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Delusions versus reality in some physics problems: theory and experiment

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Contents

1. Introduction 235
2. Theory of magnetron generation 238
2.1 Three riddles of the magnetron. The myth of the magnetron stationary regime
3. Ionization waves in gas discharge plasmas (strata) 243
4. Stochastic resonance 245
5. Turbulence in unbounded flows 247
5.1 Hydrodynamic and acoustic waves; 5.2 Basic properties of turbulent jets
6. Conclusion 254
References 254

Abstract. The important question of the relation between theory and experiment in different physical problems is discussed. A number of examples, both widely and little known, are used to show that some physical theories, considered by many as correct because they corresponded to many experimental facts and to the existing level of science, have been found to be false. One of the important reasons for this is that it is very difficult to distinguish between the causes and consequences of phenomena observed in experiments. The best example is the study of turbulent flows, where the causes and consequences are often erroneously interchanged in relation to the properties and development mechanisms of turbulence. At the same time, some counterexamples are described where phenomena that are impossible from the standpoint of universally accepted theoretical concepts turn out to be reality in special cases.

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A prejudice which is preserved even now is the belief that facts on their own, without unbiased theoretical reasoning, can and should lead to scientific progress.

A Einstein [1]

The most dangerous of errors is partly distorted truth.

G K Lichtenberg [2]

1. Introduction

This review presents a selection of examples from various branches of physics that illustrate myths in science. The myths can be divided into mythical theories and myths about people made famous through their contributions to science (according to Dal', the latter are unreal; they are legends [3]). As regards personal myths, many of them are widely known: Archimedes jumping out of the tub and crying 'Eureka!'; an apple falling onto Newton's head; the periodic table seen by Mendeleev as a dream in his sleep; Leonardo da Vinci, who knew everything beforehand; and so on. The annotation to book [4] states that the authors "uncover many intriguing secrets demonstrating clearly how a scientific legend arises and how intimately linked are the two seemingly contrasting entities, science and mythology." In this review, we choose several examples to illustrate the other aspect of myths in science — the mythical theories — capable, for some time, of explaining experiments, the foundation of physical research. The Introduction presents some examples, both widely and less known, that vividly demonstrate the mythology in science. The fields from which these examples are taken do not belong to the authors' expertise, and are therefore sketched only briefly. In the main body of the review, we collect examples that we consider deserving closer scrutiny. Examples from relatively distant fields show that delusions about the nature of physical phenomena can appear in any of them.

The word combination 'myth and reality' is contained in the titles of Refs [5, 6]. In Ref. [5], some widely used statements pertaining to the explanation of the generation of so-called 'coherent structures' in turbulent flows are called myths, in contrast to the author's own, allegedly true, views. However, as is to become apparent in what follows, the latter are frequently also myths. Such myths are not rare in works on turbulent flows. In Section 5 of this review, we present false views on some physical processes in turbulent jets and also suggest a rigorous treatment of these processes that follows from theoretical results.

We note that recently, after a set of very important and interesting discoveries in physics, the number of myths has only increased. Some of them are linked to the fundamental discovery of dynamical chaos and the view of turbulence as a manifestation of such chaos. Some recent myths are described in Ref. [6].

It is a generally shared view that experiment lies at the heart of physical research. Experiment on its own, however, does not necessarily provide a true understanding of observed phenomena. This is witnessed by a fair number of incorrect concepts dominating over long time spans without contradicting the experimental data known at that time. As mentioned elsewhere, one rationale for the dominance of false ideas is the frequent lack of the possibility to distinguish in experiment between the cause and consequence of the phenomenon being studied. We present several examples of such false concepts, correcting them with a treatment that follows from a theory that causes no doubts in our opinion.

Classical examples are furnished by theories that were widespread in their time and long-lived, like the caloric, used for the description of thermal phenomena, and the aether, introduced for the description and explanation of electromagnetic phenomena. The caloric and aether were considered as certain hypothetical material substances possessing special properties. Achieving agreement with ever growing experimental evidence required attributing unusual properties to the caloric, such as weightlessness, the highest elasticity among known substances, the capability of penetrating into the smallest pores of material bodies and expanding them. Improved in that way, the theory was in satisfactory agreement with experiments on thermal phenomena known then but failed to explain the transformation between mechanical and thermal energies [7]. Namely, owing to experiments by J P Mayer and Count Rumford exploring this transformation, it was ultimately proven that thermal phenomena are caused not by the caloric but by the chaotic motion of atoms and molecules.

The situation with the aether theory evolved similarly. The concept was introduced by J Maxwell based on the analogy between electric current and the vortical flow of fluids. The aether was viewed as an elastic medium characterized by a certain stress tensor. In that way, electromagnetic waves were identified with elastic (i.e., acoustic) waves. Maxwell was able to explain all magnetic and electric phenomena known at that time with systems of wheels and gears representing a chain of vortices in a hypothetical fluid, with small gears transferring motion from one vortex to another. The model of vortices was intuitively appealing to most researchers, even without a mathematical apparatus. Moreover, the model explained the rotation of the polarization plane by a magnetic field and gave the value of the speed of perturbation propagation in this strange medium that was exactly equal to the speed of light. Maxwell's stress tensor allowed explaining many other phenomena involving light, for instance, the radiation pressure of light.

The theory of electromagnetic phenomena based on the aether reached its culmination in works by H Lorentz and H Poincaré, who suggested an explanation (including the known Lorentz transformation) for the unsuccessful attempts to discover the motion of bodies relative to the aether. However, ideas about the aether proper as a substance and its properties were becoming rather controversial. The main contradiction pertained to the fact that the transverse character of electromagnetic waves required considering the aether as a solid body. It was not clear then why this body does not oppose the motion of other bodies through it. One had to admit that the interaction between the aether and other bodies exists in optical phenomena, whereas it is fully absent in mechanical phenomena. In fact, the only remaining role of the aether was that of carrying electromagnetic waves. These difficulties, together with the establishment of new field concepts in the electromagnetic theory, led Einstein to revisit classical concepts of space and time and to reject the needless substance (the aether).

One more example showing how long-lived myths in science are is the history of answering the question of why the sky is blue. Leonard Euler was the first to try to answer it. He understood that air is composed of molecules but presumed that these have their own color. Euler argued that the molecules strongly absorb blue rays (while, e.g., red rays are absorbed weakly), and hence the sky shines in blue light. Euler wrote about this in letter 32, "Of the azure color of the heavens" in *Letters of Leonard Euler on Different Subjects in Natural Philosophy Addressed to a German Princess*: "...the air is loaded with a great quantity of small particles, which are not perfectly transparent, but which, being illuminated by the rays of the sun, receive from them a motion of vibration, which produces new rays proper to these particles; or else they are opaque, and become visible to us from being illumined. Now the color of these particles is blue... ." In reality, the molecules composing the air are transparent in visible light and absorb only in ultraviolet. Thus, Euler's explanation is in error, but it persisted for almost a hundred years. This is truly one of the most long-lived myths.

Only in 1871 did John William Rayleigh quantitatively show that the blue appearance of the sky comes not from the absorption of sunlight followed by emission of its certain part, but from light scattering. The scattering phenomenon in optically turbid media was first explored by John Tyndall in 1869 and is called the Tyndall effect. He was the first to observe that white light becomes bluish upon scattering and argued that the blue color of the sky is caused by scattering on dust particles abundant in the Earth's atmosphere.

The Rayleigh solution is based on dimensional analysis. The problem is formulated as follows. Let a particle with a linear size l scatter sunlight with a wavelength λ and amplitude A. The amplitude of the scattered wave decreases with the distance to the particle. Let this amplitude be S at the distance r from the particle. The dependence of S on the rest of the parameters is what is sought. Next, it is taken into account that all variables have the same dimension of length L. Then, according to dimensional considerations, the amplitude S is expressed as a product of other variables raised to some powers: $S = CA^{\alpha} l^{\beta} r^{\gamma} \lambda^{\delta}$, where C is a dimensionless constant and α, β, γ, and δ are unknown exponents. Hence follows a rather unusual equation for dimensions, $[L] = [L]^{\alpha}[L]^{\beta}[L]^{\gamma}[L]^{\delta}$, or $\alpha + \beta + \gamma + \delta = 1$. To proceed further, the dimensional analysis has to be complemented with physical considerations. First, we recall that the amplitude of scattered light is inversely proportional to the distance from the particle, i.e., $\gamma = -1$. Second, the amplitude of scattered light is proportional to that of incident light, and hence $\alpha = 1$. It then follows that $\delta = 1 - \beta$ and $S = C(A/r)l^{\beta}\lambda^{1-\beta}$, or $S = C(A\lambda/r)(l/\lambda)^{\beta}$. Further, Rayleigh argues that judging by the dynamics of the process, the ratio of amplitudes of incident and scattered light varies in proportion to the volume of the scattering particle. Therefore, $S/A \sim l^{\beta} \sim l^3$ and $\beta = 3$. The final result is $S = CAl^3/(r\lambda^2)$. The intensity of scattered light is proportional to its amplitude S squared, i.e., $I \sim \lambda^{-4} \sim \omega^4$, where ω is the frequency. The last formula received support from quantitative measurements of scattered light.

After Rayleigh's work, it became apparent that in clear skies, blue light is scattered more strongly than red light, while the maximum intensity of scattered light falls into the blue part of the spectrum. At sunset and sunrise, the direct sun rays pass through a thicker layer of air than when the sun is at the zenith. The thicker atmospheric layer scatters short-wave rays more strongly, reducing their intensity in direct light. Thus, predominantly long-wave radiation — red rays — penetrates to the Earth's surface, such that we see a red sun at sunset and sunrise. In Euler's theory, the scattering was absent, and hence rose skies were not mentioned, nor was the color of the sky at sunrise and sunset explained.

Initially, Rayleigh, following Tyndall, assumed that the scattering of light in the atmosphere originates from liquid droplets, dust, or solid particles whose size is small compared to the wavelength of the light. This is apparently not the case. In reality, the amount of dust or extraneous particles is reduced with the altitude above the Earth's surface; but then the intensity of scattering from these parts of the atmosphere should also be reduced, which, as is well known, does not happen. Later, Rayleigh recognized that scattering is caused not by particles but by air molecules. Such scattering took the name of Rayleigh, or molecular scattering.

In 1907, Leonid Isaakovich Mandelshtam showed that Rayleigh scattering cannot be explained by scattering on chaotically moving molecules if they are large in number in a volume small compared to the cube of the wavelength and distributed in space uniformly. Scattered waves then mutually annihilate and only direct rays remain. In 1907, Marian Smoluchowski demonstrated that molecular scattering owes its existence to thermal fluctuations of the refractive index. Finally, in 1910, Einstein formulated the theory of scattering in gases and liquids based on the ideas of Smoluchowski. Interestingly, the Rayleigh formula remained true all the time; only its physical interpretation changed.

We briefly consider two more examples, which are much more recent than those considered above. One such highly interesting example pertains to explaining the randomness of the process of generation and collapse of gas bubbles (termed cavitation). It is known that this process has a random character. Continuous spectra of sound emission were observed in experiments with acoustic cavitation [8, 9], with subharmonics showing up against the continuous background. Because a large number of bubbles form and collapse at cavitation, it seemed natural to explain the randomness by this large number. This was the accepted view for many years, although the presence of subharmonics remained unexplained. Only considerably later, in special experiments designed by Lauterborn [10], which dealt with a single bubble in a liquid, was it observed that spectra are the same as in the case of many bubbles. It was thus shown that the origin of randomness is not the number of bubbles but the so-called dynamic chaotization related to instability (see, e.g., Ref. [11]).

The second quite interesting example is suggested by the treatment of the phenomenon of turbulence. Beginning from works by Reynolds [12], it was assumed that turbulence in liquids and gases can be fully described by the Navier–Stokes equation [13, 14]. From the standpoint of the physics of oscillations, this implied that turbulence was considered a self-oscillating process. This was explicitly mentioned by G S Gorelik, who, according to the memoirs of S M Rytov [15], argued in one conversation that "turbulence with its 'self-excitation' threshold, a characteristic hysteresis respectively accompanying its generation and decay upon increasing and reducing the speed of background flow, with a governing role of nonlinearity in its developed (stationary) state, presents a self-oscillatory process. Its specific feature is that it occurs in a continuous system, i.e., a system with an extremely large number of the degrees of freedom." In essence, L D Landau shared the same viewpoint. According to him, turbulence is generated as follows. First, the equilibrium corresponding to a laminar flow becomes unstable and oscillations at a particular frequency are excited. For the amplitude of these oscillations, Landau, relying on physical ideas, wrote a phenomenological equation that coincides with the reduced Van der Pol equation for the amplitude of self-oscillations [16].[1] "With the Reynolds number increased further," Landau wrote, "newer and newer periods appear in sequence. As regards the emerging motions, they involve smaller and smaller scales." As a result, according to the Landau hypothesis, oscillations at multiple incommensurate frequencies are established, i.e., a quasi-periodic motion sets in. It is now known that an attractor in the form of multi-dimensional torus corresponds to such oscillations in the phase space. If the number of excited frequencies is large, such motion greatly resembles a chaotic one, and hence well-developed turbulence can be considered a random process. Although the Landau theory is phenomenological and does not directly follow from the equations of fluid dynamics, it caused no doubt for a long time and was supported by almost all researchers of turbulence.

Landau's theory was further developed in the works of Stuart [17, 18], who proposed an approach to compute the coefficients appearing in the Landau equation based on an approximate solution of the Navier–Stokes equations. However, the approximate solution taken by Stuart in the form of a traveling wave with a fixed wave number[2] $A(\epsilon t)\exp[\mathrm{i}(\omega t - kx)]$ is incorrect from the physical standpoint in unbounded flows. Indeed, this solution describes a spatially periodic wave with the amplitude slowly varying in time. Strictly speaking, such a solution holds only for a periodic flow with the period $L = 2\pi n/k$, where n is an integer, i.e., for a flow with feedback. The solution assumed by Stuart, as well as the Landau theory, does not take the convective character of the instability of laminar flows into account and hence cannot be applied to unbounded flows.[3]

In the 1970s, prompted by the discovery of dynamic chaos, the view of the generation of turbulence as an abrupt development of a strange attractor in the phase space of some

[1] Landau neither used the term 'self-oscillations' nor cited the works of Van der Pol.

[2] Landau assumed the same wave form.

