METHODOLOGICAL NOTES

Is it possible to see the infinite future of the Universe when falling into a black hole?

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257

258

259

261

Contents

- 1. Introduction
- 2. Free fall upon a Schwarzschild black hole
- 3. A fall under the horizon
 - References

<u>Abstract.</u> A possibility to see the infinite future of the Universe by an astronaut falling into a black hole is discussed and ruled out.

1. Introduction

Black holes are considered to be quite usual objects in modern astrophysics. There is convincing observational evidence for their existence (see, for example, review [1]). According to the common point of view, there is a black hole at the galactic center, and black holes reside in quasars and cause their bright emission due to the 'eating' of infalling stars and interstellar gas. In contrast to supermassive black holes in galactic nuclei and quasars with a mass of millions of times the Sun, there are less massive black holes which are observed in binary systems due to their interaction with the companion star. However, when attempting a theoretical description of a black hole in the context of General Relativity, some disagreements appear, both in special and popular literature. Because of this, the aim of our notes consists in examining some of these discrepancies.

Here we shall follow Einstein's general relativity. In alternative theories, for example, in the field theory of gravitation [2], there can be no black holes at all.

There can be static, rotating, and charged black holes. They are described by the Schwarzschild (1916) [3], Kerr [4], Reissner–Nordström [5] (charged nonrotating), and Kerr– Newman [6] (charged rotating) metrics, respectively. Yet the common point of view is that the charge of a black hole can be

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Uspekhi Fizicheskikh Nauk **179** (3) 279–283 (2009) DOI: 10.3367/UFNr.0179.200903d.0279 Translated by K A Postnov; edited by A Radzig neglected if it was produced from the core collapse of a star consisting of ordinary nucleons and electrons [7].

Consider the best studied case of a static black hole. What would an astronaut falling into such a black hole see? In all textbooks in which general relativity is considered (see, for example, Ref. [8]), one can read that there are two frames of reference. The first frame (call it A) is related to the Earth; the second one (B) is related to the astronaut falling upon the black hole. In the first frame of reference, the astronaut will forever approach the surface of the black hole (the horizon at the Schwarzschild radius of the black hole) but never reach it. In the second frame, the astronaut will reach the Schwarzschild radius in a finite time interval and cross the black hole horizon, but any signal produced by him can never reach an observer on the Earth. And here such a nonnaïve physicist as Yuval Ne'eman asks a naïve question: "How can B be allowed his (or her) frame of reference, in the equalitarian regime of covariance, if we can claim in all finality that B will never cross the Schwarzschild radius, in our spacetime reality?" [9]. Similarly, the collapsing star will never cross its Schwarzschild radius in frame A. Next, Ne'eman asks how one can "add to eternity A the extra half-hour B spends inside the black hole." He calls the emerging situation 'surrealism' - the hypothesis for the existence of different realities, one of which is not only unavailable but also impossible for another. Let us attribute this observation to the problem of the correct 'philology' and accept that the brave astronaut is capable of passing from one reality to another.

The situation discussed by Ne'eman is usually described in terms of the complete and incomplete frames of reference. For example, frame A is incomplete since one cannot describe there events inside the black hole, while frame B in the Kruskal–Szekeres coordinates [10] is complete. This answer, of course, was known to Ne'eman, but apparently was not fully satisfactory to him. The time of the astronaut inside a black hole is in no way related to our time on the Earth, and, as was mentioned above, it can no way be 'added' to it. Furthermore, there is a purely mathematical problem related to the singularity of the very transform of passage from A to B at the Schwarzschild radius, which has been discussed by theoreticians ever since the appearance of the Schwarzschild black hole solution [11].

Let us agree, however, with the commonly accepted opinion with regard to the astronaut's crossing the Schwarzschild radius. Let us ask: What will he see when approaching a black hole? In the popular literature [12] (see also Ref. [13]), a very attractive picture for future tourists to the galactic nucleus is suggested: the astronaut can see all the future of the Universe. "A spacecraft with astronauts approaching a black hole will appear to the Earth's observer as braking its motion but never crossing the black hole horizon. If the situation is reversed and we analyze it from the point of view of the astronaut lingering near the horizon, then the rate of events in the external Universe is extremely accelerated: virtually in one moment of his time the astronaut will see the infinitely long development of events in the external Universe. He will see how our Sun expands to become a red giant, how the Earth evaporates from the hot solar rays when sliding over upper layers of the dying Sun's atmosphere, how the outer hydrogen envelop detaches from the Sun that ultimately turns into a white dwarf — in short, the astronaut will see the future of our Universe!" The astronaut will observe all that over a finite time interval in the frame of reference B. Is that the case?

