amplitudes, the dynamic part of the set of equations (15) takes the form

$$\text{rot }\mathbf{H}(\mathbf{r}, \omega) = -ik_0\mathbf{D}(\mathbf{r}, \omega) + \frac{4\pi}{c} \mathbf{j}_{\text{ext}}^{\text{e}}(\mathbf{r}, \omega) ,$$
$$\text{rot }\mathbf{E}(\mathbf{r}, \omega) = ik_0\mathbf{B}(\mathbf{r}, \omega) - \frac{4\pi}{c} \mathbf{j}_{\text{ext}}^{\text{m}}(\mathbf{r}, \omega) , \qquad (21)$$

where external magnetic currents are introduced for convenience and symmetry in writing; the Fourier transforms of the inductions and field strengths are connected by the linear matter relations $\mathbf{D}(\mathbf{r}, \omega) = \varepsilon(\mathbf{r}, \omega)\mathbf{E}(\mathbf{r}, \omega)$ and $\mathbf{B}(\mathbf{r}, \omega) = \mu(\mathbf{r}, \omega)\mathbf{H}(\mathbf{r}, \omega)$; the electric permittivity and magnetic permeability for isotropic and homogeneous media are $\varepsilon = \varepsilon' + i\varepsilon''$ and $\mu = \mu' + i\mu''$; and $k_0 = \omega/c$ is the wave number in the vacuum. Just as in the case of mechanical problem (20), the fluctuational nature of the electromagnetic field is taken into account by introducing external sources of fields $\mathbf{j}_{\text{ext}}^{\text{e}}(\mathbf{r}, \omega)$ and $\mathbf{j}_{\text{ext}}^{\text{m}}(\mathbf{r}, \omega)$ with assigned statistics into the right-hand sides of Eqns (21). Just as in the mechanical problem, the expression for the strength of sources of the fluctuating field follows from the condition of the thermodynamic equilibrium of the system and is determined by the fluctuation–dissipation theorem.

With chosen boundary conditions, matter equations, and random sources, the set of equations (21) uniquely determines spectral amplitudes of the field strengths and inductions. It is obvious that the second moments of the fluctuating electromagnetic field are also determined by the second moments of the external currents. Rytov [21] obtained the general form of the FDT for continuous systems whose state is described by random fields. He also showed that the correlation matrices of the spectral amplitudes of fields and external currents are uniquely determined by the linearized macroscopic Maxwell equations.

Subsequently, the theory of thermal electromagnetic fluctuations developed by Rytov and Levin [7] acquired a completed elegant form. The authors obtained the fundamental generalized Kirchhoff law without any restrictions on the relation between the wavelength of the fluctuating field and the characteristic spatial scale of the problem, from which the classical variant of the Kirchhoff law follows directly. Based on the reciprocity theorem applied to the field of a point dipole and the fluctuation field in question, it was shown that the spatial correlation functions of the components of fluctuation fields are expressed through the mixed losses of fields from two point dipoles. For example, for the components of the fluctuating electric field in spatial directions specified by unit vectors $\mathbf{l}_1$ and $\mathbf{l}_2$, we obtain

$$\langle E_{l_1}(\mathbf{r}_1)E_{l_2}^*(\mathbf{r}_2) \rangle = \frac{2\Theta(\omega, T)}{\pi} Q_{\text{ee}}(\mathbf{r}_1, \mathbf{l}_1; \mathbf{r}_2, \mathbf{l}_2) , \qquad (22)$$

where $Q_{ee}(\mathbf{r}_1, \mathbf{l}_1; \mathbf{r}_2, \mathbf{l}_2)$ are the mixed losses of the fields of two electric dipoles, one of which ($\mathbf{d}_1 = -\mathbf{l}_1/i\omega$) is located at the point $\mathbf{r}_1$ and is oriented along $\mathbf{l}_1$ and the other ($\mathbf{d}_2 = -\mathbf{l}_2/i\omega$) is located at the point $\mathbf{r}_2$ and oriented along $\mathbf{l}_2$. Such dipoles correspond to the unit electric point current $\mathbf{j}_0^e = \mathbf{l}\delta(\mathbf{r} - \mathbf{r}') = -i\omega\mathbf{d}\delta(\mathbf{r} - \mathbf{r}')$ at $\mathbf{r} = \mathbf{r}'$. These losses are written as

$$Q_{ee}(\mathbf{r}_1, \mathbf{l}_1; \mathbf{r}_2, \mathbf{l}_2)$$
$$= -\frac{i\omega}{16\pi} \int_V d^3 r \left[E_i^{e1} E_j^{e2*} (\varepsilon_{ij} - \varepsilon_{ji}^*) + H_i^{e1} H_j^{e2*} (\mu_{ij} - \mu_{ji}^*) \right],$$
$$(23)$$

where $E_i^{ek} = E_i^{ek}(\mathbf{r}, \mathbf{r}_k; \mathbf{l}_k)$ and $H_i^{ek} = H_i^{ek}(\mathbf{r}, \mathbf{r}_k; \mathbf{l}_k)$ are the fields of the kth ($k = 1, 2$) point electric dipole and the integration is performed over the volume of the body that generates the thermal field. The formulas for the components of the fluctuating magnetic field $\langle H_{l_1}(\mathbf{r}_1) H_{l_2}^*(\mathbf{r}_2) \rangle$ and for the mixed product $\langle E_{l_1}(\mathbf{r}_1) H_{l_2}^*(\mathbf{r}_2) \rangle$ are written similarly, but the losses must then be calculated using the fields $E_i^{m1,2} = E_i^{m1,2}(\mathbf{r}, \mathbf{r}_{1,2}; \mathbf{l}_{1,2})$ and $H_i^{m1,2} = H_i^{m1,2}(\mathbf{r}, \mathbf{r}_{1,2}; \mathbf{l}_{1,2})$ of the magnetic point dipoles $\mathbf{m}_1 = -\mathbf{l}_1/i\omega$ and $\mathbf{m}_2 = -\mathbf{l}_2/i\omega$ corresponding to the unit magnetic current $\mathbf{j}_0^m = \mathbf{l}\delta(\mathbf{r} - \mathbf{r}') = -i\omega\mathbf{m}\delta(\mathbf{r} - \mathbf{r}')$.

We note that the generalized Kirchhoff law expressed, in particular, by formula (22) allows calculating the spectral characteristics of the fluctuating field radiated by a heated body of an arbitrary shape into cold space; i.e., it describes a nonequilibrium situation, just as the classical Kirchhoff law (1). An essential difference between them is that the theory considered here allowed finding spectral characteristics of thermal fields in the near zone, i.e., in the zone of quasistationary random fields, which could not be done using the classical law in principle, because it was obtained in the approximation of geometrical optics.

In the case of the complete thermodynamic equilibrium, when the temperatures of all the bodies are equal, the authors of the theory showed, using the complex Lorentz lemma, that the losses are determined simply by the fields of point sources, or by the Green's functions of the problem,

$$Q_{ee}(\mathbf{r}_1, \mathbf{l}_1; \mathbf{r}_2, \mathbf{l}_2) = -\frac{1}{2} \operatorname{Re}\left(\mathbf{l}_1 \mathbf{E}^{e2}(\mathbf{r}_1)\right) = -\frac{1}{2} \operatorname{Re}\left(\mathbf{l}_2 \mathbf{E}^{e1}(\mathbf{r}_2)\right),$$
$$(24)$$

and generalized Kirchhoff law (22) acquires the simple form

$$\langle E_{l_1}(\mathbf{r}_1) E_{l_2}^*(\mathbf{r}_2) \rangle = -\frac{\Theta(\omega, T)}{\pi} \operatorname{Re}\left(\mathbf{l}_1 \mathbf{E}^{e2}(\mathbf{r}_1)\right)$$
$$= -\frac{\Theta(\omega, T)}{\pi} \operatorname{Re}\left(\mathbf{l}_2 \mathbf{E}^{e1}(\mathbf{r}_2)\right). \quad (25)$$

Analogous formulas are obtained for correlation functions of the magnetic strength components of the thermal field and for mixed correlation functions.

We note that formula (24) can easily be obtained from Poynting's theorem, which implies that the dissipated power in the entire space, created by the field of the regular current $\mathbf{j}_0 = \mathbf{l}\delta(\mathbf{r} - \mathbf{r}')$, is written as

$$Q(\mathbf{r}, \mathbf{l}) = -\int d^3 r' \langle \mathbf{j}_0(\mathbf{r}, \mathbf{r}') \mathbf{E}(\mathbf{r}') \rangle = -\frac{1}{2} \operatorname{Re}\left(\mathbf{l}\mathbf{E}(\mathbf{r})\right). \quad (26)$$

Hence, both when the system is in complete thermodynamic equilibrium with the surrounding space and when the system is in incomplete equilibrium, when only the body under consideration is heated, spectral characteristics of the thermal fluctuation field are determined by the Green's functions of the corresponding regular problem. The theory under consideration allows calculating correlation functions of the thermostimulated field at any distance from the surface of the thermal source and obtaining comprehensive information about the spectral characteristics of both the traveling and quasistationary waves of this fluctuating field.

3.2 The Green's function method
In modern physics, a very important and fruitful approach is based on the concept of Green's functions [22–24]. In solid-state theory, a Green's function defined in terms of averaged differently ordered field operators has been introduced. It turned out that the Fourier transform of the Green's function determines the spectrum of quasiparticle excitations in a solid body. It is natural that for describing electromagnetic excitations in the framework of the already developed general procedure, it made sense to develop an appropriate theory by analogy. The role of field operators in the theory of electromagnetic fluctuations [24] is played by the operators of electromagnetic field potentials φ and $\mathbf{A}$. The key concept of the theory is the retarded Green's function

$$iG_{ik}^R(\mathbf{r}_1, \mathbf{r}_2; t_1, t_2)$$
$$= \begin{cases} \langle \hat{A}_i(\mathbf{r}_1, t_1) \hat{A}_k(\mathbf{r}_2, t_2) - \hat{A}_k(\mathbf{r}_2, t_2) \hat{A}_i(\mathbf{r}_1, t_1) \rangle, & t_1 > t_2, \\ 0, & t_1 < t_2. \end{cases}$$
$$(27)$$

In the case where the Green's function depends only on the difference $t = t_1 - t_2$, its Fourier transform is written as

$$G_{ik}^R(\mathbf{r}_1, \mathbf{r}_2; \omega) = \int_0^\infty dt \exp(i\omega t) G_{ik}^R(\mathbf{r}_1, \mathbf{r}_2; t). \quad (28)$$

This theory is based on the general theory of the response of a macroscopic system to external action and on the Kubo theorem [18]. The general expression for the interaction Hamiltonian of the electromagnetic field and the medium has the form

$$\hat{V} = -\frac{1}{c} \int d^3 r \, \hat{\mathbf{j}}(\mathbf{r}, t) \, \hat{\mathbf{A}}(\mathbf{r}, t) + \int d^3 r \, \hat{\rho}(\mathbf{r}, t) \, \hat{\varphi}(\mathbf{r}, t), \quad (29)$$

where $\hat{\mathbf{j}}$ and $\hat{\rho}$ are operators of the current density and of the charge density of the medium, and the integration is taken over the entire volume. For convenience, the gauge $\varphi = 0$ is used in this theory, which does not affect the observed values, i.e., the field strengths. In this gauge, it is possible to write interaction operator (29) with the classical external current as

$$\hat{V} = -\frac{1}{c} \int d^3 r \, \mathbf{j}_0(\mathbf{r}, t) \, \hat{\mathbf{A}}(\mathbf{r}, t). \quad (30)$$

It is known that for discrete dynamic variables x_i that characterize a change in some property of a system under the effect of external actions f_i, the operator of the interaction energy has the form $\hat{V} = -\sum_i f_i \hat{x}_i$. The average values of dynamic quantities $\langle x_i(t) \rangle$ are linear functionals of external forces $f_a(t)$ and their Fourier components are connected by a

simple linear relation $\langle x_i(\omega)\rangle = \sum_j \chi_{ij}(\omega) f_j(\omega)$, where $\chi_{ij}(\omega)$ are the generalized susceptibilities. In the case of spatially distributed quantities x_i and f_i, the operator of interaction is written as

$$\hat{V} = -\sum_i \int \mathrm{d}^3 r \, f_i(\mathbf{r}, t) \hat{x}_i(\mathbf{r}, t) \,. \tag{31}$$

Comparing (30) and (31) and assuming that the external forces f_a are components of the external current $\mathbf{j}_0$, we see that components of the vector potential $\mathbf{A}/c$ are to be chosen as the dynamic quantities x_a.

In the framework of the theory of linear response, we can write

$$\langle x_i(\mathbf{r}, t)\rangle = \int \mathrm{d}^3 r' \int \mathrm{d}t' \, \chi_{ij}(\mathbf{r}, \mathbf{r}'; t - t') f_j(\mathbf{r}', t') \,, \tag{32}$$

whence the mean response for the Fourier components, as in the discrete case, is expressed through the susceptibility of the system:

$$\langle x_i(\mathbf{r}, \omega)\rangle = \sum_i \int \mathrm{d}^3 r' \, \chi_{ij}(\mathbf{r}, \mathbf{r}'; \omega) f_j(\mathbf{r}', \omega) \,. \tag{33}$$

In turn, the susceptibility, according to Kubo, is expressed through the mean values of the commutator of the operators $\hat{x}_a(\mathbf{r}, t)$ in the Heisenberg representation:

$$\chi_{ij}(\mathbf{r}, \mathbf{r}'; \omega)$$
$$= \frac{\mathrm{i}}{\hbar} \int_0^\infty \mathrm{d}t \, \exp{(\mathrm{i}\omega t)} \left\langle \hat{x}_i(\mathbf{r}, t) \hat{x}_j(\mathbf{r}', 0) - \hat{x}_j(\mathbf{r}', 0) \hat{x}_i(\mathbf{r}, t) \right\rangle \,. \tag{34}$$

Comparing formula (34) with the definition of the retarded Green's function (27)–(28), we obtain

$$G_{ik}^{\mathrm{R}}(\mathbf{r}, \mathbf{r}'; \omega)$$
$$= -\mathrm{i} \int_0^\infty \mathrm{d}t \, \exp{(\mathrm{i}\omega t)} \langle \hat{A}_i(\mathbf{r}, t) \hat{A}_k(\mathbf{r}', 0) - \hat{A}_k(\mathbf{r}', 0) \hat{A}_i(\mathbf{r}, t)\rangle \,, \tag{35}$$

which shows that the components of the tensor $-G_{ik}^{\mathrm{R}}(\mathbf{r}, \mathbf{r}'; \omega)/\hbar c^2$ should be regarded as generalized susceptibilities. Therefore, taking (32) into account, we can write the mean value of a component of the vector potential of the fluctuating field as

$$\langle A_i(\mathbf{r}, \omega)\rangle = -\frac{1}{\hbar c} \int \mathrm{d}^3 r' \, G_{ik}^{\mathrm{R}}(\mathbf{r}, \mathbf{r}'; \omega) j_k(\mathbf{r}', \omega) \,. \tag{36}$$

But the mean field determined by the vector potential satisfies the set of equations (14) or (15). Writing the mean vector potential without angular brackets in what follows, taking into account that $\mathbf{B}(\omega) = \mathrm{rot}\,\mathbf{A}(\omega)$ and $\mathbf{E}(\omega) = \mathrm{i}(\omega/c)\mathbf{A}(\omega)$ in the chosen gauge, and using the relations $B_i(\omega) = \mu_{ik}(\omega) H_k(\omega)$ and $D_i(\omega) = \varepsilon_{ik}(\omega) E_k(\omega)$, we can obtain the equation for the retarded Green's function. In the notation in [24], this equation takes the form

$$\left[\mathrm{rot}_{im}(\mu_{mn}^{-1}\,\mathrm{rot}_{nl}) - \frac{\omega^2}{c^2}\,\varepsilon_{il}\right] G_{lk}^{\mathrm{R}}(\mathbf{r}, \mathbf{r}'; \omega) = -4\pi\hbar\delta_{ik}\delta(\mathbf{r} - \mathbf{r}') \,, \tag{37}$$

where $\mathrm{rot}_{il} = e_{ikl}\,\partial/\partial x_k$, $(\mathrm{rot}\,\mathbf{A})_i = \mathrm{rot}_{il}\,A_l$, and, as usual, summation over repeated indices is understood. Just as in Rytov's theory, finding the Green's function from a concrete boundary value problem and using the FDT for the generalized susceptibilities $-G_{ik}^{\mathrm{R}}(\mathbf{r}, \mathbf{r}'; \omega)/\hbar c^2$, it is possible to find the spectral density of correlations, for example, of the symmetrized correlation function of components of the vector potential

$$(A_i^{(1)} A_k^{(2)})_\omega = \frac{\mathrm{i}\Theta(\omega, T)}{\hbar\omega} \left[G_{ik}^{\mathrm{R}}(\mathbf{r}_1, \mathbf{r}_2; \omega) - G_{ki}^{\mathrm{R}*}(\mathbf{r}_2, \mathbf{r}_1; \omega)\right] \,. \tag{38}$$

Recalling the definition of the fields in terms of the vector potential, we can immediately write the spectral densities of the correlations of the field components $(E_i^{(1)} E_k^{(2)})_\omega = (\omega^2/c^2)(A_i^{(1)} A_k^{(2)})_\omega$ and $(B_i^{(1)} B_k^{(2)})_\omega = \mathrm{rot}_{il}^{(1)}\,\mathrm{rot}_{km}^{(2)}\,(A_l^{(1)} A_m^{(2)})_\omega$. If the medium is magnetically inactive, then $G_{ik}^{\mathrm{R}}(\mathbf{r}_1, \mathbf{r}_2; \omega) = G_{ki}^{\mathrm{R}}(\mathbf{r}_2, \mathbf{r}_1; \omega)$, and Eqn (38) acquires the simpler form

$$(A_i^{(1)} A_k^{(2)})_\omega = -\frac{2\Theta(\omega, T)}{\hbar\omega}\,\mathrm{Im}\,G_{ik}^{\mathrm{R}}(\mathbf{r}_1, \mathbf{r}_2; \omega) \,, \tag{39}$$

i.e., it is expressed through the imaginary part of the Green's function. The apparent difference from the results of Rytov's theory [see (25)] is related to the different definition of the Green's functions and Fourier transforms in [7, 24]. In particular, the Helmholtz equation for the vector potential or for the Hertz vector that follows from the set of equations (21) contains the dipole $\mathbf{d} = \mathbf{i}\mathbf{l}/\omega$ (or $\mathbf{m} = \mathbf{i}\mathbf{l}/\omega$) in the right-hand side, which corresponds to the unit current in Rytov's theory.

3.3 Agarwal theory
In the 1970s, Agarwal [25] proposed a theoretical method of calculating properties of thermostimulated fields of solids. This method, just as that considered in Section 3.2, is based on the theory of linear response, the Kubo theorems, and, naturally, the Maxwell equations. Agarwal used another form of the interaction Hamiltonian:

$$V = -\int \mathrm{d}^3 r \left(\mathbf{P}_0(\mathbf{r}, t)\,\mathbf{E}(\mathbf{r}, t) + \mathbf{M}_0(\mathbf{r}, t)\,\mathbf{H}(\mathbf{r}, t)\right) \,, \tag{40}$$

where $\mathbf{P}_0(\mathbf{r}, t)$ and $\mathbf{M}_0(\mathbf{r}, t)$ are the external polarization and magnetization, respectively, related to the external current in (29) and (30) as

$$\mathbf{j}_0(\mathbf{r}, t) = \frac{\partial \mathbf{P}_0(\mathbf{r}, t)}{\partial t} + c\,\mathrm{rot}\,\mathbf{M}_0 \,. \tag{41}$$

Hamiltonian (40) follows from (29) if we take into account that $\rho_0(\mathbf{r}, t) = -\mathrm{div}\,\mathbf{P}_0(\mathbf{r}, t)$ and use the formulas of the vector analysis $\mathrm{div}\,(\varphi\mathbf{P}_0) = \varphi\,\mathrm{div}\,\mathbf{P}_0 + \mathbf{P}_0\,\mathrm{grad}\,\varphi$ and $\mathrm{div}\,[\mathbf{M}_0 \times \mathbf{A}] = \mathbf{A}\,\mathrm{rot}\,\mathbf{M}_0 + \mathbf{M}_0\,\mathrm{rot}\,\mathbf{A}$.

It directly follows from (33) that the susceptibility is determined by the first functional derivative

$$\frac{\delta\langle x_i(\mathbf{r}, \omega)\rangle}{\delta f_j(\mathbf{r}', \omega)} = \chi_{ij}(\mathbf{r}, \mathbf{r}'; \omega) \,. \tag{42}$$

With Hamiltonian (40) written as in (31) and the fields therefore expressed through the field susceptibilities via formulas like (33), it is possible to define four types of

response functions:

$$\chi_{ij}^{EE}(\mathbf{r}, \mathbf{r}'; \omega) = \frac{\delta E_i(\mathbf{r}, \omega)}{\delta P_{0j}(\mathbf{r}', \omega)},$$

$$\chi_{ij}^{EH}(\mathbf{r}, \mathbf{r}'; \omega) = \frac{\delta E_i(\mathbf{r}, \omega)}{\delta M_{0j}(\mathbf{r}', \omega)},$$

$$\chi_{ij}^{HE}(\mathbf{r}, \mathbf{r}'; \omega) = \frac{\delta H_i(\mathbf{r}, \omega)}{\delta P_{0j}(\mathbf{r}', \omega)}, \qquad (43)$$

$$\chi_{ij}^{HH}(\mathbf{r}, \mathbf{r}'; \omega) = \frac{\delta H_i(\mathbf{r}, \omega)}{\delta M_{0j}(\mathbf{r}', \omega)},$$

which, according to Kubo theorem (35), are related to the appropriate symmetrized correlation functions like (7) and their Fourier transforms (8). Using these Fourier transforms and the definitions

$$\langle [A(0), B(t)] \rangle = \frac{\mathrm{i}}{\hbar} \langle B(t)A(0) - A(0)B(t) \rangle, \qquad (44)$$

$$\langle \{A(0), B(t)\} \rangle = \frac{1}{2} \langle B(t)A(0) + A(0)B(t) \rangle, \qquad (45)$$

as well as the relation [18]

$$\int_{-\infty}^{+\infty} \mathrm{d}t \, \exp{(\mathrm{i}\omega t)} \langle [A(0), B(t)] \rangle$$

$$= \frac{\omega}{\mathrm{i}\Theta(\omega, T)} \int_{-\infty}^{+\infty} \mathrm{d}t \, \exp{(\mathrm{i}\omega t)} \langle \{A(0), B(t)\} \rangle, \qquad (46)$$

we obtain the connection between the spectral density of the symmetrized correlation function and the generalized susceptibility $\chi_{ab}^{AB}(\omega) = \chi_{ab}'^{AB}(\omega) + \mathrm{i}\chi_{ab}''^{AB}(\omega)$.

For the spectral densities of the components of the fields of their symmetrized correlation functions (45), we have

$$\gamma_{ij}^{EE}(\mathbf{r}, \mathbf{r}'; \omega) = \frac{2\mathrm{i}\Theta(\omega, T)}{\omega} \operatorname{Re} \chi_{ij}^{EE}(\mathbf{r}, \mathbf{r}'; \omega),$$

$$\gamma_{ij}^{HH}(\mathbf{r}, \mathbf{r}'; \omega) = \frac{2\mathrm{i}\Theta(\omega, T)}{\omega} \operatorname{Re} \chi_{ij}^{HH}(\mathbf{r}, \mathbf{r}'; \omega), \qquad (47)$$

and for the mixed spectral densities, we obtain

$$\gamma_{ij}^{EH}(\mathbf{r}, \mathbf{r}'; \omega) = -\frac{2\Theta(\omega, T)}{\omega} \operatorname{Im} \chi_{ij}^{EH}(\mathbf{r}, \mathbf{r}'; \omega),$$

$$\gamma_{ij}^{HE}(\mathbf{r}, \mathbf{r}'; \omega) = -\frac{2\Theta(\omega, T)}{\omega} \operatorname{Im} \chi_{ij}^{HE}(\mathbf{r}, \mathbf{r}'; \omega). \qquad (48)$$

To find susceptibilities (43), it is necessary to solve the appropriate electrodynamic problems by expressing the fields through the external polarization and magnetization.

The boundary value problem is to be solved for system (15) with the external polarization and magnetization:

$$\operatorname{rot} \mathbf{E} = -\frac{1}{c} \frac{\partial}{\partial t} (\mathbf{B} + 4\pi \mathbf{M}_0), \qquad \operatorname{div} (\mathbf{B} + 4\pi \mathbf{M}_0) = 0,$$

$$\operatorname{rot} \mathbf{H} = \frac{1}{c} \frac{\partial}{\partial t} (\mathbf{D} + 4\pi \mathbf{P}_0), \qquad \operatorname{div} (\mathbf{D} + 4\pi \mathbf{P}_0) = 0. \qquad (49)$$

Writing the solution of the set of equations (49) in form (32), for example, in terms of the Green's function of a concrete problem, and using definitions (43) and formulas (47) and (48), we can find the spectral characteristics of fluctuating

electromagnetic fields in the problem under consideration, all of whose properties are determined by its geometry and electrodynamic characteristics.

To conclude this section, we note that the above-described theoretical methods of finding spectral characteristics of the thermal electromagnetic field are in fact identical, because they are based on the solution of the set of macroscopic Maxwell equations with given boundary and linear matter conditions. In each approach, a long-wavelength part of the emission is considered, with the wave vectors of the fields satisfying the condition $ka \ll 1$, where a is the interatomic spacing in the medium. In this frequency range, all spectral characteristics of fluctuating electromagnetic fields are expressed in terms of phenomenological concepts, the dielectric permittivity and magnetic susceptibility of the medium $\varepsilon(\omega)$ and $\mu(\omega)$. Therefore, the use of one theory or another in practice is in many respects a matter of taste.

We note, however, that the theoretical approaches described in Sections 3.2 and 3.3 allow finding characteristics of random fields only under the conditions of complete thermodynamic equilibrium in the system. To consider fields under the conditions of partial equilibrium, when one part of the system is heated and radiates into colder surroundings, the contribution that corresponds to the blackbody radiation scattered by this heated part of the system must be subtracted from the obtained equilibrium solution (see, e.g., [24, Section 77, Problem 3]). In fact, it is precisely in this way that the classical Kirchhoff law (1) was obtained, where the fraction of the intensity of blackbody radiation reflected by the body was subtracted from the intensity of the equilibrium radiation. Rytov's theory gives a prescription for finding the characteristics of a fluctuating electromagnetic field already in the case where the heated body radiates into the cold space. The characteristics of the equilibrium field are found from the generalized Kirchhoff law as a particular, simpler case. The solutions obtained in the framework of different theoretical approaches are, of course, identical.

4. Spectral characteristics of thermally stimulated fluctuating electromagnetic fields of plane-layered bodies

4.1 Spectral characteristics of the fluctuating electromagnetic field of a half-space

We first recall the problem, which is well known from any classical course of mechanics, about the motion of an oscillator under the action of an external force. It follows from its solution that the displacement of the oscillator experiences a sharp increase (resonance) at the eigenfrequency of the system. The amplitude and width of the resonance are determined by dissipative processes in the system.

In our case, the dynamics of the system are determined by the set of Maxwell equations in the vacuum and in a medium like (14) or (15) with external random currents specified by the fluctuation–dissipation theorem (the so-called Langevin approach). We note that in this case, solutions must obey some given boundary conditions. It follows from these equations that the possible class of fields is excited by the sum of the chosen external currents and induced currents specific for this system. Instead of considering induced currents, we can speak about an induced polarization, since they are unambiguously connected with one another.

