

# The fundamental constants in physics<sup>1</sup>

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**Abstract.** We discuss the fundamental constants of physics in the Standard Model and possible changes of these constants on the cosmological time scale. Grand Unification of the strong, electromagnetic, and weak interactions implies relations between the time variation of the fine structure constant  $\alpha$  and of the QCD scale  $\Lambda_c$ . A change in  $\alpha$  of  $10^{-15}$ /year, as seen by an astrophysical experiment, implies a time variation in  $\Lambda_c$  of at least  $10^{-15}$ /year. An experiment in quantum optics at the MPQ in München, which was designed to seek a time variation in  $\Lambda_c$ , is discussed.

## 1. The Standard Model

The Standard Model consists of (a) the gauge theory of strong interactions, quantum chromodynamics (QCD) [1], and (b) the gauge theory of electroweak interactions, based on the gauge group  $SU(2) \times U(1)$  [2].

QCD is an unbroken gauge theory, based on the gauge group  $SU(3)$  acting in the internal ‘color’ space. The basic fermions of the theory are six quarks, which form color triplets. Gluons, the eight massless gauge bosons, are  $SU(3)$  octets. Interactions between quarks and gluons are dictated by the gauge properties of the theory. Quarks and gluons are coupled by the vertex

$$g_s \bar{q} \gamma_\mu \frac{\lambda_i}{2} q A_i^\mu,$$

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where  $q$  are the quark fields and  $A_i^\mu$  are the eight gluon fields. The eight  $SU(3)$  matrices are denoted by  $\lambda_i$ . The strength of the coupling constant is given by  $g_s$ .

QCD is a non-Abelian gauge theory. There is a direct coupling of gluons among each other. There are both a trilinear coupling proportional to  $g_s$  and a quadrilinear coupling, proportional to  $g_s^2$ . It is assumed that the QCD interaction leads to a confinement of all colored quanta, in particular of quarks and gluons. But this has thus far not been proven. Replacing the continuous space–time continuum by a lattice, one can solve QCD field equations with a computer. The results confirm the confinement hypothesis.

The experimental data are in very good agreement with QCD [3]. Quantum chromodynamics has the property of asymptotic freedom. The strength of the quark–gluon interaction tends to zero on a logarithmic scale at high energies. At low energies, the interaction strength is large. Therefore, the confinement property of QCD might indeed be true.

The equations for  $\alpha_s = g_s^2/4\pi$  describing the renormalization of the coupling are

$$\begin{aligned} \mu \frac{\partial \alpha_s}{\partial \mu} &= -\frac{\beta_0}{2\pi} \alpha_s^2 - \frac{\beta_1}{4\pi^2} \alpha_s^3 - \dots, \\ \beta_0 &= 11 - \frac{2}{3} n_f, \\ \beta_1 &= 51 - \frac{19}{3} n_f \end{aligned} \quad (1)$$

where  $n_f$  is the number of relevant quark flavors.

Since the interaction is weak at high energies, quarks and gluons appear nearly as pointlike objects at small distances. This has been observed in experiments of the deep inelastic scattering of electrons, muons, and neutrinos on nuclear targets.

The strong coupling constant at high energies is small, but not zero. Therefore, violations of the scaling behavior of the cross sections are expected. This has been seen in many experiments. The value of the QCD coupling constant  $\alpha_s = g_s^2/4\pi$  depends on the energy. In an analysis of scaling violations [3], the value

$$\alpha_s(M_Z^2) \approx 0.1187 \pm 0.002 \quad (2)$$

has been found, where  $M_Z$  is the mass of the Z boson and  $M_Z \approx 91.2$  GeV.

We can express  $\alpha_s(\mu)$  as a function of the QCD scale  $\Lambda_c$ :

$$\alpha_s(\mu)^{-1} \approx \left( \frac{\beta_0}{4\pi} \right) \ln \left( \frac{\mu^2}{\Lambda_c^2} \right),$$

$$\beta_0 = \left( 11 - \frac{2}{3} n_f \right). \quad (3)$$

Experiments give the value

$$\Lambda_c \approx 217_{-23}^{+25} \text{ MeV}. \quad (4)$$

The electroweak gauge theory is based on the gauge group  $SU(2) \times U(1)$ . Hence, there are three W bosons, related to the  $SU(2)$  group, and a B boson, related to the  $U(1)$  group. The left-handed quarks and leptons are  $SU(2)$  doublets, the right-handed leptons and quarks are singlets. Parity is maximally violated.

The gauge invariance of the  $SU(2) \times U(1)$  model is broken by the ‘Higgs’ mechanism [4]. The masses of the gauge bosons are generated by a spontaneous symmetry breaking. Goldstone bosons appear as longitudinal components of the gauge bosons. The standard Higgs mechanism involves a self-interacting complex doublet of scalar fields. In the process of symmetry breaking, the neutral component of the scalar doublet acquires a vacuum expectation value  $V$ , which is determined by the Fermi constant of the weak interactions. Therefore, the vacuum expectation value is known from the experiments, if the theory is correct:

$$V \cong 246 \text{ GeV}. \quad (5)$$

This energy sets the energy scale for the electroweak symmetry breaking. Three massless Goldstone bosons are generated, but they are absorbed to give masses to the  $W^+$ ,  $W^-$ , and Z bosons. One component of the complex doublet is not absorbed. This is the Higgs boson, thus far a hypothetical particle. It would be the only elementary scalar boson in the Standard Model. There are hopes to find this particle with the new LHC accelerator at CERN (starting in 2009).

The electroweak model involves two neutral gauge bosons, which are mixtures of the  $W_3$  and B bosons, the Z boson, and a photon. The associated electroweak mixing angle  $\Theta_w$  is a fundamental parameter that has to be fixed by experiment. It is expressed in terms of the Z mass, the Fermi constant, and the fine structure constant  $\alpha$ :

$$\sin^2 \Theta_w \cos^2 \Theta_w = \frac{\pi \alpha (M_Z)}{\sqrt{2} G_F M_Z^2}. \quad (6)$$

Experiments yield the value  $\sin^2 \Theta_w \approx 0.231$ .