A similar statement can be read in the translator's notes to book [14], 'explaining' to the reader considerations of the author, Stephen Hawking (!). The same picture for an observer sitting on the surface of a collapsing star is suggested in the popular book [14]: "It appears to such an observer that the time in the external space runs at a growing rate and instantly reaches the very 'end of all times'." Unfortunately, we must disappoint future astronauts and popular book readers. The astronaut falling upon a black hole is never seeing the infinite future of our Universe! To clarify this, let us write out several formulas.

2. Free fall upon a Schwarzschild black hole

Consider free fall upon a static noncharged black hole in Schwarzschild coordinates in which the metric has the form

$$ds^{2} = \left(1 - \frac{r_{g}}{r}\right)c^{2} dt^{2} - \frac{dr^{2}}{1 - r_{g}/r} - r^{2}(d\vartheta^{2} + \sin^{2}\vartheta d\varphi^{2}).$$
(1)

Here, $r_g = 2Gm/c^2$ is the gravitational radius of the black hole, and c is the speed of light.

Radial geodesics in metric (1) satisfy the equations (see Ref. [16])

$$\left(\frac{\mathrm{d}r}{c\,\mathrm{d}\tau}\right)^2 = \frac{r_{\mathrm{g}}}{r} + \varepsilon^2 - 1\,, \qquad \frac{\mathrm{d}t}{\mathrm{d}\tau} = \frac{\varepsilon}{1 - r_{\mathrm{g}}/r}\,,\tag{2}$$

where $\varepsilon = \text{const.}$ For timelike geodesics, τ is the proper time of a moving particle, and ε is the specific energy: a particle with the rest mass m_0 possesses total energy $\varepsilon m_0 c^2$ in the gravitational field (1).

If the particle's fall starts from rest at some distance $r_0 > r_g$ then clearly (see the first formula in Eqn (2) at $dr/d\tau = 0$) $\varepsilon = \sqrt{1 - r_g/r_0}$ and, hence (after dividing the first equation by the square of the second one and extracting the root)

$$\frac{\mathrm{d}r}{c\,\mathrm{d}t} = -\left(1 - \frac{r_{\mathrm{g}}}{r}\right) \left[1 - \frac{1 - r_{\mathrm{g}}/r}{1 - r_{\mathrm{g}}/r_{0}}\right]^{1/2}.$$
(3)

Integrating (3) yields the following expression for the time $t - t_0$ of free fall from the point r_0 (a particle at rest) at the



Figure 1. The dependences x(t) and $x(\tau)$ for a massive particle falling upon a black hole and a photon (the thick solid curve).

instant of time t_0 to a point with coordinate $r < r_0$:

$$t - t_{0} = \frac{r_{g}}{c} \left\{ \sqrt{x_{0} - 1} \left[(2 + x_{0}) \arctan \sqrt{\frac{x_{0} - x}{x}} + \sqrt{x(x_{0} - x)} \right] + 2 \ln \left(\sqrt{(x_{0} - 1) \frac{x}{x_{0}}} + \sqrt{1 - \frac{x}{x_{0}}} \right) - \ln |x - 1| \right\},$$
(4)

where $x_0 = r_0/r_g$, and $x = r/r_g$. The free-fall time obviously increases logarithmically in $r - r_g$ with no limit for $x \to 1$, i.e., $r \to r_g$.

It might be possible to assume that, during this infinite Schwarzschild time, the light rays from events that are arbitrarily remote in space and time could catch up with the freely falling astronaut. Let us make sure, however, that this is not the case. It should be noted, first of all, that the proper time of the astronaut falling upon a black hole is finite. Indeed, for the proper time $\tau - \tau_0$ of motion from r_0 to the point with radial coordinate *r* we obtain from Eqn (2):

$$\tau - \tau_0 = \frac{1}{c} \int_r^{r_0} \frac{\mathrm{d}r}{\sqrt{\varepsilon^2 - 1 + r_{\rm g}/r}} \,. \tag{5}$$

If the free fall occurs from the state at rest, then one has

$$\tau - \tau_0 = \frac{r_0}{c} \sqrt{\frac{r_0}{r_g}} \left(\arctan \sqrt{\frac{r_0}{r} - 1} + \sqrt{\frac{r}{r_0} - \frac{r^2}{r_0^2}} \right).$$
(6)

Notice that time interval (6) is exactly the same as the appropriate free-fall time in Newton gravitational theory!

