FROM THE HISTORY OF PHYSICS

The Einstein formula: $E_0 = mc^2$. "Isn't the Lord laughing?"

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Contents

Abstract. The article traces the way Einstein formulated the relation between energy and mass in his work from 1905 to 1955. Einstein emphasized quite often that the mass m of a body is equivalent to its rest energy E_0. At the same time, he frequently resorted to the less clear-cut statement of the equivalence of energy and mass. As a result, Einstein's formula $E_0 = mc^2$ still remains much less known than its popular form, $E = mc^2$, in which E is the total energy equal to the sum of the rest energy and the kinetic energy of a freely moving body. One of the consequences of this is the widespread fallacy that the mass of a body increases when its velocity increases and even that this is an experimental fact. As wrote the playwright A N Ostrovsky "Something must exist for people, something so austere, so lofty, so sacrosanct that it would make profaning it unthinkable."

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1. Introduction

The formula $E = mc^2$ is perhaps the most famous formula in the world. In the minds of hundreds of millions of people it is firmly associated with the menace of atomic weapons. Millions perceive it as a symbol of relativity theory. Numerous authors popularizing science keep persuading their readers, listeners, and viewers that the mass of any body (any particle) increases, as prescribed by this formula, when its velocity increases. And only a small minority of physicists — those who specialize in elementary particle physics — know that Einstein's true formula is $E_0 = mc^2$, where E_0 is the energy contained in a body at rest, and that the mass of a body is independent of the velocity at which it travels.

Most physicists familiar with special relativity know that in it, the energy E and momentum $\mathbf{p}$ of a freely moving body are related by the equation $E^2 - \mathbf{p}^2 c^2 = m^2 c^4$, where m is the mass of the body. Alas, not all of them realize that this formula is incompatible with $E = mc^2$. But an even smaller number of people know that it is perfectly compatible with $E_0 = mc^2$, because E_0 is the value assumed by E when $\mathbf{p} = 0$. This article is written for those who do not want to be lost in three pines* of the above three formulas and who wish to attain a better understanding of relativity theory and its history.

When Einstein first introduced the concept of rest energy in 1905 and discovered that the mass of a body is a measure of

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* "To be lost in three pines" in Russian is equivalent to "loose one's way in broad daylight" in English. (*Author's note to English version of the article*.)

the energy contained in it, he felt so amazed that he wrote in a letter to a friend: "for all I know, God Almighty might be laughing at the whole matter and might have been leading me around by the nose." In what follows, we see how throughout his life Einstein returned again and again to this same question.

We shall see how the formula $E_0 = mc^2$ made its way through Einstein's writings. Also, how he carefully emphasized that the mass of a body depends on the amount of energy it contains but never stated (in contrast to his popularizers!) that mass is a function of the body's velocity. Nevertheless, it is true that he never once rejected the formula $E = mc^2$ that is believed to be 'his formula' and in the mass psyche is an icon of modern physics.

To my reader: If you feel bored with following the meticulous analysis and collation of texts, please jump to the Epilogue and the adjacent sections, where I have tried to briefly describe the results of the analysis without going into technicalities. It is possible that after you do so, reading about Einstein's many attempts to clarify the relation between energy and mass will become more interesting and compelling.

When writing this review, I used Einstein's historically first ever collected works [1]. (This four-volume edition was published in Russian in 1965 – 1967.) Where possible, I also used the multivolume Princeton Collected Papers. (Volumes with 'all papers and documents' by Einstein [2] and their translations into English [3] began to appear in Princeton in 1987. In 2007, ten volumes were published, of which five (1, 5, 8, 9, 10) contain his correspondence until 1920 and five (2, 3, 4, 6, 7) contain his works until 1921).

2. Prologue. The years 1881 – 1904

It is well known that the principle of relativity dates back to Galileo [4] and Newton [5], and that the theory of relativity was constructed in the papers of Lorentz, Einstein, Poincaré, and Minkowski [6].

The notion of velocity-dependent mass was born in the years preceding the creation of the theory of relativity and in the first years after its creation.

It was molded in the papers of Thomson [7], Heaviside [8], Searle [9], Abraham [10], and also Lorentz [11] and Poincaré [12], who tried hard to have Maxwell's equations of electromagnetism to agree with the equations of Newton's mechanics. These publications stimulated the experiments of Kaufmann [13] and Bucherer [14, 15]. They used formulas of Newton's nonrelativistic mechanics to process their experimental data and concluded that mass increases with increasing velocity.

It was the matter not only of formulas as such but also of the very spirit, the very foundations of the nonrelativistic physics in which mass is a measure of inertia of a body. It was difficult to comprehend, at the borderline between the 19th and 20th centuries, that these foundations were being replaced by a more general base: the measure of inertia of a body is not its mass but its total energy E equal to the sum of rest energy and kinetic energy. The fact that energy E entered with a factor $1/c^2$ prompted people to interpret E/c^2 as the mass. In fact, the progress in relativity theory, achieved mostly through the efforts of Einstein, Minkowski, and Noether, showed that it was necessary to connect mass not with total energy but only with rest energy.

3. 1905 — annus mirabilis

In 1905, Einstein published his three ground-breaking, fundamental papers dealing with the properties of light and matter [16 – 18].

In [16], he introduced the concept of the quantum of energy of light and, using this concept, explained the photoelectric effect, which had been experimentally discovered not long before that. (The value of the Planck constant h — the quantum of action — had been established earlier, see [19].)

In [17], Einstein considered almost the entire set of consequences of the principle of relativity and of the finite speed of light. Thus he derived in §8 the formula for the transformation of the energy of light in the transition from one inertial reference frame to a different one that moves at a velocity v relative to the former:

$$\frac{E'}{E} = \frac{1 - (v/V) \cos \phi}{\sqrt{1 - (v/V)^2}} \, .$$

Here, V is the velocity of light and ϕ is the angle between the direction of motion of light and that of the observer. Then in §10 he obtained the expression for the kinetic energy of the electron:

$$W = \mu V^2 \left\{ \frac{1}{\sqrt{1 - (v/V)^2}} - 1 \right\},$$

where μ is the mass of the electron and v is its velocity.

(Furthermore, in §10, Einstein derived expressions for the so-called longitudinal m_l and transverse m_t masses of the electron that Abraham and Lorentz had earlier introduced and he obtained:

$$m_l = \frac{\mu}{\left(\sqrt{1 - (v/V)^2} \right)^3} \, ,$$

$$m_t = \frac{\mu}{1 - (v/V)^2} \, .$$

The second of these expressions differs from Lorentz's m_t and is wrong, and later Einstein never insisted on it.)

As regards the formulas for the kinetic energy W of an electron and for a photon energy E', he applied both these formulas in the next paper [18] when deriving the relation between mass and energy.

