Physics – Uspekhi **52** (3) 263 – 266 (2009)

METHODOLOGICAL NOTES

PACS numbers: **01.65.+g**, **03.30.+p**

Para-Lorentz transformations

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DOI: 10.3367/UFNe.0179.200903e.0285

Abstract. **In special relativity, events that are simultaneous in one inertial reference frame (IRF) are not so in another. 75 years ago, L I Mandelstam showed that the absolute simultaneity of events in different IRFs is achieved by using infinitely fast signals instead of light to synchronize clocks. 25 years later, F R Tangherlini showed in an independent study how special coordinates and time transform in this situation from one IRF to another. Although different IRFs enter on different footing in this case (the observer's frame being the privileged one), the Tangherlini transformations are still capable of describing the known experimental special relativity tests.**

In 1933–1934, L I Mandelstam gave a lecture course on the physical foundations of special relativity [1]. These lectures were so important in essence, deepness, and comprehensibility [2] that a large audience, including not only postgraduate students but also well-known scientists, attended them [3]. After the death of Mandelstam, these lecture were reconstructed from notes made by S M Rytov, G S Gorelik, M A Divilkovsky, M A Leontovich, and Z G Libin and using notes and drafts of Mandelstam himself, edited by Rytov and published in 1950 [1]. The second edition appeared in 1972 [4].

In his lecture notes, Mandelstam devoted much attention to the principle of causality and simultaneity of events from the points of view of observers in different inertial reference frames (IRFs). We note that although Einstein's special relativity (SR) [5] was recognized by most physicists at that time, there were quite a large number of scientists who either cast doubts on its validity or even strongly criticized SR, mainly for two reasons: (1) the lack of experimental tests of time dilation in a moving IRF relative to the observer at rest, and (2) the incomplete understanding (or nonacceptance) of the relativity of simultaneity in different IRFs. The first reason disappeared in 1938, when experiments by Ives and Stilwell [6] confirmed the presence of the relativistic (transverse) Doppler effect,[1] after which the number of opponents of SR drastically decreased. As regards the second reason, the opponents of SR almost immediately after its formulation started trying to substitute the Lorentz transformations (LTs) by other types of transformations[2] relating space and time coordinates in the observer's IRF at rest (K) with those in an IRF in motion (K'). During the last 100 years, several dozen incorrect para-Lorentz transformations have been suggested, which, in contrast to LTs, could not describe the results of all known experimental tests of SR. A detailed critical analysis of most incorrect para-Lorentz transformations is not the purpose here. Nevertheless, there is an exception, the so-called Tangherlini transformations (TTs) [10], which are correct and are considered in what follows.

Is it possible to suggest transformations that would be different from LTs but at the same time would be correct? On March 10, 1934, during a polemic with opponents of SR and in relation to the formulation of the causality principle in SR and the problem of simultaneity of events in different IRFs, Mandelstam noted: "Thus, the requirement that the causality not be violated during the determination of simultaneity can be uniquely satisfied[3] ... if there existed a signal moving with an infinite velocity, the requirement not to violate causality would yield the unique condition and this requirement would be universal in all [reference] frames ... Thus it must be understood that there should be no such signal that could interact... if I have a process by which it is impossible to affect ..., this process does not violate the causality principle... Many people have tried to introduce this concept of simultaneity, which, as they thought, does not depend on definition but is the consequence of there being some apriori simultaneity..."[4] [1, p. 196, 197], [4, p. 182, 183]. Then Mandelstam considered the possibility of synchronizing distant clocks in different IRFs using the phase velocity of a signal, which can be arbitrarily large. If the phase velocity tends to infinity, the absolute simultaneity in all IRFs occurs

[1] Unfortunately, Ives himself remained an opponent of SR until the end of his life. Regarding his paper [6], V L Ginzburg noted as early as 1940: "... Note, incidentally, that the author of the cited paper (Ives) tries to explain his results using pre-Einsteinian concepts, which is a total anachronism" [7].

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Received 18 April 2008, revised 16 December 2008
Uspekhi Fizicheskikh Nauk **179** (3) 285 – 288 (2009)
DOI: 10.3367/UFNr.0179.200903e.0285
Translated by K A Postnov; edited by A M Semikhatov

[2] In [8], it was suggested to call such transformations para-Lorentz. As far as we know, German physicist Max Abraham [9] was the first to try to introduce them.

