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Dark energy and universal antigravitation

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Editor's note

The paper "Dark energy of cosmic vacuum" by A D Chernin was received by the Editors of *Physics – Uspekhi* on 27 December 2006. It should be noted that the dark energy problem, which emerged in the modern sense about 10 years ago, is one of the most important, but still unclear, issues in modern physics and cosmology.

The paper was sent to four referees, whose comments resulted in a number of sometimes fairly acute remarks. The comments were forwarded to the author, with a suggestion to take the criticism into account and present a revised version of the paper. Now (11.12.2007) this version has been received and is published without any further changes. Sending the new version to the referees would indefinitely delay the discussion of the very important issue, and that would be unpractical. Accordingly, as mentioned, the new version is being published without changes, but it has also been sent to all the referees. Their comments on the new version will also appear in *Physics – Uspekhi* without refereeing.

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Abstract. Universal antigravitation, a new physical phenomenon discovered astronomically at distances of 5 to 8 billion light years, manifests itself as cosmic repulsion that acts between distant galaxies and overcomes their gravitational attraction, resulting in the accelerating expansion of the Universe. The source of the antigravitation is not galaxies or any other bodies of nature but a previously unknown form of mass/energy that has been termed dark energy. Dark energy accounts for 70 to 80% of the total mass and energy of the Universe and, in macroscopic terms, is a kind of continuous medium that fills the entire space of the Universe and is characterized by positive density and negative pressure. With its physical nature and microscopic structure unknown, dark energy is among the most critical challenges fundamental science faces in the twenty-first century.

1. Introduction

Since Newton's *Principia* was published, gravitational attraction has been referred to as the force that moves worlds, and nothing seemed to be able to shake this common belief. But it was found in 1998 – 1999 that a completely different force — repulsion, or antigravitation — rather than gravitational attraction drives the dynamics of the observed Universe. Antigravitation acts on outflawing galaxies and makes them move away from each other; hence, the expansion of the Universe is accelerating. The accelerated cosmological expansion was discovered in direct astronomical observations at distances of a few billion light years, almost at the edge of the observable Universe [1, 2].

Velocities and distances of scattering galaxies have been undergoing measurement for almost a century now. But the acceleration of galaxies was first measured only ten years ago

after long systematic observations, which were carried out by two independent groups of astronomers, the first of them being led by Brian Schmidt and Adam Riess [1] and the other by Saul Perlmutter [2]. Distant supernovae of a certain type (Ia) were studied. They are so bright at peak luminosity that they can be observed at very large, truly cosmological distances of several hundred and thousand megaparsecs [1 megaparsec (Mpc) is approximately equal to 3×10^{24} cm]. Studying supernovae allows finding how the galaxies where these stars burst are moving. The observations were carried out with the most sophisticated modern astronomical instruments, the Hubble Space Telescope and the largest ground-based reflectors. This allowed discovering and measuring a fine relativistic effect that modifies apparent brightness of a source depending on the redshift. This effect is due to the acceleration of motion of a luminous source and is noticeable only at large distances as the redshift (a relative stretch of wavelength in an observed spectrum of a source) becomes comparable to unity. In this way, the cosmological expansion was discovered to proceed with positive acceleration, which means that velocities of outflawing galaxies increase with time.

The acceleration implies a force that drives the motion of bodies. The force cannot be attraction of cosmic bodies to each other because mutual attraction of galaxies is only capable of decelerating their motion. What can accelerate the motion is a force with the opposite sign, which is called universal antigravitation. Its physical source is dark energy that manifests itself in the Universe only because of its property of providing antigravitation. As for the rest of the properties, it is invisible and elusive, it neither emits nor absorbs light, and it does not scatter light. Microscopic properties of dark energy are similar to those of a very special continuous medium with positive density and negative pressure. As far as dark energy's physical nature and microscopic structure are concerned, they are still completely unknown.

In its simplest (and, seemingly, most believable) interpretation, dark energy is related to the Einstein cosmological constant. The hypothesis of universal cosmic repulsion was proposed by Einstein [3] in 1917 when he first applied his just created general relativity theory to the problem of the world as a whole.

In his approach to cosmology, Einstein followed an old tradition of natural philosophy that ascribed ideal spatial (uniformity and isotropy) and temporal symmetry to the Universe. The temporal symmetry implies that eternity and invariability are intrinsic to the existence of the world. In accordance with these common beliefs, Einstein constructed a theoretical model of a uniform world, as well as the static — eternal and invariable — world. But the general relativity theory did not directly imply that the world must be static. To introduce this property into his cosmological model, Einstein had to resort to an extra assumption of a universal repulsion existing in nature, capable of balancing and compensating the universal gravitation in the Universe as a whole. Only under this condition could the world's matter and, hence, the entire Universe, be at rest.

This assumption required modifying the general relativity equations and adding an extra term. In general relativity, Einstein's antigravitation is represented and described by the only term, the cosmological constant Λ, which has the same value always and everywhere. It is easy to understand why and for what purpose Einstein needed universal antigravitation. It is more difficult to imagine how and in what way he could find that simple and natural form to fulfill his idea. Einstein himself was unable to explain how, although in paper [3] he tried to outline a 'wandering and rough way' of reasoning that led him to the idea of the cosmological constant. It is only today that we have started to realize the meaning and insight of his idea: it was a theoretical prediction unordinarily profound and audacious.

In 1922, five years after Einstein's works, A A Friedmann proved that antigravitation did not exclude the evolution of the world if only the cosmic attraction and repulsion were not required to be fully in balance. Friedmann constructed the model of an expanding Universe [4] — a cosmological model that possessed uniformity and isotropy in space (like Einstein's model), but was not static. This model is described by exact solutions of the general relativity equations and incorporates the cosmological constant as a free physical parameter. The value of the constant Λ does not follow from the theory; it is subject to measurement in certain cosmological observations.

Friedmann's theory with the value of the constant Λ following from the latest observational discoveries describes global properties of the real world very well and is completely consistent with the full set of modern astronomical data. It is the basis of the present-day cosmological 'standard model' [known as ΛCDM cosmology (CDM stands for Cold Dark Matter)].

Coming round to the background of the newest discoveries we note that Einstein highly appreciated Friedmann's theory (although not immediately). Hubble's astronomical research, which admittedly verified the expanding Universe theory, also greatly impressed Einstein. But since the real world is unstatic, why do we need the cosmological constant? Likely, Einstein lost interest in the idea of cosmological repulsion and suggested forgetting the cosmological constant before, as he said, 'sufficient empirical reasons' would favor it. In several editions of Landau and Lifshitz's *The Classical Theory of Fields* [5], we can read about the cosmological constant that "...there are no urgent and well-grounded reasons... to change the gravitation equations in this way." Says V L Ginzburg [6], "L D Landau did not want to hear anything about the Λ-term, but I failed to learn from him why." W Pauli was also seriously against the idea of the cosmological constant.

In the late 1960's, an astronomical hint of a nonzero and, moreover, positive cosmological constant was found in a peculiarity of quasar distribution with respect to redshift [7]. Later, these arguments were eliminated, but the general considerations about a possible role of the cosmological constant and particularly about adequate observational tests proposed at that time [8, 9] are still of great importance.

Interest in the cosmological constant occurred from time to time in view of the problem of the world's age. Clearly, the Universe as a whole cannot be younger than astronomical bodies inhabiting it. However, initial (much understated) estimates based on Hubble's data from 1930 – 1940s yielded the value of the world's age of approximately 2 billion years. But this is less than the geological age of the Earth. Subsequently, after a systematic error in Hubble's data was corrected, the age of the world was estimated to be 7 to 9 billion years. Astronomers, however, estimated the age of the oldest formations in our Galaxy, which are globular clusters, to be typically 12 to 15 billion years. The idea of the cosmological constant promised to solve this difficult para-

dox (see, e.g., the classic textbooks [10 – 12]). In book [13] published in 1988 on the 100-year anniversary of Friedmann's birthday, it was noted that antigravitation is capable of providing the sought value of the cosmological age if universal antigravitation is stronger than proper gravitation of matter in the modern Universe.

In the above-mentioned standard cosmological model, the world's age is taken to be equal to approximately 14 billion years. This follows from both the earliest data on dark energy [1, 2] and further observational research (see especially Refs [14, 15]). Thus, the contradiction with the estimates of the oldest stars' age is resolved, the age of the Universe and its other observable features being directly related to dark energy and its observable properties. In the standard cosmological model, the density of dark energy is specified by the cosmological constant (see Section 2) and, hence, this density is constant in time and perfectly uniform in space. Furthermore, the dark energy density has the same value in all reference frames.

Density is the main quantitative feature of dark energy. Its value was estimated in the earliest papers [1, 2]. If we take the mass of a hydrogen atom for comparison, the dark energy density value corresponds to about three hydrogen atoms in each cubic meter of space. To imagine the antigravitation force that an antigravitational medium with this density can exert, we consider two neutral hydrogen atoms put in space where nothing but dark energy is present. These atoms are subjected to two forces, Newton's force of mutual attraction and Einstein's repulsion force. It turns out that antigravitation is stronger than gravitation if the atoms are separated by a distance of more than half a meter.

According to the data in Refs [1, 2], the fraction of dark energy is 70% of the world's total density, and therefore dark energy is the main sort of energy/mass in the observable Universe. Clearly, under these conditions, antigravitation produced by dark energy is to dominate in the dynamics of the cosmological expansion. There are three more sorts of cosmic energy in nature. One of them is dark matter, whose proportion is 25% of the world's total density; it is suggested to comprise hypothetical nonrelativistic ('cold') stable elementary particles that do not partake in the strong nuclear interaction. Around 5% of the total density belongs to the 'ordinary' matter, i.e., protons, neutrons, and electrons, which constitute planets, stars, and other ordinary natural bodies; this cosmic energy is traditionally called baryons (although the electron is not a heavy particle). Finally, the forth cosmic energy is radiation, which is meant to include relic photons (and, perhaps, gravitons as well); the proportion of radiation is not greater than a few hundredths of a percent of the total density (these values are to be specified further in Section 2.6).

Modern data on the world's age and densities of the four cosmic energies follow from the combination of all the observational studies carried out in the past decade (papers [14 – 36] should be particularly mentioned). This noncontradictory, often referred to as 'concordant,' data set definitely shows that the discovery of antigravitation and dark energy [1, 2] withstands the reliability test.

In present-day astronomy and physics, both new concepts, antigravitation and dark energy, are gradually joining the most fundamental concepts of natural science. In this paper, we focus our attention on them. The paper does not claim to fully and equally embrace all theoretical and experimental results. Everywhere (except in specially mentioned cases), we follow the standard cosmological model; by dark energy, we mean an antigravitating medium described by Einstein's cosmological constant. Many issues that are beyond the scope of our considerations can be found in review articles and books [37 – 73], both in the latest papers and those that have already become classic. The number of papers on dark energy is constantly increasing, the literature on this issue is becoming too much to handle: on the Internet (at the time of writing), there are around 6,430,000 pages, not less than a million of them deserving attention. No wonder that our list of references is certainly incomplete and sometimes fragmentary.

2. The law of universal antigravitation

Einstein did not give a physical interpretation of the cosmological constant. It can only be guessed that he considered it likely as a 'geometrical' rather than 'material' term, because he inserted Λ in the left-hand ('geometrical') side of the general relativity equations. The constant Λ then implies the presence of an intrinsic space – time curvature, unrelated to the presence or absence of any mass/energy in space. The geometrical interpretation was not further significantly developed (see, however, Section 4).

2.1 Einstein – Gliner vacuum

In the mid-1960s, E B Gliner proposed a 'material' interpretation of the cosmological constant; he showed [74] that Einstein's idea is equivalent to the assumption of a perfectly uniform macroscopic medium with the density

$$\rho_{\mathrm{V}} = \frac{\Lambda}{8\pi G}\,, \tag{1}$$

where G is the Newton gravitational constant; here and in what follows, we set the speed of light to unity, $c = 1$. Density (1) does not vary in time and space and stays the same in all reference frames.

The medium with density (1) has a negative pressure p_{V}, and its equation of state (i.e., the relation between pressure and density) is

$$p_{\mathrm{V}} = -\rho_{\mathrm{V}}\,. \tag{2}$$

A medium with such an unusual equation of state is not like any 'normal' fluid or gas. Following Gliner [74], we list its most important unique properties.

(1) This medium cannot be a reference frame. Given two reference frames moving with some nonzero relative velocity, the medium with equation of state (2) is comoving to both of them. Hence, motion and rest relative to this medium cannot be distinguished. But this is the main mechanical property of the vacuum (cf. [75]). Therefore, the medium described by equations (1) and (2) is the vacuum.

(2) The medium with equation of state (2) is invariable and 'eternal.' Its energy is the absolute and invariable minimum of energy present in the world's space. This is one more compulsory property of the vacuum.

(3) The medium with the pressure defined by Eqn (2) produces antigravitation rather than gravitation. This is because, in accordance with the general relativity, gravitation is determined not only by the medium density (as assumed in Newton's theory) but also by its pressure, the 'effective gravitating density' being generally expressed by the

sum of two terms:

$$\rho_{\text{eff}} = \rho + 3p\,. \tag{3}$$

We note that the coefficients 1 and 3 of the density and pressure occur in the right-hand side of formula (3) because time is one-dimensional and space is three-dimensional.

With equation of state (2), the sum in the right-hand side of (3) turns out to be negative:

$$\rho_{\text{eff}} = \rho_{\text{V}} + 3p_{\text{V}} = -2\rho_{\text{V}} < 0\,. \tag{4}$$

Negative effective density implies 'negative' gravitation. Unlike universal gravitation, universal antigravitation does not cause bodies to be drawn to each other; on the contrary, it makes them more away from each other. If we put two test particles at relative rest at the initial instant in the vacuum, the vacuum causes them to move apart.

(4) In terms of Newtonian physics, the vacuum produces force, but is subjected (as a macroscopic medium) neither to external gravitational forces nor to antigravitation of its own. As is known, three masses are distinguished in physics: active gravitational mass, i.e., the mass producing gravitation; passive gravitational mass, i.e., the mass perceiving gravitation, 'feeling' it; and inertial mass, which enters the left-hand side of Newton's equation of motion. These three types of mass are inherent in all natural objects, in all bodies and energies, including vacuum. The just mentioned effective gravitating density is the density of the active gravitational mass. For any uniform medium, the density of the passive gravitational mass is given by $\rho_{\text{pass}} = \rho + p$. For the vacuum, this value is zero, $\rho_{\text{pass}} = \rho_{\text{V}} + p_{\text{V}} = 0$. That is why the vacuum feels neither external nor its own gravitation. This is the case, the only one in physics, where an action causes no reaction. In accordance with the equivalence principle underlying relativity, the inertial mass (its density is given by $\rho_{\text{in}} = \rho + p$) of the vacuum is also equal to zero.

(5) With the dark energy density having a constant value, the vacuum should be imagined as a medium uniformly permeating the space at every scale from cosmological to arbitrarily small. To be precise, in the range of small scales, we are allowed to speak only about fractions of a millimeter, not less, because the gravitation theory itself with its inverse-square law is experimentally verified only to submillimeter distances. To a reliable approximation, we can think that the vacuum exists and is absolutely uniform at least down to a small scale of around a few centimeters. Bodies (at least macroscopic ones) immersed in dark energy do not push it out from the volumes they occupy — the dark energy density is the same both inside and outside the bodies.

This interpretation of the cosmological constant (met, by the way, with enthusiasm 40 years ago) is becoming more common today and likely to become even more so soon. Henceforth, to be definite, we talk about a cosmic medium with density (1) and equation of state (2) as the Einstein–Gliner vacuum (the EG vacuum).

There are reasons to believe that dark energy discovered by astronomers [1, 2] is actually the energy of the EG vacuum. The final evidence has yet to be found, but the combination of all observational data on dark energy obtained from 1998–1999 is completely consistent with this possibility.

What are the alternatives to the EG vacuum? Although alternative hypotheses discussed in the literature assume deviations from Einstein's idea of the cosmological constant, they still keep a macroscopic description of dark energy as a medium with negative pressure. But the relation between pressure and density is assumed to be different from that in Eqn (2). For example, one of such hypotheses is the popular hypothesis of 'quintessence' [76–85], by which one usually means a nonvacuum medium for which the ratio of pressure to density is constant and greater than minus one (e.g., $p/\rho = -2/3$). Furthermore, the effective gravitating density can be negative as in the case of the EG vacuum. Such a medium is nonstatic: it changes in time and its density must diminish in the course of cosmological expansion. Generally, the quintessence density is nonuniform in space. From the very beginning, the advantages of this interpretation have raised doubts [86]. Another hypothesis interprets dark energy as a 'phantom energy' for which $p/\rho < -1$ [87–89]. Media for which the ratio of pressure to density is not constant are also considered. Such an example is the hypothesis of dark energy in the form of 'Chaplygin's gas' [90–92]. It has the equation of state $p = -C/\rho$, with C being a positive constant.

It must be said that the increasing precision of cosmological observations is gradually reducing the possibilities of alternative interpretations of dark energy. According to the latest observational data [15, 21, 25], the ratio of the dark energy pressure to its density is

$$w \equiv \frac{p}{\rho} = -0.97 \pm 0.09\,. \tag{5}$$

The EG vacuum described by (2) is consistent with this constraint, but, for example, a quintessence with the value $w = -2/3$ does not satisfy the constraint; there is a very narrow window left for it, $-1 < w < -0.89$. Some people believe that alternative variants will keep their appeal until the corresponding observational window becomes less than 1%.

As was mentioned in Section 1, the modern standard cosmological model includes the concept of the cosmological constant. In the ΛCDM model, the density of dark energy is given by relation (1). Moreover, all the above-described properties of the EG vacuum reveal themselves one way or another in the standard model.

2.2 Naturalness problem

While the macroscopic interpretation (and understanding) of dark energy as a vacuum with equation of state (2) can be considered satisfactory, the issue of the microscopic structure of the EG vacuum remains in abeyance. It is unknown what microscopic objects are 'carriers' of its dark energy. It is even unclear in this case whether it is worth speaking about some kinds of carriers at all, or whether different physical notions and concepts that have been unknown so far must be taken into account. But since this is a vacuum, the first question that arises is as follows: Is the EG vacuum identical to the vacuum of quantum fields (the Q-vacuum) that is well known in physics?

The vacuum as the lowest-energy state of quantum fields has been discussed since the late 1920s; its existence follows from the Heisenberg uncertainty principle. As is known (see, e.g., Ref. [93]), energies of 'zero' quantum fluctuations sum up to give the Q-vacuum energy. Formally, this energy is infinite in the standard quantum mechanics. But virtually, its value remains indefinite, which, however, does not prevent scientists from carrying out theoretical analysis and interpreting laboratory experiments. This is because in all processes and interactions (except the gravitational one), physical effects

depend not on the 'total' energy value but only on energy differences in different regions of space and/or at different instants. There is no doubt that the Q-vacuum actually exists: it is there where interactions of elementary particles occur, and its presence is revealed experimentally, in particular, in the Lamb shift of atomic spectral lines and in the Casimir effect (see, e.g., Ref. [94]).