In the simplest model of a medium represented as a set of oscillators, the polarization is proportional to the displacements of the oscillators from equilibrium positions. The displacements of oscillators under the action of an external driving force can be assumed, by analogy with classical mechanics, to be resonantly excited at the eigenmodes of the medium. The eigenmodes of the medium are bulk and surface polaritons, because the medium is always bounded in a real experiment.

Thus, based on this simplified picture, it can be assumed that the fields or, more precisely, their energy characteristics — the mean squares of the fields — must be excited predominantly at frequencies close to eigenfrequencies of a specific problem. This is completely confirmed by the exact solutions of the various problems of fluctuation electrodynamics.

4.1.1 Nonequilibrium problem. For an experimental study of thermostimulated radiation, of great importance is the problem of determining the statistical properties of the thermal field generated by a heated sample whose temperature is considerably higher that of the surrounding objects. A comparatively simple model of an optically thick sample here can be a half-space. This problem has been examined in detail by Levin and Rytov in [7]. Using the generalized Kirchhoff law, they determined all the possible statistical characteristics of the thermostimulated electromagnetic field generated by a half-space whose optical properties are characterized by an arbitrary complex dielectric function $\varepsilon(\omega)$ and a magnetic permeability $\mu(\omega)$.

We write the solutions obtained in [7] directly in terms of the Fresnel coefficients of p and s polarized waves that fall in a medium i at a certain angle θ to the normal to the planar boundary that separates the medium i from a medium j:

$$r_{ij}^{\mathrm{p}} = \frac{w_i \varepsilon_j - w_j \varepsilon_i}{w_i \varepsilon_j + w_j \varepsilon_i}, \qquad r_{ij}^{\mathrm{s}} = \frac{w_i \mu_j - w_j \mu_i}{w_i \mu_j + w_j \mu_i}, \qquad (50)$$

where $w_i = (k_0^2 \varepsilon_i \mu_i - p^2)^{1/2}$, $p = k_0 n_i \sin \theta$ is the tangential component of the wave vector of the incident wave, $k_0 = \omega/c$ is the wave number in the vacuum, and $n_i = \sqrt{\varepsilon_i \mu_i}$.

We assume that the interface is formed between a nonmagnetic half-space characterized by a dielectric constant ε_1 and the vacuum. The optical properties of this system are completely determined by the Fresnel coefficients r^{p} and r^{s}. In the notation used in [7], $q_1 = \mathrm{i} w_1 = (p^2 - \varepsilon_1 k_0^2)^{1/2}$ and $q = (p^2 - k_0^2)^{1/2}$. It can be easily shown using (50) that

$$1 - |r^{\mathrm{p}}|^2 = 2q \, \frac{q_1^*/\varepsilon_1^* + (q^*/q)(q_1/\varepsilon_1)}{|q_1/\varepsilon_1 + q|^2} \,. \qquad (51)$$

For traveling waves, we have $q = -q^*$, because $p < k_0$. Setting $x = p/k_0 = \sin \theta$, we relate the function

$$S_\varepsilon = \frac{k_0 (q_1^*/\varepsilon_1^* - q_1/\varepsilon_1)}{|q + q_1/\varepsilon_1|^2}$$

introduced in [7] to the Fresnel coefficient r^{p}:

$$S_\varepsilon = -\frac{\mathrm{i}(1 - |r^{\mathrm{p}}|^2)}{2 \cos \theta} \,. \qquad (52)$$

In the case of quasi-stationary waves, we have $q = q^*$ as a result of the condition $p > k_0$. Setting $y = q/k_0$, we write the

Fresnel coefficient in the form

$$r^{\mathrm{p}} = \mathrm{Re} \, r^{\mathrm{p}} + \mathrm{i} \, \mathrm{Im} \, r^{\mathrm{p}} = \frac{|q_1/\varepsilon_1|^2 + |q|^2}{|q_1/\varepsilon_1 + q|^2} - y S_\varepsilon \,. \qquad (53)$$

Taking into account that the S_ε function is purely imaginary, we find its connection with the Fresnel coefficient for quasi-stationary waves:

$$S_\varepsilon = -\frac{\mathrm{i} \, \mathrm{Im} \, r^{\mathrm{p}}}{y} \,. \qquad (54)$$

Quite similarly, we obtain a relation between the S_μ function in [7] and the Fresnel coefficient r^{s}.

Substituting the above relations in the corresponding formulas in [7] for the spectral densities, we obtain expressions for spectral characteristics of the fluctuating field in terms of the Fresnel coefficients.

For traveling waves, restricting the domain of the integration over p to $p < k_0$, we find

$$\langle |E_x|^2 \rangle^{\mathrm{Pr}} = \langle |E_y|^2 \rangle^{\mathrm{Pr}}$$
$$= \frac{\pi u_{0\omega}}{2} \int_0^{\pi/2} \mathrm{d}\theta \, \sin\theta \left(\cos^2 \theta \, \frac{1 - |r^{\mathrm{p}}|^2}{2} + \frac{1 - |r^{\mathrm{s}}|^2}{2} \right), \quad (55)$$

$$\langle |E_z|^2 \rangle^{\mathrm{Pr}} = \pi u_{0\omega} \int_0^{\pi/2} \mathrm{d}\theta \, \sin\theta \left(\sin^2 \theta \, \frac{1 - |r^{\mathrm{p}}|^2}{2} \right), \qquad (56)$$

$$\langle |H_z|^2 \rangle^{\mathrm{Pr}} = \pi u_{0\omega} \int_0^{\pi/2} \mathrm{d}\theta \, \sin\theta \left(\sin^2 \theta \, \frac{1 - |r^{\mathrm{s}}|^2}{2} \right), \qquad (57)$$

$$\langle |H_x|^2 \rangle^{\mathrm{Pr}} = \langle |H_y|^2 \rangle^{\mathrm{Pr}}$$
$$= \frac{\pi u_{0\omega}}{2} \int_0^{\pi/2} \mathrm{d}\theta \, \sin\theta \left(\cos^2 \theta \, \frac{1 - |r^{\mathrm{s}}|^2}{2} + \frac{1 - |r^{\mathrm{p}}|^2}{2} \right), \quad (58)$$

$$\langle E_x H_y^* \rangle^{\mathrm{Pr}} = -\langle E_y H_x^* \rangle^{\mathrm{Pr}}$$
$$= \frac{\pi u_{0\omega}}{2} \int_0^{\pi/2} \mathrm{d}\theta \, \sin\theta \cos\theta \left(\frac{1 - |r^{\mathrm{p}}|^2}{2} + \frac{1 - |r^{\mathrm{s}}|^2}{2} \right), \quad (59)$$

where $u_{0\omega} = \Theta k_0^2 / \pi^2 c$ is the spectral density of the equilibrium radiation energy.

For the spectral densities of the electric and magnetic energies and for their sum, it follows from (55)–(58) that

$$u_{e\omega}^{\mathrm{Pr}} = \frac{1}{4\pi} \sum_{i=x,y,z} \langle |E_i|^2 \rangle$$
$$= \frac{u_{0\omega}}{4} \int_0^{\pi/2} \mathrm{d}\theta \, \sin\theta \left(\frac{1 - |r^{\mathrm{p}}|^2}{2} + \frac{1 - |r^{\mathrm{s}}|^2}{2} \right), \qquad (60)$$

$$u_{m\omega}^{\mathrm{Pr}} = \frac{1}{4\pi} \sum_{i=x,y,z} \langle |H_i|^2 \rangle$$
$$= \frac{u_{0\omega}}{4} \int_0^{\pi/2} \mathrm{d}\theta \, \sin\theta \left(\frac{1 - |r^{\mathrm{s}}|^2}{2} + \frac{1 - |r^{\mathrm{p}}|^2}{2} \right), \qquad (61)$$

$$u_\omega^{\mathrm{Pr}} = u_{e\omega}^{\mathrm{Pr}} + u_{m\omega}^{\mathrm{Pr}}$$
$$= \frac{u_{0\omega}}{2} \int_0^{\pi/2} \mathrm{d}\theta \, \sin\theta \left(\frac{1 - |r^{\mathrm{p}}|^2}{2} + \frac{1 - |r^{\mathrm{s}}|^2}{2} \right). \qquad (62)$$

With the relation between the intensity I_ω and the energy density u_ω in the vacuum,

$$u_\omega = c^{-1} \int_0^{2\pi} \mathrm{d}\varphi \int_0^{\pi/2} \mathrm{d}\theta \, \sin\theta \, I_\omega \,, \qquad (63)$$

and with the relation between the equilibrium radiation intensity $I_{0\omega}$ and its energy density $u_{0\omega}$ in the vacuum ($I_{0\omega} = c u_{0\omega}/4\pi$), we immediately obtain from (62) that

$$I_\omega = I_{0\omega} \left(\frac{\left(1 - |r^{\mathrm{p}}|^2\right) + \left(1 - |r^{\mathrm{s}}|^2\right)}{2} \right). \tag{64}$$

Taking the equality of both polarizations into account and introducing the energy absorption coefficients $R^{\mathrm{p}} = |r^{\mathrm{p}}|^2$, $R^{\mathrm{s}} = |r^{\mathrm{s}}|^2$, and $R = (R^{\mathrm{p}} + R^{\mathrm{s}})/2$, the authors of [7] obtained the classical Kirchhoff law

$$I_\omega = I_{0\omega}(1 - R) \tag{65}$$

from (63). For the Poynting vector, it follows from (59) that

$$P_{\omega n} = \int_0^{2\pi} \mathrm{d}\varphi \int_0^{\pi/2} \mathrm{d}\theta \, I_{0\omega}(1 - R) \cos\theta \sin\theta \,. \tag{66}$$

The integration domain $k_0 < p < \infty$ corresponds to quasi-stationary waves; using (54), we therefore have

$$\langle |E_x|^2 \rangle^{\mathrm{Ev}} = \langle |E_y|^2 \rangle^{\mathrm{Ev}}$$
$$= \frac{\pi u_{0\omega}}{2} \int_0^\infty \mathrm{d}y \, \exp\left(-2k_0 hy\right)(y^2 \, \mathrm{Im}\, r^{\mathrm{p}} + \mathrm{Im}\, r^{\mathrm{s}})\,, \tag{67}$$

$$\langle |E_z|^2 \rangle^{\mathrm{Ev}} = \pi u_{0\omega} \int_0^\infty \mathrm{d}y \, \exp\left(-2k_0 hy\right)(y^2 + 1) \, \mathrm{Im}\, r^{\mathrm{p}}\,, \tag{68}$$

$$\langle |H_z|^2 \rangle^{\mathrm{Ev}} = \pi u_{0\omega} \int_0^\infty \mathrm{d}y \, \exp\left(-2k_0 hy\right)(y^2 + 1) \, \mathrm{Im}\, r^{\mathrm{s}}\,, \tag{69}$$

$$\langle |H_x|^2 \rangle^{\mathrm{Ev}} = \langle |H_y|^2 \rangle^{\mathrm{Ev}}$$
$$= \frac{\pi u_{0\omega}}{2} \int_0^\infty \mathrm{d}y \, \exp\left(-2k_0 hy\right)(y^2 \, \mathrm{Im}\, r^{\mathrm{s}} + \mathrm{Im}\, r^{\mathrm{p}})\,. \tag{70}$$

The cross spectral densities are purely imaginary quantities:

$$\langle E_x H_y^* \rangle^{\mathrm{Ev}} = -\langle E_y H_x^* \rangle^{\mathrm{Ev}}$$
$$= \mathrm{i}\, \frac{u_{0\omega}}{2\pi} \int_0^\infty \mathrm{d}y \, y \exp\left(-2k_0 hy\right)(\mathrm{Im}\, r^{\mathrm{s}} - \mathrm{Im}\, r^{\mathrm{p}})\,; \tag{71}$$

therefore, the Poynting vector corresponding to quasista-tionary waves is equal to zero,

$$P_{\omega n}^{\mathrm{Ev}} = \frac{c}{4\pi} \left(\langle E_x H_y^* \rangle^{\mathrm{Ev}} - \langle E_y H_x^* \rangle^{\mathrm{Ev}} + \mathrm{c.c.} \right) = 0\,. \tag{72}$$

For the spectral densities of the electric and magnetic energies and their sum, it follows from (67)–(70) that

$$u_{e\omega}^{\mathrm{Ev}} = \frac{u_{0\omega}}{4} \int_0^\infty \mathrm{d}y \, \exp\left(-2k_0 hy\right) \left[(2y^2 + 1) \, \mathrm{Im}\, r^{\mathrm{p}} + \mathrm{Im}\, r^{\mathrm{s}}\right]\,, \tag{73}$$

$$u_{m\omega}^{\mathrm{Ev}} = \frac{u_{0\omega}}{4} \int_0^\infty \mathrm{d}y \, \exp\left(-2k_0 hy\right) \left[\mathrm{Im}\, r^{\mathrm{p}} + (2y^2 + 1) \, \mathrm{Im}\, r^{\mathrm{s}}\right]\,, \tag{74}$$

$$u_\omega^{\mathrm{Ev}} = u_{e\omega}^{\mathrm{Ev}} + u_{m\omega}^{\mathrm{Ev}}$$
$$= \frac{u_{0\omega}}{2} \int_0^\infty \mathrm{d}y \, \exp\left(-2k_0 hy\right)(y^2 + 1)(\mathrm{Im}\, r^{\mathrm{p}} + \mathrm{Im}\, r^{\mathrm{s}})\,. \tag{75}$$

Hence, the total spectral energy density is

$$u_\omega = \omega_\omega^{\mathrm{Pr}} + u_\omega^{\mathrm{Ev}} = \frac{u_{0\omega}}{2} \left[\int_0^{\pi/2} \mathrm{d}\theta \, \sin\theta \, \frac{\left(1 - |r^{\mathrm{p}}|^2\right) + \left(1 - |r^{\mathrm{s}}|^2\right)}{2} \right.$$
$$\left. + \int_0^\infty \mathrm{d}y \, \exp\left(-2k_0 hy\right)(y^2 + 1)(\mathrm{Im}\, r^{\mathrm{p}} + \mathrm{Im}\, r^{\mathrm{s}}) \right]\,. \tag{76}$$

4.1.2 Equilibrium problem. In the case of complete thermodynamic equilibrium, when the temperatures of the sample and its surroundings are equal, finding the spectral densities of thermostimulated fields requires solving the problem of the field of a unit point source in a given geometry or, in other words, finding the Green's function of the corresponding problem. This problem has been considered by many authors and the solution is well known (see, e.g., Refs [7, 26–30]).

According to Rytov's theory, to find the spectral density of a component of the fluctuation field at a given point of space, a unit point source is to be placed at that point and its field at the same point is then to be found. We use a solution in the form presented in [27,28], but change some notation for convenience. The field of a unit point source located at a distance h from the half-space is expressed at the source-point as

$$\mathbf{E} = \frac{2}{3} \mathrm{i} k_0^3 \boldsymbol{\mu} + \frac{\mathrm{i} k_0^2}{2\pi} \int \mathrm{d}\mathbf{p} \, w_0^{-1} (\hat{\mathbf{s}}\hat{\mathbf{s}}\, r^{\mathrm{s}} + \hat{\mathbf{p}}_{0+}\hat{\mathbf{p}}_{0-}\, r^{\mathrm{p}}) \, \boldsymbol{\mu} \exp\left(2\mathrm{i}w_0 h\right)\,, \tag{77}$$

$$\mathbf{H} = \frac{2}{3} \mathrm{i} k_0^3 \mathbf{m} + \frac{\mathrm{i} k_0^2}{2\pi} \int \mathrm{d}\mathbf{p} \, w_0^{-1} (\hat{\mathbf{s}}\hat{\mathbf{s}}\, r^{\mathrm{p}} + \hat{\mathbf{p}}_{0+}\hat{\mathbf{p}}_{0-}\, r^{\mathrm{s}}) \, \mathbf{m} \exp\left(2\mathrm{i}w_0 h\right)\,, \tag{78}$$

where $\mathrm{d}\mathbf{p} = p \, \mathrm{d}p \, \mathrm{d}\varphi$, $w_0 = (k_0^2 - p^2)^{1/2}$, $\boldsymbol{\mu}$ and $\mathbf{m}$ are the point electric and magnetic dipoles, r^{s} and r^{p} are the Fresnel reflection coefficients, $k_0 = \omega/c$ is the wave number in the vacuum, $\hat{\mathbf{s}} = \hat{\mathbf{p}} \times \hat{\mathbf{z}}$, $\hat{\mathbf{p}} = \mathbf{p}/|\mathbf{p}|$, $\hat{\mathbf{p}}_{0\pm} = k_0^{-1}(p\hat{\mathbf{z}} \mp w_0\hat{\mathbf{p}})$, and $\hat{\mathbf{z}}$ and $\hat{\mathbf{p}}$ are the unit vectors respectively directed along the normal to the interface and along the interface; here, the well-known rule for dyadic products must be used, for example, $\hat{\mathbf{s}}\hat{\mathbf{s}}\boldsymbol{\mu} \equiv \hat{\mathbf{s}}(\hat{\mathbf{s}}\boldsymbol{\mu})$.

Substituting the real parts of (77) and (78) in Eqn (25), we obtain expressions for the spectral density and for any component of the fluctuating field.

We first consider the limit cases. The simplest case $r^{\mathrm{s}} = r^{\mathrm{p}} = 0$ corresponds to a black body. By orienting the point dipole along a selected axis, for example, along the x axis, and assuming it to be equal to $\mu_x = \mathrm{i}/\omega$, which corresponds to a unit point source, we use (25) and (77) to obtain the spectral density of the E_x component of the fluctuating electromagnetic field at the point located at any distance from the half-space with the optical characteristics of the black body,

$$\langle |E_x|^2 \rangle = \frac{2\pi}{3} u_{0\omega}\,, \tag{79}$$

and analogous expressions for other components. This implies an expression for the spectral density of the energy of the thermostimulated electromagnetic field for complete thermodynamic equilibrium in the closed cavity formed by the half-space in question and the bodies with arbitrary optical characteristics located at infinity,

$$u_\omega = \frac{1}{4\pi} \sum_{i=x,y,z} \langle |E_i|^2 \rangle + \frac{1}{4\pi} \sum_{i=x,y,z} \langle |H_i|^2 \rangle \equiv u_{0\omega}\,. \tag{80}$$

As can be expected, we obtained the Planck formula for the energy spectral density of equilibrium radiation $u_{0\omega} = \Theta k_0^2/\pi^2 c$. It is interesting to compare Eqn (80) with the expression for the energy spectral density in a nonequilibrium case (76) when the bodies located at infinity have a zero temperature. Substituting $r^s = r^p = 0$ in (76), we obtain $u_\omega = u_{0\omega}/2$, as it must be in the case of the unidirectional radiation flux from a half-space with the optical characteristics of a black body.

Another limit case is a half-space with ideal properties, $|\varepsilon| \to \infty$, where ε is the dielectric constant of the material of the half-space. As can be seen from the general expressions for Fresnel coefficients (50), the equalities $r^s = -1$ and $r^p = 1$ correspond to this case. Expressions (77) and (78) take the form

$$\mathbf{E} = \frac{2}{3} i k_0^3 \boldsymbol{\mu}$$
$$+ \frac{i k_0^2}{2\pi} \int_0^\infty \int_0^{2\pi} dp\, d\varphi\, p w_0^{-1} \left[-\hat{\mathbf{s}}(\hat{\mathbf{s}}\,\boldsymbol{\mu}) + \hat{\mathbf{p}}_{0+}(\hat{\mathbf{p}}_{0-}\,\boldsymbol{\mu}) \right] \exp\left(2i w_0 h\right), \tag{81}$$

$$\mathbf{H} = \frac{2}{3} i k_0^3 \mathbf{m}$$
$$+ \frac{i k_0^2}{2\pi} \int_0^\infty \int_0^{2\pi} dp\, d\varphi\, p w_0^{-1} \left[\hat{\mathbf{s}}(\hat{\mathbf{s}}\,\mathbf{m}) - \hat{\mathbf{p}}_{0+}(\hat{\mathbf{p}}_{0-}\,\mathbf{m}) \right] \exp\left(2i w_0 h\right), \tag{82}$$

i.e., they are distinguished only by the sign of the integrand; therefore, all calculations can be conducted, for example, for the electric field and then, introducing obvious changes in the results of these calculations, expressions for the components of the magnetic field can be obtained.

We note that the above-introduced unit vectors $\hat{\mathbf{s}}$ and $\hat{\mathbf{p}}$ are connected with the unit vectors of the Cartesian coordinate system as $\hat{\mathbf{s}} = \hat{\mathbf{e}}_y \cos\varphi - \hat{\mathbf{e}}_x \sin\varphi$, $\hat{\mathbf{p}} = \hat{\mathbf{e}}_x \cos\varphi + \hat{\mathbf{e}}_y \sin\varphi$. Substituting the last expressions in (81) and (82) and integrating over the polar angle, we find the Cartesian components of the electric field as

$$E_x = \frac{2}{3} i k_0^3 \mu_x - \frac{i k_0^2}{2} \mu_x \int_0^\infty dp\, p w_0^{-1} \left(1 + \frac{w_0^2}{k_0^2} \right) \exp\left(2i w_0 h\right), \tag{83}$$

$$E_y = \frac{2}{3} i k_0^3 \mu_y - \frac{i k_0^2}{2} \mu_y \int_0^\infty dp\, p w_0^{-1} \left(1 + \frac{w_0^2}{k_0^2} \right) \exp\left(2i w_0 h\right), \tag{84}$$

$$E_z = \frac{2}{3} i k_0^3 \mu_z + i \mu_z \int_0^\infty dp\, p^3 w_0^{-1} \exp\left(2i w_0 h\right). \tag{85}$$

We take into account at this stage that to obtain the spectral density of the field components, it is necessary to take the real part of the expressions obtained. The quasistationary part ($p > k_0$) immediately drops out from consideration because the second terms in all expressions (83)–(85) are purely imaginary in this case. Physically, this means that an ideal body without absorption cannot be a source of thermostimulated emission. The integration from zero to k_0, which can easily be performed, yields the following expressions for the spectral densities:

$$\langle |E_x|^2 \rangle = \langle |E_y|^2 \rangle$$
$$= \frac{2\pi}{3} u_{0\omega} \left[1 + \frac{3}{2} \left(\frac{\sin\left(2k_0 h\right)}{(2k_0 h)^3} - \frac{\cos\left(2k_0 h\right)}{(2k_0 h)^2} - \frac{\sin\left(2k_0 h\right)}{2k_0 h} \right) \right], \tag{86}$$

$$\langle |E_z|^2 \rangle = \frac{2\pi}{3} u_{0\omega} \left[1 + 3 \left(\frac{\sin\left(2k_0 h\right)}{(2k_0 h)^3} - \frac{\cos\left(2k_0 h\right)}{(2k_0 h)^2} \right) \right], \tag{87}$$

$$\langle |H_z|^2 \rangle = \frac{2\pi}{3} u_{0\omega} \left[1 - 3 \left(\frac{\sin\left(2k_0 h\right)}{(2k_0 h)^3} - \frac{\cos\left(2k_0 h\right)}{(2k_0 h)^2} \right) \right], \tag{88}$$

$$\langle |H_x|^2 \rangle = \langle |H_y|^2 \rangle$$
$$= \frac{2\pi}{3} u_{0\omega} \left[1 - \frac{3}{2} \left(\frac{\sin\left(2k_0 h\right)}{(2k_0 h)^3} - \frac{\cos\left(2k_0 h\right)}{(2k_0 h)^2} - \frac{\sin\left(2k_0 h\right)}{2k_0 h} \right) \right]. \tag{89}$$

We can introduce spherical Bessel functions $j_n(x) = \sqrt{\pi/2x}\, J_{n+1/2}(x)$ and rewrite Eqns (86)–(89), as was done in [7], where the same result was obtained in another way. It follows that the sum of all field components yields the energy spectral density of the equilibrium field $u_\omega = u_{0\omega}$. We note the analogy between Eqns (86)–(89) and expressions for Friedel's oscillations of electron waves in a solid body near an interface [31, 32]. However, in our case, the oscillating part is connected with the interference of incident waves whose source is located at infinity and waves that are reflected from the ideal surface. We also note that when considering the equilibrium problem with an ideal body from the standpoint of optical properties, it is necessary to assume the existence of infinitely remote bodies whose material has at least infinitely weak absorption so as to allow equilibrium to be established in the system.

Finally, we consider the general case of an equilibrium problem with arbitrary Fresnel coefficients. Passing in (77) and (78) to unit Cartesian vectors and using the method suggested in [7], we obtain

$$\langle |E_{x,y}|^2 \rangle = \frac{2\pi}{3} u_{0\omega}$$
$$+ \frac{\Theta k_0}{\pi c} \operatorname{Re} \int_0^\infty dp\, p (2w_0)^{-1} \left(r^s - \frac{w_0^2}{k_0^2} r^p \right) \exp\left(2i w_0 h\right), \tag{90}$$

$$\langle |E_z|^2 \rangle = \frac{2\pi}{3} u_{0\omega} + \frac{\Theta k_0}{\pi c} \operatorname{Re} \int_0^\infty dp\, p \left(\frac{p^2}{k_0^2} r^p \right) \exp\left(2i w_0 h\right), \tag{91}$$

$$\langle |H_z|^2 \rangle = \frac{2\pi}{3} u_{0\omega} + \frac{\Theta k_0}{\pi c} \operatorname{Re} \int_0^\infty dp\, p \left(\frac{p^2}{k_0^2} r^s \right) \exp\left(2i w_0 h\right), \tag{92}$$

$$\langle |H_{x,y}|^2 \rangle = \frac{2\pi}{3} u_{0\omega}$$
$$+ \frac{\Theta k_0}{\pi c} \operatorname{Re} \int_0^\infty dp\, p (2w_0)^{-1} \left(r^p - \frac{w_0^2}{k_0^2} r^s \right) \exp\left(2i w_0 h\right). \tag{93}$$

Summing (90)–(93), we find the energy spectral density at any distance from the surface of a half-space with arbitrary optical properties:

$$u_\omega = u_{0\omega} + \frac{\Theta k_0}{2\pi^2 c} \operatorname{Re} \int_0^\infty dp\, p w_0^{-1} \frac{p^2}{k_0^2} (r^s + r^p) \exp\left(2i w_0 h\right). \tag{94}$$

As in the nonequilibrium case, we isolate the parts that correspond to traveling and quasistationary waves. Using the same substitutions $p/k_0 = \sin\theta$ for traveling waves ($p < k_0$) and $w_0/k_0 = iy$ for quasistationary waves ($p < k_0$),

we obtain

$$u_\omega = u_{0\omega} + \frac{u_{0\omega}}{2} \, \mathrm{Re} \int_0^{\pi/2} \mathrm{d}\theta \, \sin\theta \, \sin^2\theta \, (r^s + r^p)$$

$$\times \exp\left(2\mathrm{i}k_0 h \cos\theta\right)$$

$$+ \frac{u_{0\omega}}{2} \int_0^\infty \mathrm{d}y \, \exp\left(-2k_0 h y\right)(y^2 + 1)(\mathrm{Im}\,r^p + \mathrm{Im}\,r^s). \quad (95)$$

We note that the second term in the right-hand side of Eqn (95), which, as in the case of an ideal half-space, characterizes the interference of incident and reflected radiation, tends to zero at large distances because of the presence of a strongly oscillating factor in the integrand. The interference term in the spectral density of field (76) of the nonequilibrium problem is absent because there are no waves that fall onto the surface of the half-space. The third term describes the spectrum of the quasistationary field and also tends to zero because of the presence of an exponential factor. Comparing (76) and (95), we see that the spectra of quasi-stationary waves in the equilibrium and nonequilibrium problems are identical if, naturally, the temperatures of the half-spaces in the equilibrium and nonequilibrium cases are identical.

