

REVIEWS OF TOPICAL PROBLEMS

PACS numbers: 72.25.Ba, 72.25.Hg, **75.47.–m**

Spintronics: exchange switching of ferromagnetic metallic junctions at a low current density

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DOI: 10.3367/UFNe.0179.200904b.0359

Contents

Abstract. **A review is given on the exchange switching of ferromagnetic metallic junctions under the effect of a low threshold current. A dramatic (orders of magnitude) threshold current reduction is achieved under conditions that include the dominance of the current-driven nonequilibrium spin injection, the optimum relation between spin resistances of the layers, and the application of an external magnetic field near the reorientation phase transition point.**

1. Introduction

Since the late 1980s, a permanent growth in the number of studies devoted to the analysis of the properties of nanodimensional structures on the basis of multilayer metallic ferromagnetic films with giant magnetoresistance (GMR) has been observed in many countries. These structures demonstrate uncommon physical properties in a wide range of temperatures, including those close to room temperature. The nature of the fundamental effects, such as the exchange instability and current-driven switching, has proven to be sufficiently new and interesting. These effects find application in sensors of magnetic field and current and in heads for readout information from magnetic disks and tapes; they are considered to be promising for application in the memory elements with a high density of information, in galvanic decouplings, and in biosensors. Investigations in the fields of spin transistors, logical nanoelements, magnetic neurons, and spin microprocessors are being conducted quite intensely. Estimations of the density of the arrangement of nanoelements, including memory elements, show that it can approach the 'physical' limit of the order of 100 Gbit cm^{-2}. Investigations have shown that the expected switching rate is sufficiently high; the switching time can be ~ 0.1 ns or even less. It is quite probable that in the near future, similar structures will prove to be the basis for the next generation of computing systems, which will have fundamentally new properties and substantially higher characteristics in comparison with the existing systems.

It is obvious that we are dealing here with an important scientific and technological domain, in which the main carrier of information is the spin state of a substance, and new effects that accompany the transfer of spin can be used for processing this information. A special name, spintronics, is used to designate this area.

The realm of spintronics, as understood in this sense, is very wide. For example, the authors of review [1], who discussed early works, showed that polarized light in semiconductor structures influences the spin state and leads to a number of interesting effects. The addition of ferromagnetic layers to such structures opens ways to develop new hybrid structures and new possibilities in research and applications. Substantial progress was due to the use of purely metallic film-type ferromagnetic structures (see, e.g., [2–4]), which have many useful properties, such as a high Curie temperature and radiation resistance.

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Received 5 November, 2008
Uspekhi Fizicheskikh Nauk **179** (4) 359–368 (2009)
DOI: 10.3367/UFNr.0179.200904b.0359
Translated by S N Gorin; edited by A M Semikhatov

In recent years, intense development in the field of spintronics has taken place and substantial progress has been achieved. First of all, we mention the world-recognized work on GMR, for which P Grünberg and A Fert shared the 2007 Nobel Prize in physics [5–8], and the mechanism of exchange instability and switching due to the spin-driven transfer of torque into the lattice suggested in [9, 10]. Another mechanism of the exchange instability and switching due to a current-driven spin injection was suggested somewhat later [11, 12]. We show in what follows that the spin-injection mechanism is characterized by a number of specific features that clarify an intimate relation between these seemingly different effects, the GMR and exchange switching.

The number of published works on spintronics is presently very large. In particular, many hundreds of experimental works exist. Strange as it may appear, the number of theoretical works that explain the experiments is much less. This is related to some difficulties that transpired in the course of developing the theory. At the same time, obviously, it is presently important not only to collect all available data and publications but also, at least in particular cases, to develop a theoretical approach for understanding and systematizing some experiments. This is the main purpose of this review.

On the other hand, the subject of this review is sufficiently relevant because we are speaking about a reduction in the threshold current density required to achieve the exchange instability. This problem is quite important, because the density of the threshold current in experiments is still rather large (exceeds 2×10^6 A cm^{-2} [13]). The lower the threshold, the simpler the experiments and the wider the possibility of practical application of the phenomenon of exchange instability. In this review, we discuss some new ways for radically reducing the threshold in an external magnetic field.

2. Spin-polarized current

We consider the ferromagnetic junction shown schematically in Fig. 1. It is given by a layered structure consisting of two ferromagnetic layers *1* and *2*, a layer of a nonmagnetic conductor *3*, and an ultrathin spacer (barrier layer), which is sufficiently transparent to conduction electrons and does not change the direction of their spins. A tangential magnetic field **H** can be applied to this junction. The arrows on the right-hand lateral surface of the ferromagnetic layers indicate the directions of the related magnetizations (as an example, they are shown oriented in opposite directions).

When a flux of electrons $\mathbf{j}/e$ flows through layers *1* and *2*, it becomes spin-polarized. This is related to the existence of a ferromagnetic ordering in the layers and to the presence of spontaneous magnetizations $\mathbf{M}_1$ and $\mathbf{M}_2$ in them. We assume for simplicity that both magnetizations are oriented along the z axis in equilibrium.

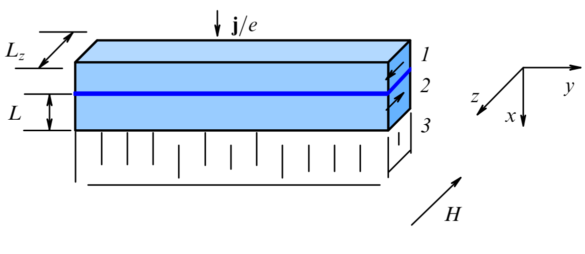

Figure 1. Magnetic junction (schematic): layer *1*, spins are pinned; *2*, spins are free; *3*, nonmagnetic conductor; layers *1* and *2* are separated by an ultrathin spacer layer.

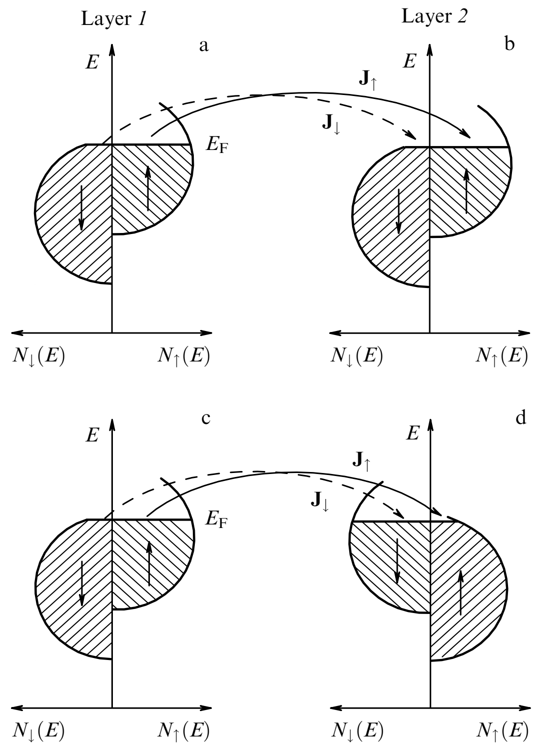

Figure 2. Spin energy subbands for conduction electrons in a ferromagnet. Hatching shows the population of the subbands.