We note that before Gliner's papers appeared, the macroscopic properties of the Q-vacuum as a continuous medium and its equation of state had probably not drawn the attention of theoreticians. Nevertheless, in the late 1920s – early 1930s, G A Gamow and W Pauli were concerned with the problem of gravitational effects of the Q-vacuum. Gamow repeatedly said that cosmological observations do not allow infinite values of the Q-vacuum density. This density has an upper bound; otherwise, the Universe would have collapsed long ago.

In 1967, Ya B Zel'dovich [95] (see also Refs [38, 39]) proposed that the sum of vacuum (formally, infinite) energies of all fields and particles could somehow provide a finite and, moreover, small value of the Q-vacuum energy. It was taken into account that the fermion and boson vacua have opposite signs of energy and, in principle, could compensate each other if there were a perfect symmetry between bosons and fermions (which was later termed supersymmetry). Then the total density of the all-field Q-vacuum would eventually be zero. But this symmetry does not have to be absolutely perfect; it can be weakly violated. Hence, the compensation of the energies is not necessarily full and, as a result, a nonvanishingly small difference between two formally infinite vacuum energies emerges (see also Ref. [43]). In this case, according to Zel'dovich's idea, it is possible to identify two vacua, the one that is described by the cosmological constant and the quantum one. So far, no one has succeeded in proving this exceptionally attractive idea either right or wrong.

Once cosmological phenomena are being considered, gravitation must be taken into account in discussions of the nature of dark energy. Unlike all the other physical interactions, only gravitation feels the entire energy, rather than its differences in different regions of space and at different instants. Unless gravitation is taken into account, it is impossible to argue whether the energy is equal to zero (when the energy zero level is chosen arbitrarily, the notion of 'absolute zero' makes no sense). On the contrary, when gravitation is taken into consideration, the energy zero level has an absolute meaning, and at least in this case, the energy of quantum zero fluctuations acquires a definite value, which it must have in cosmology.

Speculating consistently, we can imagine that the vacuum state of physical fields, in which quantum and gravitation effects could act equally, must be described by characteristic combinations of the three fundamental physical constants: the speed of light c (representing relativity), the Planck constant $\hbar$ (quantum physics) and the gravitational constant G (gravitation). If this is so, the 'natural' value of the Q-vacuum density must be the well-known Planck density, the only combination of the three constants with the required dimension:

$$\rho_{\mathrm{P}} = \frac{c^5}{(8\pi G/3)^2 \hbar} \,. \tag{6}$$

The Planck density is of the order $\sim 10^{91}$ g cm^{-3}, which is more than 120 orders of magnitude greater than the actually observed density of dark energy. Clearly, this is the upper bound for densities of any possible vacua. This gap in orders of magnitude is known as the 'naturalness problem' in theoretical physics [41, 96].

The naturalness problem is a severe test for the entire fundamental theory. If 'natural' for the fundamental theory turns out to be practically absurd, the theory itself is subject to revision. What exactly must be modified in it? A clear-cut answer to this question is unavailable so far. Many researchers believe that the answer will hardly appear until we can truly understand how gravitation can be made consistent with quantum physics. A quantum theory of gravitation does not yet exist; not everyone even agrees with the fact that the nonlinear theory of gravitation, i.e., the general relativity theory, must and may be quantized in the usual standard sense (see, e.g., papers [97, 98] and the references therein). Solving all such issues is promised by the string theory, invented exactly to combine gravitation with the rest of physics and thus to explain everything.

But until this is accomplished, shall we look for — maybe only in a purely combinatorial way, without any logic — a suitable formula instead of (6) in order to express the vacuum energy through the fundamental microscopic constants? Such attempts as made by Zel'dovich [39, 96] seemed to him to be not very successful. All the more interesting is a recent suggestion by N Arkani-Hamed et al. (see [99] and the references therein); their formula is simple and (for the first time?) gives at least the required numerical value of the dark energy density:

$$\rho_{\mathrm{V}} \sim \left(\frac{M_{\mathrm{EW}}}{M_{\mathrm{P}}}\right)^8 \rho_{\mathrm{P}} \,. \tag{7}$$

Here, $M_{\mathrm{P}} = \left[\hbar/(8\pi G/3)\right]^{1/2} \sim 10^{-6}$ g $\sim 10^{18}$ GeV is the Planck ('reduced') mass and $M_{\mathrm{EW}} \sim 10^3$ GeV ~ 1 TeV is the characteristic energy of the electroweak interaction. We can easily see that the small ratio of the two characteristic mass/energies raised to a certain power diminishes the Planck density by the required number of orders of magnitude [yet the power in (7) is unusually high].

Certainly, formula (7) does not follow from any fundamental theory; but it contains the long-discussed idea of the electroweak energy M_{EW} playing, perhaps, a central role in all of fundamental physics [93]. Whether it is true is to be ultimately verified in experiment; the general belief is that such a possibility will soon appear (this might happen even in the current year 2008) when experiments at CERN's Large Hadron Collider (LHC) with particles energy ≈ 10 TeV start.

Anyway, relation (7) is a lucky combination of only two universal energies, and if this is not just a happy arithmetical accident, formula (7) implies that the physical nature of dark energy is determined by the interaction of gravitation (represented by the Planck mass) and electroweak forces. But what kind of process is it, and where and how does it unfold? What are the expected experimental tests and observational manifestations in this case? Almost nothing is known about it yet (see, however, Section 5).

It is worth noting that among actively discussed candidates for a 'dark particle,' that is, for the carrier of dark matter, especially attractive are hypothetical elementary particles termed WIMPs (weakly interacting massive particles) whose mass is usually considered to be comparable with the electroweak mass M_{EW}. These particles had been discussed long before formula (7) appeared. The idea of

WIMPs combined with formula (7) indicates a possible relation between dark matter and dark energy. If such a relation exists, the entire 'dark sector' of cosmology is determined by two and only two fundamental physical constants, M_{P} and M_{EW}. We return to the issue of relations between cosmic energies in Section 5.

2.3 In the Newtonian language

Although Einstein's antigravitation, which came into physics along with the general relativity theory, is essentially a relativistic phenomenon, it can be described in terms of Newtonian classical mechanics in a simple way. Such a possibility is given by Friedmann's cosmological theory. It is long known [100] that its basic physical contents can be expressed in the simplest way in classical notions. It is exceptionally important that the mathematical formulas remain exactly the same as in Friedmann's theory. Only their interpretation is changed; to be precise, new names are given, but the objects themselves and relations between them remain the same as in the relativistic approach.

Friedmann's results considered in the Newtonian spirit and related to the cosmological dynamics follow from the classical conservation laws [100]. Most of all, this is the mechanical energy conservation law: the corresponding relation is derived from the general relativity theory, but has exactly the same form and meaning as in Newtonian mechanics. Friedmann's theory involves a uniform self-gravitating medium consisting of particles that move such that all distances between them change with time in accordance with the same law. Thus, the total energy E of each particle is conserved. Virtually, a particle can be represented, for example, by a galaxy (or even a cluster of galaxies) participating in the common cosmological expansion and moving away from an arbitrarily chosen center (for example, from us) with the velocity V. The kinetic energy of the particle as a whole is $K = V^2/2$ and the potential energy is $U = -GM/R$, with $R(t)$ being the distance from the particle to the center and M being the total mass inside a sphere of radius R (the energies are given per unit mass). Because Friedmann's model involves the cosmological constant Λ, speaking in modern terms, particles are assumed to be immersed in the EG vacuum of the constant density $\rho_{\mathrm{V}} = \Lambda/(8\pi G)$, this density contributing to the total mass M. The mechanical energy conservation law has a fairly classical form:

$$E = K + U\,. \tag{8}$$

We note that the potential energy U looks as if there is nothing — no matter, no vacuum with nonzero energy — outside the sphere of radius R. This implies that the potential energy of the particle [together with a force exerted upon it (see below)] is determined only by the 'internal' mass M, while the 'external' mass distribution at distances larger than R has no influence on the particle motion, irrespective of whether it ranges with no limit, even to infinity, or is finite in space.

It is known that Newton first attempted to set up the problem about gravitational forces produced by an infinite uniform matter distribution. But neither Newton nor his successors before Einstein and Friedmann succeeded in doing so; insuperable paradoxes were always impeding their attempts. And that is understood: justifying the correct approach to the problem is impossible in the framework of Newton's theory in principle.

On the other hand, the statement of the problem that is prompted by the relativistic theory seems to be natural from the standpoint of classical physics. Indeed, the main point in this approach is the possibility of eliminating infinities and restricting ourselves to considering a finite spherical volume mentally cut out of the infinite volume of the uniform matter distribution. As regards the influence of uniformly distributed external masses on motions inside the sphere of a given finite radius, to start dealing with it, we should first consider an external spherical layer of finite thickness. We can then easily verify (by direct calculation or using the Gauss theorem) that as a consequence of all-direction symmetry, the layer does not exert any force on a particle under it. It should seem that we can mentally add layer by layer up to infinity. But the argument becomes inapplicable as soon as operations with infinities are concerned; neither simple calculation nor the Gauss theorem can be used; that is why the classical theory raised paradoxes. But in the general relativity, relation (8) 'obtains itself' by straightforward integration of the equations for a uniform energy distribution, without any extra arguments, limitations, or special justification.

The mass M entering the expression for potential energy includes masses of both the gravitating matter (baryons plus dark matter) and the EG vacuum within the expanding sphere comprising the given particles,

$$M(R) = \frac{4\pi}{3}(\rho_{\mathrm{M}} + \rho_{\mathrm{V}})R^3\,, \tag{9}$$

where ρ_{M} is the matter density and ρ_{V} is the vacuum density, as above.

Evidently, the total matter mass M_{M} within the volume occupied by the given particles does not change as the volume 'comoving' with the particles expands, with the matter density decreasing as $\rho_{\mathrm{M}} \propto R(t)^{-3}$ as the radius increases. Because the vacuum density has a constant value, the total vacuum mass in the same volume increases as the volume expands:

$$M_{\mathrm{V}} = \frac{4\pi}{3}\,\rho_{\mathrm{V}} R^3 \propto R^3\,. \tag{10}$$

Because the velocity V of the particle motion is the first derivative of distance with respect to time, $\dot{R}(t)$, mechanical energy conservation law (8) takes the form

$$E = \frac{1}{2}\,\dot{R}^2 - \frac{4\pi G}{3}(\rho_{\mathrm{M}} + \rho_{\mathrm{V}})R^2\,. \tag{11}$$

Written in this form, the mechanical energy conservation law is completely equivalent to one of the two basic cosmology equations, which is called the 'first,' or dynamic, Friedmann equation.

Continuing the speculation in terms of Newton's theory, we can proceed in the usual way from the mechanical energy conservation law to forces acting on a given particle. We then have to differentiate Eqn (11) with respect to time. As a result, we obtain

$$\ddot{R}(t) = -\frac{GM_{\mathrm{M}}}{R^2} + 2\,\frac{4\pi G}{3}\,\rho_{\mathrm{V}} R\,. \tag{12}$$

The term in the left-hand side of (12) is the particle acceleration and the two terms in the right-hand side are two forces (per unit mass), the Newtonian gravitational force produced by the matter and described by the inverse-square

law,

$$F_{\mathrm{N}} = -\frac{GM}{R^2}\,, \tag{13}$$

and the Einstein antigravitational force produced by the EG vacuum

$$F_{\mathrm{E}} = +2\rho_{\mathrm{V}}\,\frac{4\pi G}{3}\,R\,. \tag{14}$$

The antigravitational force has the sign opposite to that of the Newtonian force, and linearly increases rather than decreases as the distance increases. Relation (14) is exactly the Einstein universal antigravitation law.

As we see from (11), the relativistic gravitational effect of pressure [see Eqn (3)] naturally emerges here, and it underlies the plus sign of the antigravitational force. Although dark energy is a purely relativistic object, the Newtonian approach yields an exact equation for the effective force (as in the relativistic case).

Because the gravitational force decreases and the antigravitational force increases as the distance R increases, gravitation prevails at small distances, while antigravitation does at large ones. In cosmology, R is the distance between given particles moving away from each other and is an increasing function of time. Therefore, small R correspond (for the given particles) to an early cosmological epoch and large R correspond to a late one. Gravitation is stronger with small time, antigravitation with large time.

In the infinitely distant future, the antigravitational force will fully dominate the world and the dynamics of the cosmological expansion will be governed only by Einstein force (14). In cosmology, this is called the de Sitter world. In this limit case, galaxies and their systems appear to be 'test particles' — their mutual gravitation is negligibly small compared to antigravitation produced by the vacuum. It can be easily seen that galaxy scattering then obeys the exponential law

$$R \propto \exp\frac{t}{t_{\mathrm{V}}}\,, \tag{15}$$

where t_{V} is the characteristic expansion time determined only by the vacuum density,

$$t_{\mathrm{V}} = \left(\frac{8\pi G}{3}\,\rho_{\mathrm{V}}\right)^{-1/2}.$$

If exponential law (15) for galaxy motion holds, their relative velocities in the de Sitter world are in direct proportion to relative distances with a constant proportionality factor:

$$V \equiv \dot{R} = H_{\mathrm{V}} R\,, \qquad H_{\mathrm{V}} = \frac{1}{t_{\mathrm{V}}}\,. \tag{16}$$

When applied to a fixed instant of time, relation (16) means that the observed relative velocity of two particles moving away from each other is to be in direct proportion to the distance between them.

The problem of test particles moving in a uniform and invariable vacuum background (described by the cosmological constant) had been solved before Friedmann: it was studied by W de Sitter and later H Weyl; linear velocity law (16) with a constant factor H_{V} then appeared for the first time. In 1929, the law of direct proportionality between velocity and distance was found by E Hubble in observations of galaxies moving away from us, $V = H_0 R$, with H_0 being an observed quantity known as the Hubble constant.

With the discovery of dark energy and its interpretation in the spirit of the EG vacuum, it has become clear that the modern Universe is in a dynamic state, which is not too far from the asymptotic regime described by formulas (15) and (16). This is understood if we take into account that a significant proportion [70% and maybe even 80% (see Section 2.6)] of the total energy in the present-day Universe is in dark energy. Furthermore, the antigravitation effect is amplified (doubled) because the effective gravitating density of dark energy is expressed as $-2\rho_{\mathrm{V}}$ [see relations (3) and (4)]. This is because the 'universal Hubble constant' H_{V} entering relation (16) gains a particular role in the cosmology of the observable Universe (see also Section 3). A quantitative estimate using the measured value of the dark energy density (see Section 1 and Section 2.6) leads to the value

$$h_{\mathrm{V}} = \frac{H_{\mathrm{V}}}{100} = \left(\frac{8\pi G}{3}\,\rho_{\mathrm{V}}\right)^{1/2} \approx 0.60-0.64 \tag{17}$$

(in the 100 km s^{-1} Mpc^{-1} units commonly used in cosmology). This value is not very far from the Hubble constant value $h_0 = H_0/100 = 0.72 \pm 0.04$, which is determined using global cosmological observations [14, 15], and particularly close to the observed value $h_{\mathrm{S}} = 0.623 \pm 0.063$ found in [101] for the Hubble constant in the range of distances 4 – 200 Mpc.

2.4 'Thermodynamic' Friedmann equation

The main distinctive feature of the EG vacuum as a macroscopic medium is its equation of state (2) that actually causes the antigravitation effect. But how do we know this equation of state?

If we adhere, as above, to the framework of classical physics, the answer to this question can be found by using the well-known thermodynamic identity

$$\mathrm{d}\mathcal{E} = T\,\mathrm{d}S - p\,\mathrm{d}\mathcal{V}\,, \tag{18}$$

where $\mathcal{E} = \rho\mathcal{V}$ is the total internal energy of the medium in a volume $\mathcal{V}$ comoving with the particles, and T, S, p, and ρ are the temperature of the medium, its entropy, pressure, and density. For the cosmological expansion, the adiabatic condition holds well from the earliest periods of the existence of the Universe [10]. Hence, we can set $\mathrm{d}S = 0$ in Eqn (18) and apply this equation to each of the four energies (baryons, dark matter, radiation, and dark energy) separately.

In the case of the EG vacuum, we must follow its 'mechanical' definition, according to which it is a medium with a density that is constant both in space and time in any reference frame. Therefore, $\mathcal{E} = \rho_{\mathrm{V}}\mathcal{V}$, which results in $\mathrm{d}\mathcal{E} = \rho_{\mathrm{V}}\,\mathrm{d}\mathcal{V}$. Then Eqn (18) immediately gives $p_{\mathrm{V}} = -\rho_{\mathrm{V}}$. In this simple way, we can derive equation of state (2) of the vacuum.

Thermodynamic identity (18), which is usually referred to as the internal energy conservation law, follows from the general relativity theory if the adiabatic condition $\mathrm{d}S = 0$ is satisfied. In cosmology, identity (18) is the 'second,' or thermodynamic, Friedmann equation; along with the 'first' equation (11), it is the mathematical basis of the entire dynamics of the expanding Universe. In other words, the Friedmann cosmology in the Newtonian interpretation is based on two conservation laws, the mechanical energy and the internal energy conservation laws.

2.5 Zero gravitation instant

In the first relativistic cosmological model — the static world model [3] constructed by Einstein — there was invariable balance of gravitation and antigravitation. In terms of the Newtonian attraction and repulsion forces in Eqns (13) and (14), this balance is described by the equation $F_{\mathrm{N}} + F_{\mathrm{E}} = 0$. The sum of the two forces being equal to zero makes world static. The matter and vacuum densities are then related by the simple and invariable equation

$$\rho_{\mathrm{M}} = 2\rho_{\mathrm{V}} \,. \tag{19}$$

In the real expanding world described by the Friedmann model, a balance of forces is also possible and is described by the same density relation (19). But this balance happens to exist for just one moment, rather than forever.

Because the vacuum density is constant and the density of the nonrelativistic matter (dark matter and baryons) reduces due to the cosmological expansion, $\rho_{\mathrm{M}} \propto R^{-3}$, Eqn (19) holds just at the instant when the initially high matter density decreases to the value $2\rho_{\mathrm{V}}$ in the course of expansion. At that instant, the acceleration of the cosmological expansion turns to zero, and the velocity has its minimum. Afterwards, the acceleration changes from negative to positive values and the velocity starts increasing.

Is it possible to determine the instant when gravitation turns to zero from cosmological observations? Yes, if we manage to track the acceleration time behavior and find the sign-change instant. It is not very difficult to measure galaxy-scattering velocities observationally. These are determined from redshifts in galaxy spectra resulting from light propagating in the expanding Universe (in the approximation of velocities not too close to the 'speed of light in the vacuum' c, it is described as the Doppler effect). Both the phenomenon itself and the respective value (as we mentioned in Section 1) $z = (\lambda - \lambda_0)/\lambda_0$, with λ being the registered wavelength of a given spectral line and λ_0 being the wavelength of the line known from laboratory experiments, are referred to as redshift. If velocities are small compared to the speed of light, the relation between the redshift and the galaxy velocity is simple, $V \approx z$. (We recall that in the present paper, 'the speed of light in the vacuum' is considered to be unity, and therefore the galaxy velocity is given as a fraction of the speed of light c.) From 1910 to the 1920s, the redshift values measured by Slipher and Hubble for galaxies moving away from us were not greater than 0.03, and hence this simplest approximation was applicable. The empiric Hubble law establishing the linear dependence of velocity on distance was found exactly in this approximation. Redshift is also used in cosmology as a measure of time: the greater z is, the larger the distance is, and accordingly, the farther astronomical observation can go into the world's history.