[3] See, e.g., Refs [19, 20] for the instability types in wave systems.

dynamic variables [21, 22] began to spread. The authors of Refs [21, 22] considered a finite-dimensional phase space and meant dynamical chaos by turbulence. Apparently, similarly to the works by Stuart and Landau, these works cannot be applied to unbounded flows. Unfortunately, such a view of turbulence became popular (see, e.g., Refs [13, 14, 23–27]). We note that dynamical chaos is a general feature of dynamical systems with the dimension exceeding two (see Ref. [23]), whereas turbulence in the strict sense of this word (i.e., the turbulence in hydrodynamic systems) presents a particular example of random waves. To avoid confusion, the notions of turbulence and dynamical chaos should be distinguished.

In Sections 2–5, we consider other examples of long-lived myths in science in more detail.

We note that the opposite situation is also encountered when phenomena that are impossible according to commonly accepted views are discovered in experiments or numerical simulations. One of the reasons might be that mathematical theorems proving the nonexistence of certain phenomena involve the remark 'in the generic case,' while an actual physical problem might deal not with generic but with a particular case for which the conditions of the theorem are not satisfied. For example, there exists a mathematical theorem stating that quasiperiodic oscillations with the number of incommensurate frequencies equal to or exceeding three are unstable. In mathematical language, this theorem is formulated as the statement that a 'three-dimensional torus is unstable': it either breaks down or develops a resonance. In physical terms, the development of a resonance pertains to the phenomenon of synchronization. For example, if we consider three oscillators with incommensurate frequencies that are coupled only very weakly, the oscillations at all three frequencies can persist in this system. Meanwhile, this theorem laid the basis for the critique of the Landau theory. Based on the material above, it seems plausible to assume that the Landau theory can in principle work in a circular tube, where the conditions of spatial periodicity for each mode are trivially maintained.

Another, no less important reason is that in physical systems, on changing a certain parameter toward a prescribed value, the limit is frequently absent, i.e., the limit value depends on which side the limit is approached from. It is now well known that just this can explain the existence of chaotic solutions in dynamical systems when the source of perturbations tends to zero, together with the uncertainty in the initial conditions. [4] This also explains the absence of wave breaking in solutions of Burgers' equation when the viscosity tends to zero.

A very interesting example of the reality of 'impossible' systems is furnished by quasicrystals, discovered in 1984 [28]. Reference [28] experimentally proved the existence of a metal alloy (of aluminum and manganese) with unusual properties. The alloy is formed via rapid cooling after melting (at the rate 10^6 K s^{-1}). Exploring this alloy by electron diffraction, it was discovered that it exhibits all the properties of a crystal showing a diffraction pattern composed of bright and regularly located spots. This pattern, however, can be obtained only in the presence of a fivefold (or icosahedral) symmetry, strictly forbidden for a crystal on geometrical grounds because pentagons cannot tile the space. The name 'quasicrystal' was attributed to such alloys later (see Ref. [29]). Experiments by Shechtman and colleagues and many other groups proved that there exist ideally homogeneous substances in which the fivefold symmetry is preserved in microscopic subdomains several nanometers in size. In searching for an explanation for these results, physicists recalled the mathematical discovery of tiles made by British theorist Rogers Penrose in 1974. These tiles represent aperiodic regular structures formed by rhombuses of two types with the internal angles 36° and 72° [30]. These tiles, now called Penrose tiles, turned out to be planar analogs of quasicrystals. The role of Penrose rhombuses is played by icosahedra [31], which can densely fill the three-dimensional space.

[4] According to the uniqueness theorem, such solutions are impossible.

The emerging model relies on the concept of a basis element. An internal icosahedron formed by aluminum atoms is surrounded by an external one formed by manganese atoms. The basis element contains 42 atoms of aluminum and 12 atoms of manganese. On solidification, the basis elements become rapidly connected through octahedral bridges. The icosahedron faces are equilateral triangles. The formation of an octahedral manganese bridge occurs when two such triangles (one for each cell) are sufficiently close and parallel to each other. A quasicrystal with icosahedral symmetry forms as a result. Certainly, the discovery of quasicrystals does not disprove the foundations of crystallography. The author of Ref. [29] in this respect writes: "The concept of the quasicrystal presents a fundamental interest as it generalizes and completes the definition of a crystal. A theory based on this concept replaces the long-lived idea of a 'structural unit repeated in space in a strictly periodic way,' with a key notion of long-range order. This concept extended crystallography, whose newly acquired richness we are only beginning to comprehend. Its significance in the world of minerals could be placed in line with adding irrational numbers to the set of rational numbers in mathematics."

2. Theory of magnetron generation

A magnetron is one of the first and most widespread microwave generators in which electrons moving in crossing static electric and magnetic fields interact with a high-frequency electromagnetic field.

Viewed from the perspective of its design, a modern multi-resonant magnetron consist of three main components (see, e.g., [32, p. 202]): (1) a cathode, (2) an anode block containing resonant contours, and (3) an output loop directing the microwave energy to the load. Experimental and theoretical investigations of magnetron led, beginning from early work, to many contradictions and paradoxes, and to the discovery of a set of unexplained phenomena. One of the first and seemingly the only attempt to rigorously clarify these questions and refute the main myth of the existence of stationary regimes of magnetron generation was undertaken by L A Vainshtein and A S Roshal in lectures on microwave electronics given at the 2nd winter school for engineers organized in Saratov in 1972 [33]. Unfortunately, this work, published in the proceedings of the school, remained unknown to the majority of physicists. Below, we largely follow Ref. [33] and its adaptation in Ref. [32].

The most difficult problem in the theory of the magnetron (as well as other microwave generators) was and remains a correct and complete account for the space charge. As a rule, it requires using modern computational facilities, but even numerical simulations do not fully reveal all aspects of the phenomena related to the space charge. And yet, these phenomena, in addition to obvious physical interest, are also important from the practical standpoint because they determine the limits of the device performance. In a magnetron, a strong space charge in the spokes[5] leads to their collapse, i.e., sets the maximum output of the device. A high noise level in the frequency spectrum of the signal generated by a magnetron is also related to the space charge.

In what follows, we consider magnetron generators of the pulse type, for which the space charge plays the main role. Following Ref. [33], we consider three possible states of a magnetron: (1) a nongenerating magnetron (the magnetic field exceeds the critical one), (2) a magnetron at the beginning of generation (transition from a pre-generation to the generation regime), and (3) a magnetron in a developed generation regime (the microwave field is sufficiently strong).

2.1 Three riddles of the magnetron. The myth of the magnetron stationary regime

We refer to a space charge as strong if its density $\rho \sim \rho_{\rm cr}$, where $\rho_{\rm cr}$ is the critical space charge density,

$$\rho_{\rm cr} = \frac{eH^2}{4\pi mc^2}\,, \tag{1}$$

e and m are the charge and mass of the electron, c is the speed of light, and H is the magnetic field strength [Eqn (1) rewritten in absolute units is to be derived in what follows]. A weak space charge is defined by the condition $\rho \ll \rho_{\rm cr}$.

The physical meaning of $\rho_{\rm cr}$ consists in the following. We represent Eqn (1) in the form

$$\frac{\rho_{\rm cr} mc^2}{2e} = \frac{H^2}{8\pi}\,. \tag{2}$$

The right-hand side of Eqn (2) contains the magnetic energy density or the magnetic field pressure. As is known, the magnetic field focuses beams propagating along it, opposing their transverse spread under the action of forces due to the space charge (for cylindrical beams, this happens at $\rho < \rho_{\rm cr}/2$, and at $\rho < \rho_{\rm cr}$ for plane beams). The magnetic field in a magnetron acts similarly, compensating repulsion forces in the electron spokes of a working magnetron. Relation (1) can be obtained by setting the plasma frequency of the electron cloud $\omega_{\rm p} = \sqrt{4\pi\rho e/m}$ determined by the density equal to the cyclotron frequency $\Omega = eH/mc$, which corresponds to the magnetic field in the magnetron. A charge density strongly exceeding $\rho_{\rm cr}$ cannot exist in a magnetron except for very thin layers.

Intriguing phenomena have been discovered in experimental and theoretical studies of magnetrons. The main ones are listed below.

- Riddle one: The existence in the pre-generation regime (even in a planar magnetron) of the experimentally measured electric current, which cannot be explained within the existing analytic approaches.
- Riddle two: The impossibility of explaining the experimentally observed soft self-excitation of cylindrical magnetrons in the framework of a theory neglecting the space charge.
- Riddle three: Why is the theoretical search for a stationary generation regime in magnetrons not always successful? Moreover, does such a regime exist at all?

We consider these riddles in more detail. The first is related to the behavior of the magnetron when the magnetic field exceeds the critical value and generation does not occur. In the absence of a space charge and with the boundary effects neglected, the situation is straightforward: electrons return to the cathode following cycloidal trajectories in a planar magnetron, or more complex trajectories in the cylindrical geometry. The processes become more involved when the space charge is strong. However, if we assume that the electrons turn back after having reached some plane $y = d$ in which $\mathrm{d}y/\mathrm{d}\tau = 0$ (y is the transverse coordinate) and that the electric field equals zero at the cathode, the problem yields to an analytic solution [34, 35]. We proceed from the equations of motion

$$\frac{\mathrm{d}^2 x}{\mathrm{d}\tau^2} - \Omega\,\frac{\mathrm{d}y}{\mathrm{d}\tau} = f_x\,, \tag{3}$$

$$\frac{\mathrm{d}^2 y}{\mathrm{d}\tau^2} + \Omega\,\frac{\mathrm{d}x}{\mathrm{d}\tau} = f_y\,, \tag{4}$$

where $f_x = (e/m)E_x$, $f_y = (e/m)E_y$, and E_x and E_y are the components of the electric field.

Let $f_x = 0$ and $\mathrm{d}x/\mathrm{d}\tau = 0$ at $y = 0$. It then follows from Eqn (3) that

$$\frac{\mathrm{d}x}{\mathrm{d}\tau} = \Omega y\,. \tag{5}$$

Substituting the expression given by Eqn (5) in Eqn (4), we find

$$\frac{\mathrm{d}^2 y}{\mathrm{d}\tau^2} + \Omega^2 y = f_y\,. \tag{6}$$

We assume that $f_y = 0$ at $y = 0$, i.e., that the regime of current limited by the space charge is realized at the cathode. Below the plane $y = \mathrm{const}$, the charge per unit surface equals $2j_y^{(0)}\tau(y)$, where $j_y^{(0)}$ is the density of the current entering from the cathode to the interaction domain and $\tau = \tau(y)$ is the time it takes an electron to travel from the cathode ($y = 0$) to a given value of y. The factor 2 in the formula for the charge takes accounts for the fact that half of the charge is created by the electrons coming from the cathode, while the other half is created by the electrons traveling to the cathode. Because we assume that $E_y = 0$ at the cathode, it follows for a given y that

$$E_y = 4\pi j_y^{(0)}\tau(y)\,, \tag{7}$$

and hence

$$f_y = 8\pi\,\frac{e}{m}\,j_y^{(0)}\tau(y)\,. \tag{8}$$

Integrating Eqn (6) with the right-hand side given by (8) and with the initial conditions $y = \mathrm{d}y/\mathrm{d}\tau = 0$ at the instant of leaving $\tau = 0$, we find

$$y = y(\tau) = \frac{d}{2\pi}\,(\Omega\tau - \sin\Omega\tau)\,, \tag{9}$$

where

$$d = \frac{16\pi^2}{\Omega^3}\,\frac{e}{m}\,j_y^{(0)}\,. \tag{10}$$

[5] It should be remembered that the electron cloud in a generating magnetron forms a pattern resembling a rotating wheel. The near-cathode part forms the bush of the wheel connected to the anode by moving 'spokes' — the electron structures grouped owing to the self-phasing mechanism in the region of maximum deceleration of electrons by a tangential field.

For the initial conditions $x(0) = x_0$ and $|\mathrm{d}x/\mathrm{d}\tau|_{\tau=0} = 0$, we have

$$x = x(\tau) = x_0 + \frac{d}{2\pi}\left(\frac{(\Omega\tau)^2}{2} - 1 + \cos\Omega\tau\right). \qquad (11)$$

For $\Omega\tau = 0$, the value of y is d, and Eqns (9) and (10) are applicable if $0 \leqslant \Omega\tau \leqslant 2\pi$. Because the electrons turn back after having reached $y = d$, the total travel time is $4\pi/\Omega$, while in the absence of a space charge, it equals the cyclotron period $2\pi/\Omega$.

The surface charge density in the entire electron layer below the plane $y = d$ is

$$\sigma_{\mathrm{c}} = \frac{4\pi j_y^{(0)}}{\Omega} = \rho_{\mathrm{c}} d$$

and

$$\rho_{\mathrm{c}} = \frac{4\pi j_y^{(0)}}{\Omega d} = \frac{m\Omega^2}{4\pi e} = \frac{eH^2}{4\pi mc^2}\,,$$

which coincides with Eqn (1).

We note that the quantities $j_y^{(0)}$ and d are known to us; they depend on the anode voltage. The voltage between the cathode and the external boundary of the electron layer is

$$U_{\mathrm{e}} = -\int_0^d E_y\,\mathrm{d}y = -\int_0^{2\pi/\Omega} E_y\,\frac{\mathrm{d}y}{\mathrm{d}\tau}\,\mathrm{d}\tau = -2\pi\sigma_{\mathrm{c}} d\,, \qquad (12)$$

and that between the layer boundary and the anode is expressed as

$$U - U_{\mathrm{e}} = -4\pi\sigma_{\mathrm{c}}(D - d)\,, \qquad (13)$$

because $E_y = 4\pi\sigma_{\mathrm{c}} = \mathrm{const}$. Then,

$$U = -2\pi\rho_{\mathrm{c}} d(2D - d)\,, \qquad (14)$$

which leads to a quadratic equation for d,

$$d^2 - 2Dd - \frac{U}{4\pi\rho_{\mathrm{c}}} = 0\,.$$

Hence, it follows that the thickness of the electron layer adjacent to the cathode is given by

$$d = D - \sqrt{D^2 + \frac{U}{4\pi\rho_{\mathrm{c}}}} = D\left(1 - \sqrt{1 - \frac{U}{U_{\mathrm{c}}}}\right), \qquad (15)$$

where $U_{\mathrm{c}} = -2\pi\rho_{\mathrm{c}} D^2$ ($\rho_{\mathrm{c}} < 0$) is the critical voltage. The charge density $\rho(y)$ in the layer $0 < y < d$ is not constant, and the mean density

$$\rho_{\mathrm{c}} = \frac{1}{d}\int_0^d \rho(y)\,\mathrm{d}y \qquad (16)$$

is exactly equal to the critical one.

