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LETTERS TO THE EDITORS

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On the Hartman paradox, electromagnetic wave tunneling, and supraluminal velocities

(comment on the paper "Tunneling of electromagnetic waves: paradoxes and prospects" by A B Shvartsburg)

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Abstract. Some paradoxes are discussed concerning the interpretation of the passage of particles and electromagnetic waves (pulses) across potential barriers and through nonuniform media, in particular, those with frequency dispersion and therefore dissipation. It is emphasized that a rigorous nonstationary approach does not entail any supraluminal velocities for the transfer of physical substances, although supraluminal velocities are indeed possible for several kinematically defined velocities, e.g., for the group velocity.

The Hartman paradox [1, 2] consists in the fact that the quantum particle tunneling velocity, defined in terms of the time of phase delay ('phase tunneling time' τ_{p}), may exceed the speed of light c. This paradox emerged long before the publication of Hartman's paper (see review [2]). Similar to the Hartman paradox is the Einstein–Podolsky–Rosen paradox [3–5] and its related supraluminal 'velocities of information transfer.' Supraluminal velocities also emerge in other papers of the past century and especially in recent works concerned with the tunneling of electromagnetic waves [2] and the propagation of pulses through different dispersive media and structures [6–14], including active structures, left-hand media, and photonic crystals (PCs) [15]. Thus, different paradoxes occur. Meanwhile, as stated by R Feynman, real paradoxes never occur in physics: "The paradox is only a conflict between reality and your feeling what reality ought to be" [16, p. 58]. Such a conflict often occurs due to substitution of concepts or when certain concepts are illegitimately applied to phenomena. Although it is implied in review [2] that the occurrence of 'supraluminal velocities' and sources may not lead to a higher-than-c energy or information transfer velocity, this circumstance is not stressed and the corresponding explanations of the paradox are not given.

Physical processes are subdivided into stationary (periodic in time) and nonstationary (arbitrarily time dependent). Single-frequency (harmonic in time) and multifrequency processes are also among the stationary ones, as are static (time-independent) processes. Naturally, there are no strictly stationary processes in nature; however, the notions true only for stationary processes are often applied to the nonstationary ones.

Specifically, we consider the tunneling of a quantum particle across a potential barrier. The particle is assigned certain values of energy E and momentum, i.e., the wave function in the momentum representation describes a plane wave to the left and to the right of the barrier and two counterpropagating plane waves inside the barrier. According to quantum mechanics, the particle coordinate to the left of the barrier is equiprobable throughout a semi-infinite interval (the same applies to the domain to the right of the barrier) and the phase of the wave function is defined up to $2\pi n$, and it is therefore meaningless to speak of τ_{p}-based 'tunneling time' at all. It is always possible to choose the barrier profile in such a way as to satisfy the equality $\tau_{\mathrm{p}} = 0$, i.e., to have the infinite tunneling 'velocity.' Only the tunneling probability is meaningful. This signifies that it is possible to acquire statistics on discovering particles to the left and right of the barrier for a large number of experiments, and that averaging over the ensemble for the same initial states of the particles at the left for $t = -\infty$ would yield the transmission and reflection probabilities.

Similarly, it makes no sense to speak about the transmission velocity for a stationary electromagnetic wave (harmonic signal) through a filter: it merely changes the amplitude and shifts the phase. The phase shift implies that we obtain the same value of the signal at a forward or backward distance on the time scale equal to an (arbitrary) integer number of periods, i.e., this process is inherently not a 'signal.' A periodic process is infinite in time, and its past is perfectly defined, while its future is perfectly predictable. Furthermore, for a periodic process, there is no current (instantaneous) time. As soon as current time is introduced (and, accordingly, the instantaneous spectrum), an aperiodic process is implied (the future is not defined). In dynamics, the wave function of a particle 'incident on a barrier' is a wave packet whose momenta are distributed over some (strictly speaking, infinite) domain, and the coordinate is also 'blurred,' i.e., defined with some probability density.

