

REVIEWS OF TOPICAL PROBLEMS

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Reflection and refraction at the boundary of a medium with negative group velocity

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Contents

Abstract. **Mandelshtam's views on the role of informal physical arguments in deriving the geometric law of refraction are discussed. It is shown that Sivukhin's theorem allows reconciling different approaches to justifying the choice of the refracted wave vector. Wave reflection and refraction are considered for a periodically heterogeneous half-space. It is shown that the analogy between the Snell – Descartes refraction law and the properties of a wave propagating in a chain of discrete interacting elements and in a periodically heterogeneous space is incorrect. The principal role of the homogeneity of interfacing media is stressed. The only case that corresponds to the 'purely' negative refraction is that of a homogeneous refracting medium in which both the dielectric constant and the magnetic permeability are negative.**

1. Introduction

It is common knowledge that the reflection and refraction laws cannot be derived mathematically from the boundary conditions for the electric field strength but require additional physical reasoning. The problem has a long history and has recently become particularly important because of a new issue, the electrodynamics of media with negative dielectric permittivity and magnetic permeability and media with a discrete structure or, in other words, with negative refraction [1]. For such media, the 'additional physical reasons' play a decisive and unexpected role. In 1944, in one of his lectures, L I Mandelshtam noted the possibility of a negative group velocity for electromagnetic waves and remarked on a close relation of the velocity sign to the direction of phase variation. He also derived the refraction law for a homogeneous medium with negative group velocity [2]. In 23 years, these considerations ceased to be purely 'academic and tutorial' and acquired intriguing physical and practical importance. In media with negative dielectric permittivity and magnetic permeability, new features have been predicted for the Doppler effect, Cherenkov radiation, and Fresnel formulas; specific optical devices have been proposed based on such media, and so on [1, 3]. Starting with pioneering work [4], several composite materials have been created with negative group velocity in the microwave and optical spectral ranges (see, e.g., [5]) and the unusual properties of those materials have been demonstrated.

In spite of the impressive success, we belive that some principal physical problems related to certain peculiarities of refraction on the boundary of a medium with a negative group velocity have not been fully explained. The present review is devoted to this problem.

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2. The Snell – Descartes – Fresnel problem

We consider the key point of the problem, which we concisely call the Snell – Descartes – Fresnel (SDF) problem. Figure 1 illustrates its essence. The plane $z = 0$ is an interface for two media 1 and 2 with the effective permittivities and permeabilities ε_1, μ_1 and ε_2, μ_2. The incident and reflected waves propagating in medium 1 has a frequency ω and the respective wave vectors $\mathbf{k}_1$ and $\mathbf{k}_1'$; in medium 2, the transmitted wave propagates with a wave vector $\mathbf{k}_2$:

$$\mathbf{E}\exp\left[-\mathrm{i}(\omega t-\mathbf{k}_1\mathbf{r})\right],\quad \mathbf{R}\exp\left[-\mathrm{i}(\omega t-\mathbf{k}_1'\mathbf{r})\right],$$

$$\mathbf{D}\exp\left[-\mathrm{i}(\omega t-\mathbf{k}_2\mathbf{r})\right]. \tag{1}$$

The frequencies ω and projections k_x of the wave vectors onto the interface plane are exactly equal for all three waves due to the linear and uniform character of boundary conditions [6]. The projections k_{1z}' and k_{2z}, which are perpendicular to the interface plane, can be found from

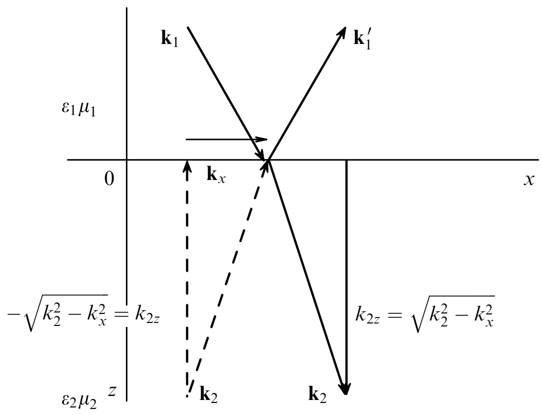

Figure 1.

the relations

$$k_x^2 + k_{1z}'^2 = k_1^2 = \varepsilon_1\mu_1\left(\frac{\omega}{c}\right)^2,$$
$$k_x^2 + k_{2z}^2 = k_2^2 = \varepsilon_2\mu_2\left(\frac{\omega}{c}\right)^2, \tag{2}$$

which express the condition that transverse plane waves (1) may be nonzero solutions of the Maxwell equations. For the refracted and reflected waves, it follows from Eqns (2) that

$$k_{1z}' = \pm\sqrt{k_1^2 - k_x^2}\,, \quad k_{2z} = \pm\sqrt{k_2^2 - k_x^2}\,. \tag{3}$$

From the mathematical standpoint, the two possible signs in Eqns (3) mean that these relations do not contradict equalities (2) at either of the signs. From the physical standpoint, different signs in relations (3) correspond to a phase increase either along the z axis $(+)$ or in the opposite direction $(-)$. For the reflected wave, the choice of sign is obvious: both the incident and reflected waves propagate in the same medium, and k_{1z} and k_{1z}' can only differ in sign; the phases of the incident and reflected waves should vary in opposite directions. In the incident wave, the component $k_{1z} > 0$ is fixed; hence, we have $k_{1z}' = -k_{1z} < 0$, that is, k_{1z}' is directed from the interface plane toward medium 1.

