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Generation of large-scale eddies and zonal winds in planetary atmospheres

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Contents

Abstract. The review deals with a theoretical description of the generation of zonal winds and vortices in a turbulent barotropic atmosphere. These large-scale structures largely determine the dynamics and transport processes in planetary atmospheres. The role of nonlinear effects on the formation of mesoscale vortical structures (cyclones and anticyclones) is examined. A new mechanism for zonal wind generation in planetary atmospheres is discussed. It is based on the parametric generation of convective cells by finite-amplitude Rossby waves. Weakly turbulent spectra of Rossby waves are considered. The theoretical results are compared to the results of satellite microwave monitoring of the Earth's atmosphere.

1. Introduction

One of the major tasks of atmospheric physics is the description of elements of global atmospheric circulation, which determine the weather and climate in vast regions of the planet. This task has long attracted and continues to attract the attention of many researchers, as witnessed by numerous reviews and monographs (see, for example, Refs [1 – 18]). Despite this particular attention, it is still far from being completed. Dynamical processes in planetary atmospheres which are dominated by the Coriolis force, have a common feature seen as the spontaneous generation of zonal winds (flows following latitude circles with respect to the planet's rotation axis) and vortices. Discussion of new achievements in exploring large-scale structures in planetary atmospheres is the focus of this review.

The general circulation of the Earth's atmosphere and oceans on large temporal and spatial scales is characterized by synoptic vortices of Rossby waves (cyclones and anticyclones) and zonal winds which are tightly interconnected. The physical nature of Rossby waves is linked to their large horizontal length and dominance of the Coriolis force, so that the Kibel (Rossby) number $\mathrm{Ki} \equiv v/fL \ll 1$ is small and the Ekman number $\mathrm{Ek} \equiv \nu/fL^2 \ll 1$ is very small. Here, v and L are the typical velocity and length scales in the plane perpendicular to the rotation axis, f is the Coriolis parameter, and ν is the kinematic viscosity. The turbulent motion existing in the atmosphere at large Reynolds numbers ($\mathrm{Re} = \mathrm{Ki}/\mathrm{Ek} \gg 1$) drives large-scale vortices and zonal jets. According to the Taylor – Proudman theorem, the motion in these conditions presents the superposition of a two-dimensional motion in the plane perpendicular to the rotation axis and a uniform motion along this axis.

Characteristic scales of synoptic motions greatly exceed the height of the atmosphere. This allows one to describe these motions as waves in the β-plane approximation. These are the Rossby waves named after Carl-Gustaf Rossby [19, 20], the American meteorologist of Swedish origin, as recognition for his contribution to the theory of synoptic waves made in the 1930 – 1940s. They constitute the branch of synoptic scale waves with the length comparable to the Rossby – Obukhov (deformation) radius in the atmosphere

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or ocean. The Rossby – Obukhov radius reaches approximately 8000 km in the Earth's atmosphere at a latitude of 15°, decreasing with increase in latitude to approximately 2000 km at midlatitudes, while it is reduced to 50 km in the open ocean. This exceeds by far the height of the atmosphere or the depth of the ocean. In the atmospheres of the giant planets Jupiter and Saturn, the Rossby – Obukhov radius is of order 6000 km, which is also much larger than the atmospheric depth but much less than the radii of these planets. The other distinct feature of giant planets is their well-expressed periodic structure of zonal winds in the meridian direction [20, 21]. The zonal wind velocity amplitude reaches about 100 m s^{-1} on Jupiter, and 200 m s^{-1} on Saturn.

Atmospheric masses are captured by long-lived synoptic eddies (cyclones and anticyclones) and transferred by them over large distances. This property determines the importance of eddies in the dynamics of mean daily pressure, temperature, wind velocity, etc. In the Earth's atmosphere, cyclones and anticyclones have spatial scales ranging from hundreds of kilometers to several thousand kilometers, and persist from several days to several weeks. The synoptic eddies of the Earth's atmosphere drift with a characteristic speed of 5 – 10 m s^{-1}, which is comparable to the velocity of fluid particles inside the eddies. The substance circulation within the eddies is slower than the planetary rotation. Because of their slow rotation and nearly horizontal (quasi-two-dimensional) motion, Rossby vortices are very distinct from small-scale (as compared with the atmospheric height) vortices such as tornadoes or the eyes of typhoons. The latter execute essentially three-dimensional motion and rotate faster than the planet. Discussing them is beyond the scope of this review which deals with large-scale synoptic structures moving nearly horizontally.

The motion of fluid in Rossby waves is analogous to the motion of ions in plasma drift waves (see, for example, Refs [8 – 10, 21 – 23]). The analogy between Rossby and drift waves, based on the similarity between the Coriolis force in a rotating fluid and the Lorentz force in a magnetized plasma, stimulates the exchange of ideas and methodical approaches. The Lorentz force in a magnetized plasma plays the same role as the Coriolis force in Rossby waves. The eddies of Rossby waves and zonal winds observed in the atmosphere can be considered as models of wave processes in the magnetized plasma, and vice versa. The anomalous transfers of heat and particles across the magnetic field in a fusion plasma are attributed to drift waves. That is why exploring these waves also receives special attention. A numerical experiment [24] has pointed to an important feature of the drift turbulence in a magnetized plasma — the intense excitation of convective cells which substantially contribute to transport processes. Sagdeev and his collaborators [25] were the first to develop a theory for the nonlinear generation of convective motions in plasmas, driven by the interaction of drift waves.

In recent years, progressively increasing attention has been given to studies on the generation of large-scale zonal structures which have a pronounced effect on transport processes in the atmosphere [26 – 28], magnetized plasmas [29 – 31], and a number of astrophysical objects, including galactic disks and some others [32]. Currently there exist two main approaches to the problem of zonal wind generation. The first one relies on three-dimensional thermal convection [33 – 36]. The second approach suggested very recently is based on a parametric instability of small-scale Rossby waves. According to the second approach, the energy transfer from small-scale Rossby waves into large-scale zonal wind structures is mediated by eddy Reynolds stresses averaged over small spatial and temporal scales. This process follows the paradigm of inverse turbulent cascade in the theory of two-dimensional anisotropic turbulence, the build up of large-scale structures out of turbulent chaos. Laboratory experiments and numerical simulation [37 – 40], assisted by analytical studies [29, 41 – 48], give witness in favor of the parametric mechanism of zonal wind generation in a two-dimensional barotropic atmosphere. The present review discusses this model of zonal wind generation in the shallow atmosphere approximation.

The existence of shear zonal winds in the atmosphere and jets in the ocean provides a framework for the generation of frontal synoptic eddies. According to meteorological observations of planetary atmospheres, an unstable zonal wind generates so-called meanders (so named for a river with a very convoluted path in Anatolia, with the old name Meander). Meanders resemble a geometrical ornament in the form of an irregular sinusoidal curve with the size of separate loops ranging from several hundred to several thousand kilometers. Meanders cut off from zonal currents evolve into cyclonic and anticyclonic eddies. Meandering and the subsequent formation of cyclones and anticyclones in the field of atmospheric zonal jets is fully analogous to the generation of large-scale eddies at sea, which emphasizes the similarity of eddy formation physics. Meanders shed off the Gulf Stream have a characteristic size of 300 – 400 km and develop into cold cyclonic (warm anticyclonic) eddies to the right (left) of the main jet [12]. The generation of meanders and the subsequent separation of eddies is the result of the Rayleigh (Kelvin – Helmholtz) instability in a shear flow.

Observations constitute the main source of information about atmospheric circulation. Early observations were ground-based. Later, they were augmented by wind, pressure, and temperature measurements from balloons. The airborne meteorology and then meteorological satellites have essentially facilitated observing the processes in the Earth's atmosphere. Instruments carried by satellites work in visual, infrared, and radio wave ranges and provide observations of synoptic eddies and winds (flows), measure distributions of their temperature and air humidity, and estimate the magnitude and direction of winds (see, for example, Refs [49 – 51]). The efficiency of satellite meteorology is increasing owing to an increase in the number of satellites, but also due to the growth in the number of onboard instruments and their quality. Access to the data provided by geostationary and low-orbit satellites is continuously improving. The system of archiving meteorological observational data is providing users with fast and efficient access to the satellite data. This creates favorable conditions for exploring atmospheric dynamics which are uniformly monitored over the globe.

The contribution of laboratory experiments to today's theory of vortex structures and Rossby wave turbulence, as well as to studies of the analogy between Rossby and plasma drift waves is indisputable [9 – 11, 15, 37 – 43]. Atmospheric circulation is simulated in rotating vessels of cylindrical or annulus geometry. Laboratory experiments have aided in examining fundamental properties of Rossby waves and establishing a common physical view of Rossby and drift waves.

In circumstances where applying direct analytical methods faces insurmountable obstacles, the value of numerical

simulation is strongly increasing. In principle, the full atmospheric dynamics can be investigated in the framework of the complete system of hydrodynamical equations. However, it is unlikely that the complete system of equations subject to appropriate initial and boundary conditions will be solved in the foreseeable future because of its unwieldy character and complexity. This explains why there is interest in investigating both analytically and numerically the simplified (model) equations which explicitly take into account the main effects. Meteorological observations suggest (as was already mentioned by J G Charney in his classical paper [52]) that motion in synoptic eddies respects the following principles: it is quasihydrostatic over the height of the atmosphere, quasi-two-dimensional, nearly adiabatic, and quasigeostrophic. In this review we limit ourselves to hydrostatic vertical motion and purely two-dimensional synoptic scale motions in a horizontal plane. The influence of vertical velocity and variations in the atmospheric parameters in the vertical direction will be neglected. This approximation allows the original system of equations to be essentially simplified. Following it, simplified equations describing the most important processes in atmospheric dynamics have been obtained. V D Larichev and G M Reznik, studying Rossby waves in the framework of the nonlinear Charney–Obukhov equation [53], have shown that the vector nonlinearity (nonlinearity of the type $[\nabla a, \nabla b]_z$, where a and b are some scalar functions, the subscript z corresponds to the z-component of the vector product, and the z-axis coincides with the normal to the β-plane) occurring in this equation can play a localizing role. It compensates for dispersive spreading of a wave package, similarly to the scalar nonlinearity in the Korteweg–de Vries equation. Nonlinear stationary vortex structures form in a fluid as a result of such compensation.