Quasiperiodic processes are considered quite frequently, especially in the theory of pulse propagation [17–20]. A pulse is a nonstationary wave, and various velocities may be introduced for it. For a stationary electromagnetic wave, it

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is possible to introduce two velocities: the velocity of energy transfer $\mathbf{v}_e(\omega)$ and the phase velocity $\mathbf{v}_p(\omega) = \mathbf{k}\omega/k^2$ (we do not consider the pulse transfer velocity). An electromagnetic wave is the energy wave carried by photons with the speed of light c. The energy velocity $\mathbf{v}_e$ in a medium is defined by the combined action of absorption and reradiation (possibly delayed) of photons by the particles of the medium in different directions, i.e., $\mathbf{v}_e$ depends on the material dispersion and the inhomogeneity properties of the medium. That is why $\mathbf{v}_e$ cannot exceed c in magnitude, whereas the phase velocity $\mathbf{v}_p$ is a kinematic velocity. No transferable material substance corresponds to this velocity, which may exceed c in modulus.

It is noteworthy that there exist dispersive media (to be more precise, dispersion laws) for which $\mathbf{v}_e(\omega) = \mathbf{v}_p(\omega)$ in a monochromatic wave. Examples are provided by metals at low frequencies, distilled water, and sea water [21].

The energy wave velocity (the velocity of energy transfer) $\mathbf{v}_e$ may also be determined by the structure. In hollow perfectly conducting waveguides, for instance, $\mathbf{v}_e$ is defined by the spatio-spectral group of two waves (for the H_{10} wave of a square waveguide), of several or an infinite number of waves (for other waves and waveguides), which propagate with a speed c at different angles to the side walls and experience perfectly elastic reflections from them. In PCs and slow-wave structures (SWSs), $\mathbf{v}_e$ is determined by rereflections from the layers or other periodic inclusions.

It is customary to introduce the group velocity $v_g = |\mathbf{v}_g| = [\partial k_z'(\omega)/\partial\omega]^{-1}$ proceeding from the one-dimensional dispersion law $k_z(\omega) = k_z'(\omega) - \mathrm{i}k_z''(\omega)$ (here, $k_z'(\omega)$ is the phase constant and $k_z''(\omega)$ is the damping constant), unreservedly assuming that $v_e = v_g$. Of course, there are papers containing indications that v_g is meaningless in dissipative and active media and structures. Mathematically, v_g may formally be introduced, but there are no grounds to do this for a monochromatic wave because there is no spectral group of waves. Here, a substitution of notions occurs, which results in paradoxes [15, 22]. In particular, for several media and structures, it is possible that $v_g > c$ [2, 9–12, 15, 21–24].

Stokes introduced the group velocity as the beating propagation velocity of two monochromatic processes with infinitely close frequencies. Naturally, this is a kinematic velocity, which does not correspond to the transfer of any physical substance. Meanwhile, in a wealth of papers, textbooks, and monographs, the group velocity is identified with the energy transfer velocity without reservations.

This may bring up the question: What about the Leontovich–Lighthill theorem [23, 25–29] stating that $\mathbf{v}_e = \mathbf{v}_g$ in conservative systems with a Hamiltonian that is quadratic in generalized coordinates and momenta? There is no doubt that the energy of a monochromatic wave is transferred with a group velocity along the axes of hollow waveguides and loss-free SWSs (but not along any direction), in loss-free PCs without material dispersion, and in collisionless plasmas along the direction of $\nabla_{\mathbf{k}}\omega(\mathbf{k})$. However, there are no perfectly conducting materials or loss-free materials in nature, and in a transient case, this is not strictly fulfilled even without dissipation [19]. Dispersion is always associated with losses, as losses are associated with dispersion [30]. That is why the notion of low losses is introduced, whereby it is supposedly legitimate to use v_g. But even in this case, it is possible that $v_g > c$ in the opacity bands of waveguides, PCs, and SWSs [15, 24], which acquire a low transmittance in the low-loss case, as well as in regions with anomalous dispersion in different media [15, 31, 32]. Furthermore, when losses tend to zero, $v_g \to \infty$ in these bands. The same occurs in active media [10–12, 22].