For the wave vector of the refracted wave, the problem is more complicated. In a somewhat more general setting, i.e., not restricting ourself to plane waves (1), we can assert the following. Due to the uniformity of media 1 and 2, the existence of a solution of the Maxwell equations or, in other words, a normal mode $f(t - z/v)$ propagating in a certain direction inevitably implies that a wave $f(t + z/v)$ with the opposite direction of propagation can exist in principle. The SDF problem is special in that it involves an incomplete set of possible normal waves. In accordance with the problem setting (we find a reflection from the interface only), only one of the two possible normal waves in (3) propagates in medium 2, and the question is which case is realized. If medium 2 is heterogeneous or is a finite-thickness layer, both normal waves with the opposite propagation directions are involved in the consideration and no physical complications occur. Indeed, a solution of the Maxwell equations involves a linear combination of all normal waves, and the boundary conditions uniquely fix their amplitudes, and hence the problem is solved mathematically without invoking 'additional physical reasoning.' The need in such reasoning arises from the SDF problem setting itself.

It is pertinent to note that Mandelshtam repeatedly stressed the principal difference between reflection from an interface of two media and from a finite-width layer, that is, from two boundaries ([7], years 1932 and 1940). To make the different solutions of these two different problems look like a 'tutorial paradox,' one of the problems was implicitly substituted for the other in [7].

There are three ways to argue how the sign of k_{2z} is chosen in (3). Somehow or other, they are related to energy considerations.

I. The energy in a refracted wave must move away from the interface toward medium 2, the energy propagates at a group velocity u, and the wave vector is directed along the group velocity (if $u > 0$), that is, from the interface toward medium 2. Hence, we have $k_{2z} > 0$. If $u < 0$, then $k_{2z} < 0$ [2, 6].

II. Let medium 2 have a weak absorption. For the field energy to pass to the energy of the medium, the Poynting vector must be directed toward the domain in which the absorption occurs, that is, from the interface toward medium 2. The Poynting vector and the wave vector are parallel, and hence $k_{2z} > 0$, i.e., the wave vector projection is directed from the interface toward medium 2.

III. Let the radiation source residing in medium 1 emit a pulse whose shape is an arbitrarily long step function. It is known that the leading edge of the pulse propagates without refraction and reflection [6]. As the emitted step-like pulse excites a polarization of the media, a virtually stationary regime is established at a finite distance from the source. The analysis of this regime leads to the conclusion [8] that the wave vector of the refracted wave is directed from the radiation source and from the interface plane toward medium 2. This argument is the most consistent, but also the most complicated. It involves the group velocity because the process is not stationary and the radiation is not completely monochromatic.

We emphasize that in arguments I and III, a strict correlation is implied between the directions of the wave vector and of the group velocity, whereas in argument II, we use the correlation between the wave vector and the Poynting vector. Sivukhin has shown that both approaches are physically and formally identical. In 1957, he derived a fundamental relation between the (time) average of the energy density $\bar{w}$ of an electromagnetic field and the average value of the Poynting vector $\bar{\mathbf{S}}$, the wave vector $\mathbf{k}$, and the group velocity u [9]. This relation is rather important for our purposes and we recall the basic points of the argument in [9]. The average energy density $\bar{w}$ in a medium is considered the result of energy storage when a radiation flux passes through the medium. We assume that no energy dissipation occurs. The initial relation is

$$\bar{w} = \frac{1}{16\pi}\left(\frac{\mathrm{d}\omega\,\varepsilon(\omega)}{\mathrm{d}\omega}\,|\mathbf{E}|^2 + \frac{\mathrm{d}\omega\,\mu(\omega)}{\mathrm{d}\omega}\,|\mathbf{H}|^2\right), \tag{4}$$

where $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic field strengths. For plane waves, the equality

$$\varepsilon|\mathbf{E}|^2 = \mu|\mathbf{H}|^2 \tag{5}$$

holds. In view of (5) and of the Poynting vector expression for a plane wave

$$\bar{\mathbf{S}} = \frac{c}{8\pi}\,\frac{\sqrt{\varepsilon\mu}}{\mu}\,|\mathbf{E}|^2\,\frac{\mathbf{k}}{k} = \frac{2}{8\pi}\,\frac{\sqrt{\varepsilon\mu}}{\varepsilon}\,|\mathbf{H}|^2\,\frac{\mathbf{k}}{k}\,, \quad \varepsilon\mu > 0\,, \tag{6}$$

Eqn (4) becomes [9]

$$\bar{w} = \frac{1}{16\pi\mu\omega}\left(\mu\omega\,\frac{\mathrm{d}\omega\,\varepsilon(\omega)}{\mathrm{d}\omega} + \varepsilon\omega\,\frac{\mathrm{d}\omega\,\mu(\omega)}{\mathrm{d}\omega}\right)|\mathbf{E}|^2$$
$$= \frac{c^2}{16\pi\mu\omega}\,\frac{\mathrm{d}(k^2)}{\mathrm{d}\omega}\,|\mathbf{E}|^2 = \frac{c^2}{16\pi\varepsilon\omega}\,\frac{\mathrm{d}(k^2)}{\mathrm{d}\omega}\,|\mathbf{H}|^2 = \frac{\bar{\mathbf{S}}\mathbf{k}}{uk}\,,\ \ \frac{1}{u} = \frac{\mathrm{d}k}{\mathrm{d}\omega}\,, \tag{7}$$

which is the statement of Sivukhin's theorem. The factor $1/u$ in (7) is the time lapse during which the energy density $\bar{w}$ is accumulated along the unit length due to the flux density $\bar{\mathbf{S}}$.