As can be seen from synoptic maps, isobars do not coincide with isotherms in the real atmosphere. Such an atmosphere is called baroclinic, and is distinct from a barotropic atmosphere where the pressure is only dependent on density and isobars coincide with isotherms. The latter atmosphere yields a simpler analysis. Most of this review deals with Rossby waves in a barotropic atmosphere, and only one section discusses Rossby waves in a horizontally baroclinic atmosphere. A multilayer model frequently used to describe vertically baroclinic atmospheres cannot be applied to describe a horizontally baroclinic atmosphere. Long-wave perturbations in the horizontally baroclinic atmosphere were addressed in Refs [54–57] which show that in this case the atmosphere can be unstable, similarly to the vertically baroclinic one.

Progress on the theory of weakly turbulent Kolmogorov spectra of small-scale Rossby waves (with scales much shorter than the Rossby–Obukhov radius) is related in Refs [58, 59]. The weakly turbulent Kolmogorov spectra of large-scale Rossby waves and the locality of Rossby wave spectra were studied in Refs [60–62].

The goal of this review is to present the current state of research related to nonlinear Rossby waves, with a particular focus on the generation of large-scale eddies and zonal winds. A part of the review is dedicated to results of laboratory, numerical, and observational studies of Rossby waves in planetary atmospheres. This is of an auxiliary character and mainly considers the results which either support or reject theoretical constructions. Section 2 has to be considered as an introduction to the theory of nonlinear Rossby waves and zonal jets. It also presents the main hydrodynamical equations. Mechanisms of generating nonlinear vortex structures by a zonal wind are addressed in Section 3. The evolution of weak turbulent spectra of the Kolmogorov type due to interactions of Rossby waves is considered in Section 4. Section 5 deals with a mechanism responsible for the generation of large-scale zonal wind structures. It is attributed to the parametric instability of small-scale Rossby wave fluctuations in a turbulent atmosphere. Section 6 concludes.

2. Shallow water equations

2.1 Shallow water approximation

In order to describe the basic features characterizing the motion of large-scale structures, including synoptic eddies and zonal winds, in a rotating atmosphere or ocean one uses the so-called shallow water (thin layer) approximation. In this approximation, the atmosphere (or the ocean) is usually considered as a layer of homogeneous incompressible fluid rotating around an axis perpendicular to the fluid layer with angular speed $\Omega \sin\theta$ (with Ω being the Earth's angular rotation rate, and θ the local latitude).

From the hydrostatic balance along the rotation axis (the z-axis), an expression for the pressure at the underlying surface follows:

$$p(x, y) = \rho g H\,. \tag{1}$$

Here, g is the acceleration due to gravity, ρ is the constant density, $H = H_0 + \tilde{H}$ is the depth of fluid, and $\tilde{H}$ is the deviation of layer thickness from the equilibrium value H_0. In this approximation, the equations of continuity and motion of an ideal fluid are reduced to the shallow water equations

$$\frac{\mathrm{d}}{\mathrm{d}t} H = 0\,, \tag{2}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} \mathbf{v} = -g\nabla H + f[\mathbf{v}, \mathbf{e}_z]\,, \tag{3}$$

where $\mathbf{v}$ is the velocity, $\mathrm{d}/\mathrm{d}t = \partial/\partial t + \mathbf{v}\nabla$ is the convective derivative with respect to time, $f = 2\Omega\sin\theta$ is the Coriolis parameter (a doubled projection of the angular rotation speed on the local vertical), and $\mathbf{e}_z$ is the unit vector perpendicular to the (x, y) plane. Applying operator rot_z to Eqn (3) and using Eqn (2) one obtains the freezing-in condition for the generalized vorticity in the barotropic atmosphere (Ertel's theorem):

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{rot}_z\,\mathbf{v} + f}{H}\right) = 0\,. \tag{4}$$

If the relative vorticity $\mathrm{rot}_z\,\mathbf{v}$ has the same sign as the Coriolis parameter (positive, with counterclockwise circulation, in the northern hemisphere or negative, with clockwise circulation, in the southern hemisphere), the Coriolis force is directed from the center of the area under consideration. Such motions are called cyclones, and are characterized by low pressure at their centers. Flows with high pressure at their centers and the relative vorticity opposite to the planetary vorticity are called anticyclones.

For small Kibel (Rossby) numbers, omitting the small inertial term on the left-hand side of Eqn (3), one obtains the condition of geostrophic equilibrium, according to which the Coriolis force is balanced by the pressure gradient. In this approximation, the velocity of motion of perturbations in the

atmosphere is $\mathbf{v} \approx \mathbf{v}_{\mathrm{g}}$, where

$$\mathbf{v}_{\mathrm{g}} = \frac{1}{f\rho}\,[\mathbf{e}_z, \nabla p] \tag{5}$$

is referred to as the geostrophic velocity or the gradient wind velocity. A surprising property of motion in rapidly rotating fluids follows from the geostrophic approximation (5): it is executed along isobars, not perpendicular to them.

Rossby [19, 20] called attention to the fact that for synoptic scale waves in the quasigeostrophic approximation the correction to the geostrophic balance due to inertial effects can be of the same order as the correction due to the meridional variation of the Coriolis parameter (the so-called β-effect):

$$f \approx f_0 + \beta y + \frac{Ay^2}{2}\,, \qquad A = -\frac{f_0}{R^2}\,, \qquad |f_0| \gg |\beta y|\,,$$

where $\beta \equiv \partial f/\partial y \approx 2\Omega\cos\theta/R$, and R is the radius of the planet. Considering the weak perturbations we assume $|\tilde{H}| \ll H_0$ and $|\tilde{p}| \ll p_0$, where $p = p_0 + \tilde{p}$, p_0 is the equilibrium pressure, and $\tilde{p}$ is the perturbation. Replacing the fluid velocity with the geostrophic velocity (5), we transform the potential vorticity conservation equation (4) to the approximate form

$$\frac{\partial}{\partial t}(\hat{p} - r_{\mathrm{R}}^2\nabla_\perp^2\hat{p}) - v_{\mathrm{R}}\left(1 + \hat{p} + A\,\frac{y}{\beta}\right)\frac{\partial}{\partial x}\,\hat{p} - r_{\mathrm{R}}^4 f_0\{\hat{p}, \nabla_\perp^2\hat{p}\} = 0\,. \tag{6}$$

Here, $\hat{p} \equiv \tilde{p}/p_0$ is the dimensionless pressure perturbation, $r_{\mathrm{R}} = c_{\mathrm{s}}/f_0$ is the deformation radius (the Rossby – Obukhov radius), $c_{\mathrm{s}} = (p_0/\rho_0)^{1/2}$ is the isothermal speed of sound, $v_{\mathrm{R}} = r_{\mathrm{R}}^2\beta$ is the Rossby velocity, and

$$\{A, B\} \equiv [\nabla A, \nabla B]_z = \frac{\partial A}{\partial x}\frac{\partial B}{\partial y} - \frac{\partial A}{\partial y}\frac{\partial B}{\partial x}$$

is the Poisson brackets for scalar fields. The term proportional to $\hat{p}\,\partial\hat{p}/\partial x$ in Eqn (6) is referred to as the scalar nonlinearity, while the term with the Poisson bracket stands for the vector (or vortex) nonlinearity.

The scalar nonlinearity exceeds the vector one in large-scale eddies of a size comparable to the intermediate-geostrophic radius

$$r_{\mathrm{IG}} = \left(\frac{r_{\mathrm{R}}^2 f_0}{\beta}\right)^{1/3}.$$

Equation (6) with the scalar nonlinearity term was first derived in Ref. [63], which considered the influence of this term on the formation of large-scale (in excess of the deformation radius) circular eddies. This term is responsible for the cyclone – anticyclone asymmetry [15]. The term proportional to $Ay\,\partial\hat{p}/\partial x$ takes into account the meridional change in the velocity of Rossby wave propagation on large scales comparable to r_{IG}. The contribution from this term leads to the phenomenon known as twisting [9, 10]. Different portions of an eddy propagate at different velocities, which destroys the eddy on a time scale comparable to $f_0/\beta v_{\mathrm{R}}$. Introducing generalized (potential) vorticity

$$q = r_{\mathrm{R}}^2\nabla_\perp^2\hat{p} - \hat{p} + \frac{\beta y}{f_0} - \frac{y^2}{2R^2}\,, \tag{7}$$

Eqn (6) can be rewritten in the following form

$$\frac{\mathrm{d}q}{\mathrm{d}t} = 0\,, \tag{8}$$

where $\mathrm{d}/\mathrm{d}t = \partial/\partial t + \mathbf{v}_{\mathrm{g}}\nabla = \partial/\partial t + (\rho_0 f_0)^{-1}\{\tilde{p},\ \}$. Equation (8) implies conservation of generalized (potential) vorticity in a barotropic atmosphere in the intermediate-geostrophic approximation. When exploring Rossby vortices in the Earth's atmosphere one may neglect the scalar nonlinearity and the meridional dependence of the Rossby wave velocity. In this approximation, Eqn (6) is rearranged to the form

$$\frac{\partial}{\partial t}(\hat{p} - r_{\mathrm{R}}^2\nabla_\perp^2\hat{p}) - v_{\mathrm{R}}\,\frac{\partial}{\partial x}\,\hat{p} - r_{\mathrm{R}}^4 f_0\{\hat{p}, \nabla_\perp^2\hat{p}\} = 0\,. \tag{9}$$

Equation (9) corresponds to the conservation of potential vorticity q without the last term on the right-hand side of Eqn (7) (the quasigeostrophic potential vorticity). It is known as the Charney – Obukhov equation. A modified form of this equation, taking into account the vertical inhomogeneity of the atmosphere, was used by J G Charney [52] and A M Obukhov [2] to derive a principal scheme of weather forecasting. A solution to Eqn (9) possesses the symmetry: $\hat{p}(x, y, t) = -\hat{p}(-x, y, t)$. In opposition to the Korteweg – de Vries equation, which yields solutions in the form of one-dimensional solitons, the Charney – Obukhov equation does not support nonlinear one-dimensional or axisymmetric structures.

