Optical properties of graphene and IV-VI semiconductors

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Abstract. The frequency dispersion of the dynamic conductivity of graphene, of a multilayer graphene, and of IV-VI semiconductors is considered as a function of the temperature and carrier density in the range of frequencies that are higher than the carrier relaxation rate but are lower than the conduction band width. A narrow gap and the linearity of the electron spectrum, which are common features of these materials, are responsible for a singularity of the dielectric function (logarithmic in the real part and step-like in the imaginary part) at the threshold of direct interband transitions and, accordingly, for an anomalously large permittivity in IV-VI semiconductors. The calculated and measured dielectric functions are in a very good agreement. The graphene transmittance in the optical range is frequency-independent and its departure from unity vields the value of the fine structure constant. The difference in dimensionality, which is equal to three for semiconductors and to two for graphene, manifests itself in the different character of plasmons and of electromagnetic waves existing for high doping (or in conditions of the field effect) near the absorption threshold.

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1. Introduction

The boom raised by graphene (a monoatomic graphite layer; see reviews [1-4]) during the last two to three years suggests also considering its three-dimensional analogue: IV-VI semiconductors, which have a narrow gap $\epsilon_g \sim 0.1~eV$ between the conduction band and the valence band. (In graphene, this gap is even narrower, however.) Because the conduction band and the valence band are of opposite parity here, the excitation spectrum is linear in a broad energy interval, $\varepsilon_g \ll \varepsilon \ll \varepsilon_{at}$, where $\varepsilon_{at} \sim 5$ eV is on the atomic scale. This leads to interesting features in different responses. In particular, discussed and observed earlier were the singularities of the magnetic susceptibility in PbTe [5, 6] as well as of the permittivity [7] and magnetic susceptibility [8] in bismuth, whose spectrum also exhibits a narrow gap. Of prime interest is a comparison of the permittivities in graphene and narrowband semiconductors, whose permittivity, in contrast to the graphene permittivity,¹ has been much studied experimentally [9, 10]. This comparison allows verifying the validity of the underlying approach to graphene as a system of 'massless Dirac fermions.'

In this paper, we discuss the permittivity features of graphene and IV-VI semiconductors. The zero-temperature permittivity of narrow-gap semiconductors was previously calculated in Ref. [11] using the isotropic model. For IV-VI semiconductors, it was considered in Ref. [12] in the framework of the electronic spectrum theory, where the difference of potentials of the atoms of lead and, for instance, Te (or Se or S) was taken into account by means of the ionicity parameter. In that case, the narrow gap did not enter the final result at all. Computer calculations of the permittivity (see Refs [13, 14]) were performed as follows. First, ab initio calculations of the band structure of these semiconductors

¹ After this manuscript was submitted for publication, an observation of the dynamic conductivity of graphene in the optical range was reported [27].

were carried out. The Fermi golden rule was then used to calculate the imaginary part of the permittivity as

$$\epsilon''(\omega) \sim \int |d_{\rm vc}|^2 \delta \big[\varepsilon_{\rm c}(\mathbf{p}) - \varepsilon_{\rm v}(\mathbf{p}) - \hbar \omega \big] \, \frac{2 {\rm d}^3 p}{(2\pi)^3} \,, \tag{1}$$

which involves the dipole matrix element d_{vc} ; next, the real part was evaluated using the Kramers–Kronig relation. These calculations leave the physical features of the permittivity obscure; furthermore, they are rather involved because the integral for the real part contains singularities. We show in what follows that simple analytic results for the frequencies corresponding to the infrared and visible parts of the spectrum can be obtained using the linearity of the electron spectrum and the experimentally measured values of the gap and the interband-momentum matrix element. The frequencies under consideration are bounded above by a value of the order of 1 eV, because the linear approximation of the electron spectrum applies only to this domain.

2. Electron excitations with a linear spectrum

Figure 1 shows the typical electron spectrum of a IV-VI semiconductor for the principal directions in the Brillouin zone. In the vicinity of point L, two nearest-neighbor bands can be found using the effective Hamiltonian [15]

$$H = \begin{pmatrix} \varepsilon_{g} & H_{1} \\ H_{1}^{+} & -\varepsilon_{g} \end{pmatrix}, \quad H_{1} = \begin{pmatrix} v_{\ell} p_{z} & v_{t} p_{-} \\ v_{t} p_{+} & -v_{\ell} p_{z} \end{pmatrix}, \quad (2)$$

where $p_{\pm} = p_x \pm i p_y$, and v_t and v_{ℓ} are the interbandmomentum matrix elements, assumed to be constants. The values of these constants are almost unchanged in passing from one IV-VI semiconductor to another [16, 17]:

$$v_{\ell} = 3.8 \times 10^7 \text{ cm s}^{-1}, \quad v_{t} = 5.1 \times 10^7 \text{ cm s}^{-1}.$$
 (3)

Terms quadratic in p_j can be written in the main diagonal of H in (2), but we do not do this because the contribution of these terms to the permittivity is small in the parameter $\varepsilon_g/\varepsilon_{at}$.

The Hamiltonian eigenvalues

$$\varepsilon_{1,2}(\mathbf{p}) = \pm \left[\varepsilon_{g}^{2} + v_{\ell}^{2} p_{z}^{2} + v_{t}^{2} p_{\perp}^{2} \right]^{1/2}$$
(4)

are doubly degenerate in spin. We need the matrix elements of the velocity $\mathbf{v} = \partial H / \partial \mathbf{p}$ in the representation where the Hamiltonian is diagonal. It is easy to find the corresponding unitary transformation

$$U = \begin{pmatrix} \frac{k_z}{n_1} & \frac{k_-}{n_1} & \frac{k_z}{n_2} & \frac{k_-}{n_2} \\ \frac{k_+}{n_1} & -\frac{k_z}{n_1} & \frac{k_+}{n_2} & -\frac{k_z}{n_2} \\ a_1 & 0 & -a_2 & 0 \\ 0 & a_1 & 0 & -a_2 \end{pmatrix},$$

where $k_z = v_\ell p_z$, $k_{\pm} = v_t p_{\pm}$, $n_{1,2} = \sqrt{2\varepsilon_1(\varepsilon_1 \mp \varepsilon_g)}$, $a_{1,2} = \sqrt{(\varepsilon_1 \mp \varepsilon_g)/2\varepsilon_1}$, and the velocity matrix in this representations

$$U^{-1}\mathbf{v}U = \begin{pmatrix} \mathbf{v}_{11} & 0 & \mathbf{v}_{13} & \mathbf{v}_{14} \\ 0 & \mathbf{v}_{11} & -\mathbf{v}_{14}^* & \mathbf{v}_{13}^* \\ \mathbf{v}_{13}^* & -\mathbf{v}_{14} & -\mathbf{v}_{11} & 0 \\ \mathbf{v}_{14}^* & \mathbf{v}_{13} & 0 & -\mathbf{v}_{11} \end{pmatrix},$$