Rayleigh, Mandelshtam, and many other researchers discussed the equality $u = \bar{S}/\bar{w}$ instead of (7), that is, considered the absolute value of the group velocity to be that of the energy transfer velocity. It seems that such a statement of the problem was related to the implicitly assumed parallelism of $\bar{\mathbf{S}}$ and $\mathbf{k}$.

The spatial energy density $\bar{w}$ is positive, and hence formula (7) entails a strong correlation, $\bar{\mathbf{S}}\mathbf{k}/ku > 0$ between the directions of $\bar{\mathbf{S}}$ and $\mathbf{k}$ and the sign of the group velocity u:

1) if the group velocity is positive, then the Poynting vector and the wave vector are unidirectional;

2) if the group velocity is negative, then the Poynting vector and the wave vector are counter-directed;

3) if the Poynting vector and the wave vector are unidirectional, then the group velocity is positive;

4) if the Poynting vector and wave vector are counter-directed, then the group velocity is negative.

Thus, according to Sivukhin's theorem (7), arguments I–III necessarily give identical results. Conclusions 1–4, albeit formulated in other terms, are made in [1, 3] in discussing optical phenomena in media with negative dielectric permittivity and magnetic permeability.

Relation (7) is obtained for a uniform plane monochromatic wave in a homogeneous isotropic nonabsorbing (nonamplifying) medium. Under these conditions, the sign of the group velocity is determined only by the dispersion law for the medium. In heterogeneous media and media with discrete elements, the sign of the group velocity can be and actually is determined by other factors.

We consider the condition that leads to a negative group velocity in a homogeneous isotropic nonabsorbing medium. The relation

$$\frac{c}{u} = \frac{c\,\mathrm{d}k}{\mathrm{d}\omega} = n + \omega\,\frac{\mathrm{d}n}{\mathrm{d}\omega} = -f(\omega)\,,\quad f(\omega) > 0\,,$$

can be considered a differential equation for the refractive index $n(\omega)$, whence

$$\omega n(\omega) = g - \int_{\omega_0}^{\omega} f(\omega_1)\,\mathrm{d}\omega_1\,,\quad n(\omega) = \sqrt{\varepsilon(\omega)\,\mu(\omega)}\,, \tag{8}$$

where $f(\omega)$ is an arbitrary nonnegative function. According to (8), the refractive index $n(\omega)$ and the product $\omega n(\omega)$, which is proportional to the wavenumber, must be decreasing functions of the frequency. Not so evident might be the explicit form of the condition, a constant g minus an arbitrary increasing positive function of the frequency.

To complete the picture, we consider direct implications of expression (6) for the Poynting vector of a monochromatic plane wave in a homogeneous medium:

a) if ε and μ are positive, then the Poynting vector and the wave vector have the same directions;

b) if the Poynting vector and the wave vector have the same directions, then both the dielectric permittivity and the magnetic permeability are positive;

c) if ε and μ are negative, then the Poynting vector and the wave vector have opposite directions;

d) if the Poynting vector and the wave vector have opposite directions, then both the dielectric permittivity and the magnetic permeability are negative;

e) in a homogeneous isotropic medium without field absorption and amplification, the only reason for opposite directions of $\mathbf{k}$ and $\bar{\mathbf{S}}$ for uniform waves can be negative values of ε and μ [1, 3].

The discussion of formulas (6) and (7) clearly illustrates the importance of factors such as the continuity and homogeneity or heterogeneity of the medium and uniformity or nonuniformity of the wave, i.e., those factors that seemed universal and not crucial, but which turn out to play a very special role in the subtle problem concerning the refraction laws and group velocity. This is well illustrated by conclusion (e), which is very strong although seemingly absurd at first glance.

Mandelshtam noted that the group velocity of electromagnetic waves may be negative [2]. In this case, the energy should still propagate from the interface plane toward medium 2, and the phase should propagate in the opposite direction, that is, the wave vector $\mathbf{k}_2$ of the refracted wave should be directed from medium 2 to the interface, as is shown in Fig. 1 by the dashed vector ($k_{2z} < 0$) [2].

To illustrate the specific features of refraction in the case of media with negative group velocity, Mandelshtam considers an example of waves in the model widely used in solid state physics, a one-dimensional chain of interacting particles [2]. In such a chain, waves with both positive and negative group velocity exist. The waves running along the chain may be of different natures: mechanical, electromagnetic, or polarization waves. Numerous systems of this kind are known presently, in particular, photon crystals, periodically heterogeneous media, and fibers with periodic heterogeneity along their axis; systems of the last type are widely used in practice in fiber-optics communication systems.

