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Pauli paramagnetism and Landau diamagnetism

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Abstract. **The Landau diamagnetism of conduction electrons at sufficiently high temperatures (larger than the Landau level spacing but smaller than the Fermi energy) is considered using the analogy with Pauli paramagnetism. An expression obtained for the diamagnetic susceptibility is identical to the well-known Landau result. While the model describing free particles assumes a quadratic dispersion law, the difference between the effective mass of a carrier and electron mass is taken into account.**

1. Pauli paramagnetism

A magnetic field H brings about an energy level shift of states with different spin projections:

$$\pm \frac{\Delta}{2} = \pm \mu_{\mathrm{B}} H, \qquad \mu_{\mathrm{B}} = \frac{|e|\hbar}{2m_0 c},$$

where μ_{B} is the Bohr magneton; in a solid, a carrier may possess a different magnetic moment. Figure 1 shows the momentum dependence of energy and how the applied magnetic field affects the positions of the energy levels. In this figure, E_{F} is the Fermi energy (it does not change with magnetic field provided the density of states is constant), the dashed line is the original momentum dependence of energy, the solid lines on the left and right correspond to shifted energies for various magnetic momentum directions, and the horizontal dotted lines indicate the energies corresponding to the former value of the Fermi momentum: these energies are shifted by $\pm\Delta/2$ relative to E_{F}.

In the ground state, particles with energies higher than E_{F} (on the left in the figure) move to free states with energies of less than E_{F} (on the right). The number per unit volume of

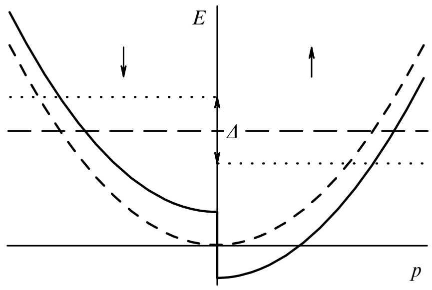

Figure 1. Spin-related energy level shifts in a magnetic field.

such particles is $\gamma\Delta/2$ (γ is the Fermi surface density of states per spin projection), and the corresponding energy decrease per particle amounts to $\Delta/2$, so that the change in the energy of the system due to the application of the magnetic field is given by

$$\frac{E(H) - E(0)}{V} = -\gamma\left(\frac{\Delta}{2}\right)^2, \qquad \Delta = 2\mu_{\mathrm{B}} H, \qquad \gamma = \frac{m p_{\mathrm{F}}}{2\pi^2\hbar^3}. \quad (1)$$

Here, m is the effective mass of the carrier. The magnetization (magnetic moment per unit volume) M is calculated in the usual way to give

$$M = -\frac{\partial}{\partial H}\frac{E(H) - E(0)}{V} = \chi_{\mathrm{P}} H,$$

where χ_{P}, the spin magnetic susceptibility, is expressed as

$$\chi_{\mathrm{P}} = \frac{\gamma\Delta^2}{2H^2} \to \frac{1}{(2\pi)^2}\left(\frac{e^2}{\hbar c}\right)\frac{v_{\mathrm{F}}}{c}\left(\frac{m}{m_0}\right)^2 \quad (2)$$

with $v_{\mathrm{F}} = p_{\mathrm{F}}/m$ being the Fermi velocity. It should be noted that although the above calculations were performed for zero temperature, the result is valid for finite temperatures as well, provided they are small compared to the Fermi energy. To see this, note the following. The Fermi surface density of states (as well as the position of the Fermi level) is unchanged by the application of a magnetic field (as long as the field is weak); hence, the temperature-dependent part of the thermodynamic potential is found to be the same as without the field. The only change is left in the energy of states in the magnetic field.

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2. Landau diamagnetism

The general — and commonly used — way of calculating the magnetic susceptibility is to first find one of the thermodynamic potentials [for example, the free energy $F(T, V, H)$] and then to apply the well-known thermodynamic relation

$$\mathrm{d}F = -S\,\mathrm{d}T - P\,\mathrm{d}V - \tilde{M}\,\mathrm{d}H$$

($\tilde{M} = MV$ is the total magnetic moment) to find the magnetic moment. However, in the limit $T \gg \hbar\omega_H$, where $\hbar\omega_H$ is the Landau level spacing, there is a very simple alternative way to obtain the required expression.

In considering spin paramagnetism, we first determined energy level shifts due to the inherent magnetic moment, and then looked at what energy decrease will result from this in equilibrium (due to particle energy redistribution). Our analysis below proceeds in a similar way.