[3] Here, Mandelstam apparently considers a (non-Einsteinian) procedure of synchronization of distant clocks proposed by H Reichenbach [11, 12]. We recall that according to Einstein [5], to synchronize clocks at points A and B, it is necessary to send a light signal from point A to point B and then reflect it back to point A. The time of arrival of the signal to point B in this case is $t_2 = t_1 + 0.5(t_3 - t_1)$, where t_1 is the time of departure of the signal from point A and t_3 is the time of return of the signal to point A. According to Reichenbach, $t_2 = t_1 + \varepsilon(t_3 - t_1)$, where $\varepsilon\,(0 < \varepsilon < 1)$ is sometimes referred to as the Reichenbach parameter. In particular, $\varepsilon = 0.5$ in SR. Within Reichenbach's clock synchronization procedure, the speed of light in the direct (c^+) and opposite (c^-) directions can be different: $c^{\pm} = c/(1 \mp \Delta)$, where $\Delta = 2\varepsilon - 1$. In SR, $\varepsilon = 0.5$, $\Delta = 0$, and $c^+ = c^- = c$. Mandelstam emphasized that Reichenbach's procedure does not violate the causality principle.

[4] Here, Mandelstam stresses that clock synchronization by infinitely fast signals leads to simultaneity in all IRFs.

and the causality principle is not violated because phase velocity does not carry either energy or information. Unfortunately, in [1, 4], the conclusion was made that it is impossible to realize this method in practice, because the corresponding velocity was considered there for a mechanical device (like the motion at the intersection of two parts of a pair of scissors), which has a finite velocity of propagation of perturbations (when one starts closing the scissors, the perturbation there propagates with the speed of sound in metal).

However, such a synchronization can still be realized using the light spot, which can have a superluminal velocity, proposed by Ginzburg [13] and considered in more detail by Bolotovsky and Ginzburg. In [13, 14], the motion across a screen of a light beam rotating with an angular velocity Ω is considered. If points A and B are at equal, sufficiently large distances R from the beam, the linear velocity of the light spot is $v = R\Omega \gg c$, where c is the speed of light in the vacuum. Of course, the spot cannot carry information with a superluminal velocity from point A to point B: photons arriving at A never come to B and hence the causality principle is not violated. Different ways of realizing the superluminal velocity of the 'spot' of arbitrary physical nature have been considered in the literature [15]. The largest velocities are achieved by spots formed by pulsar radiation [13, 14, 16]. The problems of distant clock synchronization using light spots are considered in our papers [15, 17, 18].

We note that Mandelstam discussed a second way of synchronization of distant clocks located in different IRFs, which would provide the absolute simultaneity in IRFs K and K': "... Imagine that there is one [reference] frame where somehow (for example, in the Einsteinian way) the synchronization is set... Then let there be another frame. I could arbitrarily set synchronization in that other frame such that clocks there always show the same time as in the first frame... Then synchronization is a universal concept, i.e., if in this frame something occurs simultaneously, so it should be in another frame... But then it is impossible to require that the relativity principle be valid ... When Einstein says that the relativity principle takes place in nature, this means that if you define all systems in a universal way, events [in any other frames] would occur universally" [1, p. 202, 203], [4, p. 188]. In other words, in using this method of synchronization, not all IRFs are on equal footing: the one where the primary synchronization was made becomes special (privileged).

However, Mandelstam did not address the question of the form of transformations of space and time coordinates from frame K to frame K' if distant clocks in both frames were synchronized by infinitely fast signals and how these transformations would differ from the classic LTs. Such transformations were obtained in 1958 by American physicist F R Tangherlini in his PhD thesis [10] [5] and were later called Tangherlini transformations. Direct and inverse TTs have the form [10]

$$\begin{aligned} &x' = \gamma(x - vt), \quad x = \gamma^{-1}x' + \gamma v t', \\ &y' = y, \quad y = y', \\ &z' = z, \quad z = z', \\ &t' = \gamma^{-1}t, \quad t = \gamma t', \end{aligned} \tag{1}$$

[5] The biography of Tangherlini and the history of writing and defense of his PhD thesis [10] can be found in [19]. We note that in [10], the so-called external procedure of clocks synchronization in different IRFs was also discussed, which fully coincides with the second way of synchronization proposed earlier by Mandelstam [1, 4].