The role of the sign of group velocity noted in [2] and the conclusions that follow from Eqns (6) and (7) are very important because there may be various reasons determining a correlation between the propagation directions for the phase ($\mathbf{k}$) and the flux ($\bar{\mathbf{S}}$): the discrete character of the medium, medium heterogeneity, medium amplification, or wave heterogeneity. For example, the transfer from the ordinary to the inverse population may change the sign of the derivative $\mathrm{d}n/\mathrm{d}\omega$. One more example is given by surface waves [10], in which the field amplitudes in two adjacent media decrease with the distance from the interface. The dielectric permittivity is positive in one of the media and negative in the other; the vectors $\mathbf{k}$ and $\mathbf{S}$ in the media are, correspondingly, parallel ($\varepsilon > 0$) or antiparallel ($\varepsilon < 0$).

Unfortunately, some arguments and illustrations in [2] are not fully satisfactory. It is this reason that motivated the present review. First, the Fresnel reflection is considered for monochromatic plane waves and for an interface of two homogeneous media. However, according to consequence (e) following from Eqn (6), the directions of $\mathbf{k}$ and $\bar{\mathbf{S}}$ may be different only because of negative values of ε and μ. Hence, any other direct reasons for negative group velocity are irrelevant and may only serve as analogies, distant or close. Second, in periodically heterogeneous media, such as chains of particles, normal waves are created as pairs of contra-directional running waves. Hence, the illustration in [2] with a periodically heterogeneous chain of particles is not satisfactory because waves in such a medium run in both directions and there is no choice problem. In addition, the group velocity is a good characteristic not of a normal wave but of a running wave in a chain. Finally, the problem setting of the reflection from periodically inhomogeneous half-space, which corresponds to the SDF problem, results, as we show in Section 3, in absurd consequences and is therefore incorrect. Hence, we may assert that the only substantial point in [2] is the abstract indication of a possible existence of media with negative group velocity. The example of a chain with interacting elements is not physically justified because it illustrates a relatively simple physical phenomenon via a nonexisting one, a single running wave in a periodically heterogeneous medium.

We note an important methodological point. In discussing relations (3), informal energy reasoning is first used, which then determines the mathematical and geometrical structure of the wave. In the case of a heterogeneous medium, the geometry is determined by specific features of the medium, i.e., the functions $\varepsilon(\omega, \mathbf{r})$, $\mu(\omega, \mathbf{r})$, whereas the energy properties of the field are calculated using the formulas for solving the wave equation and are therefore a consequence of medium-specific features.

3. Reflection from a periodically inhomogeneous half-space

We detail the general considerations in Section 2 for the example of a medium in the form of periodically heterogeneous half-space. We use the simplest model of weak one-dimensional harmonic heterogeneity. In essence, this model dates back to Einstein's work [11], in which fluctuations of the dielectric permittivity are presented in the form of a spatial Fourier integral for the intensity of scattered light [11, 12]. Each harmonic of this decomposition determines a harmonic heterogeneity, and hence formally coincides with our model. Mandelshtam and Brillouin later used this model, with the motion of fluctuations taken into account, for predicting the Rayleigh line splitting [12]. The classical works mentioned above differ principally from the SDF problem in that they considered a finite-size medium, whereas we are dealing with an infinite half-space.

We consider the electric field strength $E_{\perp}(x,z)$ perpendicular to the plane of incidence xz and parallel to the interface plane $z=0$. This field strength is described by the relations

$$\Delta E_{\perp}(x,z) + k^2(z)\,E_{\perp}(x,z) = 0\,, \tag{9}$$

$$k(z) = k_0 = \left(\frac{\omega}{c}\right) n_0\,, \quad z<0\,, \tag{10}$$

$$k(z) = k_0\Big\{1+\eta\cos\big[K(z-z_0)\big]\Big\}\,, \; K=\frac{2\pi}{\Lambda}\,, \; z>0\,, \tag{11}$$

where Λ is the heterogeneity period. To avoid the conventional Fresnel reflection from the interface plane, we assume the average values of the refractive index in medium 2 ($z>0$) and medium 1 ($z<0$) to be equal. We also take the resonance approximation and the approximation of small modulation depth:

$$\left|k_0 - \frac{K}{2}\right| \ll k_0\,, \quad \eta \ll 1\,. \tag{12}$$

Omitting simple calculations, which are similar to those performed in [13, 14], we give the final expressions. For simplicity, we omit the symbol $\perp$. In medium 1, the normal waves are, as usual, given by the two functions

$$\exp\big[\mathrm{i}(k_x x \pm k_z z)\big]\,,$$

with the factors

$$k_z = \sqrt{k_0^2 - k_x^2}\,.$$

It can be shown [13, 14] that in periodically heterogeneous medium (11), the normal waves can in general be constructed from the four running waves with the coordinate factors

$$\exp\left[\mathrm{i}k_x x + \mathrm{i}\left(\pm q \pm \frac{K}{2}\right) z\right], \tag{13}$$

$$q=\sqrt{\varkappa^2-\beta^2}\,, \quad \varkappa = k_z - \frac{K}{2}\,, \quad \beta=\frac{\eta K}{4}\,.$$

In a periodically heterogeneous layer of a finite thickness, the solution involves all four exponentials (13). If medium 2 is an infinite half-space, then the increasing (at $\varkappa^2<\beta^2$, $z>0$) functions should be dropped and the decreasing functions

$$\exp\left[\mathrm{i}k_x x + \mathrm{i}\left(q \pm \frac{K}{2}\right) z\right] \tag{14}$$

should be kept, where the factors are

$$q = \mathrm{i}\sqrt{\beta^2-\varkappa^2}\,, \quad \varkappa^2<\beta^2\,, \quad z>0\,.$$