There he considered 'two amounts of light,' with energy $L/2$ each, both emitted by a massive body at rest but traveling in opposite directions. In this paper, Einstein for the first time introduced the rest energy of a massive body, denoting it by E_0 before emission and by E_1 after. In view of the energy conservation law,

$$E_0 - E_1 = L \, .$$

He then looked at the same process in a reference frame moving at a velocity v relative to the body, and obtained the following expression for the difference between kinetic energies of the body before and after the act of emission:

$$K_0 - K_1 = L \left\{ \frac{1}{\sqrt{1 - (v/V)^2}} - 1 \right\} .$$

He also specially pointed out that the difference between kinetic energies contains an arbitrary additive constant C included in the expression for energy. He returned to the matter of the constant C many times during the subsequent 50 years; we discuss it later in this paper.

The left- and right-hand sides of the equality above depend on v in the same manner, as follows from the expression for W. Since the velocity v is the same before and after the emission, while the kinetic energy of the body decreased, this immediately implies that the mass of the body decreased by the amount L/V^2. From this Einstein concluded that "The mass of a body is a measure of its energy content" and remarked that it might be possible to check this conclusion in the decays of radium.

The title of the paper is noteworthy: "Does the inertia of a body depend on its energy content?". Considered together with the contents of the paper, it indicates that it was the mass that Einstein identified with the measure of a body's inertia. But this is only valid in Newton's approximation. As we know today, the measure of a body's inertia in relativity theory is its total energy E: the greater the total energy of a body, the greater its inertia. (By the 'measure of a body's inertia,' we here mean the proportionality coefficient between momentum and velocity. There is no universal proportionality coefficient between force and acceleration in relativity theory. Lorentz and Abraham had already established this when they introduced the longitudinal and transverse masses.

Einstein held to the opinion that the energy of a free body is defined in relativity theory only up to an additive constant, by analogy to potential energy in Newtonian mechanics. This may have resulted in his underestimating his own revolutionary step forward — the introduction of the concept of rest energy into physics. There is nothing special about the rest energy E_0 once energy is only defined up to C.

But as we know today, there is no place for C in the theory he created. The energy and momentum of a free particle are uniquely defined in the theory by the relation $E^2 - \mathbf{p}^2 c^2 = m^2 c^4$; we return to it more than once in what follows.

4. Have I been led around by the nose?

The discovery that mass depends on energy struck Einstein so forcibly that he wrote in a letter to his friend Conrad Habicht [20] (see also [21]):

"A consequence of the study on electrodynamics did cross my mind. Namely, the relativity principle, in association with Maxwell's fundamental equations, requires that the mass be a direct measure of the energy contained in a body; light carries mass with it. A noticeable reduction of mass would have to take place in the case of radium. The consideration is amusing and seductive; but for all I know, God Almighty might be laughing at the whole matter and might have been leading me around by the nose."

It looks as if God continues to lead the interpreters of the relativity theory by the nose much as He did in Einstein's time.

5. 1906 – 1910. Minkowski

1906
In 1906, Einstein published two papers on relativity theory: [22, 23]. In [22], he treated mass transfer by light in a hollow

cylinder from its rear face to the front. For the cylinder not to move as a whole, he imposed the condition that light with an energy E has the mass E/V^2; he thereby reproduced Poincaré's result of 1900 [12]. Presumably, he considered it inadmissible for the energy and mass carrier to have zero mass (to be massless). In [23], he considered a method for determining the ratio of longitudinal and transverse masses of the electron previously introduced by Lorentz and Abraham. As far as mass is concerned, therefore, these papers were a step back in comparison with [18].

1907
In 1907, Einstein published four papers on relativity theory: [24 – 27]. The first of these discussed the frequency of radiation from an atom. The second emphasized the difference between the relativity principle and the relativity theory. He considered his own work as dealing with the principle of relativity, which he regarded as being analogous to those of thermodynamics. As for the theory of relativity, he believed that it was yet to be constructed.

The paper that is especially significant for us here is [26], which gave the formulation of the mass – energy equivalence (see footnote in § 4): "One should note that the simplifying assumption $\mu V^2 = \varepsilon_0$ is also the expression of the principle of the equivalence of mass and energy..." (The simplifying assumption referred to here is the choice of an arbitrary constant in the expression for energy.)

The most detailed among the papers published in 1907 was [27]. It consists of five parts: (1) Kinematics (§ 1 – § 6). (2) Electrodynamics (§ 7). (3) Mechanics of a material point (electron) (§ 8 – § 10). (4) On the mechanics and thermodynamics of systems (§ 11 – § 16). (5) Relativity principle and gravitation (§ 17 – § 20). Short note [28] with corrections of misprints and elaborations belongs to this group of papers.

Of special interest for us are parts 4 and 5. In part 4, Einstein discussed the additive constant in the energy and showed that it is not included in the relation between momentum, energy, and velocity of a body. Part 5 ended with the following words:

"Thus the proposition derived in § 11, that to an amount of energy E there corresponds a mass of magnitude E/c^2, holds not only for the *inertial* but also for the *gravitational* mass, if the assumption introduced in § 17 is correct."

On the one hand, this sentence states that energy, not mass, is both the measure of inertia and the source of gravitation. But on the other hand, it can be understood to say that a photon with an energy E has both the inertial mass and the gravitational mass equal to E/c^2. This ambiguous interpretation continues to trigger heated debates.

1908
In 1908, Einstein together with J Laub published two articles on the electrodynamics of moving macroscopic bodies: [29, 30] (see also [31, 32].) Although pertaining to relativity theory, these papers are nevertheless not relevant to the problem under discussion here, the relation between energy and mass.

The talk delivered by Hermann Minkowski in 1908 [33] was an important milestone in the history of relativity theory. Minkowski was the first to propose the four-dimensional spacetime formulation of the theory. In this formulation, as we know, the mass of a particle is a quantity independent of its velocity.

It may seem paradoxical but the first paper by Lewis [34] declaring that the mass equals E/c^2 appeared at the same time. This standpoint was further developed and spread by Lewis and Tolman in [35 – 38].

1909

Einstein's paper [39] published in 1909 is not concerned with the relation between mass and energy. But we find a number of statements in his articles [40 – 42] published at the same time that shed much light on his understanding of this problem. For instance, in [42], which contains the text of Einstein's first public speech (at a congress of German natural scientists in Salzburg), he wrote:

"The first volume of the excellent textbook [1] by Chwolson which was published in 1902, contains in the Introduction the following sentence about the ether: 'The probability of the hypothesis on the existence of this agent borders extraordinarily closely on certainty.' However, today we must regard the ether hypothesis as an obsolete standpoint."