But the Hubble law tells us nothing about acceleration because the approximation under which it holds is insensitive to acceleration. Only for not small redshifts does the relation between distance and redshift depend not only on the velocity but also on the acceleration, which is the first derivative of velocity with respect to time. If one succeeds in noticing a deviation from the linear law $V = z = HR$, it becomes possible to make conclusions on acceleration from the value and sign of the deviation. If the distance for a given z is greater than expected from the linear law, then the acceleration is positive. For distances smaller than expected, the acceleration is negative. The distance is estimated from the brightness of a source. Evidently, the farther the source is, the smaller its brightness is. If the brightness appeared to be smaller than estimated from the Hubble law and other conditions are equal, the distance is larger than expected. But as just mentioned, distances that are larger than expected imply positive acceleration. Searching for this effect eventually led observers to the discovery of antigravitation [1, 2].

Systematic observations date back to the Supernova Cosmology Project in 1988. Many participants in this project came to astrophysics from physics; that is why this team, led by S Perlmutter, is, for the sake of brevity, called 'physicists.' Soon a team of 'astronomers' led (since 1996) by B Schmidt and A Riess and known as the High-z Supernova Search Team entered the competition. Strategy and observational techniques were similar in both teams; both of them used the Hubble Space Telescope and the largest ground-based instruments, including the most powerful 10-meter telescope, Keck in Hawaii.

Both teams observed supernova explosions at redshifts close to unity. At the peak of its luminosity, a supernova is very bright, for several days or weeks emitting as much light as a whole galaxy and sometimes more. Therefore, supernovae can be visible at large distances, exactly where we expect considerable deviations from the Hubble law. Depending on their spectra, supernovae are divided into two types. Supernovae with bright hydrogen lines are referred to as type II, and the ones that lack hydrogen are called type I. As a rule, type I supernovae are brighter. They, in turn, are divided into subtypes Ia and Ib. The spectra of the former involve distinct silicon absorption lines, and the spectra of the latter contain helium lines. Type-Ia supernovae are commonly believed to emerge after a catastrophic thermonuclear explosion of a carbon-and-oxygen white dwarf. Other supernova types are related to a gravitational collapse of a supermassive star core.

Cosmology research involves type-Ia supernovae. As noted by Pskovskiy [102] thirty years ago, they likely suit cosmological observations better than the others. First, type-Ia supernovae are very bright (their magnitude is -19), trailing only the biggest galaxies (-22) and quasars (-25). Second, their proper luminosity in the peak maximum can be reconstructed from the slope of the observed light curve (which is the dependence of brightness on time). Third, there are reasons to assume that explosions that occurred at different moments of cosmological time should not differ dramatically (i.e., the cosmological evolution of object of this kind should not be essential). Finally, supernovae of this type are theoretically studied quite well [103]. All of this allows observers to use type-Ia supernovae — with all reservations and precautions — as 'standard candles.'

The first to publish results in 1998 were the 'astronomers,' who had data on 16 explosions of the required type supernovae at comparably large redshifts at their disposal [1]. The 'physicists' published paper [2] a year later; they had their own data on 42 supernovae (only two of them were also on the list of the'astronomers'). The results of both teams were the same: the observed brightness of distant supernovae is systematically lower than expected from the model with zero (needless to say, negative) acceleration. This indicates that real distances to remote light sources regularly deviate from the Hubble law and are larger than expected. Hence, the cosmological expansion occurs with positive acceleration. Therefore, outflawing galaxies are driven by antigravitation, which must be stronger than matter's gravitation.

On the basis of these observations, a rough assessment of gravitation – antigravitation equality redshift was made [1, 2]. This is how the value $z_V \approx 0.7$, at which zero-gravitation condition (19) is satisfied, appeared. For $z > z_V$, i.e., before the zero-gravitation instant, the expansion of the Universe had been decelerating; after this instant, the acceleration became positive. Given the value z_V, the present-day matter density $\rho_M(z=0)$, and the law telling how this density depends on time, $\rho_M \propto R^{-3} \propto (1+z)^3$, Eqn (19) yields the dark energy density

$$\rho_V = \frac{1}{2}\,\rho_M(z=0)(1+z_V)^3\,. \tag{20}$$

Substituting the matter (baryons plus dark matter) density value in the present era $\rho_M(z=0) \approx 0.3 \times 10^{-29}$ g cm^{-3} in (20), we obtain the dark energy density value $\rho_V \approx 0.7 \times 10^{-29}$ g cm^{-3}. (In Section 1, this value was already given in the expression through the mass of a hydrogen atom.)

It is assumed in these estimates that dark energy is exactly the EG-vacuum energy rather than, e.g., quintessence or phantom energy.

Using the Friedmann cosmological model with known densities of the matter and dark energy, we can find the distance corresponding to the redshift z_V; it appears to be 7–8 billion light years [or $(2-3) \times 10^3$ Mpc], that is, half of the distance $R_H = 1/H_0$ known as the Hubble radius. Accordingly, it is found that the zero acceleration occurred when the world was 7–8 billion years old. Because the age of the modern Universe is around 14 billion years (after measurements in [14, 15]), the history of the Universe happens to be divided into two almost equal parts — during the first half, the gravitation of dark matter, baryons, and radiation dominated, and the second half is an era of dominance of the EG-vacuum antigravitation.

The discovery of antigravitation and dark energy first raised numerous questions and objections. Can the effect of extra weakening of supernova brightness be explained by some other physical factors? Possibly by cosmic dust absorbing the light? The question was answered in 2001, when, using the Hubble Space Telescope, the 'astronomers' found a required-type supernova at the redshift 1.7, which set the record at that time. If the dust absorption is the point, the most distant source is expected to have the greatest lack of brightness. But if the absorption is not essential, this source should be expected to have a surplus of brightness, as opposed to a lack of it. Because its redshift 1.7 is significantly higher than $z_V = 0.7$, the source is observed in the state it had when gravitation dominated the world and the Universe was decelerating rather than accelerating. This allowed carrying out the critical experiment. When the energy flux from the supernova was measured, it turned out that there was exactly a surplus rather than a lack of apparent brightness. With this fact, the objection of dust absorbing the light was discarded. Eventually, it became clear to skeptics that observers can reasonably answer any sensible question.

Observers continue their work [17–25]. The data available by early 2008 included more than 200 supernovae at redshifts ranging from 0.3 to 1.8. New data are consistent with the earliest results [1, 2] and definitely confirm them. However, both 'physicists' and 'astronomers' are aware that many things need improving. Most often mentioned is the evolution effect, which is the dependence of the proper supernova brightness on the age of the world. Most likely, this effect is rather small, but as far as such precise measurements at the limit of observational capacity are concerned, everything that can potentially influence the result must be taken into account as thoroughly as possible. An interesting new suggestion is to study the evolution effect in observations of supernovae at redshifts in the range $1.5 < z < 3$ [104], where the dark energy action is negligibly small and the evolution effect can be seen clearly. The empirical procedure of reconstructing the maximal proper luminosity of a supernova from the light curve profile also needs to be improved.

Today, making the measurements of the ratio w of the dark energy pressure to its density more precise is properly considered the key observational problem. If the parameter w is constant, it can be determined from the redshift value z_V at the zero-gravitation instant:

$$z_V = \left[\frac{(1-3w)\rho_V}{\rho_M}\right]^{-1/3w} - 1\,.$$

According to observers, to increase the precision of w measurements by 2 or 3 times (i.e., up to 5–3%), we need several thousand, rather than several hundred, supernovae at redshifts ranging from $z \approx 0.5$ to $z \approx 2$.

New projects of extensive orbital and ground-based observations are aimed at this problem. One of the most interesting is the project SNAP (SuperNova Acceleration Probe), which is being developed under Perlmutter's direction; among its participants are 2006 Nobel Prize winners G Smoot and J Mather. A SNAP mission carrying a specially designed telescope with a 2-meter mirror is supposed to be put into orbit at the beginning of the next decade. Up to a thousand distant supernovae are expected to be discovered each year using this tool.

2.6 Data of the WMAP mission

The presence of dark energy was independently confirmed in 2003 and later in 2006 after analyzing extensive systematic observations of the cosmic microwave background anisotropies by the WMAP mission (Wilkinson Microwave Anisotropy Probe) [14–16]. Mild variations (at the level of a few thousandths of a percent) in the microwave background temperature are an 'imprint' of the initial pre-galactic structure of the Universe, which later developed into galaxies and systems of galaxies. The evolution of these variations and their observed quantitative features depend on the physical parameters of the Universe as a whole and in particular on its geometry.

The most important (and regarded as the most reliable) result of the WMAP is the fact that the world's total density ρ_0 is close to the critical density $\rho_c = (3/8\pi G)H_0^2 = (1 \pm 0.1) \times 10^{-29}$ g cm^{-3} (where, as before, $H_0 = 72 \pm 0.4$ km s^{-1} Mpc^{-1} is the Hubble constant). The two densities may be precisely equal. According to Refs [14, 15], their ratio, known as the density parameter, is

$$\Omega = \frac{\rho_0}{\rho_c} = 1.015 \pm 0.020\,. \tag{21}$$

Result (21) follows from geometrical considerations and is based on real measurements of angular distances in the world's space. These measurements became possible due to highly precise detection of the microwave background anisotropy (see Refs [105, 106] for more details). The measurements showed that the 3-dimensional space (the one

in which galaxies are moving) is almost flat, or Euclidean, and maybe exactly flat. According to Friedmann's theory, this must imply that the total density of the world is equal to the critical density. Thus, studying geometrical features of the mild microwave background anisotropy gave the result that had been the cosmologists' dream.

Friedmann's theory also tells us that cosmological expansion in the flat world proceeds in the parabolic mode: the total mechanical energy of galaxy motion is zero. Therefore, the total energy E in the first Friedmann equation (11) can be set to be negligibly small (or even exactly zero) compared to the kinetic energy and the absolute value of the potential energy. It then turns out that the observable Universe can be very well described with the simplest variant of the theory: simplest in the sense of both geometry and dynamics. This is accepted in the standard cosmological model, where the 3-dimensional isotropic space is considered flat and the expansion dynamics parabolic.

According to the WMAP data, if the cosmic density is equal to the critical density, the fraction of dark energy is 70–80% of the total density. This is because the contributions of other energies (baryons, dark matter, and radiation) are limited a priori with the very severe upper bound of 30–20% of the critical density, which is known from a number of other independent cosmological requirements.

Recently, it has become common to say that 'the era of precise measurement' has come in cosmology. The reader can imagine the precision of cosmological measurements, for example, looking at formulas (5) and (21). We now quote the WMAP data [15] on the energy contents and age of the world with error bars. The densities of dark energy (subscript V), dark matter (subscript D), baryons (subscript B), and radiation (subscript R) are given in units of the critical density:

$$\Omega_{\mathrm{V}} = \frac{\rho_{\mathrm{V}}}{\rho_{\mathrm{c}}} = 0.75 \pm 0.05\,, \tag{22}$$

$$\Omega_{\mathrm{D}} = \frac{\rho_{\mathrm{D}}}{\rho_{\mathrm{c}}} = 0.23 \pm 0.07\,, \tag{23}$$

$$\Omega_{\mathrm{B}} h^2 = \frac{\rho_{\mathrm{B}}}{\rho_{\mathrm{c}}} = 0.022 \pm 0.001\,, \tag{24}$$

$$\Omega_{\mathrm{R}} h^2 = \frac{\rho_{\mathrm{R}}}{\rho_{\mathrm{c}}} = 7 \times 10^{-5}\,, \tag{25}$$

with h being the Hubble constant measured in units of 100 km s^{-1} Mpc^{-1}: $h = H/100 = 0.72 \pm 0.04$.

Data (22)–(25) relate to the modern state of the Universe, whose age from the WMAP data is (in billions of years)

$$t_0 = 13.7 \pm 0.3\,. \tag{26}$$

In the WMAP observations, the densities of the four energies, the Hubble constant, and the world's age are not measured directly and separately. All the numbers in (21)–(26) are a result of a complicated processing of the cosmic microwave background observations. They are found as a result of seeking such values of these quantities that allow consistently relating them to each other and to the measured features of the anisotropy, consistently with theoretical constraints. In fact, numeral values (21)–(26) mean that in cosmology, it is possible to make both ends meet within the error bars indicated in (21)–(26). It is particularly important that the WMAP results are consistent with supernova observations [1, 2, 17–25] and other cosmological observations (see, in particular, Refs [107, 108]).

Analysis of the WMAP results [14–16] published in the spring of 2006 is in progress, and censure would be appropriate. For example, all systematic errors might not have been taken into account in Refs [14–16], and hence the real precision of the measurements may be somewhat lower than was announced. One of the obvious sources of the obstinate systematic error is the complex and ambiguous procedure of subtracting the Milky Way emission contributing to the measured background anisotropy (see Section 4).

3. Dark energy in the nearby Universe

As far as is known from some of Einstein's fragmentary statements, he believed that the cosmological constant describes the phenomenon of a global scale, which can come into play only in the dynamics of the general cosmological expansion. Neither Einstein's works nor further cosmological literature indicates that antigravitation is capable of acting and even dominating gravitation on the local scale that is much smaller than the cosmological one. Soon after the discovery of dark energy, it was noted [109, 110] that antigravitation is actually capable of driving galaxies' motion almost in the entire range of cosmological distances, both at the global, 'genuinely' cosmological scales and at scales of just a few megaparsecs, practically everywhere as soon as the regular cosmological expansion in accordance with the Hubble law is actually observed. In particular, antigravitation also dominates in our nearest galactic environment at the distance of just 1–2 Mpc from the Milky Way.

3.1 Hubble–Sandage paradox

As is known, observational cosmology began from research on a comparably modest spatial scale. By 1929, when the Hubble law was discovered, astronomers had had velocities of 36 galaxies at their disposal, measured by Slipher in 1910–1925. The velocities did not exceed 10 thousand kilometers per second. Hubble measured the distances of 24 galaxies and discovered that none of them was further than 2 Mpc from us [111]. Later, A Sandage found an error in Hubble's measurements, the distances underestimated by almost 10 times (see books [112–114] for more details). The real distances were up to 17–18 Mpc. But the error of the distance measurements turned out to be systematic, i.e., the same for all the galaxies, and the Hubble law remained valid after the error was corrected.

From the very beginning, Slipher's and Hubble's observations were viewed as a direct astronomical confirmation of Friedmann's theory, but a crucial fact was overlooked. The fact is that at scales from a few megaparsecs to a few dozen megaparsecs, the cosmic matter is distributed extremely nonuniformly. It clusters in separate clumps — galaxies, groups, and clusters of galaxies — that are chaotically scattered in space. Under these conditions, the Friedmann model does not apply because the condition of the uniform distribution of matter, which is one of the basic assumptions of the expanding Universe theory, is not satisfied. Since the 1960s, astronomers have known that galaxies in the Universe are indeed uniformly distributed, but only on average, over very large scales exceeding the 'cell of uniformity' of size $\approx 100-300$ Mpc. Only at scales considerably larger than this one can the Universe be described by the Friedmann model.

How then are Friedmann's theory and Slipher's and Hubble's observations related? Do these observations make sense in cosmology?

The first to notice the problem and look into it deeply was again Sandage, once Hubble's collaborator and a disseminator of Hubble's scientific traditions in astronomy. Sandage claimed (first rather cautiously in 1972 [115] and then in a more pronounced way in 1986 [116] and 1999 [117]) that Hubble's discovery raises a number of difficult questions. The most important of them is the following: With prominent nonuniformities and clearly chaotic distribution of matter at distances up to 20 Mpc, how can the regular 'expansion flow' governed by the law of direct proportion between velocity and distance exist?

Theoretician S Weinberg also noted this fact. In the popular book *The First Three Minutes* [118], Weinberg writes, "Actually, a look at Hubble's data leaves me perplexed how he could reach such a conclusion.... In fact, we would not expect any neat relation of proportionality between velocity and distance for these 18 galaxies — they are all much too close.... It is difficult to avoid the conclusion that... Hubble knew the answer he wanted to get."

It is hardly possible to tell what exactly Hubble knew or did not know in 1929 (see, however, again [112–114]). But just one "look at Hubble's data," at his velocity–distance diagram regularly reproduced in numerous books and papers (see, e.g., Fig. 2 in paper [110] in *Physics–Uspekhi*) leaves no doubt: velocities do increase as distance increases. Sandage, who has been observing galaxy motion (including the scales Hubble observed) for many years, has no doubt that the Hubble law actually applies in the range of distances where it was first discovered. The linear dependence of velocity on distance in the original Hubble diagram has repeatedly been confirmed in increasingly precise observations. Observers deliberately emphasize that deviations from the linear dependence are relatively small and velocity scatter around the direct proportionality law is very mild. According to his measurements, Sandage estimates this scatter, termed velocity dispersion, to be 50–70 km s^{-1}; in principle, if we take the extent of nonuniformity of matter distribution in the same volume into account, the velocity dispersion could amount to both 100 and 150 km s^{-1}.

In spite of the nonuniform and chaotic distribution of galaxies within the cell of uniformity, the galaxy outflaw is very regular here; this is, says Sandage, "a surprisingly noiseless and cold flow." Henceforth, we use the notion of 'the Hubble expansion flow' introduced by Sandage, and by that we mean the regular recession, an outflaw of galaxies obeying the Hubble law.

In 2006, Sandage and his colleagues [101] marked the end of the extensive program of research of the Hubble flow with observations of type-Ia supernovae, which had been carried out by them with the Hubble Space Telescope and the best ground-based tools [119–125] for 15 years. The results are as follows: (1) In a wide range of spatial scales — from 5 to 200 Mpc — the regular expansion flow in accordance with the Hubble law $V = H_S R$ is traced distinctly; and (2) the Hubble constant H_S for the given distance range is everywhere the same within the error, $H_S = 62.3 \pm 6.3$ km s^{-1} Mpc^{-1} (we cited these data in Section 2).

As a result, the regular kinematics of galaxies appear not to be disturbed by the significant irregularity of their distribution within the cell of uniformity, with the Hubble constant H_S measured in the cell of uniformity being close to the value of the global quantity H_0 given by the WMAP observations. But how is that possible? In 1999, Sandage said, "We are still left with the mystery" [117].

3.2 Regular flow

A year later, in 2000, the Hubble–Sandage paradox was basically resolved [109]: it is due to the vacuum dark energy. Dark energy (if it is the EG vacuum) has an ideally uniform density and dominates everywhere outside large matter clumps, which results in nearly the whole world being almost uniform.

Indeed, if we adhere to the standard cosmological model and the interpretation of dark energy as the EG vacuum, we should expect that not only very distant galaxies observed at the edge of the Universe but also the nearby galaxies once observed by Hubble move not in empty space but in a uniform background of the cosmic vacuum. Dark energy is the physical factor that relates the local and global properties of the world, creating a common dynamic background in the Universe [109, 110, 126–129]. Vacuum efficiently smoothes the influence of such nonuniformities as groups, clusters, and even superclusters of galaxies, and that is why the overall energy/mass distribution in the Universe turns out to be more uniform than we had expected before looking only at galaxies. Thus, the contradiction between regular motion of galaxies and their nonuniform distribution within the cell of uniformity is resolved: with the dark energy background dominating, the overall mass/energy distribution turns out to be also regular.