Therefore, the solutions of wave equation (9) in media 1 and 2 take the form

$$E(x,z) = \exp(\mathrm{i}k_x x)\big[E\exp(\mathrm{i}k_z z) + R\exp(-\mathrm{i}k_z z)\big]\,, \quad z<0\,, \tag{15}$$

$$E(x,z) = \exp(\mathrm{i}k_x x + \mathrm{i}qz)\left\{\exp\left[\frac{\mathrm{i}K(z-z_0)}{2}\right]B + \exp\left[-\frac{\mathrm{i}K(z-z_0)}{2}\right]C\right\}, \; z>0\,, \tag{16}$$

$$C = \frac{q-\varkappa}{\beta}\,B = -\frac{\beta}{q+\varkappa}\,B\,. \tag{17}$$

The wave amplitudes R and B are found from the system of equations consisting of Eqn (17) and the continuity conditions for field strengths at the interface $z=0$:

$$B_1 + C_1 = E + R\,,$$
$$\left(\frac{K}{2}+q\right)B_1 - \left(\frac{K}{2}-q\right)C_1 = k_z E - k_z R\,, \tag{18}$$

$$B_1 = B\exp(-\mathrm{i}\varphi)\,, \quad C_1 = C\exp(\mathrm{i}\varphi)\,, \quad \varphi=\frac{Kz_0}{2}\,. \tag{19}$$

Eliminating either R or B_1 from Eqns (17) and (18), we arrive at the relations

$$B = \frac{2k_z}{2k_z+q-\varkappa}\,E\exp(\mathrm{i}\varphi) - \frac{q+\varkappa}{2k_z+q-\varkappa}\,C\exp(2\mathrm{i}\varphi)\,, \tag{20}$$

$$R = \frac{q - \varkappa}{2k_z + q - \varkappa}\, E + \frac{K}{2k_z + q - \varkappa}\, C \exp(\mathrm{i}\varphi)\,.$$

Relations (20) are interpreted in the standard way [6]: the incident wave (E) transmitted through the interface and the wave with an amplitude C reflected from it form a wave with the amplitude B. The reflected wave R is formed as a result of the reflection of the incident wave E and the transmission of the C wave. In view of conditions (12), that is, the inequalities $K,\ k_z \gg |q|,\ |\varkappa|,\ \beta$, we conclude that the transmission coefficients are close to unity and the reflection coefficients are small. Then we find

$$B = \frac{2k_z}{2k_z + q - \varkappa - \beta \exp(2\mathrm{i}\varphi)}\, E\,, \quad C = \frac{q - \varkappa}{\beta}\, B\,, \qquad (21)$$

$$R = \frac{q - \varkappa}{\beta}\, \frac{2k_z - q - \varkappa - \beta \exp(-2\mathrm{i}\varphi)}{2k_z + q - \varkappa - \beta \exp(2\mathrm{i}\varphi)}\, E \exp(2\mathrm{i}\varphi)\,. \quad (22)$$

Thus, in a periodically heterogeneous half-space, the field is described by two plane waves (16) with the amplitudes B and C and the wave vectors

$$\begin{aligned} \mathbf{k}_+ &= k_x \mathbf{e}_x + \left(\frac{K}{2} + q\right)\mathbf{e}_z \\ &= k_x \mathbf{e}_x + \left(\frac{K}{2} + \sqrt{\left(k_z - \frac{K}{2}\right)^2 - \beta^2}\right)\mathbf{e}_z\,, \\ k_z &= \sqrt{k_0^2 - k_x^2}, \end{aligned} \qquad (23)$$

$$\begin{aligned} \mathbf{k}_- &= k_x \mathbf{e}_x - \left(\frac{K}{2} - q\right)\mathbf{e}_z \\ &= k_x \mathbf{e}_x - \left(\frac{K}{2} - \sqrt{\left(k_z - \frac{K}{2}\right)^2 - \beta^2}\right)\mathbf{e}_z\,. \end{aligned} \qquad (24)$$

The parameter β^2 in Eqns (13) and (14) is related to the spatial Fresnel reflection from medium heterogeneities. The combination $\pi\eta/2$, in accordance with the calculation in [14], is equal to the amplitude reflection coefficient from the heterogeneity period. The spatial reflection plays the role of a factor damping wave oscillations similarly to friction in pendulum vibrations. At normal incidence ($k_x = 0$),

$$\begin{aligned} \mathbf{k}_+ &= \left(\frac{K}{2} + \sqrt{\varkappa^2 - \beta^2}\right)\mathbf{e}_z\,, \\ \mathbf{k}_- &= \left(-\frac{K}{2} + \sqrt{\varkappa^2 - \beta^2}\right)\mathbf{e}_z\,, \quad \varkappa = k_0 - \frac{K}{2}\,. \end{aligned} \qquad (25)$$