Then: "…the inertial mass of a body decreases upon emission of light… Energy and mass appear as equivalent quantities the same way that heat and mechanical energy do… The theory of relativity has thus changed our views on the nature of light insofar as it does not conceive of light as a sequence of states of a hypothetical medium but rather as something having an independent existence just like matter."

1910

In 1910, A Einstein and L Hopf discussed the application of probability theory to the analysis of the properties of radiation [43, 44].

At the same time, Einstein published in a French journal a major review of relativity theory [45] devoted mostly to the transformations of spatial coordinates and time but also briefly outlining Minkowski's ideas about the four-dimensional world. Only at the end of this paper did he mention that "…the mass of any arbitrary body *depends on the quantity of energy* it contains… Unfortunately, the change of mass W/c^2 is so slight that one cannot hope for its detection by experiment for the time being."

Einstein did not stipulate that by "energy W contained in a body" he meant rest energy.

6. 1911 – 1915.
On the road to General Relativity Theory

1911

In 1911, Einstein published three papers on the theory of relativity: [46 – 48].

In [46], he discussed the propagation of light in a gravitational field, starting with the assumption that a photon with energy E has an inertial and a gravitational mass, both of which are equal to E/c^2, and he calculated that the angle of deflection of light by the Sun's gravitational field would be 0.83 arc second — which is half the correct value that he would later derive (in 1915) using general relativity. (I should remark that the same "half value" had already been obtained and published by Soldner in 1804 (see [49, 50]). But Einstein was not aware of it: Soldner's paper was totally forgotten soon after its publication.)

[1] "The Course of Physics" by O D Chwolson (volumes 1 and 2) was published in Russian in 1897; its German translation appeared in 1902.

At the end of review paper [47] devoted mostly to clocks and rods in relativity theory, Einstein mentioned uniting the law of conservation of mass with the law of conservation of energy. "However odd this result might seem, still, in a few special cases, one can unequivocally conclude from empirically known facts, and even without the theory of relativity, that the inertial mass increases with energy content." Perhaps this sentence refers to experiments of Kaufmann and Bucherer. But this would suggest that he believed that mass increases with increasing kinetic energy and therefore with increasing velocity.

A short note [49] discussed the contraction of the length of a moving rod.

1912

Einstein's papers of this period [51 – 55] were mostly attempts to create a more general relativity theory that would embrace gravitation. Only lectures [51] dealt with special relativity.

His statements made during 1912 again display the above-mentioned ambiguity in the interpretation of mass as the equivalent of rest energy, on the one hand, and as a measure of inertia, on the other.

We find there a statement that m should be considered to be a characteristic constant of a 'material point' (massive point-like body), which does not vary as a function of the object's motion. On the other hand, it is also stated that the energy of a free particle is defined only up to an arbitrary additive constant. Nevertheless, mc^2 equals the rest energy (see the discussion of equation (28′) in [51]).

1913 – 1914

In paper [56] co-authored with M Grossmann, Einstein continued to discuss the proportionality between the inertial and gravitational masses, which had been measured with high accuracy in experiments by Eötvös, and he discussed the dependence of the speed of light c on the gravitational potential.

In 1914, Einstein published a short note expounding his point of view regarding the concept of mass [57]. A manuscript with a synopsis of his lectures on special relativity theory dates back to the same period [58].

In [57], he discussed the contribution of the gravitational field to the gravitational and inertial masses of a body and came to the conclusion that the inertia of a closed system is entirely determined by its rest energy.

Paper [58] gave an expression for the energy – momentum 4-vector and the relation $E_0/c^2 = m$, which would appear again only in 1921. We note that m was referred to in [58] as rest mass (Ruhemasse), which seems to imply that the mass of a body at rest is not the same as when the body moves.

1915

The year 1915 was marked by the completion of general relativity theory, in paper [59]. In fact, already in his preceding paper [60], Einstein had derived formulas that described two most important effects of this theory: the precession of Mercury's perihelion and the deflection of light by the gravitational field of the Sun. The secular motion of Mercury's perihelion (about 40″ per century), which could not be explained in terms of the influence of the known bodies in the solar system, was established by Le Verrier in 1859. Einstein calculated that general relativity theory predicted secular precession as 43″.

But the true world fame came from the prediction of the angle of deflection of light by $1.7''$ after it had been confirmed by the British expedition that observed the solar eclipse in 1919.

7. 1917. Cosmological constant

1917

A book was published in 1917 to popularize relativity theory [61]. It dealt mostly with the joint transformation of space and time coordinates. However, § 15 mentioned the kinetic energy of a material point, which now equaled not $mv^2/2$ but $mc^2/\sqrt{1 - v^2/c^2}$, and therefore incorporated both its kinetic energy proper and its rest energy.

Then we read this:

"Before the advent of relativity, physics recognized two conservation laws of fundamental importance, namely, the law of the conservation of energy and the law of the conservation of mass; these two fundamental laws appeared to be quite independent of each other. By means of the theory of relativity they have been united into one law."

And even though an attentive reader concludes from the text that follows that Einstein was speaking of $E_0 = mc^2$, a slightly less attentive reader might guess that $E = mc^2$ was meant. The fact that at times Einstein treated rest energy as part of kinetic energy did not help to clarify matters.

The most famous among Einstein's papers published in 1917 was called "Cosmological Considerations on the General Theory of Relativity" [62]. There Einstein formulated for the first time the possibility of a non-vanishing energy density of the vacuum; he denoted it by the letter λ. This energy density is the same at every point in the Universe. It is essentially a completely delocalized energy, spread over the entire Universe.

Einstein introduced this cosmological constant — the so-called λ-term — in order to be able to describe a stationary Universe in general relativity. It soon became clear, however, that a stationary solution cannot be achieved in this manner.

In 1922, Friedmann, while reading this paper by Einstein, advanced his theory of the expanding Universe [63, 64]. Einstein first dismissed Friedmann's arguments [65], but then accepted them [66]. In 1929, Hubble published the first observational data [67] supporting the expansion of the Universe.

In 1945, Einstein published the second edition of his book "The Meaning of Relativity" with a special addendum "On the Cosmological problem" devoted to the theory of the expanding Universe [68]. At the turn of the 1970s – 1980s, a model of the exponentially fast expansion (inflation) of the early Universe was suggested [69 – 71]. According to this model, the effective cosmological term forms when the Universe is created, due to a nonzero mean vacuum value of a special scalar field, which later transforms into high-energy particles. In 1998 – 1999, two groups of observers measuring the luminosity and spectra of supernovas came to a conclusion that the rate of the expansion of the Universe is increasing [72, 73] (see also [74].) The available data indicate that ordinary matter contains only 4% of the energy of the Universe, that about 24% is contained in the particles of the so-called dark matter whose nature is as yet unknown, and about 70% of the entire energy of the Universe is usually referred to as dark energy and attributed to Einstein's cosmological constant λ.