Now consider the radial motion of a light ray. From the condition ds = 0, we get

$$\frac{\mathrm{d}r}{c\,\mathrm{d}t} = \pm \left(1 - \frac{r_{\mathrm{g}}}{r}\right),\tag{7}$$

which implies the photon propagation time from r_0 to r:

$$t - t_{\rm s} = \frac{r_0 - r}{c} + \frac{r_{\rm g}}{c} \ln \left| \frac{r_0 - r_{\rm g}}{r - r_{\rm g}} \right|,\tag{8}$$

where t_s is the time of the photon start. Thus, the photon propagation time in the Schwarzschild coordinates increases logarithmically in $r - r_g$ as $r \rightarrow r_g$.

Figure 1 plots the coordinate x (in units of the Schwarzschild radius) as a function of time t for a massive particle and a photon (the thin and thick solid lines, respectively) starting their motion at the point with $r_0 = 4r_g$. The dependence of the coordinate on the proper time τ of the massive particle is shown by the dashed line.

Subtracting expression (8) from formula (4) gives the answer to the following question: At which instant of time t_s should a light signal be sent from point r_0 in the radial direction to catch up with the freely falling 'observer' at a value of the Schwarzschild radius $r < r_0$, who started their motion with zero initial velocity from point r_0 at some instant of time $t_0 < t_s$? The answer follows as

$$t_{s} - t_{0} = \frac{r_{g}}{c} \left[(2 + x_{0})\sqrt{x_{0} - 1} \arctan \sqrt{\frac{x_{0} - x}{x}} + \sqrt{x_{0} - x} \left(\sqrt{(x_{0} - 1)x} - \sqrt{x_{0} - x}\right) + 2\ln \left(\sqrt{\frac{x}{x_{0}}} + \sqrt{\frac{x_{0} - x}{(x_{0} - 1)x_{0}}}\right) \right].$$
(9)

Proceeding in expression (9) to the limit $x \to 1$, i.e., $r \to r_g$, we find how late the light can be emitted from the starting point of the freely falling massive observer to be detected before the observer crosses the horizon:

$$t_{s} - t_{0} = \frac{r_{g}}{c} \left[(2 + x_{0})\sqrt{x_{0} - 1} \arctan \sqrt{x_{0} - 1} + 2\ln 2 - \ln x_{0} \right]$$
(10)

Thus, the limit is finite and before crossing the black hole horizon there is no possibility of seeing the infinite future events occurring near the starting point of the free fall.

In Newtonian theory, the corresponding expression for $t_s - t_0$ takes the form

$$I_{s} - I_{0} = \frac{r_{g}}{c} \left[x_{0}^{3/2} \arctan \sqrt{\frac{x_{0} - x}{x}} + \sqrt{x_{0}x(x_{0} - x)} - (x_{0} - x) \right].$$
(11)

At large values of $x_0/x = r_0/r \ge 1$, both formulas (9) and (11) give the same result

$$t_{\rm s} - t_0 \sim \frac{\pi}{2} \frac{r_0}{c} \sqrt{\frac{r_0}{r_{\rm g}}}$$
 (12)

Let us consider another possible case of an astronaut falling upon a black hole and seeing the future of the Universe. Instead of freely falling upon the black hole, the astronaut 'lingers' in some close orbit and rotates about it [17]. Here, the situation can be similar to the twins paradox: in a short time interval, the astronaut will be able to observe processes occurring over a rather long period of time in the vicinity of the Earth. But this is not the infinite future of the Universe! In addition, note that circular orbits with a radius smaller than $r = 3r_g$ cease to be stable [18]. The velocity of travel in the last marginally stable circular orbit equals c/2, and hence the Lorentz time dilation here is insignificant.

Let us go back to the question of the infinite time the astronaut needs to approach the black hole horizon from the point of view of the Schwarzschild's observer on the Earth and the astronaut's finite proper time before crossing the horizon. Does that mean the 'relativity of history'? Could it be that there is no unique history of the astronaut's motion towards the black hole? If one understands history as a world line, it is clear that it is unique and has a finite length. Another 'history' possesses infinite length — the world line of clocks of the Schwarzschild's observer. So there is no relativity of history!