The plots of $k_\pm(\omega) \mp K/2$ as functions of $\varkappa$ are presented in Fig. 2. In the frequency interval

$$\left(1 - \frac{\eta}{2}\right)\frac{Kc}{2n_0} < \omega < \left(1 + \frac{\eta}{2}\right)\frac{Kc}{2n_0}\,, \quad -\beta \leqslant k_0 - \frac{K}{2} \leqslant \beta\,, \qquad (26)$$

the 'friction' is so high that the radical in Eqns (23)–(25) becomes imaginary and solutions (16) take the form of heterogeneous waves that exponentially decay with increasing z (the circle in Fig. 2). For a tilted propagation ($k_x \neq 0$), we obtain the inequalities

$$-\beta \leqslant \sqrt{k_0^2 - k_x^2} - \frac{K}{2} \leqslant \beta\,, \quad k_0^2 \leqslant k_x^2 + \left(\frac{K}{2} \pm \beta\right)^2 \qquad (27)$$

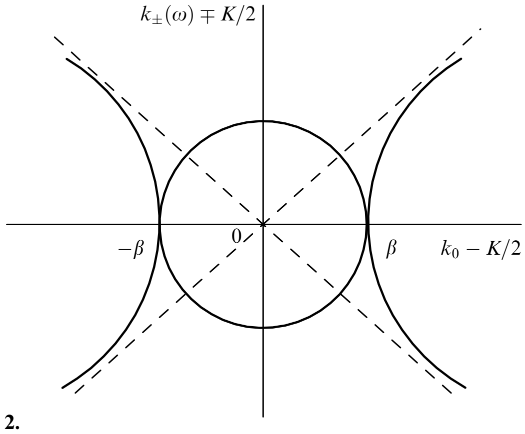

Figure 2.

instead of (26). In this case, the domain of heterogeneous waves is shifted compared to interval (26) to higher frequencies, and its width Δk_0 is less than that at normal incidence:

$$\begin{aligned} k_0 &= \frac{\sqrt{(K/2 - \beta)^2 + k_x^2} + \sqrt{(K/2 + \beta)^2 + k_x^2}}{2} \\ &\approx \sqrt{\left(\frac{K}{2}\right)^2 + k_x^2} > \frac{K}{2}\,, \\ \Delta k_0 &= \sqrt{\left(\frac{K}{2} + \beta\right)^2 + k_x^2} - \sqrt{\left(\frac{K}{2} - \beta\right)^2 + k_x^2} \\ &\approx \frac{2\beta K}{\sqrt{K^2 + 4k_x^2}} < 2\beta\,. \end{aligned}$$

Interval (26) is often called the forbidden gap, by analogy with the range considered in solid state physics for electrons in a periodic potential. The term 'band of heterogeneous waves' (BHW) seems preferable because heterogeneous waves from interval (27) play an important role in numerous optical problems and are not forbidden, hypothetically or actually. The parameters of the potentials in solid bodies are such that electron waves are quickly damped. It is not so in optics. In particular, damped waves essentially determine the spectral properties of filters based on periodically heterogeneous media [13]; such waves are present in solution (16).

We can write the expressions for the phase and group velocities of the running components of field (16), although they are not very interesting in general. The amplitudes of the wave vectors $\mathbf{k}_+$ and $\mathbf{k}_-$ are given by the formulas

$$k_\pm(\omega) = \sqrt{k_x^2 + \left(\pm\frac{K}{2} + q\right)^2}\,, \quad q = \sqrt{\left(k_z - \frac{K}{2}\right)^2 - \beta^2}. \qquad (28)$$

In a counter-propagating wave, k_- is slightly smaller than k_+ for the wave running in the positive direction. The phase velocities for the 'running' terms in (16) are expressed as

$$v_\pm = \frac{\omega}{k_\pm(\omega)} = \frac{\omega}{\sqrt{k_x^2 + (K/2 \pm q)^2}}\,. \qquad (29)$$

In approximation (12), the phase velocity has almost the same value as in a homogeneous medium. It is slightly greater for the counter-propagating wave. According to (17), we have $q = 0$, $|C/B| = 1$, that is, a purely standing wave at the BHW boundary. At a distance from the BHW boundary, $q \neq 0$ and $|C/B| \neq 1$, i.e., the field in medium 2 acquires a 'running' property.

The group velocities in approximation (12) are equal to

$$u_{\pm}(\omega) = \pm \frac{u_0 q}{\varkappa} = \pm u_0 \frac{\sqrt{(k_z - K/2)^2 - \beta^2}}{k_z - K/2}\,, \quad \frac{1}{u_0} = \frac{\mathrm{d}k_0}{\mathrm{d}\omega}\,. \tag{30}$$

The signs of waves with the amplitudes C (lower sign) and B are different. At the BHW boundary, the group velocity vanishes (see Fig. 3). At positive u_0, the branches $k_{\pm}(\omega)$ decreasing with frequency correspond to the negative group velocity (see Fig. 3).

We consider the amplitude properties of the field. According to equality (17), the oppositely directed waves with amplitudes B and C in field (16) cannot exist separately from each other. The zero amplitude of one of them inevitably implies the absence of the other, and the presence of one of the waves implies that the other is also present. Hence, as we stressed above, speaking of phase and group velocities for each of the running waves is not very substantive from the physical standpoint, to say the least. Formulas (29) and (30) follow tradition rather than being a necessity for analyzing the problem under consideration.