Figure 1. Electron spectrum of PbS [14]. GGA is the generalized gradient approximation.

where

$$\mathbf{v}_{11} = \frac{\partial \varepsilon_1}{\partial \mathbf{p}} ,$$

$$\mathbf{v}_{13} = -\frac{2\{\varepsilon_g [v_\ell^2 p_z \mathbf{e}_z + v_t^2 (p_x \mathbf{e}_x + p_y \mathbf{e}_y)] + i\varepsilon_g v_t^2 (p_x \mathbf{e}_y - p_y \mathbf{e}_x)\}}{n_1 n_2} ,$$

$$\mathbf{v}_{14} = \frac{2v_\ell v_t \varepsilon_1 (p_- \mathbf{e}_z - p_z \mathbf{e}_-)}{n_1 n_2} ,$$
(5)

and \mathbf{e}_j are the unit vectors aligned with the coordinate axes of point L.

The electron spectrum of graphene [18] is shown in Fig. 2. In graphene, the gap has the spin-orbit origin and does not exceed 0.1 K. We neglect it from the very beginning and consider two bands degenerate in spin, described by the Hamiltonian

$$H(\mathbf{p}) = v_0 \begin{pmatrix} 0 & \mathrm{i}p_{\pm} \\ -\mathrm{i}p_{\mp} & 0 \end{pmatrix}$$
(6)

in the neighborhood of K points of the Brillouin zone. The Hamiltonian eigenvalues are $\varepsilon_{1,2} = \pm v_0 p$ and the velocity matrix in the representation diagonalizing the Hamiltonian is given by

$$\mathbf{v} = \frac{v_0}{p} \begin{pmatrix} \mathbf{e}_x p_x + \mathbf{e}_y p_y & \mathbf{i} (\mathbf{e}_x p_y - \mathbf{e}_y p_x) \\ -\mathbf{i} (\mathbf{e}_x p_y - \mathbf{e}_y p_x) & -\mathbf{e}_x p_x - \mathbf{e}_y p_y \end{pmatrix},$$
(7)

where $p = (p_x^2 + p_y^2)^{1/2}$. The value of v_0 has been measured: $v_0 = 10^8 \text{ cm s}^{-1}$.





We note that the form of Hamiltonians (2) and (6) is imposed by the C_{3v} symmetry of points L and K and is therefore unrelated to any model assumptions.

3. General expression for conductivity dispersion in a band metal

The general expression for the electron conductivity in a metal can be obtained by writing the current operator

$$j_i(x) = e\tilde{\psi}^+(x')v_{x'x}^i\tilde{\psi}(x) - \frac{e^2}{c}\,\tilde{\psi}^+(x')(m^{-1})_{x'x}^{ij}\,\tilde{\psi}(x)A_j\,,\ (8)$$

where $x' \to x$, and the velocity $v_{x'x}^i$ and effective mass $(m^{-1})_{x'x}^{ij}$ operators emerge in expanding the Hamiltonian in terms of the vector potential A_j of the applied electric field. The tilde in the notation for the operator $\tilde{\psi}$ signifies that it should be taken in the interaction representation with the perturbation

$$V = -\frac{e}{c} \int \psi^{+}(x') v^{i}_{x'x} \psi(x) A_{i}(x) \,\mathrm{d}^{d+1}x \,, \tag{9}$$

where the dimensionality d of integration is equal to 2 for graphene and to 3 for IV – VI semiconductors. By expanding the first term in expression (8) in terms of interaction (9), we standardly obtain the retarded correlator of four ψ -operators. At finite temperatures, the Fourier component of this correlator (in the coordinate difference and imaginary time variables)

$$\mathcal{P}(\omega_l, \mathbf{k}) = T \sum_{\mathbf{p}\omega_n} \operatorname{Tr} \left\{ v^i \mathcal{G}(p_+) v^j \mathcal{G}(p_-) \right\}$$

is expressed in terms of the temperature Green's function

$$\mathcal{G}(p) = \left[\mathrm{i}\omega_n - H(\mathbf{p})\right]^{-1},$$

where we use the notation $p_{\pm} = (\omega_n \pm \omega_l/2, \mathbf{p} \pm \mathbf{k}/2)$, the summation is performed over the fermionic frequencies $\omega_n = 2\pi T(n + 1/2)$, and the trace is taken over the Hamiltonian band index. For instance, for graphene, this trace is given by

$$\operatorname{Tr} \left\{ v^{i} \mathcal{G} v^{j} \mathcal{G} \right\} = v_{11}^{i} \mathcal{G}_{11} v_{11}^{j} \mathcal{G}_{11} + v_{22}^{i} \mathcal{G}_{22} v_{22}^{j} \mathcal{G}_{22} + v_{12}^{i} \mathcal{G}_{22} v_{21}^{j} \mathcal{G}_{11} + v_{21}^{i} \mathcal{G}_{11} v_{12}^{j} \mathcal{G}_{22}$$

in the representation diagonalizing the Hamiltonian and hence the Green's function. The summation over the frequencies ω_n is now easy to perform. For instance, for the product of the Green's functions with distinct band indices, we find

$$T\sum_{\omega_n} \mathcal{G}_{11}(p_+)\mathcal{G}_{22}(p_-) = \frac{f\left[\varepsilon_1(\mathbf{p}_-)\right] - f\left[\varepsilon_2(\mathbf{p}_+)\right]}{\mathrm{i}\omega_l - \varepsilon_2(\mathbf{p}_+) + \varepsilon_1(\mathbf{p}_-)},$$

where $f(\varepsilon)$ is the Fermi function with the chemical potential μ . Passing to the real frequency of the external field is achieved, as usual, by analytic continuation from the discreet set of frequencies $\omega_l = 2\pi l T$, which can be done in this case by the simple replacement $i\omega_l \rightarrow \omega + i\delta$, where δ is an infinitely small positive quantity.

