Jupiter (1963) were conducted. As a result of this work, the astronomical unit was measured with high accuracy, and a new theory of the motion of the inner planets of the Solar System was developed and confirmed by measurements. The radar survey of Venus, carried out in 1983 – 1984 by the aboard integrated radar system of automatic interplanetary probes Venus-15 and Venus-16, was an outstanding world-class achievement; it produced an image of 115 mln km^2 of the northern part of the planet with a resolution of 1 km. An analysis of the unique data allowed the creation and subsequently publication of the first *Atlas of the Venusian Surface* in the history of science (Moscow: MIIGAiK, 1989). Its editor-in-chief was Academician V A Kotel'nikov (1961 – 1989, IRE).

Along with working on scientific problems and teaching, Vladimir Aleksandrovich was doing a great deal of science administration. In 1969 – 1988, V A Kotel'nikov was acting President, Vice President, then First Vice President of the USSR Academy of Sciences (AS), and headed a number of Learned Councils of the USSR AS and then Russian Academy of Sciences, as well as some interdisciplinary scientific and technical councils and commissions; he combined all this with systematic daily work at the IRE. He was doing a great deal of work on organizing and supervising long-term exploratory and fundamental research projects at the Academy, and coordinated the research of numerous organizations in the country that specialized in various fields of modern radio engineering and electronics. By realizing his enormous scientific potential and accumulated experience, possessing his phenomenal capacity for work, and through his innate responsibility for any assignment he was given, he was able to produce results with maximum efficiency.

In 1987, Vladimir Aleksandrovich resigned from the directorship of the IRE and in 1988 from the vice-presidency of the USSR AS; still heading the learned councils and taking part in the life of the Institute, he returned to theoretical work in radiophysics.

At the age of 88 to 89, he published his last papers, which completed the circle of his work in radiophysics (1996 – 1997).

As in his younger days, he worked on these papers without assistance and published them almost on the eve of his 90th birthday. The problem he was solving was the inverse of the one he treated in his earlier publications. In those papers he determined the properties that a signal needs to have for it to be transmitted through a given channel; now he reversed it: how to select the properties of a channel in order to best transmit a given signal. As in his youth, he was again far ahead of his time. These days these results have enjoyed great popularity. Radio electronics in the past prohibited the possibility to change the channel, so one had to shape the signal. Nowadays, the channel can be selected in such a manner that it can transmit the signal in an optimal manner, and on top of that, it 'cleans' the signal too, filtering out the noise that would make it impossible to properly decode the signal at the output. These are essentially adaptive channels. These were his last scientific publications. And to top it all, he turned to quantum mechanics.

Vladimir Aleksandrovich became interested in quantum mechanics already in his youth. His creative path began (in 1927) when radio engineering was coming of age, and he just loved it, and quantum mechanics was starting to blossom and provided the major excitement for the scientific intelligentsia; these people hotly discussed the quantum mechanics papers appearing in journals. No wonder that the wave of interest in this 'mysterious' field took hold of the young Vladimir Kotel'nikov.

He started buying books on quantum mechanics that began to appear in the USSR and browsed through them — there was not enough time to do serious reading. Vladimir Aleksandrovich later remembered that each time he was left with a feeling of dissatisfaction as he felt "unable to comprehend this quantum mechanics to the very bottom." He dreamed of "some day figuring it all out."

At last he "got a bit of free time" and tackled the subject. He did not regard himself as a specialist in the field and tended to look at his new project as a "hobby for an old man."

He began by carefully reading the available books on 'classical' quantum mechanics. He decided to shun all 'alternative trends' in order to avoid undesirable influences; he wanted to see what he could produce himself. His 'square one' was the Schrödinger equation. By the end of 2003 he was ready to discuss the obtained results with specialists, but time ran out for him. V A Kotel'nikov died on February 11, 2005. The 97th year of his life ended with a nearly complete but unpublished work *Model Nonrelativistic Quantum Mechanics*; the drafts were published in 2008.

In this manuscript Vladimir Aleksandrovich presented nonrelativistic quantum mechanics (based on the Schrödinger equation) in terms of classical probability and classical concepts of the existence of trajectory of a particle and a field acting on it (see Appendix). The theory that he developed is an example of so-called theories of hidden parameters on which Luis de Broglie, D Bohm, and some others worked in the 20th century. Vladimir Aleksandrovich was unaware of the results published by these authors. He independently reproduced the entire logic of the theory of hidden parameters, introduced his own terminology and notation, and generated all the basic results of nonrelativistic quantum mechanics in his own terms. By this we mean wave packet spreading, analysis of the two-slit experiment and quantum interference, construction of the theory of stationary states, the theories of the hydrogen atom and oscillator, the theory of nonstationary states and quantum transitions, and the explanation of tunneling effect.

We who worked in the Kotel'nikov Institute of Radioengineering and Electronics, RAS loved and respected Vladimir Aleksandrovich. We consider it our unwavering duty to sustain the creative atmosphere that he built in the Institute, and strive to follow his principles in our work.