Formula (95) contains the basic result of Planck's theory of heat radiation, created at the beginning of the 20th century: in the approximation of geometric optics, where the wavelengths of thermal radiation are much shorter than the characteristic length of the problem ($k_0 h \gg 1$), the spectral density of the equilibrium radiation energy is described by the Planck formula $u_\omega = u_{0\omega} = \Theta k_0^2/(\pi^2 c)$, irrespective of the optical properties of the material of the bodies that form a closed cavity.

4.1.3 Analytic approximations. In solving applied problems, it is frequently convenient to use approximate expressions for the spectral densities of the various components of the thermostimulated field. This section is devoted to finding various approximations for the quasistationary part of the field.

We first note that the leading contribution to expressions (67)–(70), (73)–(76), and (95) is determined by the exponential factors in their integrands. Therefore, the range of the integration variable y, which gives the leading contribution to the integrals, is determined by the relation $k_0 h y \sim 1$, whence $k_0 h \sim y^{-1}$. Thus, we can consider two limit cases: $y \to \infty$ ($y \gg 1$) and $y \to 0$ ($y \ll 1$). For this, it is necessary to find the limit expressions for the Fresnel coefficients.

First, we consider the case where $y \to \infty$ ($y \gg 1$), assuming that $p^2 \gg k_0^2$ and $p^2 \gg |\varepsilon(\omega)|k_0^2$ for any value of $\varepsilon(\omega)$. This is a quasistatic case, in which we can neglect the delay effects as $c \to \infty$. Because $y \to \infty$, we have $k_0 h \ll 1$. We consider the case where $k_0 h \ll |\sqrt{\varepsilon(\omega)}|^{-1}$. Thus, we find expressions for the spectral densities of the field components at those distances from the body surface for which $h \ll (k_0 |\sqrt{\varepsilon(\omega)}|)^{-1} \ll k_0^{-1}$. The Fresnel coefficients as functions of the variable $y = [(p/k_0)^2 - 1]^{1/2}$ are written as

$$r^s = \frac{\mathrm{i}y - \sqrt{\varepsilon(\omega) - y^2 - 1}}{\mathrm{i}y + \sqrt{\varepsilon(\omega) - y^2 - 1}},$$

$$r^p = \frac{\mathrm{i}y\varepsilon(\omega) - \sqrt{\varepsilon(\omega) - y^2 - 1}}{\mathrm{i}y\varepsilon(\omega) + \sqrt{\varepsilon(\omega) - y^2 - 1}}. \quad (96)$$

Expanding expressions (96) into power series in the small parameter y^{-1}, we obtain

$$r^s \approx \frac{\varepsilon(\omega) - 1}{4} \, y^{-2} + O(y^{-4}),$$

$$r^p \approx \frac{\varepsilon(\omega) - 1}{\varepsilon(\omega) + 1} + \varepsilon(\omega) \, \frac{\varepsilon(\omega) - 1}{(\varepsilon(\omega) + 1)^2} \, y^{-2} + O(y^{-4}). \quad (97)$$

Because we are considering the case $y \gg 1$, we can keep only the leading contribution in the integration variable y in Eqns (67)–(70). Substituting (97) in these expressions, we obtain the sought estimates of the spectral densities of all field components:

$$\langle |E_x|^2 \rangle = \langle |E_y|^2 \rangle \approx \frac{\pi u_{0\omega}}{4(k_0 h)^3} \frac{\varepsilon''(\omega)}{|\varepsilon(\omega) + 1|^2},$$

$$\langle |E_z|^2 \rangle \approx \frac{\pi u_{0\omega}}{2(k_0 h)^3} \frac{\varepsilon''(\omega)}{|\varepsilon(\omega) + 1|^2}, \quad (98)$$

$$\langle |H_x|^2 \rangle = \langle |H_y|^2 \rangle \approx \frac{\pi u_{0\omega}}{2k_0 h} \left(\frac{\varepsilon''(\omega)}{8} + \frac{\varepsilon''(\omega)}{|\varepsilon(\omega) + 1|^2} \right),$$

$$\langle |H_z|^2 \rangle \approx \frac{\pi u_{0\omega}}{8k_0 h} \varepsilon''(\omega). \quad (99)$$

The spectral density of field energy (75) in this approximation is written as

$$u_\omega = \frac{u_{0\omega}}{2} \left(\frac{\varepsilon''(\omega)}{2(k_0 h)^3 |\varepsilon(\omega) + 1|^2} + \frac{\varepsilon''(\omega)}{k_0 h |\varepsilon(\omega) + 1|^2} + \frac{\varepsilon''(\omega)}{8k_0 h} \right). \quad (100)$$

Here, we used the integral

$$\int_0^\infty \mathrm{d}y \, y^n \exp\left(-ay\right) = \frac{\Gamma(1 + n)}{2^{n+1}} \, a^{n+1}, \quad (101)$$

where n is a natural number and $\Gamma(1 + n) = n!$ is the gamma function.

It follows from Eqns (98)–(99) that the ratio of the electric part of energy u_ω^E to the magnetic part u_ω^H is

$$\frac{u_\omega^E}{u_\omega^H} \approx \frac{4}{(k_0 h)^2 |\varepsilon(\omega) + 1|^2}; \quad (102)$$

in particular, this ratio is much greater than unity at the frequency of the surface Coulomb polariton that satisfies the dispersion equation $\mathrm{Re}\,\varepsilon(\omega) = -1$.

The next approximation refers to the case $y \gg 1$, $y \ll |\sqrt{\varepsilon(\omega)}|$, which is realized under the condition $|\varepsilon(\omega)| \gg 1$ for distances that satisfy the inequality $(k_0 |\sqrt{\varepsilon(\omega)}|)^{-1} \ll h \ll k_0^{-1}$. Expanding Fresnel coefficients (96) into power series first in the small parameter $|\varepsilon|^{-1}$ and then in the small parameter y^{-1}, we obtain

$$r^s \approx -1 + 2\mathrm{i}y\sqrt{\varepsilon^{-1}(\omega)}, \qquad r^p \approx 1 + 2\mathrm{i}y^{-1}\sqrt{\varepsilon^{-1}(\omega)}, \quad (103)$$

whence

$$\mathrm{Im}\,r^s = 2y \, \mathrm{Re}\sqrt{\varepsilon^{-1}(\omega)}, \qquad \mathrm{Im}\,r^p = 2y^{-1} \, \mathrm{Re}\sqrt{\varepsilon^{-1}(\omega)}. \quad (104)$$

If we introduce the depth of the field penetration into the material as

$$\delta = \left(k_0 \, \mathrm{Im}\sqrt{\varepsilon(\omega)}\right)^{-1}, \quad (105)$$

then for 'good' metals in the approximation of the normal skin effect, where $\varepsilon(\omega) \approx i4\pi\sigma/\omega$, the depth of the skin layer is $\delta = c[(2\pi\sigma\omega)^{-1}]^{1/2}$ and

$$\operatorname{Im} r^{s} = y k_0 \delta, \qquad \operatorname{Im} r^{p} = y^{-1} k_0 \delta. \qquad (106)$$

Substituting (104) in (67)–(70), we obtain the following expressions for the spectral densities of the components of the thermal field in the approximation in question:

$$\langle |E_x|^2 \rangle = \langle |E_y|^2 \rangle \approx \frac{\pi u_{0\omega}}{2(k_0 h)^2} \operatorname{Re} \sqrt{\varepsilon^{-1}(\omega)}, \qquad (107)$$

$$\langle |E_z|^2 \rangle \approx \frac{\pi u_{0\omega}}{2(k_0 h)^2} \operatorname{Re} \sqrt{\varepsilon^{-1}(\omega)}, \qquad (108)$$

$$\langle |H_z|^2 \rangle \approx \left(\frac{3\pi u_{0\omega}}{4(k_0 h)^4} + \frac{\pi u_{0\omega}}{2(k_0 h)^2} \right) \operatorname{Re} \sqrt{\varepsilon^{-1}(\omega)}, \qquad (109)$$

$$\langle |H_x|^2 \rangle = \langle |H_y|^2 \rangle \approx \frac{3\pi u_{0\omega}}{8(k_0 h)^4} \operatorname{Re} \sqrt{\varepsilon^{-1}(\omega)}, \qquad (110)$$

and the total spectral density of energy (75) in this approximation is written as

$$u_\omega \approx u_{0\omega} \left(\frac{3}{8(k_0 h)^4} + \frac{1}{2(k_0 h)^2} \right) \operatorname{Re} \sqrt{\varepsilon^{-1}(\omega)}. \qquad (111)$$

In this approximation, the ratio of the electric part of energy u_ω^E to the magnetic part u_ω^H is

$$\frac{u_\omega^E}{u_\omega^H} \approx (k_0 h)^2, \qquad (112)$$

which clearly indicates that in the range of distances from the surface we consider here, the energy of the thermostimulated field is accumulated in its magnetic part.

We finally obtain asymptotic formulas under the condition that $y \to 0$ ($y \ll 1$), which corresponds to distances $h \gg k_0^{-1}$ from the surface. Expanding Fresnel coefficients (96) into power series in the small parameter y at an arbitrary value of $\varepsilon(\omega)$, we find

$$r^{s} \approx -1 + 2iy \sqrt{(\varepsilon(\omega) - 1)^{-1}},$$
$$r^{p} \approx -1 + 2iy\varepsilon(\omega) \sqrt{(\varepsilon(\omega) - 1)^{-1}}, \qquad (113)$$

whence

$$\operatorname{Im} r^{s} = 2y \operatorname{Re} \sqrt{(\varepsilon(\omega) - 1)^{-1}},$$
$$\operatorname{Im} r^{p} = 2y \operatorname{Re} \left[\varepsilon(\omega) \sqrt{(\varepsilon(\omega) - 1)^{-1}} \right]. \qquad (114)$$

In Eqns (67)–(70), we neglect terms involving y^2, which are small in comparison with terms of the order of unity, and substitute (114) in these formulas, which gives the following expressions for the spectral densities of the thermal field components at large distances ($h \gg k_0^{-1}$) from the surface:

$$\langle |E_x|^2 \rangle = \langle |E_y|^2 \rangle \approx \frac{\pi u_{0\omega}}{4(k_0 h)^2} \operatorname{Re} \sqrt{(\varepsilon(\omega) - 1)^{-1}}, \quad (115)$$

$$\langle |E_z|^2 \rangle \approx \frac{\pi u_{0\omega}}{2(k_0 h)^2} \operatorname{Re} \left[\varepsilon(\omega) \sqrt{(\varepsilon(\omega) - 1)^{-1}} \right], \qquad (116)$$

$$\langle |H_z|^2 \rangle \approx \frac{\pi u_{0\omega}}{2(k_0 h)^2} \operatorname{Re} \sqrt{(\varepsilon(\omega) - 1)^{-1}}, \qquad (117)$$

$$\langle |H_x|^2 \rangle = \langle |H_y|^2 \rangle \approx \frac{\pi u_{0\omega}}{4(k_0 h)^2} \operatorname{Re} \left[\varepsilon(\omega) \sqrt{(\varepsilon(\omega) - 1)^{-1}} \right]. \qquad (118)$$

It follows from Eqns (115)–(118) that the total spectral density of the thermal field energy is

$$u_\omega \approx \frac{u_{0\omega}}{2(k_0 h)^2} \left\{ \operatorname{Re} \sqrt{(\varepsilon(\omega) - 1)^{-1}} \right.$$
$$\left. + \operatorname{Re} \left[\varepsilon(\omega) \sqrt{(\varepsilon(\omega) - 1)^{-1}} \right] \right\}. \qquad (119)$$

In this approximation, the ratio of the electric part of energy u_ω^E to the magnetic part u_ω^H is equal to unity, as it must be at large distances from the surface.

4.1.4 Spectral density of states. As is known [33], the number of proper field oscillations with wave numbers in an interval Δk_0 per unit element of a sufficiently large ($k_0 h \gg 1$) volume $V \sim h^3$ can be represented as

$$\Delta \rho = \rho_k^0 \Delta k_0 = \frac{k_0^2 \Delta k_0}{\pi^2} = \rho_\omega^0 \Delta \omega = \frac{\omega^2 \Delta \omega}{\pi^2 c^3}, \qquad (120)$$

where $\rho_\omega^0 = \omega^2/(\pi^2 c^3) = k_0^2/(\pi^2 c)$ is the spectral density of oscillators of a free field in the vacuum or the spectral density of states of a free field (with two independent polarization states).

The Planck law for the spectral density of the equilibrium radiation energy in the vacuum can then be represented as the mean energy of an oscillator $\Theta(\omega, T)$ at a temperature T times the spectral density of states ρ_ω^0:

$$u_{0\omega} = \Theta \rho_\omega^0 = \frac{\Theta k_0^2}{\pi^2 c}. \qquad (121)$$

There are various ways [34–37] to introduce the local density of states to take the effect of interfaces between the media into account in concrete problems.

Using (120) and (121), by analogy with the case of a free field, we introduce a local density of the thermostimulated electromagnetic field generated by a half-space, which in the general case depends on spatial coordinates. For a nonequilibrium field, it follows from Eqn (76) that

$$\rho_\omega = \rho_\omega^0 \left\{ \frac{1}{4} \int_0^{\pi/2} d\theta \, \sin \theta \, \left[(1 - |r^p|^2) + (1 - |r^s|^2) \right] \right.$$
$$\left. + \frac{1}{2} \int_0^\infty dy \, \exp(-2k_0 h y)(y^2 + 1)(\operatorname{Im} r^p + \operatorname{Im} r^s) \right\}. \quad (122)$$

The spectral density of states of an equilibrium field follows from a comparison of (95) with (121):

$$\rho_\omega = \rho_\omega^0 \left\{ 1 + \frac{1}{2} \operatorname{Re} \int_0^{\pi/2} d\theta \, \sin \theta \, \sin^2 \theta \, (r^s + r^p) \right.$$
$$\times \exp(2ik_0 h \cos \theta)$$
$$\left. + \frac{1}{2} \int_0^\infty dy \, \exp(-2k_0 h y)(y^2 + 1)(\operatorname{Im} r^p + \operatorname{Im} r^s) \right\}. \quad (123)$$

The formulas for the corresponding spectral densities of states also directly follow from the formulas for the components of the fluctuating field obtained above.

First, we consider the local density of states of the quasi-stationary part of the thermal field in (122) and (123), because they are identical in the equilibrium and nonequilibrium problems. As can be seen from these formulas, the local density of states of a quasistationary field can be represented as the sum $\rho_\omega^p + \rho_\omega^s$ of states of p- and s-polarized waves, where

$$\rho_\omega^{p(s)} = \frac{\rho_\omega^0}{2} \int_0^\infty \mathrm{d}y \, \exp\left(-2k_0 h y\right) \left(y^2 + 1\right) \mathrm{Im} \, r^{p(s)}. \quad (124)$$

To illustrate the characteristic features of the density of states of the thermostimulated field generated by a half-space, we use two substantially different models of the dielectric constant for the description of the optical properties of the material, namely, the Drude model and the oscillatory model:

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + \mathrm{i}\nu)}, \qquad \varepsilon(\omega) = \varepsilon_\infty + \frac{(\varepsilon_0 - \varepsilon_\infty)\omega_{TO}^2}{\omega_{TO}^2 - \omega^2 - \mathrm{i}\gamma\omega}, \quad (125)$$

where ω_p is the plasma frequency, ν is the electron relaxation frequency, ε_0 and ε_∞ are the respective dielectric constants at low and high frequencies, ω_{TO} is the frequency of the transverse optical phonon, and γ is the anharmonic decay constant. For calculations, we take the value of parameters corresponding to aluminum, which is a typical good metal ($\omega_p \approx 1.2 \times 10^5$ cm^{-1}, $\nu \approx 10^3$ cm^{-1}), and to zinc selenide ZnSe ($\omega_{TO} \approx 200$ cm^{-1}, $\gamma = 0.02\omega_{TO}$, $\varepsilon_0 = 9.06$, and $\varepsilon_\infty = 5.8$).

Figure 1 displays the variations of the density of states of p- and s-polarized waves of the thermostimulated field as functions of the frequency at various distances h (indicated in the figure caption) from the surface of the aluminum half-space, which were calculated using Eqn (124). A characteristic feature is the existence of a strong resonance in the density of p states (Fig. 1a) of the thermal field at small distances from the surface. This resonance corresponds to a nonradiative Coulomb plasmon, as can be seen from Eqn (97). From the corresponding dispersion equation, it follows that the frequency of this plasmon is $\omega_{QP} = \omega_p/\sqrt{2}$.

Of great interest is the dynamics of the spectrum transformation with increasing distance. As follows from Eqn (124), the largest contribution comes from two factors, exponential and resonant, in the integrand for the Fresnel coefficient. Because the most substantial contribution comes from the parameters related by the condition $2k_0 h y \sim 1$, an increase in h leads to a decrease in the range of wave numbers that give the main contribution to the integral, which, in turn, leads to a smoothening of the resonance at the frequency of the Coulomb plasmon and to a shift of the maximum in the density of states toward lower frequencies. As follows from Eqn (97), there are no such resonances in the s states (see Fig. 1b). In principle, in the s state, there can exist a resonance related to the maximum in the imaginary part of the dielectric constant; but in the Drude model, this maximum is related to the pole at the zero frequency and does not manifest itself in the frequency range shown in the figure. It is seen from Fig. 1a, which displays the total density of states of the thermostimulated field in the near-field zone, that the main contribution to
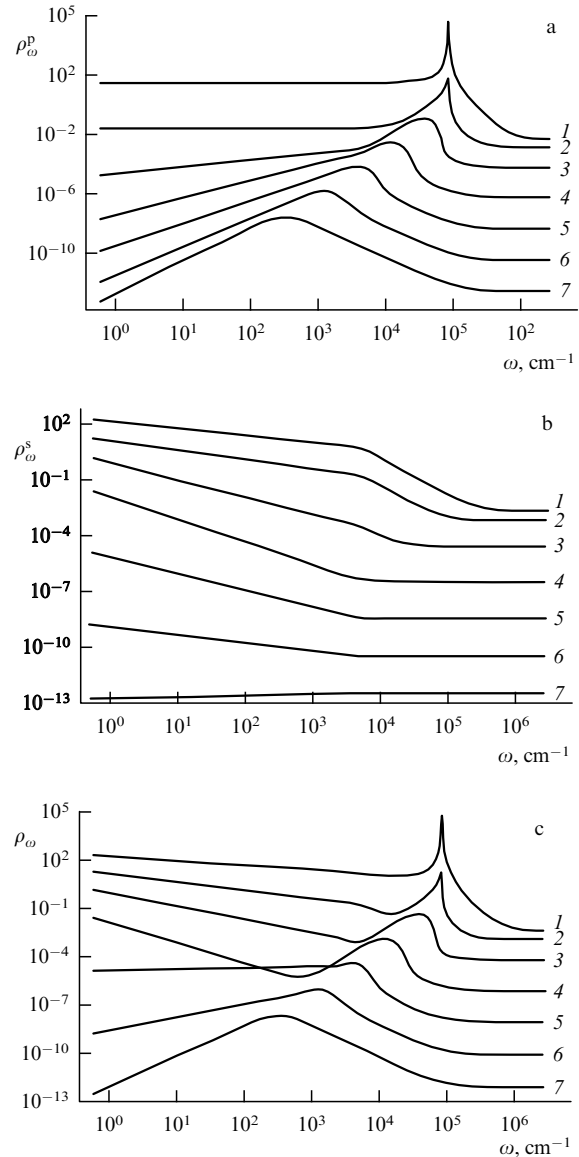

Figure 1. Frequency dependence of the density of states of p- and s-polarized waves of a thermostimulated field at various distances h from the surface of an aluminum half-space: (a) density of p states; (b) density of s states; and (c) total density of states; (*1*) $h = 10^{-7}$ cm, (*2*) $h = 10^{-6}$ cm, (*3*) $h = 10^{-5}$ cm, (*4*) $h = 10^{-4}$ cm, (*5*) $h = 10^{-3}$ cm, (*6*) 10^{-2} cm, and (*7*) $h = 10^{-1}$ cm.

the density of states of the quasistationary part of the field generated by a good conductor comes from the p-polarization states of the field.

A substantially different spectrum of the thermal field is characteristic of the zinc selenide ZnSe, as is seen from Fig. 2. Just as in the case of aluminum, the density of states is substantially modified as the distance h from the surface increases. At small distances, the peak corresponding to a Coulomb phonon–polariton is clearly pronounced. Unlike Al, in the case of ZnSe in the s-polarization state, a peak corresponding to the maximum of the imaginary part of the dielectric constant is observed at the frequency of the transverse phonon–polariton, but the height of this peak is many orders of magnitude less than the density of p states. An essential feature is also the formation of a dip in the density of states at frequencies corresponding to the negative dielectric constant.
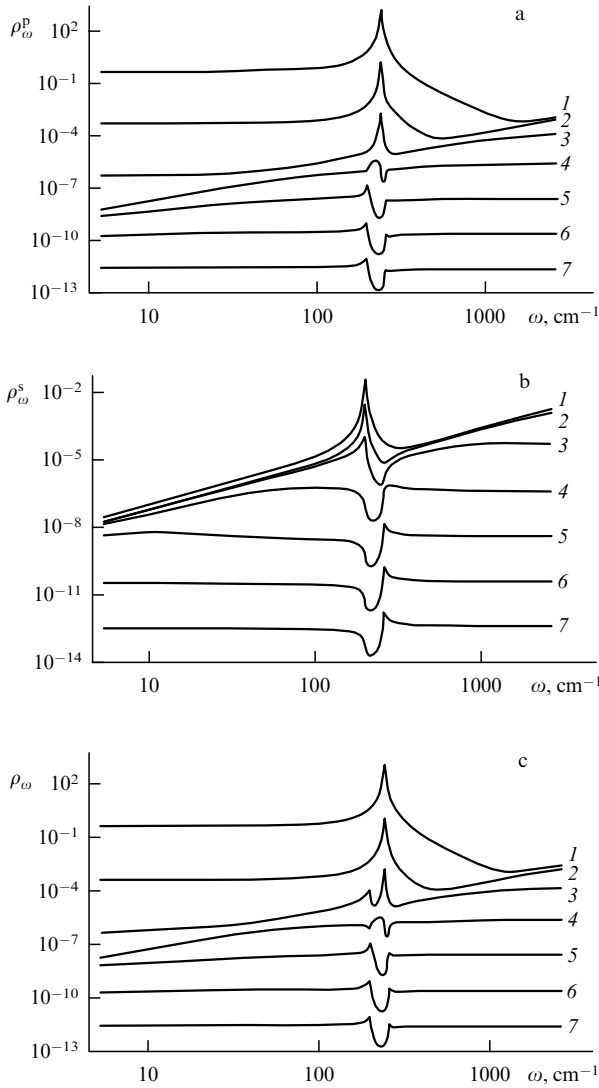
Figure 2. Frequency dependence of the density of states of p- and s-polarized waves of a thermostimulated field at various distances h from the surface of a ZnSe half-space. For the notation, see Fig. 1.
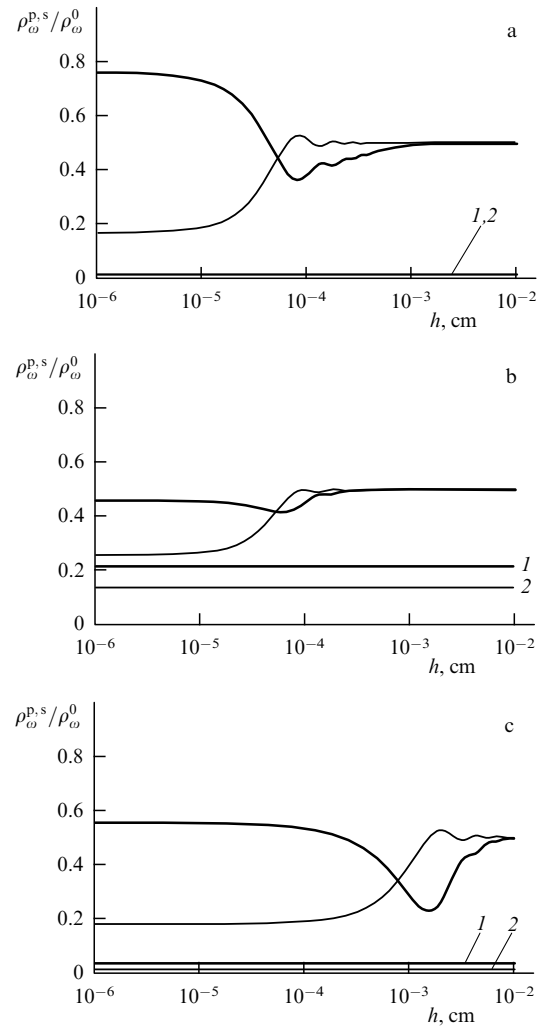

Figure 3. Normalized densities of states of p-polarized (bold lines) and s-polarized (thin lines) traveling waves of thermal fields generated by Al and ZnSe half-spaces corresponding to the equilibrium and nonequilibrium problems as functions of the distance h from the surface at the respective frequencies 5300 and 530 cm^{-1} for Al (a) and ZnSe (b) half-spaces. Figure 3c displays the same dependence for ZnSe at the frequency 212 cm^{-1}, which falls into the range of negative values of the dielectric constant. Straight lines *1* (p states) and *2* (s states) correspond to the nonequilibrium problem.

We now consider the spectral density of states for traveling waves of the thermostimulated field. As follows from Eqns (122) and (123), for the p-polarized and s-polarized states, we have

$$\rho_\omega^{p,s} = \frac{\rho_\omega^0}{2} \int_0^{\pi/2} d\theta \, \sin\theta \, \frac{1 - |r^{p,s}|^2}{2} \tag{126}$$

for the nonequilibrium problem and

$$\rho_\omega^{p,s} = \frac{\rho_\omega^0}{2} \left[1 + \mathrm{Re} \int_0^{\pi/2} d\theta \, \sin\theta \, \sin^2\theta (r^{p,s}) \exp\left(2ik_0 h \cos\theta\right) \right] \tag{127}$$

for the equilibrium problem. Figure 3 displays the normalized density of states of p- and s-polarized traveling waves of thermal fields produced by heated half-spaces of Al and ZnSe corresponding to the equilibrium and nonequilibrium problems in accordance with Eqns (126) and (127). The straight lines in all the graphs represent the density of states of p-polarized (*1*) and s-polarized (*2*) traveling waves of thermal fields produced by the heated half-spaces of Al and ZnSe in

cold surroundings. In this case, a permanent spatially uniform flow of waves is generated.

Quite a different picture is observed in the case of the equilibrium problem if an opposite flow of waves exists and, as a result, a characteristic picture arises (see Fig. 3) of the density of p-polarized states (solid curve) and s-polarized states (thin curve) near the surface. In the equilibrium case, the total density of states is modified at large distances into the equilibrium one. Figures 3a and 3b show the densities of states of traveling waves of thermal fields as functions of the distance h from the surface at the respective frequencies 5300 and 530 cm^{-1} for the Al and ZnSe half-spaces. Figure 3c displays a similar dependence for ZnSe at the frequency 212 cm^{-1}, which falls into the range of negative values of the dielectric constant.