The twofold increase in the electron residence time in the interaction domain compared to the time of cycloidal motion in the absence of a space charge manifests itself in the fact that the angular speed of electron rotation in the magnetic field (equal to Ω for a single electron) decreases under the action of a space charge according to the formula

$$\Omega_{\mathrm{p}} = \Omega\left(1 - \frac{\omega_{\mathrm{p}}^2}{2\Omega}\right) = \Omega\left(1 - \frac{\rho}{2\rho_{\mathrm{c}}}\right). \qquad (17)$$

Formula (17) is strictly valid only for $\rho \ll \rho_{\mathrm{c}}$. However, it shows that at $\rho \sim \rho_{\mathrm{c}}$, the angular speed is noticeably smaller than at $\rho \ll \rho_{\mathrm{c}}$. The electron structures in which $|\rho| > |\rho_{\mathrm{c}}|$ cannot be sustained by the magnetic field. Analogous results were obtained for a cylindrical magnetron in Refs [36, 37].

The main element of the theories mentioned above is that they consider a two-stream state of the electron cloud in a closed, nongenerating magnetron. By analogy with the motion in the absence of a space charge, it is presumed that the electron cloud is composed of two streams of electrons, moving from the cathode to the outer boundary and returning from it.

Later, works by Brillouin [38] appeared, proposing exact stationary solutions of the Poisson equation and the equations of motion for both planar and cylindrical magnetrons and showing that the electron cloud in a closed magnetron can be found in a single-stream state. In this case, electrons move parallel to the cathode at speeds that correspond to the electrons emitted by the cathode (the motion follows straight or circular trajectories in planar or circular magnetrons, respectively).

For the planar magnetron, it is assumed that (1) the charge density in the near-cathode layer is constant, (2) $E_y = 0$ at the cathode, and (3) electrons move parallel to the cathode with the speed $\mathrm{d}x/\mathrm{d}\tau = \Omega y$ [see Eqn (5)].

Because $\mathrm{d}y/\mathrm{d}\tau = \mathrm{d}^2y/\mathrm{d}\tau^2 = 0$ in this case, Eqn (4) yields

$$\frac{\mathrm{d}x}{\mathrm{d}\tau} = \frac{1}{\Omega} f_y\,. \qquad (18)$$

From the first and second assumptions, it follows that

$$E_y = 4\pi\rho y\,, \qquad f_y = 4\pi\,\frac{e}{m}\,\rho y\,, \qquad (19)$$

where ρ is the constant charge density in the layer. Equation (18) can be written, taking Eqn (19) into account, in the form

$$\frac{\mathrm{d}x}{\mathrm{d}\tau} = \frac{\omega_{\mathrm{p}}^2}{\Omega}\,y\,. \qquad (20)$$

Then, using the third assumption, we find from Eqn (20) that $\omega_{\mathrm{p}}^2 = \Omega^2$ and $\rho = \rho_{\mathrm{c}}$, i.e., the density has to be critical. The electron layer thickness d in a planar magnetron is defined by the same equation (10) as for the two-stream state.

Unfortunately, attempts by Brillouin to prove the validity of the model distinguished with simplicity and clarity turned out to be unsuccessful and obscure, and the question of the possibility of a single-stream state remained open for a long time. [6]

The question as to which state, the single or double stream, is observed must be resolved experimentally. Investigations showed that neither charge distribution is realized. It was found experimentally that even for magnetrons with planar electrodes, irregular oscillations of electron clouds occur at very different frequencies. The most apparent and clearly observed phenomenon, incompatible with the theory neglecting the space charge, turns out to be the anode current. It can readily be measured and, although it decreases with an increase in the magnetic field, this occurs more gently than suggested by computations taking the distribution of initial velocities into account.

To explain the results of anode current measurements in the magnetic fields several times greater than the critical value, one had to assign temperatures of about a million

[6] We note that the mechanism of the single stream state was clarified soon after the appearance of the Brillouin paper, but the results of that work became available much later (see Ref. [39]).

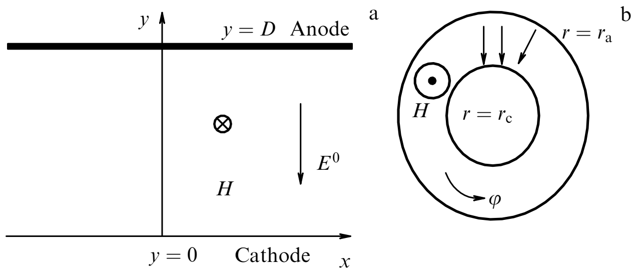


Figure 1. Schematic of planar (a) and cylindrical (b) magnetrons.

degrees to electrons, which is by many orders of magnitude larger than the cathode temperature.

The beginning of generation in the magnetron is its second riddle. It is known that the phasing mechanism in the magnetron is only effective when the electrons and field are in synchrony. In a planar magnetron (Fig. 1a), an electron drifts under the action of a uniform static fields $E^{(0)}$ and H in the direction of the x axis at the speed $v_0 = cE^{(0)}/H$.

If a slow electromagnetic wave propagates in the direction of the x axis, with the wave dependence on the coordinate x and time t written in a complex form as $\exp[\mathrm{i}(kx - \omega t)]$, its phase velocity $u = \omega/k$ should be close to v_0. The degree of the closeness in the stationary regime of generation is defined in the framework of the theory neglecting the space charge influence [40] by the inequality $|\omega - \omega_0| T \leqslant 1$, where $\omega_0 = kv_0$ is the frequency at which the speed of the slow wave with the same wave number k is exactly equal to v_0 and T is the time it takes electrons to travel from the cathode to the anode. Because the displacement of electrons from the cathode to the anode is associated with their drift in the field of a synchronous wave, the time T is inversely proportional to the amplitude of this wave: the smaller the amplitude, the closer the values of ω and ω_0 (u and v_0) should be. This suggests that the generation in a planar magnetron unfolds as follows. First, a wave with a small amplitude appearing as a result of fluctuations at a frequency nearly coincident with ω_0 forms spokes, whence its amplitude increases. As the amplitude increases, the frequencies ω and ω_0 can depart from each other.

In a cylindrical magnetron (Fig. 1b), this simple scenario of beginning the generation is not realized because the electric field $E^{(0)}$ and consequently the drift velocity v_0 are the inversely proportional to the radial coordinate r, namely, $v_0 = v\bar{r}/r$, where v is the velocity at $r = \bar{r}$. The field of a synchronous wave rotating in the positive azimuth (coordinate φ) direction is now expressed by the factor $\exp[\mathrm{i}(n\varphi - \omega t)]$. Hence, the linear phase speed on a circle with the radius r is $u = vr/\bar{r}$. Correspondingly, the full synchrony $v_0 = u = v$ is achieved for only a particular value $r = \bar{r}$. The absence of full synchrony of the wave and the electrons in the cylindrical magnetron implies, for example, that for the ratio $r_a/r_c = 1.5$, the spokes are not formed at amplitudes of the microwave field only an order of magnitude smaller than the maximum amplitude that corresponds to the stationary generation. This is corroborated by numerical simulations: a wave with a sufficiently small amplitude 'ushered' in the interaction space of a cylindrical magnetron is not able to form spokes and noticeably augment its amplitude, whatever its frequency and angular speed.

Admittedly, the theory neglecting the effect of the space charge does not describe the self-excitation processes of cylindrical magnetrons. And yet, they are easily excited in experiments, which is explained by the effect of the space charge. To understand and assess this effect is to solve the second magnetron riddle.

The third riddle of magnetrons is related to their behavior in the well-developed generation regime with a large space charge, which essentially affects the motion of electrons.

The essence of the problem is that the theoretical search for stationary regimes was in some cases not reaching the goal. In other cases, the stationary regimes were rather the result of inexact computations or limitations set by the chosen model. This pertains to models in which the self-consistent problem for a magnetron was solved by stripping electrons of at least some degrees of freedom and assuming, for example, that electron spokes have one fixed form or another, or that the electron motion obeys drift equations, and so on. Certainly, the results thus obtained were helpful to a degree, especially when they were confirmed experimentally. However, they did not permit penetrating deeper into the mechanism of magnetron functioning, and in particular, estimating the performance limits of the instrument.

In 1965, paper [45] appeared, devoted to the numerical study of transient processes in a magnetron, which was destined to become classical. This work shattered the myth of the existence of a stationary generation regime in magnetrons. It was shown that it is never realized. The electron streams and the anode current pulsate, and the characteristic pulsation time is markedly smaller than the oscillation equilibration time in the resonator. Therefore, even if a stationary state could exist, it would do so for the resonance field of the volume resonator, but not for the electron cloud or the space charge field.

How did numerical experiment contribute to solving these riddles?

Computations revealed that if the emission current is sufficiently large and the voltage is set gently, electrons emitted during the setup phase follow trajectories close to those of the single-stream state. After this phase, the supply of new electrons from the cathode to the electron cloud is practically halted. The physical reason for the fact that the two-stream state is not realized pertains to its strong instability due to the presence of two inter-penetrating streams. When velocity components in the direction normal to the cathode become small and the cloud state becomes close to a single-stream one, the two-stream instability is largely eliminated (these questions are analyzed in sufficient detail in appendices 2 and 3 in Ref. [42]).

It is noteworthy that the single-stream state is also unstable with respect to perturbations in the form of a traveling wave. The instability stems from the presence of electrons with different velocities. However, such electrons are separated in space, and hence their interaction is weaker and perturbations grow more slowly than in the two-stream state.

As a result, the single-stream state is hardly realizable either: the nonsymmetric perturbations ever present in actual practice grow and transform the regular ('laminar') motion of electrons into an irregular ('turbulent') one. The irregularity emerges through the existence of a multitude of growing oscillations with different frequencies but roughly the same increments. As these oscillations reach finite amplitudes, they begin to interact nonlinearly. As a result, the electron cloud in a nongenerating magnetron remains symmetric only in the statistical sense. The electron cloud pulsates ('boils') and is therefore confined by the magnetic field less efficiently, such that electrons can partly hit the anode. This partly solves the first magnetron riddle: a noticeable anode current in a closed magnetron already exists in a two-dimensional model if the space charge is taken into account and boundary effects are neglected.

We note that the presence of small-scale pulsations does not exclude the existence of large-scale structures, for instance, space-charge waves.

If one of the space charge oscillation modes is in synchrony with oscillations of the resonance system and both frequencies and phase speeds of the fields are close to each other, the instability develops differently. In a magnetron, the structures of the distribution of the space charge field and waves in the transmitting line are essentially different: the potential and the field are related to the modulation of the electron layer and, in particular, to the periodicity of the boundary, and hence they decay with increasing the distance from the boundary; the field of a synchronous wave decays with the distance from the anode, i.e., in the opposite direction. This model gives a qualitative answer to the second riddle of magnetrons. The self-excitation of a cylindrical magnetron owes its existence to the interaction of two synchronously rotating waves, the wave of the space charge and the wave in the resonance system, which rotate as rigid bodies, such that synchrony between them is never destroyed. Insofar as the amplitude of oscillations in the resonator is small, the distribution of the field is close to the distribution of the space-charge wave field. Electrons begin to move toward the anode, tending to form expanding spokes (Fig. 2). But the amplitude of resonance oscillations grows and determines the field distribution and motion of electrons to an ever increasing extent.

In the argument just proposed, it was tacitly assumed that the generation begins with a single-stream state of the electron cloud. This is probably not so, because the turbulent ('boiling') electron layer can also be assumed to support space-charge waves with a phase velocity of the same order. Any theoretical evidence proving this is absent, but there are experimental observations witnessing the existence of such waves.

The regime of developed generation in magnetrons is studied noticeably better than the excitation process. Interacting with the tangent field, the electrons channel their energy into it and thus support the generation. The electron spokes deform under the action of the space charge field and, as already mentioned, are nonstationary (pulsating) even in the regime of equilibrated oscillations. The anode current oscillates around its mean value because of this nonstationarity. The characteristic pulsation time turns out to be much smaller than the time it takes oscillations in the resonator to set in. Correspondingly, the amplitude of a synchronous wave in the regime of sustained oscillations is virtually constant, because it is given by the result of natural time averaging (see lecture 2 in Ref. [42]).

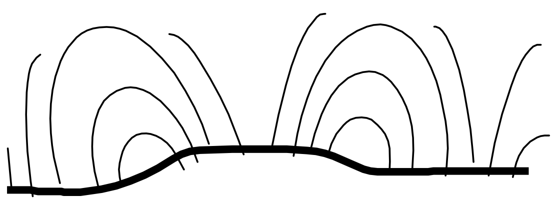


Figure 2. Equipotential lines of a space charge near the outer layer boundary.

The pulsations of the spokes share the origin with those of the electron cloud in a closed magnetron: they are caused by instability of dense electron formations. The only difference is that the instability of the spokes does not typically proceed to the end because a synchronous wave with a sufficiently large amplitude removes electrons from the interaction space relatively rapidly. The removed electrons are replaced by new ones, and hence accumulation of perturbations occurs to a lesser extent.

The pulsations of the spokes demonstrate that they represent a complex oscillating system connected to another oscillating system, the volume resonator. The properties of the electron oscillating system are still studied insufficiently. The understanding of the transition from the pre-generation to the generation regime is most obscure. In powerful pulse devices, the orbital motion of electrons is suppressed to a large extent and electrons tend to drift along equipotential lines of the resultant field, forming a laminar stream. But the laminar spokes are also unstable with respect to a wave traveling along a spoke (see appendix 3 in Ref. [42]). Each spoke emitting from the cathode to the anode and formed by the field of a synchronous wave can be treated as a peculiar ray system of the magnetron type capable of amplifying and generating oscillations even in the absence of resonance or a retarding system. In real conditions, the spokes interact among themselves, with the electron cloud located near the cathode, and with fields of different oscillations in the resonator, and react to random and periodic perturbations. The outcome is irregular oscillations superimposed on the stationary generation regime.