In the modern Universe, the motion of galaxies is driven not only by the Newtonian force of their mutual gravitation but also by the antigravitational force exerted by the vacuum dark energy. Moreover, there is much more dark energy than matter (dark matter and baryons) in the present-day Universe. This last statement refers to the global and certain local scales and hence, generally speaking, antigravitation is stronger than gravitation almost everywhere in the world. If we imagine the limit case where matter gravitation is taken to be negligible compared to vacuum antigravitation, we come to the asymptotic picture mentioned in Section 2: galaxies and their systems move as test particles in a perfectly uniform dark energy background and their motion obeys the Hubble law [see relations (16) and (17)]. The rate of galaxy scattering on all scales is then given by the common 'universal Hubble constant' $H_V = 1/t_V = [(8\pi G/3)\,\rho_V]^{1/2} \approx 60-64$ km s^{-1} Mpc^{-1}.

As we have mentioned (see Section 2), this ideal picture stands not too far from reality, from the dynamic state the observed Universe is currently experiencing and, importantly, at all spatial scales from a few megaparsecs. For this reason, the dynamic effect of dark energy naturally explains the two astronomical facts that have seemed mysterious up to now: (1) regularity of the expansion flow inside the cell of uniformity and (2) the same expansion rate at local and global scales.

This resolves the paradox that had been overshadowing Hubble's discovery [and Hubble himself (see Section 3.1)]. As we understand now, it is the dark energy of the uniform Universe vacuum that actually lies behind Hubble's discovery and makes sense of it for cosmology. Cosmological effects are present not only at scales of a few thousand megaparsecs exceeding the cell of uniformity size but also deep inside the cell.

The new understanding of the Hubble flow dynamics suggests that dark energy can be independently studied and measured in every place where a regular outflaw of galaxies is observed. Assuming that the real world is actually close to the above-described picture and reversing our reasoning, we can find the local dark energy density required for the regular expansion flow observed by Sandage et al. [101] in the distance range 4 – 200 Mpc to exist. We assume that the measured value of the Hubble constant within the cell of uniformity, $H_S = 62.3 \pm 6.3$ km s^{-1} Mpc^{-1} [101], is completely fixed by the local dark energy density. To distinguish the local and global densities, we reserve the notation $\bar{\rho}_V$ for the local density. Then the asymptotic value of the Hubble constant defined by this density becomes $\bar{H}_V = (8\pi G\bar{\rho}_V)^{1/2}$, and identifying H_S and $\bar{H}_V$, we find

$$\bar{\rho}_V = \rho_V \left(\frac{H_S}{H_V}\right)^2 = (1 \pm 0.2)\,\rho_V\,. \tag{27}$$

In formula (27), the global density ρ_V and the corresponding universal Hubble constant H_V are used just for ease of comparison; in fact, result (27) is fully independent of the global cosmological data. This result implies that the local dark energy density is close or even equal to the global one.

The precision in estimate (27) is influenced by two facts: (1) the precision of the measurement of H_S itself, which is close to 10%, and (2) the extent to which the observed state of the Universe is close to the perfect asymptotic picture. The first source of error is reflected in relation (27). As regards the second, its contribution is measured a posteriori as the ratio of the average density of the gravitating matter (dark matter and baryons) to the dark energy density $\bar{\rho}_V$ found in (27); this ratio is 0.2 – 0.3. Hence, the actual precision in estimate (27) is not better than 40 – 50%.

The observed Hubble expansion flow served as the natural 'measuring set-up' (with the size 4 – 200 Mpc) for estimating the local dark energy density. Nature itself calibrated the 'set-up' and set the absolute zero of energy: the 'set-up' is driven by the gravitational interaction, thus measuring the total energy rather than its differences. Certainly, the same applies to the flow of distant galaxies at global scales, which is studied in papers [1, 2].

We only add that our interpretation of the Hubble – Sandage paradox is now shared by Sandage and his colleagues; as they say, "No viable alternative to vacuum energy is known at present. The quietness of the Hubble flow lends support for the existence of vacuum energy" [101].

3.3 In the vicinity of the Milky Way

Already the first of Hubble's data obtained in 1929 show [after correcting the systematic error (see Section 3.1)] that the regular flow of galaxies starts at distances 1 – 2 Mpc from us. Understandably, this closest part of the flow is especially interesting. In the past ten years, the Hubble flow at distances 1 – 7 Mpc has become an object of thorough observational research carried out by Karachentsev's team [130 – 143] using the Hubble Space Telescope (almost 200 orbital periods) and other large instruments, including the 6-meter telescope of the Special Astrophysics Observatory of the Russian Academy of Sciences. For more than two hundred of the closest galaxies, Karachentsev et al. carried out velocity measurements with error not greater than 1 – 2 km s^{-1} and estimated distances with error not greater than 8 – 10%. These extensive, systematic, and high-precision observations first allowed a clear insight into the kinematics of the Hubble expansion flow around the Milky Way and the Local Group of galaxies, to which our Galaxy belongs.

Along with the Galaxy, the Local Group comprises another giant galaxy, the Andromeda Galaxy. Each of the two giant galaxies has an extended massive halo filled with dark matter. The galaxies are moving toward each other at the speed 120 km s^{-1}; in the present era, the distance between them is 0.7 Mpc. The Local Group also includes about fifty relatively small dwarf galaxies moving in the gravitational potential well mainly provided by the gravitation of the two giant galaxies. The group as a whole is gravitationally bound and quasi-stationary. According to Karachentsev, its total mass is estimated as $M_{LG} = (1.3 \pm 0.3) \times 10^{12} M_\odot$ (we note that this sets the precision record in mass estimates in the astronomy of galaxies and their systems).

Around the Local Group, up to distances of 3 Mpc from its mass center, 22 (dwarf) galaxies are observed. They are moving in different directions away from the group center and form the local Hubble flow. The flow obeys the Hubble law: the velocity of the flow is in proportion to the distance starting from distances of 1.5 – 2 Mpc from the mass center of the Local Group.

The velocity – distance diagram in Fig. 1 displays the observational data by Karachentsev et al. Points stand for galaxies of the Local Group and of the local flow with measured values of their line-of-sight velocities and distances from the mass center of the group. The points on the diagram form a pattern consisting of two distinguished parts. The internal part is the Local Group, and the external pattern is the local flow. The galaxies of the flow have only positive velocities, all of them are moving away from us. As the distance increases, their velocities change from values around zero to about 200 km s^{-1}. The galaxies of the Local Group have both positive and negative velocities in the range ± 150 km s^{-1}. Their mean radial velocity is close to zero.

The measured value of the local Hubble constant H_L is 72 ± 6 km s^{-1} Mpc^{-1}. The velocity dispersion is rather small, just about 30 – 40 km s^{-1}, which is smaller than was accepted before. Therefore, the initial part of the Hubble flow is definitely regular, or 'noiseless and cold' (in Sandage's terms). These results are consistent with those of Sandage's team [101, 119 – 125] and with research carried out by Teerikorpi (Tuorla observatory, Finland) and his colleagues for the first several megaparsecs of the local Hubble flow.

3.4 Local cosmology

We discuss the dynamics of the nearby volume of the world. The Local Group and the Hubble flow around it are immersed in uniformly distributed dark energy, if this energy is the EG vacuum. The galaxies of the flow barely interact with each other, and their total mass is much smaller (at least by a hundred times) than that of the Local Group, and hence the dwarfs of the flow can be considered 'test particles.' The dwarf galaxies are moving on a static dynamic background, which is provided by the Newtonian attraction to the Local Group and the Einsteinian repulsion caused by dark energy from the Group. This local volume with the radius 3 Mpc is (more or less) isolated from the rest of the galactic environment and the external influence can be neglected in the first approximation. One more simplification is acceptable: considering the dynamics of the flow, the Local Group can be regarded as a spherical mass (see Section 3.5 for more precise treatments).

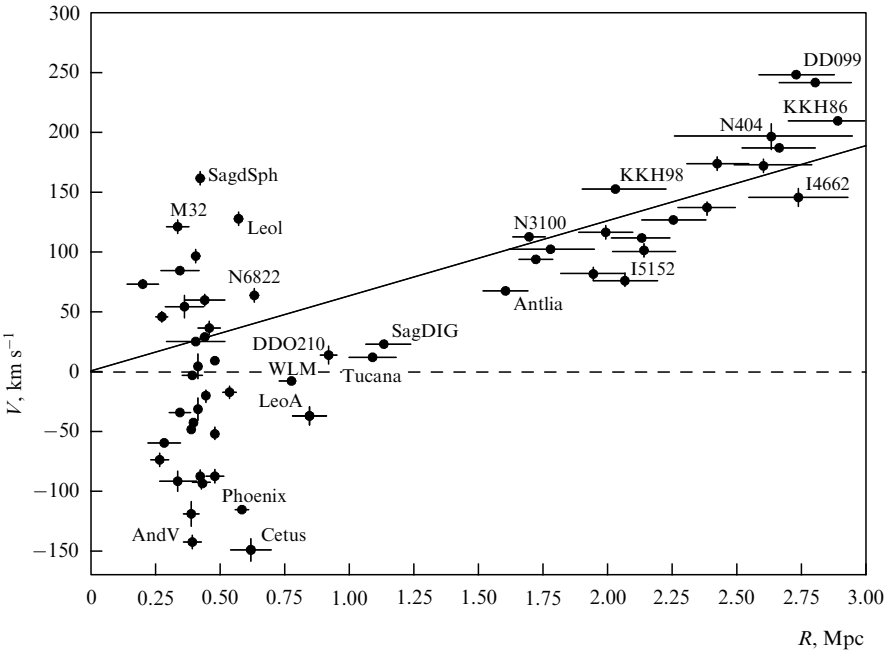

Figure 1. Velocity – distance diagram for galaxies at distances up to 3 Mpc. Each point corresponds to a galaxy with measured values of distance and line-of-sight velocity in the reference frame related to the center of the Local Group. The data were obtained by Karachentsev and his collaborators in 2002 – 2007 in observations with the Hubble Space Telescope. The diagram shows two distinct structures, the Local Group and the local flow of expansion. The galaxies of the Local Group occupy a volume with the radius up to 1.2 – 1.3 Mpc and move both away from the center (positive velocities) and toward the center (negative velocities). These galaxies form a gravitationally bound quasi-stationary system. The galaxies of the local flow are located outside the group and all of them are moving away from the center (positive velocities). Recession velocities increase as the distance increases, in accordance with the Hubble law. The straight line is the theoretical dependence $V = H_V R$; for the region outside the group, it corresponds to a radial motion of galaxies under the action of antigravitation of dark energy, the gravitation of the group being negligible. It can be seen that the galaxies of the flow 'feel' this asymptotic behavior and, generally, follow it well, even at rather small distances.

This idealized picture of the dynamic background on which the dwarf galaxies of the local flow are moving can be quantitatively described if we use the well-known result of the general relativity theory, the exact spherically symmetric solution for a point mass with the cosmological constant included, and apply it to the spatial region around the Local Group. This solution, known as the Schwarzschild – de Sitter solution, is given by the static metric

$$\mathrm{d}s^2 = g_{00}\,\mathrm{d}t^2 - r^2(\sin^2\theta\,\mathrm{d}\phi^2 + \mathrm{d}\theta^2) - g_{11}\,\mathrm{d}r^2\,, \tag{28}$$

$$g_{00}(r) = \frac{1}{g_{11}(r)} = 1 - \frac{2GM}{r} - \frac{8\pi G}{3}\,\rho_V\,r^2\,, \tag{29}$$

where r is the distance to the center of the mass M.

When the gravitation/antigravitation fields can be considered weak (this is the case in which we are interested now), deviations from the Galilei metric are small and, in this approximation, are given by the Newtonian gravitational potential $U(r)$. In the first order, following the general rule (see, e.g., Ref. [5]), we obtain from (28) and (29) that

$$g_{00}^{1/2} \approx 1 + U\,, \qquad U(r) = -\frac{GM}{r} - \frac{4\pi G}{3}\,\rho_V\,r^2\,. \tag{30}$$

Differentiating the potential $U(r)$ with respect to the coordinate r, we find a sum of the Newtonian attraction force, which is produced by the mass M, and the Einsteinian repulsion force produced by the EG vacuum in the region outside the mass M (which we consider to be the mass of the Local Group):

$$F(r) = -\frac{GM}{r^2} + \frac{8\pi G}{3}\,\rho_V\,r\,. \tag{31}$$

The close resemblance between formula (31) and relations (13) and (14) that occurred in the cosmological problem described in Section 2 is quite obvious. But in cosmology, the two forces stemmed from the nonstatic Friedmann solution, while here they originate from the static Schwarzschild – de Sitter solution. There is another difference. In the cosmological problem, forces (13) and (14) are defined in the space uniformly filled with matter and dark energy, whereas in the local problem, the space is only filled with dark energy, while all the matter is concentrated in the volume of the central mass M.

In the limit of large distances, the influence of the central mass becomes negligibly small; then metric (28), (29) along with the field of forces is determined in Newtonian approximation (31) only by the vacuum, and the Schwarzschild – de Sitter space – time turns into the de Sitter world, where $g_{00}(r) = 1/g_{11}(r) = 1 - (8\pi G/3)\,\rho_V\,r^2$. We recall that the de Sitter world is also a static asymptotic regime of the Friedmann world at large times (see Section 2). It turns out that the global space – time and the local gravitation field have the same static asymptotic form in the limit of the single domination of the vacuum. In this limit, the galaxy outflaw occurs in accordance with the law of direct proportionality (16) applicable to both cases, with the universal Hubble constant H_V. Asymptotic properties of our local cosmological model clearly relate it to the global cosmology. This relation is due to dark energy and is impossible without it.

It follows from relation (31) that in the local problem, there is a distance at which the sum of the gravitation and antigravitation forces is equal to zero; the distance is

$$r_{\mathrm{V}} = \left(\frac{3}{8\pi}\frac{M}{\rho_{\mathrm{V}}}\right)^{1/3} . \tag{32}$$

The quantity r_{V} is 'the zero-gravitation radius.' It is a close analog of the zero-gravitation moment in the global cosmology (see Section 2). But what happens in time in the global cosmology takes place in space in the local one. Indeed, in the Universe as a whole, gravitation vanishes for an instant of cosmic (proper) time simultaneously in the entire comoving space, but in the local cosmology, gravitation is absent only on a sphere of radius r_{V}, albeit during the entire period of the existence of the Local Group of galaxies (its age is estimated to be 12 – 13 billion years, which is only 1 – 2 billion years less than the age of the world).

If we assume that the dark energy density has the same value in our vicinity as at the largest distances (which must be the case with the EG vacuum), then the zero-acceleration radius can be easily estimated. For the vacuum density value given by observers (see Section 2.6) and for the Local Group mass value indicated in Section 3.3, we obtain $r_{\mathrm{V}} = 1.1 - 1.3$ Mpc. Hence, even in our closest galactic vicinity, antigravitation of dark matter dominates once we go beyond a distance of just 1.5 Mpc.

Evidently, the gravitationally bound system of the Local Group of galaxies can exist only within the region of radius $r < r_{\mathrm{V}}$, where gravitation dominates. This is the case; the observed size of the Local Group (see Fig. 1) actually satisfies this condition. Outside the Group, at distances $r > r_{\mathrm{V}}$, the Hubble flow starts; the dwarf galaxies in the flow are moving in the region of the antigravitation dominance and their dynamics are mainly governed by dark energy. According to the above, at these (and larger) distances, the flow tends to become regular. We can easily see from formula (31) that as the distance increases, radial trajectories in the range $r > r_{\mathrm{V}}$ are 'attracted' toward the trajectory $V = H_{\mathrm{V}} r$ with the universal Hubble constant H_{V}, which depends just on the (local) density of dark energy.

These considerations suggest a way of detecting and measuring dark energy in the local volume. Indeed, from formula (32) for the zero-gravitation radius, we can see the following: if we know from independent data where the zero-gravitation sphere is located and if, furthermore, the mass M of the Local Group is measured, then the dark energy density is immediately found as

$$\rho_{\mathrm{V}} = \frac{3}{8\pi}\frac{M}{r_{\mathrm{V}}^3} . \tag{33}$$

The fundamental physical constant — the dark energy density — is expressed in relation (33) through quite humble astronomical quantities, the mass of the Local group and the initial radius of the Hubble flow.

The mass of the Local Group can be considered known (see above). But how do we know the r_{V} value? The limits within which it lies can be recognized on the observational velocity-distance diagram (see Fig. 1). Indeed, the Local Group is located inside a sphere of radius r_{V} (see Section 3.3), and its border is defined by the distance beginning from which no galaxies with negative velocities occur in the velocity – distance diagram. Leo A, the farthest negative-velocity galaxy away from the Local Group center, is located at the distance that does not exceed 1 Mpc (within an error bar); approximately at the same distance is the galaxy DDO210 with a small positive velocity. Thus, 1 Mpc can be taken as a tentative lower bound for the radius r_{V}. This yields an upper bound for the local dark energy density $\bar{\rho}_{\mathrm{V}}$, $\bar{\rho}_{\mathrm{V}}/\rho_{\mathrm{V}} \leqslant 1.6$.

As an upper bound of r_{V}, we can take the mean distance, e.g., to five galaxies with positive velocities that are on the right of the galaxy DDO210 in the diagram. This mean distance is 1.5 Mpc. It yields the smallest possible value of the local density of dark energy, $\bar{\rho}_{\mathrm{V}}/\rho_{\mathrm{V}} \geqslant 0.5$. Taking the precision of measurements of the Local Group mass (30%) into account, we obtain a reliable range of the local density values: $0.3 < \bar{\rho}_{\mathrm{V}}/\rho_{\mathrm{V}} < 2$.

As we see, the local dark energy density ranging in the obtained bounds does not differ much from (and may even be equal to) the global density value. However, exact numbers are not that important so far. What is important is an indication of dark energy existing in our vicinity, which follows both from general considerations and, more formally, from the lower bound on the dark energy density.

An independent estimate of the local dark energy density in the region of the local flow can be obtained if the same considerations (see above) about the observed flow state being close to the asymptotic state described by the law $V = H_{\mathrm{V}} r$ are applied to the flow. Then [as in formula (27)], identifying the local Hubble constant H_{L} with $\bar{H}_{\mathrm{V}}$, we obtain the estimate of the dark energy density at the scale of 1 – 3 Mpc:

$$\bar{\rho}_{\mathrm{V}} = \rho_{\mathrm{V}}\left(\frac{H_{\mathrm{L}}}{H_{\mathrm{V}}}\right)^2 = 1.4(1 \pm 0.2)\,\rho_{\mathrm{V}} . \tag{34}$$

Here, as in relation (27), the indicated uncertainty of the estimate is related to the errors of observational detection of the local Hubble constant.