It is clearly seen that in this case, as in the case of Al, the level of the flux of nonequilibrium radiation is very low, because the ZnSe crystal reflects in this spectral range quite well.
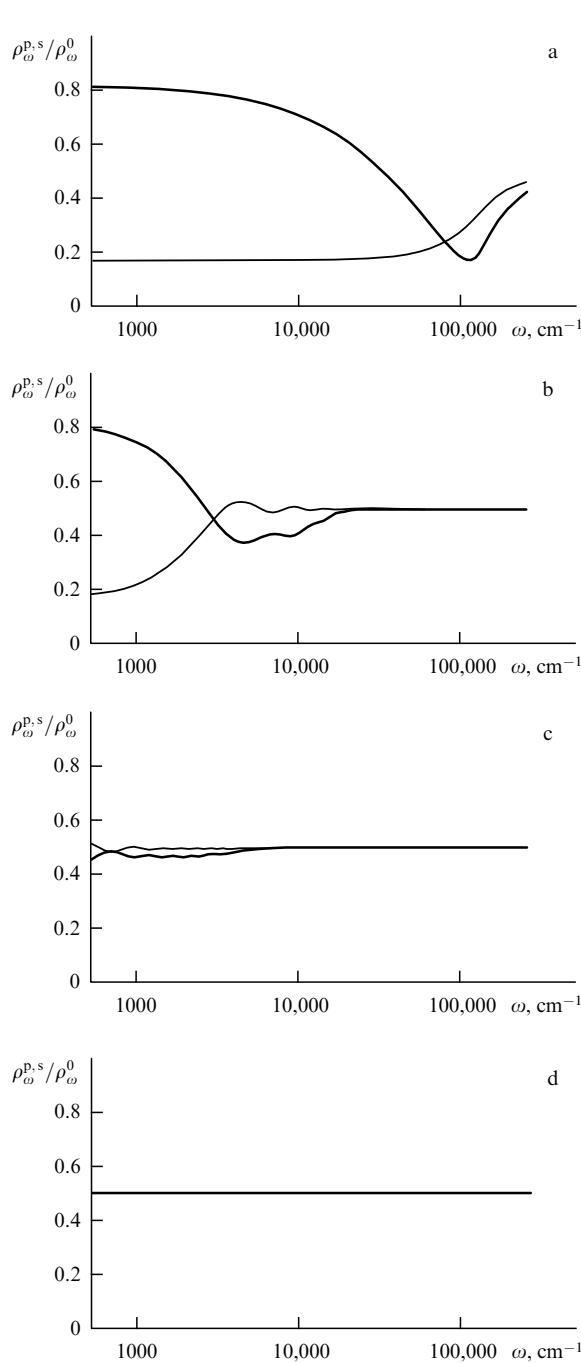
Figure 4. The normalized spectral density of p states (bold lines) and s states (thin lines) at the distance $h = 1 \times 10^{-7}$ cm (a), $h = 1 \times 10^{-4}$ cm (b), $h = 1 \times 10^{-3}$ cm (c), and $h = 1 \times 10^{-1}$ cm (d) for an Al half-space in the equilibrium problem.
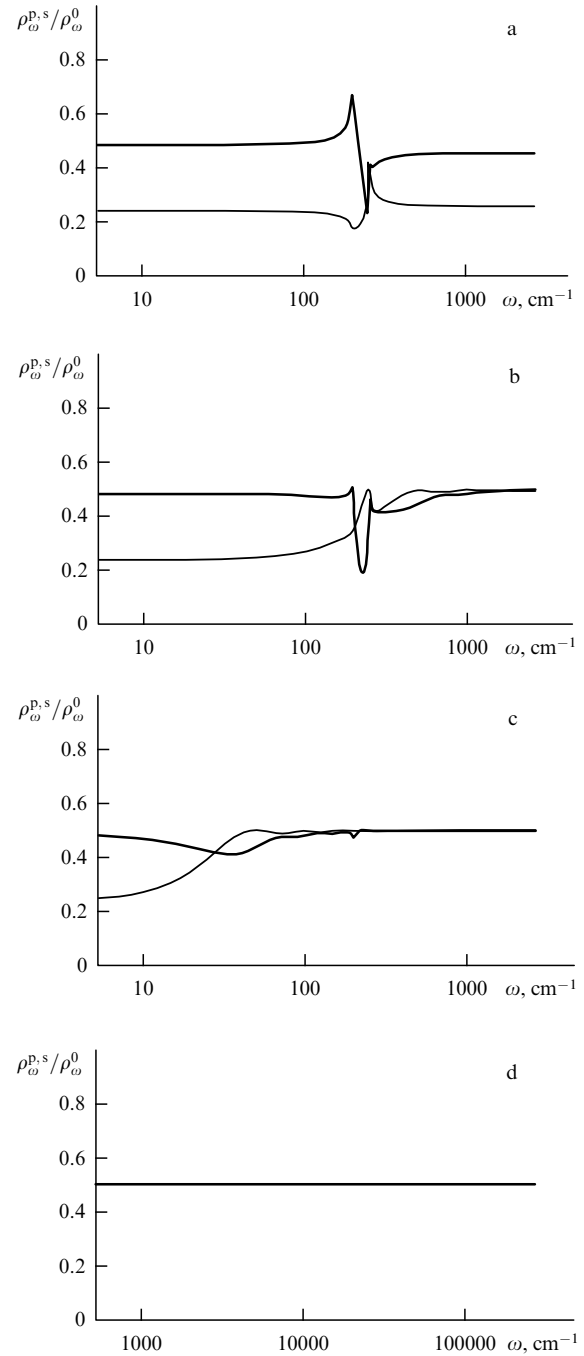
Figure 5. The normalized spectral density of p states (bold lines) and s states (thin lines) at the distance $h = 1 \times 10^{-7}$ cm (a), $h = 1 \times 10^{-3}$ cm (b), $h = 1 \times 10^{-2}$ cm (c), and $h = 1$ cm (d) for a ZnSe half-space in the equilibrium problem.

Thus, using the general theory of equilibrium thermal fields, we verified that at large distances from the surface in a closed volume formed by a material with any properties, an equilibrium density of states exists. This result was already obtained from simple thermodynamic considerations at the beginning of the 20th century in the course of the development of quantum theory.

Figures 4 and 5 display the transformation of the spectrum of traveling waves in a given spectral range depending on the distance h from the surface of Al and ZnSe half-spaces. The normalized spectral densities of p-polarized and s-polarized states are calculated via Eqn (127) for various values of the distance h to the surface, which are indicated in the figure captions.

It follows from Figs 4 and 5 that in the case of a thermodynamic equilibrium at large distances from the surface in a closed volume, a radiation spectrum that exactly corresponds to the Planck law is formed, as it should be in accordance with Eqn (127).

To conclude this section, we make one very important remark.

As is known [44], the pole of the Fresnel coefficient r^{p} determines the dispersion equation for surface eigenmodes of a boundary value problem. On the other hand, the same

coefficient is involved as a factor in the integrand for the spectral density and density of local states of the thermal field. In the equilibrium case, they are determined by the Fourier transform of the Green's function of the corresponding regular boundary value problem. Hence, the poles of the Green's function determine the spectrum of eigenstates of the thermostimulated field, and we note the complete analogy with the results of the modern theory of condensed media [24], according to which the poles of the Fourier transform of the Green's function of a solid body determine the spectrum of quasiparticle excitations. This indicates once again the fruitfulness of the formalism of Green's functions in modern physics.

4.1.5 Spatial correlation of thermal fields. In some problems, it is necessary to know not only the spectral characteristics of a random field at a chosen point of space but also the spatial correlation functions that characterize statistical relations between fields at different points of space. We give some of these explicitly expressed in terms of the Fresnel coefficients. The corresponding problem for dipoles located at points $\mathbf{r}_1 = (0, 0, h)$ and $\mathbf{r}_2 = (L, 0, h + H)$ has been solved in [7]. Using transformations (52) and (54), we obtain, for example, from [7, Eqn (8.5)],

$$
\langle E_x(\mathbf{r}_1) E_x^*(\mathbf{r}_2) \rangle = \frac{\pi u_{0\omega}}{2} \int_0^{\pi/2} d\theta \, \sin\theta \, \exp\left(i k_0 H \cos\theta\right)
$$

$$
\times \left[\left(\cos^2\theta \, \frac{1 - |r^p|^2}{2} + \frac{1 - |r^s|^2}{2} \right) J_0(k_0 L \sin\theta) \right.
$$

$$
\left. - \left(\cos^2\theta \, \frac{1 - |r^p|^2}{2} - \frac{1 - |r^s|^2}{2} \right) J_2(k_0 L \sin\theta) \right]
$$

$$
+ \frac{\pi u_{0\omega}}{2} \int_0^\infty dy \, \exp\left[-k_0 \, y(2h + H)\right]
$$

$$
\times \left[(y^2 \operatorname{Im} r^p + \operatorname{Im} r^s) J_0\left(k_0 L \sqrt{1 + y^2}\right) \right.
$$

$$
\left. - (y^2 \operatorname{Im} r^p - \operatorname{Im} r^s) J_2\left(k_0 L \sqrt{1 + y^2}\right) \right], \tag{128}
$$

$$
\langle E_y(\mathbf{r}_1) E_y^*(\mathbf{r}_2) \rangle = \frac{\pi u_{0\omega}}{2} \int_0^{\pi/2} d\theta \, \sin\theta \, \exp\left(i k_0 H \cos\theta\right)
$$

$$
\times \left[\left(\cos^2\theta \, \frac{1 - |r^p|^2}{2} + \frac{1 - |r^s|^2}{2} \right) J_0(k_0 L \sin\theta) \right.
$$

$$
\left. + \left(\cos^2\theta \, \frac{1 - |r^p|^2}{2} - \frac{1 - |r^s|^2}{2} \right) J_2(k_0 L \sin\theta) \right]
$$

$$
+ \frac{\pi u_{0\omega}}{2} \int_0^\infty dy \, \exp\left[-k_0 \, y(2h + H)\right]
$$

$$
\times \left[(y^2 \operatorname{Im} r^p + \operatorname{Im} r^s) J_0\left(k_0 L \sqrt{1 + y^2}\right) \right.
$$

$$
\left. + (y^2 \operatorname{Im} r^p - \operatorname{Im} r^s) J_2\left(k_0 L \sqrt{1 + y^2}\right) \right], \tag{129}
$$

$$
\langle E_z(\mathbf{r}_1) E_z^*(\mathbf{r}_2) \rangle = \pi u_{0\omega} \int_0^{\pi/2} d\theta \, \sin\theta \, \exp\left(i k_0 H \cos\theta\right)
$$

$$
\times \sin^2\theta \, \frac{1 - |r^p|^2}{2} \, J_0(k_0 L \sin\theta)
$$

$$
+ \pi u_{0\omega} \int_0^\infty dy \, \exp\left[-k_0 \, y(2h + H)\right]
$$

$$
\times (y^2 + 1) \operatorname{Im} r^p \, J_0\left(k_0 L \sqrt{1 + y^2}\right). \tag{130}
$$

In a similar way, we obtain spatial correlation functions for the components of a magnetic field and the cross functions of spatial correlation.

Just as in Section 4.1.3, we can find analytic expressions for the spatial correlation functions. Restricting ourselves to only very small distances from the surface, $2h + H \ll \left(k_0 |\sqrt{\varepsilon(\omega)}|\right)^{-1} \ll k_0^{-1}$, we consider the quasistatic approximation $c \to \infty$, neglecting the contribution from traveling waves. Because of the condition $y \gg 1$, we keep only the top powers of y in Eqns (128)–(130). From Eqn (128) for comparatively small lateral distances $k_0 L \approx k_0(2h + H) \ll 1$ between the chosen points, corresponding to the quasistatic approximation, we obtain

$$
\langle E_x(\mathbf{r}_1) E_x^*(\mathbf{r}_2) \rangle = \frac{\pi u_{0\omega} \varepsilon''(\omega)}{|\varepsilon(\omega) + 1|^2} \int_0^\infty dy \, \exp\left[-k_0 y(2h + H)\right]
$$

$$
\times y^2 \left[J_0(k_0 L y) - J_2(k_0 L y) \right]
$$

$$
\approx \frac{\pi u_{0\omega} \varepsilon''(\omega)}{|\varepsilon(\omega) + 1|^2} \, \frac{2\left[1 - 2L^2/(2h + H)^2\right]}{\left[k_0(2h + H)\right]^3 \left[1 + L^2/(2h + H)^2\right]^{5/2}}. \tag{131}
$$

Expression (131) transforms, of course, into the corresponding expression in (98) if $L = H = 0$. Normalizing (131) to the corresponding expression in (98), for the vertical

$$
CS_{x\perp}(H) \equiv \langle E_x(\mathbf{r}_1 = \{0, 0, h\}) \, E_x^*(\mathbf{r}_2 = \{0, 0, h + H\}) \rangle
$$

and lateral

$$
CS_{x\parallel}(L) \equiv \langle E_x(\mathbf{r}_1 = \{0, 0, h\}) \, E_x^*(\mathbf{r}_2 = \{L, 0, h\}) \rangle
$$

correlations separately, we obtain

$$
CS_{x\perp}(H) = \frac{1}{(1 + H/2h)^3}, \qquad CS_{x\parallel}(L) = \frac{1 - L^2/2h^2}{(1 + L^2/4h^2)^{5/2}}. \tag{132}
$$

From relation (130) in the same approximation, we have

$$
\langle E_z(\mathbf{r}_1) E_z^*(\mathbf{r}_2) \rangle
$$

$$
\approx \frac{\pi u_{0\omega} \varepsilon''(\omega)}{|\varepsilon(\omega) + 1|^2} \, \frac{2\left[1 - L^2/2(2h + H)^2\right]}{\left[k_0(2h + H)\right]^3 \left[1 + L^2/(2h + H)^2\right]^{5/2}}. \tag{133}
$$

Correspondingly,

$$
CS_{z\perp}(H) = \frac{1}{(1 + H/2h)^3}, \qquad CS_{z\parallel}(L) = \frac{1 - L^2/8h^2}{(1 + L^2/4h^2)^{5/2}}. \tag{134}
$$

It follows from (132) and (134) that in the quasistatic approximation, the spatial correlation length of the thermal field is of the order of the distance from the surface.

To investigate the spatial correlation of the field components at large lateral distances $(k_0 L \gg 1)$, we should use expressions for Bessel functions at large values of the argument, $k_0 L y \gg 1$. The corresponding expressions for spatial correlation functions can be obtained quite easily. We also note that using the approximations of the Bessel coefficients in (104) or (106), the characteristic scales of spatial correlation of fields can be estimated in other cases based on simpler analytic expressions.

In recent years, special attention has been given to the investigations of spatial correlations of thermal fields at the

frequencies of surface phonon–polaritons and plasmon–polaritons. Here, we mention only Refs [7, 38–40]. We note that the spatial correlation length at surface eigenfrequencies can reach several tens of wavelengths, substantially exceeding the correlation length of blackbody radiation, which is of the order of the Wien wavelength in free space. The physical reason for this is that the source of fields at the frequencies of surface excitations is a collective coherent excitation of a surface charge (induced polarization) or surface lattice vibrations at the interface between the media. Such a spatially correlated excitation on the surface occurs in spite of the spatial delta-correlated coupling of Langevin current sources in the bulk of the material, which are fictitious sources according to the fluctuation–dissipation theorem. The spatial coherence of the induced surface current is transferred to the induced random thermostimulated field at the eigenfrequencies of the problem.

We also mention the investigation of the degree of polarization of the near field of thermal sources in Ref. [41].

4.2 Spectral characteristics of the fluctuating electromagnetic field of a half-space covered with a plane-parallel layered structure

The next important problem directly connected with experimental investigations and practical applications is that of finding spectral characteristics of the thermostimulated field produced by a half-space covered with a film of a finite thickness. The nonequilibrium problem, in which the heated body radiates into cold surroundings, has been solved in [42] using the generalized Kirchhoff law; the film was assumed to have an arbitrary thickness h_f and its material and the material of the substrate were characterized by arbitrary complex dielectric constants and magnetic permeabilities. In addition, the authors of Ref. [42] determined the spectral properties of spatial derivatives of fields of all orders. The solution obtained in the case where the sample is in contact with the vacuum ($\varepsilon = \mu = 1$) is written, using the notation similar to that adopted in [7], as

$$\langle |E_z|^2 \rangle = \frac{i\Theta}{\pi c} \int_0^\infty dp\, p \exp\left[-(q + q^*)h\right] \frac{p^2}{k_0^2}\, S_{f\varepsilon} I_{f\varepsilon}^+\,, \quad (135)$$

$$\langle |E_x|^2 \rangle = \langle |E_y|^2 \rangle = \frac{i\Theta}{2\pi c} \int_0^\infty dp\, p \exp\left[-(q + q^*)h\right]$$

$$\times \left(\frac{|q|^2}{k_0^2} S_{f\varepsilon} I_{f\varepsilon}^+ + S_{f\mu} I_{f\mu}^+\right)\,, \quad (136)$$

where

$$S_{f\varepsilon} = k_0 \left(\frac{q_f^*}{\varepsilon_f^*} \frac{1 - r_{f1}^{p\,*} \exp\left(-2q_f^* h_f\right)}{1 + r_{f1}^{p\,*} \exp\left(-2q_f^* h_f\right)} - \right.$$

$$\left. - \frac{q_f}{\varepsilon_f} \frac{1 - r_{f1}^{p} \exp\left(-2q_f h_f\right)}{1 + r_{f1}^{p} \exp\left(-2q_f h_f\right)}\right) \left|q + \frac{q_f}{\varepsilon_f}\right|^{-2}\,, \quad (137)$$

$$I_{f\varepsilon}^+ = \left|\frac{1 + r_{f1}^{p} \exp\left(-2q_f h_f\right)}{1 - r_{f1}^{p} r_{f0}^{p} \exp\left(-2q_f h_f\right)}\right|^2\,, \quad (138)$$

r_{f1}^{p} and r_{f0}^{p} are the respective Fresnel coefficients of the film–substrate and film–vacuum interfaces, ε_f and ε_1 are the respective dielectric constants of the film and substrate, and $q_f = (p^2 - \varepsilon_f k_0^2)^{1/2}$. The functions $S_{f\mu}$ and $I_{f\mu}^+$ are obtained from Eqns (137) and (138) using the replacement $r^p \to r^s$.

Using permutations $S_{f\varepsilon} \vee S_{f\mu}$ and $I_{f\varepsilon}^+ \vee I_{f\mu}^+$ in Eqns (135) and (136), we obtain formulas for spectral densities of the magnetic field strength components ($\langle |H_z|^2 \rangle$ and $\langle |H_{x,y}|^2 \rangle$).

For the spectral density of the thermal field energy generated by the structure under consideration, we have

$$u_\omega = \frac{i\Theta}{4\pi^2 c} \int_0^\infty dp\, p \exp\left[-(q + q^*)h\right]$$

$$\times \left(\frac{|q|^2 + p^2}{k_0^2} + 1\right) \left(S_{f\varepsilon} I_{f\varepsilon}^+ + S_{f\mu} I_{f\mu}^+\right)\,. \quad (139)$$

Here, as earlier, we consider nonmagnetic materials ($\mu_f = \mu_1 = 1$).

We write the solutions obtained in terms of only the Fresnel coefficients. For traveling waves ($q = -q^*$), introducing the notation $p/k_0 = \sin\theta$, we immediately obtain

$$\frac{k_0 q_f / \varepsilon_f}{|q + q_f / \varepsilon_f|^2} = \frac{r_{f0} - r_{f0}^* - \left(1 - |r_{f0}|^2\right)}{4i\cos\theta}\,. \quad (140)$$

Substituting Eqn (140) and the complex conjugated equation in (137) and taking into account that $r_{f0} = -r_{0f}$, we obtain

$$S_{f\varepsilon} I_{f\varepsilon}^+ = -\frac{i\left(1 - |r_{0f1}^{p}|^2\right)}{2\cos\theta}\,, \quad (141)$$

where r_{0f1}^{p} is the Fresnel coefficient of the vacuum–film–substrate three-layer structure:

$$r_{0f1}^{p} = \frac{r_{0f}^{p} + r_{f1}^{p} \exp\left(-2q_f h_f\right)}{1 + r_{0f}^{p} r_{f1}^{p} \exp\left(-2q_f h_f\right)}\,. \quad (142)$$

For a quasistationary field ($q = q^*$), using the notation $y = q/k_0$, we obtain

$$\frac{k_0 q_f / \varepsilon_f}{|q + q_f / \varepsilon_f|^2} = \frac{\left(1 - |r_{0f}|^2\right) - (r_{0f} - r_{0f}^*)}{4y}\,. \quad (143)$$

Substituting (143) in (137), we find

$$S_{f\varepsilon} I_{f\varepsilon}^+ = -\frac{i\,\mathrm{Im}\, r_{0f1}^{p}}{y}\,. \quad (144)$$

Quite similarly, we can obtain relations for the s waves using the replacements $S_{f\varepsilon} \to S_{f\mu}$, $I_{f\varepsilon}^+ \to I_{f\mu}^+$, and $r_{0f1}^{p} \to r_{0f1}^{s}$ in Eqns (141) and (144).

For the subsequent consideration, it is useful to change notation, in particular, to use r_{123}^{p} instead of r_{0f1}^{p} for the three-layer structure under consideration. Correspondingly, in the problem of a four-layer structure, we have coefficients r_{1234}^{p}, etc.

Substitution of Eqns (141) and (144) in formulas for the spectral components (135) and (136) allows writing these in the form that is nominally identical to Eqns (55), (56), (67), and (68):

$$\langle |E_x|^2 \rangle^{Pr} = \langle |E_y|^2 \rangle^{Pr}$$

$$= \frac{\pi u_{0\omega}}{2} \int_0^{\pi/2} d\theta\, \sin\theta \left(\cos^2\theta\, \frac{1 - |r_{123}^{p}|^2}{2} + \frac{1 - |r_{123}^{s}|^2}{2}\right)\,, \quad (145)$$

$$\langle |E_z|^2 \rangle^{Pr} = \pi u_{0\omega} \int_0^{\pi/2} d\theta\, \sin\theta\, \sin^2\theta\, \frac{1 - |r_{123}^{p}|^2}{2}\,, \quad (146)$$

$$\langle |E_x|^2 \rangle^{\mathrm{Ev}} = \langle |E_y|^2 \rangle^{\mathrm{Ev}}$$

$$= \frac{\pi u_{0\omega}}{2} \int_0^\infty \mathrm{d}y \, \exp\left(-2k_0 hy\right) \left(y^2 \, \mathrm{Im}\, r_{123}^{\mathrm{p}} + \mathrm{Im}\, r_{123}^{\mathrm{s}}\right), \quad (147)$$

$$\langle |E_z|^2 \rangle^{\mathrm{Ev}} = \pi u_{0\omega} \int_0^\infty \mathrm{d}y \, \exp\left(-2k_0 hy\right)(y^2 + 1) \, \mathrm{Im}\, r_{123}^{\mathrm{p}} . \quad (148)$$

For the spectral densities of the magnetic field components, expressions that are quite similar to (57), (58), (69), and (70) are obtained, but with the Fresnel coefficients r_{123}^{p} and r_{123}^{s} corresponding to a three-layer structure. A similar replacement should be made in the formulas for other spectral characteristics. For example, the formula for the spectral energy density of the thermal field generated by a coated half-space is written as

$$u_\omega = u_\omega^{\mathrm{Pr}} + u_\omega^{\mathrm{Ev}}$$

$$= \frac{u_{0\omega}}{2} \Bigg[\int_0^{\pi/2} \mathrm{d}\theta \, \sin\theta \, \frac{\left(1 - |r_{123}^{\mathrm{p}}|^2\right) + \left(1 - |r_{123}^{\mathrm{s}}|^2\right)}{2}$$

$$+ \int_0^\infty \mathrm{d}y \, \exp\left(-2k_0 hy\right)(y^2 + 1)(\mathrm{Im}\, r_{123}^{\mathrm{p}} + \mathrm{Im}\, r_{123}^{\mathrm{s}}) \Bigg], \quad (149)$$

where

$$r_{123}^{\mathrm{p,s}} = \frac{r_{12}^{\mathrm{p,s}} + r_{23}^{\mathrm{p,s}} \exp\left(-2q_2 h_2\right)}{1 + r_{12}^{\mathrm{p,s}} r_{23}^{\mathrm{p,s}} \exp\left(-2q_2 h_2\right)} ; \quad (150)$$

here, in the new notation, h_2 is the thickness of the film, $q_2 = (p^2 - \varepsilon_2 k_0^2)^{1/2}$, ε_2 is the dielectric constant of the film material, ε_3 is the dielectric constant of the half-space material, and $\varepsilon_1 = 1$ is the dielectric constant of the vacuum. A similar procedure of replacing the Fresnel coefficients of a two-layer film by the coefficients of a three-layer one was used in [32] for solving a somewhat different problem. However, we emphasize that we obtain an exact result in our problem.

Analogous formulas can be derived based on the results in Ref. [43] for spectral densities of the thermostimulated field components and for the spectral energy density in the case of a four-layer structure consisting of two films with different, in the general case, thicknesses h_2 and h_3, whose material is characterized by different dielectric constants ε_2 and ε_3, applied onto a half-space with a dielectric constant ε_4. In this case, all properties are described by Fresnel coefficients $r_{1234}^{\mathrm{p,s}}$ of the vacuum (1)–film (2)–film (3)–substrate (4) structure as follows:

$$r_{1234}^{\mathrm{p,s}} = \frac{r_{12}^{\mathrm{p,s}} + r_{234}^{\mathrm{p,s}} \exp\left(-2q_2 h_2\right)}{1 + r_{12}^{\mathrm{p,s}} r_{234}^{\mathrm{p,s}} \exp\left(-2q_2 h_2\right)} , \quad (151)$$

where

$$r_{234}^{\mathrm{p,s}} = \frac{r_{23}^{\mathrm{p,s}} + r_{34}^{\mathrm{p,s}} \exp\left(-2q_3 h_3\right)}{1 + r_{23}^{\mathrm{p,s}} r_{34}^{\mathrm{p,s}} \exp\left(-2q_3 h_3\right)} . \quad (152)$$

For a five-layer structure, the Fresnel coefficients $r_{12345}^{\mathrm{p,s}}$ of the vacuum (1)–film (2)–film (3)–film (4)–substrate (5) structure are written as

$$r_{12345}^{\mathrm{p,s}} = \frac{r_{12}^{\mathrm{p,s}} + r_{2345}^{\mathrm{p,s}} \exp\left(-2q_2 h_2\right)}{1 + r_{12}^{\mathrm{p,s}} r_{2345}^{\mathrm{p,s}} \exp\left(-2q_2 h_2\right)} , \quad (153)$$

where

$$r_{2345}^{\mathrm{p,s}} = \frac{r_{23}^{\mathrm{p,s}} + r_{345}^{\mathrm{p,s}} \exp\left(-2q_3 h_3\right)}{1 + r_{23}^{\mathrm{p,s}} r_{345}^{\mathrm{p,s}} \exp\left(-2q_3 h_3\right)} , \quad (154)$$

$$r_{345}^{\mathrm{p,s}} = \frac{r_{34}^{\mathrm{p,s}} + r_{45}^{\mathrm{p,s}} \exp\left(-2q_4 h_4\right)}{1 + r_{34}^{\mathrm{p,s}} r_{45}^{\mathrm{p,s}} \exp\left(-2q_4 h_4\right)} . \quad (155)$$

Thus, the main structural element in the formula for the Fresnel coefficient of a multilayer system is the Fresnel coefficient of a three-layer structure of form (150). Indeed, any multilayer structure is one of the vacuum (1)–film (2)–complex half-space (3) type, where the complex half-space is the usual uniform half-space coated with a multilayer structure that is in turn a kind of three-layer structure, etc.