In Refs [43, 44], the results of numerical simulations of the process of self-modulation of the space charge are given. Qualitatively, the process can be described as follows. A high-frequency field of the oscillating system, superimposed on a homogeneous static electron stream, leads first to the formation of convexity on which new layers of electrons gradually wind, such that a local rotating bush is formed. When the upper layers of this bush are elevated and approach the anode sufficiently close, they are shed off the bush under the action of the high-frequency field of the resonance system and gradually, in the form of bundles, in pulses reach the anode. After this, the local bush is gradually recovered and the sketched process is repeated. Any irregularity in this process pertains to the fact that the formation time of the local bush is not related to its travel time from one resonator system slit to another. Therefore, the shedding process starts each time for a new initial state of the bush. Correspondingly, different charge portions are torn off the bush. This chaotic self-modulation can serve as one of the factors leading to the existence of a high noise level in the frequency spectrum of the signal generated by magnetrons.

We note that a set of studies is available (see, e.g., Ref. [45]) in which powerful computer facilities were used to directly simulate a magnetron based on the system of self-consistent Maxwell–Vlasov equations in three dimensions. However, questions related to the characteristics of the developed generation regime (and those pertaining to the first two riddles) remain. It must be admitted that the integral characteristics of a magnetron can be computed fairly well, whereas devices are frequently constructed using the similarity theory.

3. Ionization waves in gas discharge plasmas (strata)

As is known, the excitation of so-called ionization waves (strata) in a low-temperature plasma is the consequence of ionization instability [11, 46, 47]. The main role in its development is played by the dependence of the ionization rate on the temperature and concentration of electrons.

The excitation of strata in a gas discharge plasma was first observed at the end of the 19th century. Rich experimental evidence has been accumulated since then (see, e.g., reviews [46, 48–50]). In an overwhelming number of experiments, a single wave with an anomalous dispersion and with the phase velocity directed from the anode to the cathode (its group velocity is directed from the cathode to anode) was observed. Correspondingly, practically all theoretical works dealing with the instability of the stationary plasma state have sought a spatial increase in perturbations with a periodic dependence on the coordinate. We note that the same approach was and frequently continues to be used in nearly all works devoted to plasma instabilities (see, e.g., Ref. [51]) and turbulence theory [13, 14, 52]. However, the presence of boundaries should lead to the existence of an oppositely directed wave. Accounting for boundary conditions also allows writing a characteristic equation for wave eigennumbers of the excited waves. Treatment of the problem taking the oncoming wave into account permits assessing the degree of correctness of the traditional approach that ignores this wave. A rigorous analysis of the problem indicates that the instability is of the convective type and its excitation requires feedback through either the oncoming wave or the supplying circuit. Thus, views that feedback could be neglected in describing ionization waves should be considered mythical. The stability condition of the stationary plasma state with due regard for feedback through the oncoming wave and supplying circuit was first investigated in Refs [53, 54]. In particular, they explain why the oncoming wave was not observed in many experiments.

Below, starting with the hydrodynamic model, we show where this result comes from. The hydrodynamic model comprises continuity equations for the charge of electrons and ions, the energy conservation for electrons (the energy of ions is supposed to be equal to zero), and the Poisson equation for the electric current. In a widely used approximation of plasma quasineutrality,[7] the system of equations can be brought to the form [46]

$$\frac{\partial n}{\partial t}-\frac{\partial}{\partial x}\left(D_{\mathrm a}\,\frac{\partial n}{\partial x}\right)-\mu_{\mathrm i}\left[\frac{\partial n}{\partial x}\frac{\partial T}{\partial x}+\gamma_1\,\frac{\partial}{\partial x}\left(n\,\frac{\partial T}{\partial x}\right)\right]$$
$$=n\big(Z(n,T)-Z_0(T)\big)\,,$$
$$\frac{3}{2}\,e\,\frac{\partial(nT)}{\partial t}-e\gamma\,\frac{\partial}{\partial x}\left(\mu_{\mathrm e}nT\,\frac{\partial T}{\partial x}\right)$$
$$=-\left(\zeta\,\frac{\partial T}{\partial x}-E\right)j-enH(n,T)\,, \qquad (21)$$
$$j=en\mu_{\mathrm e}\left(E-\frac{T}{n}\frac{\partial n}{\partial x}-\gamma_1\,\frac{\partial T}{\partial x}\right),$$

where n is the concentration of electrons (and ions) at the tube axis, e is the electron charge, T is the temperature of electrons in electronvolts, $\mu_{\mathrm e}$ and $\mu_{\mathrm i}$ are the mobilities of electrons and ions, $D_{\mathrm a}=\mu_{\mathrm i}T$ is the coefficient of electron diffusion related to the ion mobility (it is termed the coefficient of ambipolar diffusivity), $Z(n,T)$ is the effective ionization frequency, $Z_0(T)=(2{,}4/R_0)^2\mu_{\mathrm i}T$ is the quantity inverse to the diffusion lifetime of electrons and ions due to recombination at the tube wall, R_0 is the tube radius, E is the longitudinal component of the electric field strength, j is the current density, $H(n,T)$ are electron energy losses per unit time due to collisions, and ζ,γ_1, and γ are the kinetic coefficients, whose values depend on the shape of the electron distribution function over velocities (for the Maxwell distribution, $\zeta=2$, $\gamma_1=1/2$, and $\gamma=1$).

Equations (21) form a closed system if the current is known. If this is not the case, this system has to be augmented by Ohm's law for a closed circuit:

$$\mathcal{E}=jR_{\mathrm i}S+\int_0^L E\,\mathrm{d}x\,, \qquad (22)$$

where $\mathcal{E}$ is the electromotive force of the power supply, $R_{\mathrm i}$ is its internal resistance, and S and L are the effective cross section and the length of the positive column in the gas discharge tube.

To facilitate understanding the physical principles underlying the relevant processes, we choose the simplest boundary conditions expressing the absence of electron temperature and concentration perturbations at the boundaries of the positive column,

$$n(0,t)=n(L,t)=n_0\,,\qquad T(0,t)=T(L,t)=T_0\,, \qquad (23)$$

where n_0 and T_0 are the stationary values of n and T, determined from the equations following from Eqns (21) and (22),

$$Z(n_0,T_0)=Z_0(T_0)\,,\qquad \mu_{\mathrm e}E_0^2=H(n_0,T_0)\,,$$
$$j_0=\frac{\mathcal{E}-E_0L}{R_{\mathrm i}S}\,,\qquad j_0=en_0\mu_{\mathrm e}E_0\,.$$

For subsequent calculations, it is convenient to eliminate the electric field E from Eqns (21) and (22) and pass to the dimensionless variables

$$\xi=\frac{E_0}{T_0}\,x\,,\qquad \tau=\frac{\mu_{\mathrm i}E_0^2}{T_0}\,t\,,\qquad N=\frac{n-n_0}{n_0}\,,$$
$$U=\frac{T-T_0}{T_0}\,,\qquad J=\frac{j-j_0}{j_0}\,.$$

The equations obtained can be conveniently written, taking boundary conditions (23) into account, in the form

$$\frac{\partial N}{\partial\tau}-\eta_T U=(1+U)\,\frac{\partial^2 N}{\partial\xi^2}+\gamma_1(1+N)\,\frac{\partial^2 U}{\partial\xi^2}$$
$$+(1+\gamma_1)\,\frac{\partial N}{\partial\xi}\frac{\partial U}{\partial\xi}+\eta(U,N)-\eta_T U\,,$$
$$\gamma\,\frac{\partial^2 U}{\partial\xi^2}+\frac{\partial N}{\partial\xi}=\alpha\,\frac{1+J}{(1+N)(1+U)}\,\frac{\partial U}{\partial\xi} \qquad (24)$$
$$+\left(\frac{N(N+2)-J}{(N+1)^2}-\frac{\gamma}{1+N}\,\frac{\partial U}{\partial\xi}\right)\frac{\partial N}{\partial\xi}$$
$$-\frac{\gamma}{1+U}\left(\frac{\partial U}{\partial\xi}\right)^2+\frac{(N-J)(N+J+2)}{(N+1)^2(U+1)}+h(U)\,,$$

$$J=\frac{R}{(R_{\mathrm i}+R)l}\int_0^l\left(\frac{(1+J)N}{1+N}+\frac{N-U}{1+N}\,\frac{\partial N}{\partial\xi}\right)\mathrm{d}\xi\,, \qquad (25)$$

[7] The quasineutrality condition implies that concentrations of electrons and ions are equal.

where

$$\eta_T = \left(\frac{2.4T_0}{E_0 R_0}\right)^2 \left(\frac{T_0}{Z_0} \frac{\partial Z}{\partial T}\bigg|_{n=n_0,\, T=T_0} - 1\right), \qquad \alpha = \zeta - \gamma_1\,,$$

$$h(U,N) = \frac{H - H_0}{H_0(1+U)}\,,$$

$$\eta(U,N) = \frac{Z - Z_0(1+U)}{\mu_{\rm i} E_0^2}\, T_0(1+N)\,,$$

$R = E_0 L / j_0 S$ is the discharge resistance with respect to the direct current, and $l = E_0 L / T_0$ is the dimensionless length of the positive column. The left-hand sides of Eqns (24) contain linear conservative terms, while their right-hand sides contain nonconservative and nonlinear terms.

To find the self-excitation conditions of the strata, Eqns (24) and (25) should be linearized. Neglecting the dependence of H on the electron concentration, we obtain

$$\frac{\partial N}{\partial \tau} - \eta_T U = \frac{\partial^2 N}{\partial \xi^2} + \gamma_1 \frac{\partial^2 U}{\partial \xi^2} + \eta_n N\,, \qquad (26)$$

$$\gamma \frac{\partial^2 U}{\partial \xi^2} + \frac{\partial N}{\partial \xi} = \alpha \frac{\partial U}{\partial \xi} + 2(N - J) + h_T U\,,$$

$$J = \frac{R}{(R_{\rm i} + R)l} \int_0^l N\, {\rm d}\xi\,, \qquad (27)$$

where

$$\eta_n = \left(\frac{2.4T_0}{E_0 R_0}\right)^2 \frac{n_0}{Z_0} \frac{\partial Z}{\partial n}\bigg|_{n=n_0,\, T=T_0}, \qquad h_T = \frac{T_0}{H(T_0)} \frac{\partial H}{\partial T}\bigg|_{T=T_0}.$$

A solution of Eqn (26) can be represented as a sum of two terms, a term describing traveling waves and an in-phase component related to modulation of the discharge current, i.e.,

$$N(\xi,\tau) = \left(\sum_{j=1}^{4} C_j \exp(\beta_j \xi) + C_0\right) \exp(p\tau)\,, \qquad (28)$$

$$U(\xi,\tau) = \left(\sum_{j=1}^{4} V_j C_j \exp(\beta_j \xi) + V_0 C_0\right) \exp(p\tau)\,,$$

where β_j are complex-valued roots of the dispersion equation

$$p = \eta_n + \beta^2 + \frac{(\beta - 2)(\eta_T + \gamma_1 \beta^2)}{\alpha\beta + h_T - \gamma\beta^2}\,, \qquad (29)$$

$$V_j = \frac{p - \eta_n - \beta_j^2}{\eta_T + \gamma_1 \beta_j^2}\,, \qquad j = 0, \ldots, 4\,, \qquad \beta_0 = 0\,,$$

V_j are the weighted coefficients of electron temperature and concentration amplitudes,

$$C_0 = \frac{J_0}{(p - \eta_n) h_T / 2\eta_T + 1}\,, \qquad (30)$$

and J_0 is the current modulation amplitude ($J = J_0 \exp(p\tau)$). Substituting Eqn (28) in Eqn (27) and taking Eqn (30) into account, we find the relation between the coefficient C_0 and the other coefficients C_j:

$$C_0 = \frac{R}{R_{\rm i} l} \left[1 + \frac{(R_{\rm i} + R) h_T}{2 R_{\rm i} \eta_T} (p - \eta_n)\right]^{-1} \sum_{j=1}^{4} \frac{C_j}{\beta_j} \left(\exp(\beta_j l) - 1\right). \qquad (31)$$

Dispersion relation (29) allows finding complex wave numbers β_j or, more precisely, the relation between them and the frequency p of excited waves. We emphasize that the wave frequencies are still unknown. They can be computed from the characteristic equation obtained upon substitution of the expressions found for wave numbers in the boundary conditions. The presence of oncoming waves is necessary in this situation in order to satisfy all the boundary conditions. Finding an exact analytic solution is difficult in the general case. To proceed, we limit ourselves to the case of traveling strata in plasmas of inert gases, in which the parameter η_T is typically large [46], and hence the small parameter $\varepsilon = \eta_T^{-1/4} \ll 1$ can be introduced. In this case, as follows from the results obtained below, the condition

$$\frac{|p - \eta_n|}{\eta_T} \sim \varepsilon \qquad (32)$$

holds in the vicinity of the stratum excitation boundary.

Under condition (32), one of the roots of dispersion equation (29) is of the order of unity, while the other roots are markedly larger. The smallest root is

$$\beta_1 = 2 + \frac{(p - \eta_n) h_T}{\eta_T} + o(\varepsilon)\,, \qquad (33)$$

and the root that is next in amplitude is

$$\beta_2 = -\frac{\eta_T}{\gamma(p - \eta_n)} + o(\varepsilon)\,. \qquad (34)$$

The two remaining roots are related to standing strata [11] and present no interest to us.

Substituting Eqn (28) in the boundary conditions and taking Eqn (30) into account, we obtain a system of homogeneous equations for the coefficients C_j. Setting the determinant of that system equal to zero yields the characteristic equation. It can be readily demonstrated that this equation can be split, up to terms of the order ε, into two independent equations. One of them, of the form

$$\exp\left[(\beta_2 - \beta_1) l\right] - 1 + \frac{1}{\beta_1 l} \left(\frac{(R_{\rm i} + R)\beta_1}{2R} - 1\right)^{-1} \times \left[1 - \exp(-\beta_1 l)\right]\left[\exp(\beta_2 l) - 1\right] = 0\,, \qquad (35)$$

specifies the conditions of excitation of traveling strata.