That is why the group velocity is introduced with several reservations in the modern literature. It is assumed that it can be used in transmission windows, where losses are negligible. However, when considering oscillator models of media (both classical and quantum) [32], it is easily seen that an infinite oscillator frequency separation corresponds to perfect transmission windows of final width. In this case, the dispersion vanishes, i.e., $\mathbf{v}_g = \mathbf{v}_p = \mathbf{v}_e$. In dispersion theory, v_g is commonly introduced as the first approximation in using asymptotic techniques for calculating the spectral intervals that correspond to the pulse propagation [17–19]. Higher approximations may also be considered by taking the distortion of the pulse shape during propagation into account.

This approach is asymptotic (i.e., not rigorous [18]). In this case, once again, a substitution of notions commonly occurs: the pulse propagation velocity is identified with v_g. Several velocities may be introduced for a pulse: the precursor velocity, the velocity of the leading edge, the velocity of the trailing edge, the travel velocity of the envelope peak, the velocity of the pulse as a whole [20], the velocity of energy transfer at each specific point of the pulse, the velocity of signal transfer by the pulse, etc. It is meaningless to speak about the pulse velocity without concretization [20, 33, 34], for instance, about the velocity of a cloud, each part of which moves with its own velocity. The velocity V_e of motion of a pulse as a whole may also be defined in several ways, for instance, spectrally.

Let $S(\omega)$ be the spectral energy density of a pulse, i.e., let the energy $E(t)$ have the form

$$E(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S(\omega)\exp(\mathrm{i}\omega t)\,\mathrm{d}\omega\,. \tag{1}$$

Pulse energy (1) is a function of time, which decreases in dissipative media and increases in active ones, and $S(\omega) = S^*(-\omega)$ may be defined in terms of the field intensities (the pulse shape) and the parameters of the medium. The energy flux in the range $\mathrm{d}\omega$ is $\mathbf{v}_e(\omega)|S(\omega)|\,\mathrm{d}\omega$, where $\mathbf{v}_e(\omega)$ is the spectral energy transfer velocity at a frequency ω. Therefore,

$$\mathbf{V}_e(t) = \frac{1}{2\pi E(t)}\int_{-\infty}^{\infty} \mathbf{v}_e(\omega)\big|S(\omega)\big|\,\mathrm{d}\omega = \frac{1}{\pi E(t)}\int_0^{\infty} \mathbf{v}_e(\omega)\big|S(\omega)\big|\,\mathrm{d}\omega\,. \tag{2}$$

The energy density in 'supraluminal pulses' in active media increases with time, and invariably $V_e < c$ in this case. We emphasize that the energy velocity $\mathbf{v}_e(\mathbf{r}, t)$ defined by Umov is also a function of the coordinates of a point in space, and therefore expression (2) defines the global velocity.

Let a plane pulse propagate along the z coordinate. Relation (2) is applicable to diffraction from the inhomogeneities of the medium. In this case, for plane monochromatic waves, there is an analogy to quantum mechanics: the passage of a particle with an energy E across a barrier or a well with a potential $U(z)$ corresponds to diffraction by a dielectric layer with the permittivity $\varepsilon(z) \sim 1 - U(z)/E$. In the diffraction of the pulse, a part of the pulse is reflected and travels with a negative velocity; in this case, there may be two or more maxima in the time dependence, depending on the function $U(z)$. When a particle passes across a barrier, it makes sense to speak only about the rate of probability density transfer or of the velocity of probability density peaks; in this case, it is well to bear in mind that the Schrödinger equation is a first-order equation in time, i.e., is not relativistically invariant.