It seems quite obvious that an n-layer structure with an arbitrary number of layers is a new half-space whose optical properties are characterized by a composite Fresnel coefficient of the general form

$$r_{123\ldots n}^{\mathrm{p,s}} = \frac{r_{12}^{\mathrm{p,s}} + r_{234\ldots n}^{\mathrm{p,s}} \exp\left(-2q_2 h_2\right)}{1 + r_{12}^{\mathrm{p,s}} r_{234\ldots n}^{\mathrm{p,s}} \exp\left(-2q_2 h_2\right)} , \quad (156)$$

where

$$r_{234\ldots n}^{\mathrm{p,s}} = \frac{r_{23}^{\mathrm{p,s}} + r_{34\ldots n}^{\mathrm{p,s}} \exp\left(-2q_3 h_3\right)}{1 + r_{23}^{\mathrm{p,s}} r_{34\ldots n}^{\mathrm{p,s}} \exp\left(-2q_3 h_3\right)} , \quad (157)$$

$$r_{345\ldots n}^{\mathrm{p,s}} = \frac{r_{34}^{\mathrm{p,s}} + r_{45\ldots n}^{\mathrm{p,s}} \exp\left(-2q_4 h_4\right)}{1 + r_{34}^{\mathrm{p,s}} r_{45\ldots n}^{\mathrm{p,s}} \exp\left(-2q_4 h_4\right)} , \quad (158)$$

$$\cdots$$

$$r_{(n-3)(n-2)(n-1)n}^{\mathrm{p,s}} = \frac{r_{(n-3)(n-2)}^{\mathrm{p,s}} + r_{(n-2)(n-1)n}^{\mathrm{p,s}} \exp\left(-2q_{n-2} h_{n-2}\right)}{1 + r_{(n-3)(n-2)}^{\mathrm{p,s}} r_{(n-2)(n-1)n}^{\mathrm{p,s}} \exp\left(-2q_{n-2} h_{n-2}\right)} , \quad (159)$$

$$r_{(n-2)(n-1)n}^{\mathrm{p,s}} = \frac{r_{(n-2)(n-1)}^{\mathrm{p,s}} + r_{(n-1)n}^{\mathrm{p,s}} \exp\left(-2q_{n-1} h_{n-1}\right)}{1 + r_{(n-2)(n-1)}^{\mathrm{p,s}} r_{(n-1)n}^{\mathrm{p,s}} \exp\left(-2q_{n-1} h_{n-1}\right)} , \quad (160)$$

$q_j = (p^2 - \varepsilon_j k_0^2)^{1/2}$, ε_j is the dielectric constant of the jth layer, and h_j is its thickness. Therefore, all formulas for the spectral densities of the thermal field generated by a plane-parallel layered medium look quite similar, with the only difference that they contain different composite Fresnel coefficients $r_{123\ldots n}^{\mathrm{p,s}}$, for example,

$$\langle |E_x|^2 \rangle^{\mathrm{Pr}} = \langle |E_y|^2 \rangle^{\mathrm{Pr}}$$

$$= \frac{\pi u_{0\omega}}{2} \int_0^{\pi/2} \mathrm{d}\theta \, \sin\theta \left(\cos^2\theta \, \frac{1 - |r_{123\ldots n}^{\mathrm{p}}|^2}{2} + \frac{1 - |r_{123\ldots n}^{\mathrm{s}}|^2}{2} \right), \quad (161)$$

$$\langle |E_z|^2 \rangle^{\mathrm{Pr}} = \pi u_{0\omega} \int_0^{\pi/2} \mathrm{d}\theta \, \sin\theta \, \sin^2\theta \, \frac{1 - |r_{123\ldots n}^{\mathrm{p}}|^2}{2} , \quad (162)$$

$$\langle |E_x|^2 \rangle^{\mathrm{Ev}} = \langle |E_y|^2 \rangle^{\mathrm{Ev}}$$

$$= \frac{\pi u_{0\omega}}{2} \int_0^\infty \mathrm{d}y \, \exp\left(-2k_0 hy\right) \left(y^2 \, \mathrm{Im}\, r_{123\ldots n}^{\mathrm{p}} + \mathrm{Im}\, r_{123\ldots n}^{\mathrm{s}}\right), \quad (163)$$

$$\langle |E_z|^2 \rangle^{\mathrm{Ev}} = \pi u_{0\omega} \int_0^\infty \mathrm{d}y \, \exp\left(-2k_0 hy\right)(y^2 + 1) \, \mathrm{Im}\, r_{123\ldots n}^{\mathrm{p}} , \quad (164)$$

and similarly for the spectral densities of the magnetic field components.

The spectral density of the thermal field energy in the general case is written as

$$u_\omega = u_\omega^{Pr} + u_\omega^{Ev}$$

$$= \frac{u_{0\omega}}{2} \left[\int_0^{\pi/2} d\theta \, \sin\theta \, \frac{\left(1 - |r_{123\ldots n}^p|^2\right) + \left(1 - |r_{123\ldots n}^s|^2\right)}{2} \right.$$

$$\left. + \int_0^\infty dy \, \exp\left(-2k_0 hy\right)(y^2 + 1)(\mathrm{Im}\, r_{123\ldots n}^p + \mathrm{Im}\, r_{123\ldots n}^s) \right].$$

$$(165)$$

We recall that the denominators of Fresnel coefficients (150), (151), (153), and (156) determine the dispersion relations for the eigenmodes of the relevant structures. For example, from the condition of the vanishing [44] of the denominator of the simple three-layer structure in Eqn (150), the following well-known dispersion relation [45] is obtained:

$$r_{12}^{p,s} r_{23}^{p,s} \exp\left(-2q_2 h_2\right) = -1. \qquad (166)$$

As is known [46, 47], in the case of p waves, Eqn (166) describes both surface and waveguide excitations in a three-layer structure. For example, in the frequency range where $q_2 = i\xi$, neglecting dissipation, we can write the respective dispersion relations for the p-polarized and s-polarized waveguide modes:

$$\xi h_2 = \arctan \frac{\varepsilon_2 q_3}{\varepsilon_3 \xi} + \arctan \frac{\varepsilon_2 q_1}{\xi} + l\pi, \quad l = 0, 1, 2 \ldots \quad (167)$$

$$\xi h_2 = \arctan \frac{q_3}{\xi} + \arctan \frac{q_1}{\xi} + l\pi, \quad l = 0, 1, 2 \ldots . \quad (168)$$

It follows from Eqns (167) and (168) that there exists some film thickness $h_{2\,min}$ that is critical for the excitation of these modes. Indeed, assuming, for example, that $l = 0$ in (167) and that $h_{2\,min}$ is reached at $\xi_{max} = (\varepsilon_2 k_0^2 - p_{min}^2)^{1/2}$, where $p_{min} = k_0$, we find

$$h_{2\,min} = \frac{c}{\omega \left[\varepsilon_2(\omega) - 1\right]^{1/2}} \arctan \left(\frac{\varepsilon_2(\omega)}{\varepsilon_3(\omega)} \sqrt{\frac{1 - \varepsilon_3(\omega)}{\varepsilon_2(\omega) - 1}} \right).$$

$$(169)$$

For a four-layer structure, the dispersion equation follows from Eqns (151) and (152):

$$r_{12}^{p,s} \frac{r_{23}^{p,s} + r_{34}^{p,s} \exp\left(-2q_3 h_3\right)}{1 + r_{23}^{p,s} r_{34}^{p,s} \exp\left(-2q_3 h_3\right)} \exp\left(-2q_2 h_2\right) = -1. \quad (170)$$

The spectral density of states of the thermal electromagnetic field is introduced just as in Section 4.1.4. For example, for a three-layer structure, the density of the p and s states of the quasistationary part of the spectrum is determined by formulas (124), in which the appropriate Fresnel coefficient (150) must be used:

$$\rho_\omega^{p,s} = \frac{\rho_\omega^0}{2} \int_0^\infty dy \, \exp\left(-2k_0 hy\right)(y^2 + 1) \, \mathrm{Im}\, r_{123}^{p,s}. \quad (171)$$

To graphically illustrate the results obtained, we numerically calculated the density of states for the vacuum (1)–ZnSe film (2)–Al half-space (3) structure using Eqns (150)
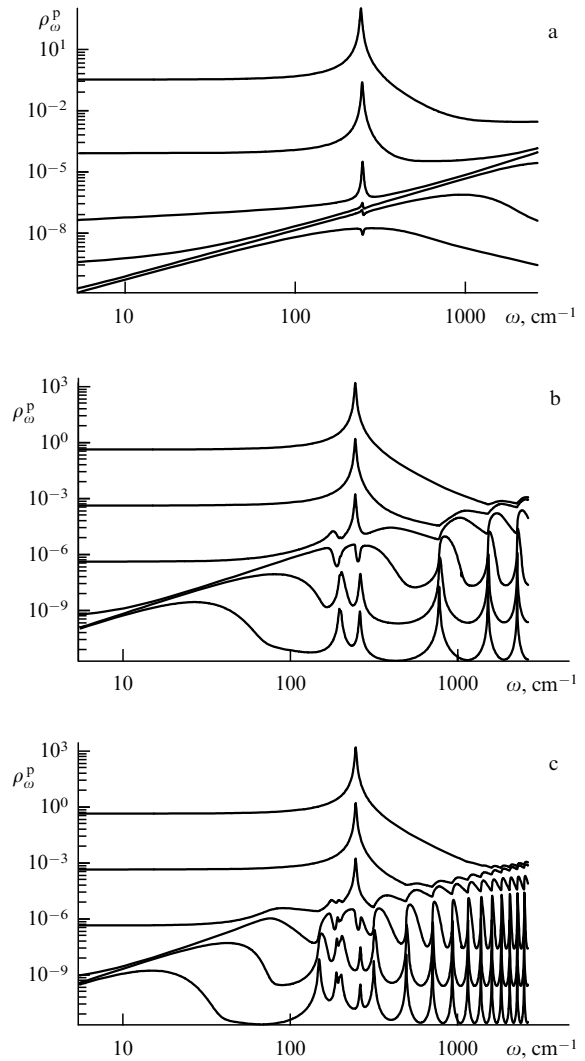

Figure 6. Spectral density of p states of the thermostimulated field of a vacuum–ZnSe-film–Al-half-space structure as a function of frequency at various distances h from the surface of the structure depending on the film thicknesses h_2: 1×10^{-6} cm (a), 3×10^{-4} cm (b), and 1×10^{-3} cm (c). The curves corresponding to the smallest distance from the surface ($h = 1 \times 10^{-6}$ cm) are uppermost in the graphs; the curves corresponding to the largest distance from the surface $h = 1 \times 10^{-1}$ cm are lowermost; between these, curves corresponding to $h = 1 \times 10^{-5}$ cm, 1×10^{-4} cm, 1×10^{-3} cm, and 1×10^{-2} cm are located from top down in the order of increasing distance.

and (171). Figure 6 displays the spectral density of p states depending on the frequency of the thermostimulated field generated by such a structure, at different distances h from the surface at various thicknesses of the film, namely, at $h_2 = 1 \times 10^{-6}$ cm, $h_2 = 3 \times 10^{-4}$ cm, and $h_2 = 1 \times 10^{-3}$ cm (Figs 6a, 6b, and 6c, respectively). The curves corresponding to the smallest distance from the surface ($h = 1 \times 10^{-6}$ cm) are uppermost in the graphs; the curves corresponding to the largest distance from the surface are lowermost; between these, curves corresponding to $h = 1 \times 10^{-5}$ cm, 1×10^{-4} cm, 1×10^{-3} cm, and 1×10^{-2} cm are located from top down in the order of increasing distance. Compared to the spectrum of states for the half-space, the spectrum is much more complex. Along with a strong resonance at small distances, corresponding to the Coulomb polariton (243 cm^{-1}), resonances related to waveguide modes are

observed. These resonances appear when the film thickness exceeds some minimum thickness at a fixed frequency. As the film thickness increases, the waveguide states fill the entire spectral range; they are located both above the longitudinal phonon–polariton frequency and below the transverse phonon–polariton frequency, which determine the region of negative values of the dielectric constant (200–250 cm^{-1}). This convincingly follows from Fig. 6. We note that at some distances from the surface, the waveguide resonances become equal to surface resonances in magnitude.

For convenience, we obtain approximate expressions for the spectral density of the field or for the spectral density of states. Here, we consider only the quasistatic approximation at $y \gg |\sqrt{\varepsilon_{2,3}}| \gg 1$, which corresponds, as we saw, to small distances from the surface, $k_0 h \ll |\sqrt{\varepsilon_{2,3}}|^{-1} \ll 1$. In this case, restricting ourselves to only the top powers of y in (171), we obtain

$$\rho_\omega^p \approx \frac{\rho_\omega^0}{2} \int_0^\infty dy \, \exp(-2k_0 h y) \, y^2 \, \mathrm{Im} \, r_{123}^p, \qquad (172)$$

where the Fresnel coefficient in the approximation in question has the form

$$r_{123}^p \approx \left[\frac{\varepsilon_2 - 1}{\varepsilon_2 + 1} + \frac{\varepsilon_3 - \varepsilon_2}{\varepsilon_3 + \varepsilon_2} \exp(-2k_0 h_2 y) \right]$$
$$\times \left[1 + \frac{\varepsilon_2 - 1}{\varepsilon_2 + 1} \frac{\varepsilon_3 - \varepsilon_2}{\varepsilon_3 + \varepsilon_2} \exp(-2k_0 h_2 y) \right]^{-1}. \qquad (173)$$

Hence, assuming that $k_0 h_2 y \gg 1$, we have

$$r_{123}^p \approx \frac{\varepsilon_2 - 1}{\varepsilon_2 + 1} + \frac{\varepsilon_3 - \varepsilon_2}{\varepsilon_3 + \varepsilon_2} \frac{4\varepsilon_2}{(\varepsilon_2 + 1)^2} \exp(-2k_0 h_2 y). \qquad (174)$$

Substituting (174) in (172), we integrate to obtain

$$\rho_\omega^p \approx \frac{\rho_\omega^0}{2} \left\{ \frac{1}{4} \, \mathrm{Im} \, \frac{\varepsilon_2 - 1}{\varepsilon_2 + 1} (k_0 h)^{-3} \right.$$
$$\left. + \mathrm{Im} \left(\frac{\varepsilon_3 - \varepsilon_2}{\varepsilon_3 + \varepsilon_2} \frac{\varepsilon_2}{(\varepsilon_2 + 1)^2} \right) [k_0(h + h_2)]^{-3} \right\}. \qquad (175)$$

It can be seen from Eqn (175), that along with a low-frequency Coulomb resonance (~ 243 cm^{-1}), whose frequency is a root of the equation $\varepsilon_2(\omega) = -1$, the density of states involves a well-pronounced high-frequency Coulomb resonance related to the film (2)–substrate (3) interface (at $\sim 46{,}000$ cm^{-1}), which is determined by the equation $\varepsilon_3(\omega) = -\varepsilon_2(\omega)$. The frequency of this resonance is lower than that of the Coulomb resonance of the vacuum (1)–substrate (3) interface ($\sim 84{,}700$ cm^{-1}), which is a root of the equation $\varepsilon_3(\omega) = -1$. In Fig. 6, the high-frequency region is not shown. It follows from Eqn (175) that in the absence of a foreign film, i.e., at $\varepsilon_2 = \varepsilon_3$, the second term in the right-hand side of Eqn (175) vanishes.

We note that the approximate expressions for the Fresnel coefficient $r_{123}^{p,s}$ of a three-layer structure can be obtained in various approximations by expanding it into a power series in y; this allows finding analytic expressions for the spectral density or for the density of states of a quasistationary thermal field for a wide range of distances to the surface, film thicknesses, and electromagnetic properties of the materials.
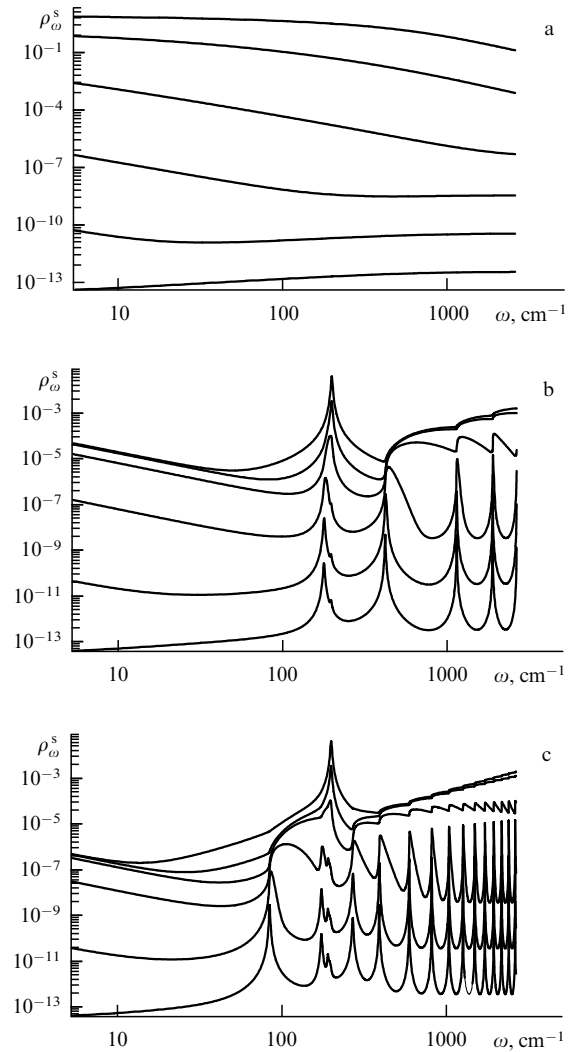

Figure 7. Spectral density of s states of the thermostimulated field of a vacuum–ZnSe-film–Al-half-space structure as a function of frequency at various distances h from the surface of the structure depending on the film thickness h_2. The distances from the surface and the film thicknesses are the same as in Fig. 6.

Figure 7 demonstrates the density of electromagnetic s states of a thermostimulated field generated by an Al half-space coated with a ZnSe film as a function of the frequency. The calculations were performed for the same set of distances from the surface and the same thicknesses of the ZnSe film as in Fig. 6. The results of numerical calculations show how the spectrum is transformed with increasing the film thickness. For thicknesses much greater than the wavelength of the range under investigation, the spectrum of the structure transforms into the spectrum of the thermal field of the ZnSe half-space.

When studying a structure of an Al film on a ZnSe half-space with the same geometric parameters, we see that in the chosen spectral range, all resonance features in the density of states disappear because the metal efficiently shields the substrate.

The most important conclusion that follows from the analysis of the spectral density of states of a thermostimulated field is the existence of clearly pronounced characteristic resonances whose frequencies are unambiguously related to the eigenmodes of the system under consideration.

4.3 Results of the experimental study of resonance states of the thermally stimulated electromagnetic field of semiconductor films on metallic substrates

Optical methods of studying solids yield rich information on the electronic (excitonic, plasmonic) and vibrational (phononic) excitations of both the volume and surface of crystal layers. In the last three decades, a number of new results have appeared, which have enriched the existing concepts on the interaction of electromagnetic radiation with condensed media. An important role in the experimental investigations is played by the study of spectra of the thermostimulated radiation of solids [48–60]. In these works, the experimental methods concerning the registration of the thermal radiation of samples were also described.

The investigation of the optical properties of solids is based on measurements of the spectral intensity of light emitted from a sample. When measuring radiation spectra, a sample is placed in a thermostat instead of a standard source of infrared (IR) radiation such that the radiation angle can easily be varied by rotating the thermostat without readjusting the spectrometer. In this case, no rotating or moving mirrors are necessary to measure the angular dependence of radiation, i.e., it is much simpler to investigate polaritons by detecting thermal radiation than by using reflection spectra. The procedure of calibrating devices and the details of measurements have been described in sufficient detail in Refs [54, 55]. Figure 8 displays spectra of thermostimulated p-polarized radiation of ZnSe films of various thicknesses on an aluminum mirror and schematically shows the density of single-phonon states (for optical phonons) of a ZnSe single crystal. In addition to the experimental spectra, Fig. 8 also depicts calculated spectra that were obtained without any fitting parameter. The radiation spectra were calculated via the Kirchhoff law using Fresnel coefficient (150) for a three-layer structure and assuming that the energy reflection coefficient is $R = |r_{123}^{\mathrm{p}}|^2$. In this case, to assign the dielectric functions for Al and ZnSe film, the Drude formula and oscillatory model (125) with the corresponding parameters were used.

The experimental data shown in Fig. 8 yield several interesting results.

First, p-polarized light radiation is observed in thin films not only at the transverse optical phonon frequency but also at the frequency of the longitudinal optical phonon, which is forbidden by the selection rules for bulk crystals [50–55]. Second, in a bulk crystal, the absorption and emission of light occur not at the frequencies of single-phonon states of the crystal belonging to the range $\omega_{\mathrm{TO}} < \omega < \omega_{\mathrm{LO}}$ but in the range $\omega_{\mathrm{ex}} > \omega > \omega_{\mathrm{LO}}$ and $\omega < \omega_{\mathrm{TO}}$, where no single-phonon states exist [54, 55]. Third, the radiation in the ranges $\omega_{\mathrm{ex}} > \omega > \omega_{\mathrm{LO}}$ and $\omega < \omega_{\mathrm{TO}}$ proves to be close to unity, i.e., close to that of the ideal black body.

The investigation of nonradiative states or quasistationary thermal fields that are inseparably linked with the field source (to the right of the light line in the graph of the frequency dependence on the wave number) has some specific features. The useful signal is typically detected in experiments in the far radiation range. This concerns experiments with a prism of a frustrated total internal reflection over the sample, with a diffraction grating on the sample surface, with a near-field optical microscope, etc. Therefore, the information comes from traveling waves (in the far-field zone), which are the product of the decay of a quasistationary state. In fact, a transformation of fields
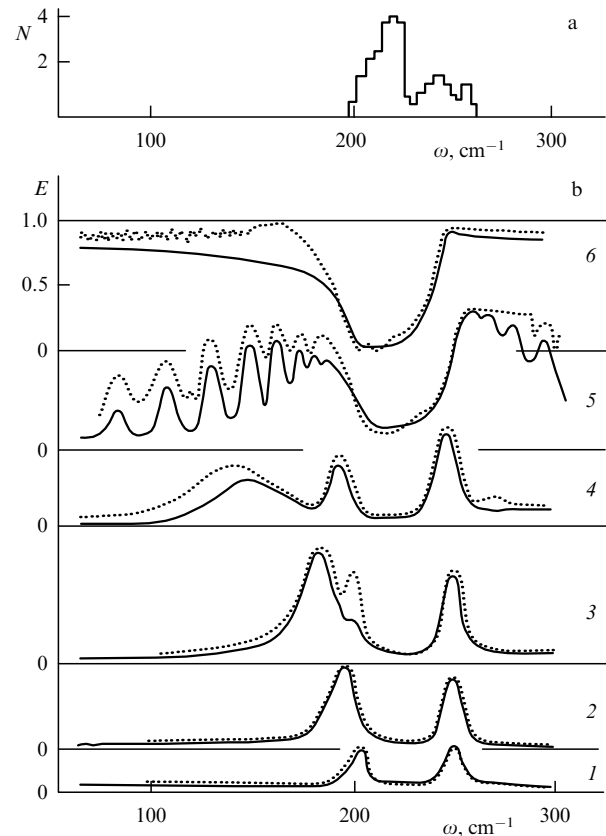

Figure 8. (a) Density of optical phonons in ZnSe. (b) Spectra of p-polarized thermostimulated radiation of ZnSe films of various thicknesses d on an Al substrate (normalized to the blackbody radiation spectrum): $d = 1\ \mu\mathrm{m}$ (*1*), $d = 2\ \mu\mathrm{m}$ (*2*), $d = 3\ \mu\mathrm{m}$ (*3*), $d = 5\ \mu\mathrm{m}$ (*4*), $d = 56\ \mu\mathrm{m}$ (*5*), and $d = 5\ \mathrm{mm}$ (*6*). Points correspond to experimental data; solid curves, to the results of calculations without any fitting parameter.

occurs in the experiment. In this sense, the methods of optical measurements resemble the experimental methods of atomic physics or the physics of cosmic rays, where the information on a particle is obtained from decay products or from the results of the destruction of a medium produced by the particle under study.

We emphasize that spectral investigations of thermostimulated fields allow studying resonances in a system under conditions of an indestructible experiment. The positions of the resonances in the radiation spectrum, their widths, and their dependence on the parameters that are controlled during the experiment, such as the temperature, the composition and conditions of the sample preparation, the radiation angle detection, and the magnitude of external fields, all this information is a basis for the explanation of the structure of matter and the laws of interaction of fields of various natures.

A detailed analysis of vibrational eigenmodes of thin films on metallic substrates [50, 52–55] shows that the probabilities of absorption of a p-polarized field with a p-polarized vibration of the surface mode of the first ($\omega_2 \approx \omega_{\mathrm{LO}}$) and second ($\omega_1 \approx \omega_{\mathrm{TO}}$) types are respectively proportional to $k_0 d \sin^2 \varphi$ and $(k_0 d)^3$. A direct calculation of the thermostimulated radiation intensity of a ZnSe film on Al at a given solid angle in a fixed direction using the Kirchhoff formula and the Fresnel coefficient for a three-layer structure gives qualitatively the same result (see Fig. 9). We recall that for
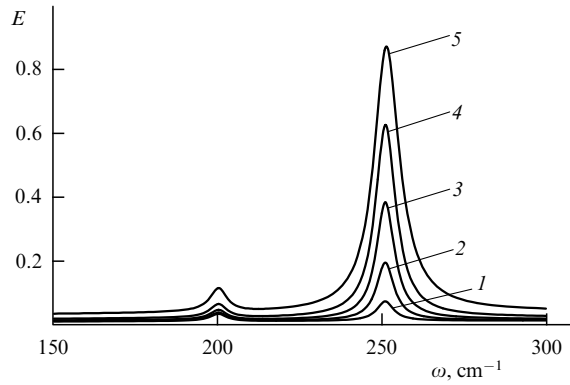
Figure 9. Normalized spectrum of p-polarized thermostimulated radiation of a ZnSe film 0.4 μm thick on an aluminum mirror in the spectral range 150–300 cm^{-1} as a function of the angle from the normal to the sample: 15° (1), 30° (2), 45° (3), 60° (4), and 75° (5).
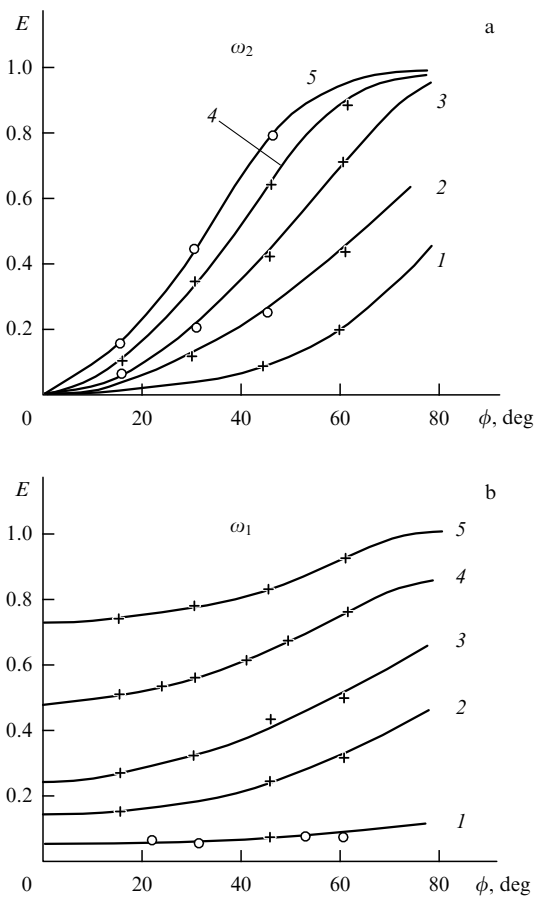

Figure 10. Peak emissivity of ZnSe films on Al for the emission bands $v_2 \approx v_{LO} = 250$ cm^{-1} (a) and $v_1 \approx v_{TO} = 200$ cm^{-1} (b) as a function of the emission angle at different film thicknesses d: 0.3 (1), 0.5 (2), 1 (3), 1.6 (4), and 2 μm (5). Various symbols correspond to experimental values; solid curves, to calculation results.