We note that the electroweak mixing angle is also related to the mass ratio  $M_W/M_Z$ . If we neglect radiative corrections, we find

$$\sin^2 \Theta_w = 1 - \frac{M_W^2}{M_Z^2}, \quad M_Z = \frac{M_W}{\cos \Theta_w}. \quad (7)$$

In the Standard Model, the interactions depend on 28 fundamental constants. These are:

- the gravity constant  $G$ ,
- the fine-structure constant  $\alpha$ ,
- the coupling constant  $g_w$  of the weak interactions,

- the coupling constant  $g_s$  of the strong interactions,
- the mass of the W boson,
- the mass of the Higgs boson,
- the masses of the three charged leptons  $m_e$ ,  $m_\mu$ , and  $m_\tau$ ,
- the neutrino masses  $m(\nu_1)$ ,  $m(\nu_2)$ , and  $m(\nu_3)$ ,
- the masses of the six quarks  $m_u$ ,  $m_d$ ,  $m_c$ ,  $m_s$ ,  $m_t$ , and  $m_b$ ,
- four parameters describing the flavor mixing of the quarks, and six parameters describing the flavor mixing of the leptons, measured by neutrino oscillations.

In physics, we are dealing with the laws of nature, but little thought is given to the boundary conditions of the Universe, related directly to the Big Bang. We do not know at the moment what role is played by the fundamental constants, but these constants could form a bridge between the boundary conditions and the local laws of nature. Thus, they would be accidental relics of the Big Bang.

Some physicists believe that at least some of the fundamental constants are just cosmic accidents, fixed by the dynamics of the Big Bang. The constants are therefore arbitrary, depending on the details of the Big Bang. Obviously, there is no way to calculate the fundamental constants in this case.

Some fundamental constants might be cosmic accidents, but it is unlikely that this is the case for all fundamental constants. New interactions to be discovered, for example, with the new LHC accelerator at CERN might offer a way to calculate at least some of the fundamental constants.

We do not understand either why the fundamental constants are constant in time. Small time variations are indeed possible, and even suggested by astrophysical experiments. In the theory of superstrings, we expect time variations of the fundamental constants, in particular, of the fine structure constant, of the QCD scale  $\Lambda_c$ , and of the weak interaction coupling constant [5, 6].

If the fundamental constants are found to be changing in time, then they are not just numbers, but dynamic quantities that change according to some deeper laws that we have to understand. These laws would be truly fundamental and may even point to the way to a unified theory including gravity.

## 2. Fundamental constants and the Standard Model

The Standard Model of particle physics is the theory of phenomena observed in particle physics. However, it depends on 28 fundamental constants. There is no way to calculate these constants within the Standard Model.

The most famous fundamental constant is the fine structure constant  $\alpha$ , introduced in 1916 by Arnold Sommerfeld:

$$\alpha = \frac{e^2}{\hbar c}. \quad (8)$$

The electromagnetic coupling  $e$  enters into this constant, as do the constant of quantum physics  $\hbar$  and the speed of light  $c$ . Sommerfeld realized that  $\alpha$  is a dimensionless number, close to the inverse of the prime number 137. Experiments give the value  $137.03599911(46)$  for  $\alpha^{-1}$  [3].

In 1936, Heisenberg proposed the relation

$$\alpha = 2^{-4} 3^{-3} \pi, \quad (9)$$

which gives  $\alpha^{-1} = 137.51$ . In 1971, Wyler [7] published the following expression for  $\alpha$ :

$$\alpha = \frac{9}{8\pi^4} \left( \frac{\pi^5}{24 \cdot 5!} \right)^{1/4}, \quad (10)$$

which gives  $\alpha^{-1} = 137.03608$ .

Feynman wrote about the fine structure constant [8]: “It has been a mystery ever since it was discussed more than fifty years ago, and all good theoretical physicists put this number up on their wall and worry about it. Immediately you would like to know where this number for a coupling comes from: is it related to  $\pi$  or perhaps to the base of the natural logarithms? Nobody knows. It’s one of the greatest mysteries of physics: a magic number that comes to us with no understanding by man... .”

In quantum field theory, the strength of an interaction is not a fixed constant, but a function of the energy involved. The ground state of a system is filled with virtual pairs of quanta, e.g., with  $e^+e^-$ -pairs in QED. Thus, an electron is surrounded by  $e^+e^-$ -pairs. The virtual electrons are repelled by the electrons, the virtual positrons are attracted. The electron charge is partially screened by virtual positrons. At relatively large distances, the electron charge is smaller than at distances less than  $\lambda_c$ . The dependence on the energy is described by the renormalization-group equations of Gell-Mann and Low [9]:

$$\frac{d}{d \ln(q/M)} e(q) = \beta(e), \quad (11)$$

where

$$\beta(e) = \frac{e^3}{12\pi^2} + \text{higher-order terms}. \quad (12)$$

In QED, not only virtual  $e^+e^-$  pairs but also the  $\mu^+\mu^-$  and  $\tau^+\tau^-$  pairs, as well as quark–antiquark pairs must be included. It follows that the fine structure constant  $\alpha$ , at the mass of the Z boson, should be the inverse of 128, in good agreement with the experimental data found with the LEP accelerator [3].

Another fundamental parameter of the Standard Model is the proton mass. In QCD, the proton mass is a parameter that can be calculated as a function of the QCD scale parameter  $A_c$  and of the light-quark masses. The QCD scale parameter has been determined in many experiments:

$$A_c = 217 \pm 25 \text{ MeV} \quad (13)$$

( $A_c$  is defined in the modified minimal subtraction ( $\bar{\text{MS}}$ ) scheme for five quark flavors).

The QCD theory gives a very clear picture of mass generation. In the limit where the quark masses are neglected, the nucleon mass is the confined field energy of the gluons and quarks. It can be written as

$$M(\text{nucleon}) = \text{const } A_c. \quad (14)$$

The coefficient const has been calculated using the lattice approach to QCD. It is about 3.9, predicting the nucleon mass in the limit  $m_q = 0$  to be about 860 MeV. The observed nucleon mass (about 940 MeV) is higher, due to the contributions of the mass terms of the light quarks u, d, s, which in reality are not massless.