A typical energy spectrum of conduction electrons in a ferromagnet is displayed in Fig. 2. The vertical arrows indicate the directions of spins of conduction electrons in each energy subband. In Figs 2a–2c, the total magnetization vector is directed downward and in Fig. 2d, upward. The horizontal axes correspond to the densities of states $N_\uparrow(E)$ and $N_\downarrow(E)$ at an energy level E. The Fermi energy corresponds to a level from which the electrons pass from layer *1* into layer *2*. The transitions themselves are shown by the curved arrows above; the spin fluxes are designated by vectors $\mathbf{J}_\uparrow$ and $\mathbf{J}_\downarrow$.

3. Giant magnetoresistance. Spin injection

The simplest assumption is that the spin of an electron is conserved in $1 \leftrightarrow 2$ transitions. This can be due to both the small thickness of the spacer and the weakness of spin scattering in it. In any case, this assumption frequently suffices for describing the results of experiments [14]. Precisely such transitions with the conservation of spin are shown in Fig. 2.

We see that in the case of the parallel orientation of the magnetizations $\mathbf{M}_1$ and $\mathbf{M}_2$ shown in Figs 2a and 2b, the current is transferred mainly by spin-up electrons. At the Fermi level, the number of such electrons is greater in layer *1*, and the number of sites for them is greater in layer 2. Conversely, in the case of the antiparallel orientation of $\mathbf{M}_1$ and $\mathbf{M}_2$ shown in Figs 2c and 2d, the situation for the passage of the current is less favorable. The number of spin-up electrons is large in layer *1*, but the number of sites for them in layer *2* is small; the situation for the spin-down electrons is analogous. It can therefore be expected that the resistance of the sample is greater for the antiparallel (AP) orientation and is less for the parallel (P) orientation. The difference between these resistances $\Delta R \equiv R(\mathrm{AP}) - R(\mathrm{P})$ depends on the external magnetic field because its relative orientation, as can be seen from Fig. 2, affects the populations of the spin subbands. It is important that the magnetoresistance ΔR can be large. In modern experiments, the relative value of ΔR can exceed several tens of percent [14–16]:

$$\frac{\Delta R}{R(\mathrm{P})} \geqslant 0.1\,; \tag{1}$$

the maximum value can reach hundreds percent [4] at room temperature. If we recall that the magnetoresistance previously observed in metals is less than 0.02%, it is understandable why the value in (1) is called 'giant.' It is necessary here to understand that this magnetoresistance is related not to a change in the effective mean free path of charge carriers but to a change in the energy-band structure of conduction electrons under the effect of a magnetic field. This effect finds wide applications in magnetic-field sensors of various purposes, e.g., in the readout heads of information-storage devices [5].

We now focus attention on another effect, which is called spin injection; it is especially well visible in Figs 2a and 2b. The electron flux $\mathbf{J}_\uparrow$ is predominant here and increases the concentration of spin-up electrons to the right of the barrier. We note that the electron concentration in this case cannot change in view of the local electroneutrality in the metal, but the concentrations of spins can change quite strongly. In this respect, the effect under consideration differs from the charge injection in semiconductors investigated by Shockley [17]. The effect of spin injection was first proposed and discussed by Aronov [18]. The level of spin injection can be relatively high, which allows proposing some interesting consequences, such as the effect of spin filtration [4], the effect of negative effective spin temperature [19, 20], and the effect of a significant decrease in the threshold current upon exchange switching in ferromagnetic nanojunctions [21]. In this review, only the last effect, which seems the most relevant to us, is discussed in more detail.

4. Current-driven sd exchange switching

In Refs [9, 10], it was shown theoretically for the first time that the spin-polarized current in a ferromagnetic junction (analogous to that shown in Fig. 1) can induce an instability of magnetic fluctuations. An increase in fluctuations due to this instability leads to the development of nonlinear effects, one of which consists in the inversion of the magnetization direction, i.e., in the 'switching' of the sample. There can also exist other scenarios of instability development, but we now consider only the switching.

At the first stage, we should qualitatively understand the instability mechanism suggested in [9, 10]. For this, we consider Fig. 3, which displays the layers of a junction and the directions of the magnetization vectors of the lattice and of the conduction electrons. In layer *1*, these respective magnetizations are $\mathbf{M}_1$ and $\mathbf{m}_1$; in layer *2*, they are $\mathbf{M}$ and $\mathbf{m}$. The flow of electrons $j/e > 0$ is also shown.

The magnetizations $\mathbf{M}_1$ and $\mathbf{M}$ are noncollinear in general. Therefore, as the electron flow passes from layer *1* into layer *2*, the vector $\mathbf{m}$ starts precessing about the vector $\mathbf{M}$; this is depicted by ovals in Fig. 3. Because the electron velocities in layer *1* are characterized by a statistical

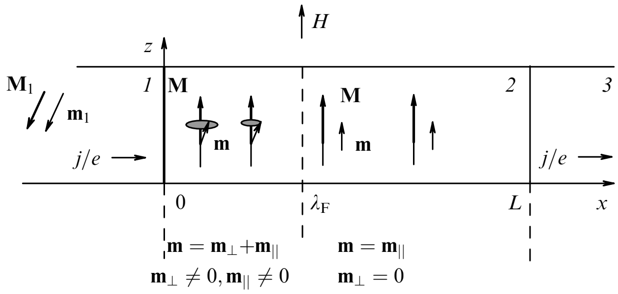

Figure 3. Schematic of processes that occur in the layers of the magnetic junction.

scatter, the electron spin precession phases become random after the electrons pass a certain distance in layer *2* (about a quantum wavelength of electrons at the Fermi level, λ_F). This leads to the disappearance of transverse components, and the vectors $\mathbf{m}$ and $\mathbf{M}$ become collinear, which is also shown in Fig. 3.

A question arises: where did the transverse component of magnetization pass to? The dissipative processes were ignored. Therefore, it only remains to assume that this component passed into the magnetic lattice, inducing nonequilibrium processes in it. At a sufficiently large current density, which is estimated as $j > j_{\mathrm{th}} \geqslant 10^6 - 10^7$ A cm^{-2} at room temperature, these processes can lead to an instability of magnetic fluctuations and to switching. Such was the main idea in Refs [9, 10]. This idea was soon confirmed in [14] and in many subsequent works. The process itself was called the transfer of the current-induced sd exchange torque, or spin–torque transfer. As we see, the threshold j_{th} of such a process can be rather large, which is not always acceptable.