Formula (8) also contains the second, 'diamagnetic' term. Instead of calculating it, we use the fact that the current must vanish when the vector potential is constant in time and space. In the present case, in the absence of a permanent magnetic field, this implies that the current must vanish at the zero frequency. Therefore, to determine the total current, we can simply subtract from the first expression its value at $\omega = 0$. As a result, we find the conductivity

$$\sigma_{ij}(\omega, k) = 2ie^{2} \sum \left\{ \frac{v_{11}^{i} v_{11}^{j} \left\{ f \left[\varepsilon_{a}(\mathbf{p}_{-}) \right] - f \left[\varepsilon_{a}(\mathbf{p}_{+}) \right] \right\}}{\left[\varepsilon_{a}(\mathbf{p}_{+}) - \varepsilon_{a}(\mathbf{p}_{-}) \right] \left[\omega - \varepsilon_{a}(\mathbf{p}_{+}) + \varepsilon_{a}(\mathbf{p}_{-}) \right]} + 2\omega \frac{v_{1a}^{i} v_{a1}^{j} \left\{ f \left[\varepsilon_{1}(\mathbf{p}_{-}) \right] - f \left[\varepsilon_{2}(\mathbf{p}_{+}) \right] \right\}}{\left[\varepsilon_{2}(\mathbf{p}_{+}) - \varepsilon_{1}(\mathbf{p}_{-}) \right] \left\{ (\omega + i\delta)^{2} - \left[\varepsilon_{2}(\mathbf{p}_{+}) - \varepsilon_{1}(\mathbf{p}_{-}) \right]^{2} \right\}} \right\},$$

$$(10)$$

where summation (integration) is performed with respect to the two-dimensional quasimomentum \mathbf{p} for graphene and with respect to the three-dimensional quasimomentum for IV-VI semiconductors, and, in addition, with respect to the band index *a* (summation over the spin has led to the factor 2 in front of the sum).

The first term in formula (10) is the intraband contribution; all quantities in it refer to the same energy band. In the semiclassical limit $kv \ll (T, \mu)$, it coincides with the ordinary Drude–Boltzmann expression; it is well known that taking the collision frequency into account then amounts to replacing the infinitesimal quantity δ with the collision frequency τ^{-1} , i.e., to the substitution $\omega \rightarrow \omega + i\tau^{-1}$. The second term in formula (10) describes interband transitions, and $a \neq 1$ in this term. The sum of the squares of the matrix elements involved here is the squared dipole moment and is found with the help of expressions (5) and (7).

We emphasize that formula (10) is quite general in character because no specific expressions for the electron spectrum were used in its derivation [19].

In the optical range, the spatial dispersion of the conductivity is insignificant and we can take $k \rightarrow 0$ in formula (10). With the aid of formulas (5), we then express the squared matrix elements in (10) as

$$\begin{split} (v_{11}^x)^2 &= \frac{v_t^4 p_x^2}{\varepsilon_1^2} \,, \\ v_{13}^x v_{31}^x + v_{14}^x v_{41}^x &= v_t^2 \left(1 - \frac{v_t^2 p_x^2}{\varepsilon_1^2} \right) \,. \end{split}$$

Integration in formula (10) is conveniently performed by introducing polar angles and the energy variable ε in accordance with (4):

$$p_{z} = \sqrt{\varepsilon^{2} - \varepsilon_{g}^{2}} \frac{\cos \theta}{v_{\ell}} ,$$

$$p_{x} = \sqrt{\varepsilon^{2} - \varepsilon_{g}^{2}} \frac{\sin \theta \cos \varphi}{v_{t}} ,$$

$$p_{y} = \sqrt{\varepsilon^{2} - \varepsilon_{g}^{2}} \frac{\sin \theta \sin \varphi}{v_{t}} .$$

In integrating over the angles, the nondiagonal elements of the conductivity tensor vanish, as they must in the case of the C_{3v} symmetry of the L point. We thus obtain the intraband conductivity

$$\sigma_{xx}^{\text{intra}}(\omega) = \frac{-ie^2}{3\pi^2 v_\ell \omega} \int_{\varepsilon_g}^{\varepsilon_{\text{at}}} \left[f'(\varepsilon) + f'(-\varepsilon) \right] (\varepsilon^2 - \varepsilon_g^2)^{3/2} \, \frac{\mathrm{d}\varepsilon}{\varepsilon} \quad (11)$$

and the interband conductivity

-. 2

$$\sigma_{xx}^{\text{inter}}(\omega) = \frac{2\mathrm{i}e^{2}\omega}{3\pi^{2}v_{\ell}} \\ \times \int_{\varepsilon_{g}}^{\varepsilon_{\text{at}}} \left[f(-\varepsilon) - f(\varepsilon) \right] \frac{\left(\varepsilon^{2} - \varepsilon_{g}^{2}\right)^{1/2}}{\left(\omega + \mathrm{i}\delta\right)^{2} - 4\varepsilon^{2}} \left(1 + \frac{\varepsilon_{g}^{2}}{2\varepsilon^{2}} \right) \mathrm{d}\varepsilon \,. \tag{12}$$

The last integral diverges at the upper limit, where the linear spectrum expansion cannot be used. But the leading contribution to the integral originates from the domain $\varepsilon_g \ll \varepsilon \ll \varepsilon_{at}$, and we therefore cut off the integral at $\varepsilon = \varepsilon_{at}$ with logarithmic accuracy. From Fig. 1, which shows the electron spectrum of the semiconductors under consideration, we can see that the cut-off parameter ε_{at} should be equal to about 8 eV.

There are four L points in the Brillouin zone, and their contributions should be summed. It must then be taken into account that the conductivity component σ_{zz} referred to the axes related to point L is different from σ_{xx} in that v_{ℓ} is replaced by v_t^2/v_{ℓ} . By rotating the coordinate axes at each L point to the common axes and summing over these points, we find the total conductivity. Only the diagonal, and equal, conductivity components $\sigma(\omega)$ are nonzero. They can be obtained by the replacement

$$\frac{1}{v_\ell} \to \frac{8}{3v_\ell} + \frac{4v_\ell}{3v_t^2} \equiv \frac{1}{v}$$

in formulas (11) and (12). From the measured values in (3), we calculate the quantity $v = 1.12 \times 10^7$ cm s⁻¹ common to all

semiconductors. Finally, the permittivity $\epsilon(\omega)$ is related to the conductivity as

$$\epsilon(\omega) = \epsilon_0 + \frac{4\pi i \sigma(\omega)}{\omega} , \qquad (13)$$

where ϵ_0 is the lattice (phonon) contribution.