The solution of Eqn (35) can be represented in the form

$$\beta_2 = \psi_0(k) + {\rm i}k\,, \qquad (36)$$

where k is a real wave number taking a discrete set of eigenvalues and $\psi_0(k) \ll k$ is the increment of the wave spatial growth. It weakly depends on the wave number k and is largely determined by the ratio $R_{\rm i} l / R$. Setting $\beta_2 \approx {\rm i}k$ in Eqns (33) and (34), we find

$$\beta_1 = 2 + {\rm i}k_1\,, \qquad (37)$$

where $k_1 = h_T / \gamma k$. A wave corresponding to the wave number β_2 is the basic ionization wave. The other wave, with the wave

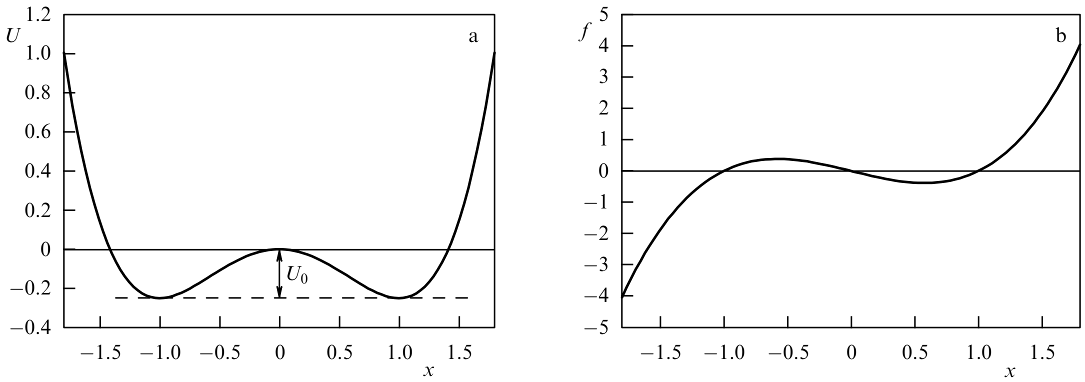


Figure 3. The potential $U(x) = -x^2/2 + x^4/4$ (a) and the corresponding force $f(x) = -x + x^3$ (b).

number β_1, strongly decays with the distance in the direction from the anode to the cathode. The existence of this wave was first predicted theoretically in Refs [53, 54] and then observed experimentally by many researchers. When the discharge takes power from the source with a very high internal resistance, such that the current in the circuit remains virtually constant, it is precisely this wave that provides feedback leading to the self-excitation of strata. However, if the resistance is sufficiently small, the influence of this wave is negligible. That is why it escaped the attention of many researchers.

Substituting Eqn (36) in dispersion relation (29), we find the complex eigenfrequencies p associated with eigenvalues k:

$$p = \eta_n - k^2 - \frac{2\eta_T}{\gamma k^2}\left(a + \frac{\psi_0}{2}\right) + \frac{\mathrm{i}\eta_T}{\gamma k}\,, \tag{38}$$

where $a = 1 - \alpha/2\gamma$. It follows from Eqn (38) that the quantity $\delta = \mathrm{Re}\, p$ can be positive, i.e., the self-excitation of oscillations is possible. The maximum value of δ,

$$\delta = \eta_n - 2\left[\frac{\eta_T(2a + \psi_0)}{\gamma}\right]^{1/2} \sim \varepsilon^{-2}\,, \tag{39}$$

is attained at

$$k = k_0 = \left[\frac{\eta_T(2a + \psi_0)}{\gamma}\right]^{1/4} \sim \varepsilon^{-1}\,,$$

$$\omega = \mathrm{Im}\, p = \left[\frac{\eta_T^3}{\gamma^3(2a + \psi_0)}\right]^{1/4} \sim \varepsilon^{-3}\,.$$

Hence, it follows that condition (32) used in our computations is satisfied.

4. Stochastic resonance

One example, this time related to recent times, is the explanation of the so-called stochastic resonance. The concept of 'stochastic resonance' was first introduced in 1981 because of the need to explain a close-to-periodic sequence of Earth's glaciation epochs (with the period T approximately equal to 100,000 years) [55–57]. One of the reasons behind this sequence can be the observed periodic variation in the Earth's orbit eccentricity with nearly the same period. But this variation is very small and, taken alone, cannot result in a significant change in the climate. However, having added some noise to the derived equations, the authors of the works cited above found that such changes become possible in a certain range of noise amplitudes. In this manner, the phenomenon called stochastic resonance was discovered. Although the question of whether this explanation corresponds to the real causes of Earth's glaciation epochs has not yet been ultimately solved, the phenomenon of stochastic resonance has become widely known [58, 59]. It is observed in systems of very different physical natures, including turbulent jets [60].

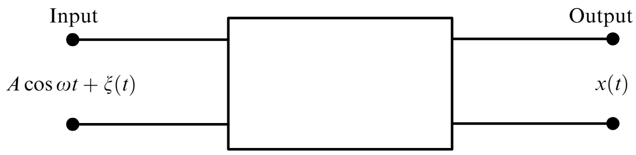


Figure 4. Schematic of a four-pole network.

The first researchers of stochastic resonance simulated the sequence of glaciation–deglaciation phases in the Earth's climate using an equation for the Earth surface temperature in the form of an equation of motion of a light particle in a bistable potential field perturbed by a weak input periodic signal and noise:

$$\dot{x} + f(x) = A\cos\omega t + \xi(t)\,, \tag{40}$$

where x is the temperature, $A\cos\omega t$ is the weak periodic perturbation caused by variations in the Earth's orbit eccentricity at the frequency ω, $f(x) = \mathrm{d}U(x)/\mathrm{d}x = -x + x^3$, $U(x) = -x^2/2 + x^4/4$ is a symmetric two-well potential, and $\xi(t)$ is the white noise with intensity K, with $\langle\xi(t)\xi(t+\tau)\rangle = K\delta(\tau)$. The potential $U(x) = -x^2/2 + x^4/4$ and the force $f(x)$ corresponding to it are given in Fig. 3, where U_0 denotes the potential barrier height for the transition of a particle from one well to another. We note that from the standpoint of the four-pole network theory, the perturbation $A\cos\omega t$ in Eqn (40) can be considered an input signal and the variable x an output signal. A schematic of a respective four-pole network is given in Fig. 4.

As follows from Ref. [61], the solution of Eqn (40) contains discrete components (odd harmonics of the frequency ω) and continuous components enforced by the noise. The ratio of the amplitude of the first signal harmonic B at the output to the signal input amplitude A is called the amplitude transformation coefficient[8] and is denoted by $Q(K)$.

[8] This definition of the transformation coefficient is standard and generally accepted.

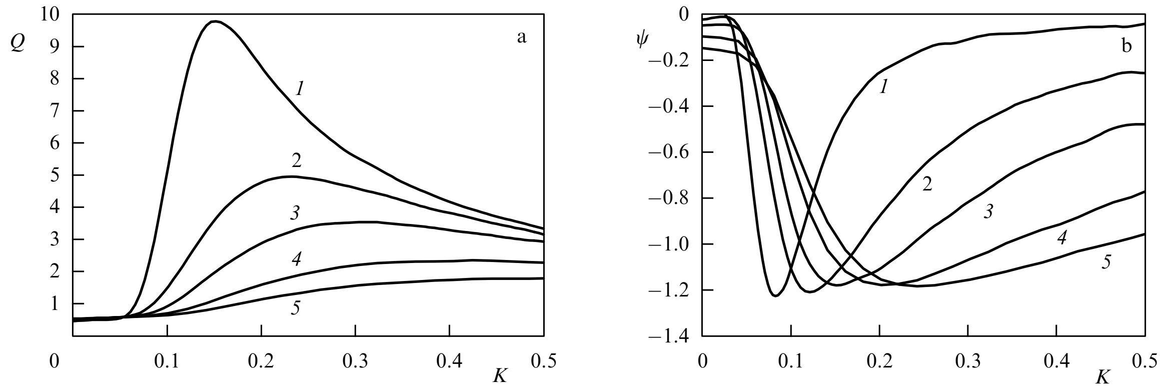


Figure 5. The dependences of Q (a) and ψ (b) on the noise intensity K at $A = 0.1$ and $\omega = 0.01$ (*1*), 0.05 (*2*), 0.1 (*3*), 0.2 (*4*), and 0.3 (*5*) found from numerical solution of Eqn (40) with $f(x) = -x + x^3$. The results are adapted from Ref. [64], which describes how they were obtained.

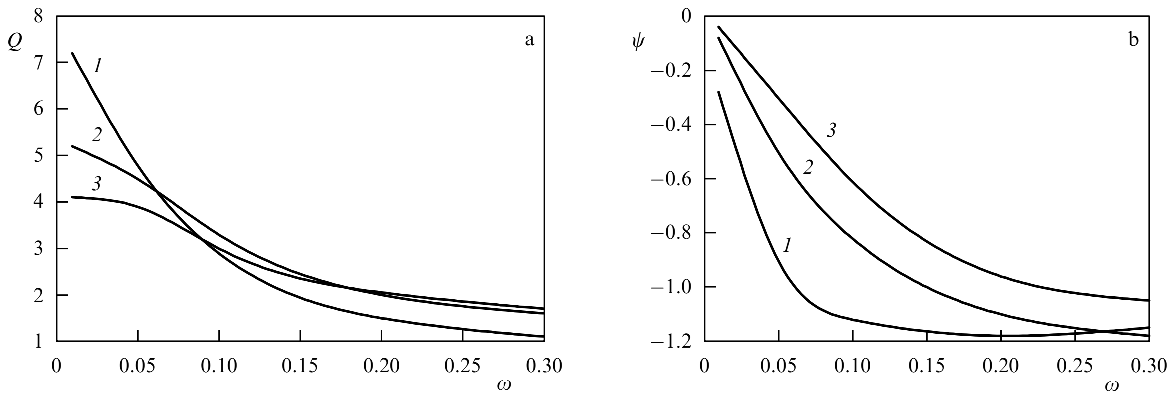


Figure 6. Numerical dependences of Q (a) and ψ (b) on the frequency ω for Eqn (40) at $A = 0.1$, $K = 0.2$ (*1*), $K = 0.3$ (*2*), and $K = 0.4$ (*3*). (Adapted from Ref. [64].)

Numerical and approximate analytic solutions of Eqn (40) show that the transformation coefficient $Q(K, \omega, A)$ and the phase shift $\psi(K, \omega, A)$ are nonmonotonic functions of the noise intensity K (Fig. 5). The magnitude of Q reaches a maximum at a certain value of K, and this value increases with the frequency of the signal. The dependence of Q on K at a fixed frequency and amplitude of the signal resembles a resonance dependence of the amplitude of an oscillator on the forcing frequency. Because K defines the mean frequency of jumps from one well to another under the action of noise, with the jumps related to random transitions through the potential barrier U_0, it is natural to assume that the maximum in the dependence of Q on K occurs when the resonance condition is satisfied between the signal frequency and mean frequency of noise-induced transitions from one stable state to another.[9] In the case considered, this condition corresponds to the equality between the signal period $T = 2\pi/\omega$ and the doubled mean time T_{tr} of the first transition through the potential barrier,

$$T_{\text{tr}} = \frac{\pi}{\sqrt{2}} \exp\left(\frac{2U_0}{K}\right). \tag{41}$$

In English-based literature, the expression for T_{tr} is called the Kramers formula, although the first work on computing T_{tr} by Pontryagin, Andronov, and Vitt [62] was published in 1933, prior to paper [63] by Kramers, which only appeared in 1940.

It is apparent, however, that the assumption formulated above, although supported by a simple logical consideration, remains mythical. If it were true, the magnitude of Q would reach a maximum not only under the variation of the noise intensity K that determines the mean transition frequency but also under the variation of ω (the input signal frequency). It is well known that as ω increases, the value of Q monotonically decreases (Fig. 6).

In works that followed, it was shown that the cause of stochastic resonance is the change under the action of noise in the system elasticity and the damping coefficient with respect to the reaction to the input signal [64–66]. Noteworthy is the fact that the noise-driven variations in the system effective parameters have been known for a long time (see, e.g., Refs [67, 68]). The stochastic resonance, however, has been addressed from such a perspective only in the works cited above.

Thus, the observational data frequently fail to provide an unambiguous explanation of the phenomenon observed, i.e., fail to suggest a relevant theory based on these data. Only a rigorous theory, albeit with a limited applicability, based on rigorous mathematics can allow deciding which of the two explanations is correct.

The nonmonotonic dependence of Q on K prompted an idea that the phenomenon of stochastic resonance can be used

[9] We note that just this hypothesized resonance condition has brought about the name 'stochastic resonance' [55–57].

to increase the signal-to-noise ratio. This is indeed so in the case of a harmonic signal. But for a harmonic signal, other, more efficient methods of signal retrieval from a noisy background are known, for example, those based on synchronous detection. For a nonharmonic signal, the question of the possibility of increasing the signal-to-noise ratio with the help of a stochastic resonance remains open because both the transformation coefficient Q and the phase shift ψ are strongly dependent on the frequency, which would inevitably lead to the input signal distortion. We also note that the mere notion of the signal-to-noise ratio is defined in many works ambiguously. For example, review [59] suggests the following definition: "The signal-to-noise ratio (SNR) is defined as the ratio of spectral power densities of the signal to the noise at the frequency of the signal... For a harmonic input signal, this definition in experimental conditions corresponds to the ratio of the amplitude of the modulation signal spectral line above the noise background to that of the noise background in the spectrum of the output signal." However, as follows from Ref. [61], the spectral signal density represents a set of δ-functions, and hence the amplitude of the spectral line at the signal frequency tends theoretically to infinity. Thus, if one follows this definition, then the signal-to-noise ratio is theoretically always equal to infinity. For a numerical procedure, the amplitude of the spectral line depends on the realization length, the discretization frequency, averaging parameters, and other factors. Similar definitions can be found in other papers. It then follows from the material presented above that using a stochastic resonance for retrieving a complex signal out of noise is not feasible, but a solution to the task of discriminating a signal based on stochastic resonance is apparently possible and deserves attention.