We now return to Fig. 3. After the termination of precession near a point $x \geqslant \lambda_F \sim 1$ nm, the flow of injected nonequilibrium spins continues, although the vectors $\mathbf{m}$ and $\mathbf{M}$ become collinear. The typical diffusion length for spins in metals at room temperature is $l \geqslant 30$ nm. Therefore, if the thickness L of layer *2* is small, e.g., $L \sim 5-10$ nm $< l$, then the spins almost uniformly populate the two energy subbands over almost all of this layer. All spins interact with the lattice as a result of the sd exchange and produce a nonequilibrium addition to the effective magnetic field $\Delta\mathbf{H}_{\mathrm{eff}}$. If the energy of the sample increases in this case, then, at a sufficiently high current density $j > j'_{\mathrm{th}}$, a switching (inversion of magnetization) occurs, the energy decreases, and the sample passes into a new stable stationary state with an inverse magnetization $\mathbf{M}$.

By its nature, this switching mechanism, just as the spin-related torque, is also a result of the action of an sd exchange. However, this is another manifestation of the sd exchange. The idea of such an effect of spin-injection switching seems to have been first proposed and substantiated in Refs [11, 12]. The calculation performed in [21] showed (see Section 8) that under specific conditions, the threshold j'_{th} can be lower than j_{th} by an order of magnitude. For example, the value of j'_{th} can be less than $\leqslant 10^5$ A cm^{-2} or even much less.

5. The dynamics of magnetizations m and M in the presence of current

We proceed to a quantitative examination of processes at the junction during the passage of an electric current through it. Layer *1* has a pinned lattice magnetization $\mathbf{M}_1$ and a free magnetization of electrons $\mathbf{m}_1$. In layer *2*, both vectors $\mathbf{M}$ and

m are free. A detailed theory in a form convenient for future discussion was developed in Refs [22–24]. Other variations of the theory are presented in Refs [25, 26]. Below, we discuss the structure of the theory in [24] and the basic possibilities of the realization of effects following from this approach.

For simplicity, the junction is considered to be infinite and uniform in the yz plane, and all the related quantities are assumed to be dependent only on the x coordinate (see Fig. 3). The interaction of conduction electrons (s electrons) with the lattice (d electrons) is described using the standard sd exchange energy

$$U_{\rm sd} = -\alpha_1 \int_{-L_1}^{0} \mathbf{m}_1(x')\,\mathbf{M}_1\,{\rm d}x' - \alpha \int_0^L \mathbf{m}(x')\,\mathbf{M}(x')\,{\rm d}x', \tag{2}$$

where $\alpha_1 \sim \alpha \sim 10^4$ are dimensionless parameters of the sd exchange for respective layers *1* and *2*, and L_1 and L are the thicknesses of layers *1* and *2* (typically, $L_1 \gg L$). The barrier layer at $x = 0$ is thin and does not influence the processes in question. In principle, such conditions can easily be realized experimentally. The nonmagnetic layer *3* is located at $x > L$ and serves to close the electric circuit.

The dynamics of the magnetizations is described by the following equations:

(1) the continuity equation for mobile electrons

$$\frac{\partial \mathbf{m}}{\partial t} + \frac{\partial \mathbf{J}}{\partial x} + \gamma\alpha[\mathbf{m}\times\mathbf{M}] + \frac{\mathbf{m}-\bar{\mathbf{m}}}{\tau} = 0\,, \tag{3}$$

where the flux of spins of mobile electrons $\mathbf{J}$ is equal to

$$\mathbf{J} = \frac{\mu_{\rm B}}{e}\,(j_\uparrow - j_\downarrow)\,\hat{\mathbf{M}}\,, \tag{4}$$

$j_\uparrow$ and $j_\downarrow$ are the partial electric current densities for the spin-up and spin-down electrons, $\hat{\mathbf{M}} = \mathbf{M}/M$ is the unit vector of the lattice magnetization, τ is the spin relaxation time, $\mu_{\rm B}$ is the Bohr magneton, e is the electron charge, γ is the gyromagnetic ratio, and t is the current time;

(2) the Landau–Lifshitz–Gilbert (LLG) equation for the lattice magnetization $\mathbf{M}$

$$\frac{\partial \hat{\mathbf{M}}}{\partial t} + \gamma[\hat{\mathbf{M}}\times\mathbf{H}_{\rm eff}] - \varkappa\left[\hat{\mathbf{M}}\times\frac{\partial\hat{\mathbf{M}}}{\partial t}\right] = 0\,, \tag{5}$$

where $\varkappa$ is the Gilbert damping parameter (a typical estimate for it at room temperature is $\varkappa \sim 3\times10^{-2}$. The effective magnetic field

$$\mathbf{H}_{\rm eff} = \mathbf{H} + \mathbf{H}_{\rm a} + A\,\frac{\partial^2\mathbf{M}}{\partial x^2} + \mathbf{H}_{\rm d} + \mathbf{H}_{\rm sd} \tag{6}$$

includes the external magnetic field $\mathbf{H}$, the anisotropy field $\mathbf{H}_{\rm a}$, the demagnetizing field $\mathbf{H}_{\rm d}$, the sd exchange field $\mathbf{H}_{\rm sd} = -\delta U/\delta\mathbf{M}$, which is obtained by variational differentiation of energy (2), and the effective field of intralattice exchange with a typical value of the constant $A \sim 10^{-12}$ cm^2.

To find a solution of the set of equations (3) and (5), it is necessary to explicitly calculate the derivative $\mathbf{H}_{\rm sd} = -\delta U/\delta\mathbf{M}$. To do this, it is necessary to express the vector $\mathbf{m}$ through the vector $\mathbf{M}$ from Eqn (3) and to substitute the result in the expression for energy (2). This can most easily be done using the smallness of the electron relaxation time $\tau \sim 3\times10^{-13}$ s. The small value of τ ensures the fulfillment of the condition $\omega\tau \ll 1$ and the possibility of neglecting the time derivative in Eqn (3). The meaning of such an approximation is that the electrons 'follow' the lattice vibrations. At the same time, the characteristic frequency of the sd exchange $\omega_{\rm sd} \equiv \gamma\alpha M$ at the typical values of the parameters $\alpha \sim 2\times10^4$ and $M \sim 10^3$ G is estimated as $\omega_{\rm sd} \sim 3\times10^{14}$ s^{-1}. Therefore, the condition $\omega_{\rm sd}\tau \sim 10^2 \gg 1$ is satisfied. With the above conditions, Eqn (3) is essentially simplified and can be easily solved for $\mathbf{m}$. Such a solution was obtained in Refs [22–24]. Using it, we obtain the nonequilibrium (current-dependent) addition to the effective field as

$$\Delta\mathbf{H}_{\rm sd} = h_{\rm sd}\hat{\mathbf{M}}_1 l\delta(x-0)\,, \tag{7}$$

where $l \sim 3\times10^{-6}$ cm is the mean spin free path in typical ferromagnetic metals at room temperature. The part of the $\mathbf{H}_{\rm sd}$ field that is not included in (7) is proportional to the vector $\hat{\mathbf{M}}$ and is omitted from the LLG equation. This part of the field is not considered in what follows.