Thus, we have considered the simplest case of a nonrotating, noncharged black hole and the motion of an observer up to the horizon only. But, perhaps, when falling under the horizon towards the singularity (which, as is well known, cannot be observed from outside the horizon) all the future history of the world can become available to the brave astronaut who left this world forever? Or maybe one can see the future in the case of charged or rotating black holes?

To explore these possibilities, a more complicated analysis of the global structure of solutions describing black holes is needed. The interested reader should familiarize himself with books [7, 16, 19, 20] for more detail. Here we just want to bring up some basic facts which are relevant to the problems considered.

3. A fall under the horizon

Formula (1) becomes senseless when the infalling observer crosses the horizon at $r = r_g$, which is related to the inadequacy of the coordinate system. However, there are Kruskal–Szekeres coordinates which allow one to write out the Schwarzschild solution both outside and inside the black hole horizon.

Let us introduce new coordinates u, v, such that

$$u = \sqrt{x - 1} \exp\left(\frac{x}{2}\right) \cosh\frac{ct}{2r_{\rm g}}, \qquad (13)$$
$$v = \sqrt{x - 1} \exp\left(\frac{x}{2}\right) \sinh\frac{ct}{2r_{\rm g}},$$

where $x \equiv r/r_g > 1$. Here, clearly, the inequalities $u > |v| \ge 0$ are valid. In these coordinates, the Schwarzschild metric (1) takes the form

$$\mathrm{d}s^2 = r_\mathrm{g}^2 \left[\frac{4}{x \exp x} (\mathrm{d}v^2 - \mathrm{d}u^2) - x^2 (\mathrm{d}\vartheta^2 + \sin^2\vartheta \,\mathrm{d}\varphi^2) \right]. \tag{14}$$

Next, let us assume that the coordinates u, v are changing from $-\infty$ to $+\infty$, and x > 0 is a function of u, v given by the following equation

 $u^2 - v^2 = (x - 1) \exp x.$ (15)

This is the Kruskal–Szekeres coordinate system. In these coordinates, the world lines can be conveniently depicted in spacetime both inside and outside the black hole horizon (Fig. 2).

Here, one considers an 'eternal' black hole which actually has two singularities (their equation is $v^2 - u^2 = 1$; they are shown by the upper and lower heavy hyperbolas in Fig. 2) hidden from the external observer under the horizon. The second (bottom) singularity is absent for black holes that originated from stellar collapses.

Mapping (13) covers only one-fourth of the (u, v)-plane: region I in Fig. 2. In region II, where v > |u| and 0 < x < 1, we define

$$r \equiv r_{\rm g} x$$
, $t \equiv 2 \, \frac{r_{\rm g}}{c} \operatorname{artanh} \frac{u}{v}$. (16)



Figure 2. Free fall upon a black hole in the Kruskal-Szekeres coordinates.

The inverse transformation takes the form

$$u = \sqrt{1 - x} \exp\left(\frac{x}{2}\right) \sinh\frac{ct}{2r_{\rm g}}, \qquad (17)$$
$$v = \sqrt{1 - x} \exp\left(\frac{x}{2}\right) \cosh\frac{ct}{2r_{\rm g}}.$$

In region II, one has $0 < r < r_g$, $t \in (-\infty, +\infty)$, and the Kruskal–Szekeres metric (14) takes the form of the Schwarzschild metric (1). However, now (inside the horizon) the coordinate *r* becomes timelike, and *t* becomes spacelike! Therefore, by denoting $\eta = r/c$, l = ct, where $\eta \in (0, r_g/c)$, $l \in \mathbb{R}$, we write out the metric inside the horizon in the form

$$\mathrm{d}s^2 = \frac{c^2 \,\mathrm{d}\eta^2}{r_{\mathrm{g}}/(c\eta) - 1} - \left(\frac{r_{\mathrm{g}}}{c\eta} - 1\right) \mathrm{d}l^2 - (c\eta)^2 (\mathrm{d}\vartheta^2 + \sin^2\vartheta \,\mathrm{d}\varphi^2) \,. \tag{18}$$

The spacetime described by metric (18) is quite unusual. The space of this 'universe', i.e., the surface $\eta = \text{const}$, is spherically symmetric but anisotropic. The direction along the *l*-axis is preferential. Surfaces ($\eta = \text{const}$, l = const) represent S² spheres. However, the coordinate *l* is not radial. It takes all real values and metric (18) does not depend on it. The topology of spatial cross sections $\eta = \text{const}$ is the R¹ × S² topology. From outside the black hole it appears that the space inside the black hole has a finite volume, but inside the black hole it turns out that there is a world line of infinite length (a cylinder of infinite length). It is exactly in connection with this remarkable property that we must agree with Ne'eman's note about 'different realities'. The reality inside the black hole it, although it can be understood!