The proton mass can be decomposed as

$$M_p = \text{const } A_c + \langle p | m_u \bar{u}u | p \rangle + \langle p | m_d \bar{d}d | p \rangle + \langle p | m_s \bar{s}s | p \rangle + c_{\text{elm}} A_c. \quad (15)$$

The last term describes the electromagnetic self-energy. It is proportional to the QCD scale  $A_c$ . Calculations give [10]

$$c_{\text{elm}} A_c \approx 2.0 \text{ MeV}. \quad (16)$$

The up-quark mass term contributes about 20 MeV to the proton mass, the d-quark mass term about 19 MeV. Thus, the d-contribution to the proton mass is about as large as the u-contribution, although there are two u-quarks and only one d-quark in the proton. This is because the d mass is larger than the u mass.

In chiral perturbation theory, the u and d masses can be estimated as [11]

$$m_u \approx 3 \pm 1 \text{ MeV}, \quad m_d \approx 6 \pm 1.5 \text{ MeV}. \quad (17)$$

These masses are normalized at the scale  $\mu = 2 \text{ GeV}$ . We note that quark masses are not the masses of free particles but are dynamical quantities. They depend on the energy scale  $\mu$  relevant for the discussion.

The mass of the strange quark can also be estimated in the chiral perturbation theory [11]. At  $\mu = 2 \text{ GeV}$ , the result is

$$m_s \approx 103 \pm 20 \text{ MeV}. \quad (18)$$

The strange quark mass is about 20 times larger than the d-mass. Although there are no valence s-quarks in the proton, the  $\bar{s}s$  pairs contribute about 35 MeV to the proton mass, i.e., more than the  $\bar{u}u$  or  $\bar{d}d$  pairs, due to the large ratio  $m_s/m_d$ . Heavy quarks, e.g., c-quarks, contribute at most  $\sim 1 \text{ MeV}$  to the nucleon mass [12].

We can decompose the proton mass as follows, leaving out the contribution of the heavy quarks:

$$\begin{aligned} M_p &= 938 \text{ MeV} \\ &= (862 + 20 + 19 + 35 + 2) \text{ MeV} \\ &\quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \\ &\quad \text{QCD} \quad \text{u-quarks} \quad \text{d-quarks} \quad \text{s-quarks} \quad \text{QED}. \end{aligned} \quad (19)$$

The masses of the heavy quarks c and b can be estimated by considering the spectra of particles containing c or b quarks, e.g., the charm mesons or the B mesons, with the result [3]

$$m_c = 1.15 - 1.35 \text{ GeV } (\bar{\text{MS}} \text{ mass}), \quad m_b = 4.1 - 4.4 \text{ GeV } (\bar{\text{MS}} \text{ mass}). \quad (20)$$

The dark corner of the Standard Model is the sector of the fermion masses. There are six quark masses, three charged fermion masses, three neutrino masses, four flavor-mixing parameters of the quarks, and six flavor-mixing parameters of the leptons (if neutrinos are Majorana particles). These parameters make up 22 of the 28 fundamental constants.

What are the fermion masses? We do not know. They might also be due to a confined field energy, but in that case the quarks and leptons would have to have a finite radius, as

in composite models. The masses would be generated by a new interaction. The experiments give a limit on the internal radius of leptons and quarks, which is of the order of  $10^{-17}$  cm [13].

In the Standard Model, the masses of the leptons and quarks are generated spontaneously, like the W and Z masses. Each fermion couples with a certain strength to the scalar Higgs boson via a Yukawa coupling. A fermion mass is then given by

$$m(\text{fermion}) = gV, \quad (21)$$

where  $V$  is the vacuum expectation value of the Higgs field. For the electron, this Yukawa coupling constant must be very small, because  $V$  is about 246 GeV:

$$g(\text{electron}) = 0.00000208. \quad (22)$$

Nobody understands why this coupling constant is so small. The problem of fermion masses remains to be solved. It seems to be the most fundamental problem we are facing at the present time. New experiments at the LHC and at the International Linear Collider (ILC) might clarify the issue.

If we are interested only in stable matter, as, for example, in solid state physics, then only seven fundamental constants enter:

$$G, A_c, \alpha, m_e, m_u, m_d, m_s. \quad (23)$$

The s-quark mass has been included because the  $\bar{s}s$  pairs contribute about 40 MeV to the nucleon mass. These seven constants describe atoms and molecules.

It is possible that there exist relations between the fundamental constants. The relations that seem to work very well are those between the flavor mixing angles and the quark masses, which were predicted some time ago [13]:

$$\Theta_u = \sqrt{\frac{m_u}{m_c}}, \quad \Theta_d = \sqrt{\frac{m_d}{m_s}}. \quad (24)$$

Similar relations can be derived for neutrino masses and the associated mixing angles [14].

These relations are obtained if both for u-type and d-type quarks, the following mass matrices are relevant (texture 0 matrices):

$$M = \begin{pmatrix} 0 & A & 0 \\ A^* & C & B \\ 0 & B^* & D \end{pmatrix}. \quad (25)$$

It would be interesting to know whether such mass matrices are indeed realized in nature.

### 3. Time variation of the fine structure constant

Recent observations in astrophysics [15, 16] indicate that the fine structure constant  $\alpha$  depends on cosmic time. Billions of years ago, it was smaller than it is today. A group of researchers from Australia, the UK, and the USA analyzed the spectra of distant quasars using the Keck telescope in Hawaii. They studied about 150 quasars, some of them about 11 billion lightyears away. The redshifts of these objects varied between 0.5 and 3.5. This corresponds to ages varying between 23% and 87% of the age of our Universe. They studied the spectral lines of iron, nickel, magnesium, zinc, and

aluminum. It was found that  $\alpha$  is not constant:

$$\frac{\Delta\alpha}{\alpha} = (-0.58 \pm 0.11) \times 10^{-5}. \quad (26)$$

With the ages of the observed quasars taken into account, the conclusion is that in the linear approximation, the absolute magnitude of the relative change of  $\alpha$  must be

$$\left| \frac{d\alpha/dt}{\alpha} \right| \approx 1.2 \times 10^{-15} \text{ year}^{-1}. \quad (27)$$

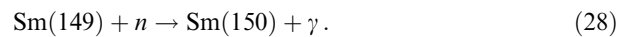
But recent observations of quasar spectra performed by different groups seem to rule out a time variation of  $\alpha$  at the level given above [17, 18].