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Appendix
Model Nonrelativistic Quantum Mechanics. Considerations*

V A Kotel'nikov

INTRODUCTION

Quantum mechanics considers the motion of very small bodies such as elementary particles. Experiments have

* Below, the Introduction and Chapters 1 and 2 are presented. The full text of the work was published in 2008 (Moscow: Fizmatlit, 2008), 72 pages (in Russian).

shown that this motion does not always obey the laws of classical mechanics. Quantum mechanics has turned out to be more complicated and counter-intuitive than classical mechanics. In 'classical' quantum mechanics, particles have no apparent images. They have no trajectories, cannot have definite positions and velocities simultaneously, etc. The motion of particles is determined by many rules that do not always rigorously follow from the basic laws like, for instance, in the classical mechanics of macroscopic bodies and in electrodynamics.

All this complicates learning and applying quantum mechanics, especially for those who are more inherent in figurative thinking.

In this work, a figurative model of quantum mechanics is proposed, which is in agreement with the accumulated experimental data and makes the quantum mechanics of small bodies more evident and more logically rigorous.

Chapter 1

CONSTRUCTING THE MODEL

1.1 Basic statements of nonrelativistic quantum mechanics

The basic statement of classical mechanics is as follows: the state of a particle with mass m considered as a pointlike body is determined by its position given by the radius vector $\mathbf{r}$ and its velocity $\mathbf{V}$. Knowing these parameters for a certain instant of time t and the external force that acts on the particle, we can find, using Newton's laws, all parameters of the particle motion for any instant of time.

In quantum mechanics, this turns out to be different. As experiments and their analysis have shown, the state of a particle at a given instant of time t cannot be fully described by the values of $\mathbf{r}$ and $\mathbf{V}$.

The basic statement of nonrelativistic quantum mechanics, confirmed by experiments, is as follows: if one does not take into account the spin of a particle, the state of the particle at some instant of time t is fully described by some complex function (the wavefunction) in three-dimensional space:

$$\psi(\mathbf{r}, t) = a(\mathbf{r}, t) \exp\left(\mathrm{i}\beta(\mathbf{r}, t)\right), \tag{1.1}$$

where $a(\mathbf{r}, t)$ and $\beta(\mathbf{r}, t)$ are real. The function $a(\mathbf{r}, t)$ determines the probability that the particle at some instant of time t resides at a certain point in space. For instance, the probability that at the point in time t the particle will be found within a small volume $\mathrm{d}q$ containing the end point of the radius vector $\mathbf{r}$ is given by

$$\mathrm{d}P = a^2(\mathbf{r}, t)\, \mathrm{d}q\,. \tag{1.2}$$

The function $\beta(\mathbf{r}, t)$ determines the dynamical state of the particle.

In the absence of a magnetic field, knowing $\psi(\mathbf{r}, t)$ at the initial instant of time, the mass m of the particle, and the external field forces $\mathbf{F}_\mathrm{o}(\mathbf{r}, t)$ acting on it, one can find $\psi(\mathbf{r}, t)$ for other points in time using the Schrödinger equation. For the nonrelativistic case, which will be the only case considered here, and in the absence of the spin and the magnetic field, it is written as follows:

$$\mathrm{i}\hbar\, \frac{\partial\psi(\mathbf{r}, t)}{\partial t} = -\frac{\hbar^2}{2m}\, \nabla^2\psi(\mathbf{r}, t) + U(\mathbf{r}, t)\, \psi(\mathbf{r}, t)\,, \tag{1.3}$$

where $\hbar = 1.05 \times 10^{-27}$ erg s is the Planck constant, and $U(\mathbf{r}, t)$ is the force function of the field acting on the particle. At the same time, the force acting on the particle equals

$$\mathbf{F}_\mathrm{o} = -\nabla U(\mathbf{r}, t)\,. \tag{1.4}$$

Knowing $\psi(\mathbf{r}, t)$ and m, one can, using the rules of quantum mechanics, find the other parameters of the particle's motion.

One has to use probabilistic parameters here because completely identical experiments on registering small particles never have similar outcomes. The coordinates of the particles are registered within a certain range of results, and one can only talk about the probability that the particle is found at one position or another.

The rule given above is the basic statement of nonrelativistic quantum mechanics.

1.2 Velocity of a particle in model nonrelativistic quantum mechanics

In commonly used quantum mechanics, it is claimed that a particle cannot be at a certain position and simultaneously have a certain velocity. However, quantum mechanics considers the parameters of a particle taken from the different realizations of a process with the same wavefunction.

Let us try to construct a model that would correspond to the above-given basic statement of quantum mechanics and hence to the experimental evidence but at the same time imply a certain trajectory of the particle, as is the case in macroscopic mechanics. While constructing the model, we will consider the position and the velocity of a particle for the same realization, in which position and velocity can exist simultaneously. To this end, let us first find the velocity and the acceleration of the particle if it moves according to the basic statement of quantum mechanics, i.e., it satisfies the Schrödinger equation (1.3).