In a magnetic field, motion transverse to the field is quantized [1], giving rise to the Landau levels

$$\epsilon_n = \hbar\omega_H\left(n + \frac{1}{2}\right). \qquad (3)$$

Now suppose the magnetic field is $H/2$, half of that before, and look at Fig. 2 with Landau levels shown both for H (right) and for $H/2$ (left). It is seen that the Landau levels doubled in number for the latter case, so that each of the levels (3) is in a sense split into two. For example, instead of $\epsilon_0 = \hbar\omega_H/2$, there are two levels $\epsilon_0/2$ and $3\epsilon_0/2$ or, more generally, instead of the level ϵ_n there appear levels $\epsilon_n \pm \epsilon_0/2$. The doubling of levels corresponds to the fact that in a field $H/2$ the number of states at a Landau level is half that in a field H. The splitting of the initial level is $\delta = \epsilon_0$. As a result, we arrive at the same picture as in the previous case: half of the states shift upwards, and the other half downwards. The diagram in Fig. 1 relates to each one of the Landau levels (with the replacement $\Delta \to \delta$; the energy depends on the field-direction component of the momentum), with the shifted curves corresponding to $H/2$. The fact that the moment is one-dimensional should not cause any confusion, because for temperatures $T \gg \hbar\omega_H$ the total density of states remains the same as in the absence of a magnetic field.

Using Eqn (1), we obtain the following equation for the energy difference upon the establishment of equilibrium:

$$\frac{E_0(H/2) - E_0(H)}{V} = -\gamma\left(\frac{\delta}{2}\right)^2, \qquad \delta = \epsilon_0 = \frac{\hbar\omega_H}{2}. \qquad (4)$$

The required energy difference $E_0(H) - E_0(0)$ can be written out as

$$E_0(H) - E_0(0) = \left(E_0(H) - E_0\left(\frac{H}{2}\right)\right) + \left(E_0\left(\frac{H}{2}\right) - E_0\left(\frac{H}{4}\right)\right) + \left(E_0\left(\frac{H}{4}\right) - E_0\left(\frac{H}{8}\right)\right) + \ldots.$$

Employing expression (4) for various values of the magnetic field we obtain

$$\frac{E_0(H) - E_0(0)}{V} = \gamma\left(\frac{\delta}{2}\right)^2\left\{1 + \frac{1}{4} + \frac{1}{16} + \ldots\right\} = \gamma\left(\frac{\delta}{2}\right)^2\frac{4}{3}. \qquad (5)$$

Notice here that this (orbital) energy increases — a fact which corresponds to the phenomenon of diamagnetism. Then diamagnetic susceptibility χ_L is written out in accordance with Eqn (4) as

$$\chi_\mathrm{L} = -\frac{4}{3}\frac{\gamma\delta^2}{2H^2} \to -\frac{1}{3}\frac{1}{(2\pi)^2}\left(\frac{e^2}{\hbar c}\right)\frac{v_\mathrm{F}}{c}. \qquad (6)$$

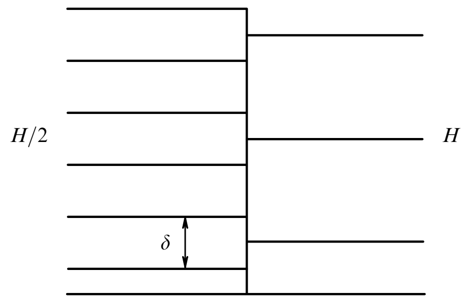

Figure 2. Landau levels in a magnetic field.

This result agrees exactly with Landau's formula [1]. For free electrons ($m = m_0$) this susceptibility is three times smaller than and opposite in sign to spin susceptibility. In a lattice, however, this relation may be different — if for no other reason than because of the difference between the effective mass of a carrier and free electron mass.

It should be emphasized that Landau quantization results in a set of discrete levels in the plane normal to the magnetic field, whereas for spins (for either projection) the spectrum is continuous. Therefore, both our analogy with the spin case and the conclusions we came to are valid only for sufficiently high temperatures ($T \gg \hbar\omega_H$), for which this difference is of no significance.

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Reference

1. Landau L D *Sobranie Trudov* (Collected Works) Vol. 1 (Ed. E M Lifshitz) (Moscow: Nauka, 1969) p. 47; *Z. Phys.* **64** 629 (1930)