8. 1918 – 1920. Noether

1918

In 1918, the brilliant paper of Emmy Noether was published [75], in which she proved, among other things, that the dynamic conservation laws are implied by the symmetry properties of space – time. We know that conservation of energy is a consequence of the uniformity of time, and that conservation of momentum is a consequence of the uniformity of space. Angular momentum is conserved as a result of the isotropy of space: physics remains unchanged if coordinate axes undergo rotation in the planes xy, yz, zx. Similarly, Lorentz invariance follows from the fact that physics remains unchanged under pseudo-Euclidean rotations in the planes xt, yt, zt. Einstein wrote very enthusiastically about this discovery of Noether in a letter to Hilbert [76]:

"Yesterday I received a very interesting paper by Ms. Noether about the generation of invariants. It impresses me that these things can be surveyed from such general point of view. It would not have harmed the Göttingen old guard to have been sent to Miss Noether for schooling. She seems to know her trade well!"

Soon after that Einstein sent for publication a paper [77] on the conservation of energy in general relativity, which presented a statement that the energy of a closed system plays the role of both inertial and gravitational mass.

1919

Among the publications of 1919, I need to specially mention a short note "A test of the general theory of relativity" [78] on the discovery of the deflection of light rays by attraction of the Sun and an article in *The Times* entitled "What is the theory of relativity?" [79]. Among other things, Einstein wrote:

"The most important upshot of the special theory of relativity concerned the inertial masses of corporeal systems. It turned out that the inertia of a system necessarily depends on its energy-content, and this led straight to the notion that inert mass is simply latent energy. The principle of the conservation of mass lost its independence and became fused with that of the conservation of energy."

1920

In 1920, Einstein prepared a draft manuscript of an extensive popular article "Fundamental ideas and methods of the theory of relativity, presented in their developments." Einstein worked on this article as an invited publication in *Nature*, but it was never published [80].

At the same time, Einstein's letter appeared in a Berlin newspaper, "My response. On the anti-relativity company" [81]. The letter opens with the words: "Under the pretentious name "Arbeitsgemeinschaft deutscher Naturforscher," a variegated society has assembled whose provisional purpose of existence seems to be to degrade, in the eyes of nonscientists, the theory of relativity as well as me as its originator."

Then Einstein wrote: "...I have good reasons to believe that motives other than the striving for truth are at the bottom of this business. [...] I only answer because well-meaning circles have repeatedly urged me to make my opinion known.

First, I want to note that today, to my knowledge, there is hardly a scientist among those who have made substantial contributions to theoretical physics who would not admit that the theory of relativity in its entirety is founded on a logical basis and is in agreement with experimental facts which to

date have been reliably established. The most important theoretical physicists — namely, H A Lorentz, M Planck, A Sommerfeld, M Laue, M Born, J Larmor, A Eddington, P Debye, P Langevin, T Levi-Civita — support the theory, and most of them have made valuable contributions to it. [...]

I have been accused of running a tasteless advertising campaign for the theory of relativity. But I can say that all my life I have been a friend of well-chosen, sober words and of concise presentation."

9. 1921.
"The Meaning of Relativity"

In 1921, Einstein was invited to Princeton and delivered there a course of lectures that make up the book "The Meaning of Relativity" [82]. In this book, he described for the first time, with maximum exposure to the public and unambiguously, what he understood by the equivalence of energy and mass. His equations $(41)–(43)$ give expressions for the components of the energy – momentum 4-vector of a body in terms of its mass and velocity. Equation (44) gives an expression for the energy of a body in terms of its mass: $E_0 = mc^2$. In equation (45), he gave an expression for energy at a low velocity q: $E = m + mq^2/2 + 3mq^4/8 + \ldots$ (in units in which $c = 1$.) The text between equations (44) and (45) reads: "Mass and energy are therefore essentially alike; they are only different expressions of the same thing. The mass of a body is not constant; it varies with changes in its energy." Then follows a footnote about energy release in radioactive decays: "The equivalence of mass at rest and energy at rest which is expressed in equation (44) has been confirmed in many cases during recent years. In radio-active decomposition the sum of the resulting masses is always less than the mass of the decomposing atom. The difference appears in the form of kinetic energy of the generated particles as well as in the form of released radiational energy."

Three aspects deserve our attention in these statements. First, while giving a clear definition of mass in the equations as a velocity-independent quantity, the term "mass at rest" is used for it, which implies that mass depends on velocity. Second, there is no explicit statement that mass changes only when the energy of a body changes, but not its velocity. Third, the ambiguous statement that mass and energy are "only different expressions of the same thing," even though mass is a relativistic invariant, i.e., a four-dimensional scalar, while energy is the fourth component of a four-dimensional vector. It is possible that these rather imprecise words accompanying perfectly precise formulas are the reason why many readers still fail to see in [82] a clear-cut statement in favor of $E_0 = mc^2$ and against $E = mc^2$.

A small popular-science brochure deserves being mentioned here: "Relativity theory" [83], whose author, I Leman, expressed his gratitude to Einstein for valuable advice. He spoke of his awe for the profundity and elegance of Minkowski's ideas and emphasized the enormous amounts of energy stored in matter as its mass.

10. 1927–1935

1927

In 1927, several conferences were dedicated to the bicentennial of the death of Isaac Newton. Einstein marked the occasion with a number of publications. He wrote in [84]:

"Newton's teaching provided no explanation for the highly remarkable fact that both the weight and the inertia of a body are determined by the same quantity (its mass). The remarkableness of this fact struck Newton himself."

By 1927, mostly through the work of Einstein, it became clear that the inertia and the weight of a moving particle are determined not by its mass but by its energy E and the quantity $p_\mu p_\nu/E$, where p_μ is energy – momentum vector. In the Newtonian limit, both are reduced to the rest energy, i.e., to mass. Such is the simple explanation provided by relativity theory of the equality of the inertial and gravitating masses in Newtonian mechanics.

However, we see that Einstein continued to use the old nonrelativistic terminology.

1928

In the paper "Fundamental concepts of physics and their most recent changes" [85], Einstein formulated his attitude to the problem of causality in quantum mechanics, saying that

"Thus the field theory shook the fundamental concepts of time, space and matter. But upon one column of the edifice it made no assault: on the hypothesis of causality. From some single condition of the world at a given time, all other previous and subsequent conditions uniquely follow based upon the laws of of nature.

Today, however, serious doubts have emerged about the law of causality thus understood. This is not to be charged to the craving for new sensations on the part of the learned, but to the momentum of facts which seem irreconcilable with a theory of strict causality. It seems at this time as if the field, considered as a final reality, does not make proper allowance for the facts of radiation and atomic structure. We reach here a complication of questions with which the modern generation of physicists is struggling in a gigantic display of intellectual power."