In the limit case $(T, \varepsilon_g) = 0$, we obtain a very simple result,

$$\epsilon(\omega) = \epsilon_0 + \frac{e^2}{3\pi\hbar v} \left[-\frac{4\mu^2}{\omega^2} + \ln\frac{4\epsilon_{\rm at}^2}{|\omega^2 - 4\mu^2|} + i\pi\theta(\omega - 2\mu) \right]$$
(14)

The first term in square brackets is the Drude–Boltzmann intraband contribution and the other two (the logarithm and the θ function) result from interband transitions. At the absorption threshold $\omega = 2\mu$, the real part of the permittivity has a logarithmic singularity. It is cut off by the temperature. Calculations show that the following change should be made at low but finite temperatures:

$$\omega^2 - 4\mu^2 \rightarrow |\omega^2 - 4\mu^2| + 4\omega T.$$

If the carrier collision frequency v plays a greater role than the temperature, but is low in comparison with the chemical potential, the corresponding change is of the form

$$\begin{split} \omega^2 - 4\mu^2 &\to \left[(\omega^2 - 4\mu^2)^2 + (2\omega v)^2 \right]^{1/2} \\ \theta(\omega - 2\mu) &\to \frac{1}{2} + \frac{1}{\pi} \arctan\left(\frac{\omega - 2\mu}{v}\right). \end{split}$$

Simple formulas are also obtained for pure semiconductors at zero temperature, when the conduction band is empty and the valence band is filled. The imaginary part of the permittivity is nonzero for $\omega > 2\varepsilon_g$:

$$\epsilon''(\omega) = \frac{e^2}{3\hbar\nu\omega^3} \left(\omega^2 + 2\varepsilon_g^2\right) \sqrt{\omega^2 - 4\varepsilon_g^2} \,. \tag{15}$$

A similar expression was obtained in Ref. [11]. It is clear from formulas (14) and (15) that the imaginary part of the permittivity approaches the constant value $\epsilon_0'' = e^2/3\hbar v$ for $\omega > 2 \max{(\mu, \varepsilon_g)}$. Using the value of v given above, we find $\epsilon_0'' = 6.5$ for all semiconductors under discussion.

In the case where there are no carriers and $\mu = 0$, the expression for the real part of the permittivity can be written, with logarithmic accuracy in the parameter $1/\ln(\varepsilon_{at}/\varepsilon_g)$, as

$$\epsilon'(\omega) = \epsilon_0 + \frac{2e^2}{3\pi\hbar\nu} \ln \frac{\epsilon_{\rm at}}{\max\left\{\epsilon_{\rm g}, \omega/2\right\}} \,. \tag{16}$$

This formula and Eqn (14) imply that the real part of the dielectric function contains a large logarithm in comparison with the imaginary part. Furthermore, unlike the imaginary part, the real part depends on the specific material via ε_g and decreases for $\omega > 2 \max{(\mu, \varepsilon_g)}$. When the frequency ω is low in comparison with the energy gap $2\varepsilon_g$, the expression

$$\epsilon_{\infty} = \epsilon_0 + \frac{e^2}{3\pi\hbar v} \left(2\ln\frac{2\varepsilon_{\rm at}}{\varepsilon_{\rm g}} - \frac{5}{3} \right) \tag{17}$$

can be derived for the real part; up to the terms $2 \ln 2 - 5/3$, i.e., with logarithmic accuracy, it coincides with expression (16) for those frequencies.



Figure 3. Real (a) and imaginary (b) parts of the dielectric function of a IV-VI semiconductor; the carrier density (with the corresponding chemical potential value $\mu = 1491$ K for T = 0 K assumed to be large in comparison with the gap width ε_g) and the carrier collision frequency (in K) are indicated.

The phonon frequencies in IV–VI semiconductors are of the order of 100 K, i.e., 10^{-2} eV, while the gap $2\varepsilon_g \sim 0.15$ eV. By ϵ_{∞} , in accordance with the notation, we must therefore mean the high-frequency permittivity and set $\epsilon_0 = 1$ in this case. Formula (17) with $\epsilon_g = 0.075$ eV, $\epsilon_{at} = 8$ eV, and $v = 1.12 \times 10^7$ cm s⁻¹ yields $\epsilon_{\infty} = 19.8$.

Figure 3 shows the dispersion of the dielectric function in the infrared range, plotted using the above formulas for a typical value $\varepsilon_g = 0.1 \text{ eV}$. The plots can be compared with the numerical simulation data in Ref. [14] and the available measurement data given in Fig. 4. It follows that the value $\epsilon_0'' = 6.5$ and the maximal value $\epsilon'_{max} \simeq 20-25$ of the real part obtained with the help of expression (12) in the interval between the frequency $2\varepsilon_g \simeq 0.15 \text{ eV}$ of the transition under consideration and the next transition frequency $\simeq 1.2 \text{ eV}$ are quite well reproduced both by numerical simulations and by extrapolation of the measured values from the higherfrequency side, $\sim 0.5-0.8 \text{ eV}$, where $\epsilon''(\omega)$ saturates at a plateau. Furthermore, we can see that the real part $\epsilon'(\omega)$ decreases, in agreement with expression (16), when the frequency exceeds the threshold value $2\varepsilon_g$.

It is generally assumed that one of the most reliable ways to determine the gap in the energy spectrum is by its measurement from the optical absorption threshold. The resultant data are interpreted, in particular, as a 10%reduction of the gap in IV–VI semiconductors with increasing the temperature from liquid helium to room temperature. The calculations made here permit estimating the effect of carriers on these data. It is seen from Fig. 5 that the presence



Figure 4. Real (a) and imaginary (b) part of the dielectric function of IV - VI semiconductors calculated by different numerical techniques (GGA, the so-called generalized gradient approximation; LDA, the local-density approximation) and measured in Ref. [14].



Figure 5. (a) Chemical potential (in units of $\varepsilon_g = 0.075 \text{ eV}$) as a function of temperature for three values of the carrier density (the horizontal line shows the bottom of the conduction band) and (b) imaginary part of the dielectric function as a function of frequency (in units of $2\varepsilon_g$) for different temperatures in a IV–VI semiconductor. The $2\varepsilon_g$ gap in the energy spectrum was assumed to be temperature-independent; however, the variation of the curves in form with increasing the temperature may lead to a conclusion about the narrowing of the gap if it is related to the absorption threshold.



Figure 6. Normal-incidence reflectivity for a IV - VI semiconductor; the parameter values are the same as in Fig. 3.

of a moderate number of carriers, $\sim 10^{18}$ cm⁻³, can imitate the temperature variation of the gap.