Proceeding from the dispersion law $k_z(\omega)$, it is possible to introduce an infinite number of quantities like v_g with the dimension of speed. One may consider the velocity of signal transfer by a pulse, the velocity of motion of the leading edge, and the velocity of energy motion in the pulse, which are commonly identified but do not coincide in general. In particular, the velocity of signal transmission is defined by detection and depends on the sensitivity of the detector, its response time [2, 18], and the velocity of the leading edge, while the velocity of energy motion is different in different parts of the pulse: it is highest in the precursor and lowest in the tail [34], while in the middle part, it is not equal to the velocities at the edges. The velocities of wave fronts may also be defined in different ways, for instance, as the velocities of the peaks of envelope derivatives. The envelope itself should be defined in terms of the analytical signal [18]; however, this approach is usually hard to realize, and therefore methods of averaging, like the moving average, are typically used.

It is quite frequently stated that a pulse moves as a whole with a velocity v_g when the spectral width $\Delta\omega$ of the signal is small in comparison with the carrier frequency ω_0 (a quasiharmonic process). However, in reality, the spectrum of a pulse is always infinite (albeit with a low energy at very high and sometimes low frequencies), and spreading inevitably occurs at long distances. That is why in this case, too, v_g is no more than the first approximation to the velocity of a wave front in the absence of dissipation, which is inevitably present. There is no way of satisfying the condition $\Delta\omega \ll \omega_0$ rigorously (it can be satisfied only approximately, when the energy of the frequencies $|\omega - \omega_0| > \Delta\omega$ is neglected). However, in passing to the limit $\Delta\omega \to 0$, we obtain a monochromatic wave, in which the velocity of energy motion is in no way related to v_g [21] and there is no spectral wave group! It is noteworthy that in waveguides, there is a spatio-spectral group of waves that travel (and transfer energy) with a velocity c at an angle to the axis.

Therefore, the equality $v_g = v_e = c^2/v_p$ is a consequence of geometric (kinematic) relations. In a collisionless plasma, charged particles oscillate with the phase shift $-\pi/2$ relative to the field of a plane wave and reradiate energy in both directions, i.e., there are forward and backward energy fluxes with the velocity c. This is clearly seen from the one-dimensional integral equation (IE) for the electric field $E_x(z)$ with the one-dimensional scalar Green's function (GF) $G(z-z') = -\mathrm{i}\exp(-\mathrm{i}k_0|z-z'|)/(2k_0)$ and the polarization current $J_{px} = -\mathrm{i}\varepsilon_0\omega_p^2 E_x(z)/\omega$ in the integrand. For $\omega > \omega_p$, the forward flux prevails over the backward one, and the total flux moves with the velocity v_g. For $\omega = \omega_p$, both fluxes cancel, the energy does not propagate (when the plasma structure is infinite in size), and the field ceases to depend on z and acquires an oscillatory character. Specifically, we have the IE

$$E_x(z) = \frac{\mathrm{i}\omega_p^2}{2c\omega}\int_{-L}^{L} \exp\left(-\mathrm{i}k_0|z-z'|\right) E_x(z')\,\mathrm{d}z'. \qquad (3)$$

In this case, with infinitely low losses $k_0 = \omega/c - \mathrm{i}\delta$, $\delta \to 0$, and infinite layer thickness $L \to \infty$, the condition $E_x(z) = \text{const}$ implies that $\omega = \omega_p$. For all z, the particles oscillate in phase and radiate waves equally in both directions with the velocity c, and the resultant flux is zero. A wave with the phase velocity $v_p = c/(\omega^2 - \omega_p^2)^{1/2}$ satisfies Eqn (3). When $\omega < \omega_p$, Eqn (3) has a damped solution $E_x(z) = E_0 \exp[-(\omega_p^2 - \omega^2)^{1/2} z/c]$, and the fluxes are again compensated in this case. The situation is different in a collisional plasma, in which there is always an energy flux. The group delay time can no longer be used in the tunneling across a plasma layer.

Another substitution of notions is encountered quite often: the energy velocity is defined in terms of the Poynting vector and the energy density (which is entirely correct); but for the energy density, the formula introduced by Brillouin is used, which involves differentiation of the permittivity with respect to time [30] and fits v_e to v_g. It is pertinent to note that this result (like the introduction of v_g) was obtained in the first approximation and on the basis of asymptotic expansions (in the sense of Ref. [18]), which are not necessarily convergent.