Suppose that at some instant of time t the particle is at a point with radius vector $\mathbf{r}$ and has velocity $\mathbf{V}(\mathbf{r}, t)$. Let us find the probability that during a time interval $t, t + \mathrm{d}t$ the particle will cross a small area $\mathrm{d}\mathbf{S}$ (see Fig. 1).[1] During the time interval $\mathrm{d}t$, the particle moves by $\mathbf{V}\,\mathrm{d}t$. It will cross the area $\mathrm{d}\mathbf{S}$ if at some instant of time t it was at a distance of $-\lambda\mathbf{V}\,\mathrm{d}t$ $(0 < \lambda < 1)$ from one of the points of this area or, in other words, if at time t it was within a domain of volume $\mathrm{d}q = \mathbf{V}\,\mathrm{d}\mathbf{S}\,\mathrm{d}t$ adjacent to this area. According to formula (1.2), the probability of this event is $\mathrm{d}P_{\mathrm{d}S} = a^2\mathbf{V}\,\mathrm{d}\mathbf{S}\,\mathrm{d}t$.

Thus, $\mathrm{d}P_{\mathrm{d}S}$ is the probability of the particle crossing of the area $\mathrm{d}\mathbf{S}$ during the time interval $t, t + \mathrm{d}t$. For $\mathbf{V}\,\mathrm{d}\mathbf{S} < 0$, the particle will cross the area $\mathrm{d}\mathbf{S}$ in the opposite direction, and in this case $\mathrm{d}P_{\mathrm{d}S}$ will be negative.

Let us choose some volume q bounded by a closed surface S. The probability that the particle will escape from this volume, i.e., will cross the surface S, within the time interval $t, t + \mathrm{d}t$ is, according to the Ostrogradsky – Gauss theorem, given by

$$P_- = \mathrm{d}t \oint_S a^2\mathbf{V}\,\mathrm{d}\mathbf{S} = \mathrm{d}t \int_q \nabla(a^2\mathbf{V})\,\mathrm{d}q\,.$$

The probability that at the point in time t the particle resided within the volume q is, according to formula (1.2),

[1] Unfortunately, figures are absent both in the manuscript and the published work (Moscow: Fizmatlit, 2008). (*Editor's note.*)

equal to

$$P_t = \int_q a^2 \, \mathrm{d}q \, .$$

The probability that the particle will stay within volume q at time $t + \mathrm{d}t$ can be expressed as

$$P_{t+\mathrm{d}t} = \int_q \left(a + \frac{\partial a}{\partial t} \, \mathrm{d}t \right)^2 \mathrm{d}q = \int_q \left(a^2 + 2a \frac{\partial a}{\partial t} \, \mathrm{d}t \right) \mathrm{d}q \, .$$

Here, we omitted the term with $\mathrm{d}t^2$ as an infinitesimal of higher order of magnitude.

Evidently, the event 'the particle is within volume q at some instant of time t' will be necessarily succeeded by either the event 'the particle stays within volume q at some instant of time $t + \mathrm{d}t$' or the event 'the particle leaves domain q within the time interval $t, t + \mathrm{d}t$.' Therefore, one finds

$$P_t = P_{t+\mathrm{d}t} + P_- \, ,$$

or

$$P_{t+\mathrm{d}t} - P_t = -P_- \, .$$

From this equality it follows that

$$\int_q \frac{\partial a^2}{\partial t} \, \mathrm{d}q = -\int_q \nabla(a^2 \mathbf{V}) \, \mathrm{d}q \, ,$$

and, since this equality should be valid for any q, one has

$$\frac{\partial a^2}{\partial t} = -\nabla(a^2 \mathbf{V}) \, . \tag{1.5}$$

Now, let us find the value of $\partial a^2/\partial t$ according to Schrödinger equation (1.3). We substitute $\psi(\mathbf{r}, t)$ from formula (1.1) into Eqn (1.3). Then we arrive at

$$\mathrm{i}\hbar \left(\frac{\partial a}{\partial t} + \mathrm{i}a \frac{\partial \beta}{\partial t} \right) \exp(\mathrm{i}\beta) = -\frac{\hbar^2}{2m} \left[\nabla^2 a + 2\mathrm{i} \nabla a \, \nabla \beta + \mathrm{i}a \nabla^2 \beta - \alpha (\nabla \beta)^2 \right] \exp(\mathrm{i}\beta) + Ua \exp(\mathrm{i}\beta) \, . \tag{1.6a}$$

Cancelling both sides of the above equation by $\hbar \exp(\mathrm{i}\beta)$ and setting the imaginary parts equal, we find that

$$\frac{\partial a}{\partial t} = -\frac{\hbar}{2m} \left[2\nabla a \, \nabla \beta + a \nabla^2 \beta \right] .$$