The idea that the fundamental constants have a cosmological time dependence is not new. In the 1930s, Dirac [19] discussed a time variation of the Newton constant  $G$ . Dirac argued that the gravity constant should vary by about a factor of two during the lifetime of the universe. The present limit on the time variation of  $G$  is  $\dot{G}/G = (1 \pm 5) \times 10^{-14} \text{ year}^{-1}$  [20]. According to Dirac's hypothesis, the time variation of  $G$  should be about  $10^{-10} \text{ year}^{-1}$ , in conflict with the quoted limit. In the 1950s, Landau discussed a possible time variation of the fine structure constant  $\alpha$  in connection with the renormalization of the electric charge [21].

French nuclear physicists discovered that about 1.8 billion years ago, a natural reactor existed in Gabon, West Africa, close to the river Oklo. About 2 billion years ago, uranium-235 was more abundant than it is today (about 3.7%). Today, it is only 0.72%. The water of the Oklo served as a moderator for the reactor. The natural reactor operated for about 100 million years.

Isotopes of the rare-earth elements, for example, Samarium, were produced by the fission of uranium. The distribution of the isotopes observed today is consistent with the calculation, assuming that the isotopes were exposed to a strong neutron flux.

The reaction of Samarium with neutrons is especially interesting [22–24]:



The very large cross section of this reaction (about  $91 \pm 6$  kb) is due to a nuclear resonance just above the threshold. The energy of this resonance is very small:  $E = 0.0973$  eV. The position of this resonance cannot have changed in the past 2 billion years by more than 0.1 eV. We suppose  $\alpha$  has changed during this time. The energy of the resonance depends, in particular, on the strength of the electromagnetic interaction. Nuclear physics calculations give

$$\frac{\alpha(\text{Oklo}) - \alpha(\text{now})}{\alpha(\text{now})} < 10^{-7}. \quad (29)$$

The relative change in  $\alpha$  must be less than  $10^{-16}$  per year, as estimated by Damour and Dyson [23]. This conclusion is correct only if no other fundamental parameters have changed in the past two billion years. If other parameters, like the strong-interaction coupling constant, have also changed, the constraint mentioned above does not apply.

The Oklo constraint for  $\alpha$  is not consistent with the astrophysical observation for the relative changes in  $\alpha$  of the order  $10^{-15}$  per year. However, if other parameters also changed in time, there would be a rather complicated

constraint for a combination of these parameters, but there would be no inconsistency.

Recently, a time change of the mass ratio

$$R = \frac{M(\text{proton})}{m(\text{electron})} \quad (30)$$

was also found. The light from a pair of quasars 12 billion light years away from the earth was observed [25–27]. This light was emitted when the universe was only 1.7 billion years old. The study of the spectra by independent groups revealed that the mass ratio  $R$  has changed in time:

$$\frac{\Delta R}{R} \approx (2 \pm 0.6) \times 10^{-5}, \quad \frac{\Delta R}{R} \approx (2.6 \pm 3) \times 10^{-6}. \quad (31)$$

Taking the lifetime of 12 billion years into account, the change in  $R$  would be  $10^{-15}$  per year.

#### 4. Grand Unification and time variation

In the Standard Model, we have three basic coupling constants. The gauge group of the Standard Model is  $SU(3)_c \times SU(2) \times U(1)$ . The three gauge interactions are independent of each other.

Since 1974, the idea has been discussed that the gauge group of the Standard Model is a subgroup of a larger simple group. The three gauge interactions are embedded in the Grand Unified Theory (GUT). Grand Unification implies that  $\alpha_3$ ,  $\alpha_2$ , and  $\alpha_1$  are related. They can be expressed in terms of the unified coupling constant  $\alpha_{\text{un}}$  and the unification energy scale  $A_{\text{un}}$ .

The simplest Grand Unification theory is based on the gauge group  $SU(5)$  [28]. The quarks and leptons of one generation can be described by two  $SU(5)$  representations. We consider the 5-representation of  $SU(5)$ . After the breakdown of  $SU(5)$  to  $SU(3) \times SU(2) \times U(1)$ , we obtain

$$\begin{aligned} 5 &\rightarrow (3, 1) + (1, 2), \\ \bar{5} &\rightarrow (\bar{3}, 1) + (1, 2). \end{aligned} \quad (32)$$

The 5-representation contains a color triplet, which is a singlet under  $SU(2)$ , and a color singlet (an  $SU(2)$ -doublet):

$$(\bar{5}) = \begin{pmatrix} \bar{d}_r \\ \bar{d}_g \\ \bar{d}_b \\ \nu_e \\ e^- \end{pmatrix}. \quad (33)$$

The representation with the next higher dimension is the 10-representation, which is an antisymmetric second-rank tensor. The 10-representation decomposes as

$$(10) \rightarrow (3, 2) + (\bar{3}, 1) + (1, 1). \quad (34)$$

In terms of the lepton and quark fields of the first generation, we can write the 10-representation (an antisymmetric  $5 \times 5$ -matrix) as

$$(10) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \bar{u}_b & -\bar{u}_g & -\bar{u}_r & -\bar{d}_r \\ -\bar{u}_b & 0 & \bar{u}_r & \bar{u}_g & -\bar{d}_g \\ \bar{u}_g & -\bar{u}_r & 0 & -\bar{u}_b & -\bar{d}_b \\ u_r & u_g & u_b & 0 & e^+ \\ d_r & d_g & d_b & -e^+ & 0 \end{pmatrix}. \quad (35)$$

Combining these two representations, we find the lepton and quarks of one generation:

$$\bar{5} + 10 \rightarrow (3, 2) + 2(\bar{3}, 1) + (1, 2) + (1, 1). \quad (36)$$

For the first generation, we have

$$\bar{5} + 10 \rightarrow \begin{pmatrix} u \\ d \end{pmatrix}_L + \bar{u}_L + \bar{d}_L + \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L + e_L^+. \quad (37)$$

The second and third generation are analogous. The unification based on the gauge group  $SU(5)$  has a number of interesting features:

(1) The electric charge is quantized:

$$\text{tr } Q = 0 \rightarrow Q(d) = \frac{1}{3} Q(e^-). \quad (38)$$

(2) At some high mass scale  $A_{\text{un}}$ , the gauge group of the Standard Model becomes the  $SU(5)$  group, and there is only one gauge coupling. The three coupling constants  $g_3$ ,  $g_2$ , and  $g_1$  for  $SU(3)$ ,  $SU(2)$ , and  $U(1)$  must be of the same order of magnitude, related to each other by algebraic constants.