The expressions for the Poynting vectors $\bar{\mathbf{S}}_1$ and $\bar{\mathbf{S}}_2$ in media 1 and 2 are instructive:

$$\bar{\mathbf{S}}_1 = \frac{c^2}{8\pi\omega\mu}\,|\mathbf{E}|^2 \left\{ \mathbf{e}_z k_z \left[1 - \frac{|q-\varkappa|^2}{\beta^2} \right] + \mathbf{e}_x k_x \left[1 + \frac{|q-\varkappa|^2}{\beta^2} + 2\,\frac{q-\varkappa}{\beta}\cos 2k_z z \right] \right\}, \tag{31}$$

$$\bar{\mathbf{S}}_2 = \frac{c^2}{8\pi\omega\mu}\,|\mathbf{E}|^2 \left\{ \mathbf{e}_z\,\frac{K}{2} \left[1 - \frac{|q-\varkappa|^2}{\beta^2} \right] + (\mathbf{e}_x k_x + \mathbf{e}_z q) \left[1 + \frac{|q-\varkappa|^2}{\beta^2} + 2\,\frac{q-\varkappa}{\beta}\cos K(z - z_0) \right] \right\}. \tag{32}$$

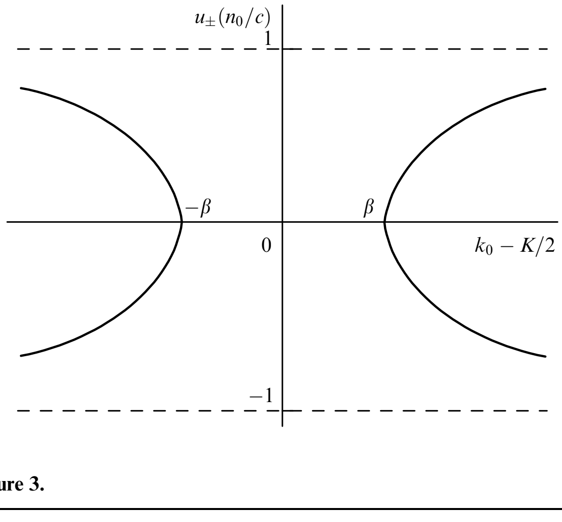

Figure 3.

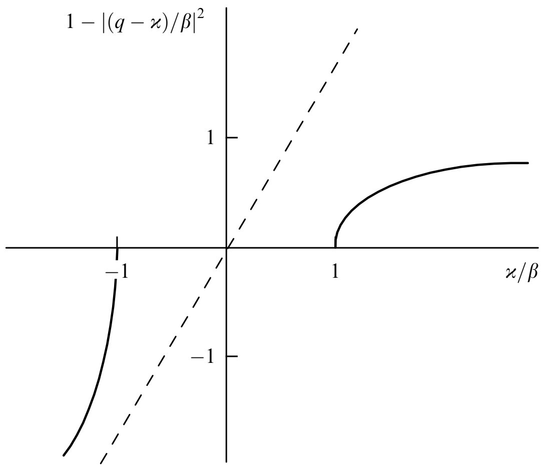

Figure 4.

The components $\bar{S}_{1z}$ and $\bar{S}_{2z}$ involve the characteristic quantity $1 - |(q-\varkappa)/\beta|^2$. Because $K \gg q$, just this quantity is dominating in $\bar{S}_{2z}$. It can by easily shown that

$$1 - \frac{|q-\varkappa|^2}{\beta^2} = 2\sqrt{\varkappa^2 - \beta^2}\;\frac{\varkappa - \sqrt{\varkappa^2 - \beta^2}}{\beta^2}\,, \quad \varkappa^2 > \beta^2\,. \tag{33}$$

According to (33) (see Fig. 4, where the dashed line is the asymptote for $\varkappa < 0$), we have

$$1 - \frac{|q-\varkappa|^2}{\beta^2} > 0\,, \quad \varkappa > \beta\,;$$
$$1 - \frac{|q-\varkappa|^2}{\beta^2} < 0\,, \quad \varkappa < -\beta\,. \tag{34}$$

Hence, depending on the sign of $\varkappa = k_z - K/2$, the total flux propagates either from medium 1 to medium 2 ($\varkappa > 0$) or in the opposite direction ($\varkappa < 0$). Within the BHW, the total flux equals zero because the waves propagating in opposite directions have equal intensities in both media.

There is no need for a more detailed analysis of formulas (33) and (34) because the calculation result given above contradicts the energy conservation law: according to Eqn (22), the inequality $|q-\varkappa|^2/\beta^2 > 1$ implies that the reflected wave is more intensive than the incident one.

Thus, from the methodological standpoint, the properties of wave amplitudes in the reflection from a periodically heterogeneous half-space do not allow using the arguments similar to those presented in Section 2 in arguments I – III concerning the reflection from a homogeneous medium. Additional physical arguments of type I – III are inapplicable in the case of reflection from a heterogeneous medium. In relation to physics, the absurdity of conclusions (31) – (34) seems to be related to the incorrect setup of the problem of reflection from an infinite periodically heterogeneous medium. In the transition process of forming a wave reflected from such a medium, this wave acquires energy similarly to the case of total reflection in an optically less dense medium, which makes the situation for an infinite medium uncertain. The problem can be posed correctly only for a periodically heterogeneous layer of a finite thickness, where, in addition to waves of type (14), the complete set of exponentials (13) exists [13].