Further, multiplying both sides by $2a$, after some algebraic transformations we obtain

$$2a \frac{\partial a}{\partial t} = \frac{\partial a^2}{\partial t} = -\frac{\hbar}{2m} \left[4a \nabla a \, \nabla \beta + 2a^2 \nabla^2 \beta \right] , \tag{1.6b}$$

or

$$\frac{\partial a^2}{\partial t} = -\frac{\hbar}{m} \nabla(a^2 \nabla \beta) \, . \tag{1.6}$$

Setting equal the real parts in Eqn (1.6a) and cancelling by $a \exp(\mathrm{i}\beta)$, we find

$$-\hbar \frac{\partial \beta}{\partial t} = \frac{\hbar^2}{2m} \left[-\frac{\nabla^2 a}{a} + (\nabla \beta)^2 \right] + U \, . \tag{1.7}$$

We see that equations (1.5) and (1.6) will coincide if we assume

$$\mathbf{V}(\mathbf{r}, t) = \frac{\hbar}{m} \nabla \beta(\mathbf{r}, t) \, . \tag{1.8}$$

Hence, *if at some instant of time t a particle is at a point with radius vector* $\mathbf{r}$*, its velocity should correspond to equation* (1.8) *in order that the Schrödinger equation and relation* (1.2) *should be satisfied.*

1.3 Forces in model nonrelativistic quantum mechanics

Let us now find the forces that should act on the particle to provide these velocities. For this, let us find the acceleration of the particle from the velocities we obtained. If a particle moves along a certain trajectory, so that one should take into account the variation of $V(\mathbf{r}, t)$ due to both $\mathbf{r}$ and t, its acceleration and velocity are known to be related by the equation (see Appendix 1)

$$\frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t} = \frac{1}{2} \nabla(\mathbf{V}^2) - \mathbf{V} \times (\nabla \times \mathbf{V}) + \frac{\partial \mathbf{V}}{\partial t} \, . \tag{1.9}$$

According to Eqn (1.8), the particle velocity $\mathbf{V}$ is a gradient; therefore, the vector product $\nabla \times \mathbf{V} = 0$ and, hence, one has

$$\frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t} = \frac{1}{2} \nabla(\mathbf{V}^2) + \frac{\partial \mathbf{V}}{\partial t} \, . \tag{1.10}$$

Note that there is a difference between $\mathrm{d}\mathbf{V}/\mathrm{d}t$ and $\partial \mathbf{V}/\partial t$. The derivative $\mathrm{d}\mathbf{V}/\mathrm{d}t$ defines the acceleration of the particle corresponding to its motion along the trajectory, while $\partial \mathbf{V}/\partial t$ is the partial derivative of $\mathbf{V}(\mathbf{r}, t)$ with respect to time, with $\mathbf{r}$ considered constant.

If we assume that the particle motion satisfies Newton's law, then the existence of acceleration requires a force acting on the particle:

$$\mathbf{F} = m \frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t} = \frac{m}{2} (\mathbf{V}^2) + m \frac{\partial \mathbf{V}}{\partial t} \, , \tag{1.11}$$

or, taking into account formula (1.8), we obtain

$$\mathbf{F} = \frac{m}{2} \nabla \left(\frac{\hbar^2}{m^2} (\nabla \beta)^2 \right) + m \frac{\hbar}{m} \frac{\partial \nabla \beta}{\partial t} = \frac{\hbar^2}{2m} \nabla (\nabla \beta)^2 + \hbar \frac{\partial}{\partial t} \nabla \beta \, . \tag{1.12}$$

The expression on the right-hand side of Eqn (1.12) can also be obtained from relation (1.7). Indeed, calculating the gradients of the left-hand and right-hand parts of equality (1.7), we arrive at

$$-\hbar \frac{\partial \nabla \beta}{\partial t} = \frac{\hbar^2}{2m} \left[-\nabla \frac{\nabla^2 a}{a} + \nabla (\nabla \beta)^2 \right] + \nabla U \, ,$$

or

$$\frac{\hbar^2}{2m} \nabla (\nabla \beta)^2 + \hbar \frac{\partial}{\partial t} \nabla \beta = \frac{\hbar^2}{2m} \nabla \frac{\nabla^2 a}{a} - \nabla U \, .$$

Taking this into account, we can rewrite expression (1.12) as

$$\mathbf{F} = \frac{\hbar^2}{2m} \nabla \frac{\nabla^2 a}{a} - \nabla U = \mathbf{F}_\mathrm{q} + \mathbf{F}_\mathrm{o} = m \frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t} \, . \tag{1.13}$$

Here,

$$\mathbf{F}_{\mathrm{o}} = -\nabla U\,, \tag{1.14}$$

according to condition (1.4), is the external field force acting on the particle, and

$$\mathbf{F}_{\mathrm{q}} = \frac{\hbar^2}{2m}\,\nabla\,\frac{\nabla^2 a}{a} \tag{1.15}$$

is an additional force that should act on the particle to provide its motion according to the Schrödinger equation and hence there is an agreement with the experimental results. This force is determined by the modulus of the wavefunction $a(\mathbf{r}, t)$.