Taking the Fourier representation [$\hat{p} = p_{\mathbf{k}}\exp(-\mathrm{i}\omega_{\mathbf{k}}t + \mathrm{i}\mathbf{kr})$, where $p_{\mathbf{k}}$ is the Fourier amplitude and $\omega_{\mathbf{k}}$ and $\mathbf{k}$ are the frequency and wavevector, respectively], one obtains the dispersion relation

$$\omega_{\mathbf{k}} = -\frac{k_x v_{\mathrm{R}}}{1 + k^2 r_{\mathrm{R}}^2} \tag{10}$$

from the linearized Charney – Obukhov equation. The x- and y-axes of the adopted coordinate system are directed eastward and to the nearest pole, respectively. In the conditions of the Earth's atmosphere, the Rossby – Obukhov radius is comparable to the Earth's radius and the dispersion relation can be used in the approximate form

$$\omega_{\mathbf{k}} = -\frac{k_x v_{\mathrm{R}}}{k^2 r_{\mathrm{R}}^2}\,. \tag{11}$$

It is inferred from equations (10) and (11) that the phase velocity of the Rossby waves, $\omega_{\mathbf{k}}/k_x < 0$, is directed westward.

The Charney – Obukhov equation possesses two integrals being conserved (integrals of motion). They are (up to a dimensional coefficient) the energy integral

$$W = \int\left[\hat{p}^2 + r_{\mathrm{R}}^2(\nabla\hat{p})^2\right]\mathrm{d}^2x\,, \tag{12}$$

and enstrophy integral

$$H = \int\left[(\nabla\hat{p})^2 + r_{\mathrm{R}}^2(\nabla^2\hat{p})^2\right]\mathrm{d}^2x\,. \tag{13}$$

The integration in formulas (12) and (13) is carried over an arbitrary 'fluid' domain, the points of which move with the velocity $\mathbf{v}$.

2.2 Rossby waves in a barotropic atmosphere with a zonal wind. Stability of zonal flows

Rossby waves propagating in a barotropic atmosphere with a zonal wind — a stationary shear flow directed along the x-axis and varying in the meridional direction, $U(y)$ — obey, taking into account the viscous dissipation, the equation

$$\left(\frac{\partial}{\partial t}+U(y)\,\frac{\partial}{\partial x}\right)\nabla_\perp^2\,\hat{p}+\beta\,\frac{\partial}{\partial x}\,\hat{p}-\frac{\partial^2 U}{\partial y^2}\,\frac{\partial}{\partial x}\,\hat{p}$$
$$+\,r_{\mathrm{R}}^2 f_0\{\hat{p},\nabla_\perp^2\,\hat{p}\}=\nu\nabla_\perp^4\,\hat{p}\,. \tag{14}$$

Here, ν is the kinematic viscosity. Equation (14) represents the generalization of the Charney–Obukhov equation (9) for small-scale Rossby waves ($r_{\mathrm{R}}^2\nabla_\perp^2\gg 1$) to the case of an atmosphere with a zonal wind ($U\neq 0$) and with due regard for the viscosity effects.

The shear flows in hydrodynamics are frequently unstable. The presence of the third term proportional to $\mathrm{d}^2U/\mathrm{d}y^2$ on the left-hand side of Eqn (14) can contribute to the instability of the shear flow. In a linear approximation, for small perturbations of the form $\hat{p}(\mathbf{r},t)=\hat{p}(y)\exp(-\mathrm{i}\omega t+\mathrm{i}k_x x)$, Eqn (14) transforms into the Orr–Sommerfeld equation [64]

$$-\,\mathrm{i}\nu\left(\frac{\partial^2}{\partial y^2}-k_x^2\right)^2\hat{p}+(\omega-k_xU)\left(\frac{\partial^2}{\partial y^2}-k_x^2\right)\hat{p}$$
$$+\,k_x(U''-\beta)\,\hat{p}=0\,. \tag{15}$$

Here, $U''\equiv\mathrm{d}^2U/\mathrm{d}y^2$. Neglecting the contribution of viscosity effects, Eqn (15) is simplified to

$$\hat{p}''-k_x^2\,\hat{p}+\frac{k_x(U''-\beta)}{\omega-k_xU}\,\hat{p}=0\,, \tag{16}$$

where $\hat{p}''\equiv\mathrm{d}^2\hat{p}/\mathrm{d}y^2$. Equation (16) is the modification of the well-known Rayleigh equation [65] for $\beta\neq 0$. The stability of a plane-parallel shear flow obeying Eqn (16) was the subject of thorough research initialized by works [65–67]. The fulfilment of the condition

$$U''(y_{\mathrm{c}})-\beta=0 \tag{17}$$

at some point $y=y_{\mathrm{c}}$ of the shear flow is the necessary condition of instability. When it is fulfilled, differential equation (16) may remain regular even if the resonance condition $\omega=k_xc$, where $c=U(y_{\mathrm{c}})$, holds at some point within the shear flow. For such vibrations, Rayleigh equation (16) assumes the form

$$\hat{p}''-k_x^2\,\hat{p}+F(y)\,\hat{p}=0\,. \tag{18}$$

Here, $F(y)=(U''-\beta)/\big[c-U(y)\big]$. Equation (18) has a discrete set of eigenfunctions $\hat{p}^{(n)}$, eigenvalues $k_x^{(n)}$, and frequencies $\omega^{(n)}=k_x^{(n)}c$, provided $F(y)>0$. This inequality is the sufficient condition of flow instability. The sufficient condition reduces to satisfying the inequality [68]

$$F(y_{\mathrm{c}})=-\frac{U'''(y_{\mathrm{c}})}{U'(y_{\mathrm{c}})}>0\,. \tag{19}$$

In the Earth's atmosphere, β commonly greatly exceeds the quantity U'' in large-scale zonal flows and thus wind remains stable most of the time. However, periodically wave breaking events occur such that the condition $\beta\approx U''$ is satisfied in some vicinity of the layer $y=y_{\mathrm{c}}$. This triggers instability over some period of time, after which the zonal wind rearranges and becomes stable again.

2.3 Rossby waves in a horizontally baroclinic atmosphere

Let the temperature T, potential temperature Θ, and pressure in a horizontally baroclinic atmosphere be expanded as sums of equilibrium values and weak perturbations thereof, $\tilde{T}(t,x,y)$, $\tilde{\Theta}(t,x,y)$, and $\tilde{p}(t,x,y)$:

$$T=T_0+\tilde{T}(t,x,y)\,,\qquad \Theta=\Theta_0+\tilde{\Theta}(t,x,y)\,,$$
$$p=p_0+\tilde{p}(t,x,y)\,. \tag{20}$$

In this case, from Eqn (6) it follows that [56, 57]

$$\frac{\partial}{\partial t}(\hat{p}-r_{\mathrm{R}}^2\nabla_\perp^2\,\hat{p})-v_{\mathrm{R}}\left(1+\hat{T}+A\,\frac{y}{\beta}\right)\frac{\partial}{\partial x}\,\hat{p}$$
$$-\,r_{\mathrm{R}}^4 f_0\{\hat{p},\nabla_\perp^2\,\hat{p}\}+r_{\mathrm{R}}^2 f_0\{\hat{p},\hat{\Theta}\}=0\,, \tag{21}$$

where the parameters $\hat{T}=\tilde{T}/T_0$, $\hat{p}=\tilde{p}/p_0$, and $\hat{\Theta}=\tilde{\Theta}/\Theta_0$ stand for dimensionless perturbations of the temperature, pressure, and potential temperature. The potential temperature is a unique function of the entropy and is connected to the temperature and pressure via the relationship

$$\Theta=T\left(\frac{p_0}{p}\right)^{(\gamma-1)/\gamma}.$$

In this section, the notation $r_{\mathrm{Rs}}=c_{\mathrm{s}}/f_0$ for the Obukhov scale is introduced, with $c_{\mathrm{s}}=(\gamma p_0/\rho_0)^{1/2}$ being the adiabatic speed of sound. Introducing the potential vorticity in the form (7), Eqn (21) can be rearranged as

$$\frac{\mathrm{d}q}{\mathrm{d}t}=r_{\mathrm{Rs}}^2 f_0\{\hat{p},\hat{\Theta}\}-v_{\mathrm{R}}\,\hat{T}\,\frac{\partial}{\partial x}\,\hat{p}\,. \tag{22}$$

Here, $\mathrm{d}/\mathrm{d}t=\partial/\partial t+\mathbf{v}_{\mathrm{g}}\nabla$, as in the preceding section. In such an atmosphere, the potential vorticity is not conserved, in contrast to the barotropic atmosphere. This makes possible spontaneous generation of vortices from nonclosed streamlines in the horizontally baroclinic atmosphere.