This problem was solved twenty years later in Feynman's two papers on quantum electrodynamics (see below), but Einstein failed to notice it. This may have been caused by Einstein's belief that all of quantum physics violated causality.

1929

In his article for the Encyclopedia Britannica [86], Einstein described the four-dimensional spacetime continuum but wrote not a word about Minkowski and the energy–momentum four-dimensional space.

In his speech at the ceremony in honor of the 50th anniversary of Planck's presentation of his doctoral dissertation — at which Einstein received the Planck Medal — he returned to the problem of causality in quantum mechanics. He wrote that even though he was deeply convinced that theory would not stop at the subcausality level and would ultimately reach the supercausality in the sense discussed by him earlier, he was impressed by the work of the younger generation of physicists on quantum mechanics, and that he regarded this theory as a correct one. He only mentioned that restrictions resulting in the *statistical* nature of its laws should be eliminated with time [87].

1934 – 1935

On December 29, the Pittsburgh Post-Gazette published an interview with Einstein under the heading "Atom energy hope is spiked by Einstein" [88].

In December 1934, Einstein read to the joint session of the American Mathematical Society, the American Physical

Society, and the American Society for the Advancement of Science a lecture entitled "Elementary derivation of the equivalence of mass and energy." This lecture was published in 1935 in the Bulletin of the American Mathematical Society [89].

The challenge Einstein set himself was to prove that mass and energy are equivalent, on the basis of only three assumptions:

"In the following considerations, except for the Lorentz transformation, we will depend only on the assumption of the conservation principles for impulse and energy."

In its first pages, Einstein introduces the velocity 4-vector, and by multiplying it by mass m, obtains the 4-vector whose spatial components — in his opinion — can naturally be regarded as momentum and the time component as energy:

"Here it is natural to give it directly the meaning of energy, hence to ascribe to the mass-point in a state of rest the *rest-energy m* (with the usual time unit, mc^2)."

Of course, …in no way is it shown that this impulse satisfies the impulse-principle and this energy the energy-principle…

Furthermore, it is not perfectly clear as to what is meant in speaking of the *rest-energy*, as the energy is defined only to within an undetermined additive constant…

What we will now show is the following. If the principles of conservation of impulse and energy are to hold for all coordinate systems which are connected with one another by the Lorentz transformations, then impulse and energy are really given by the above expressions and the presumed equivalence of mass and rest-energy also exists."

And he undertook to prove that conservation laws indeed hold for the 4-momentum that he considered. To achieve this, he calculated the energies and momenta of two particles before and after their collision in different Lorentz reference frames and concluded:

"The rest-energy changes, therefore, in an inelastic collision (additively) like the mass. As the former, from the nature of the concept, is determined only to within an additive constant, one can stipulate that E_0 should vanish together with m. Then we have simply $E_0 = m$, which states the principle of equivalence of inertial mass and rest-energy."

It is worthy of note here that in this lecture, Einstein never mentioned Noether's theorem [75], which implies that the conservation of the 4-momentum and the Lorentz invariance follow from symmetry properties of the Min-kowski space-time. He preferred to derive the properties of the 4-momentum by considering two-body collisions in the three-dimensional space and to independently assume the Lorentz invariance and conservation of energy and momentum.

On May 4, 1935, he published an obituary in The New York Times entitled "The late Emmy Noether" [90], where he spoke of his high opinion of her contributions to mathematics but failed to mention her theorem that is of such importance in physics. A self-consistent presentation of conservation laws on the basis of the symmetries of space – time in the spirit of Noether was given for the first time in 1941 by L D Landau and E M Lifshitz in their "Field theory" (see below.)

In the same year, 1935, another famous paper was published [91], written in collaboration with N Rosen and B Podolsky on the interpretation of measurements in quantum mechanics.

11. 1938 – 1948. Atomic bomb

1938
In 1938, the famous science-popularizing book was published, "The Evolution of Physics" [92], written by Einstein and his young assistant Leopold Infeld. The authors often returned to the concept of mass on its pages. The section "One clew remains" in chapter I "The rise of the mechanical view" introduced the concepts of inertial and gravitating masses and described their equality as a thread leading the way to general relativity. In the section "Relativity and mechanics" of chapter III "Field, relativity," the authors introduced the concept of rest mass: "A body at rest has a definite mass, called *rest mass*." Then they wrote: "radiation traveling through space and emitted from the sun contains energy and therefore has mass;" and a bit later: "According to the theory of relativity, there is no essential distinction between mass and energy. Energy has mass and mass represents energy. Instead of two conservation laws we have only one, that of mass-energy. This new view proved very successful and fruitful in the further development of physics." One might justly think that this statement is an adequate 'verbal' equivalent of the formula $E = mc^2$ and is incompatible with the formula $E_0 = mc^2$.

In the section "General relativity and its verification" of the same chapter III, we read that the elliptical orbit of Mercury precesses, completing a full cycle around the Sun in three million years. This precession of Mercury's perihelion is caused by relativistic properties of the gravitational field. The next section says this:

"We have two realities: matter and field. [. . .] But the division into matter and field is, after the recognition of the equivalence of mass and energy, something artificial and not clearly defined. Could we not reject the concept of matter and build a pure field physics?"

The creation of relativistically invariant quantum electro-dynamics at the junction of the 1940s and 1950s, and later of the quantum field theory of the electroweak and strong interactions, as well as various models of the so-called grand unification of all interactions, can be regarded as the implementation of Einstein's dream of a unified field theory. However, all these theories are based not only on the theory of relativity but also on quantum mechanics, whose probabil-istic interpretation was unacceptable to Einstein, who insisted that "God does not play dice." It was owing precisely to quantum mechanics that matter was not expelled from these theories but rather became their foundation. This is seen especially clearly in the language of Feynman diagrams, in which real particles (including photons) represent matter and virtual particles represent force fields (see below). The concluding chapter IV entitled "Quanta" is a story about quantum mechanics. The section "The quanta of light" tells the reader that light consists of grains of energy — light quanta, or photons. The section "The waves of matter" emphasizes the similarity between photons and electrons in the combination of wave and corpuscular properties. "One of the most fundamental questions raised by recent advances in science is how to reconcile the two contradictory views of matter and wave." The authors are just a stone's throw from conceding that the photon is just as much a particle of matter as the electron is. However, at the end of book, they say:

"Matter has a granular structure; it is composed of elementary particles, the elementary quanta of matter. Thus,

the electric charge has a granular structure and — most important from the point of view of the quantum theory — so has energy. Photons are the energy quanta of which light is composed.''

Light is therefore identified with energy and becomes an antithesis of matter. Could it be that this identification and this opposition constitute one of the roots of the mass – energy confusion?

1939
On August 2, 1939, Leo Szilard persuaded Einstein to write the famous letter to President F D Roosevelt warning that ''...the element uranium may be turned into a new and important source of energy...'' [88].