The analysis of the velocity – distance diagram shown in Fig. 1 suggests that a fairly well estimate can be obtained by comparing $\bar{H}_{\mathrm{V}}$ with the quantity $\langle H \rangle$, the mean value of the ratio V/r over all the galaxies of the flow. According to the data in the figure, $\langle H \rangle = 59 \pm 11$, with both the averaging error and the errors of observational measurements of distances and velocities taken into account. Then, identifying $\langle H \rangle$ and $\bar{H}_{\mathrm{V}}$, we obtain the local density

$$\bar{\rho}_{\mathrm{V}} = \rho_{\mathrm{V}}\left(\frac{\langle H \rangle}{H_{\mathrm{V}}}\right)^2 = 0.9(1 \pm 0.4)\,\rho_{\mathrm{V}} . \tag{35}$$

The estimates in (34) and (35) are consistent with each other within 40 – 50%; they also fit well into the range between the upper and lower bounds for the local density. Finally, we can conclude that dark energy is likely present in the local volume and its density here is equal to the global density, with an accuracy of at least 40 – 50%.

3.5 Little Bang

The simple model of the local Hubble flow described in Section 3.4 is confirmed (and improved whenever necessary) by computer simulations of the flow dynamics. In simulations [153 – 155], the Local Group is regarded not as a spherical mass but as a dynamic system consisting of two comparable gigantic masses drawing toward each other. Hence, finding the trajectories of the galaxies of the local Hubble flow

requires solving the three-body problem. This problem is too complicated for analytic integration and, as is known, has no general solution. But in computer simulations, it is rather simple and convenient, especially if a third particle can be considered a test particle, as in our case. Thus, computer simulation of the local Hubble flow reduces to integrating the restricted three-body problem in the uniform background of antigravitating dark energy.

As expected, computer simulations have shown that the gravitational field of the Local Group is actually neither spherical nor static. Nonsphericity is a result of the presence of two comparable gravitating masses in the group, while time dependence emerges because these masses are moving toward each other. The zero-gravitation surface is not a sphere any more, and just the line-of-sight component of acceleration, rather than the total acceleration, vanishes on this surface. But these deviations from the simple picture described in Section 3.4 are not too great, not greater than 10–20%. Therefore, the simple theory still holds as a quite good approximation.

Increasing the precision of computer simulations is a secondary issue. The important point is to construct a quantitative picture of the origin and evolution of the local Hubble flow. One of the features of this picture is that the dwarf galaxies forming the flow were initially located in the volume of the Local Group in the gravitational potential well of the two giant galaxies. In the past, in the volume with the diameter around 2 Mpc, the Local Group apparently contained a great number of small galaxies and subgalactic fragments existing along with the two main galaxies of the group 12–13 billion years ago. Most of them did remain in this volume, with many of them having been merged by the two giant galaxies; the others could leave the Local Group, slipping away from its gravitational potential well.

The possibility of galaxies being thrown out from the Local Group was previously considered by Valtonen et al. in the model they called the Little Bang [156], as opposed to the Big Bang that gave birth to the global expansion of the Universe. A number of observational considerations in favor of a chaotic initial state of the Local Group are given in papers [157, 158]. Indeed, in our computer simulations, many of the dwarf galaxies randomly distributed at the initial instant within the volume of the Local Group and having both positive and negative random velocities relative to its center of mass actually left the group. Having moved away to distances of 1–1.5 Mpc, they later flew outside the surface of zero line-of-sight acceleration and entered the region of antigravitation dominance. In different versions of computer simulations for ensembles of two or three dozen particles in each, the local Hubble constant value was typically in the range from 60 to 80 km s^{-1} Mpc^{-1} and the final (in the present-day state of the flow) velocity dispersion was 15–30 km s^{-1}.

It is worth noting that the presence of the vacuum significantly facilitates the exit of small galaxies from the Local Group. Indeed, in order to move from the Local Group to infinity, a small galaxy must have a total energy exceeding the threshold energy required for leaving. In the model without the vacuum, the threshold energy is obviously zero. But in the model with the vacuum, the potential barrier is lower, with the negative threshold energy $E_0 = -3GM/(2r_V)$. If the vacuum is present, a small galaxy moving inside the group can take the energy $E \geqslant E_0$ more easily and leave the system. Using relation (30), we can easily obtain the threshold energy E_0 and see that the maximum of the particle potential energy is located at the distance $r = r_V$ from the group center. After entering the region $r > r_V$, the galaxy that left the group gains positive acceleration and increases its velocity as it moves farther away from the Local Group. Moreover, the smallest velocity in the flow formed by such galaxies, defined by the condition $E = E_0$, marks the lower bound of the flow on the velocity–distance diagram. This minimal velocity is expressed as

$$v_{\min}(r) = \left(\frac{2GM}{r_V}\right)^{1/2}\left[\frac{r}{r_V}+\frac{1}{2}\left(\frac{r}{r_V}\right)^2-\frac{3}{2}\right]^{1/2}. \qquad (36)$$

This implies the possibility of a critical test: if the accepted model is valid, velocities of the galaxies of the local flow satisfy the condition $v > v_{\min}(r)$. Using the data shown in Fig. 1, we can verify that this constraint is satisfied in the observed local flow.

The general approach developed for the local Hubble flow can be applied to studying other observed flows at scales of a few megaparsecs. Recently, extensive observational data on two close groups (Cen A and M81/M82) with the Hubble flows surrounding them were obtained in [159–166]. These groups and the expansion flows around them reproduce the main structural and dynamic features of the Local Group and flow. In particular, the condition of the minimal velocity is satisfied for them [166–168].

Hubble already knew that there are numerous small groups of galaxies around us and the Local Group is a typical representative. This is also confirmed in computer simulations of the cosmological evolution. A typical feature of these simulations [169–174] is the formation of groups of galaxies with expansion flows around them. Systems of this kind could be termed 'Hubble cells.' Serious studies of them have just started. The main physical characteristic of a Hubble cell is the zero-gravitation radius, which emerges owing to dark energy. Just the Hubble cells should perhaps be considered the 'structural blocks' of the nearby Universe. If this is the case, the cosmic structure is a 3-dimensional web covering almost the entire world's space and built almost exclusively of the Hubble cells whose scales (the zero-gravitation radii) are in the relatively narrow range of several megaparsecs (for groups) to ten megaparsecs (for clusters). Centers of mass of individual cells keep the initial momentum of the cosmological Big Bang, while their mutual recession in the past 7–8 billion years has been accelerated by universal antigravitation.

The Hubble cells emerge in the expanding Universe during the common process of formation of cosmic structures from individual galaxies to clusters and superclusters of galaxies. The main mechanism of the process is the gravitational instability [11, 12]. The presence of dark energy in the world introduces an essential feature into this picture: gravitational instability can develop only when and where (mainly dark) matter gravitation is stronger than antigravitation produced by dark energy. Therefore, the initial linear stage of gravitational instability must end no later than antigravitation of dark energy starts dominating in the Universe as a whole [175]. This condition applies to all scales and shows again that a dynamic role of dark energy and antigravitation is crucial not only on the global cosmological scale but also on all smaller scales where galaxies and their systems emerge and evolve.

4. Size and dimension of space

We discuss the Einstein cosmology of 1917 again. Besides the hypothesis of antigravitation and the postulate of a static world, there is another crucial concept in it, the concept of a finite and closed Universe. Very recently, for the first time in the history of cosmology, objective observational data have appeared indicating that the real Universe may actually be finite. If this is confirmed, Einstein will have been right about this, just as he appeared to be right about universal antigravitation.

4.1 What do Einstein and Friedmann say about the world's topology?

In Einstein's cosmological model, the 3-dimensional space is non-Euclidean. It has a constant (the same at each point and each instant of time) positive curvature. Such space is similar to the ordinary 2-dimensional sphere; the 2-dimensional space of the sphere also has a constant positive curvature. The sphere has a finite area, but has no 2-dimensional boundaries. Its 3-dimensional analogue actually used by Einstein is called a hypersphere; the hypersphere has a finite 3-dimensional volume and has no 3-dimensional boundaries.

Why did Einstein believe that the space of the Universe must be finite? The general relativity theory yields no implications regarding whether the world is finite or infinite. This is because the theory is based on differential geometry as its mathematical apparatus. But differential geometry describes only local properties of space. They are local in the sense that they refer to each spatial point and its small vicinity. But in the cosmological reality, these 'vicinities' are regions of the size exceeding that of the cell of uniformity. Differential geometry can argue whether the geometry of a given region in the isotropic world is the Euclidean, Lobachevsky, or hyperspherical geometry. This is related to local properties of space rather than its structure as a whole, which is to be studied in topology, not in differential geometry.

In his popular scientific book *The World as Space and Time*, Friedmann [176] draws the reader's attention to the fact that the general relativity theory can discuss only differential geometry of the world; there is no topology in it. Topology does not follow from differential geometry; the latter just imposes certain constraints on the former. Here are two examples. A plane and a 2-dimensional cylinder are Euclidean surfaces. But on a plane, dimensions are unbound in all directions, whereas distances transverse to the cylinder axis are finite and not larger than 2π multiplied by the cylinder radius. The second example is the 2-dimensional Lobachevsky space, which as a whole may have the shape of a hyperboloid (a gramophone tube) or may be a saddle. A hyperboloid and a saddle-like surface differ dramatically both in shape and overall structure. For example, for a hyperboloid, all dimensions perpendicular to its axis are finite, while the dimensions of a saddle-like surface are infinite in all directions. But the differential geometry is the same in both cases, being a 2-dimensional space with constant negative curvature.

Friedmann's remark likely had not only a pedagogical but also a polemic sense. Friedmann felt it necessary to contradict Einstein and emphasize that the concept of finite hypersphere is an arbitrary extra hypothesis following in no way from general relativity as such.

By the way, the Friedmann models are often called open (Lobachevsky's space) and closed (hypersphere). Moreover, it is sometimes said that in the former case, the world's volume is infinite, while in the latter it is finite. But this statement is not always valid. Different topologies of such spaces are possible, determining whether the spaces are open or closed, finite or infinite. That is why one ought not be misled by the words 'closed model' and 'open model' in relation to 3-dimensional space topologies in these models: in reality, nothing is known about the topologies and about the total volumes of these spaces.

So far there is no 'topological general relativity theory,' i.e., a theory of space – time that would associate the topology of the world with the physical processes in it. Perhaps it is a matter for the future. But nothing now prevents theoreticians from trying to imagine a possible topology of the Universe, with Einstein having been first to do it, or observers from searching for manifestations of this topology in the real properties of the world. In principle, there are many interesting and diverse mathematically acceptable variants of the global structure of the world as a whole (see, e.g., Refs [177 – 180]).

As far as the hypersphere in Einstein's cosmology is concerned, this case seems to be so simple and even natural that, for example, it is given without any comment in *The Classical Theory of Fields* [5] — "the volume of the positive-curvature space" is $2\pi^2 a^3$, with a being the curvature radius of the space, and it is stated that "the positive curvature space turns out to be 'closed in itself' — with finite volume, but clearly having no boundaries."

Says Friedmann [176], "the metric [i.e., differential geometry — *Remark by* A D Ch] of the world is not enough to solve the problem of finiteness of the Universe. For the solution, extra theoretical and experimental research is required." It seems that the era of this research has come and the world's topology is becoming the most conceptual and maybe the only truly fundamental geometric issue in cosmology.

4.2 Power spectrum

Topological research in cosmology is developing along two directions. In the first area, the 'phantom effect' is being studied: in a topologically closed space with a finite volume, a double or even multiple image of one object can be observed. In this case, light from the source can reach an observer in different ways; for example, having traveled around the closed world, it can come from the direction that is opposite to the direction to the source. Thus, an observer may be able to see the same object in two antipodal directions. The search for such effects has been carried out for a long time, but there have not been any definite or undoubted results so far.

The other area is related to studies of the microwave background. It is these studies that have recently given hope of success. In late 2003, *Nature* published a paper by J-P Luminet et al. (Paris-Meudon observatory) arguing that "after two millennia of speculations" about whether our universe is finite or infinite, a real possibility of solving this issue empirically has finally emerged. It was demonstrated in that paper that some peculiarities in the microwave background anisotropy clearly indicate that the volume of the Universe is finite.

It is known that microwave radiation is coming to us from all directions and is remarkably isotropic and uniform. But there is a mild anisotropy in the microwave radiation at the

level of a few thousandths of a percent, which is due to cosmic medium perturbations of an amplitude that was once small and later developed into galaxies and other large-scale cosmic structures (as we mentioned in Section 2). The anisotropy is observed in the celestial sphere as mild variations of the microwave background. The level of deviations from the average value depends on the angular scale on which they are observed. The result of the observations can be quantitatively expressed in terms of a sum of spherical harmonics, each harmonic entering with its own amplitude. The set of these amplitudes is referred to as the anisotropy power spectrum. There is a well-developed theory taking the details and specifics of the problem into account (see, e.g., Refs [105, 106]). The power spectrum contains certain information about the general physical parameters of the Universe and particularly about its geometry, both differential and global; the information about topology is contained in the low-frequency range of the spectrum (which is not surprising), corresponding to the largest angular and spatial scales.

The lowest harmonic in the power spectrum — the monopole — corresponds to oscillations of the sphere as a whole; the monopole is assigned the wave number $l = 0$; its amplitude is the microwave radiation temperature averaged over the entire sphere (which is 2.727 K). The dipole with the wave number $l = 1$ is attributed to the angular scale 180°. But the dipole in the microwave background can hardly be measured because it is overridden by a much more intense dipole of a completely different origin, caused by our motion (of the entire Solar system) relative to the microwave radiation. The lowest of the measurable monopoles is a quadrupole with the wave number $l = 2$ and the angular scale 90°, followed by an octupole ($l = 3$, the angle is 60°) and other higher harmonics.

At present, the most complete observational data on the microwave radiation anisotropy spectrum were obtained by the WMAP mission [14, 15] and published in 2003 – 2006. These data somewhat contradict the theoretical expectations based on the assumption that the volume of the 3-dimensional comoving space is infinite. Namely, the measured quadrupole appeared to be 5 – 7 times weaker than expected for infinite space; the octupole is weaker by 30%; for higher harmonics, considerable lack of power was not found.

4.3 Poincaré space

Luminet et al. [181, 182] proposed interpreting the power deficiency at the low harmonics as an observational manifestation of the Universe space being finite rather than infinite. According to them, the lack of power at the largest angular and spatial scales means that these scales simply do not fit the Universe. The authors presented a specific model of a finite-space world, which is well consistent with the complete set of the WMAP data, including the microwave background anisotropy power spectrum.

In this model. space has a constant positive curvature, as in Einstein's model and the 'closed' Friedmann model [4] of 1922. Of course, Luminet's model is not static; it is a model of the Universe expanding in accordance with Friedmann. Its key physical characteristic is the density parameter Ω. According to Friedmann, for the comoving space to have positive curvature, the parameter Ω must be greater than unity; furthermore, its present-day value is strictly fixed in Luminet's model as $\Omega(t) = 1.013$. This value close to unity does not contradict the concordant cosmological data or, most importantly, the WMAP data, according to which the tolerance for the modern value of the density parameter is $\Omega = 1.015 \pm 0.020$ (see Section 3).

The topology of the new model differs from the simple topology of Einstein's hypersphere. With the same differential geometry as in Einstein's and Friedmann's models, Luminet's model has a nontrivial topology, its space being multiply connected. It is assumed to be the so-called Poincaré dodecahedral space. This is a 'tighter' space than the closed hypersphere space: if the curvature radii are equal, its volume is 120 times smaller than the volume of the hypersphere in Einstein's model. The Poincaré space is built of 120 blocks, which are regular twelve-face polyhedrons (dodecahedrons) with spherical faces, these faces satisfying a certain (nontrivial) identification condition. The dodecahedrons fill the 3-dimensional volume without overlaps or gaps.

We note that this is a rather complicated topological construction, although it has been a favorite topic for professionals for a long time. Here we do not go into further details; a deeper insight into the Poincaré space can be obtained at http://www.geometrygames.org/CurvedSpaces.

It must be particularly pointed out that Luminet's model can be valid only in the presence of dark energy in the Universe. The relation between the Poincaré space and dark energy emerges because, as we have mentioned, the density parameter in this space must be close to unity; if there were no dark energy, the parameter value would be 0.2 – 0.3, which is incompatible with the positive spatial curvature.

With the above density parameter value, the maximal distances $R_{\mathrm{U}}(t_0)$ in the Poincaré space are different in different directions, but the differences are not too big at present:

$$0.82 R_0(t_0) < R_{\mathrm{U}}(t_0) < 1.03 R_0(t_0)\,, \tag{37}$$

with $R_0(t_0) = 1/H_0 \approx 1.2 \times 10^{28}$ cm being the present-day value of the Hubble radius (given that $H_0 = 72\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ in accordance with the WMAP data [15]).

With good precision, the size of the finite space in Luminet's model is equal to the present-day Hubble horizon radius. Because the two quantities behave differently as time increases in the course of cosmological expansion, their equality in the present era is the fact indicating the special and distinguished character of the present era. On the other hand, the observation itself of effects of spatial finiteness using the microwave radiation power spectrum appears to be possible only when the size of the world does not dramatically differ from the Hubble radius [181]. With these quantities differing significantly, the topological effect cannot be observed: it would be recognized neither much earlier before nor much later after the present era.

Can Luminet's model be verified in observations? Yes, but it can be discarded if observations more precise than current ones yield a density parameter value differing from the one that is strictly fixed in the Poincaré space; if this is the case, it is not obvious that another, maybe more complicated, topological construction can be found that would satisfy new and more rigorous requirements. But if Luminet's idea is confirmed, thus proving that the world's space is finite, it will become, as some people believe (see, e.g., Ref. [183]), one of the most outstanding discoveries in the whole history of science.

But at the present stage of work, we should first verify that the initial observational data are reliable: Is there actually a significant lack of power at the quadrupole

harmonic in the microwave radiation anisotropy spectrum? We note that there is an insufficient number of 'observational points' in the area of the lowest multipoles, raising doubts about whether the WMAP data are reliable enough. For example, the participants in the project themselves admit [15, 16] that the contribution of our Galaxy (and maybe of the Solar system) to the observed anisotropy spectrum at long wavelengths must be subtracted more carefully than this has been done before. Not everything is fine here, as can best be seen from the presence of an unexpected correlation (which is clearly unrelated to cosmology) between the quadrupole and octupole axes and the Galactic and maybe even ecliptical plane (which was already named 'the axis of evil').

The subtraction of foreground sources is a difficult problem likely having no rigorous or unambiguous solution. There is, however, hope that things may be cleared up by the new Planck mission that will be carrying out observations of the microwave background anisotropy at many wavelengths, which will help to solve the problem correctly. (Planck is planned to be put into orbit in 2008). In this research, a great role can be played by special observations on ground-based radio-astronomical instruments, especially on the largest radio-telescope RATAN-600 (Yu N Parijskij's project Cosmological Gene).

We emphasize again that, today, Luminet's model satisfies all necessary observational tests and is consistent with the entire set of cosmological data; it is no worse than the standard ΛCDM model. In a sense, Luminet's model is even richer than the standard model because it answers the question about the power of the lowest multipoles in the microwave radiation anisotropy spectrum, which is left without an answer in models with infinite space. Therefore, with all reservations about the necessity of further verifications, it can be said that Luminet's model presently gives a full description of the observed properties of the real world.