The real atmosphere is characterized by large-scale gradients of potential temperature and pressure. In general, they are oriented meridionally. Let us assume that the equilibrium potential temperature and pressure contain large-scale equilibrium inhomogeneities $\kappa_\Theta y$ and $\kappa_p y$ in the meridional direction:

$$\hat{\Theta}_0=1+\kappa_\Theta y\,,\qquad \hat{p}_0=1+\kappa_p y\,, \tag{23}$$

whereas the scale of weak perturbations $\tilde{\theta}(t,x,y)$ and $\tilde{p}(t,x,y)$ is essentially smaller than κ_Θ^{-1} and κ_p^{-1}, respectively. In agreement with Eqn (20), writing the dimensionless potential temperature and pressure as $\hat{\Theta}=\hat{\Theta}_0(y)+\tilde{\theta}(t,x,y)$ and $\hat{p}=\hat{p}_0(y)+\tilde{p}(t,x,y)$, one obtains from the law of conservation of the potential temperature (which is an unambiguous function of the entropy), viz.

$$\frac{\mathrm{d}}{\mathrm{d}t}\,\Theta=0 \tag{24}$$

and the potential vorticity conservation equation (21), neglecting the scalar nonlinearity [55], the following equa-

tions

$$\frac{\partial}{\partial t}\,\tilde{\theta} + \kappa_\Theta\,\frac{\partial\tilde{p}}{\partial x} - \kappa_p\,\frac{\partial\tilde{\theta}}{\partial x} + \{\tilde{p},\tilde{\theta}\} = 0\,, \tag{25}$$

$$\begin{aligned}&\frac{\partial}{\partial t}(\tilde{p} - r_{\mathrm{Rs}}^2\nabla_\perp^2\tilde{p}) - (v_{\mathrm{R}} - f_0 r_{\mathrm{Rs}}^2\kappa_\Theta)\,\frac{\partial}{\partial x}\,\tilde{p}\\ &\quad + f_0 r_{\mathrm{Rs}}^2\kappa_p\left(r_{\mathrm{Rs}}^2\,\frac{\partial}{\partial x}\,\nabla^2\tilde{p} - \frac{\partial}{\partial x}\,\tilde{\theta}\right)\\ &\quad - f_0 r_{\mathrm{Rs}}^4\{\tilde{p},\nabla^2\tilde{p}\} + f_0 r_{\mathrm{Rs}}^2\{\tilde{p},\tilde{\theta}\} = 0\,.\end{aligned} \tag{26}$$

This is the system of nonlinear equations describing a horizontally baroclinic atmosphere with equilibrium linear gradients of pressure and potential temperature. The system of equations (25) and (26) conserves the energy integral

$$E \propto \int\left[\tilde{p}^2 + r_{\mathrm{Rs}}^2(\nabla\tilde{p})^2 - \frac{\kappa_p}{\kappa_\Theta}\,\tilde{\theta}^2\right]\mathrm{d}^2x\,. \tag{27}$$

It is of interest to compare it with the energy integral (12). Expression (27) shows that the energy is not always a positive definite quantity. It may become negative if gradients of the potential temperature and pressure have the same sign. In this case, the atmosphere can be unstable. In the linear approximation, the system of equations (25) and (26) implies the following dispersion equation

$$\begin{aligned}\omega_\pm = &-\frac{k_x}{2(1+k^2r_{\mathrm{Rs}}^2)}\\ &\times\left[v_{\mathrm{R}} - f_0 r_{\mathrm{Rs}}^2(\kappa_\Theta - \kappa_p) + f_0 r_{\mathrm{Rs}}^4 k^2(\kappa_p^* + \kappa_p) \pm D^{1/2}\right],\end{aligned} \tag{28}$$

where

$$\kappa_p^* = \frac{\kappa_p}{\gamma}\,, \tag{29}$$

$$\begin{aligned}D = k_x^2\Big\{&\left[v_{\mathrm{R}} - f_0 r_{\mathrm{Rs}}^2(\kappa_\Theta - \kappa_p^*) - f_0 r_{\mathrm{Rs}}^4 k^2(\kappa_p^* - \kappa_p)\right]^2\\ &- 4f_0^2 r_{\mathrm{Rs}}^6 k^2\kappa_\Theta\kappa_p^*\Big\}\,.\end{aligned} \tag{30}$$

According to equality (30), the radicand in the dispersion equation can get negative for some value of k if κ_Θ and κ_p have the same sign. This condition of horizontally baroclinic instability coincides with the other one, which may be derived from the condition that the energy integral (27) is positive definite.

3. Vortex structures

3.1 Dipole vortices

To explore stationary waves traveling along the x-axis with the velocity u in a shallow barotropic atmosphere, the Charney – Obukhov equation (9) can be cast in the form

$$\left\{\nabla^2\hat{p} - \Lambda\hat{p},\,\hat{p} + \frac{y}{b}\right\} = 0 \tag{31}$$

on introducing the variable $\eta = x - ut$. Here, constants Λ and b are defined as follows:

$$\Lambda = \frac{1}{r_{\mathrm{R}}^2}\left(1 + \frac{v_{\mathrm{R}}}{u}\right), \qquad b = \frac{r_{\mathrm{R}}^2 f_0}{u}\,. \tag{32}$$

The equation

$$\nabla^2\hat{p} - \Lambda\hat{p} = F\left(\hat{p} + \frac{y}{b}\right), \tag{33}$$

where F is an arbitrary function of its arguments, provides a particular solution to the nonlinear Eqn (31). Examining solutions to equation (33), we select F to be a linear function, so that

$$\nabla^2\hat{p} - \Lambda\hat{p} = C\left(\hat{p} + \frac{y}{b}\right), \tag{34}$$

where C is an arbitrary constant. Performing the analysis of vortex structures described by Eqn (34), we will also use, together with the Cartesian coordinates x and η, polar coordinates $r = (x^2 + \eta^2)^{1/2}$ and $\vartheta = \arctan(\eta/x)$. Following Ref. [53], we imagine that the vortex consist of internal and external parts separated by some boundary $r = a$, where a is a constant termed the vortex radius. In the outer part of the vortex $C = 0$, while in the inner part $C \neq 0$. A particular solution to Eqn (39) is a so-called dipole vortex

$$\hat{p}(r,\vartheta) = \Phi(r)\cos\vartheta\,. \tag{35}$$

In this case, the function $\Phi(r)$ in the outer part of the vortex (for $r > a$) is given by

$$\Phi(r) = \Phi(a)\,\frac{K_1(r\beta/a)}{K_1(\beta)}\,. \tag{36}$$

Inside the vortex (for $r < a$), this function assumes the form

$$\Phi(r) = \Phi(a)\left[\left(1 - \frac{\beta^2}{\gamma^2}\right)\frac{r}{a} - \frac{\beta^2}{\gamma^2}\,\frac{J_1(r\gamma/a)}{J_1(\gamma)}\right]. \tag{37}$$

The constants β and γ are linked to Λ and C by the relationships

$$\beta^2 = a^2\Lambda \quad\text{and}\quad \gamma^2 = -a^2(\Lambda + C)\,. \tag{38}$$

It is apparent that for $\hat{p}$ to be bounded at infinity ($r \to \infty$), the inequalities $\beta^2 > 0$ and $\Lambda > 0$ must hold. According to Eqn (32), vortices propagating eastward can have, in general, any velocity ($u > 0$), whereas those propagating westward should move faster than the Rossby velocity. From the continuity conditions for p, $\partial p/\partial r$, $\nabla^2\hat{p}$, and $\partial\nabla^2\hat{p}/\partial r$ at the vortex boundary ($r = a$), the vortex parameter matching condition follows:

$$\frac{K_2(\beta)}{\beta K_1(\beta)} = -\frac{J_2(\gamma)}{\gamma J_1(\gamma)}\,. \tag{39}$$

An approximate solution of dispersion equation (39) has the form $\beta \approx 3.9 + 1.2\gamma^2/(3.4 + \gamma^2)$. In a dipole vortex with this matching condition, both the energy [see Eqn (12)] and enstrophy [see Eqn (13)] of the vortex are conserved.

3.2 Vortex streets

Let us turn to the description of nonlinear vortex structures generated in the atmosphere by a zonal wind. There is a close physical analogy between the resonant interaction of Rossby waves with zonal winds, described by Eqn (14), and the resonant interaction in a system of self-gravitating bodies

(see, e.g., Ref. [32]) or oscillations in plasmas [69]. This analogy was first addressed in Refs [70–72]. It allows the resonance phenomena occurring in media that are, at first glance, completely different to be considered in the same framework. For waves interacting with plane-parallel flows, nonlinear effects show themselves primarily in the vicinity of the resonance layer. It is therefore possible to assume that the fluid vibrations are linear outside the resonance layer, while nonlinearity governs the structure of a solution within the layer. The solutions of the Rayleigh equation are found as the limit of solutions of the Orr–Sommerfeld equation for perturbations and an infinitesimal viscosity ($\nu \to 0$). The solutions of inviscid (Rayleigh) equations are not, however, unique and only accounting for finite viscosity allows selecting one of them.