In solving Eqn (3) and deriving formula (7), boundary conditions of two types have been used.

(1) The conditions of quasi-equilibrium for the electron transfer through the boundaries of layers *1* $\leftrightarrow$ *2* and *2* $\leftrightarrow$ *3*. These conditions are reduced to the continuity of the chemical potential; the contacting layers can have noncollinear magnetizations, i.e., $\cos\chi = (\hat{\mathbf{M}}_1\hat{\mathbf{M}}(0)) \leqslant 1$. Such a problem was solved recently in Ref. [27], where the unknown boundary conditions at $x = 0$ were obtained in the form

$$N_1\Delta m_1(-0) = N_2\Delta m_2(+0)\cos\chi \quad \text{at} \quad \frac{j}{e} > 0\,, \tag{8}$$

$$N_1\Delta m_1(-0)\cos\chi = N_2\Delta m_2(+0) \quad \text{at} \quad \frac{j}{e} < 0\,, \tag{9}$$

where

$$N_{1,2} = \frac{1}{2\mu_{\rm B}}\left(\frac{1}{g_{\uparrow1,2}} + \frac{1}{g_{\downarrow1,2}}\right). \tag{10}$$

The quantities $g_{\uparrow1,2}$ and $g_{\downarrow1,2}$ are the electron densities of states on the Fermi surface in layers *1* and *2*.

Analogously, at the boundary $x = L$, have

$$N_2\Delta m_2(L-0) = N_3\Delta m_3(L+0)\,. \tag{11}$$

(2) The continuity conditions for the longitudinal part of the spin flux of electrons, namely,

$$\big(\mathbf{J}(-0)\,\hat{\mathbf{M}}(+0)\big) = \big(\mathbf{J}(+0)\,\hat{\mathbf{M}}(+0)\big) \quad \text{at} \quad \frac{j}{e} > 0\,, \tag{12}$$

$$\big(\mathbf{J}(+0)\,\hat{\mathbf{M}}_1\big) = \big(\mathbf{J}(-0)\,\hat{\mathbf{M}}_1\big) \quad \text{at} \quad \frac{j}{e} < 0\,, \tag{13}$$

and the conditions

$$\big(\mathbf{J}(L-0)\,\hat{\mathbf{M}}(L-0)\big) = \big(\mathbf{J}(L+0)\,\hat{\mathbf{M}}(L-0)\big) \tag{14}$$

for any sign of j/e.

Expression (7) involves a new exchange field $h_{\rm sd}$, which depends on the current direction. In the case of the forward current ($j/e > 0$), we obtain

$$h_{\rm sd} = \mu_{\rm B}\alpha n Q_1\,\frac{j}{j_{\rm D}}\,\frac{\lambda\nu_1(\nu^* - \cos^2\chi) + 2b\nu^*\cos\chi}{(\nu^* + \cos^2\chi)^2}\,, \tag{15}$$

where n is the concentration of charge carriers in layer *2*, Q_1 is the coefficient of current polarization in layer *1* (it is selected to be close to 1), $j_D = enl/\tau$ is the characteristic 'diffusion' current, $\lambda = L/l \sim 0.1$, $b = \alpha_1 \tau_1 M_1 / \alpha \tau M$, and $\cos\chi = (\hat{\mathbf{M}}_1 \hat{\mathbf{M}}(0))$. The parameters $v_1 = Z_1/Z_2$ and $v^* = Z_1/Z_3 + \lambda Z_1/Z_2$ characterize the relations between the spin resistances of the layers; by definition (see, e.g., [23]), the spin resistance of the ith layer is $Z_i = l_i \rho_i / (1 - Q_i^2)$, where l_i, ρ_i, and Q_i are the spin diffusion length, resistance, and current polarization coefficient in the ith layer.

In the case of the backward current ($j/e < 0$), we obtain

$$h_{\rm sd} = -\mu_{\rm B} \alpha n Q_1 \left| \frac{j}{j_{\rm D}} \right| \frac{\lambda v_1 (1 - v^* \cos^2\chi) - 2bv^* \cos\chi}{(1 + v^* \cos^2\chi)^2} \,. \quad (16)$$

6. Boundary conditions in lattice dynamics

The next step is to solve LLG equation (5) for the lattice magnetization $\mathbf{M}$. The solution should satisfy the boundary conditions that express the continuity of the total spin fluxes at the boundaries of the layers $x = -L_1$, 0, L, and $+\infty$, and account for the fluxes of both electron spins pinned in the lattice and spins of mobile electrons, which, according to the idea of the spin-transfer torque [9, 10], can pass into the lattice. The explicit analytic expressions for these fluxes can be derived directly from Eqns (3) and (5). They follow from the divergent terms in (3), (5), and (7). A detailed derivation of the boundary conditions is described in Refs [22–24].

In Eqns (3) and (5), the following divergent terms can be identified.

(1) From Eqn (3), the vector $\mathbf{J}$ can be separated, which is defined by Eqn (4) and represents the flux of spins of mobile electrons. This flux can be considered longitudinal, because it is parallel to the magnetization $\hat{\mathbf{M}}$.

(2) From Eqn (5), we can extract the vector

$$\mathbf{J}_{\rm M} = a \left[\hat{\mathbf{M}} \times \frac{\partial \hat{\mathbf{M}}}{\partial x} \right] , \quad (17)$$

where $a = \gamma A M$, which gives the flux density of the lattice magnetization. This flux is transverse, because it is perpendicular to $\hat{\mathbf{M}}$.

(3) From equation (5), with the help of expression (7) for the effective field, one additional flux $\mathbf{J}_{\rm sd}$ can be isolated, which is defined by

$$\gamma [\mathbf{M} \times \mathbf{H}_{\rm sd}] = \frac{\partial \mathbf{J}_{\rm sd}}{\partial x} \,, \quad (18)$$

where

$$\mathbf{J}_{\rm sd}(x) = \gamma h_{\rm sd}\, l \left[\mathbf{M}(0) \times \hat{\mathbf{M}}_1 \right] \theta(x - 0) \,,$$

and $\theta(x-0) = 1$ for $x > 0$ and $\theta(x-0) = 0$ for $x < 0$. It is obvious that this flux is also transverse.