4.4 Extra dimensions

It is known that a classic representative of German philosophy Kant was the first to point out the relation between Newton's universal gravitation law and the fact that our space is 3-dimensional. Only in a 3-dimensional space is the force of mutual attraction of two masses in inverse proportion to the squared distance between them. If, for example, the space were 4-dimensional, this would be the inverse-cube rather than inverse-square law. In a 10-dimensional space, the gravitational force is in inverse proportion to the ninth power of distance. Generally, in a space with the number of dimensions D, the gravitational force decreases in inverse proportion to distance to the power $D-1$. We add that the form of Einstein's universal antigravitation law [see formulas (14) and (31)] is independent of the number of dimensions: with any number, the antigravitational force is in direct proportion to distance.

In the 1920s, P Ehrenfest [184] found that the three dimensions of space is the fact underlying the existence itself of the world or real things. For example, if the number of spatial variables were four rather than three, there would be no closed orbits of planets and the Solar system would not be able to form. In the 1970s, L E Gurevich and V M Mostepanenko [185] extended this analysis to quantum mechanics and proved that there would be no closed orbits of electrons in atoms and the atomic structure of matter would be impossible.

We can see from these examples that the number of spatial dimensions is an exceptionally important attribute of nature. There is no doubt that the same applies to the number of temporal dimensions. But how can it be independently verified that our Universe actually has the number of spatial and temporal dimensions that is evident and familiar to us? Are there any extra hidden dimensions in space and/or time that we have not noticed in our world so far?

It is certainly clear that in accordance with Ehrenfest's result, at the scale of planet systems, space remains effectively 3-dimensional in order for planet systems with closed orbits to exist. The sizes of extra dimensions, if any, must be very small compared to orbit sizes. Actually, there is a much more stringent upper bound for the size of extra dimensions. Experiments in the laboratory show that no deviations from the Newton inverse-square law are revealed at distances amounting to 0.1 – 001 cm [186 – 189].

As regards other interactions, rather than gravitation, they do not feel extra dimensions at much smaller scales. Indeed, the existence itself of atoms implies, in accordance with Gurevich and Mostepanenko's result, that, for example, the Coulomb law, which is an electrostatic analogue of Newton's law, is valid at least at scales of the electron orbits of atoms.

This last fact, however, may not mean that space is 3-dimensional at microscopic scales. Perhaps all the fields and interactions in nature except gravitation(!) 'live' on a 3-dimensional hypersurface in a multidimensional space. This is the situation with the string theory, where spaces with six and seven extra dimensions having a size close to the characteristic Planck length $L_{\mathrm{P}} \sim 10^{-33}$ cm are considered. A 3-dimensional hypersurface of this kind is called brane (originating from the word 'membrane,' which is specially kept for the case of a 2-dimensional manifold). (For various aspects of multidimensional theories with microscopic spatial dimensions, see, e.g., papers [190 – 195].)

In 1998 – 1999 (by chance, in the years when the earliest observational indications of dark energy appeared), new original ideas on the possible character of extra spatial dimensions were expressed. Arkani-Hamed and his colleagues [196 – 198] suggested that, along with extra microscopic dimensions of the string theory and most likely regardless of whether they exist, there can be extra dimensions in nature of a relatively large macroscopic size. They assumed that the extra dimensions are circles of a constant radius R_*. The finite size implies compactness of the space of the extra dimensions. Moreover, in the three 'ordinary' directions, the world's space can stretch infinitely far; there are no constraints on that. But if the real 3-dimensional space is also compact as, for example, is the Poincaré space in Luminet's model discussed above, this would imply that the world is closed in itself and isolated in all the spatial dimensions from cosmological to macroscopic ones.

Actually, the assumption of extra macroscopic dimensions is underlain by a radically new formulation of the question about the nature of gravitation (and of antigravitation as well, as we see in what follows), about the place that gravitation occupies among the other fundamental interactions. Gravitation is known to be much weaker than the other interactions, the electromagnetic and weak (electroweak) ones as well as the strong interaction. For example, the Newtonian force of gravitational attraction between two electrons is 42 orders of magnitude weaker than the force of their electrostatic repulsion. The relative weakness of the

attraction is determined by the small value of the Newton gravitational constant G.

For convenient comparison with the other interactions, it is helpful to express the gravitational constant through the Planck mass (see Section 2.2):

$$\frac{8\pi G}{3} = M_{\mathrm{P}}^{-2} . \tag{38}$$

Henceforth, we use the 'natural system of units' conventionally accepted in fundamental physics; in this system, not only the speed of light but also the Planck constant is set to unity: $\hbar = c = 1$. Then all dimensional quantities can be expressed in energy units: $1\ \mathrm{eV} = 1.8 \times 10^{-33}\ \mathrm{g} = 5.1 \times 10^{4}\ \mathrm{cm}^{-1} 1.5 \times 10^{15}\ \mathrm{s}$.

The Planck mass $M_{\mathrm{P}} \approx 10^{-6}\ \mathrm{g} \approx 10^{18}\ \mathrm{GeV}$ is closer to the masses being weighed on a druggist's scale than the masses of elementary particles. It is often considered to be the most fundamental mass/energy in physics. Indeed, the Planck mass, as well as the Planck density (see Section 2), is a combination of the three physical constants, which 'represent' relativity (c), the quantum nature of the world ($\hbar$), and space–time geometry (G). For a particle with the mass M_{P}, the Compton wavelength equals the Schwarzschild radius (if the particle is smaller than this radius), and hence the Planck mass sets the scale at which gravitation is believed to lose its classical, nonquantum character.

One way or another, the mass M_{P} yields the characteristic energy of gravitational interaction; its huge value (compared to masses of elementary particles) results in the weakness of gravitation: gravitation is that weak, because the energy M_{P} is that high.

If the characteristic energy of gravitation is compared to that of the other interactions, the nearest to the Planck energy would be the energy of the electroweak interaction (it was also mentioned in Section 2); it is the energy at which the electromagnetic and weak interactions unite, $M_{\mathrm{EW}} \sim 10^{3}\ \mathrm{GeV} = 1\ \mathrm{TeV}$. The energy M_{EW} is 15 orders of magnitude lower than the Planck mass. The respective coupling constant defined (like G) by the inverse square of the characteristic energy is 30 orders of magnitude higher than the gravitational constant. The huge gap between the Planck mass M_{P} and the mass M_{EW}, which is measured by the dimensionless number

$$X = \frac{M_{\mathrm{P}}}{M_{\mathrm{EW}}} \sim 10^{15} , \tag{39}$$

has no explanation in the fundamental theory and is one of the most acute problems in physics, termed the 'hierarchy problem.' The search for new approaches to this problem became a motivation for the hypothesis of extra spatial dimensions.

The main idea in [196–198] is that gravitation actually exists not in the 3-dimensional but in a multidimensional space, and the characteristic energy of gravitation in this 'genuine' space is not the Planck mass but some quantity M_*, which can be much smaller than the Planck mass. This resolves the hierarchy problem if the energy M_* is comparable to M_{EW}. To be specific, one can assume that $M_* = xM_{\mathrm{EW}}$, with the parameter x of the theory being a dimensionless quantity that is neither too small nor too large. And again (see Section 2), the characteristic quantity M_{EW} emerges, this time in the picture of a multidimensional world. Here it also plays a central role.

In a D-dimensional space with extra dimensions, the gravitational constant G_{D} is not equal to the Newton constant and is determined not by the Planck mass but by the new 'genuine' energy of gravitation:

$$G_D = M_*^{1-D} . \tag{40}$$

Accordingly, the gravitational force produced by some mass M and exerted on another mass located at a distance R in this space takes the form (per unit mass)

$$F_D = M_*^{1-D} M R^{1-D} . \tag{41}$$

The size R_* of extra dimensions is assumed to be similar for all dimensions. Then at distances exceeding R_*, the effect of extra dimensions vanishes and the gravitational force (per unit mass) takes the usual form $M_{\mathrm{P}}^{-2} M R^{-2}$. But at $R = R_*$, both forces are equal, and the 'matching condition' yields the relation

$$(M_* R_*)^{D-1} = (M_{\mathrm{P}} R_*)^2 . \tag{42}$$

This yields the size of extra dimensions:

$$R_* = \left(\frac{M_{\mathrm{P}}}{M_*}\right)^{2/(D-3)} M_*^{-1} = \left(\frac{M_{\mathrm{P}}}{M_*}\right)^{(D-1)/(D-3)} M_{\mathrm{P}}^{-1} . \tag{43}$$

With one extra dimension ($D = 4$),

$$R_* = \left(\frac{M_{\mathrm{P}}}{M_*}\right)^{3} M_{\mathrm{P}}^{-1} \approx x^{-3}\, 10^{48} M_{\mathrm{P}}^{-1} . \tag{44}$$

If $x \sim 1$, this length is comparable to the size of the Solar system ($\sim 10^{16}$ cm), and therefore, under the obvious considerations developed above, the case of one extra dimension is excluded.

According to [196–198], space with two extra dimensions ($D = 5$) plays a unique role. In this case,

$$R_* = \left(\frac{M_{\mathrm{P}}}{M_*}\right)^{2} M_{\mathrm{P}}^{-1} \approx x^{-2}\, 10^{32} M_{\mathrm{P}}^{-1} . \tag{45}$$

This length amounts to $0.1x^{-2}$ [cm] and is compatible with the above-cited experimental bounds ($R_* < 0.1 - 0.001$ cm) if the parameter of the theory $x \geqslant 1$. With $D = 5$, we see that the extra dimensions have lengths that are 30 orders of magnitude greater than the Planck length M_{P}^{-1} and more or less comparable with our familiar everyday lengths.

4.5 'Genuine' constants and dark energy

In multidimensional physics, the 'ordinary' physical constants, which include the Planck quantities and the Newton gravitational constant, turn out to be derivatives of two basic quantities M_* and R_* and look like '3-dimensional shadows' of genuinely fundamental multidimensional constants. With $D = 5$, the Planck mass and hence the Newton constant are expressed through the two 'genuine' constants in an unsophisticated way:

$$M_{\mathrm{P}} = M_*^2 R_* , \tag{46}$$

$$G = M_{\mathrm{P}}^{-2} = M_*^{-4} R_*^{-2} . \tag{47}$$

Incidentally, expressions (46) and (47) imply that if $D = 5$, then the large hierarchical number is just a product of two

new 'genuine' constants:

$$X \equiv \frac{M_{\rm P}}{M_*} = M_* R_* \sim 10^{15} \,. \tag{48}$$

In this sense, the hierarchy does not actually vanish; a large dimensionless number X remains in the theory, although in a new form.

In Section 2, we met the hierarchy phenomenon while talking about the ratio of two energies $M_{\rm EW}/M_{\rm P}$ and about its possible relation to dark energy density. The dark energy density in ordinary 3-dimensional space (i.e., on the 3-dimensional brane of the 'genuine' space) is expressed from formula (7) in terms of a power of the hierarchical number, $\rho_{\rm V} \sim X^{-8} M_{\rm P}^4$. In the units accepted here, the observed density value is $\rho_{\rm V} \sim 10^{-120} M_{\rm P}^4$; therefore, with $X \sim 10^{15}$, this relation gives the correct order of magnitude for the dark energy density [that is why formula (7) was constructed].

Using the expression for the hierarchical number in the case $D = 5$, we find that the dark energy density in the 3-dimensional world is completely determined by the size of extra dimensions [199]:

$$\rho_{\rm V} \sim R_*^{-4} \,. \tag{49}$$

This formula is built such that the quantity $1/R_*$ yields the mass of dark energy in a spatial volume with the size R_*. Then the density is found by dividing this mass by the volume R_*^3. If $R_* \sim 10^{-3}$ cm, the density $\rho_{\rm V} \sim 10^{-120} M_{\rm P}^4$. A remarkable feature of relation (49) is that the dark energy density is independent of the 'genuinely fundamental mass' and the hierarchy vanishes in this case.

We also note that expression (49) reminds us that the relation known from the quantum Casimir effect — the force of attraction between two parallel conducting plates per unit surface (having the dimension of energy density) — is given by a similar formula (see, e.g., Ref. [94]), $\rho_{\rm CAS} \sim d^{-4}$, with d being the size of the small gap between the plates.

If dark energy (taken as the EG vacuum) is assumed to fill the full 5-dimensional space, it is easy to see that its 5-dimensional density in this space depends just on the size of the extra dimensions:

$$\rho_{\rm V5} \sim R_*^{-6} \,. \tag{50}$$

As regards the Einstein antigravity force, in a 5-dimensional space it takes the form

$$F_{\rm E5} \propto M_*^{-4} \rho_{\rm V} R \,, \tag{51}$$

keeping the linear dependence on distance as expected.

Using (51) and the formulas given above, we can find, for example, the expression for the zero-gravitation radius in the 5-dimensional space. Instead of formula (32) (see Section 3), we now obtain

$$r_{\rm V} = \left(\frac{16\pi^3}{5} M R_*^6 \right)^{1/5} . \tag{52}$$

This radius does not exceed the size of extra dimensions R_* when the mass M is small:

$$M \leqslant M_{\rm V} \sim 10^{-17} M_* \,. \tag{53}$$

The mass $M_{\rm V}$ is 11 orders of magnitude lower than the electron mass. This implies that for all known elementary particles (needless to say, macroscopic bodies), the values of their zero-gravitation radius are far beyond the extra macroscopic dimensions (in the 5-dimensional world).

We recall that quantum nongravitational fields are not supposed to 'live' in the extra dimensions; they and their zero fluctuations exist only on the 3-dimensional brane. For the considerations underlying formulas (49) – (53), this must imply that the vacuum of the 5-dimensional space is produced not by physical fields; it is of a different, nonquantum nature. In multidimensional physics, as the above speculations hint, the dark energy of the vacuum results from the presence of extra dimensions in the world and its density depends just on the number and size of these dimensions and bears no relation to the quantum fields on the brane. Then the observed EG vacuum (as a 3-dimensional shadow of the 'genuinely fundamental' 5-dimensional vacuum) has a 'geometrical' rather than a 'material' nature.

In multidimensional physics, the Einstein cosmological constant Λ is also just a shadow of the genuinely fundamental constants; if $D = 5$, the relation between them is as follows:

$$\Lambda = 8\pi R_*^{-4} M_{\rm P}^{-2} = (M_* R_*)^{-4} R_*^{-2} \,. \tag{54}$$

There is a good prospect of experimental verification of the new view of space dimensions proposed in Refs [195 – 197]. Laboratory experiments with gravitation are in full swing aimed at sub-millimeter distances, at which the 5-dimensional space will become available if the extra dimensions actually exist. By the way, if the number of extra dimensions is 3 or more, their lengths are so small (10^{-7} cm and less) that experimental verification would be much more difficult or even impossible.

The hypothesis of extra macroscopic dimensions promises a new physics at energies close to the 'genuinely fundamental' energy $M_* \sim M_{\rm EW} \sim 1$ TeV. Among other effects, it predicts the birth of gravitons and maybe black holes in experiments at the Large Hadron Collider [200 – 203]. We do not discuss this vast topic here and just restrict ourselves to giving references to reviews in *Physics – Uspekhi* [200, 201].

5. Internal symmetry in cosmology

According to Aristotle, who is sometimes called the first physicist, everything in the world consists of four 'basic elements.' They are earth, water, fire, and air. As far as is known, it was not discussed in the days of Aristotle how much water or fire there was in the Universe or how the amounts of the elements related to each other. Since 1998 – 1999, there are also exactly four elements, or cosmic energies, in modern cosmology, forming everything in the world. The contribution of each energy to the total energy of the world is measured quite precisely (see Section 2). We recall that dark energy contributes about 70 – 80%, dark matter 15 – 25%, baryons about 5%, and radiation a few hundredths of a percent to the total energy of the Universe.

The measured percentage corresponds to the present-day state of the world. In the course of the evolution of the Universe, the relative contribution of each energy has been changing as a result of the general cosmological expansion. For example, the share of the vacuum was close to zero in the early Universe in the era of primordial nucleosynthesis,

when the world was a few minutes old, while the radiation share was almost 100%. In the distant future, the contribution of dark energy will be almost 100%, while the contributions of the three nonvacuum energies will tend to zero.

An apparently accidental recipe for the cosmic mixture, which is also changing with time, may seem unnatural, terribly complicated, weird, and even absurd; such definitions are wandering about scientific and popular literature (see, e.g., the paper called "Absurd universe" [204]). But actually, the seemingly arbitrary set of numbers is underlain by a simple rule independent of time, which is a specific symmetry [110, 205, 206].

5.1 The four energies

Of the four energies, we were mostly talking about dark energy, this being the main topic of the article. We now give a brief overview of the three nonvacuum energies.

As has been noted, ordinary matter consists of nonrelativistic protons, neutrons, and electrons; this type of cosmic energy is commonly called baryonic. Not everything is clear about this ordinary matter. The main question is why there are protons and neutrons, but the same number of antiprotons and antineutrons is not observed. According to one of the general physical laws, there should be equality between particles and antiparticles. The same applies to electrons, whose antiparticles, positrons, are very rare in the natural environment.

The imbalance in favor of baryons may have emerged in the early Universe in the era of extremely high temperatures when these particles were relativistic. Under such conditions, there was an equal number of particles and antiparticles. But if the symmetry between them was not perfect, but rather weakly violated, a little surplus of baryons over antibaryons could have been formed at some point. The hypothetical process of formation of 'extra' baryons is referred to as cosmic baryogenesis. Later, when the temperature of the cosmic medium decreased as a result of cosmological expansion, annihilation of most baryons and antibaryons occurred. But there were no counterparts with which the extra baryons could annihilate, and therefore they have survived in the Universe until now. As a result, an initially small surplus of particles over antiparticles turned into practically one-hundred percent dominance of baryons over antibaryons. This way of solving the problem was outlined by Sakharov [207] and Kuz'min [208]. In papers [207, 208], the necessary conditions for efficient baryogenesis were discovered and a number of crucial features of the process were studied; but a complete and final solution to the problem has not yet been achieved [209].

The physics of radiation is known much better. Radiation is a residue, or relic, of the once dense and extremely hot state of matter at the early stages of the evolution of the Universe. The existence of relic radiation was predicted by Gamow in the 1940s and 1950s, and was later confirmed in observational discoveries. The radiation consists of photons (and probably gravitons [210]) that were in thermodynamic equilibrium with matter and were also extremely hot in the distant past of the Universe. Then, in the course of cosmological expansion, radiation cooled down to the very low temperature observed at present, which is around 3 degrees above absolute zero. In addition, the photons themselves have not disappeared and their total number has survived until now. There are plenty of these particles. In the present era, there are about 500 relic photons per cubic centimeter of space. Radiation fills the entire volume of the Universe almost perfectly uniformly.

The number of nonrelativistic baryons is also conserved as the world expands; presently, there are only two particles for every ten cubic meters of space on average. The ratio of the number of photon to the number of baryons is a large dimensionless 'baryon number' $B \sim 10^9$. Mostly because of the obscure situation with baryons (see above), the physical nature of this number is one of the most difficult mysteries of cosmology and microphysics. While baryons and antibaryons remained ultrarelativistic, the number of the former and the latter was equal to that of photons. The inverse value, $1/B \sim 10^{-9}$, gives a quantitative measure of the weak violation of the symmetry between particles and antiparticles in the early Universe. The baryon number is also a measure of entropy per baryon [10]; for this reason, among other things, it influenced the rate of helium generation in primordial nucleosynthesis when the world was a few minutes old.