The choice of a valid branch of a multivalued solution to the Rayleigh equation in the vicinity of the branch point $y = y_c$ follows the Lin contour selection rule [67] which is analogous to the contour selection rule in plasma physics (the Landau detour [69]). The existence of resonance points with the respective path-tracing rules determines a mechanism of energy exchange between waves (perturbations) and the mean flow. It is not immediately connected with viscous dissipation and exists even in an ideal fluid. Fluid particles 'trapped' by the wave in the resonance layer $y = y_c$ move along the x-axis in the westward direction with the velocity $U(y_c) = v_R$. From Eqn (14) it follows that for a quasistationary state under the conditions of resonance the following equation is valid:

$$\{\hat{p}, \nabla^2 \hat{p}\} = 0\,. \tag{40}$$

The solution to this equation is written out as

$$\nabla^2 \hat{p} = F(\hat{p})\,, \tag{41}$$

where $F(\hat{p})$ is an arbitrary function of its argument. Equation (41) implies a particular case of Eqn (34) and coincides with the condition of vorticity conservation in an incompressible inviscid fluid, $\nabla^2 \psi = F(\psi)$ (ψ is the streamfunction), which follows from the Navier–Stokes equation. The solution of Eqn (41) was obtained in Ref. [73] by matching asymptotic expansions on both sides of the singular point $y = y_c$ with a simultaneous account of weak viscosity and nonlinearity effects. This solution [74] corresponds to the function

$$F(\hat{p}) = k^2 K \exp\left(-\frac{2\hat{p}}{K}\right). \tag{42}$$

The meaning of parameters k and K will be clarified below. For this function $F(\hat{p})$, Eqn (41) has the solution

$$\hat{p} = K \ln\left[C \cosh kx + (C^2 - 1)^{1/2} \cos ky\right], \tag{43}$$

where the parameter K characterizes the amplitude of the vortex street, and $2\pi/k$ sets the size of vortices along the y-axis. Expressions for the velocity components follow from Eqns (5) and (43):

$$v_x = v_R k r_R K \frac{C \sinh ky}{C \cosh ky + (C^2 - 1)^{1/2} \cos kx}\,, \tag{44}$$

$$v_y = v_R k r_R K \frac{(C^2 - 1)^{1/2} \sin kx}{C \cosh ky + (C^2 - 1)^{1/2} \cos kx}\,. \tag{45}$$

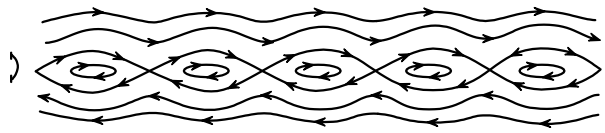

Figure 1. Streamlines in the form of 'cat eyes' in the vicinity of the resonance level (in a reference frame moving with the wave).

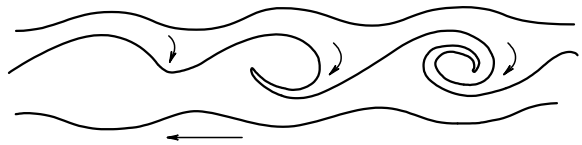

Figure 2. Development of meanders in the vicinity of the resonant level [76]. Viscosity modifies them into a structure of the 'cat eye' type.

Here, C is a constant larger than one. If $C = 1$, solution (45) describes a zonal flow (the x- and y-axes are directed to the east and north, respectively):

$$v_x = v_R k r_R K \tanh ky\,, \qquad v_y = 0\,. \tag{46}$$

Solutions (44) and (45) with closed streamlines resembles 'cat eyes' and were first obtained by Kelvin. Figure 1 illustrates the trajectories of fluid particles in a vortex street of the cat eye type. Such structures are typical for phase space portraits of nonlinear resonance. In plasma physics, cat eye structures occur when one considers phase space trajectories of particles in the field of a monochromatic plasma wave [75].

Numerical simulation of the temporal evolution of instability in flows of ideal fluids, made with due account for nonlinear effects at the resonance level [76], indicated that the entity of spiral vortices growing with time, meanders, develop in the vicinity of $y = y_c$. Figure 2 illustrates schematics of this development for a zonal flow [76]. Numerical modeling of perturbation evolution in the framework of the Orr–Sommerfeld equation or Eqn (14) is the subject of numerous works. In particular, Refs [77–82] explore the dynamics of a shear flow in a barotropic atmosphere in the nonlinear phase before quasistationary state is reached in the region of instability saturation. The weak viscosity of the atmosphere facilitates the formation of vortex streets of the cat eye type from the meanders. Figures 3 and 4 show schematically numerical results [81] which illustrate particle trajectories in stationary nonlinear structures generated by zonal shear flows in a barotropic atmosphere (or ocean). Vortex streets resembling cat eyes are formed in the vicinity of resonance regions where the flow velocity is compared with the local Rossby velocity ($U = c$).

Satellite observations contribute to studies of the formation and subsequent evolution of vortex streets in zonal winds. To interpret such observations we have selected the near-equatorial part of the Pacific Ocean with the largest zonal extent during the summer–autumn period of the most active tropical cyclogenesis spanning the months from August to November. The east–west-directed zonal wind in the near-equatorial Earth's atmosphere is most unstable during this season.

By way of example, Figs 5 and 6 display fragments of the Earth's microwave brightness temperature field at 19.35 GHz from the electronic Global-Field collection compiled in the

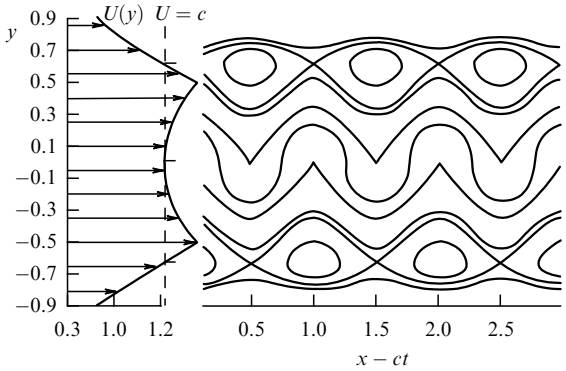

Figure 3. Trajectories of fluid particles in the vicinity of the resonance layer in appropriate shear flows [81].

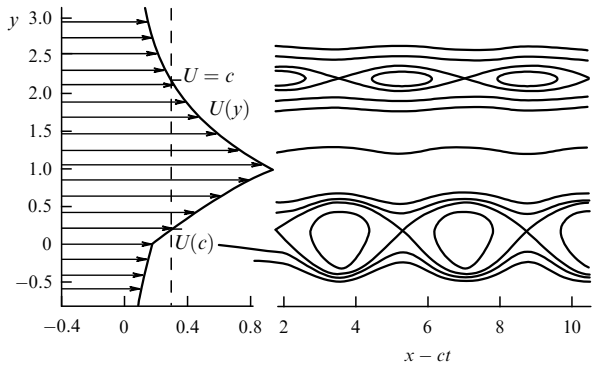

Figure 4. Trajectories of fluid particles in the vicinity of the resonance layer in particular shear flows [81].

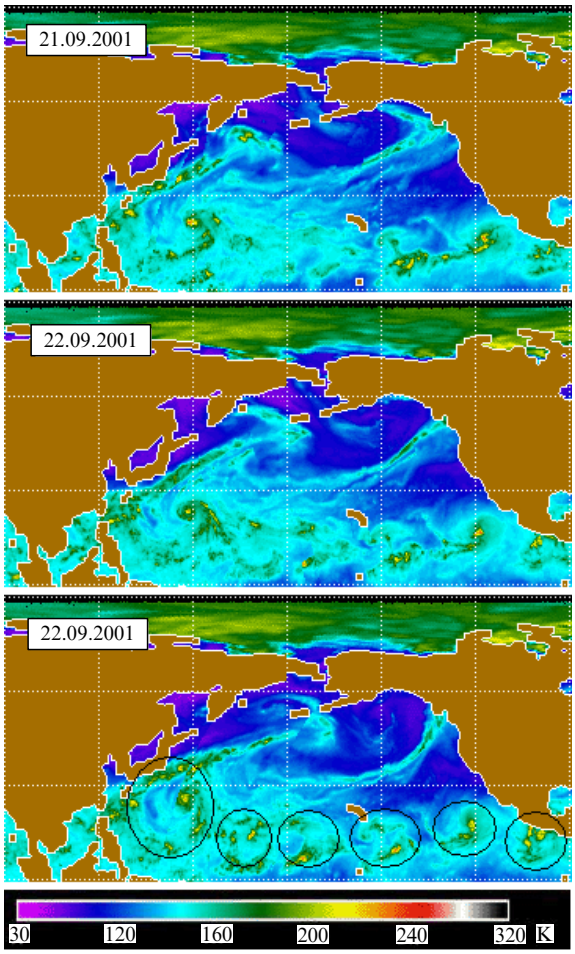

Figure 5. A chain of vortices in an eastern zonal flow over the North Pacific, formed on 21 – 22 September 2001 near 15° N.