The continuity of the total spin flux at the boundary surface $x = 0$ is expressed by the equality

$$\mathbf{J}(+0) - \mathbf{J}(-0) + \mathbf{J}_{\rm M}(+0) - \mathbf{J}_{\rm M}(-0) + \mathbf{J}_{\rm sd}(+0) - \mathbf{J}_{\rm sd}(-0) = 0 \quad (19)$$

and represents a boundary condition that is necessary to uniquely solve Eqn (5), i.e., to find the vector $\mathbf{M}$. Condition (19) can be substantially simplified. First, layer *1* has a pinned lattice, which indicates the disappearance of the flows $\mathbf{J}_{\rm M}(-0) = \mathbf{J}_{\rm sd}(-0) = 0$. Second, the calculations can be simplified by separately considering the projections of (19) onto the vector $\hat{\mathbf{M}}(+0)$ [see (12)] and onto the plane perpendicular to vector $\hat{\mathbf{M}}(+0)$ for the forward current ($j/e > 0$)

$$\left[\hat{\mathbf{M}}(+0) \times \left[\mathbf{J}(-0) \times \hat{\mathbf{M}}(+0) \right] \right] = \mathbf{J}_{\rm M}(+0) + \mathbf{J}_{\rm sd}(+0) \,, \quad (20)$$

and onto vector $\hat{\mathbf{M}}_1$ (13) and the plane perpendicular to $\hat{\mathbf{M}}_1$, for the backward current ($j/e < 0$)

$$\left[\hat{\mathbf{M}}_1 \times \left[\mathbf{J}(+0) \times \hat{\mathbf{M}}_1 \right] \right] = -\mathbf{J}_{\rm M}(+0) - \mathbf{J}_{\rm sd}(+0) \,. \quad (21)$$

On the other boundary surface of layer *2*, at $x = L$, the continuity condition for the total spin flux takes the form

$$\mathbf{J}(L+0) - \mathbf{J}(L-0) - \mathbf{J}_{\rm sd}(L-0) = 0 \,, \quad (22)$$

which gives condition (14) for the longitudinal component and

$$\mathbf{J}_{\rm M}(L-0) = 0 \quad (23)$$

for the transverse component.

Thus, we have presented different types of boundary conditions:

• continuity conditions for chemical potentials (8), (9), and (11);

• continuity conditions for longitudinal spin fluxes (12)–(14);

• continuity conditions for transverse spin fluxes (20), (21), and (23).

These conditions allow considering different linear and nonlinear problems in magnetic junctions and, in particular, the problem of switching. Both channels of the sd exchange interaction (spin-transfer torque and spin-injection effective field) can be simultaneously described from this standpoint. The triple vector products in formulas (20) and (21) show how the longitudinal spin fluxes of mobile electrons can be transformed into the transverse spin fluxes of the lattice, which was the idea suggested in [9, 10].

7. Macrospin approximation

For application to specific problems, the above boundary conditions should be expressed explicitly through the sought magnetization; this can be done using the above current densities (4), (17), and (18), as well as the solutions of Eqn (3) for the function $\mathbf{m}(x)$ and expressions (15) and (16) for the exchange field $h_{\rm sd}$ calculated earlier. Taking the vector product of the boundary conditions by $\hat{\mathbf{M}}(+0)$ transformed this way, we obtain

$$\left. \frac{\partial \hat{\mathbf{M}}(x)}{\partial x} \right|_{x=+0} = -p \left[\hat{\mathbf{M}}(+0) \times \left[\hat{\mathbf{M}}_1 \times \hat{\mathbf{M}}(+0) \right] \right] + k \left[\hat{\mathbf{M}}_1 \times \hat{\mathbf{M}}(+0) \right] , \quad (24)$$

$$\left. \frac{\partial \hat{\mathbf{M}}(x)}{\partial x} \right|_{x=L-0} = 0 \,. \quad (25)$$

The parameters p and k in condition (24) are proportional to the current density and describe the influence of the two above mechanisms of the sd exchange interaction of electron spins with the lattice, i.e., the spin injection and the spin–

torque transfer. These parameters calculated for the forward current ($j/e > 0$) take the form

$$p = \frac{\mu_B \gamma \alpha \tau Q_1}{a} \, \frac{j}{e} \, \frac{\lambda v_1 (v^* - \cos^2\chi) + 2bv^* \cos\chi}{(v^* + \cos^2\chi)^2}\,, \tag{26}$$

$$k = \frac{\mu_B Q_1}{aM} \, \frac{j}{e} \, \frac{v^*}{(v^* + \cos^2\chi)^2}\,. \tag{27}$$

For the backward current ($j/e < 0$), the calculation gives

$$p = \frac{\mu_B \gamma \alpha \tau Q_1}{a} \left|\frac{j}{e}\right| \frac{\lambda v_1 (v^* \cos^2\chi - 1) + 2bv^* \cos\chi}{(1 + v^* \cos^2\chi)^2}\,, \tag{28}$$

$$k = \frac{\mu_B Q_1}{aM} \, \frac{j}{e} \, \frac{v^* \cos^2\chi}{(1 + v^* \cos^2\chi)^2}\,. \tag{29}$$

The application of this strategy to specific problems frequently requires extensive computations. But in experiments, junctions with a very narrow operating layer *2*, such that $L \ll l, \sqrt{AM/H_a}$, have been of main interest so far; the parameters in the right-hand side of this inequality are the spin diffusion length and the domain-wall thickness, which are usually of the order of $\sim 3 \times 10^{-6}$ cm. It is assumed that the magnetization changes only a little over the thickness of the layer, and hence the expansion

$$\hat{\mathbf{M}}(x) = \hat{\mathbf{M}}(+0) + \hat{\mathbf{M}}'(+0)\,x + \frac{1}{2}\,\hat{\mathbf{M}}''(+0)\,x^2 + \ldots \tag{30}$$

holds, where the primes denote derivatives with respect to the coordinate x. Now, it is necessary to rewrite LLG equation (5) such that a time-dependent function $\hat{\mathbf{M}}(+0)$ would appear instead of the unknown function $\hat{\mathbf{M}}(x)$. We differentiate Eqn (30) with respect to x, set $x = L$, and use boundary conditions (24) and (25), with the result

$$\begin{aligned}\hat{\mathbf{M}}''(+0) &= -L^{-1}\,\hat{\mathbf{M}}'(+0) \\ &= L^{-1}\Big\{p\Big[\hat{\mathbf{M}}(+0) \times \big[\hat{\mathbf{M}}_1 \times \hat{\mathbf{M}}(+0)\big]\Big] \\ &\quad - k\big[\hat{\mathbf{M}}_1 \times \hat{\mathbf{M}}(+0)\big]\Big\}\,.\end{aligned} \tag{31}$$