The radius of the sphere S² (it is equal to $c\eta$) decreases with time and vanishes at $\eta = 0$, which corresponds to the Schwarzschild singularity. This singularity is not a space point inside the black hole but represents the disruption of time for all world lines inside it. The Schwarzschild singularity is spacelike, and for any observer under the horizon it is located in the *future*. It is impossible for an observer to 'see' the singularity inside the black hole before their own catastrophic destruction. The opposite statement in Refs [12, 13] is erroneous.

In the Kruskal-Szekeres coordinates, radial geodesics along which the light propagates are represented by straight lines inclined by 45° to the coordinate axes. So, the radial light cone in the Kruskal-Szekeres coordinates has the same form as in special relativity. This property allows one to easily establish the causal link between events using graphical representation in the Kruskal-Szekeres coordinates [19]. Let us take advantage of this property to answer the question of which light signals catch up with the infalling observer under the event horizon. The world line of the observer who started the free fall from point r_0 at the instant of time t_0 is shown by the line BHF in Fig. 2. The point F corresponds to the world line disruption at the singularity. The causal past of the event F is shown in gray. In the Kruskal-Szekeres coordinates, the surfaces of constant radius r are shown by hyperbolas with asymptotes $u = \pm v$, and surfaces of constant time t are represented by straight lines passing through the origin of the coordinates. Therefore, as indicated in Fig. 2, until the tragic ruin of the free-falling observer at the singularity at the instant F, light rays emitted from the point **B** of a free-fall origin no later than the time t_s corresponding to the line OS can catch up with the observer. Therefore, during the free fall inside the black hole up to the singularity the observer cannot see the infinite future!

It should be noted, however, that in a rotating black hole described by the Kerr metric or in a charged black hole (the Reissner-Nordström metric and Kerr-Newman metric), a phenomenon formally shows up that could be described as the possibility of an observer seeing all the future of the Universe external to the black hole. In addition to the event horizon, as in the Schwarzschild metric, here a new horizon, the Cauchy horizon, appears. The Cauchy horizon inside a black hole is the boundary for prediction of the evolution of physical fields from initial data in the external Universe. The future of an astronaut crossing such a horizon is unpredictable from their past. In the Kerr metric, the Cauchy horizon represents a null (lightlike) surface. The astronaut can approach this horizon after crossing the first one (the event horizon). So, as shown in several textbooks (see, for example, Ref. [16, § 38]), at the instant of crossing the Cauchy horizon surface, "...the person will witness, in a flash, a panorama of the entire history of the external world in infinitely blue shifted rays." Nevertheless, as stated in Ref. [7, § 12.2], the infinite violet shift means such a large energy concentration that "would lead to the reconstruction of the spacetime and to the emergence of the true singularity of the spacetime."

Does that mean that inside such a black hole it is impossible to see the infinite future of the external Universe? So far, there is no definite answer to this question. To analyze this problem, it is insufficient to consider the Kerr solution only. An analysis of the evolution of the singularity under the action of radiation falling into the black hole is in order. For example, in papers [21, 22] and in the new English edition of V P Frolov and I D Novikov's book [7], it is shown that if one takes into account gravitational perturbations to the Reissner-Nordström or Kerr metric, the Cauchy horizon surface becomes singular — a new singularity emerges, which is different from both the spacelike and timelike singularities in respective Schwarzschild and Kerr metrics. This singularity belongs to the class of 'weak' singularities [23]. It was argued in paper [22] that "the tidal deformation associated with the singularity is so small that it cannot damage the object, and, in some conditions, it cannot even be detected before the curvature becomes infinite. This reopens the question whether a journey through the Cauchy horizon of black holes is possible." However, if one takes into account the inverse effect of external fields, for example, the massless scalar field in some model problems [24, 25], the null singularity can, under certain conditions, evolve into a strong spacelike singularity. The case is also possible where two singularities, the null and strong spacelike ones, exist simultaneously.

Even if this singularity remains the null one, the question of whether the astronaut can cross it lacks clear answer. When a strong spacelike singularity is present, the astronaut will be destroyed by tidal forces. All these results were obtained for several model problems that allow simple mathematical solutions. What the astronaut sees inside a real rotating black hole, which cannot be described by the Kerr metric any more, is unclear.

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