5. Turbulence in unbounded flows

5.1 Hydrodynamic and acoustic waves

It is well known that both hydrodynamic and acoustic waves exist and interact in jets. Just this fact offers a possibility to control the level of turbulence through the effect of acoustic waves [69]. It is seemingly less known that acoustic and hydrodynamic waves represent two wave solutions of the same equations of fluid dynamics in a moving medium. We demonstrate this for a two-dimensional unbounded fluid satisfying the Euler and continuity equations. Assuming that the fluid moves in the direction of the x axis with a speed U_0, we have

$$\begin{gathered}\frac{\partial u}{\partial t}+(U_0+u)\,\frac{\partial u}{\partial x}+v\,\frac{\partial u}{\partial y}=-\frac{1}{\rho_0+\rho}\,\frac{\partial p}{\partial x}\,,\\ \frac{\partial v}{\partial t}+(U_0+u)\,\frac{\partial v}{\partial x}+v\,\frac{\partial v}{\partial y}=-\frac{1}{\rho_0+\rho}\,\frac{\partial p}{\partial y}\,,\\ \frac{\partial \rho}{\partial t}+(U_0+u)\,\frac{\partial \rho}{\partial x}+(\rho_0+\rho)\,\frac{\partial u}{\partial x}+v\,\frac{\partial \rho}{\partial y}+(\rho_0+\rho)\,\frac{\partial v}{\partial y}=0\,,\end{gathered}\tag{42}$$

where p_0 and ρ_0 are the pressure and the density in a stationary state and u, v, p, and ρ are respective deviations of the velocity components, pressure, and density from their stationary values.

The system of three equations (42) contains four unknown fields. The relation between two of them (p and ρ) is provided by thermodynamics. Assuming that the processes are adiabatic, we express p through ρ as

$$p=p_0\left[\left(\frac{\rho}{\rho_0}+1\right)^{\gamma}-1\right],\tag{43}$$

where γ is the adiabatic process exponent. Substituting Eqn (43) in Eqn (42) and limiting ourselves to terms linear in the perturbation, we obtain the equations

$$\begin{gathered}\frac{\partial u}{\partial t}+U_0\,\frac{\partial u}{\partial x}+\frac{a^2}{\rho_0}\,\frac{\partial \rho}{\partial x}=0\,,\qquad \frac{\partial v}{\partial t}+U_0\,\frac{\partial v}{\partial x}=0\,,\\ \frac{\partial \rho}{\partial t}+U_0\,\frac{\partial \rho}{\partial x}+\rho_0\,\frac{\partial u}{\partial x}=0\,,\end{gathered}\tag{44}$$

where $a=\sqrt{\gamma p_0/\rho_0}$ is the speed of sound relative to the fluid at rest.

Solutions of Eqns (44) are sought in the form of traveling waves of a frequency ω propagating along the x axis:

$$\begin{gathered}u=A\cos(\omega t-kx)\,,\qquad v=B\cos(\omega t-kx)\,,\\ \rho=C\cos(\omega t-kx)\,,\end{gathered}\tag{45}$$

where k is the wave number. Substituting Eqn (45) in Eqns (44), we obtain a system of equations for the unknown amplitudes A, B, and C. The condition that its determinant is zero gives the dispersion equation

$$(\omega-kU_0)\left[(a^2-U_0^2)k^2+2\omega U_0 k-\omega^2\right]=0\,.\tag{46}$$

It can be readily seen that Eqn (46) describes two types of waves, a hydrodynamic wave and two acoustic waves. For the hydrodynamic wave, which always propagates in the direction of fluid motion,

$$k=\frac{\omega}{U_0}\,,\tag{47}$$

whereas for the oppositely directed acoustic waves,

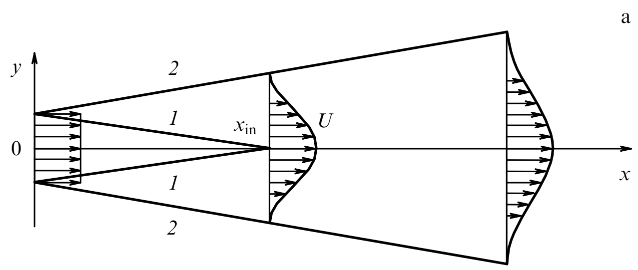
$$k_{1,2}=\pm\frac{\omega}{a\pm U_0}\,.\tag{48}$$

The difference in the propagation speed of two oppositely directed acoustic waves is related to the Doppler effect. It follows from Eqn (48) that the speed of the wave propagating downstream can be much larger than that of the upstream wave.

According to Eqns (44), the amplitudes of the longitudinal component of velocity and of the density (A and C) are equal to zero in the hydrodynamic wave, and only the transverse velocity amplitude B differs from zero. This implies that in contrast to acoustic waves, the hydrodynamic wave is transverse. For acoustic waves, on the contrary, the amplitudes of the transverse velocities are equal to zero, while the amplitudes of the longitudinal component are related to the amplitudes of density perturbations by the known formula

$$A_{1,2}=\pm\frac{a}{\rho_0}\,C_{1,2}\,.\tag{49}$$

It follows from Eqns (47) and (48) that in the linear approximation adopted here, the hydrodynamic wave, similarly to the acoustic waves, shows no dispersion and has the speed equal to that of the fluid. But experiments with jets in which the hydrodynamic wave is usually identified with the motion of vortices indicate that the speed of this wave



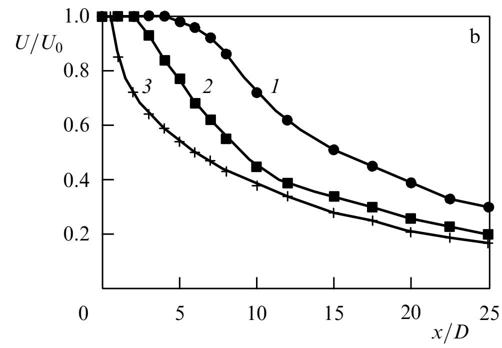


Figure 7. (a) Schematic of a free jet illustrating the deformation of the mean velocity profile and broadening of the boundary layer: lines *1* and *2* schematically show the internal and external boundaries of the boundary layer. (b) Experimental dependences of the normalized mean velocity U/U_0 at the jet axis at the relative distance x/D from the nozzle for three levels of turbulence at the nozzle exit, $\epsilon_u(0) = 0.015, 0.093$, and 0.209 (curves *1*, *2*, and *3*, respectively).

constitutes 0.5–0.7 of the fluid speed. The reasons reside in both nonlinear effects and jet divergence.

5.2 Basic properties of turbulent jets

On leaving a nozzle, a jet in a fluid[10] always noticeably diverges in the transverse direction. This is caused by viscosity, owing to which neighboring layers of the fluid are entrained in motion. The mean velocity profile changes accordingly. If the velocity profile is close to that at the nozzle exit, it gradually becomes bell-shaped (Fig. 7a). Close to the jet axis, the jet velocity first slowly decreases with the relative distance from the nozzle x/D, where D is the nozzle diameter, and the decrease becomes stronger further downstream. Admittedly, the marked reduction in the mean velocity begins at the progressively smaller values of x/D, the greater the level of turbulent pulsations at the nozzle exit. This is well illustrated by Fig. 7b, which shows experimental dependences of the normalized mean velocity U/U_0 on x/D for three values of the turbulence level at the nozzle exit,

$$\epsilon_u(0) = \frac{\sqrt{\overline{u(0)^2} - U_0^2}}{U_0} \,,$$

where U_0 and $u(0)$ are respectively the mean velocity and the longitudinal component of the full velocity vector at the nozzle exit [70].

The fluid layer where the mean velocity varies essentially is called the *boundary* or *mixing* layer. Interestingly, vortices typical of turbulence are created just in this layer and have a size of the order of the layer thickness. Downstream from the nozzle, the thickness of the boundary layer increases. At a certain distance $x = x_{\rm in}$ (Fig. 7a), the thickness of the internal part of the boundary layer reaches half of the jet width, whereupon the boundary layer spans the entire jet. The jet part $x \leqslant x_{\rm in}$ is termed the initial part. Within the initial part, the jet velocity changes but slightly. This initial part is most interesting for many practical purposes.

The vortices formed in the boundary layer exhibit a high degree of order and are therefore called 'coherent structures.'[11] Within the initial jet part, this order might be related to the resonant character of the spectra of velocity and pressure pulsations. Experiments show that the frequency $f_{\rm m}$ corresponding to the spectral density maximum decreases downstream from the nozzle [11, 69, 72–76][12] and this decrease can be approximately described by a power law with an exponent depending on the distance to the jet axis [76]. We note that at the jet axis, ${\rm St_m} = f_{\rm m}D/U_0$ decreases much more slowly, while the intensity of velocity pulsations is much less than in the boundary layers. The dependences of ${\rm St_m}$ on x/D were already obtained by different techniques in the 1970s, mainly in the work by Petersen [73] (Fig. 8). It was found that ${\rm St_m}$ is approximately inversely proportional to x/D.

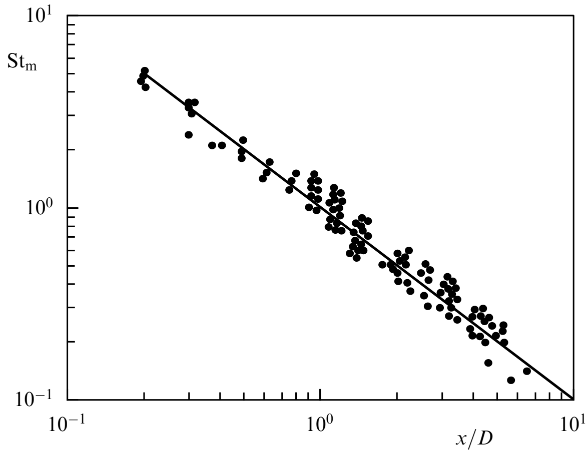


Figure 8. Dependence of the Strouhal number corresponding to the spectral density maximum ${\rm St_m}$ on the relative distance to the nozzle along a line shifted one radius from the jet axis (adapted from Ref. [73]).

A widespread misconception in the theory of turbulent jets is related to the explanation of these shifts in spectral maxima with the distance to the nozzle. In virtually all papers (see, e.g., Refs [5, 70, 72–75, 77–82]), the shifts are attributed to vortex merging, which reduces the frequency twofold. According to the views advocated by the authors of these works, as two vortices merge, a feedback is established through an acoustic wave generated by a strong vortex perturbation in the merger region and propagating upstream. The presence of such a wave finds support in

[10] The discussion here encompasses both liquids and gases.

[11] We note that the term 'coherent structures' is multifaceted. For example, a book by A Scott [71] entitled *Nonlinear Science: Emergence and Dynamics of Coherent Structures* was recently published. However, it offers no definition of coherent structures.

[12] In works on turbulent jets, frequencies f are commonly measured in terms of Strouhal numbers, ${\rm St} = fD/U_0$, where D is the characteristic nozzle diameter.

many experiments (see, e.g., Ref. [78]). This presence is manifested through side frequencies in the spectra of turbulent pulsations in the mixing layer near the nozzle, which are shifted from the maximum by the frequency that corresponds to the frequency of the spectral maximum in one of the merger regions. For the feedback to be efficient, the resonance condition

$$x_i\left(\frac{f_i}{v_\mathrm{h}}+\frac{f_i}{v_\mathrm{a}}\right)=N \qquad (50)$$

should be satisfied. Here, x_i is the coordinate of the section where the ith merger occurs, f_i is the frequency of the spectral maximum in this section, v_h is the velocity of the hydrodynamic wave, v_a is the speed of the acoustic wave, and N is an integer. Condition (50) states that an integer number of acoustic and hydrodynamic waves should fit the length from the nozzle exit to the location of the ith merging. In support of their views, the authors of the works mentioned above point to a satisfactory agreement of the frequencies f_i with experimental data if the values $v_\mathrm{h}=0.6U_0$ and $N=2$ are assumed. Why it should be $N=2$ and not 5 or 10 is not explained by the authors of these views. The gradual evolution of spectra observed in experiments contradicts the discrete evolution dictated by Eqn (50), but is explained via the statistical scatter of merger regions. This is explicitly formulated, for example, in book [69]: "The gradual, not stepwise, character of the curve $\mathrm{St_m}(x/D)$ bears witness that locations where the coherent structures are generated, merge, or break up are subject to stochastic scatter." However, as is known, any statistical scatter should have its causes. This question is not discussed by the authors of these works.

All these arguments could be considered logical if it were not for the results of the theory dealing with generation of turbulent pulsations at small distances from the nozzle [83]. To treat this generation theoretically, one uses the approximation of an incompressible fluid satisfying the Navier–Stokes equation, which is solved with the Krylov–Bogolyubov asymptotic method. In this approximation, the acoustic wave is totally neglected. The method is applicable to that part of the jet where turbulent pulsations remain small. As a result, the evolution of the spectral density of the stream function of a hydrodynamic wave is estimated. The shift of the spectral maximum observed in experiments is predicted by this theory even in the first (linear) approximation. This implies that the merging of vortices and induced acoustic waves, which are nonlinear effects, cannot be governing the observed evolution of spectra. In other words, the above fairly common explanation of the reasons for the observed spectral evolution is a myth. The theory maintains that the shifts in spectra described above should be explained not by discrete mergers but by a gradual jet expansion. Thus, theoretical results discard the myth enjoying wide popularity that the turbulence in jets develops via feedback involving an acoustic wave, i.e., the myth that turbulence represents self-oscillations. We remark that the development of feedback through an acoustic wave induced by the scattering of hydrodynamic waves on irregularities can be fairly strong and indeed lead to self-oscillations if a screen is placed across the jet such that the jet hits it. In the presence of such a screen, the jets, called impact jets [69, 84–87], generate strong acoustic waves. The mechanism through which self-oscillations are excited in such jets pertains to global instability. In ordinary jets, the feedback through acoustic waves is insufficient to trigger self-oscillations.

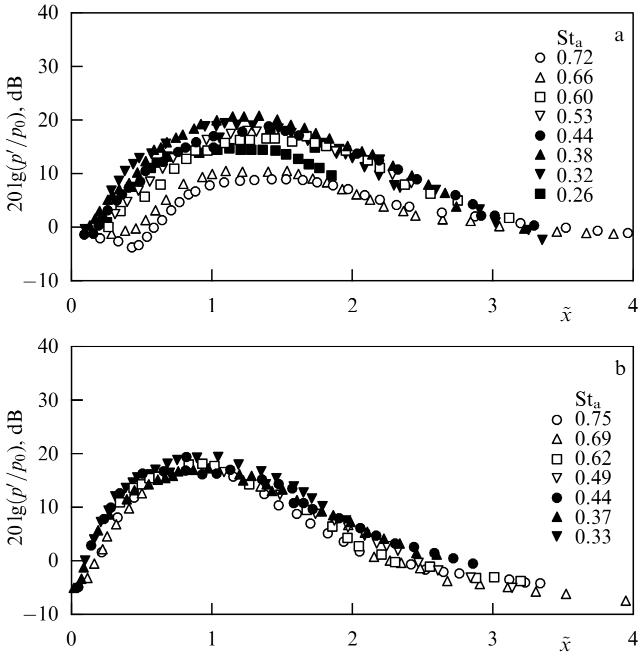


Figure 9. Experimental dependences of the relative pressure amplitude in a circular jet on the relative distance to the nozzle $\tilde{x}=(x/D)\,\mathrm{St_a}$ during longitudinal acoustic excitation at frequencies corresponding to the Strouhal numbers $\mathrm{St_a}$ given on the right side of panels (a) along the jet axis and (b) in the middle of the boundary layer. $\mathrm{Re}=2.6\times10^5$, the sound pressure level at the nozzle exit section is 125 dB close to the nozzle edge.