A rigorous analysis invites derivation of higher-order corrections and investigation of the convergence of the series obtained. In transmission windows (where there is no dispersion), $v_g = v_p = v_e$ and Brillouin's result is trivial. The introduction of $v_e(t)$ therefore calls for a rigorous determination of the energy density per unit volume. It is a function of time and is defined by the free energy of the field and matter [21, 29, 33]. Therefore, it is necessary to consider the entire prehistory of the field–matter interaction and to take both the accumulated field energy and the accumulated energy of matter particles into account [21, 32]. This cannot be done for purely monochromatic processes: they should be approximated by quasimonochromatic processes, for instance, with amplitudes smoothly increasing up to a specified magnitude. For a monochromatic process, v_e may be rigorously introduced by a limiting process starting from a quasimonochromatic process [21] with the inclusion of its prehistory. Interestingly, for an electromagnetic field in empty space, which is void of moving particles, the dependence of energy on the prehistory is naturally neglected [35]. Ordinarily, the field energy is transferred (when there are no matter fluxes). That is why in the transmission window of a collisionless plasma, the inclusion of the particle oscillation energy yields $v_e = v_g < c$ [32], while the inclusion of only the field part of the energy leads to the incorrect result $v_e = v_p > c$.

It follows from the aforesaid that the group delay time τ_g cannot be used as the tunneling time in stationary processes either. For low-loss quasimonochromatic processes, it may be used approximately [18] and with great caution. Naturally, wave packets are to be considered in the case of tunneling. To such a packet, there corresponds a particle with an uncertainty in momentum and coordinate, while the packet velocity is the probability transfer rate. This also brings up the question: What is meant by the tunneling velocity? It may be defined as the probability transfer rate from the velocity of the peak of the squared wave function (of the peak of the squared envelope). In the electrodynamics of continuous media (and of active media, in particular), the velocity of the envelope peak, which may be much higher than c [6], is a kinematic characteristic unrelated to energy transfer. The envelope itself defined in terms of an analytical signal may have a supraluminal precursor [18]. This does not violate the causality principle because the envelope is not a signal and is not bound to obey this principle [18]. In this case, a discontinuity travels with the speed of light, while the real Sommerfeld–Brillouin precursor propagates at a velocity slightly lower than c.

So, what is the way to solve the problems involving the tunneling of electromagnetic waves (pulses) through nonuniform structures? The answer is simple: these problems pertain to nonstationary electrodynamics and should be solved by its methods, i.e., on the basis of the solution of transient Maxwell equations with the inclusion of matter dispersion (temporal and spatial). This approach is also pursued, in particular, in the works of the author of Ref. [2] (see, e.g., Refs [36, 37]). There are several numerical approaches, for example, finite-difference, finite-element, and variational. More suited to this purpose are the methods of spatio-temporal IEs and integro-differential equations based on the spatio-temporal tensor GFs [33, 34, 38]. These functions satisfy the causality condition, with the consequence that the solution of the equations also satisfies it.

This signifies that supraluminal velocities cannot occur for real quantities (the velocity of field energy and momentum transfer, the velocity of motion of the field front, etc.) defined in terms of these fields. The highest possible velocity of discontinuity motion is c. In a stationary case, it makes sense to speak about the energy (and momentum) transfer rate across some small nonclosed surface, but not about the tunneling velocity. As regards the transient quantum 'supraluminal' tunneling, i.e., the tunneling of wave packets, there are special features [39] arising from nonlocality. In particular, it is meaningless to formulate the problem in terms of the time of particle residence inside the barrier [39], i.e., also on its transit velocity. In this case, the situation is similar to the possible 'mathematical' supraluminal envelope of a pulse, whose energy (and hence the signal and all perturbations) is transferred with subluminal velocities, while the envelope may have a nonzero value ahead of the discontinuity [18].

In the introduction of different velocities, it should therefore be clearly defined what precisely is implied by this.

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