The only term in (5) that contains the second derivative with respect to x can be rewritten, using (31), as follows:

$$\begin{aligned}a\big[\hat{\mathbf{M}}(+0) \times \hat{\mathbf{M}}''(+0)\big] &= \frac{a}{L}\Big\{p\big[\hat{\mathbf{M}}(+0) \times \hat{\mathbf{M}}_1\big] \\ &\quad + k\Big[\hat{\mathbf{M}}(+0) \times \big[\hat{\mathbf{M}}(+0) \times \hat{\mathbf{M}}_1\big]\Big]\Big\}\,.\end{aligned} \tag{32}$$

Substituting Eqn (32) in (5) and using the vector $\hat{\mathbf{M}}(+0)$ instead of $\hat{\mathbf{M}}(x)$, we obtain

$$\begin{aligned}&\frac{d\hat{\mathbf{M}}(+0)}{dt} - \varkappa\left[\hat{\mathbf{M}}(+0) \times \frac{d\hat{\mathbf{M}}(+0)}{dt}\right] + \gamma\big[\hat{\mathbf{M}}(+0) \times \mathbf{H}'\big] \\ &\quad + \frac{ap}{L}\,\big[\hat{\mathbf{M}}(+0) \times \hat{\mathbf{M}}_1\big] \\ &\quad + \frac{ak}{L}\Big[\hat{\mathbf{M}}(+0) \times \big[\hat{\mathbf{M}}(+0) \times \hat{\mathbf{M}}_1\big]\Big] = 0\,,\end{aligned} \tag{33}$$

where $\mathbf{H}' = \mathbf{H} + \mathbf{H}_a + \mathbf{H}_d$; in the simplest situation, $\mathbf{H}_a = \beta M\mathbf{n}\big(\hat{\mathbf{M}}(+0)\,\mathbf{n}\big)$ and $\mathbf{H}_d = -4\pi M\hat{\mathbf{x}}\big(\hat{\mathbf{M}}(+0)\,\hat{\mathbf{x}}\big)$; β and $\mathbf{n}$ respectively characterize the energy and the direction of the anisotropy axis and $\hat{\mathbf{x}}$ is the unit vector in the direction of the x axis.

Equation (33) describes the dynamics of the uniform magnetization inside layer *2*. The layer behaves like a single large magnetic moment, frequently called 'macrospin.' Such a representation is, naturally, only approximate, but it corresponds to the situation realized in some experiments and, furthermore, it substantially facilitates calculations. For the first time, a similar idea was introduced by Slonczewski in [9] for only one mechanism, that of spin-related torque. Here, a more general idea, which was first developed in [28], is given, which, in addition to the spin-transfer torque, also involves the spin injection, i.e., a term proportional to the parameter p in (33).

8. Current-induced exchange instability of magnetization

We show how Eqn (33) leads to the instability of magnetization at a sufficiently high current, which exceeds the threshold value. Let the initial stationary magnetization be $\hat{\mathbf{M}} = (0,\,0,\,\pm 1)$. The initial magnetization is therefore directed along the z axis. We consider small harmonic fluctuations of the magnetization $\Delta\hat{M}_x$ and $\Delta\hat{M}_y \sim \exp(-i\omega t)$ and linearize equations with respect to them. This yields a dispersion relation in the form

$$\omega^2 + 2iv\omega - w = 0\,, \tag{34}$$

where

$$w = \frac{1}{1 + \varkappa^2}\left[\left(\Omega_x + \frac{ap}{L}\,\hat{\hat{M}}_z\right)\left(\Omega_y + \frac{ap}{L}\,\hat{\hat{M}}_z\right) + \left(\frac{ak}{L}\right)^2\right], \tag{35}$$

$$v = \frac{\varkappa}{1 + \varkappa^2}\left[\frac{1}{2}\,(\Omega_x + \Omega_y) + \frac{a}{L}\left(p + \frac{k}{\varkappa}\right)\hat{\hat{M}}_z\right], \tag{36}$$

$$\begin{aligned}&\Omega_x = \gamma\big(H\hat{\hat{M}}_z + H_a + 4\pi M\big)\,, \quad \Omega_y = \gamma\big(H\hat{\hat{M}}_z + H_a\big)\,, \\ &p = p\big(\hat{\hat{M}}_z\big)\,, \quad k = k\big(\hat{\hat{M}}_z\big)\,.\end{aligned} \tag{37}$$

The relevant parameters are typically estimated as follows:

$$\Omega_x \approx 4\pi\gamma M \sim 10^{11}\ \mathrm{s}^{-1}, \quad \Omega_y \approx \gamma(H + H_a) \sim 10^9\ \mathrm{s}^{-1},$$

$$\varkappa \sim 3 \times 10^{-2} \ll 1\,.$$

The general condition for instability, $\mathrm{Im}\,\omega > 0$, requires the fulfillment of at least one of two inequalities: either $v < 0$ or $w < 0$. The first inequality depends on the damping parameter $\varkappa$. It describes the instability caused by the action of the spin-related torque. The second inequality is independent of dissipation and describes the instability caused by spin injection.

With the above estimates of the parameters, the condition $v < 0$ is approximately reduced to the inequality

$$1 + \frac{2ak}{\varkappa L\Omega_x}\,\hat{\hat{M}}_z < 0\,, \tag{38}$$

and the condition $w < 0$, to the inequality

$$1 + \frac{ap}{L\Omega_y}\,\hat{\hat{M}}_z < 0\,. \tag{39}$$

We substitute expressions (26)–(29) for the parameters p and k in (38) and (39). It follows from (26) and (28) that the sign of p is independent of the current direction; however, it depends on the relation between the spin resistances of the layers. For condition (39) to be satisfied, it is necessary that $\hat{\hat{M}}_z = -1$ at $p > 0$ and that the AP orientation be realized. Accordingly, at $p < 0$, the condition $\hat{\hat{M}}_z = 1$ should be satisfied and the P orientation should be realized. The orientations that are opposite to those above are stable due to the mechanism of spin injection at any current.

As regards the parameter k, the situation is different. According to Eqns (27) and (29), the sign of k always coincides with the sign of the electron flux j/e. Therefore, for condition (38) to be satisfied, it is necessary that $\hat{\hat{M}}_z = -1$ in the case of the forward current, when $j/e > 0$, and that $\hat{\hat{M}}_z = 1$ in the case of the backward current, when $j/e < 0$. Hence, if the mechanism of spin-transfer torque is operative, then only the AP orientation of magnetization can be unstable for the forward current, and only the P orientation, in the case of the backward current.