As regards dark matter, the earliest hints of its existence emerged in the early 1930s; valid results were obtained in the 1970s (see review [211]). In the 1980s, a hypothesis saying that dark matter is a gas of nonrelativistic neutrinos and antineutrinos of all the sorts was discussed. Later, it became clear that the neutrino mass is too small to accomplish this. Now it is clear that none of the known elementary particles can play the role of the dark matter carrier. Dark matter remains beyond the scope of the standard model of particle physics. This model does not include anything like that, and the existence of dark particles was and is a mystery in it. Dark matter still remains elusive in direct physical experiments in spite of ongoing efforts in this area. But it is reliably known that the mass/energy of dark matter is 4 – 6 times greater than that of baryons. Dark particles fill huge volumes around galaxies, groups, and clusters and form dark coronae, or halos.

According to a widely accepted opinion, the best candidates for the role of dark matter carriers would be yet unknown elementary particles with rather high masses. They have already been named WIMPs (see Section 2). Unlike protons and neutrons, these particles do not feel the strong nuclear forces, but like electrons they are involved in the electroweak interaction. Dark particles are considered stable and are conserved in the course of cosmological expansion. For example, these particles may be the lower-mass supersymmetric partners of such particles as the photon or graviton (the latter being believed more plausible); then dark particles would be fermions and, in accordance with the rule conventionally accepted in physics, would be called a photino or gravitino.

Other hypothetical particles are also considered for this role; these are superlight (axions) [212, 213] and supermassive (with mass close to the Planck mass) [214 – 218] particles, and neutrinos of the fourth generation (they are not forbidden by Schwarzman's criterion [219] if their mass is big enough for them to be nonrelativistic in the era of primordial nucleosynthesis). These could also be hypothetical mirror particles, which have been suspected to exist (from independent considerations) since the mid-1950s [93]. The main question is why nature gives an overwhelming fraction of its nonvacuum energy to these particles.

As we can see, not much is known about cosmic energies. Important questions concerning their physical nature remain mostly unanswered. However, each energy can be described

macroscopically as a medium with certain values of density and pressure. The densities of the cosmic energies are measured in observations (see Section 2.6). The relation between density and pressure, i.e., the equation of state, is also known for each medium. Baryons and dark matter are nonrelativistic (at least after the early era of nucleosynthesis); therefore, their pressure is small compared to the energy density and can be taken to be exactly zero. Radiation is a relativistic medium, its pressure being one third of the energy density. In the case of the vacuum, as we know, the pressure is negative and equal to the dark energy density taken with the minus sign.

Knowing the equation of state of a given cosmic energy, we can determine how it behaves in the course of the expansion of the Universe. This is given by one of the most general laws of nature — the internal energy conservation law appearing in cosmology as the second Friedmann equation (see Section 2). It follows from this law that the total number of particles in a given expanding volume does not change with time (as we have said). But this is clear because if the particles are stable, then there is always the same number of particles in a 'comoving' volume. This last statement is valid for particles of all three nonvacuum energies, i.e., baryons, photons and neutrinos, and dark particles. As regards the dark energy of the vacuum, there are no (real) particles in it and its density turns out to be a conserved quantity; the EG vacuum does not change during cosmological expansion at all.

5.2 Symmetry

The availability of the physical characteristics of cosmic energies, which are conserved in time, allows formulating the recipe for the cosmic mixture, not in percentage, which changes as the Universe expands, but in terms of constant values:

$$A_{\mathrm{V}} \sim A_{\mathrm{D}} \sim A_{\mathrm{B}} \sim A_{\mathrm{R}} \sim 10^{60\pm1} M_{\mathrm{P}}^{-1} . \tag{55}$$

Here, each of the four constants, which are called Friedmann integrals, represents a certain cosmic energy: the dark energy of the vacuum (A_{V}), dark matter (A_{D}), baryons (A_{B}), and radiation (A_{R}). The Friedmann integrals are approximately equal (within two orders of magnitude); their numerical value is given in 'natural units' (see Section 4) with $c = \hbar = 1$.

The place of the Friedmann integrals in cosmology can be seen from the first Friedmann equation (11) expressing the mechanical energy conservation law (see Section 2). If all four cosmic energies are taken into account in Eqn (11), it takes the form

$$\dot{R}^2 = \left(\frac{A_{\mathrm{V}}}{R}\right)^{-2} + \frac{A_{\mathrm{B}}}{R} + \frac{A_{\mathrm{D}}}{R} + \left(\frac{A_{\mathrm{R}}}{R}\right)^{2} . \tag{56}$$

Equation (56) is the equation of the standard cosmological model (ΛCDM) with a flat 3-dimensional space and parabolic ($E = 0$) dynamics. Here, $R(z)$ is the scale factor (a function of time or the redshift z), in proportion to which all cosmological distances change:

$$R(z) = R_0(1+z)^{-1} , \qquad R_0 = 3 \times 10^{60} M_{\mathrm{P}}^{-1} . \tag{57}$$

The present-day value ($z = 0$) of the scale factor with normalization accepted here is close to the value of the Hubble radius $R(z=0) = R_0 \sim H_0^{-1}$. The expanding region of the world having the size $\sim R(z)$ is often called 'our cosmic domain' or the Metagalaxy.

The Friedmann integrals are integrals in the proper sense of the word. They are constants of integration emerging after solving the second Friedmann equation (18). We recall that Eqn (18) expresses the internal energy conservation law, the cosmic energies being macroscopically interpreted as media with a certain relation between pressure and density. If $\mathrm{d}S = 0$, Eqn (18) for each individual cosmic energy has the form

$$\frac{\dot{\rho}}{\rho(1+w)} = -3\,\frac{\dot{R}}{R} , \tag{58}$$

where ρ is the density of a given energy and w is the ratio of the pressure of this energy to its density, with $w = -1, 0, 0, 1/3$ for dark energy (the EG-vacuum), dark matter, baryons, and radiation, respectively. For an energy with given ρ and w, the integral of Eqn (58) is

$$A = \left(M_{\mathrm{P}}^{-2} \rho R^{3(1+w)}\right)^{1/(1+3w)} . \tag{59}$$

Formula (59) (together with relation (57) for the scale factor) defines the Friedmann integrals.

The numerical values of all four integrals can be found if at some (no matter which) instant of time, the values of the respective energy density and scale factor are known. We use the data on the present-day density values quoted in Section 2.6. From the definition of the Friedmann integrals, we then find that the four integrals are close to each other within an order of magnitude, their numerical value being close to the present-day value of the Hubble radius, $A \sim 1/H_0 \sim 10^{28}$ cm $\sim 10^{60} M_{\mathrm{P}}^{-1}$.

Due to their origin from the internal energy conservation law, the integrals are completely independent of each other, and their values are not related by any constraints other than trivial ones. For example, in a world without dark energy (the model that was commonly accepted until recently), the integral A_{V} turns to infinity. In a 'cold world' with zero initial temperature (the model discussed before the discovery of microwave radiation), the integral A_{R} is zero. Thus, the entire range from zero to infinity is open for these four energy parameters, and, generally, they could arbitrarily differ from one another. However, the actual values of the four integrals do not span the entire infinite range allowed a priori, but are located in a rather narrow range not exceeding two orders of magnitude.

The physical meaning of the Friedmann integrals is quite transparent: they are directly related to the conserved values mentioned in Section 5.1. It is easy to see that

$$A_{\mathrm{V}} = M_{\mathrm{P}} (\rho_{\mathrm{V}})^{-1/2} , \tag{60}$$

$$A_{\mathrm{D}} = \frac{3}{4\pi}\, M_{\mathrm{P}}^{-2} M_{\mathrm{D}} , \tag{61}$$

$$A_{\mathrm{B}} = \frac{3}{4\pi}\, M_{\mathrm{P}}^{-2} M_{\mathrm{B}} , \tag{62}$$

$$A_{\mathrm{R}} \approx \left(\frac{3}{4\pi}\right)^{1/2} M_{\mathrm{P}}^{-1} N^{2/3} , \tag{63}$$

where M_{D} and M_{B} are the total masses of dark matter and baryons within the sphere of radius $R(z)$ and N is the total number of relic photons within the same sphere. Both total

masses, M_D and M_B, and the total number N of particles are values that do not vary in the course of cosmological expansion in accordance with the definition of the comoving volume (in Section 5.1, we were talking about the conservation of the number of dark particles and baryons in a comoving volume; but the respective total masses are also conserved because the mass of each nonrelativistic particle is constant).

We note that in the model of the finite closed space proposed by Luminet [181] (see Section 4), the nonvacuum integrals A_D, A_B, and A_R are expressed through the total masses of dark matter and baryons and the total number of relic photons in the whole world.

As we can see, the nonvacuum integrals have a clear interpretation. The dark-matter integral is specified by the invariable density of the EG vacuum; in this case, such is the requirement of the internal energy conservation law. Although the first integrals are expressed through extensive values and the fourth involves an intensive one, all four integrals are of the same dimension (the dimension of length) and can therefore be compared to each other.

Although the values of the Friedmann integrals were calculated using the observational data relating to the present era, the integrals themselves and approximate equality (55) between them are valid at each instant of the world's evolution — in the past, at present, and in the future, at each instant when these four energies exist in nature. The four integrals are covariant quantities. Their values are independent of the reference frame. Numerical values of these integrals found in the comoving space remain the same in any other spatial section and in the 4-dimensional space as a whole. Hence, approximate equality (55) is also covariant and valid in any reference frame.

As we can see, the energy contents of the Universe are actually not too complicated despite the first impression. The recipe for cosmic mixture (55) written in terms of the constant energy parameters of the Universe, the Friedmann integrals, seem to be neither intricate nor weird, while its physical meaning is simple and evident. Close numerical coincidence of the Friedmann integrals in (55) can hardly be considered just an arithmetic accident. Rather, this fact must be indicating some sort of regularity, a certain internal correspondence between vacuum and nonvacuum cosmic energies. We can assume that this correspondence is a kind of internal symmetry [110, 205, 206, 220].

Here, we follow the most general definition of symmetry, which "designates the type of consistency between different parts that unifies them in a whole" [221]. A symmetry is called internal if it does not involve space – time relations. A long-known physical example of internal symmetry is the symmetry unifying the proton and the neutron (in spite of the obvious differences in masses, electric charges, lifetimes, etc.) in a whole, in the doublet of nucleons with the common value of isotopic spin.

Internal symmetry in cosmology makes the energy contents of the Universe simple and ordered. It unifies the cosmic energies into a quartet, in which they are characterized by a 'charge' conserved in time, i.e., by the Friedmann integral whose numerical value is (approximately) the same for all of them. The symmetry is not perfect but weakly violated because the exact values of the four integrals are not the same; the extent of the symmetry violation can be seen from (55). Furthermore, the symmetry is stable because it is insensitive to the details of the observational data used. (The symmetry between baryons and radiation was noticed soon after the discovery of microwave radiation [222].)

As we can see from (55), dark energy naturally fits the set of cosmic energies. It is a regular member of the energy 'quartet.' In this sense, the measured density of dark energy seems to be neither too low nor too high — it is just right for the Friedmann integral representing it to have the value that is close to the numerical value of the other three integrals within an order of magnitude. If dark energy, for example, had the Planck density, it would look extremely unnatural in this set of energies [we recall the 'naturalness problem' (see Section 2)]; the respective vacuum integral would be 60 orders of magnitude smaller than the other three integrals.

5.3 Coincidence of densities

As regards the derivation of symmetry relation (55), it is an empirical fact, a result of direct analysis of observational data. The cosmological theory was only needed to introduce the general definition of Friedmann integrals (59) as constant energy parameters of the Universe. We now consider physical relations that actually underlie empirical relation (55).

We first note that the numerical value of the vacuum integral is not related to the normalization of scale factor (57) assumed in definition (59) of the Friedmann integrals. This integral is entirely independent of the scale factor. But the values of the other three integrals do depend on the normalization. With another normalization, A_D and A_B would still be close to each other (because they depend on the scale factor in the same way), but, generally speaking, not close to either A_V or A_R. Yet this does not imply that the approximate equality of all four values is completely determined by normalization (57). Indeed, a specific choice of only one parameter does not equalize four values at once unless an internal relation exists between them, independent of the normalization.

The existence of such an internal relation becomes obvious if we look again at the list of the cosmic energies in (22) – (25). These data show that in the present era, the densities of all four energies do not differ too much, their values ranging within four orders of magnitude:

$$\rho_V \sim \rho_D \sim \rho_B \sim \rho_R\,, \qquad t \sim t_0\,. \tag{64}$$

Such is the characteristic feature of our era. In the distant past and future, the ratios between the four densities were and will be far beyond a few orders of magnitude.

In addition, this is the characteristic feature of the Universe itself. No general principles predict that the cosmic energies must be close to each other in some era of cosmic evolution. The possibility of such an era is a unique property of the real world. It is revealed in observations, but, in fact, it was put in the physics of the cosmic energies 'from the very beginning,' in the way their initial internal nature was organized. It is this 'intrinsic' property of the internal organization of the quartet of cosmic energies that is revealed and fixed by symmetry relation (55) in terms of the energy parameters conserved in time.

It is worth noting that relations (64) and (55) are similar (not only in their simplicity), but are by no means equivalent. Indeed, relation (64) is valid just in one definite era (ours) of cosmic evolution, whereas relation (55) holds at any instant and is independent of time. We also note that the scatter of values in (55) is 100 times smaller than in (64).

At the empirical level, temporary relation (64) turns into 'eternal' relation (55) due to the internal energy conservation law (58) and definition (59) of the Friedmann integrals following from (58).

Furthermore, the empirical (approximate) relation for the present-day value of the scale factor is also essential:

$$R_0 \sim \frac{1}{H_0} \sim \frac{1}{H_V} \sim A_V \,. \tag{65}$$

The closeness of the values H_0 and H_V was already discussed in Section 3. This is another unique feature of the present era.[1] With that taken into account, relation (64) and definition (59) yield the relation

$$A_V \sim A_D \sim A_B \sim A_R \sim \rho_V^{-1/2} M_P \,. \tag{66}$$

In this way, instead of a relation for the densities (three of which depend on time), we obtain the relation for Friedmann integrals (55) that is already independent of time and, consequently, reflects the constant property of the energy quartet — its symmetry and internal consistency.

5.4 Hierarchy again

But what physics provides the consistency of the energies? The physics of cosmic energies can hardly be discussed until the microscopic structure of the vacuum is revealed, the issue of baryonic asymmetry of the world is resolved, and carrier particles of dark matter are found. But at the phenomenological level, something can be clarified, as it seems, already now. For this, we consider kinetic processes in the early Universe and, following paper [99], consider the mechanism of freezing-out of dark matter annihilation [110, 205, 220].

If dark matter's carriers are WIMPs, then the Universe in its present state involves both these particles and an equal number of their antiparticles. Their mutual annihilation is now practically impossible because the present-day concentration (number density) of the particles is low, but the cross section of weak processes is relatively small. Annihilation of dark particles already became impossible in the distant past of the Universe when the characteristic time of annihilation and birth of particle – antiparticle pairs turned out to be longer than the cosmological time that determines the cosmological expansion rate. In this sense, dark matter is a thermal relic of the early Universe, like the cosmological neutrinos and photons [10 – 12].

[1] A different question is why we happened to live exactly in this era, why not much earlier or later. Speculations concerning this are contained in the so-called anthropic principle. Without going into the details, we say that in accordance with this principle [223 – 225], the observed Universe is as is because it ensures physical conditions for the emergence and development of life, for the advent of complex organisms and the mind, for the existence of an observer who is capable of asking a question like this. From this standpoint, the 'uniqueness' of our era and its specific character are obvious. Indeed, our Universe is not too young, and hence there is enough carbon and oxygen prepared, which are necessary for the emergence of life and the formation of complex organisms. On the other hand, the Universe is still in its flourishing age, and hence there are stars like the Sun that are capable of providing life with the required heat and light. The very possibility of our existence appears to be limited by a number of physical conditions, and they are certainly satisfied in the era when, in accordance with the above, the radius of the Metagalaxy, our cosmic domain, is close to the value of the Friedmannian integral.

In a simple model of freezing-out of annihilation of dark particles [99], a central role of the electroweak energy scale $M_{EW} \sim 1$ TeV (often mentioned above) in fundamental physics is initially assumed. Moreover, the freezing-out process is presumed to be completely described in terms of just two fundamental energies, M_{EW} and the Planck energy $M_P \sim 10^{18}$ GeV. On the basis of this last assumption, a formula for dark energy density (7) is chosen and a value for the mass of dark particles (which must be close to 1 TeV) is assumed. Under these assumptions, the dark matter annihilation freezing-out process proceeds at temperatures 1 TeV on the order of magnitude (henceforth, not only the speed of light and the Planck constant but also the Boltzmann constant are taken to be unity). The corresponding cosmic age t_{EW} defined by the standard cosmological relation $t \sim M_P/\rho^{1/2}$ is around three picoseconds. In that era, radiation mainly contributed to the density, and therefore the density ρ in the last formula may be understood as the radiation density

$$\rho_R \sim M_{EW}^4 \,, \qquad t \sim t_{EW} \,. \tag{67}$$

The main relation of the freezing-out model follows from the condition that annihilation terminates when dark particles become nonrelativistic and the characteristic time of particle – antiparticle annihilation,

$$t_D \sim (\sigma n)^{-1} \,, \tag{68}$$

becomes equal to the cosmological time t:

$$n \sim M_{EW}^2 M_P^{-1} \rho_R^{1/2} \,, \qquad t \sim t_{EW} \,. \tag{69}$$

Here, n is the number of surviving dark particles in unit volume and $\sigma \sim M_{EW}^{-2}$ is the cross section of electroweak processes. From (69), we obtain the density of ('cold,' i.e., nonrelativistic) dark matter as

$$\rho_D = M_{EW} n \sim M_{EW}^5 M_P^{-1} \,, \qquad t \sim t_{EW} \,. \tag{70}$$

Equations (67) and (70) relate cold dark matter and radiation in the era of freezing-out of dark particles. As can be seen from these two equations, the density of radiation in that early era was considerably higher than the density of dark matter, $\rho_R/\rho_D \sim M_P/M_{EW} = X \sim 10^{15}$. The ratio of these densities was changing in the course of further cosmological expansion:

$$\frac{\rho_R(z)}{\rho_D(z)} \sim \frac{X(1+z)}{1+z_{EW}} \,, \tag{71}$$

where z_{EW} is the redshift at the instant $t = t_{EW}$. Following the standard cosmology, we obtain the temperature $T_{EW} \sim 1$ TeV $\sim 10^{16}$ K, which corresponds to the redshift

$$1 + z_{EW} \sim \frac{T_{EW}}{T_R} \sim 3 \times 10^{15} \,. \tag{72}$$

Here, $T_R \approx 3$ K is the present-day temperature of microwave radiation. Then, following (71) and taking (72) into account, we obtain

$$\rho_R \sim \rho_D \,, \qquad t \sim t_0 \,. \tag{73}$$

In accordance with (72), $z_{\rm EW} \sim X$, and hence the last relation can be extended using formulas (67) and (70) again:

$$\rho_{\rm R} \sim \rho_{\rm D} \sim X^{-8} M_{\rm P}^4 , \qquad t \sim t_0 . \tag{74}$$

Relation (74) is the desired result. Now it is clear when, how, and why the internal consistency between dark matter and radiation could emerge. It emerged in the first picoseconds of the existence of the Universe thanks to physical processes driven by competition between the electroweak interaction (represented by the energy $M_{\rm EW}$) and gravitation (the Planck mass $M_{\rm P}$). This physics provides the equality of the two respective densities in the present era and, consequently, the equality of two Friedmann integrals in empirical symmetry relation (55).