Figure 11. Peak emissivity of ZnSe films on Al as a function of the film thickness for the emission bands $v_2 \approx v_{LO} = 250$ cm^{-1} (a) and $v_1 \approx v_{TO} = 200$ cm^{-1} (b) at various emission angles: $\varphi = 15°$ (1), $\varphi = 30°$ (2), $\varphi = 45°$ (3), and $\varphi = 60°$ (4). The symbols correspond to experimental values; solid curves, to calculation results.

ZnSe, the longitudinal and transverse optical phonon frequencies are $\omega_{LO} \approx 250$ cm^{-1} and $\omega_{TO} \approx 200$ cm^{-1}.

Figure 10 displays the peak dependences of radiation of a ZnSe film on Al depending on the angle between the normal to the sample surface and the direction of observation at various film thicknesses. Figure 10a shows the dependence for the mode $\omega_2 \approx 250$ cm^{-1} and Fig. 10b, for the mode $\omega_1 \approx 200$ cm^{-1}. Figure 11 demonstrates the corresponding dependences on the film thickness at fixed radiation angles. Very good agreement with the calculation results is observed.
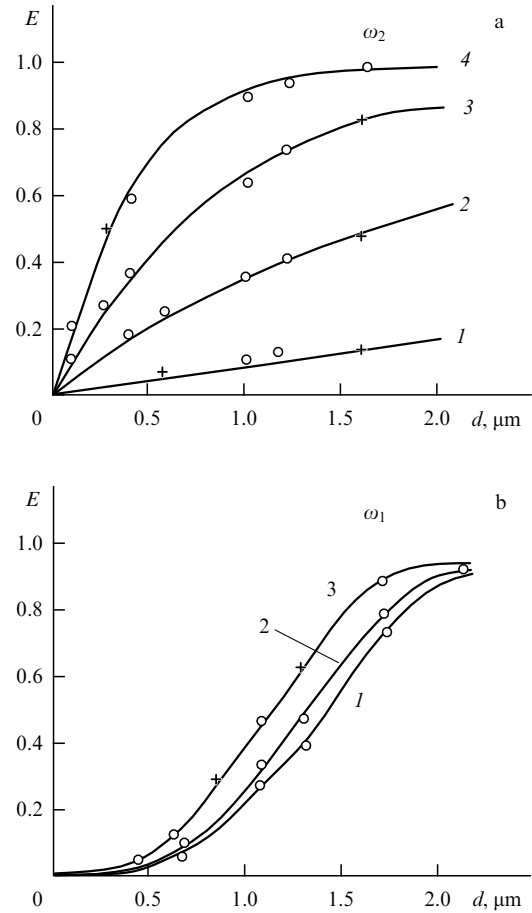
We emphasize that the absorption peak observed at a frequency close to that of the longitudinal phonon is related to the transverse eigenstate of a thin film. The longitudinal optical phonons (modes), as follows from the analysis in Refs [53–55], do not interact with the transverse electromagnetic field at all.

It follows from the data obtained that the radiation peak at the frequency $\omega_2 \approx \omega_{LO}$ belongs to the radiative surface polariton localized at the film–vacuum interface. However, it is impossible to determine from this series of experiments to which type of vibrations (a bulk phonon or the surface phonon of a film–metal interface) the radiation peak at the frequency $\omega_1 \approx \omega_{TO}$ belongs.

As follows from theoretical calculations, the unambiguous assignment of the emission band at the frequency ω_1 to transverse vibrational states of the film bulk or to the second surface polariton is possible only based on their different responses to the action of the electron subsystem of the metallic substrate.

Figure 12 presents spectra of the thermostimulated radiation of vacuum–ZnSe film–metal structures with thick metallic layers (aluminum, chromium, titanium), which differ in the magnitude of Im $\varepsilon_M(\omega)$. On all these metallic sub-
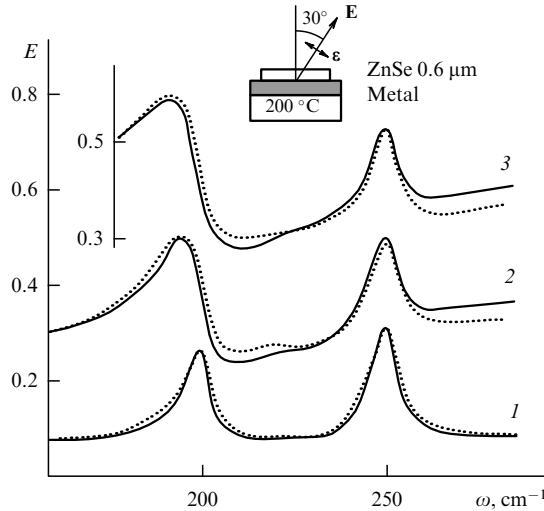
Figure 12. Spectra of p-polarized emissivity of ZnSe films 0.6 μm thick on metallic substrates of aluminum (*1*), chromium (*2*), and titanium (*3*). The dots correspond to experimental values; solid curves, to calculation results.

Figure 13. Spectra of p-polarized emissivity of a vacuum–ZnSe-film–Si-film–Al structure at various thicknesses *d* of the Si film: 0 μm (*1*), 1 μm (*2*), and 3 μm (*3*). The ZnSe film thickness is 0.6 μm. Points are experimental data; solid curves, calculation results. The experimental points for spectra *1* and *2* are shifted by 0.06 and those for spectrum *3*, by 0.13 (correction for background radiation).

strates, ZnSe films were deposited simultaneously by thermal evaporation in the vacuum.

It follows from Fig. 12 that the low-frequency ω_1 band of the p-polarized radiation is broadened with decreasing the conductivity of the metallic substrate and is shifted into the lower-frequency part of the spectrum. All this in totality means that the ω_1 band in the p-polarized thermostimulated radiation of vacuum–ZnSe-film–metal structures is related to the radiative surface (interfacial) polariton of the film–metal interface rather than to transverse vibrational states of the film bulk [50–55]. Figure 12 also shows calculated spectra of the radiation of these sandwiches. Because the frequencies of plasmons and the frequencies of electron collisions in them are not known for chromium and titanium, we used the Hagen–Rubens relation $R_M(\omega) \approx 1 - \sqrt{2\omega/\pi\sigma}$, where $\sigma = \sigma(\omega)$ is the conductivity of the substrate.

The best agreement between the experimental and calculated spectra was reached at $\sigma_{Al} = 8000\ \Omega^{-1}\ cm^{-1}$ ($R_{Al} = 0.94$), $\sigma_{Cr} = 500\ \Omega^{-1}\ cm^{-1}$ ($R_{Cr} = 0.76$), and $\sigma_{Ti} = 250\ \Omega^{-1}\ cm^{-1}$ ($R_{Ti} = 0.67$). As can be seen from Fig. 12, the experimental results agree well with the theoretical ones. Their small (insignificant) difference in the frequency range $\omega > 260\ cm^{-1}$ for ZnSe films on Cr and Ti seems to be related to the fact that the conductivity of these substrates at high frequencies is only poorly described by the Hagen–Rubens equation.

Thus, the investigations performed show that light absorption in thin films of wideband semiconductors on metallic substrates is determined not by vibrational states of the bulk of the film but by radiative surface states (polaritons of the film–vacuum and film–substrate interfaces) [54, 55]. The dipole moments of transverse optical phonons seem to undergo strong metallic quenching, because the total dipole moment of a transverse phonon, which is parallel to the film surface, along with its image in the metal, is close to zero in thin films. The image forces of dipole moments in metal, which are responsible for the transverse vibrational states in the film bulk, can be weakened by separating the film from the metal or, to be more exact, by creating a dielectric interlayer between the metal and the film. For example, Fig. 13 displays
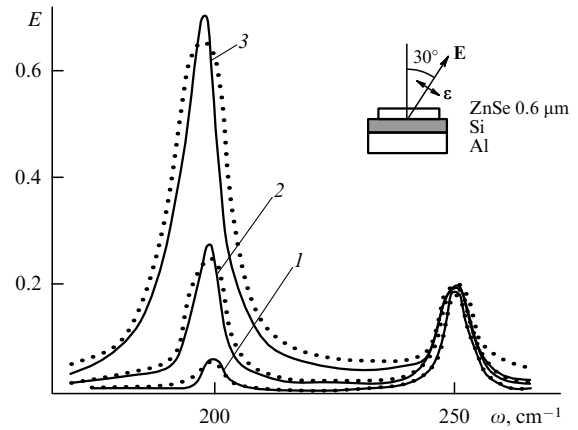
the spectra of the thermostimulated radiation of a vacuum–ZnSe-film–Si-film–Al structure [51].

Silicon has no dipole-active states in the IR range of the spectrum; therefore, its radiative capacity is very small and unselective. As can be seen from Fig. 13, the spectra calculated via the formula $A = E = 1 - |r_{1234}^p|^2$ with r_{1234}^p given by (151) (where ε_1 is the dielectric constant of the vacuum, ε_2 is the dielectric constant of the ZnSe film of thickness h_2, ε_3 is the dielectric constant of the silicon film of thickness h_3, and ε_4 is the dielectric constant of the aluminum substrate) agree with the experimental spectra quite well. There is a rapid increase in the radiation intensity and in the half-width of the low-frequency band with increasing the interlayer thickness and a small shift of this band into the lower-frequency range. The intensity, half-width, and frequency of the maximum of the high-frequency band remain unchanged. As follows from Fig. 13 and from a theoretical analysis in [50–55], the metallic quenching of the absorption ability is characteristic of only states with dipole moments parallel to the sandwich plane, whereas states with dipole moments perpendicular to the film plane do not interact with the conduction electrons of the metallic substrate. At the frequency $\omega_1 \approx \omega_{TO}$ and a small thickness of the silicon interlayer, we seem to be dealing with both an interfacial polariton and strongly weakened transverse vibrational states of ZnSe bulk states. These states, strictly speaking, are not transverse optical phonons of the film bulk, as was assumed earlier, because the half-width Γ of the absorption bands for these states substantially depends on both the interlayer thickness and on the film thickness and at a large thickness of the interlayer, is several times greater than the half-width of the imaginary part of the dielectric constant γ of bulk ZnSe. Such a difference is related to the radiative channel of the decay of these states, whose probability γ_r in thin films is additive with the probability of the anharmonic decay γ, i.e., $\Gamma \approx \gamma + \gamma_r$. Similar results were previously observed in the investigation of the spectra of reflection–absorption (RA) and thermal radiation of LiF films [56] and RA spectra of CdS films [60]. In these spectra, a large increase in the intensity of absorption bands is also observed at the ω_1 frequency; their
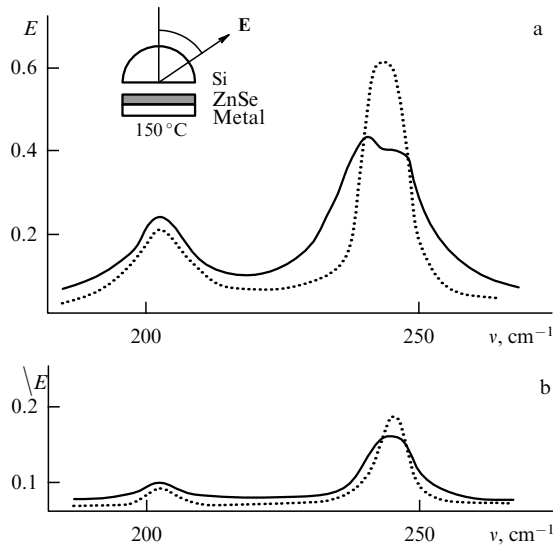
Figure 14. Spectra of emissivity (absorption) of an Si-prism–gap–ZnSe-film (1 μm)–aluminum-substrate structure with the gap equal to 12 μm ($\varphi = 18°$) (a) and 3 μm ($\varphi = 30°$) (b).

Figure 15. Dispersion of TM surface polaritons at $q_\parallel > k_0$ of a ZnSe film on a metallic substrate: v_{S+} (*1*) and v_{S-} (*2*) branches for a ZnSe film on Al; v_{S+} (*3*) and v_{S-} (*4*) branches for a ZnSe film on Cr; v_{S-} branch (*5*) for a ZnSe film on a thin (< 0.1 μm) aluminum mirror. Solid curves are calculation results; symbols, experimental data [49, 55].

half-width increases upon the replacement of metallic substrates by insulating ones.

The investigations performed showed that in thin films of wideband semiconductors on metallic substrates, light is absorbed and radiated by only radiative surface polaritons of the film–vacuum and film–substrate interfaces. Vibrational states of the film bulk (longitudinal and transverse optical phonons) do not absorb and do not radiate p-polarized light. Transverse optical phonons of the film can manifest themselves in the absorption and thermal radiation spectra if the film is separated from the metallic substrate by an insulating interlayer.

The experimental spectra of thermostimulated radiation of surface polaritons (nonradiative quasistationary modes) of ZnSe films on various metallic substrates have been obtained using a silicon semicylinder mounted at a certain distance over the sandwich investigated [48, 49, 52–55]. Figure 14 depicts the experimental and calculated spectra of the p-polarized radiation of surface polaritons of a 1 μm ZnSe film on an aluminum substrate at two radiation angles and various gaps between the film and the semicylinder, which were specified by special frames made of mylar films. The radiation angle determines the vector of surface polaritons $q_\parallel = k_0 n_{Si} \sin \varphi$, where n_{Si} is the refraction index of the prism material and k_0 is the wave vector of light in the vacuum. At $\varphi > \varphi_c$ and $q_\parallel > k_0$, we can reconstruct (by varying the radiation angle) the dispersion law of quasistationary (nonradiative) modes from the thermostimulated radiation spectra in the regime of inverted frustrated total internal reflection (FTIR).

Figure 14 shows thermostimulated radiation spectra of a silicon-semicylinder–gap–film–substrate structure at the radiation angle 20°, which is greater than the critical angle $\varphi_c = 17°$. The magnitude of the gap, which was 12 μm, was selected such that the disturbance of the quasistationary surface phonon by the silicon prism was minimum and did not lead to a shift of the emission band. The exponentially decreasing field of the surface polariton extends into the vacuum to a distance of the order of $q_\parallel^{-1}$; therefore, the thickness of the gap should be no less than $q_\parallel^{-1}$ in order that
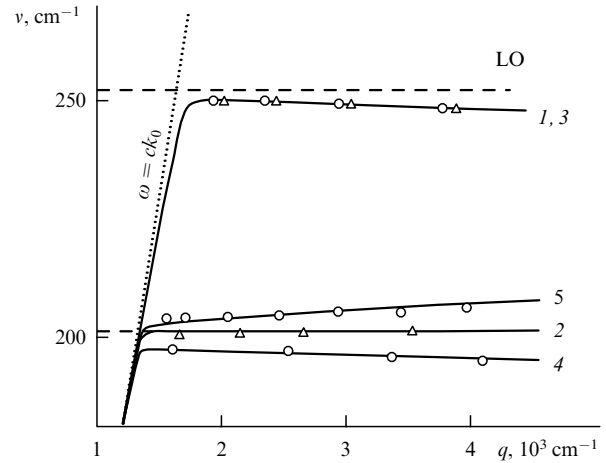
the disturbance not be too large. Figure 14b corresponds to the case where the radiation is directed at the angle 30° and the gap is about 3 μm. The thicknesses of the ZnSe and Al films were about 1 μm; in this case, the thickness of the aluminum mirror was much greater than the thickness of the skin layer in the frequency range of interest. Numerical calculation of the emissivity spectra of a structure with a silicon prism was performed based on the Kirchhoff formula with the use of the Fresnel coefficient r_{1234}^p for a four-layer structure (151). The calculated spectra of the system radiation normalized to the radiation of the ideal blackbody model measured under identical conditions agree with the experimental data qualitatively well.

We emphasize that the high-frequency peak in the radiation spectra (see Fig. 14) corresponds to the frequency of a weakly disturbed quasistationary polariton at a surface Coulomb phonon (~ 243 cm^{-1}), which corresponds to the equation $\varepsilon_2(\omega) = -1$.

Figure 15 displays dispersion relations (reconstructed from experimental data [48, 49, 52–55]) of surface TM (curves *1* and *3*) and interfacial (*2*, *4*, and *5*) quasistationary polaritons of a silicon-semicylinder–gap–ZnSe-film–substrate structure.

To demonstrate the degree of the disturbance of the nonradiative field of surface phonon–polaritons by the prism, Fig. 16 shows the spectral radiative capacity of a ZnSe film (1 μm) on an Al mirror in the regime of inverted FTIR at various gaps between the film and the prism (radiation angle $\phi = 20° \pm 2°$).

The resonances in curve *5* in Fig. 16 result from the interaction between the first and second transverse modes of a thin film with the field of a surface polariton of the metallic surface. In other words, the appearance of such 'radiative tails' is related to the disturbance of the initially nonradiative plasmon–polariton of the metal–vacuum interface by the thin film. It is seen from Fig. 16 that the disturbance of the nonradiative polaritons of the entire vacuum–film–metal structure by the prism increases with decreasing the distance between the prism and the structure. Therefore, to obtain the dispersion curves of surface phonon–polaritons (see Fig. 15),
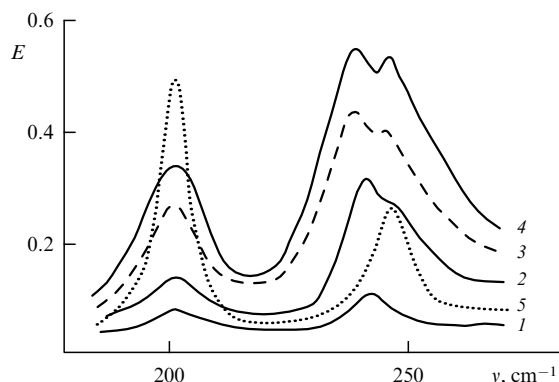
Figure 16. Emissivity of a ZnSe film (1 μm) on an Al mirror in the spectral range of p-polarized 'nonradiative' surface polaritons in the regime of inverted FTIR at various gaps between the film and the prism (emission angle $\phi = 20° \pm 2°$): 20 μm (*1*), 12 μm (*2*), 6 μm (*3*), 0 μm (*4*) (mechanical contact), and (*5*) emission of the film without an FTIR prism.

the prism should be placed as far from the surface as possible.

Thus, experimental investigations of the spectral composition of thermostimulated radiation of solids allow studying the eigenstates of the sources of a thermal field in much detail. In particular, in the structures of the FTIR-prism–gap–metal and semiconductor-film–metal types, the initially nonradiative surface plasmon of the metal transforms into a set of radiative states whose electromagnetic field is determined by all dipole-active excitations of each material of the structure and, in addition, depends on the real geometry of the experiment. The resonances observed in the spectra are very sensitive to specific features of the geometry and to the electrodynamic properties of the materials involved. One of the most important consequences of the experimental and theoretical investigations of optical properties of crystals [48–60] is that in crystals of finite dimensions, the absorption of light is a multistage process; a photon does not generate a phonon directly (in one step); the photon first transforms into an intermediate state (radiative polariton), and then this polariton interacts with the entire set of dipole-active states of the sample. As a result of this interaction, the polariton transforms into a phonon or into a multiphonon excitation, an exciton, a free electron (hole), or a plasmon of the metallic substrate. The absorption of light in this case is determined by the probabilities of the radiative and anharmonic decay of the polariton. The polariton of a multilayer thin-film structure is a mixed state of various dipole-active excitations of each of the materials of the structure.

Correspondingly, thermostimulated radiation of crystals of finite dimensions also arises as a result of a multistage transformation of mechanical fluctuations into dipole-active states of the structure, which generate their own radiative polariton states that are characterized by a finite probability of radiative decay, which in turn generates a traveling radiation field of the sample. It is natural that along with the intrinsic thermal radiation of the sample, nonradiative eigenstates — thermostimulated surface polaritons — arise as a result of fluctuations.

We also note that if the film has a thickness greater than a certain threshold value, resonances appear in the spectrum of the thermostimulated electromagnetic field, which are related to interference modes, and in the spectra of the

radiation part of the field, and waveguide modes appear in the spectra of the nonradiative part of the field. In experiments, the conditions for the appearance of interference modes and possible consequences for various physical phenomena have been investigated in much detail [54, 55, 58]. In particular, the absorption, thermostimulated radiation, and luminescence at the frequencies of interference modes, a sharp increase in the light absorption at the frequencies of local vibrations of impurities of the crystal lattice of the film when they coincide with the frequency of the interference mode, an increase in the intensity of the spectra of spontaneous Raman light scattering (RLS) when the frequency of the incident and (or) scattered light coincides with the frequency of the interference mode — all these phenomena, which are clearly observed in the spectra of films, indicate that the interference modes are real states of the electromagnetic field in thin-film structures. The interference modes are uniformly broadened and the half-widths of their bands in the spectra of absorption, thermo-stimulated radiation, and luminescence are mainly determined by their radiation lifetimes. This radiation lifetime under certain conditions is extremely small compared to the lifetimes of the majority of known oscillators; it can be as small as a few tens of femtoseconds [59]. The investigation of resonances in spectra corresponding to nonradiative wave-guide modes has not yet been performed.

5. Some applications of the theory of fluctuating electromagnetic fields

5.1 Dispersion interaction between bodies. Lifshitz problem

In the mid-1950s, E M Lifshitz published a series of papers [61–63] devoted to the theory of molecular forces of attraction between condensed media that fill two half-spaces with plane-parallel boundaries. In these works, he developed a theory of dispersion interaction forces via a fluctuating electromagnetic field between macroscopic bodies in complete thermodynamic equilibrium. The interaction force per unit area of the surface was calculated as the mean normal-to-the-surface component of the Maxwell stress tensor. The solution obtained is valid at any temperature and any complex dielectric constant characterizing the electrodynamic properties of the interacting bodies.

Lifshitz's work led to a continuous flow of investigations that have not stopped to date, in which the method suggested was perfected and modernized; new alternative methods have also been proposed, leading to the same result. A vast literature (including numerous Russian-language works) devoted to this problem exists. The main results concerning the theory of van der Waals forces and the related theory of Casimir forces [70] can be found in many reviews and monographs, e.g., [64–69]. Here, we only consider general relations and the simplest asymptotic regimes.

5.1.1 Dispersion interaction between bodies in the complete thermodynamic equilibrium. The spectral density of the Maxwell stress tensor contains all the spectral densities of the components of the thermostimulated field in a vacuum plane-parallel layer between two half-spaces. Therefore, the problem can be completely solved in terms of the fluctuating electrodynamics based on the Rytov theory, as was done in [61–63].

We first write, in terms of the Fresnel coefficients, the expression in [62] for the interaction force per unit surface that acts between two half-spaces separated by a planar vacuum layer of thickness l:

$$F(l) = \int_0^\infty \int_0^\infty \mathrm{d}p\, \mathrm{d}\omega\, F_\omega(p, l)\,,\qquad (176)$$

where

$$F_\omega(p, l) = -\frac{\Theta(\omega, T)}{\pi^2 \omega}\, \mathrm{Im}\left[pq \left(\frac{r_{01}^{\mathrm{p}} r_{02}^{\mathrm{p}} \exp(-2ql)}{1 - r_{01}^{\mathrm{p}} r_{02}^{\mathrm{p}} \exp(-2ql)} \right.\right.$$
$$\left.\left. + \frac{r_{01}^{\mathrm{s}} r_{02}^{\mathrm{s}} \exp(-2ql)}{1 - r_{01}^{\mathrm{s}} r_{02}^{\mathrm{s}} \exp(-2ql)} \right) \right]\,,\qquad (177)$$

$q = [p^2 - (\omega/c)^2]^{1/2}$ and $r_{0i}^{\mathrm{p,s}}$ are Fresnel coefficients (50) of the interfaces between the vacuum layer and half-spaces, $i = 1, 2$.

The general expression for the interaction force is sufficiently complicated, but, under some simplifications, it has been used to obtain the leading asymptotic terms [61–64]. For example, small distances between the bodies correspond to the case where $l \ll \lambda_0$ and $l \ll l_T$, where λ_0 is the characteristic wavelength in the absorption spectra of the bodies and $l_T = c\hbar/k_{\mathrm{B}}T \sim 10^{-4}$ cm at room temperature. For nonmagnetic materials, we then have

$$F(l) \approx \frac{\hbar \omega}{8\pi^2 l^3}\,,\qquad \omega = \int_0^\infty \mathrm{d}\xi\, \frac{(\varepsilon_1(\mathrm{i}\xi) - 1)(\varepsilon_2(\mathrm{i}\xi) - 1)}{(\varepsilon_1(\mathrm{i}\xi) + 1)(\varepsilon_2(\mathrm{i}\xi) + 1)}\,.\ (178)$$

The opposite relation $l \gg \lambda_0$ corresponds to the case of large distances. An especially simple expression for the interaction force is obtained for ideal metals when the second condition, $l \ll l_T$, is satisfied; in this case, we have

$$F(l) \approx \frac{\pi^2 \hbar c}{240\, l^4}\,.\qquad (179)$$

This is the Casimir force [70], which was earlier derived from quite different considerations, namely, by summing the eigenmodes of the field in a plane-parallel layer between ideally reflecting walls. It is interesting to note that the Casimir force serves here as a particular case of the interaction force in the framework of the problem of fluctuating electrodynamics.

A third particular case occurs when the effect of temperature on the interaction between the bodies is taken into account; this case corresponds to the conditions $l \gg \lambda_0$ and $l \gg l_T$ and yields

$$F(l) \approx \frac{k_{\mathrm{B}} T}{8\pi l^3}\, \frac{(\varepsilon_{10} - 1)(\varepsilon_{20} - 1)}{(\varepsilon_{10} + 1)(\varepsilon_{20} + 1)}\,,\qquad (180)$$

where ε_{10} and ε_{20} are the electrostatic values of the dielectric constants. Hence, at large distances, just as at small ones, the dependence is inversely proportional to the cube of the distance between the bodies.

We now derive asymptotic formulas (178)–(180) from Eqns (176) and (178). Formula (178) is a typical example of a result obtained in the quasistatic approximation ($c \to \infty$). Therefore, Fresnel coefficients (50) should be taken in the form $r_{0i}^{\mathrm{p}} \approx (\varepsilon_i - 1)/(\varepsilon_i + 1)$ and $r_{0i}^{\mathrm{s}} \approx 0$, $i = 1, 2$. In addition, using that $\hbar\omega/k_{\mathrm{B}}T = 2\pi l_T/\lambda$, we write (176), in the approx-

imation where $\lambda \ll l_T$, as

$$F(l) = -\frac{\hbar}{2\pi^2} \int_0^\infty \mathrm{d}\omega \sum_{n=0}^\infty \mathrm{Im}\, (r_{01}^{\mathrm{p}} r_{02}^{\mathrm{p}})^{n+1}$$
$$\times \int_{k_0}^\infty \mathrm{d}p\, p \sqrt{p^2 - k_0^2}\, \exp\left[-2\sqrt{p^2 - k_0^2}\, l(n+1) \right]\,, \ (181)$$

where we use the formula for an infinite geometrical series

$$\left[1 - r_{01}^{\mathrm{p}} r_{02}^{\mathrm{p}} \exp(-2ql) \right]^{-1} = \sum_{n=0}^\infty (r_{01}^{\mathrm{p}} r_{02}^{\mathrm{p}})^n \exp(-2qln)\,.\quad (182)$$

Changing the variable as $y^2 = p^2/k_0^2 - 1$ and integrating over y, we obtain

$$F(l) = -\frac{\hbar}{8\pi^2 l^3} \int_0^\infty \mathrm{d}\omega \sum_{n=0}^\infty \mathrm{Im}\, \frac{(r_{01}^{\mathrm{p}} r_{02}^{\mathrm{p}})^{n+1}}{(n+1)^3}\,,\qquad (183)$$

whence, after the use of the Cauchy theorem at $n = 0$, we obtain formula (178).