The rather different values of the coupling constants  $g_3$ ,  $g_2$ , and  $g_1$  at low energies must be due to renormalization effects. This would also explain why the strong interactions are strong and the weak interactions are weak: this is related to the size of the corresponding group.

Apart from normalization constants, the three coupling constants  $g_3$ ,  $g_2$ , and  $g_1$  are equal at the unification mass  $A_{\text{un}}$ . Thus, the  $SU(2) \times U(1)$  mixing angle, given by  $\tan \Theta_w = (g_1/g_2)$ , is fixed at or above  $A_{\text{un}}$ :

$$\sin^2 \Theta_w = \frac{\text{tr } T_3^2}{\text{tr } Q^2} = \frac{3}{8}. \quad (39)$$

At an energy scale  $\mu \ll A_{\text{un}}$ , the parameter  $\sin^2 \Theta_w$  changes along with the three coupling constants:

$$\begin{aligned} \frac{\sin^2 \Theta_w}{\alpha} - \frac{1}{\alpha_s} &= \frac{11}{6\pi} \ln \left( \frac{M}{\mu} \right), \\ \frac{\alpha}{\alpha_s} &= \frac{3}{10} (6 \sin^2 \Theta_w - 1). \end{aligned} \quad (40)$$

At  $\mu = M_Z$ , the electroweak mixing angle has been measured:  $\sin^2 \Theta_w = 0.2312$ , and hence

$$\frac{\alpha}{\alpha_s} = \frac{3}{8}. \quad (41)$$

This relation can be checked by experiment. To achieve agreement between the observed values of  $g_3$ ,  $g_2$ , and  $g_1$  and the values predicted by the  $SU(5)$  theory, the unification scale must be very high, as can be easily seen. We note that

$$\begin{aligned} \ln \left( \frac{M}{\mu} \right) &= \frac{6\pi}{11} \left( \frac{\sin^2 \Theta_w}{\alpha} - \frac{1}{\alpha_s} \right), \quad \mu = M_Z, \\ \ln \left( \frac{M}{M_Z} \right) &\cong 39.9, \quad M \approx 2 \times 10^{15} \text{ GeV}. \end{aligned} \quad (42)$$

The precise values of the three coupling constants, determined by the LEP experiments [3], disagree with the  $SU(5)$  prediction. The three coupling constants do not converge to

a single coupling constant  $\alpha_{\text{un}}$  [29]. A convergence occurs if supersymmetric particles are added above the 1 TeV energy. Supersymmetry implies that for each fermion, a boson is introduced (sleptons, squarks), and for each boson, a new fermion is introduced (e.g., photinos). These new particles are not observed in experiments. It is assumed that they have a mass about 1 TeV.

The new particles contribute to the renormalization of the gauge coupling constants at high energies (about 1 TeV). A convergence of the three coupling constants takes place. Therefore, a supersymmetric version of the SU(5) theory is consistent with experiments [29].

Grand Unification theories like the SU(5) theory involve quarks, antiquarks, and leptons in the same fermion representation. Hence, the proton can decay, e.g., as  $p \rightarrow e^+ \pi^0$ . The lifetime depends on the unification mass scale. For  $A_{\text{un}} = 5 \times 10^{14}$  GeV in the SU(5) theory without supersymmetry, the proton lifetime is found to be  $10^{30}$  years. The experimental lower limit is about  $10^{33}$  years.

There is a natural embedding of an SU( $n$ ) group into SO( $2n$ ) because  $n$  complex numbers can be represented by  $2n$  real numbers. The gauge group SO(10) may be used instead of SU(5). This was discussed in 1975 by P Minkowski and the author [31]. The fermions of one generation are described by the 16-dimensional spinor representation of SO(10).

Because SU(5) is a subgroup of SO(10), we have the decomposition

$$16 \rightarrow \bar{5} + 10 + 1. \quad (43)$$

The fermions of the SU(5) theory and one additional fermion (per family) are obtained. This state is an SU(5) singlet and describes a left-handed antineutrino field. Using the leptons and quarks of the first generation, we can write the 16-representation in terms of left-handed fields as

$$(16) = \begin{pmatrix} \bar{\nu}_e & \bar{u}_r & \bar{u}_g & \bar{u}_b & \vdots & u_r & u_g & u_b & \nu_e \\ e^+ & \bar{d}_r & \bar{d}_g & \bar{d}_b & \vdots & d_r & d_g & d_b & e^- \end{pmatrix}. \quad (44)$$

A feature of the SO(10) theory is that the gauge group for the electroweak interactions is larger than in the SU(5) theory. SO(10) has the subgroup SO(6)  $\times$  SO(4). Because SO(4) is isomorphic to SU(2)  $\times$  SU(2), we find

$$\text{SO}(10) \rightarrow \text{SU}(4) \times \text{SU}(2)_L \times \text{SU}(2)_R. \quad (45)$$

The SU(4) group must contain the color group SU(3)<sub>c</sub>. The 16-representation of the fermions decomposes under SU(4) into two 4-representations. These contain three quarks and one lepton, e.g., ( $d_r, d_g, d_b$ ) and  $e^-$ . The leptons can be interpreted as the fourth color. But the gauge group SU(4) must be broken at high energies (higher than at least 1 TeV):

$$\text{SU}(4) \rightarrow \text{SU}(3) \times \text{U}(1). \quad (46)$$

At low energies, we obtain the gauge group

$$\text{SU}(3)_c \times \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1). \quad (47)$$

But the masses of the gauge bosons for the SU(2)<sub>R</sub> group must be much larger than the observed W-bosons masses related to the SU(2)<sub>L</sub> group.

In the SU(5) theory, the minimal number of fermions of the Standard Model is included. In the SO(10) theory, a new right-handed neutrino is added. This right-handed fermion is interpreted as a heavy Majorana particle. The mass of the left-handed neutrino is generated by a ‘see-saw’ mechanism [32]. Thus, in the SO(10) theory, the neutrinos are massive, while in the SU(5) theory, they must be massless. The SO(10) theory is more symmetric than the SU(5) theory. It is hard to believe that nature would stop at SU(5) if it has chosen to unify the fundamental interactions.