One more widely shared myth refers to an important property of turbulent jets related to their ability to transform weak input acoustic waves into amplifying hydrodynamic waves [88, 89]. This transformation occurs mainly at the edge of the nozzle. In this case, as is shown in Ref. [89], for a low-frequency acoustic excitation, the amplification coefficient of pressure pulsations varies nonmonotonically with the distance from the nozzle. The results in Ref. [89] pertaining to the Reynolds number $\mathrm{Re}=2.6\times10^5$ are presented in Fig. 9a for the jet axis and for the middle of the boundary flow; in the second case, the sensitivity of the dependence of the amplification factor from $x\,\mathrm{St_a}/D$ to $\mathrm{St_a}$, where $\mathrm{St_a}$ is the acoustic perturbation frequency expressed through the Strouhal number, is very weak, whereas it is much stronger in the first case. The most significant issue is that in the boundary layer region, the positions of the maximum amplification coefficient approximately coincide for different Strouhal numbers and are determined by $x/D\sim1/\mathrm{St_a}$. They are essentially different at the jet axis.

In the linear approximation, the behavior of the amplification coefficient with the distance from the nozzle, as demonstrated in Fig. 9b, can be explained based on a jet model formulated in terms of a set of resonators with eigenfrequencies changing with the distance to the nozzle and to the jet axis. Such a model corresponds to the behavior of spectra of the unperturbed jet described above. Under the assumption that the eigenfrequencies are equal to those of the spectral peaks, their behavior can be approximately assessed from the dependence of the Strouhal number $\mathrm{St_m}$ on the distance to the nozzle along the line shifted one nozzle radius away from the axis presented in Fig. 8. Figure 8 shows that in this case, the eigenfrequency is approximately inversely proportional to x/D, i.e., the condition of resonance between the forcing frequency $\mathrm{St_a}$ and the eigenfrequency is realized at

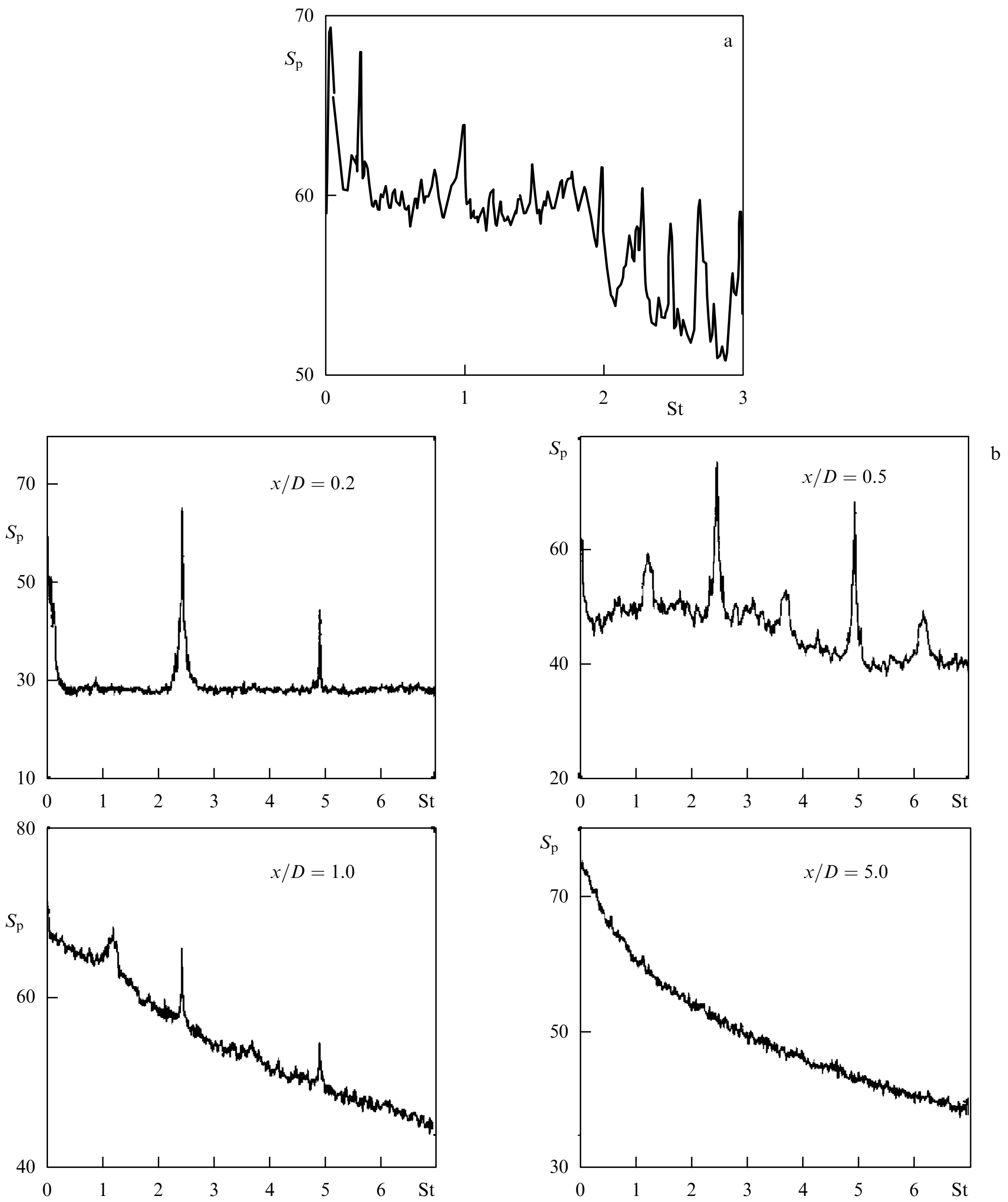


Figure 10. Spectral densities of velocity pulsations in a boundary layer of a circular jet under a low- and high-frequency acoustic excitation with the Strouhal numbers $\mathrm{St_a} = 0.25$ at $x/D = 0.5$ (a) and $\mathrm{St_a} = 2.75$ (b) for different x/D. Re = 25,000.

$x\,\mathrm{St_a}/D \approx 1$, which is observed in experiments reported in Ref. [89].

It follows from the suggested explanation that the observed dependence of the amplification coefficient on the distance to the nozzle, featuring a maximum at some point, represents, in principle, a linear effect and can be explained by the dependence of the resonance jet frequency, linked to shifts in spectral maxima, on the distance, i.e., the expansion of the jet. This is supported by the results in Ref. [90], where, by approximately solving linearized Euler equations for a slowly expanding jet, theoretical dependences were obtained that are similar to those in Fig. 9. The difference in the dependences for the jet axis and the boundary layer in Ref. [90] could be attributed to the fact that the acoustic pressure at the input section was held constant in the region of the boundary layer. Thus, it had an effect at the jet axis only some distance downstream from the nozzle. Despite this obvious explanation, one may encounter mythical conclusions drawn from the effect in the literature. Thus, book [69] argues: "The analysis of these dependences shows that the perturbations of pressure p' propagate from the shear layer to the jet axis because they start growing at the jet axis not immediately on leaving the nozzle, but some distance downstream."

The next myth pertains to the interpretation of changes in turbulent spectra caused by acoustic excitation. It is known that under the action of acoustic waves on the jet, the spectra of turbulent pulsations become drastically modified. First and foremost, this is exemplified through the appearance of a discrete component induced by acoustic excitation, together with subharmonic resonance regions. This can be seen from Fig. 10, which displays spectra of lateral velocity pulsations under the acoustic excitation at both low ($\mathrm{St_a} = 0.25$)

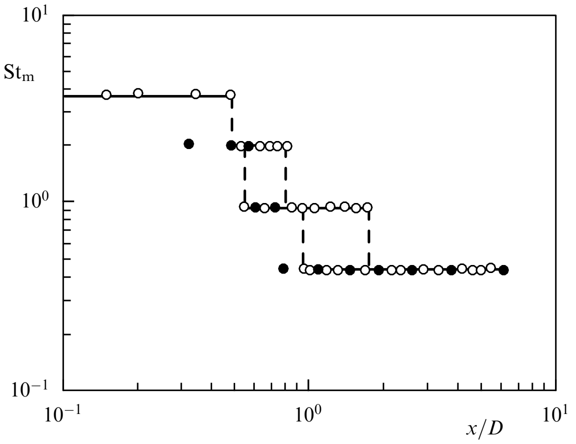


Figure 11. Dependence of the spectral density maxima on the distance to the nozzle in high-frequency acoustic excitation with the Strouhal number 3.54 along the jet axis (white circles) and along the line passing through the edge of the nozzle (black circles).

(Fig. 10a) and high ($St_a = 2.75$) (Fig. 10b) frequencies [76]. In the first case, the spectra contain discrete components of the acoustic frequency and its harmonics in a vicinity of the nozzle. The level of turbulence increases. In the second case, the spectra also contain discrete components at the action frequency and its harmonics, but only very close to the nozzle. However, at small distances downstream from the nozzle, the spectra exhibit maxima at the second and fourth subharmonics. The appearance of these maxima can be explained as a manifestation of an *n*th-order resonance [91] between the frequency of the acoustic signal and frequencies around the spectral maximum of the unperturbed jet. Further downstream, the intensities of the subharmonics and the main harmonic decrease. If the distance increases even further, the spectrum decays monotonically.

The dependence of spectral density maxima on the distance to the nozzle under acoustic excitation with the Strouhal number 3.54 is presented in Fig. 11 as found by Kibens [75]. It is readily seen that this dependence is stepwise, with clearly expressed hysteresis phenomena. It is therefore markedly different from the respective smooth dependence in the absence of acoustic influence. The stepwise character of the dependence can be interpreted as the subsequent appearance of subharmonic resonances of increasingly higher orders with the increase in x/D. It is apparent that the transition from a subharmonic resonance to that of another order can be accompanied by hysteresis if both resonances are stable over some interval of x/D. The interpretation outlined here has no relation to the widespread myth explaining the observed spectral behavior by localization of vortex region merger under acoustic excitation (see, e.g., Ref. [69]). It is known [61] that a spectrum of any system (linear or nonlinear), subject to a periodic signal and noise, should contain discrete components at frequencies that are multiples of the forcing frequency in the continuous background. This actually explains the observed behavior of spectra under acoustic excitation. The mythical character of assertions that the difference between the spectra in the presence and absence of acoustic excitation is explained by 'statistical scatter of merger locations' and their 'localization' thus becomes apparent.

Another myth is the description of the transition from a laminar to the turbulent flow in terms of the excitation of chaotic self-oscillations. As already mentioned, a laminar flow of a fluid in a tube is almost always unstable with respect to small perturbations. It can be shown that this instability is convective, i.e., propagates downstream, and hence cannot lead to self-excitation of turbulence in the absence of a global feedback. Nevertheless, as already noted, recent editions of books [13, 14] argue based on Refs [21, 22] that the turbulence represents chaotic self-oscillations, i.e., that the system giving birth to turbulence can be considered a *dynamic* one.[13] In certain cases, for example, in bounded flows[14] or jets intercepted by some obstacle (a plane screen or wedge), this is indeed so, because a global instability develops there. But in free jets, the transition to turbulence is related not to the excitation of self-oscillations but to strong amplification of weak perturbations that are always present upstream [93–96]. Because hydrodynamic waves and acoustic waves formed at flow inhomogeneities do not interact in the linear approximation, the linear feedback in jets, which could lead to self-excitation of self-oscillations, is absent. All this implies that turbulent jets, at least in some vicinity of their nozzles, cannot be considered in the framework of dynamic models.

Since the works mentioned above, a significant number of papers have appeared whose authors connect the transition to turbulence with the appearance of a low-order strange attractor, i.e., with the excitation of self-oscillations in dynamic systems [97–101]. Granted that instability in turbulent jets is of a convective type, one can conjecture that the existence of such an attractor is one of the myths.

According to the ideas formulated in Ref. [83], the turbulent character of jet flows pertains to a strong amplification of random perturbations, which are always present at the nozzle exit.[15] At some distance from the nozzle, the amplification becomes essentially nonlinear. As a result, the system evolves into a qualitatively new state whose characteristics depend on the source of perturbations only weakly. Its behavior turns out to be very similar to that which would be observed had the system underwent a certain nonequilibrium phase transition. This is supported by the existence of a highly interesting analogy between processes in turbulent jets and the behavior of a pendulum with a randomly vibrating suspension axis, which undergoes such a phase transition [11, 76, 93–96].

Because turbulent pulsations are weak at small distances from the nozzle, they can be explored using a quasilinear theory and the Krylov–Bogolyubov asymptotic method for distributed systems [47]. Such a theory is elaborated in Ref. [83]. We note that the first applications of the linear theory to the analysis of jet stability were proposed in Refs [103–106] based on the Euler equations. Seemingly, in addition to an obvious simplification of the solution, the authors of those papers used a statement shared by many works that processes in jets can be assessed in the inviscid fluid approximation. As is shown in Ref. [83], solving this task

[13] A system is termed dynamic when its state is fully determined by initial conditions (see, e.g., Ref. [92]). Random behavior is also possible for dynamic systems, but it should be linked not to the external noise but to an instability.

[14] One example can be furnished by the Couette flow in an annulus between two coaxially rotating cylinders.

[15] It is noteworthy that random sources exist everywhere in jets even in the absence of any external perturbations; these are so-called natural fluctuations [102]. Their influence, however, is noticeably less than that of perturbations at the nozzle exit and they can therefore be neglected.