1941. Landau and Lifshitz
The first Russian edition of Landau and Lifshitz's ''The theory of fields'' [93] appeared in 1941. In §10 ''Energy and momentum'' (it became §9 in subsequent editions of the volume), they introduced the energy – momentum 4-vector and its square equal to mass squared, and discussed rest energy, although did not denote it by E_0. The nonadditivity of mass in relativity theory was mentioned as the nonconservation of mass. All conservation laws in this book were consistently obtained from the symmetry properties of space – time in accordance with Noether's theorem. However, it is very unlikely that Einstein read Russian textbooks. He likewise missed the publication of the translation into English in 1951 [94].

1942
In 1942, P G Bergmann's book was published [95] with a foreword by Einstein, which said, among other things, that:

''This book gives an exhaustive treatment of the main features of the theory of relativity which is not only systematic and logically complete, but also presents adequately its empirical basis. ...Much effort has gone into making this book logically and pedagogically satisfactory, and Dr. Bergmann has spent many hours with me which were devoted to this end.''

In chapter VI, we read:
''...relativistic kinetic energy equals

$$E = \frac{mc^2}{\sqrt{1 - u2/c^2}} + E_0 \,, \qquad (6.17)$$

where E_0 is the constant of integration...

$$T = mc^2 \left[\left(1 - \frac{u^2}{c^2} \right)^{-1/2} - 1 \right]. \qquad (6.20)$$

...The quantity $mc2$ is called the 'rest energy' of a particle, while T is its 'relativistic kinetic energy.''

It is not clear to me why the integration constant C had to be denoted by E_0. Neither do I understand why the 'relativistic kinetic energy' was denoted by two different symbols E and T. Could it be that a misprint crept in and Eqn (6.17) is the total, not kinetic, energy? Immediately following it is this text:

''*Relation between energy and mass.* The ratio between the momentum and the mass, the quantity μ, is often called 'the relativistic mass' of a particle, and m is referred to as 'the rest mass.' The relativistic mass is equal to the total energy divided by c^2, and likewise the rest mass is $(1/c^2)$ times the rest energy. There exists, thus, a very close correlation between mass and energy which has no parallel in classical physics.''

We thus see that the additive constant in the expression for energy and the dependence of mass on velocity survived in this book. Also retained was the ambiguity connected with the definition of the relativistic kinetic energy, which dates back to a 1917 paper [61].

It looks as if all of this, including the use of the term ''relativistic mass,'' reflected Einstein's views.

1945
On August 6, 1945, an atomic bomb was dropped on Hiroshima; another was dropped on Nagasaki on August 9.

In September, the British magazine 'Discovery' published photographs of the first atomic test explosion on July 16, 1945 and two papers, ''The Progress of Science — We enter the New Age'' and ''The Science behind the Atomic Bomb.'' The latter mentioned, in the chronology of the atomic physics discoveries, ''1905. Einstein's special relativity theory demonstrated the equivalence of mass and energy.'' However, Einstein's photograph was not among the 25 accompanying portraits of scientists from Becquerel to Oppenheimer [96].

In September 1945, the book ''Atomic energy for military purposes'' by H D Smyth was published [97]. The Introduction said, in the section ''Conservation of mass and of energy'':

''1.2 There are two principles that have been cornerstones of the structure of modern science. The first — that matter can be neither created nor destroyed but only altered in form — was enunciated in the eighteenth century and is familiar to every student of chemistry; it has led to the principle known as the law of conservation of mass. The second — that energy can be neither created nor destroyed but only altered in form emerged in the nineteenth century...; it is known as the law of conservation of energy.

1.3 ...but it is now known that they are, in fact, two phases of a single principle for we have discovered that energy may sometimes be converted into matter and matter into energy.''

The section ''Equivalence of mass and energy'' said this:
''1.4 One conclusion that appeared rather early in the development of the theory of relativity was that the inertial mass of a moving body increased as its speed increased. This implied an equivalence between an increase in energy of motion of a body, that is, its kinetic energy, and an increase in its mass. ...He [Einstein] concluded that the amount of energy, E, equivalent to a mass, m, was given by the equation

$$E = mc^2 \,,$$

where c is the velocity of light. If this is stated in actual numbers, its startling character is apparent.''

In these passages, the following deserves our attention:
1. Matter is identified with mass.
2. The law of conservation of momentum is not mentioned, although mass conservation cannot be understood without it.
3. It is stated that mass increases with velocity.
4. The rest energy and the formula $E_0 = mc^2$ are not mentioned.

We also note that H D Smyth was the chairman of the physics department of Princeton University.

1946
On July 1, 1946, *Time* magazine had Einstein's portrait on its front cover against the background of a nuclear mushroom cloud with $E = mc^2$ written on it [88].

In 1946, Einstein published two papers on the equivalence of mass and energy: "Elementary derivation of the equivalence of mass and energy" [98] and "$E = mc^2$: The most urgent problem of our time" [99].

In the first of them, he partly changed the proof given in 1905 [17]: the body at rest does not emit radiation but absorbs it; he uses the formulas of conservation not of energy but of momentum; formulas for the transformation of energy and momentum of radiation are not used, but instead Einstein uses the known angle of aberration of stellar light caused by the motion of the Earth: $\alpha = v/c$. As a result, Einstein obtains the increment to the mass of the body $M' - M = E/c^2$, where E is the energy of the absorbed radiation, and concludes: "This equation expresses the law of the equivalence of energy and mass. The energy increase E is connected with the mass increase E/c^2. Since energy according to the usual definition leaves an additive constant free, we may so choose the latter that $E = Mc^2$."

It is obvious from the derivation that E here stands for the rest energy of the body.

Einstein does not explain why the rest energy of the body is defined up to a constant.

In his brief popular-style article [99], Einstein first described the law of the conservation of energy using the kinetic and potential energy of a pendulum as an example, and then proceeded to deal with the conservation of mass:

"Now for the principle of the conservation of mass. Mass is defined by the resistance that a body opposes to its acceleration (inert mass). It is also measured by the weight of the body (heavy mass). That these two radically different definitions lead to the same value for the mass of a body is, in itself, an astonishing fact. According to the principle — namely, that masses remain unchanged under any physical or chemical changes — the mass appeared to be the essential (because unvarying) quality of matter. Heating, melting, vaporization, or combining into chemical compounds would not change the total mass.

Physicists accepted this principle up to a few decades ago. But it proved inadequate in the face of the special theory of relativity. It was therefore merged with the energy principle — just as, about 60 years before, the principle of the conservation of mechanical energy had been combined with the principle of the conservation of heat. We might say that the principle of the conservation of energy, having previously swallowed up that of the conservation of heat, now proceeded to swallow that of the conservation of mass — and holds the field alone.