1.4 Model of a small particle in model nonrelativistic quantum mechanics

Based on the above considerations, the following model of a small particle is proposed. The model involves two components: a bulk one, the scalar field $a^2(\mathbf{r}, t)$ equal to the modulus of the wavefunction (1.1), and a pointlike one, being the particle moving in this field. The model field will be called the quasifield, the pointlike particle will be called the T-particle, and the combination of the quasifield and the T-particle will be called the quanton.

The dynamics of the quasifield are determined by Schrödinger equation (1.3). The motion of the T-particle obeys Newton's law (1.13), i.e., the T-particle moves as a pointlike particle in classical mechanics under the action of two forces: the classical one, F_{o} (1.14), and the quantum one, F_{q} (1.15). The existence of force F_{q} makes the difference between quantum and classical mechanics.

If the force F_{q} can be neglected in comparison with the external forces F_{o}, the motion is executed according to the rules of classical mechanics. The probability that at some instant of time t the particle will be found within a small volume $\mathrm{d}q$ containing the end of the radius vector $\mathbf{r}$ is given by expression (1.2). In this case, the velocity of the T-particle will be defined by expression (1.8). If exactly the same experiment is performed many times, each time the wavefunction, as well as the quasifield, will have the same form but the T-particle will take up different positions, with the probabilities given by Eqn (1.2). Correspondingly, for the different positions of the T-particle, its velocity will be defined by expression (1.8). Since the position of a T-particle does not enter the Schrödinger equation, the particle has no effect on its quasifield. A T-particle can sets up electromagnetic and other fields and act, through external forces, on other elementary particles.

We have considered the case where the elementary particle has a single wavefunction. This is the so-called pure case. The situation may be more complicated, with the elementary particle having one of several possible wavefunctions $\psi_1(\mathbf{r}, t), \psi_2(\mathbf{r}, t), \ldots, \psi_n(\mathbf{r}, t)$, with the probabilities $P_1, P_2, \ldots, P_n$. This is the so-called mixed case, when the situation should be considered separately for each wavefunction, and the results should be summed up with an account for the probabilities $P_1, P_2, \ldots, P_n$.

Nonrelativistic quantum mechanics can be constructed from the proposed model of an elementary particle, alluding to the fact that the model does not contradict the experiment. The rest, including the Schrödinger equation, can be logically derived from this model.

Chapter 2
QUASIFIELD

2.1 Let us consider the properties of the quasifield in more detail. According to Eqn (1.2), the probability that at some instant of time t the T-particle will be found within some domain q equals

$$P_q(t) = \int_q a^2(\mathbf{r}, t)\,\mathrm{d}q\,, \tag{2.1}$$

where integration is taken over this domain. Let us call $a^2(\mathbf{r}, t)$ the **density** of the quasifield at point $\mathbf{r}$ and some instant of time t, and the integral (2.1) the **amount** of the quasifield within volume q. Then, the following statement will be valid: the probability that a T-particle is found within some domain is equal to the amount of the quasifield in this domain.

The quasifield, which can be viewed as some gas or compressible fluid with the density $a^2(\mathbf{r}, t)$, neither appears nor disappears with time but only moves with the velocity $V(\mathbf{r}, t)$. Under these conditions, the quasifield must satisfy the relation

$$\mathrm{d}t\,\frac{\partial}{\partial t}\int_q a^2\,\mathrm{d}q = -\mathrm{d}t\,\oint_S a^2\mathbf{V}\,\mathrm{d}\mathbf{S}\,. \tag{2.2}$$

The left-hand integral in formula (2.2) is taken over some domain q, while the right-hand one, over the closed surface S surrounding it. During a time $\mathrm{d}t$, the amount of quasifield in domain q will be reduced by the value of the left-hand side of expression (2.2). This reduction will be only due to the field escaping through the surface S. The amount of the quasifield escaping through an element $\mathrm{d}\mathbf{S}$ of the surface during a time $\mathrm{d}t$ will be equal to $a^2\mathbf{V}\,\mathrm{d}\mathbf{S}\,\mathrm{d}t$, while the amount escaping through the whole surface will be equal to the right-hand side of equation (2.2).