Space Research Institute of the Russian Academy of Sciences (SRI RAS) [83] based on satellite monitoring data gathered within the Defense Meteorological Satellite Program (DMSP). Radiometric instruments SSM/I (Special Sensor Microwave/Imager) on board satellites of DMSP series supply global operative information about the state of the atmosphere – ocean system in the microwave range. The orbits of satellites of this series and viewing angles of instruments are such that daily data collected from a single satellite do not cover the entire globe but leave large gaps especially noticeable in low latitudes (the scan width is 1400 km, while gaps in the equatorial belt reach approximately 1200 km). As a result, an estimated 25% of the planet's surface is void of data. A special algorithm of intertrack and cross-instrument equilibration and augmentation was developed in the Laboratory for Climate Research of the SRI RAS. It supplies two full microwave images of the Earth daily (based on data from all satellites of the series to fill the gaps). A good spatial – temporal resolution of the fields of microwave brightness temperature in the Global-Field collection (two realizations at a 0.5 by 0.5 degree grid daily from 1995 to 2007) makes it possible to trace the formation and evolution of intense large-scale atmospheric eddies, together with studying their structure.

Radiometric instruments SSM/I work in a passive regime by receiving microwave radiation coming from the Earth's surface, which carries information about various physical objects. If the atmosphere were void of tracer gases and were perfectly dry and clear, the SSM/I instruments would have measured the radio brightness temperature of the surface of the world's oceans and land. Yet the atmosphere contains fractions which have resonant absorption lines in the radiowave range at certain frequencies. The radiometric SSM/I instruments register radiation on frequencies of 19.35, 22.24, 37.00, and 85.50 GHz, which carries information on the presence of water in various forms in the Earth's atmosphere. These can be water vapor, water droplets of various sizes, snow, ice crystals and some others.

The microwave brightness temperature is associated with the depth-integrated contents of the respective fraction in the atmosphere, i.e., with the water vapor content, cloud liquid water, and precipitation over water and land. Microwave brightness temperature fields on frequencies of 19.35, 22.24, and 37.00 GHz visualize the content of water vapor and water in droplets (the fields are well correlated at these frequencies). Figures 5 and 6 show fragments of the microwave brightness temperature fields of the Earth at 19.35 GHz over the North Pacific (in the Mercator projection with a superimposed 30° grid). The colorbar of brightness temperature in degrees Kelvin is also displayed there. The continents are readily recognizable. The patches of gray against the almost black background of the surface of the Pacific Ocean characterize the brightness temperature of the troposphere over this part of the ocean and, consequently, the depth-integrated content of water vapor and droplets in the troposphere. Since the droplets and vapor are 'frozen' in air streams, the fields of radio brightness temperature visualize instantaneous

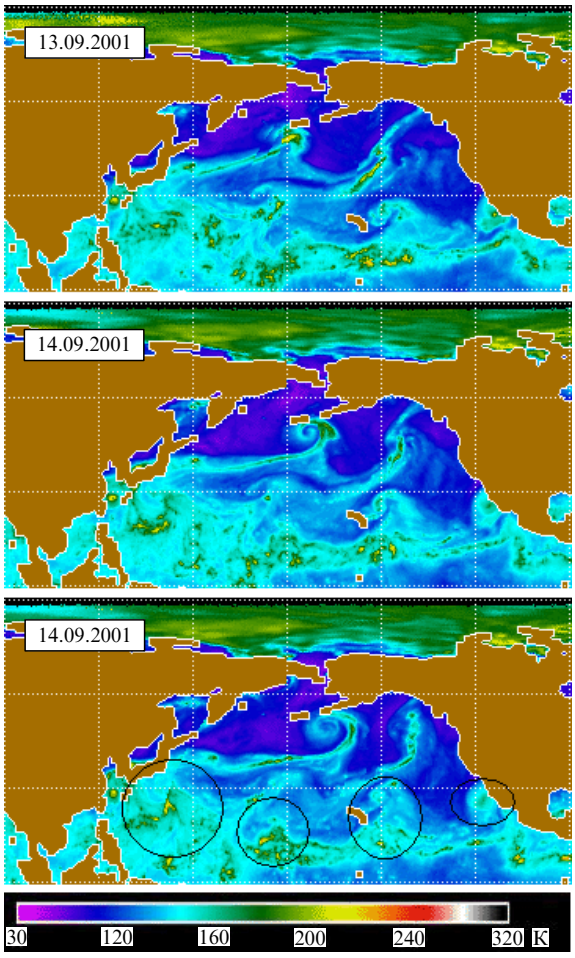

Figure 6. A chain of vortices analogous to those depicted in Fig. 5, formed on 13–14 September 2001 near 25–30° N.

imprints of atmospheric motions. In particular, they permit studying the structure and dynamics of such intense moist atmospheric flows as the equatorial zonal current (with its instabilities and eddy structures) and tropical cyclones.

Figure 5 presents three successive realizations of the brightness temperature over the Pacific, separated by 12-hour time intervals. The first fragment corresponds to the evening of 21 September 2001, while the second and third ones correspond, respectively, to the morning and evening time of the next day. Within the zonal jet which has a cyclonic shear one may distinguish a chain of five cyclonic eddies moving from east to west at a latitude of about 15° N (from 5 to 20° N). They are of approximately equal intensity except for the first, most westward one which constitutes the well-formed cyclonic eddy by that time. Its center already contains a tropical cyclone. Upon impinging on the Asia continent, the cyclonic eddy will be shifted to a higher latitude by the action of the Coriolis force and trapped in the region of westerlies at mid- and mid-high latitudes where the wind shear is anticyclonic. Later on, the direction of eddy propagation will reverse and the eddy itself will weaken. Figure 5 demonstrates the evolution of similar eddy precursors in this phase at midlatitudes.

A similar chain of vortices formed several days earlier over the same part of the Pacific at a higher latitude of about 25–30° N is displayed in Fig. 6. In the same manner as in Fig. 5, the three panels are separated by 12-hour intervals. The larger size of cyclonic eddies at this latitude with respect to those in Fig. 5 is apparently linked to a larger spatial scale of the zonal wind shear at their latitude.

Similar chains of vortices can be observed over water areas of other oceans within periods of active cyclogenesis. The number of vortices in a chain is limited by the finite sizes of ocean basins. Thus, for example, chains constituting at most of 2–3 eddies are observed over the Atlantic.

4. Kolmogorov spectra of weak turbulence

4.1 Basic equations of weak turbulence

Consider a weak turbulence evolving due to three-wave interaction and obeying the kinetic equation [75, 84]

$$\frac{\partial N_{\mathbf{k}}}{\partial t} \propto \int U(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2)\big(N_{\mathbf{k}1}N_{\mathbf{k}2} - N_{\mathbf{k}}N_{\mathbf{k}1}\,\mathrm{sign}\,\omega_{\mathbf{k}}\omega_{\mathbf{k}2} - N_{\mathbf{k}}N_{\mathbf{k}2}\,\mathrm{sign}\,\omega_{\mathbf{k}}\omega_{\mathbf{k}1}\big)\,\delta(\omega_{\mathbf{k}} - \omega_{\mathbf{k}1} - \omega_{\mathbf{k}2}) \times \delta(\mathbf{k} - \mathbf{k}_1 - \mathbf{k}_2)\,\mathrm{d}\mathbf{k}_1\,\mathrm{d}\mathbf{k}_2\,. \tag{47}$$

Here, $N_{\mathbf{k}}$ is the 'number of quanta' (or the 'wave action density'), and the integrand is defined as follows:

$$U(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2) = \big|V(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2)\big|^2\,, \tag{48}$$

where $V(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2)$ is the matrix element of interaction, which possesses some specific symmetry properties (see, for example, Refs [75, 84, 85]). A constant multiplier which depends on how $N_{\mathbf{k}}$ and $\omega_{\mathbf{k}}$ are normalized is omitted on the right-hand side of relationship (47). We assume that the matrix element $V(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2)$ is scale invariant, so that

$$V(\varepsilon_x k_x, \varepsilon_y k_y; \varepsilon_x k_{x1}, \varepsilon_y k_{y1}; \varepsilon_x k_{x2}, \varepsilon_y k_{y2}) = \varepsilon_x^u \varepsilon_y^v V(k_x, k_y; k_{x1}, k_{y1}; k_{x2}, k_{y2}) \tag{49}$$

with the similarity indices u and v. Additionally, it is assumed that the dispersive part of the wave frequency for weakly dispersive ($\omega_{\mathbf{k}} \approx k_x v_{\mathrm{R}}$) or strongly dispersive waves is also scale invariant with the similarity indices a and b. Under these assumptions, and also regarding the expression for the number of quanta as scale invariant with the similarity indices α and β, the kinetic equation for waves (47) can be represented in the following form [60]:

$$\frac{\partial N_{\mathbf{k}}}{\partial t} \propto |k_x|^{2\alpha+2u+1-a}\,|k_y|^{2\beta+2v+1-b} \tag{50}$$

or

$$\frac{\partial D_{\mathbf{k}}^{(i)}}{\partial t} + \nabla \mathbf{P}^{(i)}(\mathbf{k}) = 0\,, \tag{51}$$

where $i = 1$ or 2, and

$$D_{\mathbf{k}}^{(1)} \equiv |\Omega_{\mathbf{k}}| N_{\mathbf{k}}\,, \qquad D_{\mathbf{k}}^{(2)} \equiv |k_x| N_{\mathbf{k}}\,,$$

$$\mathbf{P}^{(1)}(\mathbf{k}) \propto |k_x|^{2(\alpha+u+1)}\,|k_y|^{2(\beta+v+1)}\left(\frac{1}{|k_y|}\,,\,\frac{1}{|k_x|}\right), \tag{52}$$

$$\mathbf{P}^{(2)}(\mathbf{k}) \propto |k_x|^{2\alpha+2u+3-a}\,|k_y|^{2\beta+2v+2-b}\left(\frac{1}{|k_y|}\,,\,\frac{1}{|k_x|}\right). \tag{53}$$

For weakly dispersive waves $D_{\mathbf{k}}^{(1)}$ has the sense of enstrophy (or the dispersive part of the energy), while $D_{\mathbf{k}}^{(2)}$

has the sense of wave energy. Accordingly, Eqn (51) with the index $i = 1$ or $i = 2$ corresponds to the law of enstrophy or energy conservation, respectively, while $\mathbf{P}^{(1)}(\mathbf{k})$ and $\mathbf{P}^{(2)}(\mathbf{k})$ are the enstrophy and energy fluxes. From the condition that the enstrophy or energy fluxes be constant, one finds the similarity indices for the number of quanta:

$$\alpha^{(1)} = -(1+u)\,, \qquad \beta^{(1)} = -(1+v)\,, \tag{54}$$

$$\alpha^{(2)} = \frac{a}{2} - \left(\frac{3}{2} + u\right), \qquad \beta^{(1)} = \frac{b}{2} - (1+v)\,. \tag{55}$$

Thus, the existence of stationary power-law type solutions of the wave kinetic equation leans upon the requirement that the dispersion and matrix element of the interaction be scale-invariant functions of wave vectors.