Solving inequalities (38) and (39) for the current and using formulas (26)–(29), we finally obtain the following instability threshold conditions:

• for the forward current ($j/e > 0$), when the spin–torque mechanism prevails, we have

$$\left(-\hat{\hat{M}}_z\right)\frac{j}{e} > \varkappa\lambda l\,\frac{2\pi\gamma M^2}{\mu_{\mathrm{B}}Q_1}\,\frac{1+\nu^*}{\nu^*}\,; \tag{40}$$

• for the backward current ($j/e < 0$), when the spin-torque mechanism prevails, we obtain

$$\hat{\hat{M}}_z\left|\frac{j}{e}\right| > \varkappa\lambda l\,\frac{2\pi\gamma M^2}{\mu_{\mathrm{B}}Q_1}\,\frac{1+\nu^*}{\nu^*}\,; \tag{41}$$

• irrespective of the current direction, when the spin injection mechanism prevails, the condition is

$$\left(-\hat{\hat{M}}_z\right)\left|\frac{j}{e}\right| > \left(1+\frac{H\hat{\hat{M}}_z}{H_{\mathrm{a}}}\right)\frac{\lambda l H_{\mathrm{a}}}{\mu_{\mathrm{B}}\alpha\tau Q_1} \times \frac{(1+\nu^*)^2}{\lambda\nu_1(\nu^*-1)+2b\nu^*\hat{\hat{M}}_z}\,. \tag{42}$$

We next discuss the numerical estimates that follow from threshold conditions (40)–(42). These estimates depend on the interaction mechanism (spin–torque transfer or spin injection) and on the relation between spin resistances. We begin with conditions (40) and (41) that are valid for the spin–torque mechanism. The modulus of the threshold current for the spin–torque mechanism, $j = (j_{\mathrm{th}})_{\mathrm{k}}$, is equal to

$$\left|\frac{j_{\mathrm{th}}}{e}\right|_{\mathrm{k}} = \frac{2\pi\gamma\varkappa M^2\lambda l}{\mu_{\mathrm{B}}Q_1}\left(1+\frac{1}{\nu^*}\right). \tag{43}$$

For $\gamma M \sim 10^{10}\ \mathrm{s}^{-1}$, $l \sim 2\times10^{-6}$ cm, $\lambda \sim 0.1$, $\varkappa \approx 3\times10^{-2}$, and $Q_1 \approx 0.3$, this yields

$$|j_{\mathrm{th}}|_{\mathrm{k}} \approx 6\times10^7\left(1+\frac{1}{\nu^*}\right)\ \mathrm{A\,cm^{-2}}\,.$$

Therefore, the threshold current is sufficiently high, which agrees with the experimental results for the spin–torque mechanism (see, e.g., [14, 15]). As is evident, such a threshold cannot be substantially decreased due to a special choice of spin resistances.

We now examine condition (42). The situation of interest for us is $\lambda\nu_1 \gg 1$ and $\nu^* \gg 1$. In this case, $\nu^*/\nu_1 = \lambda + (Z_1/Z_3)$. Condition (42) can then be simplified such that the modulus of the threshold current for the injection mechanism, $j = (j_{\mathrm{th}})_{\mathrm{p}}$, is obtained as

$$\left|\frac{j_{\mathrm{th}}}{e}\right|_{\mathrm{p}} = \frac{H_{\mathrm{a}}l}{\mu_{\mathrm{B}}\alpha\tau Q_1}\left(\lambda+\frac{Z_2}{Z_3}\right)\left(1+\frac{H\hat{\hat{M}}_z}{H_{\mathrm{a}}}\right). \tag{44}$$

After the substitution of numerical values of the parameters (in particular, $\alpha \approx 2\times10^4$, $\tau \approx 3\times10^{-13}$ s, $H_{\mathrm{a}} \sim 100$ Oe, $H = 0$, and $Z_2 \ll Z_1, Z_3$), we obtain $(j_{\mathrm{th}})_{\mathrm{p}} \approx 1.9\times 10^5\ \mathrm{A\ cm^{-2}}$. In this case, the reduction of the threshold by almost three orders of magnitude in comparison with the current $(j_{\mathrm{th}})_{\mathrm{k}}$ occurs as a result of the passage to the spin-injection mechanism and a special choice of spin resistances.

9. The effect of an external magnetic field on the exchange-switching threshold

According to Eqn (44), the threshold for spin-injection switching depends on the external magnetic field H. This dependence is of great interest. It follows that as the field $H\hat{\hat{M}}_z$ tends to $-H_{\mathrm{a}} + 0$, the last factor in (44) becomes zero, which implies the vanishing of the exchange threshold as well. The reason for this behavior is related to the reorientation phase transition caused by an increase in the external magnetic field. In approaching this threshold from below, conditions for the exchange switching of the magnetization are facilitated near it; the external field, acting together with the exchange field, favors switching. In this case, the exchange field, even if created by a comparatively small current, is sufficient for switching.

Thus, near the threshold of the magnetization reversal of the junction in a field H, the threshold for the current-driven exchange switching can be decreased considerably when approaching it from below. It is important that the most essential feature of the current-driven switching, the locality of switching, is then preserved. In a matrix of junctions capable of switching, which compose a memory unit, only the junction through which the current flows at a given moment can be switched. The other junctions (even the nearest neighbors) preserve their stability. In other words, the density of writing information remains high (as in the absence of a magnetic field).

As was shown in Section 2, the magnetization reversal of one layer of the junction by a field H leads to a substantial increase in the resistance of the junction for the electric current flowing perpendicular to the layers. This effect, as was already mentioned, is called giant magnetoresistance. It has been much studied since the 1980s, and great success was achieved in both the fundamental science and applied fields. We note that the GMR effect and the exchange switching with a low current threshold are, apparently, closely related.

To confirm this, we consider the results of experiments performed in Refs [29, 30]. In these experiments, the authors studied the magnetic tunnel structure at room temperature. The structure represented a typical magnetic junction, consisting of two ferromagnetic films (cobalt and permalloy $Ni_{20}Fe_{80}$) separated by an ultrathin (1–1.5 nm) layer of aluminum oxide Al_2O_3. The details of the structure preparation and measurements are described in a special report [30]. We assume that the relation between the spin resistances of the junction layers suggests that the parameters λv_1 and v^* are sufficiently large to allow using formula (42) with a positive denominator, as well as formula (44).

The authors of [29, 30] measured GMR (1) for different directions and values of the electric current flowing perpendicularly to the most developed boundary planes of the magnetic junction. The current was varied in the range 1–100 μA, which, with the transverse dimension of the junction about 100 μm, corresponds to the maximum current density ~ 1 A cm^{-2}. This current density is considerably less than the theoretical estimates made in the absence of the field, i.e., at $H = 0$ [see (43), (44)]. Therefore, the authors of [29] supposed that in the spacer layer of Al_2O_3, a noticeable role can be played by pores with the diameter ~ 0.1 μm, in which the current density could be much higher.