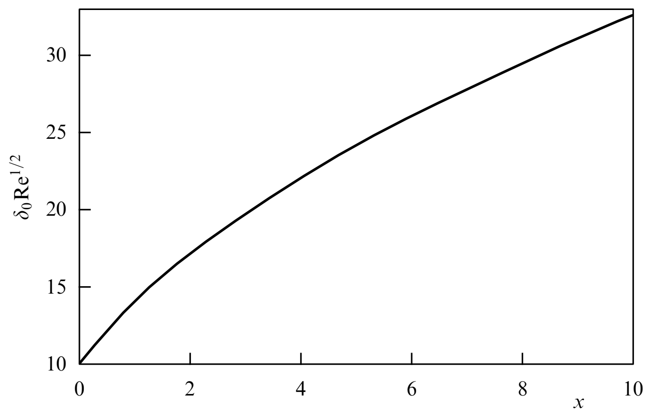


Figure 12. Dependence of the thickness of the boundary layer related to $\sqrt{\mathrm{Re}}$ on the distance to the nozzle.

based on the Navier–Stokes equation leads to results that differ essentially from those obtained by these authors.

In all classical works on the theory of turbulence, beginning from those by Reynolds (see Refs [13, 14]), a solution of the Navier–Stokes equation was represented as a sum of mean fields and their random deviations. Because the mean fields depend on the deviations through the quadratic nonlinearity of the Navier–Stokes equations, the problem of closing the equations thus obtained emerged. To avoid this problem, the authors of Ref. [83] proposed representing the solution as the sum of stationary solutions (dynamic components) and random deviations from stationary solutions (stochastic [16] components). The latter are defined by the sources at the nozzle exit.

The solution of the equations for the dynamic component, even if obtained numerically, is fairly complex. Therefore, we specified the longitudinal velocity profile based on experimental data and a set of physical arguments. It contained some unknown functions of the longitudinal coordinate x, the most important of which is the boundary layer thickness $\delta_0(x)$. It should be mentioned that the mean longitudinal velocity profile similar to ours was earlier used in Refs [103–106], where the Euler equations were solved.

To find these unknown functions, it is convenient to use conservation laws for the energy and momentum fluxes. These laws are routinely derived from the Reynolds equations for the mean values of these fluxes [70, 107] and therefore contain the so-called turbulent viscosity. In Ref. [83], they are derived from the Navier–Stokes equations. The resulting expression for the boundary layer thickness is

$$\delta_0(x) = \sqrt{\delta_{00}^2 + \frac{32q^2}{3\mathrm{Re}}\,x}\,, \tag{51}$$

where $\delta_{00} = \delta_0(0)$ is defined by conditions of fluid ejection from the nozzle, Re is the Reynolds number, and $q \approx 3$. If the jet leaves the nozzle in a laminar regime, the boundary layer can be approximately described by Blasius's function [13] and δ_{00} is proportional to $1/\sqrt{\mathrm{Re}}$. We assumed $\delta_{00} = 1/(b_0\sqrt{\mathrm{Re}})$, where $b_0 = 0.1$. The dependence of $\delta_0(x)\sqrt{\mathrm{Re}}$ at $q = 3$ and the specified value of b_0 is presented in Fig. 12. We mention that the dependence $\delta_0(x)$ found by us is essentially different from that given in Ref. [107], which contains the turbulent viscosity ν_t. Because ν_t is proportional to $\delta_0(x)$ according to the Prandtl hypothesis [108], the dependence obtained in Ref. [107] turns out to be linear. But experiments show that this dependence is essentially nonlinear. We emphasize that accounting for the nonlinearity in higher orders of the Krylov–Bogolyubov method makes the dependence $\delta_0(x)$ closer to the experimental one.

[16] Here and below, 'stochastic' is used as a synonym to 'random.' Quantities are truly stochastic when their probability can be computed.

The dynamical components of the velocity $U_d(y)$ and $V_d(y)$ and the vorticity $\Omega_d(y)$ found in that way are plotted in Fig. 13 for $x = 0$ and $x = 8$. It can be inferred that for all values of y, excluding the narrow vicinities of $y = \pm 1$, $U_d(y)$, $V_d(y)$, and $\Omega_d(y)$ are approximately constant; the values of $V_d(y)$ and $\Omega_d(y)$ are close to zero for $|y| < 1$ and are respectively close to $V_d(x, \pm\infty)$ and $\Omega_d(x, \pm\infty)$ for $|y| > 1$. We note that the presence at $|y| > 1$ of a nonzero transverse velocity component directed to the jet axis reflects the known fact of entrainment of the surrounding fluid by jets.

In Ref. [83], stochastic components were assumed to be small, i.e., proportional to some conventional small parameter ε. This permitted solving equations for stochastic components with a method similar to the Krylov–Bogolyubov method applied to distributed systems (see Refs [47, 83]).

In the first (linear) approximation, the following approximate equation was obtained for the stream function Ψ_{st1}:

$$\frac{\partial \Delta\Psi_{st1}}{\partial t} + U_d(x,y)\,\frac{\partial \Delta\Psi_{st1}}{\partial x} + V_d(x,y)\,\frac{\partial \Delta\Psi_{st1}}{\partial y} - \frac{\partial \Omega_d(x,y)}{\partial y}\,\frac{\partial \Psi_{st1}}{\partial x} + \frac{\partial \Omega_d(x,y)}{\partial x}\,\frac{\partial \Psi_{st1}}{\partial y} - \frac{2}{\mathrm{Re}}\,\Delta\Delta\Psi_{st1} = 0\,. \tag{52}$$

Because Eqn (52) is nonlinear and the jet expands slowly, the solution can be sought as a sum of traveling waves with a frequency S, a slowly varying complex wave number $Q(S, x)$, and an amplitude $f^{(S)}(\mu x, y)$:

$$\Psi_1(t,x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f^{(S)}(\mu x, y)\exp\left[\mathrm{i}\left(St - \int_0^x Q(S,x)\,\mathrm{d}x\right)\right]\mathrm{d}S\,. \tag{53}$$

Here, μ is one more conventional small parameter reflecting the slowness of jet expansion. It is convenient to set

$$Q(S,x) = \frac{S}{v_{ph}(S,x)} + \mathrm{i}\Gamma(S,x) \tag{54}$$

in Eqn (53), with $v_{ph}(S,x)$ being the phase velocity of the hydrodynamic wave with the frequency S and $\Gamma(S,x)$ the amplification coefficient related to it.

Expanding the functions $f^{(S)}(\mu x, y)$ and $Q(S,x)$ in series in the small parameter μ and substituting these expansions in Eqn (53) and then substituting $\Psi_1(t,x,y)$ in Eqn (52) and keeping only the terms of the first order in μ, we obtain the following equation for the function $f(S, \mu x, y)$:

$$\mathrm{i}\big(S - U_d(x,y)Q\big)\left(\frac{\partial^2 f}{\partial y^2} - Q^2 f\right) + V_d(x,y)\left(\frac{\partial^3 f}{\partial y^3} - Q^2\,\frac{\partial f}{\partial y}\right) + \mathrm{i}Q\,\frac{\partial \Omega_d(x,y)}{\partial y}\,f + \frac{\partial \Omega_d(x,y)}{\partial x}\,\frac{\partial f}{\partial y} - \frac{2}{\mathrm{Re}}\left(\frac{\partial^4 f}{\partial y^4} - 2Q^2\,\frac{\partial^2 f}{\partial y^2} + Q^4 f\right) = 0\,. \tag{55}$$

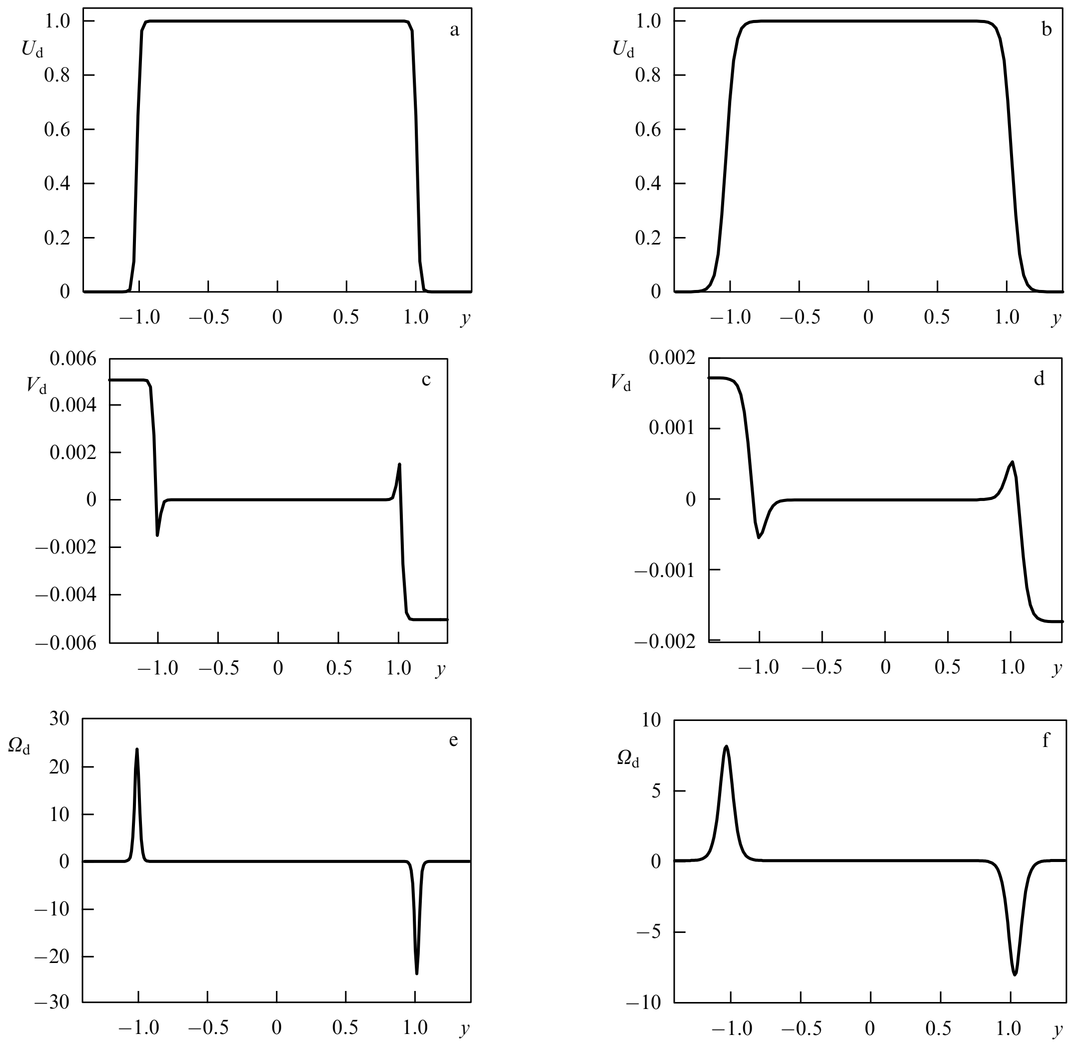


Figure 13. Dependences of $U_d(y)$, $V_d(y)$, and $\Omega_d(y)$ for $x = 0$ and $x = 8$ at $b_0 = 0.1$ and $\mathrm{Re} = 25{,}000$.

Equation (55) with the vanishing boundary conditions for f at $y = \pm\infty$ describes a boundary value problem that is not self-adjoint and in which Q plays the role of a complex eigenvalue. We note that such boundary value problems are only little explored by mathematicians.

The general solution of Eqn (55) can be represented in the form

$$f(x, y) = \sum_{j=1}^{4} C_j(x)\, f_j(x, y)\,,$$

where $C_j(x)$ are arbitrary functions of x. It is known from experiments that in a plane jet, velocity perturbations in the vicinity of the nozzle are even functions of the transverse coordinate y. Consequently, the stream function should be an odd function of y. In this case, only positive values of y can be considered.

Because Eqn (55) has constant coefficients outside the boundary layer, its solutions at $y \leqslant y_1(x)$ and $y \geqslant y_2(x)$, where $y_{1,2}(x)$ are the internal and external boundaries of the boundary layer, can be found analytically. Then they should be matched at the turning point [17] of the longitudinal velocity defined by the function $\partial f(x, y)/\partial y$. The matching condition reduces to the requirement that a certain complex-valued determinant depending on $v_{\mathrm{ph}}(S, x)$ and $\Gamma(S, x)$ is zero. This condition determines eigenvalues of the phase velocity and the related amplification factor.

Prescribing an input perturbation, we can easily compute its transformation with the distance from the nozzle. As a result of such computations, we find that the perturbation spectrum gradually shifts to the low-frequency domain downstream from the nozzle. We emphasize that these results are obtained in the linear approximation without accounting for merger events and feedback through an acoustic wave.

We note that on varying the Reynolds number in wide limits, the eigenvalues of v_{ph} and Γ vary rather weakly. Had the Euler equation been used instead of the Navier–Stokes equation, the variation would be quite noticeable. This is related to the fact that for an arbitrary Reynolds number, odd eigenfunctions of the Navier–Stokes equation represent a linear combination of two particular solutions, the fast and

[17] A value $y = y^*$ of the argument of a complex-valued function is called a turning point if the derivative of the modulus of this function with respect to y changes its sign, i.e., the modulus has an extremum at this point. Turning points can have different orders. For the turning point of the first order, the equation for the modulus of this function should be reducible to the Airy equation in some vicinity of this point.

slow ones, whereas eigenfunctions of the Euler equation are defined by only one solution, which is the slow one. Even the correction to the Euler equation defined by the Reynolds number, found, e.g., in Ref. [90], does not change the essence, because it contributes to only the slow part of the solution. Hence follows our conclusion: The claim contained in numerous papers that turbulent processes in jets can be computed based on the Euler equations is a delusion.

6. Conclusion

A set of myths pertaining to different problems of physics is considered. Having started from discussing myths on the caloric and aether, we subsequently moved to problems of microwave electronics (magnetron), plasma physics (strata), nonlinear dynamics (stochastic resonance), and fluid dynamics (turbulent jets). Turbulent jets are given special emphasis because their theory is a constellation of various myths. To be sure, we have touched only several topics that are closest to our interests; many volumes can be dedicated to discussing myths and reality in physics. The topic 'myths and reality' is multifaceted. Its surprising aspect is a metamorphosis of myths into reality. An example can be furnished by teleportation, a favorite topic of science fiction writers that is frequently mentioned today under the name of quantum information and which includes quantum cryptography and quantum computing [109].

During the preparation of this work, papers [6, 110] appeared, which bear evidence that even the newest physical results are quickly accumulating myths.

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