The SO(10) theory involves one additional free parameter related to the masses of the right-handed W bosons. Since right-handed charged currents are not observed, the masses of the associated W bosons must be rather high, 300 GeV at least [32]. There is a new parameter  $M_R$  in the SO(10) theory. It can be chosen such that the coupling constant converges at very high energies without using supersymmetry. If we choose  $M_R \sim 10^9 - 10^{11}$  GeV, convergence occurs.

The idea of Grand Unification leads to a reduction of the number of fundamental constants by one. The three gauge coupling constants of the Standard Model can be expressed in terms of the unified coupling constant  $\alpha_{\text{un}}$  at the energy  $A_{\text{un}}$  where the unification occurs. The three coupling constants  $\alpha_s$ ,  $\alpha_2$ , and  $\alpha_1$  are replaced by  $\alpha_{\text{un}}$  and  $A_{\text{un}}$ .

In the Grand Unified Theory, the three coupling constants of the Standard Model are related to each other. If, for instance, the fine structure constant shows a time variation, the other two coupling constants should also vary in time. Otherwise, the unification would not be universal in time. Knowing the time variation of  $\alpha$ , we must be able to calculate the time variation of the other coupling constants. We here investigate only the time change of the QCD coupling constant  $\alpha_s$ .

We use the supersymmetric SU(5) theory to study the time variation of the coupling constants [33, 34]. The change in  $\alpha$  is traced back to a change in the unified coupling constant at the unification energy and to a change in the unification energy. These changes are related to each other:

$$\frac{1}{\alpha} \frac{\dot{\alpha}}{\alpha} = \frac{8}{3} \frac{1}{\alpha_s} \left( \frac{\dot{\alpha}_s}{\alpha_s} \right) - \frac{10}{\pi} \frac{\dot{A}_{\text{un}}}{A_{\text{un}}}. \quad (48)$$

We consider the following three scenarios:

(1)  $A_{\text{un}}$  is kept constant,  $\alpha_{\text{un}} = \alpha_{\text{un}}(t)$ . We obtain

$$\frac{1}{\alpha} \frac{\dot{\alpha}}{\alpha} = \frac{8}{3} \frac{1}{\alpha_s} \frac{\dot{\alpha}_s}{\alpha_s}. \quad (49)$$

Using the experimental value  $\alpha_s(M_Z) \approx 0.121$ , we find the time variation of the QCD scale [33]

$$\frac{\dot{A}_c}{A_c} \approx R \frac{\dot{\alpha}}{\alpha}, \quad R \approx 38 \pm 6. \quad (50)$$

The uncertainty in  $R$  comes from the uncertainty in the determination of the strong-interaction coupling constant  $\alpha_s$ . A time variation of the QCD scale  $A_c$  implies a time change in the proton mass and of the masses of all atomic nuclei. The change in the nucleon mass during the last 10 billion years amounts to about 0.3 MeV.

In QCD, the magnetic moments of the nucleon and of the atomic nuclei are inversely proportional to the QCD scale parameters  $A_c$ . For the nuclear magnetic moments, we find

$$\frac{\dot{\mu}_n}{\mu_n} = \frac{d/dt(1/A_c)}{1/A_c} = -\frac{\dot{A}_c}{A_c} = -R \frac{\dot{\alpha}}{\alpha}. \quad (51)$$

Taking the astrophysics result for  $(\dot{\alpha}/\alpha)$ , we obtain

$$\frac{\dot{A}_c}{A_c} \approx 4 \times 10^{-14} \text{ year}^{-1}. \quad (52)$$

(2) The unified coupling constant is kept invariant, but  $A_{\text{un}}$  changes in time. In this case, we find [34]

$$\frac{\dot{\alpha}}{\alpha} \approx -\alpha \frac{10}{\pi} \frac{\dot{A}_{\text{un}}}{A_{\text{un}}} \quad (53)$$

and

$$\frac{\dot{A}_c}{A_c} \approx -31 \frac{\dot{\alpha}}{\alpha}. \quad (54)$$

The change in the unification mass scale  $A_{\text{un}}$  can be estimated based on the time variation of the fine structure constant  $\alpha$ . Thus,  $A_{\text{un}}$  is decreasing at the rate

$$\frac{\dot{A}_{\text{un}}}{A_{\text{un}}} \approx -7 \times 10^{-14} \text{ year}^{-1}. \quad (55)$$

The relative changes in  $A_c$  and  $\alpha$  are opposite in sign. While  $\alpha$ , according to Ref. [15], is increasing at the rate  $10^{-15} \text{ year}^{-1}$ , the QCD scale  $A_c$  and the nucleon mass are decreasing at the rate about  $3 \times 10^{-14} \text{ year}^{-1}$ . The magnetic moments of the nucleons and of nuclei must increase:

$$\frac{\dot{\mu}_n}{\mu_n} \approx 3 \times 10^{-14} \text{ year}^{-1}. \quad (56)$$

(3) The third possibility is that both  $\alpha_{\text{un}}$  and  $A_{\text{un}}$  are time-dependent. In this case, we find

$$\frac{\dot{A}_c}{A_c} \cong 46 \frac{\dot{\alpha}}{\alpha} + 1.07 \frac{\dot{A}_{\text{un}}}{A_{\text{un}}}. \quad (57)$$

The right-hand side involves two relative time changes:  $\dot{\alpha}/\alpha$  and  $\dot{A}_{\text{un}}/A_{\text{un}}$ . These two terms might correlate such that  $\dot{A}_c/A_c$  is smaller than about  $\pm 40 \dot{\alpha}/\alpha$ .