The reflectivity at normal radiation incidence

$$R = \left|\frac{\sqrt{\epsilon(\omega)} - 1}{\sqrt{\epsilon(\omega)} + 1}\right|^2$$

calculated by our formulas is plotted in Fig. 6. At low frequencies, the reflectivity is determined by the carriers present in the sample. The reflectivity rapidly changes (from unity to zero) in the neighborhood of the frequency $\omega_{\rm pl}$, which is approximately determined by the equation

$$\epsilon_0 - \frac{2e^2}{3\pi\hbar v} \left(\frac{2\mu^2}{\omega_{\rm pl}^2} - \ln \frac{\varepsilon_{\rm at}}{\mu} \right) = 0 \,.$$

In this range, an essential role is played by the large logarithm, which owes it origin to interband transitions. Next, the reflectivity value ~ 0.4 is determined primarily by the interband contribution. At the very threshold $\omega = 2\mu = 2982$ K, which corresponds to the carrier density 10^{19} cm⁻³, the narrow reflectivity peak, which should be observable at low temperatures and low carrier relaxation frequencies, replicates the form of the real part of the permittivity. Its observation conditions may be quantified: the temperature $T \sim 10$ K, the carrier mean free time $\tau > 10^{-13}$ s, and the carrier density $10^{18} - 10^{19}$ cm⁻³. At higher frequencies, the interband absorption (the imaginary part of the permittivity) is responsible for a certain decrease in reflectivity.

5. Dynamic conductivity of graphene

In the Brillouin zone of graphene, there are two K points at which the conduction and valence bands intersect. We sum the contributions of these points, integrate over the angle of the two-dimensional vector **p**, and pass to the variable $\varepsilon = v_0 p$ to obtain the conductivity [19, 21]

$$\sigma(\omega) = \frac{e^2 \omega}{i\pi\hbar} \left[\int_{-\infty}^{+\infty} d\varepsilon \, \frac{|\varepsilon|}{\omega^2} \frac{df(\varepsilon)}{d\varepsilon} - \int_0^{+\infty} d\varepsilon \, \frac{f(-\varepsilon) - f(\varepsilon)}{(\omega + i\delta)^2 - 4\varepsilon^2} \right].$$
(18)

The first, intraband, term can be integrated:

$$\sigma^{\text{intra}}(\omega) = \frac{2\mathrm{i}e^2 T}{\pi\hbar(\omega + \mathrm{i}\tau^{-1})} \ln\left[2\cosh\left(\frac{\mu}{2T}\right)\right],\tag{19}$$

where we wrote $\omega + i\tau^{-1}$ instead of ω to include the effect of electron damping. In this form, the intraband term coincides with the classical Drude–Boltzmann expression for the conductivity. At low temperatures, $\mu \ge T$, when the carriers are degenerate, the intraband term is of the form

$$\sigma^{\text{intra}}(\omega) = \frac{\mathrm{i}e^2|\mu|}{\pi\hbar(\omega + \mathrm{i}\tau^{-1})} \,. \tag{20}$$

In pure graphene, the chemical potential $\mu = 0$, and high-frequency conductivity (19) is proportional to the temperature. The carrier density can be changed either by doping or with the help of a constant electric field (the field effect); the chemical potential (Fig. 7) is then found from the condition

$$n_0 = \frac{2}{\pi (\hbar v_0)^2} \int_0^{+\infty} \varepsilon \big[f(\varepsilon - \mu) - f(\varepsilon + \mu) \big] \, \mathrm{d}\varepsilon \,, \tag{21}$$

where n_0 $(-n_0)$ is the density of electrons (holes). For a low temperature, the optical conductivity, as is evident from formula (19), is proportional to the square root of the density because $\mu = \hbar v_0 \sqrt{\pi n_0}$ (hereinafter, we restrict ourselves to the case $\mu > 0$ for simplicity of writing).

The interband term in expression (18) for the conductivity contains both the real part, which results from going around a pole, and the imaginary part. At zero temperature, the corresponding integral in expression (18) is easily taken:

$$\sigma^{\text{inter}}(\omega) = \frac{e^2}{4\hbar} \left[\theta(\omega - 2\mu) - \frac{i}{2\pi} \ln \frac{(\omega + 2\mu)^2}{(\omega - 2\mu)^2} \right], \quad (22)$$

where the θ function expresses the condition of interband electron transitions with the threshold at $\omega = 2\mu$. The



Figure 7. (a) Chemical potential of graphene as a function of temperature for samples with different carrier densities. (b) Scattering geometry for a multilayer system.

logarithmic singularity is cut off by the temperature (or the carrier relaxation), and the change

$$\theta(\omega - 2\mu) \rightarrow \frac{1}{2} + \frac{1}{\pi} \arctan\left(\frac{\omega - 2\mu}{2T}\right),$$

$$(\omega - 2\mu)^2 \rightarrow (\omega - 2\mu)^2 + (2T)^2$$
(23)

should be made in expression (22) for a finite temperature that is low in comparison with the chemical potential

For nondegenerate carriers, $\mu \ll T$, the interband conductivity is

$$\sigma_{2}^{\text{inter}}(\omega) = -\frac{2ie^{2}}{\pi\hbar} \begin{cases} \frac{T}{\omega} \left[\ln 2 + 6\zeta(3) \left(\frac{T}{\omega}\right)^{2} \right], & \omega \gg 4T, \\ \frac{\omega}{16T} \ln \left(\frac{4T}{\omega}\right), & \omega \ll 4T, \end{cases}$$

where $\zeta(3) = 1.20$.

For numerical calculations of the interband conductivity at finite temperatures, it is useful to introduce the notation

$$G(\varepsilon) = f(-\varepsilon) - f(\varepsilon) = \begin{cases} \theta(\varepsilon - \mu), & \mu \ge T, \\ \tanh\left(\frac{\varepsilon}{2T}\right), & \mu \ll T \end{cases}$$
(24)

for the difference of the Fermi functions in the integrand in expression (18). Subtracting $G(\omega/2)$ from and adding it to the numerator in the integrand, we note that the principal value of the resultant integral with $G(\omega/2)$ is equal to zero and then arrive at the integral of a singularity-free function:

$$\sigma^{\text{inter}}(\omega) = \frac{e^2}{4\hbar} \left[G\left(\frac{\omega}{2}\right) - \frac{4\omega}{i\pi} \int_0^{+\infty} d\varepsilon \, \frac{G(\varepsilon) - G(\omega/2)}{\omega^2 - 4\varepsilon^2} \right]. \tag{25}$$

Two main conclusions may be drawn from the results obtained (Fig. 8). First, at high frequencies, $\omega \ge (T, \mu)$, the conductivity is mostly real and independent of any parameters:

$$\sigma(\omega) = \frac{e^2}{4\hbar} \, .$$



Figure 8. Imaginary (a) and real (b) parts of the conductivity in units of e^2/\hbar for graphene with the carrier density $n_0 = 10^{11}$ cm⁻². The values $\mu = 428$, 389, and 214 K of the chemical potential for the respective temperatures 3, 100, and 300 K indicated by the curves are defined by Eqn (21). The singularity at $\omega = 2\mu$ seen at the low temperature is associated with the interband transition threshold.