If, following [99], we assume that the same hierarchy phenomenon is responsible for the nature of dark energy, then its density is given by relation (7). Comparison with (74) shows that the constant density of dark matter is close to the present densities of dark matter and radiation. This then implies that the equality of three densities arises in the present era,

$$\rho_{\rm R} \sim \rho_{\rm D} \sim \rho_{\rm V} \sim X^{-8} M_{\rm P}^4 , \qquad t \sim t_0 , \tag{75}$$

and, hence, the equality of three integrals:

$$A_{\rm R} \sim A_{\rm D} \sim A_{\rm V} \sim X^4 M_{\rm P}^{-1} \sim 10^{60} M_{\rm P}^{-1} . \tag{76}$$

The numerical value of the integrals in (76) is practically the same as the empirically found value [see (55)]. We emphasize that symmetry relation (76) emerged not in the present era but in the first picoseconds of the history of the Universe. The symmetry of the cosmic energies is eventually underlain by the fundamental (and still mysterious) hierarchy phenomenon in microphysics.

The fourth energy, which is associated with baryons, is beyond the scope of such considerations. Link to baryons, however, might have also emerged in the era of electroweak temperatures. If we assume that electroweak baryogenesis [226] did take place, this process may at least be expected to give the required baryon density in the present era and, hence, a suitable value of the baryon integral. Therefore, in the first picoseconds of the expansion, all these events in the Universe could have resulted in the 'initial conditions' that completely determined the subsequent cosmological evolution. These conditions are expressed by time-independent symmetry relation (55).

We again recall the concept of two extra macroscopic dimensions (see Section 4), which was proposed [196, 197] in an attempt to find a new approach to the hierarchy problem in fundamental physics. The point is that a phase transition from a 5-dimensional space to the 3-dimensional one, known as the compactification of extra dimensions, occurs in the same era of electroweak energies $t \sim t_{\rm EW}$. By the instant the world was a few picoseconds old, the actual Hubble radius, $\sim t_{\rm EW}$, had increased to the size of the extra dimensions, $t_{\rm EW} \sim R^*$, while the energies of particles had decreased to the value of the 'genuinely fundamental' energy $\sim M_{\rm EW}$. In this worldview, the era $t \sim t_{\rm EW}$ of electroweak energies is the beginning of the Friedmann ('3-dimensional') stage of cosmological evolution.

Using relation (49), which determines the dark energy density in terms of the size of extra dimensions, we can reveal the relation between the Friedmann integrals and the 'genuine' constants of multidimensional physics:

$$A \sim (R_* M_*)^2 R_* . \tag{77}$$

If understood literally, formula (77) means that the constant energy parameters of the observed Universe (together with the internal symmetry of the cosmic energies) stem from the physics of the extra dimensions. It is curious that relations like (77) hint at the possibility of a certain interrelation among three levels of nature: microscopic (M_*), macroscopic (R_*), and megascopic (A).

5.5 Large numbers

In cosmology, there is the large baryon number $B \sim 10^9$, which is the ratio of the number of relic photons to the number of baryons (see Section 5.1). Why is this number large? That is the question that emerged in cosmology and fundamental physics after the discovery of microwave radiation. The baryon number reflects particle properties at a microscopic level. It characterizes symmetry breaking between baryons and antibaryons. The physical nature of this number is clearly related to the baryogenesis process. What can the phenomenological symmetry of the energies tell us about this problem?

First, the baryon number can be easily expressed through the two corresponding Friedmann integrals:

$$B = \frac{A_{\rm R}}{A_{\rm B}} (A_{\rm R} M_{\rm P})^{1/2} \frac{m}{M_{\rm P}} , \tag{78}$$

where $m \sim 1$ GeV is the proton mass. The quantity B is independent of the scale factor normalization. Substituting the exact numerical values of the integrals in (78), we obtain the exact value of the baryon number. If exactness is not the goal, formula (78) can be simplified using approximate symmetry relation (55) and assuming $A_{\rm R} \sim A_{\rm B} \sim A \sim 10^{60} M_{\rm P}^{-1}$. Then we find

$$B \sim (A M_{\rm P})^{1/2} \frac{m}{M_{\rm P}} \sim X^2 \frac{m}{M_{\rm P}} \sim 10^{12} . \tag{79}$$

The value of B obtained in this way is higher (by approximately 3 orders of magnitude) than the exact one; but a more important point is not the difference but the closeness to the exact value within an order of magnitude. The question raised above can be answered as follows: the dimensionless number B is large because there are two constant energy parameters in cosmology, $A_{\rm R}$ and $A_{\rm B}$, and both of them are close to the universal parameter $A_{\rm V} \sim 10^{60} M_{\rm P}^{-1}$ within an order of magnitude.

Assuming that baryogenesis can occur in the era of electroweak temperatures (see Section 5.4), in the leading, albeit rough, approximation, the number B expressing the result of the process must be a combination of the two energies, $M_{\rm EW}$ and $M_{\rm P}$ (and the nucleon mass m?). But this is exactly the form of approximate relation (79).

Mimicking the way the large baryon number was introduced, we can introduce the 'large dark number' D as the ratio of the number of relic photons to the number of dark particles in unit volume. This number expresses the result of the dark particle annihilation freezing-out (see Section 5.3) to yield the fraction of dark particles and antiparticles that were conserved and became nonrelativistic, $D \sim 10^{12}$. We write the large dark number using the corresponding Friedmann integrals and assuming, as above, that the dark particle mass

is close to the electroweak energy:

$$D \sim \frac{A_{\mathrm{R}}}{A_{\mathrm{D}}} (A_{\mathrm{R}} M_{\mathrm{P}})^{1/2} \frac{M_{\mathrm{EW}}}{M_{\mathrm{P}}} . \tag{80}$$

Assuming again that $A_{\mathrm{R}} \sim A_{\mathrm{D}} \sim 10^{60} M_{\mathrm{P}}^{-1}$, we find

$$D \sim X \sim 10^{15} . \tag{81}$$

Within an order of magnitude, this last value is not very far from the real value of the large dark number. As expected, the hierarchy phenomenon lies behind either of the large dimensionless numbers B and D. The difference between the simple estimate and the exact values of these two numbers possibly indicates that both the symmetry of the energies and its weak violation are important in the appropriate physics.

In cosmology, there are other large dimensionless numbers, which are huge for another reason. They are extensive characteristics of the observed Universe as a whole. It is not very difficult to see that they are also expressed through the Friedmann integrals and, as a result, turn out to be powers of the hierarchy number. To illustrate, the total numbers of relic photons and of dark particles in the observed region of the Universe amount to

$$N_{\mathrm{R}} \sim X^6 \sim 10^{90} , \tag{82}$$

$$N_{\mathrm{D}} \sim X^5 \sim 10^{75} . \tag{83}$$

To summarize, the internal symmetry of the energies and its physical interpretation help to reveal relations in nature that have not been noticed before. Among them are the relation between the large dimensionless numbers in cosmology and the hierarchy phenomenon in fundamental physics [220]. To all appearances, the processes in which these relations were formed (or realized) could have occurred in the early Universe in the first picoseconds of its existence.

5.6 Dicke's problem

The 3-dimensional space directly available to us in everyday life is certainly flat, or Euclidean. But space at the scale of the entire observed Universe is also almost flat (or maybe exactly flat), which is directly indicated by observations (see Sections 2 and 4). We mean the 3-dimensional comoving space. Why does it happen to be practically flat? This question, first distinctly raised by R Dicke [227] in 1970, is the subject of the 'flatness problem.'

Here is how Dicke formulated the problem. According to Friedmann's theory, an isotropic 3-dimensional space can be flat (the curvature $k = 0$), spherical ($k > 0$), or hyperbolic ($k < 0$). Accordingly, there are three cosmological models, and if one of them is claimed to describes the real world, it must meet all necessary observational requirements. The three models are identical in almost everything other than one circumstance: the signs of the comoving isotropic space curvature differ. The choice among the three variants can be made using observational data; but carrying out direct geometrical measurements at cosmological scales with the required precision was impossible (in the days of Dicke). But it follows from Friedmann's theory that the type of geometry is related in a one-to-one way to the density parameter Ω, which is the ratio of the total energy density to the critical density. In other words, in terms of Newtonian mechanics (see Section 2), Ω is the ratio of the absolute value of the potential energy to the kinetic energy, $\Omega = |U|/K$. The flat geometry corresponds to $\Omega = 1$, while the spherical and hyperbolic ones correspond to $\Omega > 1$ and $\Omega < 1$, respectively.

We recall that according to the latest and most precise data (see Section 2), $\Omega = 1.015 \pm 0.020$ in the present Universe. Hence, the space may be exactly flat; but if it is curved, the deviation of its geometry from Euclidean, measured by the difference $|\Omega - 1|$, is quite small in the present era. In this sense, all of the three models are equally valid if only the deviations from Euclidean geometry in the models with curved space are in the range allowed by observations.

In the 1970s, when the problem was first formulated [227], the measurement precision was much lower and the density parameter was assumed to be in the range from 0.1 to 10. At that time, Dicke noted that this range implies an exceptionally fine 'initial tuning' of the Universe. Using the cosmological model that was standardly accepted at that time (without the cosmological constant), he calculated that for the density to be in the window allowed by observations, the initial values of potential and kinetic energies must be in tune with each other with a 16th decimal places if the conditions were fixed in the era of primordial nucleosynthesis when the world was a few minutes old. This extremely fine tuning of the cosmological dynamics was fairly considered by Dicke as weird and unnatural. But no other solution to the problem was proposed then.

With the discovery of dark energy, Dicke's problem is viewed in a new way. The symmetry relation $A_{\mathrm{D}} \approx A_{\mathrm{V}}$ leads to a rather severe upper bound for any possible deviations of the spatial geometry from Euclidean [206]. To show this, we consider the first (dynamical) Friedmann equation, i.e., conservation law (11) for the mechanical energy of cosmological expansion:

$$\frac{1}{2} \dot{R}^2 = \frac{4\pi}{3} \rho R^2 + E . \tag{84}$$

Here, $\rho = \rho_{\mathrm{V}} + \rho_{\mathrm{D}} + \rho_{\mathrm{B}} + \rho_{\mathrm{R}}$ is the total energy density in the world. If the 3-dimensional comoving space is non-Euclidean, then the total energy $E = \mathrm{const}$ is not zero, with

$$E = \frac{k}{2} \left(\frac{R}{a} \right)^2 , \tag{85}$$

where $k = -1, 0, +1$ for spaces with positive, zero, and negative curvature, respectively, and $a(t)$ is the curvature radius of the space if $k \neq 0$.

At the early stages of expansion, all three nonvacuum energy densities tend to infinity as time tends to zero. Therefore, in the limit of small times, the constant E can be neglected in Eqn (84). This means that in this limit case, space is actually perfectly flat and the expansion proceeds in a practically parabolic mode. But the same applies to the opposite limit, when time, along with the radius R of our cosmological domain, tends to infinity.

Indeed, at large times, the nonvacuum densities tend to zero as time tends to infinity. Therefore, in this asymptotic regime, dark energy, with its constant density, dominates. The corresponding term in Eqn (84) becomes arbitrarily large compared to the constant value E. Hence, the deviation of space from Euclidean geometry is negligible both at the earliest and latest times. In the early period, as time tends to zero, this deviation is restricted by the gravitation of radiation, while at late times, as time tends to infinity, it is restricted by antigravitation of dark energy of the EG vacuum.

In the cosmological picture used by Dicke in formulating the flatness problem, there was no antigravitation and, consequently, the deviation from the Euclidean geometry could only increase in the course of cosmological expansion.

The deviation from the Euclidean geometry is measured, as was said, by the value $|\Omega(t) - 1|$, which explicitly follows from Eqn (84). In accordance with this, the value $|\Omega(t) - 1|$ tends to zero at early and late times and must have a maximum at 'medium' times. It is easy to see that the maximum is achieved just at the instant of the gravitation–antigravitation balance at the redshift $z = z_{\mathrm{V}} \approx 0.7$, when $R(z_{\mathrm{V}}) = ((1/2)A_{\mathrm{V}}^2 A_{\mathrm{D}})^{1/3}$. At that instant,

$$|\Omega - 1|_{\max} \approx \frac{1}{2}\left(\frac{A_{\mathrm{V}}}{A_{\mathrm{D}}}\right)^{2/3}\left(\frac{R}{a}\right)^2 . \tag{86}$$

Taking the symmetry $A_{\mathrm{V}} \approx A_{\mathrm{D}}$ into account, we thus obtain the absolute upper bound for the deviation of space from Euclidean geometry at any time:

$$|\Omega - 1| \leqslant y \equiv \frac{1}{2}\left(\frac{R}{a}\right)^2 . \tag{87}$$

Any cosmological model with a non-Euclidean space ($k \neq 0$) satisfies the observational limitations of the 1970s (see above) if the constant dimensionless parameter entering relation (87) is of the order of unity, $y = (1/2)(R/a)^2 \sim 1$. This implies that no fine tuning is required and the problem in Dicke's formulation [228] is actually reduced to choosing a constant dimensionless parameter of the order of unity. That is the answer to the question raised in cosmology almost 40 years ago.

An example from modern cosmology is Luminet's model of a finite Universe with the positive-curvature space [181] considered in Section 4; the parameter y in it is not too far from unity either, $y = 0.02$. In this and any other model with the same value of the y parameter, the constraints following from the firmest WMAP data are conservatively satisfied. To see the contrast to speculations on fine tuning, it is worth comparing the modest numbers 1 and 0.02 with the awkward 10^{-16} obtained by Dicke.

To summarize, the visual picture of an almost Euclidean space of the Universe is actually underlain by the balance between gravitation of matter (dark matter and baryons) and antigravitation of dark energy [206] (see also Refs [228, 229]). This balance is controlled by the internal symmetry of the energies, which completely excludes significant deviations from Euclidean space in the present era, as well as at each instant in the past and future. The issue of the nature of the almost flat space of the Universe is not solved by the specific and fairly nonnatural choice of initial conditions with the fine tuning of potential and kinetic energies. Instead of such sort of initial conditions, we actually see a simple criterion, which is independent of time. This is what is sometimes called 'initial conditions without initial moment' [230].

We note that a solution to Dicke's problem is also proposed by the inflation theory [231, 232]; as is known, this theory deals with gigantic vacuum densities close to the Planck density and requires extrapolating the established physical laws to an unknown domain by 30 orders of magnitude of spatial scale. All this is unnecessary, however. The real measured density of dark energy suffices to account for the Dicke problem and eliminate it.

But what if the isotropic space is not approximately but exactly flat? As mentioned, this possibility does not contradict any observational data. Moreover, today's working cosmological model does use the variant of flat space (not as a rigorous result but as a simple and very good approximation to reality). Dicke's formulation of the problem does not embrace the case of exactly flat space; in this case, the above-cited new considerations [206] are not valid either; nor does the inflationary model work [224]. If the 'problem of exactly flat space' arises some day (it is, however, unclear how it could actually occur), we will have to search for entirely different approaches.

6. Conclusion

The issue of dark energy refers to the 23rd problem in Ginzburg's list, "Cosmological problem. Inflation. Λ-term and 'quintessence' (dark energy). Relation between cosmology and high-energy physics" [6, 223, 234]. From these issues, only inflation was slightly mentioned above, although a large body of literature exists on the subject. By the way, having recently held the cosmology publications record, hypothetical inflation now definitely trails observational, empirical, phenomenological, and theoretical studies of truly existing dark energy.

The main result of the cosmological research of recent decades is as follows: the existence of dark energy and the antigravitation it produces is reliably and finally verified. There is increasing evidence in favor of the Einstein cosmological constant Λ and the interpretation of dark energy as the Einstein–Gliner vacuum. This conclusion can be drawn from the entire set of the newest results in the extensive flow of cosmological literature.

Do these statements not sound too confident? Cosmology has had the reputation of a careless science for a long time; as Landau said, cosmology often makes mistakes and never doubts. This time, however, there was no lack of criticism or carefulness. The discovery of dark energy does not raise any doubts now [1, 2], because it has surely been confirmed in the subsequent numerous observations of supernovae, in studies of the microwave radiation anisotropy, in analysis of the dynamical structure of the Hubble flow — generally in the entire set of current cosmological research, both observational and empirical. Independent cosmological data consistent with each other are constantly reinforced with new results, which only increase the reliability and precision of both qualitative and quantitative results.

Further research in observational cosmology is expected to yield more precise information on the dark energy equation of state. This is probably an acute issue from the list of topical issues. To solve it, we need information not about several hundred supernovae (which had been known to astronomers by early 2008) but about several thousand stars like these. This amount of information will be available in the next decade when (and if) observations by special missions [such as SNAP or JDEM (Joint Dark Energy Mission)] start. In the more distant future, these problems will also be posed on the gigantic 42-meter optical telescope planned by the European South Observatory. An important and independent source of cosmological information is observations of microwave radiation, where we must learn how to reliably subtract the emission of the Galaxy and other foreground sources from the background. This must be greatly helped by new orbital research projects (especially Planck), as well as by ground-

based radio-astronomical observations (in particular, on RATAN-600). Ongoing investigations of kinematics and dynamics of the Hubble flow at medium and small scales will offer the increasing precision of determination of qualitative characteristics of local dark energy, this becoming possible by receiving more observational data on galaxy flows around close groups and clusters of galaxies.

On the theoretical side, deep relations between dark energy and key phenomena and processes of fundamental physics are gradually being clarified. Among other things, it is being discussed that the nature of dark energy might be governed by an interplay of gravitation and electroweak processes at a fundamental level. Thus, the concept of the central role of the electroweak energy scale emerges to prominence again (although in a different context), not only in field theory and particle physics but also in physical cosmology. As it turns out, a unique, if not crucial, role in the physics of the Universe is now played by the mysterious hierarchy phenomenon, which is equally important in the physics of the micro-world. It can hardly be considered an arithmetical accident that the dark energy density and other basic physical parameters of the world as a whole are expressed through the large hierarchical number in a quite simple way. It is also suspected that the hierarchy itself may be rooted in the extra spatial dimensions of a macroscopic size. It is widely believed that a new physics related to this set of ideas and corresponding to the electroweak scale energies may soon become subject to direct experiments at the Large Hadron Collider at CERN.

Astronomical observations using the most powerful space and ground-based telescopes, in combination with physical experiments at gigantic accelerators, are gradually uniting to form an extended front of research aimed at tackling the newest fundamental problem of natural science — revealing the physical nature and microscopic structure of dark energy.

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