2.2 Let us now find the velocity $V(\mathbf{r}, t)$ of the quasifield required in order to satisfy both relation (2.2) and the condition that the quasifield density $a^2(\mathbf{r}, t)$ correspond to the wavefunction $\psi(\mathbf{r}, t) = a(\mathbf{r}, t)\exp\big(\mathrm{i}\beta(\mathbf{r}, t)\big)$, which is the solution to the Schrödinger equation. For this, let us utilize the theorem[2] from Appendix 2, assuming in (A2.3) that

$$\psi_1 = \psi_2 = a\exp(\mathrm{i}\beta)\,.$$

Then, taking into account that $\nabla\psi_{1,2} = (\nabla a)\exp(\mathrm{i}\beta) + \mathrm{i}(\nabla\beta)a\exp(\mathrm{i}\beta)$, we obtain

$$\begin{aligned}\frac{\partial}{\partial t}\int_q a\exp(\mathrm{i}\beta)a\exp(-\mathrm{i}\beta)\,\mathrm{d}q \\ = \mathrm{i}\,\frac{\hbar}{2m}\oint_S \Big\{a\exp(-\mathrm{i}\beta)\big[(\nabla a)\exp(\mathrm{i}\beta) + \mathrm{i}(\nabla\beta)a\exp(\mathrm{i}\beta)\big] \\ - a\exp(\mathrm{i}\beta)\big[(\nabla a)\exp(-\mathrm{i}\beta) - \mathrm{i}(\nabla\beta)a\exp(-\mathrm{i}\beta)\big]\Big\}\,\mathrm{d}\mathbf{S}\,.\end{aligned}$$

[2] This theorem states: "*If $\psi_1(\mathbf{r}, t)$ and $\psi_2(\mathbf{r}, t)$ vary in time according to the same Schrödinger equation, namely*

$$\mathrm{i}\hbar\,\frac{\partial}{\partial t}\,\psi_1 = -\frac{\hbar^2}{2m}\,\nabla^2\psi_1 + U\psi_1\,, \tag{A2.1}$$

$$\mathrm{i}\hbar\,\frac{\partial}{\partial t}\,\psi_2 = -\frac{\hbar^2}{2m}\,\nabla^2\psi_2 + U\psi_2\,, \tag{A2.2}$$

then

$$\frac{\partial}{\partial t}\int_q \psi_1^*\psi_2\,\mathrm{d}q = \mathrm{i}\,\frac{\hbar}{2m}\int_S\big(\psi_1^*\,\nabla\psi_2 - \psi_2\,\nabla\psi_1^*\big)\,\mathrm{d}\mathbf{S}\,. \tag{A2.3}$$"

(*Editor's note.*)

After simplifying this expression, we get

$$\frac{\partial}{\partial t}\int_q a^2\,\mathrm{d}q = -\frac{\hbar}{m}\oint_S (\nabla\beta)\,a^2\,\mathrm{d}\mathbf{S}\,.$$

Comparing this expression with Eqn (2.2), we see that they will coincide if the quasifield velocity is assumed to be

$$\mathbf{V}(r,t) = \frac{\hbar}{m}\,\nabla\beta(r,t)\,. \tag{2.3}$$

Thus, we can assume in the model that *the quasifield cannot disappear or appear but can only move with the velocity given by expression* (2.3).

If the integration domain is assumed to be the whole space occupied by the quasifield, where $a \neq 0$, then the integral of the right-hand side of expression (2.2) will be equal to zero, since in this case $a = 0$ on the surface S.

Thus, the full amount of the quasifield of an elementary particle is always constant. This amount will be equal to the probability that the T-particle is found somewhere in the quasifield, and this probability equals unity. Hence, it follows that *the full 'amount' of the quasifield of an elementary particle is always equal to unity, i.e., the integral* (2.1) *taken over the whole field should be equal to unity*:

$$\int_Q a^2(\mathbf{r},t)\,\mathrm{d}q = 1\,. \tag{2.4}$$

Hereafter, the subscript Q of an integral means that integration is performed over the whole space where $a^2 \neq 0$.

2.3 Comparing expressions (1.8) and (2.3) we see that *the velocity of a* T-*particle is equal to the velocity of travel of the quasifield at the point where the* T-*particle is placed, i.e., the particle is entrained by the quasifield and moves together with it*.

Since the velocities of the quasifield and the T-particle are equal, their accelerations should also be equal. Therefore, an element of the field moving along some trajectory will have acceleration, according to formula (1.13), equal to

$$\frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t} = \frac{\hbar^2}{2m^2}\,\nabla\,\frac{\nabla^2 a}{a} - \frac{1}{m}\,\nabla U_0\,. \tag{2.5}$$

The second term in the right-hand side of this equation is determined by external forces, while the first term, by the forces of the field itself. It depends on the field density and its derivatives at the point where the T-particle is placed, namely, on the parameter $\nabla^2 a/a$, which will often occur further. Therefore, let us consider it in more detail.