This universal value is different from the value, also universal, obtained for the direct-current conductivity [22, 23]. Second, when there are degenerate carriers in graphene, at sufficiently low temperatures, the imaginary part of the conductivity contains a logarithmic singularity at the interband absorption threshold $\omega = 2\mu$, where the real part experiences a finite jump. This singularity smooths out with increasing the temperature, as well as due to the finiteness of the carrier mean free path. For the singularity to be observable, the collision frequency should not exceed the temperature, i.e., should be equal to 10-40 K in energy units. As we see in what follows, the occurrence of the singularity gives rise to special electromagnetic excitations near the absorption threshold.

6. Electrodynamics of graphene layers

To calculate the experimentally measured optical characteristics—the reflection and transmission coefficients—for three-dimensional semiconductors, it suffices to substitute the expressions for the permittivity given in Section 4 in the well-known Fresnel formulas. But the situation with twodimensional films of the graphene type is more complicated. We proceed from the Maxwell equations

$$\nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = \epsilon_0 \, \frac{\omega^2}{c^2} \, \mathbf{E} + \frac{4\pi i \omega}{c^2} \mathbf{j} \,, \tag{26}$$

where **j** is the conductivity current and ϵ_0 is the lattice contribution to the permittivity. We first consider a single graphene layer [19, 24] and then a stack [20] of parallel graphene layers (Fig. 7b).

6.1 Optics of a monolayer

Let the light polarized in the xz plane (the plane of incidence) be incident from a vacuum on a graphene layer at z = 0 on a substrate (z > 0) with a permittivity $\epsilon_0 = \epsilon_s$. The current in

$$j_x = \sigma(\omega)\delta(z)E_x.$$
⁽²⁷⁾

After the Fourier transformation with respect to the *x* coordinate, the Maxwell equations for two field components become

$$ik_x \frac{dE_z}{dz} - \frac{d^2 E_x}{dz^2} - \epsilon_0 \frac{\omega^2}{c^2} E_x = \frac{4\pi i\omega}{c^2} j_x,$$

$$ik_x \frac{dE_x}{dz} + \left(k_x^2 - \epsilon_0 \frac{\omega^2}{c^2}\right) E_z = 0.$$
(28)

The condition for the jump of the normal component of the induction gives

$$\epsilon_{s} E_{z} \big|_{z=+0} - E_{z} \big|_{z=-0} = 4\pi \int_{-0}^{+0} \rho(\omega, k_{x}, z) \,\mathrm{d}z \,.$$
⁽²⁹⁾

The charge density is related to current density (27) by the continuity equation

$$\rho(\omega, k_x, z) = j_x(\omega, k_x, z) \, \frac{k_x}{\omega} \, .$$

We substitute E_z from the second equation in (28) into expression (29) to find the boundary condition

$$\frac{\epsilon_s}{k_s^2} \frac{\mathrm{d}E_x}{\mathrm{d}z} \Big|_{z=+0} - \frac{1}{(k_z^i)^2} \frac{\mathrm{d}E_x}{\mathrm{d}z} \Big|_{z=-0} = \frac{4\pi\sigma(\omega)}{\mathrm{i}\omega} E_x \Big|_{z=0}, \quad (30)$$

where

$$k_{\rm s} = \sqrt{\epsilon_{\rm s} \left(\frac{\omega}{c}\right)^2 - k_x^2}, \quad k_z^i = \sqrt{\left(\frac{\omega}{c}\right)^2 - k_x^2},$$

This condition, in combination with the continuity condition of the field component E_x , permits calculating [20] the reflected (r) and transmitted (t) wave amplitudes

$$r = \frac{1-C}{1+C}, \quad t = \frac{2}{1+C},$$
 (31)

where $C = k_z^i [4\pi\sigma(\omega)/\omega + (\epsilon_s/k_s)]$.

Very simple results are obtained for free graphene in a vacuum. In this case, $\epsilon_s = 1$ and $k_s = k_z^i$, and the coefficient *C* takes values close to unity:

$$C = 1 + \frac{4\pi}{c} \,\sigma(\omega) \cos\theta\,,\tag{32}$$

where θ is the angle between the normal and the propagation direction of the incident wave. The reflected and transmitted wave amplitudes follow from Eqn (31) as

$$r = -\frac{2\pi}{c}\sigma(\omega)\cos\theta$$
, $t = 1 - \frac{2\pi}{c}\sigma(\omega)\cos\theta$. (33)

The normal-incidence transmittance $|t|^2$ calculated with the aid of Eqns (19), (25), and (31) for graphene with carrier densities 10^{10} and 10^{11} cm⁻² is plotted in Fig. 9 as a function of frequency for different temperatures. Significant at low frequencies is the intraband conductivity, which decreases with frequency. Then, at frequencies $\omega > 2\mu$, the transmittance saturates at a value determined by interband transitions and differs from unity by a value of the order of the fine



Figure 9. Normal-incidence transmittance for graphene with the carrier density $n_0 = 10^{10}$ cm⁻² (a) and $n_0 = 10^{11}$ cm⁻² (b) as a function of frequency at temperatures indicated at the curves. For graphene with the density $n_0 = 10^{11}$ cm⁻², the values of the chemical potential are 428, 389, and 294 K at the respective temperatures 3, 100, and 200 K.



Figure 10. Transmission spectra for a monolayer and bilayer of graphene in the visible range [27].

structure constant $e^2/\hbar c$:

$$|t|^2 = 1 - \frac{4\pi}{c} \operatorname{Re} \sigma(\omega) = 1 - \pi \frac{e^2}{\hbar c},$$
 (34)

where the term with a logarithmic singularity is omitted because it is proportional to the square of the fine structure constant. The effect linear in the fine structure constant was recently measured in Ref. [27] in the optical domain. The measured transmittance (Fig. 10) is in good agreement with theoretical value (34) and is independent of the frequency in a broad interval of the visible range, as predicted by the theory. Systems consisting of two [25] or more parallel planar graphene layers are presently being grown, and for them, like for three-dimensional semiconductors, the special features of interband absorption are much easier to observe.