4.2 Wave interaction matrix element

We expand the pressure $\hat{p}$ in Fourier harmonics:

$$\hat{p} = \sum_{\mathbf{k}} p_{\mathbf{k}}(t) \exp\left(\mathrm{i}\mathbf{k}\mathbf{r} - \mathrm{i}\omega_{\mathbf{k}} t\right) + \text{c.c.}, \tag{56}$$

where c.c. stands for complex conjugate expression, $p_{\mathbf{k}}(t)$ is the amplitude of the Fourier harmonic which is assumed to be a slowly varying function of time, and $\omega_{\mathbf{k}}$ is the frequency of a Fourier harmonic with the wave vector $\mathbf{k}$, defined by the linear dispersion relation (11).

The following dynamic equation corresponds to the Charney – Obukhov equation (9) [58, 59]:

$$\frac{\partial p_{\mathbf{k}}}{\partial t} \propto \sum_{\mathbf{k}_1+\mathbf{k}_2=\mathbf{k}} [\mathbf{k}_1, \mathbf{k}_2]_z \, \frac{k_2^2 - k_1^2}{1 + k^2 r_{\mathrm{R}}^2} \, p_{\mathbf{k}1} \, p_{\mathbf{k}2} \times \exp\left[-\mathrm{i}(\omega_{\mathbf{k}1} + \omega_{\mathbf{k}2} - \omega_{\mathbf{k}})t\right]. \tag{57}$$

According to Eqn (12), the spectral energy density has the form

$$W_{\mathbf{k}} \propto (1 + k^2 r_{\mathrm{R}}^2)\,|p_{\mathbf{k}}|^2\,. \tag{58}$$

The number $N_{\mathbf{k}}$ of quanta determined from the condition $N_{\mathbf{k}} \propto W_{\mathbf{k}}/\omega_{\mathbf{k}}$ is given by

$$N_{\mathbf{k}} \propto (1 + k^2 r_{\mathrm{R}}^2)^2 |p_{\mathbf{k}}|^2 |k_x|\,. \tag{59}$$

Employing relationship $N_{\mathbf{k}} \propto |C_{\mathbf{k}}|^2$, we introduce the normalized complex wave amplitude

$$C_{\mathbf{k}} \propto (1 + k^2 r_{\mathrm{R}}^2)\,|p_{\mathbf{k}}|\,|k_x|^{-1/2}\,. \tag{60}$$

Using relationship (60) and the frequency synchronism condition $\omega_{\mathbf{k}} = \omega_{\mathbf{k}1} + \omega_{\mathbf{k}2}$, one transforms dynamic equation (57) into the canonical form [75]

$$\mathrm{i}\,\frac{\partial C_{\mathbf{k}}}{\partial t} = \sum_{\mathbf{k}_1+\mathbf{k}_2=\mathbf{k}} V(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2) C_{\mathbf{k}1} C_{\mathbf{k}2} \exp\left[-\mathrm{i}(\omega_{\mathbf{k}1} + \omega_{\mathbf{k}2} - \omega_{\mathbf{k}})t\right]. \tag{61}$$

Using this, one gets the following expression for the interaction matrix element:

$$V(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2) \propto |k_x k_{x1} k_{x2}|^{1/2} \times \left(\frac{k_{x1}}{1 + k_1^2 r_{\mathrm{R}}^2} + \frac{k_{x2}}{1 + k_2^2 r_{\mathrm{R}}^2} - \frac{k_x}{1 + k^2 r_{\mathrm{R}}^2}\right). \tag{62}$$

References [58, 59, 62] derived the matrix element (62) in the framework of Hamilton's formalism. Let us separately consider short-wave ($k^2 r_{\mathrm{R}}^2 \gg 1$) and long-wave ($k^2 r_{\mathrm{R}}^2 \ll 1$) turbulence. Additionally, we limit ourselves to the case of waves with $k_y \gg k_x$. According to papers [60, 62], the main portion of energy carried by Rossby waves is contained in waves with $k_y \gg k_x$.

4.3 Short-wave turbulence

In a short-wave limit $k^2 r_{\mathrm{R}}^2 \gg 1$ and for $k_y \gg k_x$, the Rossby wave frequency and interaction matrix element behave as

$$\omega_{\mathbf{k}} \propto k_x k_y^{-2}\,, \tag{63}$$

$$V(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2) \propto |k_y k_{y1} k_{y2}|^{1/2} \left(\frac{1}{k_{x1}} + \frac{1}{k_{x2}} - \frac{1}{k_x}\right). \tag{64}$$

Hence, the exponents of scale invariance for the frequency and matrix element are $a = 1$, $b = -2$, $u = 3/2$, and $v = -1$. These exponents imply the following energy spectra:

$$W_{\mathbf{k}} \propto k_x^{-3/2} k_y^{-2}\,, \tag{65}$$

$$W_{\mathbf{k}} \propto k_x^{-3/2} k_y^{-3}\,. \tag{66}$$

Spectra (65) and (66) are linked to energy and enstrophy fluxes, respectively. Numerical simulation [21, 86] of short-wave isotropic ($k_x = k_y$) turbulence of nonlinearly interacting waves obeying the Charney – Obukhov equation yields spectral forms similar to $W_{\mathbf{k}} \propto k_\perp^{-4}$. Close to the validity limits, in isotropic approximation with $k_x \approx k_y \approx k_\perp$, it follows from expressions (65) and (66) that $W_{\mathbf{k}} \propto (k_\perp^{-7/2}, k_\perp^{-9/2})$. Thus, the numerical value of the exponent of the isotropic spectrum lies between the two anisotropic Kolmogorov spectra. An additional analysis of the spectra carried out in Refs [60, 87] shows that spectrum (65) linked to the energy flux is local, while the spectrum given by expression (66), which is linked to the enstrophy flux, is nonlocal. The nonlocality of the spectrum stems from the long-wave part with $k_x \propto k_y^3$. This part of the spectrum corresponds to zonal flows. The energy flux in spectrum (65) is directed toward larger k_x and smaller k_y. A similar behavior is observed in results of numerical simulations [21, 87]. The enstrophy flux in spectrum (66) is directed to small k_y. Reference [62] demonstrates that the nonlocality of the Kolmogorov turbulence spectra is the cause of spectrum evolution. It results in the appearance of two regions separated in the wavenumber space, which correspond to a strong zonal current and jet spectrum of small-scale turbulence.

4.4 Long-wave turbulence

The dispersive correction in the long-wave limit $k^2 r_{\mathrm{R}}^2 \ll 1$ for waves with $k_y \gg k_x$ assumes the form

$$\omega_{\mathbf{k}} \propto k_x k_y^2\,, \tag{67}$$

i.e., the frequency is scale invariant with the similarity indices

$$a = 1\,, \qquad b = 2\,. \tag{68}$$

The matrix element (62) in this limit is written out as

$$V(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2) \propto |k_x k_{x1} k_{x2}|^{1/2} (k_{x1}^3 + k_{x2}^3 - k_x^3)\,. \tag{69}$$

Hence, it follows that

$$u = \frac{3}{2}\,, \qquad v = 3\,. \tag{70}$$

The following energy spectra correspond to these indices:

$$W_{\mathbf{k}} \propto k_x^{-3/2} k_y^{-4}\,, \tag{71}$$

$$W_{\mathbf{k}} \propto k_x^{-3/2} k_y^{-3}\,. \tag{72}$$

Spectra (72) and (71) are linked to the energy and enstrophy fluxes, respectively. An analysis of these spectra [60] reveals that, similarly to the short-wave approximation, spectrum (72) related to the energy flux is local. Spectrum (71) is linked to the enstrophy flux and is nonlocal. The nonlocality of the spectrum is caused by wave interaction in the long-wave part of the spectrum with $k_x \propto k_y$. This, however, contradicts the initial assumption of $k_y \gg k_x$.