The question arises as to whether a time change in the QCD scale parameter could be observed in experiments. The mass of the proton and the masses of the atomic nuclei, as well as their magnetic moments, depend linearly on the QCD scale. If this scale changes, the mass ratio  $M_p/m_e = R$  changes as well, if the electron mass is taken to be constant. The mass ratio  $R$  seems to show a time variation; in the linear approximation, we have

$$\frac{\Delta R}{R} \approx 10^{-15} \text{ year}^{-1}. \quad (58)$$

If we take the electron mass to be constant in time, this would imply that the QCD scale  $A_c$  changes at the rate

$$\frac{\Delta A_c}{A_c} \approx 10^{-15} \text{ year}^{-1}. \quad (59)$$

The connection between a time variation of the fine structure constant and of the QCD scale discussed above is only valid if either the unified coupling constant or the unification scale depends on time, not both. If both the unification scale and the unified coupling constant are time dependent, we should use Eqn (57) instead. There might be a

cancellation between the two terms. In this case, the time variation of the QCD scale would be smaller than  $10^{-14}$  per year. If the two terms cancel exactly, the QCD scale would be constant, but this seems unlikely. Therefore, a time variation of the QCD scale of the order of  $10^{-15}$  per year is quite possible.

Can such a small time variation of  $A_c$  be observed in experiments? In quantum optics, very precise experiments with lasers can be carried out. In the next section, we describe such an experiment at the Max Planck Institute of Quantum Optics in Munich, which was designed especially to find a time variation of the QCD scale  $A_c$ .

## 5. Results from quantum optics

The hydrogen atom is a very good test object for checking fundamental theories. Its atomic properties can be calculated with a very high accuracy. The level structure of the hydrogen atom can be very accurately probed using spectroscopy methods in the visible, infrared, and ultraviolet regions. Thus, the hydrogen atom plays an important role in determining fundamental constants like the fine structure constant.

Measurements of the Lamb shift and the 2S hyperfine structure permit very sensitive tests of quantum electrodynamics. Combining optical frequency measurements in hydrogen with results from other atoms, stringent upper limits for a time variation of the fine structure constant [35] and of the QCD scale parameter can be derived.

The use of frequency combs [36] has turned high-precision frequency measurements into a routine procedure. The high accuracy of the frequency comb has opened up wide perspectives for optical atomic clock applications in fundamental physics. Frequency measurements in the laboratory have recently become competitive in terms of sensitivity to a possible time variation of the fine structure constant. Although the time interval covered by these measurements is restricted to a few years, the very high accuracy compensates for this disadvantage. Their sensitivity becomes comparable with astrophysical and geological methods operating on a billion-year time scale.

The important advantages of laboratory experiments are the variety of different systems that may be tested, the possibility of changing parameters of the experiments in order to control systematic effects, and the determination of the drift rates from the measured data. Modern precision frequency measurements deliver information about the stability of the present values of the fundamental constants, which can only be tested with laboratory measurements. But only nonlaboratory methods are sensitive to processes that occurred in the early Universe, which can be much more severe compared to the present time.

In the experiment of the MPQ group in Munich [35], the frequency of the hydrogen 1S–2S-transition was determined to be 2466061102474851(34) Hz. A comparison with the experiment performed in 1999 gives an upper limit on a time variation of the transition frequency in the time between the two measurements, 44 months apart. We find the difference as  $(-29 \pm 57)$  Hz, which is indistinguishable from zero.

The hydrogen spectrometer can be interpreted as a clock, like the cesium clock. However, the hydrogen spectrometer involves a normal transition for the determination of the flow of time. This transition depends on the electron mass and on the fine structure constant. In a cesium clock, the flow of time

is determined by a hyperfine transition, which depends not only on the fine structure constant but also on the nuclear magnetic moment.

Comparing the 1S–2S hydrogen transition with the hyperfine transition of cesium  $^{133}\text{Cs}$  allows obtaining information about the time variation of the ratio  $\alpha/\alpha_s$ . The cesium hyperfine transition depends on the magnetic moment of the cesium nucleus, and the magnetic moment is proportional to  $1/A_c$ . If  $A_c$  varies in time, the magnetic moment must also vary.

A limit for the time variation of the magnetic moment of the cesium nucleus has been obtained [35]:

$$\frac{\delta\mu_n}{\mu_n} = (1.5 \pm 2.0) \times 10^{-15} \text{ year}^{-1}. \quad (60)$$

These results are indistinguishable from zero. The limit on the time variation of  $\alpha$  is of the same order as the astrophysics result.

The result concerning the magnetic moment implies a limit on the time variation of  $A_c$ :

$$\frac{\Delta A_c}{A_c} = (-1.5 \pm 2.0) \times 10^{-15} \text{ year}^{-1}. \quad (61)$$

This result is in disagreement with our results based on the assumption that either  $\alpha_{\text{un}}$  or  $A_{\text{un}}$  changes in time. We obtained about  $10^{-14}/\text{year}$ , which is excluded by this experiment.

The result given above is consistent with no time change for  $A_c$ , but it also agrees with a small time change of the order of  $10^{-15}$  per year. If we assume that the electron mass does not change in time, such a change in  $A_c$  would agree with the astrophysics result on the time variation of the ratio  $R = M_p/m_e$  [26]. Theoretically, we would expect such a time variation if both  $A_{\text{un}}$  and  $\alpha_{\text{un}}$  change in time.

## 6. Conclusions and outlook

We have summarized our present knowledge about the fundamental constants and their possible time variation. Today, we do not know how these constants are generated or whether they depend on time. There might be relations between these constants, e.g., between the flavor mixing angles and the fermion masses, or relations between the three coupling constants, implied by the idea of Grand Unification. This would reduce the number of basic constants from 28 to a smaller number, but at least 18 fundamental constants would still exist.

A possible time variation of the fundamental constants must be rather slow, at least for those fundamental constants that are measured very precisely, i.e., the fine structure constant, the QCD scale  $A_c$ , and the electron mass. The constant of gravity  $G$  is known with a precision of  $10^{-14}$  per year. No other fundamental constants, e.g., the masses of other leptons or the masses of heavy quarks, are known with a high precision. The present limits on the time variation of the fine structure constant, the QCD scale, or the electron mass are of the order of  $10^{-15}$  per year. These limits should be improved by at least two orders of magnitude in the near future.