However, we note the following. First, the effect occurs only in the presence of an external magnetic field and only near those values at which a magnetization reversal occurs, i.e., at which a GMR is observed. In Fig. 4 borrowed from report [30], it is shown that the dependence of the GMR on the field H is modified upon the passage of a current. (We note that in this figure, and in all subsequent figures, the field is referenced to the magnetization-reversal field $\pm H_a$.) Curve *1* in Fig. 4 describes the variation of the GMR with an increase in the field H; curve *2*, with its decrease. We see that in both cases, the symmetry of the peaks becomes broken; to the right and to the left of the peaks, the slopes of the curves are different. The passage from a higher to a smaller value of $\Delta R/R$ always occurs more steeply, almost abruptly.

The last fact is very significant. It is a second effect on which we should focus attention for the interpretation of this dependence. According to formula (42), an exchange switching is possible for any direction of the current (both at $j/e > 0$ and at $j/e < 0$), but only if the magnetization orientations in the layers are antiparallel, i.e., $(-\hat{M}_z) > 0$. It follows from the experiment in [29, 30] that this condition is strictly satisfied.

According to [29, 30], the electric field leads, apart from switching, to a decrease in the GMR. This effect of decreasing GMR seems quite natural. If the electric field attempts to switch the junction, it must decrease the angle between the magnetizations of the layers. In turn, the decrease in the angle leads to a decrease in the resistance, which in this case approaches a minimum value for parallel layers. The influence of the field on the GMR is illustrated by the data in Ref. [29] presented in Figs 5 and 6.

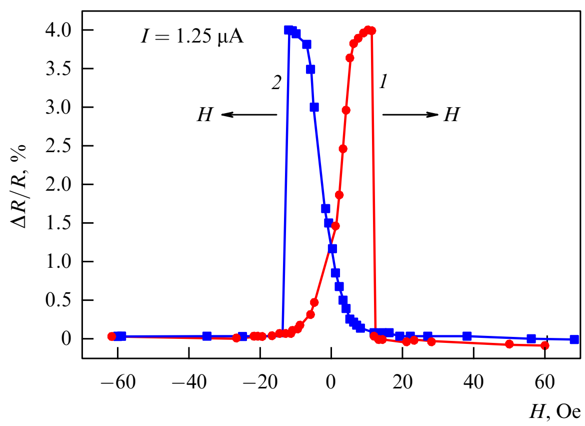

Figure 4. Relative magnetoresistance as a function of the field H, which can increase or decrease.

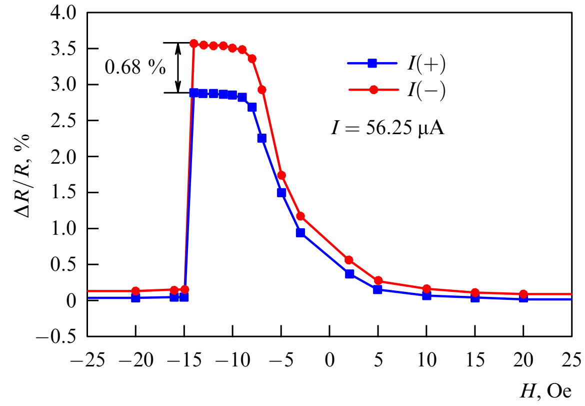

Figure 5. Variation of the relative magnetoresistance as a function of the field H depending on the current direction.

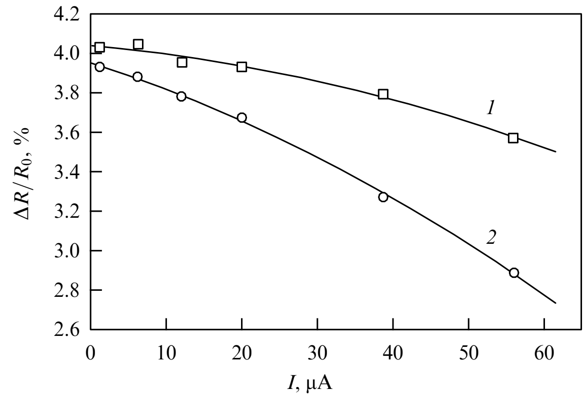

Figure 6. Maximum GMR as a function of the current with a negative (*1*) and a positive (*2*) potential at the lower ferromagnetic layer.

Thus, the experimental results in [29, 30] concerning the influence of the current on the GMR are in accordance with the conclusions of the above theory of spin-injection exchange switching in an external magnetic field near the threshold of the reorientation phase transition.

10. Conclusion

• The conduction electrons that participate in the polarized current (s electrons) interact with the lattice magnetization (d electrons) in a ferromagnetic junction via two channels: (1) via the transfer of the transverse spin (perpendicular to the magnetization) to the lattice, and (2) via the transfer to it of the longitudinal spin parallel to the magnetization. The latter can be considered a change in the population of spin energy subbands, i.e., the injection of nonequilibrium spins. This injection leads to the creation of a nonequilibrium sd exchange effective field, which, in turn, affects the dynamics of the lattice.

• In the range of problems that appear upon the imposition of a tangential magnetic field H, as well as upon the simultaneous passage of a current j perpendicular to the surfaces of the junction, the following effects are especially important: (1) a giant magnetoresistance in the reorientation phase transition in fields $H \sim H_a$; and (2) the instability of magnetic fluctuations of the sd-exchange origin at current densities that exceed a certain threshold, $j > j_{\rm th}$.

• The magnitude of the threshold current $j_{\rm th}$ depends on the parameters of the junction and on the external magnetic field. In the absence of a field, the threshold is minimum at specific ratios between the spin resistances of the layers Z_i, where $i = 1, 2, 3, \ldots$ labels the layers. In particular, the condition $Z_2 \ll Z_1, Z_3$ leads to a reduction in the threshold. The estimates for concrete samples show that in this case, the threshold can be lowered by orders of magnitude, for example, from $j_{\rm th} \sim 6 \times 10^7$ to $\sim 2 \times 10^5$ A cm^{-2}. The minimum thresholds always correspond to the predominance of the spin-injection channel of the sd exchange interaction.

• In the presence of an external magnetic field H, an additional significant decrease in the threshold current $j_{\rm th}$ is possible due to the proximity of H to the reorientation phase transition threshold. In other words, this means that the two above-mentioned effects — the GMR and the exchange instability — are closely related to each other. The presence of this relation is confirmed by some previously performed experiments. However, additional studies are required in this field.

• The connection between the GMR and the exchange instability may be interesting for applications, because the GMR effect acquires new properties and becomes very sensitive to the current, and the exchange instability effect is achieved at extremely low thresholds and can therefore in principle ensure the possibility of recording information with an extremely high density (to tens of Gbit cm^{-2}).

Acknowledgments. We are grateful to N P Laverov for the interest in our work and for the stimulating discussions, and also to Yu G Kusraev for the fruitful discussions and help. P E Z is grateful to the authors of [30] for the permission to reproduce Fig. 4. This work was supported in part by the Russian Foundation for Basic Research grants 06-02-16197 and 08-07-00290.

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