It is customary to express the equivalence of mass and energy (though somewhat inexactly) by the formula $E = mc^2 \ldots$"

What deserves our attention in this passage is not only what Einstein clarified but also what he chose not to explain: namely, that the measure of inertia in relativity theory is not mass but energy and that the quantity $p_\mu p_\nu / E$ creates and feels the gravitational field (and therefore there is nothing surprising about the equality between the inertial mass and the gravitational mass in Newtonian mechanics: both are equal to E_0/c^2), that the principle of energy conservation holds the field not alone but together with the conservation of momentum, that the energy and momentum determine the mass and its conservation and/or nonconservation jointly, and that the mass is equivalent to the rest energy.

In 1949 Einstein published "Autobiographical Notes" [100], which open with these words: "Here I sit in order to write, at the age of 67, something like my own obituary." So in fact he was writing them in 1946–1947. In these notes, Einstein made an attempt to tell us what and how he had been thinking about for many years: "For me it is not dubious that our thinking goes on for the most part without use of signs (words) and beyond that to a considerable degree unconsciously."

On the creation of general relativity he wrote:

"The possibility of the realization of this program was, however, dubious from the very first, because the theory had to combine the following things:

(1) From the general considerations of special relativity theory it was clear that the *inert* mass of a physical system increases with the total energy (therefore, e.g., with the kinetic energy).

(2) From very accurate experiments (specially from the torsion balance experiments of Eötvös) it was empirically known with very high accuracy that the gravitational mass of a body is exactly equal to its *inert* mass."

These words can be interpreted, if one so wishes, as a statement that the formula $E = mc^2$ not only follows from special (partial) relativity theory but is also the cornerstone of general relativity.

1948

In June 1948, Einstein wrote about the thorny question of mass for the last time. In a letter to L Barnett, author of the book "The Universe and Dr. Einstein", he wrote [101]:

"It is not good to introduce the concept of *mass* $M = m/\sqrt{1 - v^2/c^2}$ of a moving body for which no clear definition can be given. It is better to introduce no other mass concept than the "rest mass" m. Instead of introducing M it is better to mention the expression for the momentum and energy of a body in motion."

12. 1949. Feynman diagrams

In 1949, Feynman published "The theory of positrons" [102] and "Space–time approach to quantum electrodynamics" [103]. These papers put quantum electrodynamics into a form that was completely compatible with the symmetry of the Minkowski world. In these papers, he formulated and developed a method known as Feynman diagrams.

The external lines of the diagrams correspond to real on-shell particles: for them, $p^2 = m^2$, where p is the 4-momentum of a particle and m is its mass. The internal lines correspond to virtual particles that are off-shell: for these, $p^2 \neq m^2$. Antiparticles look like particles that move backwards in time. All particles — both massive and massless — are described in the same manner, with a single difference: $m = 0$ is assumed for the latter. (Virtual photons with positive p^2 are called timelike, and those with negative p^2, spacelike.) It goes without saying that the Feynman diagram method is based on the concept of invariant mass m that is independent of the velocity of the particle.

Feynman diagrams drastically simplified calculations for processes involving elementary particles. They unified all types of matter, both for real particles and for virtual ones that replaced fields.

F Dyson, who at the time worked with Feynman, recently recalled [104]:

"During the time that the young physicists at the Institute for Advanced Study in Princeton were deeply engaged in developing the new electrodynamics, Einstein was working in the same building and walking every day past our windows on

his way to and from the Institute. He never came to our seminars and never asked us about our work. To the end of his life, he remained faithful to his unified field theory.''

We know that his famous aphorism—"God is subtle but He is not malicious''—was engraved above the fireplace at the Institute for Advanced Study where Einstein worked. One cannot help recalling his other pronouncement: "I have second thoughts. Maybe God *is* malicious'' [105].

13. 1952 – 1955. Last years

1952
In 1952, Einstein published a new edition of his popular-science book "Relativity, The Special and the General Theory, A Popular Exposition'' [106], was first published in 1917 [61]. For this new edition, he wrote a special appendix, entitled "Relativity and the problem of space," in order "to show that space-time is not necessarily something to which one can ascribe a separate existence, independently of the actual objects of physical reality... In this way the concept of 'empty space' loses its meaning.'' With these words, Einstein was referring not only to general relativity but also to special relativity theory. The concept of virtual particles was perhaps alien to him.

1954
Einstein's foreword to Jammer's book "Concepts of space" [107] may contain a clue to what prevented Einstein from regarding the photon as a material object: "Now as to the concept of space, it seems that this was preceded by the psychologically simpler concept of place. Place is first of all a (small) portion of the earth's surface identified by a name. The thing whose 'place' is being specified is a 'material object' or body.''

From this standpoint, any particle, no matter how light, is a material object while a strictly massless particle is not.

1955
In 1895, the 16-year-old Einstein wrote his first scientific essay [108] on the propagation of light through the ether.

In 1955, in his last autobiographic notes [109], he recalled that at that time a thought experiment started to puzzle him:

"If one were to pursue a light wave with the velocity of light, one would be confronted with a time independent wave field. Such a thing doesn't seem to exist, however! This was the first childlike thought-experiment concerned with the special theory of relativity.''

Thought experiments played an important role in Einstein's research during all his life.

Einstein died on April 18, 1955. A month before his death, Leopold Infeld gave a talk in Berlin at a meeting that celebrated the 50th anniversary of relativity theory [110]. He named the dependence of mass on velocity as the first of the three experimental confirmations of special relativity theory. The baton of "relativistic mass'' was passed on to future generations.

14. Born, Landau, Feynman

Born's books
An important role in this passing of the baton belongs to Max Born's book "Einstein's theory of relativity.'' An outstanding physicist, one of the creators of quantum mechanics, Born did very much to help spread relativity theory. The first edition of his book appeared in 1920 [111] (its Russian edition was published in 1938 [112]). The next edition [113] appeared after Einstein's death in 1962 (and its translation into Russian [114] in 1964 and 1972). Unfortunately, both these editions, which greatly influenced how physics was taught in the 20th century, state without any qualifications that the increase in the mass of a body when its velocity increases is an experimental fact. It is also asserted in [115, 116].

In 1969 — a year before passing away — Born published his correspondence with Einstein [117], which lasted from 1916 till 1955. Not even one among more than a hundred letters touches on the aspect of the [in]dependence of mass on velocity. The correspondence was translated into English, its latest edition was published in 2005 [118] with a detailed foreword, which also ignored the mass controversy.