If the astrophysics experiments indicate a time variation of the order of  $10^{-15}$  per year, it does not mean that experiments in quantum optics should also give such a time

variation. It might be that until about 10 billion years after the Big Bang, the constants did vary slowly, but after that they remained constant. No theory exists thus far for a time variation, and there is no reason to believe that a time variation should be linear, i.e.,  $10^{-15}$  per year throughout the history of our Universe. If the fundamental constants do vary, the variation very close to the Big Bang would be expected to be rather large. In the first microseconds after the Big Bang, constants like  $\alpha$  or  $A_c$  might have changed by a factor 2, and we would not know.

In cosmology, time variations of fundamental parameters should be considered in more detail. Perhaps allowing a suitable time variation of the constants leads to a better understanding of the cosmic evolution immediately after the Big Bang. Allowing time variations might lead to better cosmological theories and to a better understanding of particle physics. Particle physics and cosmology together would give a unified view of the universe.

## References

1. Fritzsch H, Gell-Mann M, in *Proc. of the XV. Intern. Conf. on High Energy Physics, Chicago, 1972*; Fritzsch H, Gell-Mann M, Leutwyler H *Phys. Lett. B* **47** 365 (1973)
2. Glashow S L *Adv. Nucl. Phys.* **22** 433 (1961); Salam A, Ward J C *Phys. Lett.* **13** 168 (1964); Weinberg S *Phys. Rev. Lett.* **19** 1264 (1967); Salam A, in *Elementary Particle Theory Vol. 2* (Ed. N Svartholm) (Stockholm: Almquist and Wiksells, 1968) p. 367
3. Yao W-M et al. (Particle Data Group) *J. Phys. G: Nucl. Part. Phys.* **33** 1 (2006)
4. Higgs P W *Phys. Rev. Lett.* **13** 508 (1964); Englert F, Brout R *Phys. Rev. Lett.* **13** 321 (1964); Guralnik G S, Hagen C R, Kibble T W B *Phys. Rev. Lett.* **13** 585 (1964)
5. Taylor T R, Veneziano G *Phys. Lett. B* **213** 450 (1988)
6. Witten E *Phys. Lett. B* **149** 351 (1984)
7. Wyler A *Phys. Today* **24** (8) 17 (1971)
8. Feynman R P *QED: the Strange Theory of Light and Matter* (Princeton, NJ: Princeton Univ. Press, 1985) p. 129
9. Gell-Mann M, Low F E *Phys. Rev.* **95** 1300 (1954)
10. Craigie N S, Narison S, Riazuddin *Nucl. Phys. B* **174** 207 (1980)
11. Leutwyler H *Nucl. Phys. B Proc. Suppl.* **94** 108 (2001); Narison S *Phys. Rev. D* **74** 034013 (2006)
12. Shifman M A, Vainshtein A I, Zakharov V I *Phys. Lett. B* **78** 443 (1978)
13. Fritzsch H *Nucl. Phys. B* **155** 189 (1979)
14. Fritzsch H, Xing Z *Phys. Lett. B* **634** 514 (2006); Fritzsch H, in *Proc. of the 4th Intern. Conf. on Flavor Physics, September 24–28, 2007, Beijing, China* (in press)
15. Webb J K et al. *Phys. Rev. Lett.* **87** 091301 (2001)
16. Murphy M T, Webb J K, Flambaum V V *Mon. Not. R. Astron. Soc.* **345** 609 (2003)
17. Quast R, Reimers D, Levshakov S A *Astron. Astrophys.* **415** L7 (2004)
18. Chand H et al. *Astron. Astrophys.* **417** 853 (2004)
19. Dirac P A M *Nature* **192** 441 (1961)
20. Pitjeva E V *Pis'ma Astron. Zh.* **31** 378 (2005) [*Astron. Lett.* **31** 340 (2005)]
21. Landau L D, in *Niels Bohr and the Development of Physics* (Ed. W Pauli) (New York: McGraw-Hill, 1955)
22. Shlyakhter A I *Nature* **264** 340 (1976)
23. Damour T, Dyson F *Nucl. Phys. B* **480** 37 (1996)
24. Fujii Y et al. *Nucl. Phys. B* **573** 377 (2000)
25. Ivanchik A et al. *Astron. Astrophys.* **440** 45 (2005)
26. Reinhold E et al. *Phys. Rev. Lett.* **96** 151101 (2006)
27. King J A et al. *Phys. Rev. Lett.* **101** 251304 (2008); arXiv:0807.4366
28. Georgi H, Glashow S L *Phys. Rev. Lett.* **32** 438 (1974)
29. Amaldi U, de Boer W, Fürstenau H *Phys. Lett. B* **260** 447 (1991); Ellis J, Kelley S, Nanopoulos D V *Phys. Lett. B* **260** 131 (1991); Langacker P, Polonsky N *Phys. Rev. D* **47** 4028 (1993); Carena M, Pokorski S, Wagner C E M *Nucl. Phys. B* **406** 59 (1993)



30. Shiozawa M et al. (Super-Kamiokande Collab.) *Phys. Rev. Lett.* **81** 3319 (1998); Hayato Y et al. (Super-Kamiokande Collab.) *Phys. Rev. Lett.* **83** 1529 (1999)
31. Fritzsch H, Minkowski P *Ann. Physics* **93** 193 (1975)
32. Minkowski P *Phys. Lett. B* **67** 421 (1977); Yanagida T *Prog. Theor. Phys.* **64** 1103 (1980); Gell-Mann M, Ramond P, Slansky R, in *Supergravity* (Eds P Van Nieuwenhuizen, D Z Freedman) (Amsterdam: North-Holland, 1979); Mohapatra R N, Senjanović G (D0 Collab.) *Phys. Rev. Lett.* **44** 912 (1980)
33. Abachi S et al. *Phys. Rev. Lett.* **76** 3271 (1996); Chang D, Mohapatra R N, Gipson J M, Marshak R E, Parida M K *Phys. Rev. D* **31** 1718 (1985)
34. Calmet X, Fritzsch H *Phys. Lett. B* **540** 173 (2002)
35. Langacker P, Segrè G, Strassler M J *Phys. Lett. B* **528** 121 (2002)
36. Kolachevsky N, Alnis J, Matveev A, Udem Th, Holzwarth R, Hänsch T W "Precision measurements in atomic hydrogen", in *Optical Atomic Clock* (Ed. Y Ovchinnikov) (in press)
37. Udem Th, Holzwarth R, Hänsch T W *Nature* **416** 233 (2002)