Classical Casimir law (179) is obtained from (176) under the condition $\varepsilon_{1,2} \to \infty$. In this case, under the condition $\lambda \ll l_T$, we have

$$F(l) = -\frac{\hbar}{\pi^2} \int_0^\infty \mathrm{d}\omega\, \mathrm{Re} \int_0^{k_0} \mathrm{d}p\, p \sqrt{k_0^2 - p^2}$$
$$\times \sum_{n=0}^\infty \exp\left[-2\mathrm{i}\sqrt{k_0^2 - p^2}\, l(n+1) \right]\,.\qquad (184)$$

We note that in (176), the part of the integral over p in the range from k_0 to ∞ drops out because we should take the imaginary part of a purely real (in this approximation) expression in (177).

Changing the variable as $x^2 = 1 - p^2/k_0^2$, taking into account that

$$\int_0^1 \mathrm{d}x\, x^2 \cos\left[2k_0 l(n+1)x \right] = \frac{\sin u_n}{u_n} + \frac{2\cos u_n}{u_n^2} - \frac{2\sin u_n}{u_n^3}\,,$$
$$(185)$$

where $u_n = 2k_0 l(n+1)$, and changing the order of integration and summation, we obtain from (184) that

$$F(l) = -\frac{\hbar c}{\pi^2} \sum_{n=0}^\infty \int_0^\infty \mathrm{d}k_0\, k_0^3 \left(\frac{\sin u_n}{u_n} + \frac{2\cos u_n}{u_n^2} - \frac{2\sin u_n}{u_n^3} \right)\,.$$
$$(186)$$

The integration in (186) can be sufficiently easily performed using the stationary phase method [71], namely, using the Erdelyi lemma. For example, for the first integral in (186), we have

$$\int_0^\infty \mathrm{d}k_0\, k_0^3\, \frac{\sin u_n}{u_n} = \frac{1}{2l(n+1)}\, \mathrm{Im} \int_0^\infty \mathrm{d}k_0\, k_0^2 \exp\left[\mathrm{i}2k_0 l(n+1) \right]$$
$$\approx -\frac{1}{8l^4(n+1)^4}\,.\qquad (187)$$

As a result, we obtain

$$F(l) = \frac{3\hbar c}{8\pi^2 l^4} \sum_{n=0}^\infty (n+1)^{-4} = \frac{\pi^2 \hbar c}{240\, l^4}\,.\qquad (188)$$

Finally, formula (180) is obtained from general expression (176) for the force in the classical approximation. Namely, the mean energy of a field oscillator is equal to $\Theta(\omega) \approx k_B T$, because $\hbar\omega \ll k_B T$. In other words, we consider the spectrum region where $\lambda \gg l_T$ under the condition $|q|^{-1} \ll l \ll \lambda$. General expression (176) then becomes

$$F(l)$$
$$\approx -\frac{k_B T}{\pi^2} \, \mathrm{Im} \int_0^\infty \frac{\mathrm{d}\omega}{\omega} \int_{\omega/c}^\infty \mathrm{d}p \, pq (r_{01}^p r_{02}^p + r_{01}^s r_{02}^s) \exp(-2ql) \,. \tag{189}$$

In deriving (189), we neglected the exponential terms in the denominator of (177), which corresponds to taking only the first term in (182) into account, because $|q|l \gg 1$. In addition, we ignored the part of the integral over p in the interval from zero to k_0, since, in view of the same condition, the integrand contains a strongly oscillating function.

Using the new variable $y = q/k_0$, we see that the leading contribution to the integral comes from the values $y \gg 1$, because $k_0 l \ll 1$ and $2k_0 l y \sim 1$. In this approximation, the Fresnel coefficients, as we already established, are independent of y: $r^p \approx (\varepsilon - 1)/(\varepsilon + 1)$, $r^s \approx 0$. Introducing one more variable, $z = 2k_0 l y$, and taking the integral, we can rewrite Eqn (189) as

$$F(l) \approx -\frac{k_B T}{4\pi^2 l^3} \, \mathrm{Im} \int_0^\infty \frac{\mathrm{d}\omega}{\omega} \, r_{01}^p(\omega) \, r_{02}^p(\omega) \,. \tag{190}$$

It can be clearly seen from (190) that the leading contribution to the integral comes from the domain of low frequencies. We now use the Cauchy theorem, according to which the integral over a closed path of the integrand in (190) in the domain of its analytic behavior in the right-hand quadrant of the complex-frequency plane is zero. The first integration here is performed along the real frequency axis; the second integration is carried out along the circle of a large radius R counterclockwise in the angular range from 0 to $\pi/2$; the third integration is along the imaginary axis, on which the integrand is purely real; and the fourth integration is performed along the circle of a small radius ρ around the point $\omega = 0$ clockwise in the angular range from $\pi/2$ to 0. As a result, we obtain

$$\mathrm{Im} \int_0^\infty \frac{\mathrm{d}\omega}{\omega} \, r_{01}^p(\omega) \, r_{02}^p(\omega) = -\frac{\pi}{2} \, r_{01}^p(0) \, r_{02}^p(0) \,. \tag{191}$$

Substituting (191) in (190), we obtain (180).

We note that the representation of the interaction force in form (176)–(177) is suitable because it allows analyzing the spectral features of the force density. Such an analysis was performed in Refs [72, 73], whence it follows that a substantial contribution to the interaction comes from the electromagnetic eigenmodes of the cavity, which is due to the vanishing of the denominators in Eqn (177), e.g., $1 - r_{01}^p r_{02}^p \exp(-2ql) = 0$.

In the case of interaction of identical materials, the equation $r^p = \pm \exp(ql)$ determines two modes corresponding to symmetric and antisymmetric solutions of this equation. It was shown in Ref. [74] that the appearance of symmetric and antisymmetric modes corresponds to the removal of degeneracy upon the hybridization of surface states of isolated surfaces when they are located sufficiently close to one another. Upon splitting, the antisymmetric mode

has a higher frequency than the symmetric one. In our case, the antisymmetric mode corresponds to repulsion and the symmetric one, to attraction, according to the results in Ref. [73].

To conclude this section, we note that the authors of Refs [61–64] used the general expressions for the interaction force between two half-spaces to consider the interaction of sufficiently rarefied materials for which the dielectric constant only weakly differs from unity and for which, thus, the additivity principle can be used; they obtained formulas for the dispersion interaction between a molecule (microparticle) and a half-space, as well as between two molecules.

5.1.2 Dispersion interaction between bodies in an incomplete thermodynamic equilibrium.
The number of works that have investigated the dispersion interaction between bodies under conditions of incomplete thermodynamic equilibrium is much less than the number of works where the same processes are studied under conditions of complete thermodynamic equilibrium. The dispersion interaction between half-spaces placed in thermostats with different temperatures and separated by a plane-parallel vacuum layer was first considered in Refs [75, 76]. In these works, an expression was obtained for the spectral density of the force acting between two half-spaces with different temperatures for any thickness of the gap and any temperature difference. The material of the half-spaces can be characterized by arbitrary complex dielectric constants and magnetic susceptibilities. Various forms of writing the spectral density of the force have been given. For example, if bodies 1 and 2 are in thermostats with the respective temperatures T_1 and T_2, the spectral density of the force acting on a unit surface can be represented as [76, Eqn (9)]

$$\tilde{F}_\omega(l) = -\frac{\Theta(\omega, T_1) + \Theta(\omega, T_2)}{4\pi^2\omega} \, \mathrm{Im} \int_0^\infty \mathrm{d}p \, pq \left(\frac{\Delta}{D} + \frac{\tilde{\Delta}}{\tilde{D}}\right)$$
$$- \frac{\Theta(\omega, T_1) - \Theta(\omega, T_2)}{4\pi^2\omega} \, \mathrm{Im} \int_0^\infty \mathrm{d}p \, pq \left(\frac{\delta_1 - \delta_2}{|D|^2} + \frac{\tilde{\delta}_1 - \tilde{\delta}_2}{|\tilde{D}|^2}\right), \tag{192}$$

where

$$\Delta = \left(\frac{\beta_1}{\mu_1} + \frac{\beta_2}{\mu_2}\right) \sinh(ql) + \left(1 + \frac{\beta_1\beta_2}{\mu_1\mu_2}\right) \cosh(ql) \,,$$
$$\tilde{\Delta} = \left(\frac{\beta_1}{\varepsilon_1} + \frac{\beta_2}{\varepsilon_2}\right) \sinh(ql) + \left(1 + \frac{\beta_1\beta_2}{\varepsilon_1\varepsilon_2}\right) \cosh(ql) \,,$$
$$D = \left(\frac{\beta_1}{\mu_1} + \frac{\beta_2}{\mu_2}\right) \cosh(ql) + \left(1 + \frac{\beta_1\beta_2}{\mu_1\mu_2}\right) \sinh(ql) \,,$$
$$\tilde{D} = \left(\frac{\beta_1}{\varepsilon_1} + \frac{\beta_2}{\varepsilon_2}\right) \cosh(ql) + \left(1 + \frac{\beta_1\beta_2}{\varepsilon_1\varepsilon_2}\right) \sinh(ql) \,, \tag{193}$$
$$\delta_1 = \frac{\beta_2}{\mu_2} \left(\frac{q}{q^*} - \left|\frac{\beta_1}{\mu_1}\right|^2\right), \quad \delta_2 = \frac{\beta_1}{\mu_1} \left(\frac{q}{q^*} - \left|\frac{\beta_2}{\mu_2}\right|^2\right),$$
$$\tilde{\delta}_1 = \frac{\beta_2}{\varepsilon_2} \left(\frac{q}{q^*} - \left|\frac{\beta_1}{\varepsilon_1}\right|^2\right), \quad \tilde{\delta}_2 = \frac{\beta_1}{\varepsilon_1} \left(\frac{q}{q^*} - \left|\frac{\beta_2}{\varepsilon_2}\right|^2\right),$$

and $\beta_{1(2)} = q_{1(2)}/q$, with $q_{1(2)} = [p^2 - \varepsilon_{1(2)}\mu_{1(2)}(\omega/c)^2]^{1/2}$.

Following the general procedure adopted in this review, we rewrite (192) using the Fresnel coefficients. For this, we

directly verify that

$$\frac{\Delta}{D} = 1 + \frac{2r_{01}^s r_{02}^s \exp(-2ql)}{1 - r_{01}^s r_{02}^s \exp(-2ql)} \,,$$

$$\frac{\tilde{\Delta}}{\tilde{D}} = 1 + \frac{2r_{01}^p r_{02}^p \exp(-2ql)}{1 - r_{01}^p r_{02}^p \exp(-2ql)} \,. \qquad (194)$$

In addition, for the field of traveling waves ($p < \omega/c$), we have $q/q^* = -1$; therefore, it follows from (193) that

$$\mathrm{Im}\left[\mathrm{i}|q|\left(\frac{\delta_1 - \delta_2}{|D|^2}\right)^{\mathrm{Pr}}\right] = |q| \frac{|r_{02}^s|^2 - |r_{01}^s|^2}{\left|1 - r_{01}^s r_{02}^s \exp(-2ql)\right|^2} \qquad (195)$$

and analogous expressions follow for the p waves.

For a quasistationary field ($p < \omega/c$), we have $q/q^* = 1$; hence, it follows from (193) that

$$\mathrm{Im}\left[q\left(\frac{\delta_1 - \delta_2}{|D|^2}\right)^{\mathrm{Ev}}\right]$$

$$= 2q \exp(-2ql) \frac{\mathrm{Re}\, r_{01}^s \, \mathrm{Im}\, r_{02}^s - \mathrm{Im}\, r_{01}^s \, \mathrm{Re}\, r_{02}^s}{\left|1 - r_{01}^s r_{02}^s \exp(-2ql)\right|^2} \,, \qquad (196)$$

and analogous expressions for the p waves.

Substituting (195) and (196) in (192) and keeping only terms that depend on the distance between the bodies, we rewrite the spectral density of the force that acts on the unit area of the surface as

$$F_\omega(l, T_1, T_2) = \frac{F_\omega(l, T_1)}{2} + \frac{F_\omega(l, T_2)}{2} + \Delta F_\omega(l, T_1, T_2)\,, \quad (197)$$

where $F_\omega(l, T_1)$ and $F_\omega(l, T_2)$ are the spectral force densities corresponding to equilibrium case (176) at the respective temperatures T_1 and T_2. The third term in the right-hand side of (197), which takes the temperature difference between the bodies into account, can be represented in the form $\Delta F_\omega(l, T_1, T_2) = \Delta F_\omega^{\mathrm{Pr}}(l, T_1, T_2) + \Delta F_\omega^{\mathrm{Ev}}(l, T_1, T_2)$, first suggested in Ref. [77], where

$$\Delta F_\omega^{\mathrm{Pr}}(l, T_1, T_2)$$

$$= -\frac{\Theta(T_1) - \Theta(T_2)}{4\pi^2\omega} \int_0^{k_0} \mathrm{d}p\, p|q| \sum_{j=s,p} \frac{|r_{02}^j|^2 - |r_{01}^j|^2}{\left|1 - r_{01}^j r_{02}^j \exp(-2ql)\right|^2}\,, \qquad (198)$$

$$\Delta F_\omega^{\mathrm{Ev}}(l, T_1, T_2)$$

$$= \frac{\Theta(T_1) - \Theta(T_2)}{2\pi^2\omega} \int_{k_0}^\infty \mathrm{d}p\, pq \exp(-2ql)$$

$$\times \sum_{j=s,p} \frac{\mathrm{Re}\, r_{02}^j \mathrm{Im}\, r_{01}^j - \mathrm{Im}\, r_{02}^j \mathrm{Re}\, r_{01}^j}{\left|1 - r_{01}^j r_{02}^j \exp(-2ql)\right|^2}\,. \qquad (199)$$

We note [75, 76] that the additional terms in formulas (192) or (198) and (199) disappear in the absence of a temperature difference or in the case of the interaction of bodies with identical properties. Therefore, in considering the Casimir forces between ideal materials, taking the temperature difference into account yields nothing new. The effect of the temperature difference is also absent if $\lambda \ll l_T$ ($\hbar\omega \gg k_B T$). The difference in temperatures manifests itself at large distances under the opposite condition, i.e., if $\lambda \gg l_T$, when $\Theta(T_1) - \Theta(T_2) \approx k_B T_1 - k_B T_2$. Then the leading contribution to the integral comes from the region where $y \gg 1$ because of the conditions $k_0 l \ll 1$ and $2k_0 ly \sim 1$. Using the

same change of the integration variable in (199) as we made in deriving Eqn (190), we find an additional term, which takes the temperature difference into account in the expression for spectral force density (197) and in the expression for the total force per unit area of the surface:

$$\Delta F(l, T_1, T_2) \approx -\frac{k_B T_1 - k_B T_2}{8\pi^2 l^3}$$

$$\times \int_0^\infty \frac{\mathrm{d}\omega}{\omega}\left(\mathrm{Im}\, r_{01}^p(\omega)\, \mathrm{Re}\, r_{02}^p(\omega) - \mathrm{Re}\, r_{01}^p(\omega)\, \mathrm{Im}\, r_{02}^p(\omega)\right)\,. \qquad (200)$$

In contrast to the equilibrium case, the integrand in (200) is not analytic; therefore, we cannot perform integration on the complex plane and should use numerical methods. However, it is evident that when simple approximations for the Fresnel coefficients are known, analytic calculations can be advanced still farther.

We note that the authors of Refs [77–70] studied the problem of a dispersion interaction between an atom and a half-space placed in thermostats with different temperatures.

5.2 Energy transfer via a thermally stimulated field between two bodies located in thermostats at different temperatures

The key role of the quasistationary field in the heat transfer between bodies separated by a small gap appears to have been recognized for the first time by Rytov et al. [4–7]. They considered the case of heat transfer through a vacuum plane-parallel gap between an absorbing half-space and an almost ideal mirror located in thermostats with different temperatures. Subsequently, many authors investigated the problem of energy exchange between two half-spaces separated by a planar gap (see, e.g., Refs [80–84]). One of the most essential effects that were revealed was the phenomenon of a sharp increase in the energy transfer in the case of a small gap d between the bodies. The gap is considered small if it is of the order of a typical wavelength of the radiative fluctuation spectrum of the interacting bodies, e.g., of the order of the Wien wavelength λ_W. In other words, it was found that the leading contribution to the energy transfer process comes from the fluctuating near-field zone, similarly to how this takes place in the theory of dispersion interactions.

In the last two decades, special probe microscopes for the investigation of surface properties of solids have appeared in experimental physics. Therefore, problems directly related to the geometry of the probes have been considered [85–89]. It has been shown that the rate of energy transfer is determined by the temperature difference between the bodies, by the distance between them, and by some resonance factors. For example, in the case of a probing body of spherical shape and a planar surface, such resonances correspond to quasistatic Fröhlich and Coulomb modes.

Along with theoretical investigations of heat transfer between various objects, some experimental studies have been performed. In the case of a plane-parallel geometry, an anomalous increase in the heat flux was revealed [90] at small distances between two parallel chromium films applied onto glass substrates. A strong enhancement of heat transfer between the probing body and a planar sample has been found by many authors [91–94].

In this section, following [95], we consider the process of heat transfer between two bodies of an arbitrary shape placed in thermostats with different temperatures. For this, we take

the system of two bodies, P (probe) and S (sample), of an arbitrary shape, separated by a vacuum gap of some small width d. The bodies are characterized by complex electric and magnetic susceptibilities (ε_P and μ_P, and ε_S and μ_S, respectively). They are in thermodynamic equilibrium with their thermostats with the respective temperatures T_P and T_S. In terms of the Langevin approach [6, 7], the random stationary currents with specified statistics, which arise in such bodies, generate thermostimulated fluctuating electromagnetic fields inside and outside each body. The bodies generate and absorb these fields. The entire system is out of thermodynamic equilibrium; therefore, there exists a certain stationary heat release rate inside the body with the lower temperature.

We define the heat power released, e.g., in the probing body, as the difference $\Delta W_P = W_+ - W_-$, where W_+ is the power absorbed by the probe and W_- is the loss of power inside the sample, thereby assuming that the probe is the only factor responsible for the losses in the sample.

To determine the heat power absorbed by the probe, we consider the total differential of the internal or free energy [12] of the probe in external fluctuating electromagnetic fields $\mathbf{E}_0$, and $\mathbf{H}_0$:

$$dU_P = T_P\, dS_P - \mathbf{\mu}\, d\mathbf{E}_0 - \mathbf{m}\, d\mathbf{H}_0 \,,$$
$$dF_P = -S_P\, dT_P - \mathbf{\mu}\, d\mathbf{E}_0 - \mathbf{m}\, d\mathbf{H}_0 \,, \qquad (201)$$

where S_P is the entropy of the probe and $\mathbf{\mu}$ and $\mathbf{m}$ are the electric and magnetic dipoles induced in the probing body by an external (relative to the probe) thermostimulated fluctuation electromagnetic field of the sample. The use of the first or the second formula depends on the specific features of the experiment. The power absorbed in the probe can be found by averaging the time derivative of the corresponding energy:

$$S_P = \text{const}: \quad W_+ = \left\langle \frac{dU_P}{dt} \right\rangle = -\left\{ \left\langle \mathbf{\mu}\, \frac{d\mathbf{E}_0}{dt} \right\rangle_s + \left\langle \mathbf{m}\, \frac{d\mathbf{H}_0}{dt} \right\rangle_s \right\},$$
$$(202)$$

$$T_P = \text{const}: \quad W_+ = \left\langle \frac{dF_P}{dt} \right\rangle = -\left\{ \left\langle \mathbf{\mu}\, \frac{d\mathbf{E}_0}{dt} \right\rangle_s + \left\langle \mathbf{m}\, \frac{d\mathbf{H}_0}{dt} \right\rangle_s \right\},$$
$$(203)$$

where the subscript 's' denotes symmetrization and the angular brackets denote ensemble averaging.

We note that terms of the form $\mathbf{\mu}^{sp}\, d\mathbf{E}$ can be formally added to Eqns (201), where $\mathbf{\mu}^{sp}$ is the intrinsic fluctuation moment and $\mathbf{E}$ is the fluctuating field (induced by the moment $\mathbf{\mu}^{sp}$) that is reflected from the sample. However, the reflected field $\mathbf{E}$ carries virtually no information about the thermodynamic state of the sample, unlike its intrinsic fluctuation field $\mathbf{E}_0$. Therefore, taking such terms into account in this problem, we go beyond the precision that is used in deriving the sought formula (based on the energy balance). Roughly speaking, when ignoring this term, we neglect the temperature dependence of the Fresnel coefficients.

There is no need to consider other possible combinations of $\mathbf{\mu}^{sp}\, d\mathbf{E}_0$ and $\mathbf{\mu}\, d\mathbf{E}$ because the random quantities referring to different thermostats do not correlate and the corresponding products vanish after averaging.

In this section, we use the formal definition of the Fourier transformation of all random functions of time in the following form:

$$\mathbf{A}(t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi}\, \mathbf{A}(\omega)\exp(-i\omega t), \quad \mathbf{A} = \mathbf{\mu}, \mathbf{m}, \mathbf{E}_0, \mathbf{H}_0. \quad (204)$$

In what follows, in the formulas obtained, we consider only the electric part, because the magnetic part is obtained similarly. Representing the quantities $\mathbf{\mu}(t)$ and $\mathbf{E}_0(t)$ in form (204) and substituting them in (202) or (203), we obtain

$$W_+ = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi}\, \omega \sum_{ik} \frac{\alpha_{ik}(\omega) - \alpha_{ki}^*(\omega)}{2i} \left\langle E_{0i}(\omega)\, E_{0k}(\omega) \right\rangle, \quad (205)$$

where we use an equality that is valid for stationary processes:

$$\left\langle \mathbf{\mu}(\omega)\, \mathbf{E}_0(\omega') \right\rangle = \alpha_{ik}(\omega) \left\langle E_{0k}(\omega)\, E_{0i}(\omega) \right\rangle 2\pi\delta(\omega + \omega'), \quad (206)$$

in which $\alpha_{ik}(\omega)$ is the polarizability tensor of the probing body having the property $\alpha_{ik}(-\omega) = \alpha_{ik}^*(\omega)$ for purely real frequencies.

The power of losses in the sample can be found using the Poynting theorem:

$$W_- = -\frac{1}{2} \left\langle \int_{V_P} d\mathbf{r}' \left[\mathbf{j}_{ext}(\mathbf{r}', t)\, \mathbf{E}(\mathbf{r}, \mathbf{r}'; t) \right.\right.$$
$$\left.\left. + \mathbf{E}(\mathbf{r}, \mathbf{r}'; t)\, \mathbf{j}_{ext}(\mathbf{r}', t) \right] \right\rangle, \quad (207)$$

where V_P is the volume of the probe, $\mathbf{j}_{ext} = (\partial \mathbf{\mu}^{sp}/\partial t)\, \delta(\mathbf{r} - \mathbf{r}')$, $\mathbf{\mu}^{sp}$ is the intrinsic spontaneously fluctuating dipole moment of the probe at the point $\mathbf{r}'$, the induced field is related to the response of the sample: $E_i(\omega; \mathbf{r}, \mathbf{r}') = \beta_{ik}(\omega; \mathbf{r}, \mathbf{r}')\, \mu_k(\omega; \mathbf{r}')$, and $\beta_{ik}(\omega; \mathbf{r}, \mathbf{r}')$ is the corresponding linear response tensor.

Representing $\mathbf{\mu}^{sp}(t)$ and $\mathbf{E}(t)$ in form (204) and substituting them in (207), we obtain

$$W_- = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi}\, \omega \sum_{ik} \frac{\beta_{ik}(\omega) - \beta_{ki}^*(\omega)}{2i} \left\langle \mu_i^{sp}(\omega)\, \mu_k^{sp}(\omega) \right\rangle. \quad (208)$$

Substituting (205) and (208) in the expression for the power of heat released in the probe, $\Delta W_P = W_+ - W_-$, and using the FDT, we obtain the sought formula

$$\Delta W_P = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \left[\Theta_S(\omega, T_S) - \Theta_P(\omega, T_P) \right]$$
$$\times \sum_{ik} 2\alpha_{ik}^{ah}(\omega)\, \beta_{ik}^{ah}(\omega), \qquad T_S > T_P, \quad (209)$$

where the superscript 'ah' corresponds to the anti-Hermitian part and $\Theta_{S,P}(\omega) = (\hbar\omega/2)\coth\left[\hbar\omega/(2k_B T_{S,P})\right]$.

The FDT here was used for symmetrized quantities of the form

$$\left\langle \mu_i^{sp}\, \mu_k^{sp} \right\rangle = \frac{i\Theta(\omega)}{\omega} \left[\alpha_{ki}^*(\omega) - \alpha_{ik}(\omega) \right], \quad (210)$$

$$\left\langle E_{0i}(\omega)\, E_{0k}(\omega) \right\rangle = \frac{i\Theta(\omega)}{\omega} \left[\beta_{ki}^*(\omega) - \beta_{ik}(\omega) \right]. \quad (211)$$

Formula (209) determines the spectral density of the released power

$$\Delta W_P(\omega) = \left[\Theta_S(\omega, T_S) - \Theta_P(\omega, T_P) \right] \sum_{ik} 2\alpha_{ik}^{ah}(\omega)\, \beta_{ik}^{ah}(\omega). \quad (212)$$

We note that the formulas obtained are valid for interacting objects of arbitrary shapes, whose optical properties can be characterized by nonlocal susceptibilities.

As an example, we consider, in terms of a local approximation, the problem for an ellipsoidal probe made of an isotropic material and placed close to a half-space. In this case, as is known [12], the polarizability in the quasistatic case is equal to

$$\alpha_{ik}(\omega) = \alpha_{ii}(\omega) = \frac{V_{\mathrm{P}}}{4\pi} \frac{\varepsilon_{\mathrm{P}}(\omega) - 1}{1 + (\varepsilon_{\mathrm{P}}(\omega) - 1) n_i}, \qquad i = x, y, z\,,$$

where n_i is the depolarizing factor. In the same approximation, the linear response tensor is written as (see, e.g., Ref. [96])

$$\beta_{zz}(\omega) = 2\beta_{xx}(\omega) = 2\beta_{yy}(\omega) = \frac{1}{4d^3} \frac{\varepsilon_{\mathrm{S}}(\omega) - 1}{\varepsilon_{\mathrm{S}}(\omega) + 1}\,.$$

In this example, the anti-Hermitian parts of the susceptibilities are merely equal to their imaginary parts and in the particular case of a sphere ($n_i = 1/3$), the substitution of the local expressions in (209) yields

$$\Delta W_{\mathrm{P}} = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \frac{6a^3 \left[\Theta_{\mathrm{S}}(\omega, T_{\mathrm{S}}) - \Theta_{\mathrm{P}}(\omega, T_{\mathrm{P}})\right]}{d^3}$$
$$\times \frac{\varepsilon_{\mathrm{P}}''(\omega)\, \varepsilon_{\mathrm{S}}''(\omega)}{\left|\varepsilon_{\mathrm{P}}(\omega) + 2\right|^2 \left|\varepsilon_{\mathrm{S}}(\omega) + 1\right|^2}\,, \qquad (213)$$

where a is the radius of the sphere and d is the distance between the center of the sphere and the planar surface. This formula was first obtained in [87]. It can be clearly seen that the spectral density of the power released is determined by the temperature difference, by a given dependence on the distance, and by the poles of the denominators, which give resonance frequencies corresponding to Fröhlich modes ($\varepsilon_{\mathrm{P}}(\omega) = -2$) and Coulomb modes ($\varepsilon_{\mathrm{S}}(\omega) = -1$).