2.4 Let the origin of the coordinate system be put at our point of interest. Let us represent a as a Taylor series and limit the domain of consideration, so that the quadratic terms are sufficient. We will get

$$a(x,y,z) = \frac{1}{2}\frac{\partial^2 a}{\partial x^2}\,x^2 + \frac{1}{2}\frac{\partial^2 a}{\partial y^2}\,y^2 + \frac{1}{2}\frac{\partial^2 a}{\partial z^2}\,z^2 + \frac{\partial^2 a}{\partial x\,\partial y}\,xy + \frac{\partial^2 a}{\partial y\,\partial z}\,yz + \frac{\partial^2 a}{\partial z\,\partial x}\,zx + \frac{\partial a}{\partial x}\,x + \frac{\partial a}{\partial y}\,y + \frac{\partial a}{\partial z}\,z + a\,,$$

where the derivatives and the a function are taken at point $(0,0,0)$. Let us find the average value of a at a distance δ from the point $(0,0,0)$, understanding it as

$$\langle a_\delta\rangle = \frac{1}{6}\big[a(\delta,0,0) + a(-\delta,0,0) + a(0,\delta,0) + a(0,-\delta,0) + a(0,0,\delta) + a(0,0,-\delta)\big]\,.$$

Then we arrive at

$$a(\delta,0,0) + a(-\delta,0,0) = \frac{1}{2}\frac{\partial^2 a}{\partial x^2}\,\delta^2 + \frac{\partial a}{\partial x}\,\delta + a + \frac{1}{2}\frac{\partial^2 a}{\partial x^2}\,\delta^2 - \frac{\partial a}{\partial x}\,\delta + a = \frac{\partial^2 a}{\partial x^2}\,\delta^2 + 2a\,.$$

Expressions for $a(0,\delta,0) + a(0,-\delta,0)$ and $a(0,0,\delta) + a(0,0,-\delta)$ are obtained similarly.

Substituting these expressions into $\langle a_\delta\rangle$, we find

$$\langle a_\delta\rangle = \frac{1}{6}\left(\frac{\partial^2 a}{\partial x^2} + \frac{\partial^2 a}{\partial y^2} + \frac{\partial^2 a}{\partial z^2}\right)\delta^2 + a = \frac{1}{6}\,\delta^2\nabla^2 a + a\,.$$

Hence follows

$$\frac{\nabla^2 a}{a} = 6\,\frac{\langle a_\delta\rangle - a}{a\delta^2}\,. \tag{2.6}$$

Therefore, the quantity given by formula (2.6) shows how much the field at the center is weaker than the field in the nearest neighborhood. This quantity will be further called the **rarefaction** of the quasifield.

Note that the rarefaction is independent of the field intensity. It is also independent of the rotation of coordinate axes, since, as we know, $\nabla^2 a$ is independent of it.

In Eqn (2.5), the first term in the expression for the acceleration of the quasifield and the T-particle is directed along the rarefaction gradient of the quasifield towards larger rarefactions and is proportional to the gradient. Therefore, the quasifield will tend to move in such a way that the rarefaction will be reduced and spread uniformly over space.

Since the velocities of travel of the quasifield elements are determined, according to Eqn (1.8), by a gradient of the scalar β, then $\operatorname{rot}\mathbf{V} = 0$, i.e., the quasifield cannot have any vortices.

2.5 As we have already mentioned, the state of an elementary particle is fully determined by its wavefunction (1.1). This wavefunction also fully determines the parameters of the quasifield, such as its density a^2 and velocity of travel $\mathbf{V} = (\hbar/m)\nabla\beta$. However, the inverse is not true: it is impossible to fully determine the wavefunction knowing only the density and the velocity of the quasifield. Indeed, in this case we will only know, according to Eqn (1.8), the modulus of the wavefunction and the gradient of its argument, i.e., the derivatives of the argument with respect to the coordinates. Moreover, one can add to the argument an arbitrary function of time, which is independent of the coordinates, and the gradient will not change.

In order to define β, one should also know the derivative $\partial\beta/\partial t$. It can be found from equation (1.7), provided that U is known, since the wavefunction should satisfy the Schrödinger equation. Then we obtain

$$\beta(\mathbf{r},t) = \beta(\mathbf{r}_0,t_0) + \int_{t_0}^{t}\frac{\partial\beta(\mathbf{r}_0,t)}{\partial t}\,\mathrm{d}t + \int_{\mathbf{r}_0}^{\mathbf{r}}\nabla\beta(\mathbf{r},t)\,\mathrm{d}\mathbf{r}\,. \tag{2.7}$$

Thus, the quasifield, in combination with U, determines the wavefunction up to a constant $\beta(\mathbf{r}_0,t_0)$, which has no effect on the state of the quanton.

Similarly to the way in which not every complex function can be a wavefunction, since it has to satisfy the Schrödinger equation, not every field $a^2(\mathbf{r}, t)$ and $\mathbf{V}(\mathbf{r}, t)$ can represent a quasifield: they have to correspond to some wavefunction.