5. Zonal wind generation

Let us explore the dynamics of interaction between Rossby waves and a zonal wind in a turbulent barotropic atmosphere. Since the time scale of zonal wind variability is larger than typical periods of Rossby waves, we will employ the multiscale expansion method, assuming that the regions of Rossby wave turbulence and zonal winds are well separated in the wavenumber space. Let us represent the dimensionless perturbation of the atmospheric pressure (normalized to the equilibrium pressure) as a sum of low- and high-frequency components: $p/p_0 = \hat{p} + P$, where $P(\mathbf{R}, T)$ denotes large-scale pressure perturbations in the zonal wind, and $\hat{p}(\mathbf{r}, t; \mathbf{R}, T)$ is the pressure perturbation in small-scale Rossby waves. Here, $\mathbf{R}$ and T are the large-scale space and time coordinates, and $\mathbf{r}$ and t are the small-scale ones. Averaging Eqn (9) over short time scales, one arrives at the evolution equation for the pressure in the zonal wind:

$$\nabla_\perp^2 \frac{\partial}{\partial T} P = -f r_{\mathrm{R}}^2 \overline{\{\hat{p}, \nabla_\perp^2 \hat{p}\}}\,, \tag{73}$$

where the over-bar denotes short-time averaging. The right-hand side of Eqn (73) describes the Reynolds stresses due to Rossby wave fluctuations. The interaction of a Rossby wave package with a zonal current is described by the wave kinetic equation [75]

$$\frac{\partial N_{\mathbf{k}}}{\partial t} + \frac{\partial \omega_{\mathbf{k}}}{\partial \mathbf{k}} \frac{\partial N_{\mathbf{k}}}{\partial \mathbf{x}} - \frac{\partial \omega_{\mathbf{k}}}{\partial \mathbf{x}} \frac{\partial N_{\mathbf{k}}}{\partial \mathbf{k}} + \gamma N_{\mathbf{k}} = S\,, \tag{74}$$

where S is the right-hand side of Eqn (47). The term with γ characterizes sources and sinks of waves, which are ignored here. We seek a solution to equation (74) in the case of a zero right-hand side. Equations (73) and (74) govern the dynamics of interaction of the Rossby wave package with the zonal wind. One supposes that the Rossby wave spectrum consists of the equilibrium and modulated parts: $N_{\mathbf{k}} = N_{\mathbf{k}}^0 + \tilde{N}_{\mathbf{k}}$, $N_{\mathbf{k}}^0 \gg \tilde{N}_{\mathbf{k}}$, while the wave frequency is represented as $\omega_{\mathbf{k}} = \omega_{\mathbf{k}}^0 + k_x V$. Here, $V = f r_{\mathrm{R}}^2\, \partial P/\partial y$ is the geostrophic velocity of the large-scale zonal current stemming from the finite gradient of the large-scale pressure P, and $\omega_{\mathbf{k}}^0$ is the frequency of the wave package of small-scale Rossby waves, with $\omega_{\mathbf{k}}^0 \gg k_x V$. Assuming that

$$(\tilde{N}_{\mathbf{k}}, P) \propto \exp(-\mathrm{i}\Omega T + \mathrm{i} q Y)\,,$$

where Ω and q are the frequency and wavenumber of large-scale perturbations, we can linearize the system of equations (73) and (74). The result is written out as

$$-\mathrm{i}\Omega P = f r_{\mathrm{R}}^2 \int k_x k_y |p_{\mathbf{k}}|^2\, \mathrm{d}\mathbf{k} \tag{75}$$

and

$$\tilde{N}_{\mathbf{k}} = -\mathrm{i} q^2 r_{\mathrm{R}}^2 \frac{k_x v_{\mathrm{R}}}{\Omega - q V_{\mathrm{g}}} \frac{\partial N_{\mathbf{k}}^0}{\partial k_y}\,, \tag{76}$$

where $V_{\mathrm{g}} = \partial \omega_{\mathbf{k}}/\partial k_y$ is the component of the group velocity of the wave package. Taking into account that $\tilde{N}_{\mathbf{k}} = k^2 |p_{\mathbf{k}}|^2/\omega_{\mathbf{k}}$ and $V_{\mathrm{g}} = -2\omega_{\mathbf{k}} k_y/k^2$, one obtains from the system of equations (75) and (76):

$$\Omega = -\frac{f^2}{2} q^2 r_{\mathrm{R}}^2 \int \mathrm{d}\mathbf{k}\, \frac{\partial N_{\mathbf{k}}^0}{\partial k_y} \frac{V_{\mathrm{g}} k_x}{\Omega - q V_{\mathrm{g}}}\,. \tag{77}$$

We make use of the approximation $N_{\mathbf{k}}^0 = N^0 \delta(k_x - k_{x0}) \times W(k_y - k_{y0}, \Delta k_y)$, where $W(k_y - k_{y0}, \Delta k_y)$ is the step function, $W(k_y - k_{y0}, \Delta k_y) = \Delta k_y^{-1}$ for $|k_y - k_{y0}| < \Delta k_y/2$ and $\Delta k_y \ll k_{y0}$, and $W(k_y - k_{y0}, \Delta k_y) = 0$ for all other k_y. In this approximation, Eqn (77) leads to

$$\Omega = -\frac{f^2}{2} q^2 r_{\mathrm{R}}^4 \frac{k_{x0}}{2\Delta k_y} \times N_0 \left[\frac{V_{\mathrm{g}} - V_{\mathrm{g}}' \Delta k_y}{\Omega - q V_{\mathrm{g}} + V_{\mathrm{g}}' \Delta k_y q/2} - \frac{V_{\mathrm{g}} + V_{\mathrm{g}}' \Delta k_y}{\Omega - q V_{\mathrm{g}} - V_{\mathrm{g}}' \Delta k_y q/2} \right], \tag{78}$$

where $V_{\mathrm{g}}' \equiv \partial V_{\mathrm{g}}/\partial k_y = -2 k_y \omega_{\mathbf{k}}/k^2$. Approximating $\Delta k_y = q$, one finds from Eqn (78) the following relationship

$$(\Omega - q V_{\mathrm{g}})^2 - \left(\frac{V_{\mathrm{g}}' q^2}{2}\right)^2 = 2 f^2 q^2 k_0^2 r_{\mathrm{R}}^4 |\tilde{p}_{\mathbf{k}0}|^2\,. \tag{79}$$

Hence, one concludes that the real part of the frequency is zero ($\mathrm{Re}\,\Omega = 0$), while the increment ($\gamma \equiv \mathrm{Im}\,\Omega$) is given by

$$\gamma = \left(2 f^2 q^2 k_0^2 r_{\mathrm{R}}^4 |\tilde{p}_{\mathbf{k}0}|^2 - \frac{q^4}{k^4}\, \omega_{\mathbf{k}}\right)^{1/2}. \tag{80}$$

The expression obtained for the increment of the parametric instability is valid, strictly speaking, only at the initial, quasilinear stage of instability. However, as follows from numerical simulations [88], the magnitude of the parametric instability increment obtained in the quasilinear approximation remains unchanged at the nonlinear stage of instability development. Equation (80) provides the condition for the scales of the zonal wind structure compatible with the existence of instability:

$$0 < \left(\frac{q}{k}\right)^2_{\max} < 2\left(\frac{f}{\omega_{\mathbf{k}}}\right)^2 (k r_{\mathrm{R}})^4 |\tilde{p}_{\mathbf{k}0}|^2\,. \tag{81}$$

For a given value of k, the fastest growth is exhibited by perturbations satisfying the condition

$$\left(\frac{q}{k}\right)^2_{\max} = \left(\frac{f}{\omega_{\mathbf{k}}}\right)^2 (k r_{\mathrm{R}})^4 |\tilde{p}_{\mathbf{k}0}|^2\,, \tag{82}$$

with the maximum increment equal to

$$\gamma = \frac{f^2}{\omega_{\mathbf{k}}} (k r_{\mathrm{R}})^6 |\tilde{p}_{\mathbf{k}0}|^2\,. \tag{83}$$

For typical values of the parameters of the Earth's atmosphere at 30° latitude ($f \approx 0.8 \times 10^{-4}$ s^{-1}, $r_R \approx 4 \times 10^6$ m, $\tilde{p}_{\mathbf{k}0} \approx 10^{-2}$, $k_0 r_R \approx 2$, and $v_R \approx 3 \times 10^2$ m s^{-1}), the instability increment $\gamma \approx 2.4 \times 10^{-6}$ s^{-1}. This corresponds to the characteristic time γ^{-1} of instability development equal to ≈ 5 days. As a result of this process, a periodic structure is formed in the meridional direction with a characteristic scale $q_{\max}^{-1} \approx 3 \times 10^3$ km. These rough estimates agree with observations of zonal winds. Thus, the instability considered here can be responsible for the generation of zonal winds.

6. Conclusion

Despite the large variety of wave motions in planetary atmospheres, studying large-scale eddies and zonal winds has attracted and continues to attract the particular interest of researchers. Indeed, these structures determine the global transfer of air masses, which forms the climate and weather of vast regions on the Earth. The dynamics of large-scale nonlinear structures formed by planetary (Rossby) waves is modelled by the relatively simple Charney – Obukhov equation (9) or its modification (14) which takes account of the zonal wind. However, even this rather simple model description has permitted us to discover a set of remarkable specific features of two-dimensional motion of fluid in Rossby waves. One of the main features of these waves is their self-organization, which manifests itself through the spontaneous generation of coherent structures, such as large-scale vortices and zonal winds. The zonal wind is a self-controlled system of shear flows where the source is the modulation instability of Rossby waves in a turbulent barotropic atmosphere, and the sink is provided by the Rayleigh instability. It essentially suppresses the equator – pole transport processes. This mechanism provides an effective channel of energy transfer from the region of small-scale turbulence of Rossby waves to the region of large-scale motions which correspond to zonal winds, and contributes importantly to the regularization of atmospheric turbulence. Laboratory experiments [9 – 11, 15, 36 – 39, 90, 91], together with numerical simulation [25, 40 – 42, 82, 89, 92] and analytical studies [26 – 30, 40 – 47], witness in favor of such a mechanism of zonal wind generation by small-scale Rossby waves in a two-dimensional barotropic atmosphere.

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