where v is the velocity of motion of IRF K', relative to the privileged IRF K, along the axes X and X', and $\gamma = 1/\sqrt{1 - v^2/c^2}$ is the Lorentz factor. Tangherlini himself called (1) the absolute Lorentz transformations. It follows from (1) that the expression relating the collinear velocities V and V' in IRFs K and K', or, equivalently, the velocity addition law for TTs is [10] *

$$V' = \frac{V - v}{(1 - v^2/c^2)} \,. \tag{1*}$$

Formula (1*) is essentially different from the relativistic velocity addition law

$$V' = \frac{V - v}{(1 - vV/c^2)} \,.$$

For $V = c$, [10] we find

$$c' = \frac{c}{1 + (v/c)\cos\theta'} \,, \tag{2}$$

where the angle θ' is relative to the axis X' in K'. In the general case where light propagates in an optical medium with a refractive index n, the medium being at rest in K', the TT takes the form [10]

$$c' = \frac{c}{n + (v/c)\cos\theta'} \,. \tag{3}$$

We note that TTs correspond to the Reichenbach parameter $\varepsilon = 0.5[1 + v/c]$. Frames K and K' are on equal footing for LTs, i.e., for an observer in K, time in K' slows down by the factor of γ, and for an observer in K', the time in K also slows down by a factor of γ. For TTs, IRF K and K' are not on equal footing any more: for an observer in K, time in K' slows down by a factor of γ, but for an observer in K', time in K is stretched by a factor of γ. Nevertheless, TTs [see (2)] can describe the results of the Michelson – Morley [20, 21] and Kennedy – Thorndike [23] experiments, because, as follows from (2), the total time of light propagation in the direct and opposite directions is independent of the velocity of motion of IRF K' relative to the privileged IRF K. It can be shown that TTs can also describe the results of Sagnac experiments [23 – 25], even when the ring interferometer is embedded in an optical medium, as well as the Hoek [26] and Ragul'skii [27] experiments. TTs can also describe measurements of the relativistic Doppler effect [6] (which have been repeated many times with increasingly high accuracy; see, e.g., [28]). However, as we noted above, TTs come true only for an observer in IRF K. A brief description of the main results in [10] and, in particular, the TTs was published by Tangherlini in 1961 in Section 1.3. of [29] and in 1994 in the Appendix of his paper [30].

From the group theory standpoint, classical LTs are a representation of the rotation group (the Lorentz group) in Minkowski space in the Cartesian coordinate system through the angle $\varphi = \operatorname{arsinh}(v/c)$. If the velocity of IRF K' relative to K is parallel to the axis X, only the axes X' and icT' rotate. TTs bring us from the Cartesian frame K to a nonrectangular coordinate system corresponding to IRF K'. Here, the axis X', as in the case of LTs, rotates relative to the X axis through the angle $\varphi = \operatorname{arsinh}(v/c)$, and the icT' axis remains parallel

* *Note by the Editors*: Compared to the Russian original of this paper, the following two formulas have been changed by the author.

to the icT axis. Such IRFs are sometimes referred to as generalized IRFs [31]. As shown in the monograph by Pauli [32] (see also [31]), only in Cartesian (rectangular) coordinate systems (which are sometimes called Galilean coordinate systems), the speed of light is the physical velocity, while in all other systems, including nonrectangular ones, the speed of light is a coordinate velocity, i.e., it depends on the choice of the coordinate system. All this is related to the speed of light that follows from TTs [see (2)]. We note that only in the Cartesian (rectangular) coordinate system does the metric tensor have a diagonal form. TTs do not belong to the Lorentz group, because TTs can be represented as a product of two consecutive transformations: a Galilean transformation (GTs) and a local one for time, and, hence, as shown in [10], their product does not belong to the Lorentz group.

It is interesting to consider how the form of the transformations relating IRFs K and K' depend on the method of clock synchronization. As was shown by Mandelstam [1, 4] and Reichenbach [11, 12], the causality principle is not violated if the Reichenbach parameter ε lies in the range $0 < \varepsilon < 1$. Paper [33] by Sjödin considers more general, as compared to LTs and TTs, transformations (so-called Sjödin transformations (STs)):

$$x' = \gamma(x - vt)\,, \quad y' = y\,, \quad z' = z\,,$$

$$t' = \gamma - \xi \frac{v}{c^2}\, x + \left[1 - (1 - \xi)\,\frac{v^2}{c^2}\right] t\,, \tag{4}$$

where ξ is a dimensionless parameter that lies in the range $1 - c/v < \xi < 1 + c/v$ and is related to the Reichenbach parameter ε as

$$\varepsilon = 0.5\left[1 + \frac{v}{c}\,(1 - \xi)\right].$$

The STs differ from LTs by the form of time transformation only. For $\xi = 1$, STs become the LTs; for $\xi = 0$, they become the TTs. For arbitrary ξ (lying, however, in the allowed range), STs can describe the results of Michelson–Morley [20, 21] and Kennedy–Thorndike [22] experiments. However, it is easy to show that the correct value of the Fresnel entrainment coefficient can be obtained only for $\xi = 0$, i.e., for TTs [see (3)], and, of course, for $\xi = 1$, i.e., for LTs.