6.2 Spectroscopy of graphene superlattices

It is evident that the problem of several graphene layers may be solved analytically either when the number of layers is small, by considering the scattering by each layer separately, or in the opposite limit case of a large number of layers. In the latter case, it is possible to begin with an infinite layer sequence and then solve the problem for a film with finitely many layers, imposing the corresponding electrodynamic boundary conditions on its boundaries.

We therefore consider an unbounded system of graphene layers with a spacing d between them (Fig. 7b). This system can be regarded as a model of graphite in which d = 3.35 Å is significantly greater than the interatomic distance in the layer, and the interaction between layers emerges only due to currents flowing in the layers. This interaction is described by Maxwell equations (26). For the E_x field component, they reduce to the equation

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2} + k_{\mathrm{g}}^2 + 2k_{\mathrm{g}}\mathcal{D}\sum_n \delta(z - nd)\right)E_x = 0\,,\tag{35}$$

where $\mathcal{D} = 2i\pi\sigma(\omega)k_g/\epsilon_g\omega$ and $k_g = \sqrt{\epsilon_g(\omega/c)^2 - k_x^2}$. Because we neglect the electron hopping between the layers, the quantity $\epsilon_g \simeq 2.5$ is the lattice contribution to the permittivity of the graphene layers.

For a infinite medium, Eqn (35) has two independent solutions in the form of Bloch functions:

$$e_{1,2}(z) = \exp\left(\pm ik_z nd\right) \left\{ \sin k_g(z - nd) - \exp\left(\mp ik_z d\right) \right.$$
$$\times \left. \sin k_g \left[z - (n+1)d \right] \right\}, \quad nd < z < (n+1)d,$$

where the quasimomentum k_z is defined by the dispersion equation

$$\cos k_z d = \cos k_g d - \mathcal{D} \sin k_g d. \tag{36}$$

These solutions are nothing but electromagnetic waves in an anisotropic periodic medium.

The solution of the problem is simplified in the longwavelength limit, $k_z, k_g \ll 1/d$. In this case, it is possible to introduce the permittivity not only in the direction perpendicular to the layers, $\epsilon_{zz} = \epsilon_g \simeq 2.5$, but also in the parallel direction, $\epsilon_{xx} = \epsilon_g + 4\pi i \sigma(\omega)/\omega d$, where $\sigma(\omega)$ is the conductivity of a single layer (18).

For a sample with graphene layers, we seek the solution, as usual, in the form of transmitted and reflected waves, $\mathbf{E} \propto \exp(ik_x x \pm ik_z z)$; instead of Eqn (36), we then obtain the dispersion equation

$$k_x^2 \epsilon_{xx} + k_z^2 \epsilon_{zz} = \left(\frac{\omega}{c}\right)^2 \epsilon_{xx} \epsilon_{zz} \,. \tag{37}$$

It follows from this equation that undamped solutions at normal incidence $(k_x = 0)$ are possible only if the real part of ϵ_{xx} is positive and exceeds the imaginary part. Also, it follows from Eqns (20) and (22) that the imaginary part of the conductivity must be negative and greater than the real part. This condition is fulfilled somewhat below the absorption threshold, as can be seen in Fig. 11.

The tangential projections of the electric and magnetic fields should obey the continuity conditions at the boundaries of a film of thickness l. By solving the ordinary electrodynamic problem, for instance, for the reflection and transmission of a p-polarized wave, we find the amplitudes of the



Figure 11. Real (a) and imaginary (b) parts of the permittivity ϵ_{xx} for a graphene superlattice with the in-layer carrier density $n_0 = 10^{11}$ cm⁻² and the interlayer distance d = 3.35 Å at temperatures 10 (dashed – dotted line), 50 (solid line), and 100 K (dashed line). The values $\mu = 428$, 419, and 389 K for the respective temperatures 3, 100, and 300 K are defined by Eqn (21). The low-temperature peak in the real part is related to the interband transition threshold (a jump in the imaginary part); the $\omega = 2\mu$ threshold position is shown by the vertical straight line.



Figure 12. Normal-incidence transmission and reflection spectra of a graphene superlattice with the in-layer carrier density $n_0 = 10^{11}$ cm⁻² ($\mu = 428$ K for T = 0) at different temperatures; the interlayer distance d = 3.35 Å, the film thickness l = 100d.

reflected wave and the wave transmitted through the film:

$$r = -1 + 2k_z \frac{(k_z + k_z^i)f - (k_z - k_z^i)f^{-1}}{(k_z + k_z^i)^2 f - (k_z - k_z^i)^2 f^{-1}},$$

$$t = \frac{4k_z k_z^i}{(k_z + k_z^i)^2 f - (k_z - k_z^i)^2 f^{-1}},$$
(38)

where $f = \exp(-ik_g l)$, $k_z^i = (\omega/c) \cos \theta$, and k_z is defined by dispersion equation (37) with $k_x = (\omega/c) \sin \theta$. Figure 12 shows the calculated transmission and reflection coefficients of a graphene layer superlattice as functions of the frequency for different temperatures. The carrier density is determined either by doping or with the aid of a permanent electric field.

Two features of the reflection coefficient must be noted. First, there is a dip in the vicinity of the interband transition threshold $\omega = 2\mu$ (for 859 K in Fig. 12). With increasing the temperature (or increasing the carrier relaxation frequency), this dip is smeared out, because it exists only for a sufficiently large value of the logarithm in the imaginary part of conductivity (22). Therefore, the sharp dip in reflectivity is related to the excitation of electromagnetic waves in the superlattice. Their spectrum is given by Eqns (36) and (37). We emphasize that these excitations are different both from ordinary longitudinal plasmons in three-dimensional systems with a quadratic spectrum and a gap, and from zero-gap plasmons in two-dimensional systems, which are excited only under the conditions of total internal reflection. They resemble the waves that show up, for instance, in the neighborhood of the cyclotron resonance.

Second, after a large decrease, the reflectivity becomes substantially lower than its pre-threshold value. This is a direct result of interband absorption, i.e., of the θ -like singularity (24) in the real part of the conductivity. It is noteworthy that the features under discussion are governed by functions rapidly varying in the neighborhood of $\omega \approx 2\mu$, but unlike in the monolayer case, they are multiplied by the dimensionless factor $e^2/\hbar\epsilon_g\omega d$, which may be of the order of unity. A comparison with the monolayer in Fig. 9b shows this difference. An observation of low-temperature features in samples with a low carrier relaxation frequency is the direct way to measure the carrier mobility and density.