Along with the dipole approximation in the description of the power of heat release, we can also use a multipole description, based on the general expression for the interaction energy

$$V = -\int \mathrm{d}\mathbf{r}\, \mathbf{P}\, \mathbf{E}\,, \qquad (214)$$

where integration is performed over the volume of the probe, $\mathbf{P}$ is the specific polarization of the probe material, and $\mathbf{E}$ is an external field. It follows from this expression that

$$V = -\mu_i E_i - Q_{ij} \nabla_j E_i - Q_{ijk} \nabla_j \nabla_k E_i - Q_{ijkl} \nabla_j \nabla_k \nabla_l E_i - \ldots\,, \qquad (215)$$

where μ_i is the induced dipole, Q_{ij} is the induced quadrupole, etc. (see, e.g., Refs [67, 97]). In Eqn (215), any multipole is denoted by Q, but the order of the multipole is uniquely determined by the subscripts. The induced multipoles are expressed through an infinite set of polarizabilities as

$$\mu_i(\omega) = \alpha_{ij}^{(1,1)}(\omega) E_j + \alpha_{ij,k}^{(1,2)}(\omega) \nabla_k E_j + \ldots$$
$$Q_{ij}(\omega) = \alpha_{i,jk}^{(2,1)}(\omega) E_k + \alpha_{ij,kl}^{(2,2)}(\omega) \nabla_k E_l + \ldots \qquad (216)$$
$$\cdots\,,$$

where the polarizability $\alpha_{ij}^{(1,1)}(\omega)$ characterizes the dipole linear response of the system to the applied field, the polarizability $\alpha_{ij,k}^{(1,2)}(\omega)$ is the dipole linear response of the system to the first spatial derivative of the field, $\alpha_{i,jk}^{(2,1)}(\omega)$ is the quadrupole linear response of the system to the field, and so on, for any multipole.

We emphasize that in the case of isotropic spherically symmetrical objects in a coordinate system with the origin at the symmetry center, only the diagonal terms $\alpha_{\cdots}^{(i,i)}$ are nonzero.

The total differential of the free energy can be written, by analogy with the total differential in the dipole case (201), as

$$\mathrm{d}F_{\mathrm{P}} = -S_{\mathrm{P}}\, \mathrm{d}T_{\mathrm{P}} - \mu_i\, \mathrm{d}E_i - Q_{ij}\, \mathrm{d}E_{ji}'$$
$$- Q_{ijk}\, \mathrm{d}E_{jki}'' - Q_{ijkl}\, \mathrm{d}E_{jkli}''' - \ldots\,, \qquad (217)$$

where the prime denotes differentiation of the ith component of the field with respect to x_j, $\mathrm{d}E_{ji}' \equiv \mathrm{d}(\nabla_j E_i)$, $\mathrm{d}E_{jki}'' \equiv \mathrm{d}(\nabla_j \nabla_k E_i)$, and summation over repeated indices is understood.

We determine the heat-release power $\Delta W_{\mathrm{P}}^{(l)} = W_+^{(l)} - W^{(l)}$ for an order-l multipole. As an example, we consider the case for a quadrupole, $l = 2$. In this case, it follows from (217) that $\mathrm{d}F_{\mathrm{P}}^{(2)} = -S_{\mathrm{P}}\, \mathrm{d}T_{\mathrm{P}} - Q_{ij}\, \mathrm{d}(\nabla_j E_{i0})$ and the expression for the absorbed power is

$$W_+^{(2)} = \left\langle \frac{\mathrm{d}F_{\mathrm{P}}^{(2)}}{\mathrm{d}t} \right\rangle = -\left\langle Q_{ij}(t) \frac{\mathrm{d}(\nabla_j E_{i0})}{\mathrm{d}t} \right\rangle_{\mathrm{s}}\,. \qquad (218)$$

After a Fourier transformation [Eqn (204)] of the functions $Q_{ij}(t)$ and $\nabla_j E_{i0}(t)$ and substitution of the induced quadrupole $Q_{ij}(\omega) = \alpha_{ij,kl}^{(2,2)}(\omega) \nabla_k E_{l0}(\omega)$ from (216) in (218), we obtain

$$W_+^{(2)} = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi}\, \omega \sum_{ijkl} \frac{\alpha_{ij,kl}^{(2,2)}(\omega) - \alpha_{kl,ij}^{(2,2)*}(\omega)}{2\mathrm{i}}$$
$$\times \left\langle \frac{\partial E_{l0}(\omega)}{\partial x_k} \frac{\partial E_{i0}(\omega)}{\partial x_j} \right\rangle\,. \qquad (219)$$

To calculate the electromagnetic losses in the sample, we use the same expression (207) from the Poynting theorem, with

$$j_i^{\mathrm{ext}} = -\frac{\partial Q_{ij}^{\mathrm{sp}}(t)}{\partial t} \frac{\partial \delta(\mathbf{r} - \mathbf{r}')}{\partial x_j'}\,,$$

where $Q_{ij}^{\mathrm{sp}}(\mathbf{r}, t) = Q_{ij}^{\mathrm{sp}}(t)\, \delta(\mathbf{r} - \mathbf{r}')$. Substituting the external current in (207), we obtain losses in the quadrupole approximation:

$$W_-^{(2)} = -\left\langle \frac{\partial Q_{ij}^{\mathrm{sp}}(t)}{\partial t} \frac{\partial E_i(\mathbf{r}, t)}{\partial x_j} \right\rangle_{\mathrm{s}}\,. \qquad (220)$$

Just as in the dipole case, the linear response of the sample in the quadrupole approximation is expressed by a relation $E_i = \beta_{i,kl} Q_{kl}$, whence $\partial E_i/\partial x_j = (\partial \beta_{i,kl}/\partial x_j) Q_{kl} \in \beta_{ij,kl} Q_{kl}$. Making Fourier transformation (204) for the functions $\partial Q_{ij}^{\mathrm{sp}}(t)/\partial t$ and $\partial E_i(\mathbf{r}, t)/\partial x_j$ and substituting the expression for the spatial derivative of the field in (220), we obtain the following formula for the power of electromagnetic losses in the sample:

$$W_-^{(2)} = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi}\, \omega \sum_{ijkl} \frac{\beta_{ij,kl}(\omega) - \beta_{kl,ij}^*(\omega)}{2\mathrm{i}} \left\langle Q_{ij}^{\mathrm{sp}}(\omega)\, Q_{kl}^{\mathrm{sp}}(\omega) \right\rangle\,. \qquad (221)$$

The use of the FDT [by differentiating (211)] to the spectrum of fluctuations of spatial derivatives of the sample field in (210) and of the quadrupole moment in (221) yields the sought expression for the power of heat release in the probe

in the quadrupole approximation:

$$\Delta W_{\mathrm{P}}^{(2)} = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \left[\Theta_{\mathrm{S}}(\omega, T_{\mathrm{S}}) - \Theta_{\mathrm{P}}(\omega, T_{\mathrm{P}})\right]$$
$$\times \sum_{ijkl} 2 \left[\alpha_{ij,kl}^{(2,2)}(\omega)\right]^{\mathrm{ah}} \left[\beta_{ij,kl}(\omega)\right]^{\mathrm{ah}}. \quad (222)$$

It is obvious that a corresponding expression can be obtained for any multipole interaction.

5.3 Shift and broadening of levels of a particle in a thermally stimulated fluctuating field of a solid body. Relaxation of the excited state of a particle near a planar surface

If an atom or a molecule is placed in an electromagnetic field, its level can shift by a certain magnitude; in addition, the lifetime of excited states can change and the degeneracy can be lifted partly or completely. When a particle is located near the surface of a sample, its characteristics depend on the distance between the particle and the surface and also on the electrodynamic properties of the sample, because any body is a source of a thermostimulated electromagnetic field. As an example, we mention the shift $\delta E_a(\mathbf{r})$ of some level a or the lifetime of the excited state $\tau = \gamma_a^{-1}(\mathbf{r})$, where $\gamma_a(\mathbf{r}) = \sum_i \gamma_i(\mathbf{r})$ is the decay rate of the state a, which is the sum of decay probabilities per unit time via various relaxation channels.

The decay rate can be found in terms of classical electrodynamics by assuming that the particle located at a point $\mathbf{r}_0$ near the sample is the source of a polarization $\mathbf{P}(\mathbf{r}, t) = \boldsymbol{\mu} \exp(-\mathrm{i}\omega t)\, \delta(\mathbf{r} - \mathbf{r}_0)$, which is associated with the current density

$$\mathbf{j}(\mathbf{r}, t) = \frac{\partial P(\mathbf{r}, t)}{\partial t} = \mathbf{j}(\mathbf{r}, \omega) \exp(-\mathrm{i}\omega t)$$
$$= -\mathrm{i}\omega \boldsymbol{\mu}\, \delta(\mathbf{r} - \mathbf{r}_0) \exp(-\mathrm{i}\omega t). \quad (223)$$

According to the Maxwell equations, this current generates the field $\mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}, \omega) \exp(-\mathrm{i}\omega t)$ in the entire system under consideration. Then, as is known [12, 32], the mean power of losses in the particle–sample system is equal to

$$W = -\frac{1}{2} \operatorname{Re} \left(\int \mathbf{j}^*(\mathbf{r}, \omega)\, \mathbf{E}(\mathbf{r}, \omega)\, \mathrm{d}\mathbf{r}\right) = \frac{\omega}{2} \operatorname{Im}\left(\boldsymbol{\mu}^* \mathbf{E}(\mathbf{r}_0)\right), \quad (224)$$

where the integration is carried out over the volume occupied by the sources.

Given the power of losses, the decay rate is determined as $\gamma = W/\hbar\omega$. In view of (224), the power of losses is a function of the coordinate of the source, and hence the decay rate depends on the mutual arrangement of the particle and the sample. According to (224), it is necessary to find the field of the dipole $\mathbf{E}(\mathbf{r}_0, \omega)$ at the point of its location.

Substituting the value of the field at the dipole (77) in (224), we obtain

$$W = \frac{ck_0^4}{3} |\boldsymbol{\mu}|^2 + \frac{ck_0^3}{4\pi} \operatorname{Re} \int_0^{\infty} \int_0^{2\pi} \frac{\mathrm{d}p\, \mathrm{d}\varphi\, p}{w_0} \left[\boldsymbol{\mu}^*(\hat{\mathbf{s}}\hat{\mathbf{s}})\boldsymbol{\mu}\, r^{\mathrm{s}}\right.$$
$$\left. + \boldsymbol{\mu}^*(\hat{\mathbf{p}}_0^+ \hat{\mathbf{p}}_0^-)\boldsymbol{\mu}\, r^{\mathrm{p}}\right] \exp(2\mathrm{i}w_0 h). \quad (225)$$

Taking into account that $\hat{\mathbf{s}} = \hat{\mathbf{e}}_y \cos\varphi - \hat{\mathbf{e}}_x \sin\varphi$ and $\hat{\mathbf{p}} = \hat{\mathbf{e}}_x \cos\varphi + \hat{\mathbf{e}}_y \sin\varphi$ and using the rule of multiplication of a dyad by a vector, we obtain the power of losses in the form

given in Ref. [32]:

$$W = \frac{ck_0^4}{3} |\boldsymbol{\mu}|^2 + \frac{ck_0}{2} \operatorname{Re} \int_0^{\infty} \frac{\mathrm{d}p\, p}{w_0}$$
$$\times \left[|\mu_\perp|^2 p^2 r^{\mathrm{p}} + \frac{|\mu_\parallel|^2}{2}(k_0^2 r^{\mathrm{s}} - w_0^2 r^{\mathrm{p}})\right] \exp(2\mathrm{i}w_0 h), \quad (226)$$

where $\mu_\perp = \mu_z$ is the dipole-moment component perpendicular to the sample surface and $\mu_\parallel$ is the component parallel to the sample surface, with $|\mu_\parallel|^2 = |\mu_x|^2 + |\mu_y|^2$.

The first term in Eqn (226) determines the dipole radiation power into free space; the interpretation of the second term depends on the model of the dielectric constant and can determine the losses for the excitation of electron–hole pairs in the sample, the excitation of surface plasmon-polaritons or phonon–polaritons, the excitation of quasi-stationary non-radiative waves, etc. All these processes, as well as the simplified expressions for the power of losses and the decay rate, have been described in Ref. [32] in sufficient detail. From (226), the expression for the normalized decay rate immediately follows:

$$\frac{\gamma(h)}{\gamma_0} = 1 + \frac{1}{2\hbar\gamma_0} \operatorname{Re} \int_0^{\infty} \frac{\mathrm{d}p\, p}{w_0}$$
$$\times \left[|\mu_\perp|^2 p^2 r^{\mathrm{p}} + \frac{|\mu_\parallel|^2}{2}(k_0^2 r^{\mathrm{s}} - w_0^2 r^{\mathrm{p}})\right] \exp(2\mathrm{i}w_0 h), \quad (227)$$

where $\gamma_0 = \omega^3 |\boldsymbol{\mu}|^2 / 3c^3 \hbar$ is the rate of spontaneous decays in the vacuum; we should take a doubled dipole moment in this expression in order to obtain the correct quantum mechanical expression for the Einstein coefficient.

The rate of decay of a state into other states of a particle in a random stationary field can be found in the linear approximation of the perturbation theory [9]. If, as usual, the Hamiltonian of the interaction of a particle and the field of the sample in the dipole approximation is $V(t) = -\boldsymbol{\mu}\mathbf{E}(t)$, then we can find the probability of transition per unit time or the rate of transition from the state a into a certain set of states n allowed by selection rules in the perturbation theory,

$$\gamma_a = \hbar^{-2} \sum_n \mu_\alpha^{an} \mu_\beta^{na} J_{\alpha\beta}(\omega_{na}), \quad (228)$$

where μ_α^{na} is the matrix element of the αth component of the dipole moment operator with respect to the eigenfunctions of the unperturbed Hamiltonian and $J_{\alpha\beta}(\omega)$ is the spectral density of states, which is the Fourier transform of the correlation tensor of the stationary field $\langle E_\alpha(t') E_\beta(t'') \rangle$. Hence, the rate of decay of a particle state is proportional to the spectral density of states of the thermostimulated field of the sample at the corresponding transition frequency.

The most consistent theory of the above processes was developed by the authors of Refs [98, 99] on the basis of the theory of linear response. We here give the main results following from Ref. [99]. We consider the expression for the level shift [99b, Eqn (2.13)]:

$$\delta E_a = -\frac{1}{\pi} P \sum_n \mu_\alpha^{an} \mu_\beta^{na}$$
$$\times \int_{-\infty}^{\infty} \mathrm{d}\omega \frac{\operatorname{Im} G_{\alpha\beta}(\mathbf{r}_0, \mathbf{r}_0; \omega)}{[1 - \exp(-\hbar\omega/k_{\mathrm{B}}T)](\omega + \omega_{na})}, \quad (229)$$

from which the authors of [99b] obtained the final expression for the level shift in the low-temperature limit, i.e., at $T = 0$.

We obtain an analytic expression for the shift of the energy level at an arbitrary temperature, which is very important for the consideration of vibrational and rotational transitions. For this, we use the Sokhotsky identity and write (229) in the form

$$\delta E_a = -\frac{1}{\pi} \sum_n \mu_\alpha^{an} \mu_\beta^{na}$$

$$\times \int_{-\infty}^{\infty} d\omega \, \frac{\text{Im} \, G_{\alpha\beta}(\mathbf{r}_0, \mathbf{r}_0; \omega)}{[1 - \exp(-\hbar\omega/k_B T)](\omega + \omega_{na} + i\eta)}$$

$$- i \sum_n \mu_\alpha^{an} \mu_\beta^{na} \, \text{Im} \, G_{\alpha\beta}(\mathbf{r}_0, \mathbf{r}_0; \omega_{an}) \left[1 - \exp\left(-\frac{\hbar\omega_{an}}{k_B T}\right) \right], \tag{230}$$

where it is understood that $\eta \to 0$.

In formula (230), we transform the integral as follows:

$$\int_{-\infty}^{\infty} d\omega \, \frac{\text{Im} \, G_{\alpha\beta}(\mathbf{r}_0, \mathbf{r}_0; \omega)}{[1 - \exp(-\hbar\omega/k_B T)](\omega + \omega_{na} + i\eta)}$$

$$= \frac{1}{2i} \int_{-\infty}^{\infty} d\omega \, \frac{G_{\alpha\beta}(\mathbf{r}_0, \mathbf{r}_0; \omega) - G_{\alpha\beta}^*(\mathbf{r}_0, \mathbf{r}_0; \omega)}{[1 - \exp(-\hbar\omega/k_B T)](\omega + \omega_{na} + i\eta)}$$

$$= \frac{1}{2i} \left\{ \int_{-\infty}^{\infty} d\omega \, \frac{G_{\alpha\beta}(\mathbf{r}_0, \mathbf{r}_0; \omega)}{[1 - \exp(-\hbar\omega/k_B T)](\omega + \omega_{na} + i\eta)} \right.$$

$$\left. - \int_{-\infty}^{\infty} d\omega \, \frac{G_{\alpha\beta}(\mathbf{r}_0, \mathbf{r}_0; \omega)}{[1 - \exp(\hbar\omega/k_B T)](\omega_{na} + i\eta - \omega)} \right\}$$

$$= \frac{1}{2i} \{ J_1 - J_2 \}, \tag{231}$$

where we change the variable as $\omega = -\omega$ in the second integral and use the property of the Green's function $G_{\alpha\beta}^*(-\omega) = G_{\alpha\beta}(\omega)$ on the real frequency axis, which is valid for nonmagnetic materials.

We note that the integrands in (231) have an infinite number of simple poles on the imaginary frequency axis at the Matsubara frequencies $\xi_m = 2\pi m k_B T/\hbar$, $m = 0, 1, 2, \ldots$, as well as poles at the points $\omega = -\omega_{na} - i\eta$ and $\omega = \omega_{na} + i\eta$ in the complex frequency plane (Fig. 17).

To perform the integration in (231), we take into account that the Green's function is analytic in the upper part of the complex plane; therefore, we consider auxiliary contour integrals. For the integral J_1, the contour C_1 includes the path from $-\infty$ to ∞, with the point $\omega = 0$ bypassed clockwise from above along a small semicircle; this path is closed by a large semicircle in the upper part of the complex plane; the point $\omega = -\omega_{na} - i\eta$ is outside this contour. For the integral J_2, the contour C_2 also includes the path from $-\infty$ to ∞; the
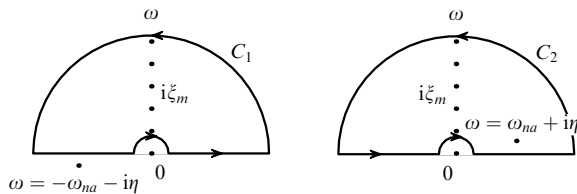
point $\omega = 0$ is bypassed clockwise above along a small semicircle; this path is also closed by a large semicircle in the upper part of the complex plane, but the point $\omega = \omega_{na} + i\eta$ is inside the contour. The integrals along such contours are equal to the infinite sum of residues (times $2\pi i$) at the simple poles $i\xi_m$ ($m = 1, 2, \ldots$) inside the integration contour for the first integral; for the second integral, we should add the residue at the point $\omega = \omega_{na} + i\eta$. By dividing these contour integrals into integrals along separate paths and calculating semiresidues taken with the opposite signs at the point $\omega = 0$ for the first and second integrals, we find the values of J_1 and J_2 in (231). As a result, substituting the values of the integrals in (230), we obtain, after the limit transition $\eta \to 0$,

$$\delta E_a = -\frac{2k_B T}{\hbar} \sum_n \omega_{na} \mu_\alpha^{an} \mu_\beta^{na} \sum_{m=0'}^{\infty} \frac{G_{\alpha\beta}(\mathbf{r}_0, \mathbf{r}_0; i\xi_m)}{\omega_{na}^2 + \xi_m^2}$$

$$- \sum_n \mu_\alpha^{an} \mu_\beta^{na} \, \text{Re} \, G_{\alpha\beta}(\mathbf{r}_0, \mathbf{r}_0; \omega_{an}) \left[1 - \exp\left(-\frac{\hbar\omega_{an}}{k_B T}\right) \right]^{-1}, \tag{232}$$

where the prime at the lower limit in the second sum ($m = 0$) implies that this term should be taken with half the weight.

As $T \to 0$, the Matsubara frequencies $\xi_m = 2\pi k_B T m/\hbar$ become 'concentrated' and the summation in (232) can be replaced by integration in accordance with the usual rule:

$$\sum_m \to \int \rho_\xi \, d\xi \, ,$$

where $\rho_\xi = (\partial\xi/\partial m)^{-1} = \hbar/2\pi k_B T$; as a result, we obtain low-temperature formula (2.22) derived in [99]:

$$\delta E_a = -\frac{1}{\pi} \sum_n \omega_{na} \mu_\alpha^{an} \mu_\beta^{na} \int_0^{\infty} d\xi \, \frac{G_{\alpha\beta}(\mathbf{r}_0, \mathbf{r}_0; i\xi)}{\omega_{na}^2 + \xi^2}$$

$$- \sum_n \mu_\alpha^{an} \mu_\beta^{na} \, \text{Re} \, G_{\alpha\beta}(\mathbf{r}_0, \mathbf{r}_0; \omega_{na}) \, \theta(\omega_{an}), \tag{233}$$

where $\theta(x)$ is the Heaviside function.

Changing the order of summation in (232) and using the expression for the dipole–dipole polarizability of the particle, which is invariant under time inversion,

$$\alpha_{\alpha\beta}^a(i\xi) = \frac{2}{\hbar} \sum_n \frac{\omega_{na} \mu_\alpha^{an} \mu_\beta^{na}}{\omega_{na}^2 + \xi^2}, \tag{234}$$

we obtain the formula

$$\delta E_a = -k_B T \sum_{m=0'} G_{\alpha\beta}(\mathbf{r}_0, \mathbf{r}_0; i\xi_m) \, \alpha_{\alpha\beta}^a(i\xi_m)$$

$$- \sum_n \mu_\alpha^{an} \mu_\beta^{na} \, \text{Re} \, G_{\alpha\beta}(\mathbf{r}_0, \mathbf{r}_0; \omega_{an}) \left[1 - \exp\left(-\frac{\hbar\omega_{an}}{k_B T}\right) \right]^{-1}, \tag{235}$$

which is valid at any temperatures. At $T = 0$, Eqn (233) becomes

$$\delta E_a = -\frac{\hbar}{2\pi} \int_0^{\infty} d\xi \, G_{\alpha\beta}(\mathbf{r}_0, \mathbf{r}_0; i\xi) \, \alpha_{\alpha\beta}^a(i\xi)$$

$$- \sum_n \mu_\alpha^{an} \mu_\beta^{na} \, \text{Re} \, G_{\alpha\beta}(\mathbf{r}_0, \mathbf{r}; \omega_{na}) \, \theta(\omega_{an}). \tag{236}$$

Figure 17. Integration contours C_1 and C_2 on the complex frequency plane for the integrals J_1 and J_2 in (231). The poles at the points $i\xi_m$ are conventionally shown on the imaginary axis.

As was shown in Refs [99], the Green's function $G_{\alpha\beta} = G^0_{\alpha\beta} + G^R_{\alpha\beta}$, which enters all the formulas, is a sum of the Green's function for free space

$$G^0_{\alpha\beta}(\mathbf{r}, \mathbf{r}_0) = \vec{F}^0_{\alpha\beta}(\mathbf{r}, \mathbf{r}_0) + 4\pi\delta_{\alpha\beta}\delta(\mathbf{r} - \mathbf{r}_0)$$

and the Green's function

$$G^R_{\alpha\beta}(\mathbf{r}, \mathbf{r}_0) = \vec{F}^R_{\alpha\beta}(\mathbf{r}, \mathbf{r}_0)$$

that determines the effect of the sample. To take this effect into account, we should use $G^R_{\alpha\beta}(\mathbf{r}, \mathbf{r}_0)$ in all the formulas containing the Green's functions. It is obvious that having found the level shift, we can determine the shift of the transition frequency under the effect of the sample, e.g., between the states a and b, as $\delta\omega_{ab} = (\delta E_a - \delta E_b)/\hbar$.

We also give the formula obtained in [99] for the rate of transition of a particle from a state a into a certain set of states $\{n\}$ at any temperature:

$$\gamma_a = 2\hbar^{-1}\sum_n \mu^{an}_\alpha \mu^{na}_\beta \operatorname{Im} G^R_{\alpha\beta}(\mathbf{r}_0, \mathbf{r}_0; \omega_{na})$$

$$\times \left[1 - \exp\left(-\frac{\hbar\omega_{na}}{k_{\mathrm{B}}T}\right) \right]^{-1}. \tag{237}$$

It can be easily shown that (237) follows from (228) by using the relation [99]

$$J_{\alpha\beta}(\mathbf{r}, \mathbf{r}'; \omega) = 2\hbar \left[1 - \exp\left(-\frac{\hbar\omega}{k_{\mathrm{B}}T}\right) \right]^{-1} \operatorname{Im} G^R_{\alpha\beta}(\mathbf{r}, \mathbf{r}'; \omega).$$

We note that the authors of [99] used the interaction Hamiltonian in the form $V' = -\boldsymbol{\mu}\mathbf{D}$, where $\mathbf{D}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}, t) + 4\pi\boldsymbol{\mu}(t)\delta(\mathbf{r} - \mathbf{r}_0)$ is the transverse displacement field; this Hamiltonian differs from the more usual interaction Hamiltonian $V'' = -\boldsymbol{\mu}\mathbf{E}$. However, as was shown in [99–102], the total Hamiltonians that include parts describing the interaction are connected by a certain unitary transformation. As a consequence of this identity, the corrections calculated in the perturbation theory are equal: $\delta E(V') = \delta E(V'')$.

To conclude this section, we note that the above-considered problems can also be solved using various other methods. We only tried to illustrate the efficiency of the application of the theory of thermostimulated fields induced in solids. We also note that in recent years, works have appeared in which such problems were considered under more complex conditions, e.g., problems of spontaneous emission of a particle near nanoobjects (see, in particular, Ref. [103]).

6. Conclusions

Thermostimulated electromagnetic fluctuations, just like Brownian motion, are a natural physical phenomenon that requires no special preparation. Its role in nature can hardly be overestimated. The problems of fluctuation electrodynamics are directly related to problems such as the van der Waals interaction of bodies, heat transfer between bodies separated by a vacuum gap, the capture of atoms, molecules, and coherent material states by electromagnetic traps, and a number of important physicochemical phenomena near the surface of condensed media, such as adsorption and desorption of atoms and molecules.

Theoretical methods for calculating the properties of fluctuating electromagnetic fields in fact amount to solving a mathematical physics problem based on the set of Maxwell equations in a chosen geometry with given boundary conditions and to the application of the fluctuation–dissipation theorem. Modern theories have allowed unambiguously separating the thermal field of a heated body into two parts, a propagating field, which can exist without a source that generates it, and a near quasistationary field, which is completely related to the heated body itself. In foreign literature, the near quasistationary field is called the evanescent field. An analysis shows that the statistical properties of these two parts of the single thermal field differ substantially. In particular, the characteristic length of the spatial correlation of traveling waves is of the order of the Wien wavelength, whereas in the case of the near field, this scale can be of the order of the distance to the body surface. We note that in the important case of thermostimulated radiation of plane-layered media, all correlation properties of fields can be expressed through physically clear electrodynamic characteristics, namely, through the Fresnel coefficients of the interfaces.

The theoretical methods of solving the problem of thermostimulated fields allow investigating both the case of a complete thermodynamic equilibrium and the practically very important case where the temperature of the body differs from the temperature of the surroundings. We note that the spectral characteristics of thermostimulated fields necessarily contain resonance features related to the intrinsic states of the body surface, the so-called surface polaritons. This allows studying the electrodynamic properties of the surface and its states.

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