2.6 If we substitute the wavefunction $\psi(\mathbf{r}, t) = a(\mathbf{r}, t) \exp\big(\mathrm{i}\beta(\mathbf{r}, t)\big)$ into Schrödinger equation (1.3) and take into account relation (2.3) for the quasifield velocity, then expressions (1.6) and (1.7) for the real and imaginary parts of the Schrödinger equation acquire a simple physical meaning. Indeed, with account for Eqn (2.3), relation (1.6) becomes

$$\frac{\partial a^2}{\partial t} = -\nabla(a^2\mathbf{V}) \,, \tag{2.8}$$

and relation (1.7), if one takes the gradients of both its sides, changes to

$$-m\,\frac{\partial \mathbf{V}}{\partial t} = -\frac{\hbar^2}{2m}\,\nabla\,\frac{\nabla^2 a}{a} + \frac{m}{2}\,\nabla V^2 + \nabla U \,. \tag{2.9}$$

Equation (2.8) is equivalent to equation (2.2) and indicates that the quasifield cannot appear or disappear but can only be displaced. Equation (2.9), taking into account Eqns (1.10) and (2.3), will be equivalent to Eqn (2.5), i.e., to the statement that acceleration of the quasifield elements is equal to the sum of the forces, the external one and the quasifield one, divided by the particle mass. Thus, the Schrödinger equation for the quasifield corresponds to the gas dynamics equation, the only difference being that the force of the quasifield self-action, denoted here by $\mathbf{F}_{\mathrm{q}}$, is essentially different from the analogous force in gas dynamics.

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Vladimir Aleksandrovich Kotel'nikov and Solar System studies

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1. Introduction

In biographically portraying the work of Academician Vladimir Aleksandrovich Kotel'nikov, one usually describes his outstanding work devoted to the fundamentals of the communication theory (the famous sampling theorem, the potential noise immunity theory, theorems in cryptography theory). His role in Soviet space programs is described to a lesser extent. Meanwhile, the contribution of Vladimir Aleksandrovich and colleagues from the organizations founded and headed by him [the Special Design Bureau of Moscow Power Engineering Institute (OKB MEI in *Russ. abbr.*) and the Institute of Radioengineering and Electronics of the Russian Academy of Sciences (IRE RAS)] to this field is also very significant. Another side of his 'space' activity is related to his positions as the Vice President of the USSR Academy of Sciences and the Head of the Intercosmos Council.

In this short report we shall briefly describe the main stages of V A Kotel'nikov's activity, who was an outstanding scientist, engineer, politician, and scientific manager.

2. V A Kotel'nikov and space radar

The development of space radar was motivated by quite practical needs. In the 1960s, the state of space facilities in the USSR and USA allowed scientists and engineers to plan scientific space missions in order to explore nearby planets: Venus and Mars. To ensure the approach of spacecraft to these planets at a distance of several hundred kilometers, one needed to know their position relative to Earth with a good accuracy. Previous astronomical observations of Solar System's bodies located them precisely only relative to each other, while the absolute values of the mutual distances had been known very crudely from the space navigation point of view, which required high accuracy to handle the spacecraft.

All distances between planets are conveniently expressed through the astronomical unit (a.u.), which is equal to the mean distance from Earth to the Sun and is estimated to be around 150 mln km. Astronomical observations had determined this value to an accuracy of about 10,000 km. This means that the distance, for example, to Venus had been known to an accuracy of several thousand kilometers. Clearly, this accuracy could not be considered as satisfactory.

Radio ranging provided the possibility of measuring the distance between Earth and a nearby planet with the required accuracy. To measure the distance with a one-kilometer accuracy, it is sufficient to send radio pulses with a duration of approximately 6 μs. The question is how powerful these pulses should be for the reflected signal to exceed the noise level in a ground-based detector. Considering that in radio ranging 'the inverse fourth power distance law' operates and interplanetary distances are at best several dozen million kilometers, it is easy to understand that antennas with an area of several thousand square meters and transmitters with a power of several dozen kilowatts are required for successful radio ranging of planets. This was very expensive and accessible only for countries with highly developed industry. So it was quite natural that planetary radar started developing in the USA, the USSR and partially in the UK.

At that time, the Remote Space Communication Center (RSCC) was constructed near the city of Eupatoria (Crimea) in the USSR. The center was designed for communications primarily with spacecraft to be sent to Venus and Mars. For this purpose, three ADU-1000 antennas (Fig. 1) were constructed: one for signal transmission, and the other two for signal reception. The radio transmitter with a power of about 10 kW operated at a wavelength of 40 cm. These characteristics fit planetary radar requirements, so the RSCC was chosen to perform the experiment.

Advances in radar facilities (increase in transmitter power and detector sensitivity, development of digital frequency-linear signal modulation, etc.) allowed a very precise measurement of the astronomical unit: $1\ \mathrm{AU} = 149{,}597{,}867 \pm 0.9$ km. Such an accuracy required knowing very precisely the speed of light, since in radio ranging one directly measures the time of radio pulse propagation, and the distance between space bodies is obtained by multiplying the delay time by the speed of light. For this reason, the XVIth General Assembly of the International Astronomical Union (1967), by analyzing the results of experiments carried out in the USSR and USA, adopted the value of $1\ \mathrm{AU} = 149{,}597{,}870 \pm 2$ km for the assumed speed of light $c = 299{,}792{,}558 \pm 1.2\ \mathrm{m\ s^{-1}}$. Such a high accuracy in determining the astronomical unit has provided successful flights of spacecrafts for planetary studies and exploration of the interplanetary space in the Solar System. Moreover, such an