7. Longitudinal plasmons

Ordinary longitudinal plasmons can exist in the electron plasma of IV–VI semiconductors, as well as of graphene. In three-dimensional semiconductors, their spectrum exhibits a gap and a quadratic dispersion; in two-dimensional graphene, they are zero-gap excitations with a square-root dependence $\omega \propto \sqrt{k}$. However, in narrow-gap materials, their frequency depends on the carrier density in an unusual way due to a significant contribution of interband transitions to the dielectric function; the expressions for the dielectric function were given in different limit cases in Section 4. We write the expression for the dielectric function for T = 0 in the range $\omega < 2\mu$, where there is no absorption caused by interband transitions.

For IV-VI semiconductors with degenerate carriers, the dielectric function defined by Eqns (11) and (12) has the form (Fig. 13)

$$\epsilon(\omega) = \epsilon_0 + \frac{e^2}{3\pi\hbar v} \left[-\frac{(\mu^2 - \epsilon_g^2)^{3/2}}{\mu b^2} + 2\ln\frac{2\epsilon_{at}}{\mu + \sqrt{\mu^2 - \epsilon_g^2}} + \frac{\epsilon_g^4/\mu b^2}{\mu + \sqrt{\mu^2 - \epsilon_g^2}} + F(\omega) \right],$$
(39)

where

$$F(\omega) = \frac{2b^4 - \varepsilon_g^2 b^2 - \varepsilon_g^4}{b^3 |\varepsilon_g^2 - b^2|^{1/2}} \\ \times \begin{cases} \arcsin\frac{b}{\varepsilon_g} - \arcsin\frac{b\sqrt{\mu^2 - \varepsilon_g^2}}{\varepsilon_g\sqrt{\mu^2 - b^2}}, & b < \varepsilon_g, \\ \ln\frac{b\sqrt{\mu^2 - \varepsilon_g^2} + \mu\sqrt{b^2 - \varepsilon_g^2}}{(b + \sqrt{b^2 - \varepsilon_g^2})\sqrt{\mu^2 - b^2}}, & b > \varepsilon_g, \end{cases}$$
(40)



Figure 13. Frequency dependence of the real part of the dielectric function (in units of the chemical potential) for T = 0 for a IV–VI semiconductor with a gap $2\epsilon_g = 870$ K. The carrier densities are indicated by the curves, the corresponding values of μ are 1112 and 3327 K.

 $b = \omega/2$. This expression coincides with formula (14) for $\varepsilon_g = 0$; as $\omega \to 0$, it gives

$$\epsilon(\omega \to 0) = \epsilon_0 + \frac{e^2}{3\pi\hbar v} \left[-4 \frac{(\mu^2 - \epsilon_g^2)^{3/2}}{\mu \omega^2} + 2 \ln \frac{2\epsilon_{at}}{\mu + \sqrt{\mu^2 - \epsilon_g^2}} + \frac{1}{3} \left[1 - \frac{(\mu^2 - \epsilon_g^2)^{3/2}}{\mu^3} \right] - 2 \left(1 - \frac{\sqrt{\mu^2 - \epsilon_g^2}}{\mu} \right) \right].$$
(41)

With the aid of the condition $\epsilon(\omega) = 0$ and the equation for the carrier density,

$$n_0 = \frac{4(\mu^2 - \varepsilon_g^2)^{3/2}}{3\pi^2\hbar^3 v_\ell v_t^2}$$

we find the plasma wave frequency as a function of the carrier density (Fig. 14). The intraband contribution to the dielectric function is proportional to μ^2 for values of the chemical potential greater than the gap, as is clear from formula (39). The plasma frequency is, roughly speaking, proportional to the square root of the charge carrier density, $\omega_{\rm pl} \sim \sqrt{n_0}$, as in ordinary plasmas. However, owing to the frequency dependence of the interband contribution, the plasma frequency is substantially higher (solid curve) than the value obtained when this frequency dispersion is neglected (dashed curve).

For graphene, the spectrum of longitudinal plasmons is found [19, 26] with the help of Eqn (35). Outside the $z \neq 0$ layer, the solutions decreasing at $\pm \infty$ are sought, respectively, as $E_x = C \exp(\mp \kappa z)$, where $\kappa = ik_g$. We substitute this in the condition

$$E'_{x}|_{z=+0} - E'_{x}|_{z=-0} = -2i\kappa \mathcal{D}E_{x}|_{z=0}, \qquad (42)$$



Figure 14. (a) Dependence of the plasma frequency on the carrier density at T = 0 for a IV-VI semiconductor with $\varepsilon_g = 870$ K; the dashed curve was plotted neglecting the frequency dependence of the permittivity. (b) Dependence of the chemical potential on the carrier density.

which follows directly from Eqn (35), and thus find the spectrum of two-dimensional plasmons:

$$-2\pi i \kappa \sigma(\omega) = \omega \,. \tag{43}$$

This equation has a real solution for ω and κ , as can be seen from expressions (19) and (22), in the low-frequency range, where the intraband conduction plays the leading role. We keep only the intraband term to obtain

$$\omega^2 = \frac{8e^2 T\kappa}{\hbar} \ln \left[2 \cosh \left(\frac{\mu}{2T} \right) \right].$$

The time delay, i.e., the term with ω^2/c^2 , can normally be neglected in the expression for κ , and hence $\kappa = |k_x|$. We emphasize this unusual dependence of the plasma frequency on the carrier density, $\omega \propto n_0^{1/4}$, at low temperatures.

8. Conclusion

It is evident that IV – VI semiconductors and graphene have much in common: a narrow gap and a broad linear region in the electron spectrum. Despite the difference in their dimensionality, this circumstance leads to similar singularities in conductivity: a logarithmic singularity in the imaginary part and a jump in the real part. These singularities are smeared out with increasing the temperature or the carrier relaxation rate. The static permittivity of the IV-VI semiconductors under consideration, as well as the transmittance in the infrared range calculated with the use of this simple model of the electron spectrum, are in excellent agreement with numerical calculations and available experimental data. The recently measured value of the transmittance for graphene in the visible range equal to $1 - \pi e^2/\hbar c$ is consistent, to a high degree of accuracy, with the value calculated in the framework of very simple ideas regarding the nature of this unique material. Other interesting properties of graphene (in the infrared range and in the variation of the carrier density or temperature) will undoubtedly attract the attention of researchers.

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