Landau and Rumer brochure
I mentioned above that the Landau – Lifshitz book "Field theory'' [93] was the first monograph on relativity theory in world literature that consistently applied the idea that the mass of a body is independent of its velocity. It is all the more incomprehensible why in their popular brochure "What is relativity theory?'' [119, 120], Landau and Rumer chose for the first introduction into the theory the statement that mass is a function of velocity and that this is an experimental fact. In the third edition of this brochure published in 1975, Yu B Rumer added "Pages of reminiscences about L D Landau,'' where he quoted a jocular characteristic of the brochure given by Landau himself: "Two con men trying to persuade the third one that for the price of a dime he would understand what relativity theory is.''

The Feynman Lectures
The magnificent lectures on physics that Feynman gave to students of Caltech in 1961 – 1964 [121] instilled a love for physics in the hearts of millions of readers around the world (see, e.g. [122]). They teach readers to think independently and honestly. Alas, these lectures never mention the Feynman diagrams that he invented in 1949 [102, 103] and which brought him the Nobel prize in 1965. Furthermore, the entire relativity theory is introduced in these lectures through the formula $E = mc^2$, not through the concept of the Lorentz-invariant mass on which Feynman diagrams are based.

Feynman states already in the first chapter that the dependence of the mass of a body on its velocity is an experimental fact, in the fourth he says that Einstein discovered the formula $E = mc^2$, and in the seventh that mass is the measure of inertia. In chapter 15, we meet the formula $m = m_0 / \sqrt{1 - v^2/c^2}$ and Feynman discusses the consequences of the "relativistic increase of mass''; in chapter 16, he derives this formula. This chapter ends with the words:

"That the mass in motion at speed v is the mass m_0 at rest divided by $\sqrt{1 - v^2/c^2}$, surprisingly enough, is rarely used. Instead, the following relations are easily proved, and turn out to be very useful: $E^2 - P^2 c^2 = M_0^2 c^4$ and $Pc = Ev/c$.'' (The original notation used by Feynman is retained in this quotation.)

Even in Chapter 17, where Feynman introduces four-dimensional space – time and uses units in which $c = 1$, he continues to speak of the rest mass m_0, not simply of the mass m.

In the course of 2007, I e-mailed a question to a number of Feynman's former students, assistants and co-authors. Not one of them was able to recall even a single occasion when Feynman used the notion of relativistic mass or the formula $E = mc^2$ in discussions he had with them. Nevertheless, several millions of readers of his lectures firmly believe that mass is a function of velocity. Why would the great physicist who gave us the language of Feynman diagrams place the notion of velocity-dependent mass at the foundation of his Feynman lectures?

Perhaps we can find an answer to this question in Feynman's Nobel lecture [123]. He described there numerous 'blind alleys' in which he had been trapped while on his way to constructing quantum electrodynamics, but still expressed the firm belief that "many different physical ideas can describe the same physical reality."

Thus he wrote about the idea of an electron moving backwards in time: "it was very convenient, but not strictly necessary for the theory because it is exactly equivalent to the negative energy sea point of view." However, without a time-reversed electron, there would be no Feynman diagrams, which introduced order and harmony into huge areas of physics.

15. Epilogue

Why is it that the weed of velocity-dependent mass is so resistant? First and foremost, because it does not lead to immediate mistakes as far as arithmetic or algebra are concerned. One can introduce additional 'quasi-physical variables' into any self-consistent theory by multiplying true physical quantities by arbitrary powers of the speed of light. The most striking example of such a 'quasi-quantity' is the so-called 'relativistic mass.' If calculations are done carefully enough, their results should be the same as in the original theory. In a higher sense, however, after the introduction of such 'quasi-quantities,' the theory is mutilated because its symmetry properties are violated. (For example, the relativistic mass is only one component of a 4-vector, while the other three components are not even mentioned.)

Some other explanations of the longevity of relativistic mass can be given here. The formula $E = mc^2$ is 'simpler' than the formula $E_0 = mc^2$ because the additional zero subscript that requires explanation is dropped. The energy divided by c^2 indeed has the dimensionality of mass. Intuition based on conventional everyday experience slips in a hint that the measure of inertia of a body is its mass, not its energy, and this prods one to 'drag' the nonrelativistic formula $\mathbf{p} = m\mathbf{v}$ into relativity theory. The same intuition suggests, with hardly less insistence, that the source of gravitation is 'our own' mass, not an 'alien' quantity $p_\mu p_\nu / E$. Everyday experience rebels particularly strongly against the idea of treating light as a type of matter.

The arguments given above may explain the 'Newtonian bias' of an ordinary person, let us say 'a pedestrian.' However, it would be too flippant to attribute them to such a great physicist as Einstein. Indeed, it was Einstein who introduced the concept of rest energy E_0 into physics and wrote about $E_0 = mc^2$ far more often than about $E = mc^2$. Still, one thing remains unexplained: why was it that during the half-century of discussing the relation between mass and energy, Einstein never once referred in either his research publications or his letters to the formula $E^2 - \mathbf{p}^2 c^2 = m^2 c^4$, which defines the Lorentz-invariant mass?

It is possible that the formulation of the total equivalence of energy and mass reflected Einstein's absolute reliance on his powerful intuition. It was without a doubt his confidence in his own intuition that resulted in his rejection of quantum mechanics. One feels that he perceived the concept of electromagnetic potential not only with his mind but with his entire body. And he 'felt' the wave function to be very much like the electromagnetic wave. His resistance to quantum mechanics prevented Einstein's world line from meeting Feynman's world line in the space of ideas — in the noosphere, so to speak. As a consequence, Einstein refused to accept the photon as a particle of matter and continued to treat it as a quantum of energy.

16. Conclusion

When shown an art exhibition in Moscow Manege in 1962, Nikita Khrushchev (1894–1971) rudely attacked the sculptures of Ernst Neizvestny. When Khrushchev died, his children requested Neizvestny to create a memorial sculpture at the grave of their father. The main part of this memorial consists of two vertical marble slabs, one white, the other black, whose protrusions penetrate each other. These slabs in a way symbolize the good and the evil.

The history of the confrontation of two concepts of mass in the 20th century resembles this sculpture. Here the light and the darkness were fighting each other in the minds of the creators of modern physics.

In the world of opinions, pluralism is considered to be politically correct. To insist on a single point of view is thought to be a manifestation of dogmatism. A good example of fruitful pluralism is the wave–particle duality in quantum mechanics. But there are cases in which a situation is ripe for establishing unambiguous terminology. The relation between energy and mass is more than ripe for this. It is high time we stopped deceiving new generations of college and high school students by inculcating into them the conviction that mass increasing with increasing velocity is an experimental fact.

Postscriptum.
In memory of J A Wheeler

This review was already completed when I received the sad news that John Archibald Wheeler, an outstanding physicist and teacher who accomplished so much for establishing the spacetime interpretation of relativity theory and of the concept of Lorentz-invariant mass, died on 13 April 2008, at the age of 96. I dedicate this paper to his memory.

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