Transformations of space coordinates and time in passing from one IRF to another can describe the results of known experimental tests of SR only if the procedure of synchronization of distant clocks in the moving IRF and that at rest corresponds to the time transformation. But even in this case, the speed of light obtained under such transformations in the moving IRF is not necessarily coincident with the physical speed of light, but can be a coordinate speed of light. The identity of the physical and coordinate speeds of light in the moving IRF occurs only for LTs.

For more than 100 years after the formulation of SR [5], most scientists believed that LTs follow directly from two postulates of SR: the equal footing of all IRFs and the equality and isotropy of the speed of light in all IRFs and its independence of the velocity of the source of emission [5]. However, as early as 1934 in lectures given by L I Mandelstam [1, 4], it was shown that this is not sufficient, and obtaining LTs also requires using the Einsteinian procedure of synchronization of distant clocks in different IRFs. If another clock synchronization procedure is used instead, other transformations can be obtained [see (4)]. The possibility of synchronizing clocks using infinitely fast signals was already considered by Reichenbach [11, 12], but only Mandelstam showed that this procedure would lead to the absolute simultaneity in all IRFs [1, 4], and Tangherlini derived the corresponding transformations [10]. We note a more important difference of TTs from LTs. For TTs, the speed of light [see (2)] is the speed of light in IRF K' as measured by the observer who stays in K if the clocks in both IRFS are synchronized by infinitely fast signals; at the same time, an observer in K' discovers that $c' = c$. For LTs in any IRF (K, K', or another), $c' = c$ and, consequently, the speed of light in K' as measured by an observer either in K or K' is always constant. The anisotropy of the coordinate speed of light c' in K' for TTs is the price paid for the absolute simultaneity in all IRFs [34]. Along with LTs, TTs can adequately describe processes in a moving IRF, but LTs are more convenient in most cases, because they leave the speed of light constant and isotropic in all IRFs.

Initially, TTs were almost unnoticed by scientists. However, after the discovery of anisotropy of cosmic microwave background radiation in 1977 [35], when it became clear that our IRF is moving in space with the velocity 360 km s^{-1} relative to some privileged IRF (in which the relic background radiation is 'mostly' isotropic and the total momentum of all bodies in the Universe is zero) and different assumptions about the possible anisotropy of the speed of light emerged, TTs turned out to be useful. The possibility of using TTs to explain the Michelson–Morley experimental results when the speed of light is anisotropic was first suggested in [36]. Recently, interest in TTs has increased due to the possible discovery by a group of scientists from Grenoble of a fairly tiny anisotropy of the speed of light [37–39]. TTs can also be useful in theoretical attempts to find a 'graceful' Lorentz-invariance violation, which sometimes are invoked in trying to interpret some exotic phenomena, for example, the origin of ultra-high-energy cosmic rays or dark-matter particles (and also dark energy), and are used in different cosmological and quantum gravity models (see, e.g., [34, 40–48]). To date, more than 100 papers are known where TTs were used or discussed. Thus, in some cases, TTs can be very convenient. We also note that some problems in the electrodynamics of moving bodies can be more conveniently solved using the Galilean transformations [49].

The author is especially grateful to F R Tangherlini for providing his PhD thesis [10] and the detailed discussion of the results. The useful discussions with V L Ginzburg and Vl V Kocharovsky are acknowledged. The author express his gratitude to E G Malykin, V I Pozdnyakova, and N V Roudik for their help. The work is partially supported by a grant of the President's Council for Support of leading Scientific Schools NSh-1931.2008.2.

Notes added by the author for the English translation. The author found a very interesting paper by English mathematician Albert Eagle [50], who, as early as 1958, i.e., 20 years before PhD thesis [10] was written, derived (although by not strictly rigorous means) part of the direct Tangherlini transformations. Paper [50] has remained virtually unnoticed (only two references to it were identified [51, 52]).

Note of the Editorial Board

The Editorial Board of *Physics–Uspekhi* considers any further discussion of the question of clock synchronization on the pages of this journal unworthy.

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