4. Conclusion

We summarize our observations. It is not correct to use waves arising in periodically heterogeneous media for a physical illustrative interpretation of waves in the SDF problem, as was done in [2]. First, in a heterogeneous medium, there are no running waves because the waves are standing and the group velocity is not a 'good' characteristic for them. Second, the choice among the normal waves is not a problem in a heterogeneous medium, although it is a key point in the SDF problem. Both the solid and dashed arrows in Fig. 1 correspond to real waves (16). Finally, the Fresnel reflection is considered for two half-spaces, whereas periodically heterogeneous media related to such a problem can only be correctly considered for a layer of finite thickness.

An important positive element in [2] was the indication that a negative group velocity may exist in principle, with the corresponding consequences for the geometrical law of refraction. Nevertheless, up to now, the hypothetic medium with negative dielectric permittivity ε and magnetic permeability μ is the only example of a homogeneous medium with negative refraction. Moreover, according to Eqn (6), no other example of plane waves in uniform media can exist. In 1957, Sivukhin wrote: "Media with $\varepsilon < 0$ and $\mu < 0$ are not known. The question whether such media can exist in principle is not clear" [9]. In the more than 50 years that have passed, the situation has not changed and, I am sure, will never change. No continuous homogeneous media with $\varepsilon < 0$ and $\mu < 0$ can exist in the optical spectrum range.

The above consideration does not mean that media with negative group velocities for monochromatic plane waves merit no attention. Quite the opposite, as we mentioned in the Introduction, a rapid progress is currently observed in the electrodynamics of periodically heterogeneous media, media with discrete elements, and media with periodically disposed elements. In such media, there are spectral intervals wherein the group velocity takes a negative value. In the optical spectrum range, these are media of a new type with unusual properties, which are quite promising from the standpoint of applications in nanotechnology. In addition to the relatively simple case of negative refraction, there may be other new linear [1] and nonlinear (see, e.g., [15]) optical effects. However, the propagation of waves in such finite-size media is a difficult independent problem, which we do not consider here.

Mandelshtam's interpretation of a possible negative refraction in the framework of the analogy with the properties of a medium comprising discrete elements had a negative effect. The fact is that all experimental and theoretical works aimed at searching for negative refraction were performed earlier and are performed now with samples that are analogous to a chain involving interacting particles, that is, comprising periodically disposed discrete elements. The negative group velocity in such samples is determined not by the negative dielectric permittivity and magnetic permeability but by the periodic heterogeneity of the medium. It would seem reasonable to develop a theoretical approach and a system of physical concepts for such media of the new type. But most of the approaches available in the literature only reduce systems with discrete elements to certain effective continuous media. For example, we note papers [16, 17], which contain interesting results, but where this tendency is well pronounced.[1] It is clear, however, that dealing with an effective refractive index is fraught with losing new, interesting, and unusual optical properties of the discrete systems.

To conclude, we join the discussion of the terminology pertaining to the problem under consideration [18]. The term 'forbidden zone' has already been discussed. At the beginning of the last century, the tradition was established to associate the fields of optics or electrodynamics with the properties of matter equations: the optics of homogeneous media, the optics of isotropic media, crystal optics, metal optics, the optics of gyrotropic media, the optics of heterogeneous media, the optics of randomly heterogeneous media, the optics of fibered media, etc. In the relevant literature, however contrary to the tradition mentioned, the term 'negative refraction medium' is used. In other words, the term used pertains to a particular, concrete phenomenon (refraction) rather than the properties of the matter equations (negative dielectric permittivity and magnetic permeability), which can and do embrace numerous phenomena considered in [1] and in the short, albeit substantial, review [3]. All this also applies to the term 'left-hand materials,' which is based on a symmetry of the vector triplet **E**, **H**, **k**. But this symmetry characterizes a very particular type of field, a linearly polarized monochromatic plane wave in a medium with negative permittivity and permeability, rather than the medium itself. Of course, such an eccentric name attracts attention to the new field; however, there is little chance of it persisting 'for ages.'

It seems preferable to follow the terminology tradition and use the terms mentioned above: 'media with negative permittivity and permeability' and 'media with discrete elements.'

The terminology tradition mentioned above is not solitary, it is closely connected to the 'general principle of physical science,' as it was emotionally and precisely formulated by the great Fresnel in 1819 [19]: "Nature seems to be aimed at doing much by smaller means; this principle is steadily confirmed in the course of developing physical sciences... If Nature is aimed at creating maximum phenomena by minimum reasons then, undoubtedly, this great problem is solved by Nature with all entirety of its laws... No doubt, it is very difficult to reveal the fundamentals of such wonderful economy, that is, the simplest origins of phenomena considered from sufficiently wide point of view. But if this general philosophical principle of physical sciences does not help us to directly cognize the truth, nevertheless, it can control human mental efforts obviating the systems that reduce the phenomenon to excessively great number of various reasons...."

[1] Paper [16], similarly to some other papers by the same authors, starts with the phrase: "Refractive index... is a key parameter in the interaction of radiation with matter." In fact, spatially heterogeneous materials with discrete periodically disposed elements are discussed in [16]. The concept of the refractive index cannot be applied to such a medium as a whole, because it is not a property the medium. One can only speak about a local value of the refractive index at a certain point in the medium or about some effective refractive index in the framework of the analogy with a continuous medium. It is common knowledge that 'interaction of radiation with matter' is determined by the parameters of matter equations, that is, the dielectric permittivity and the magnetic permeability. In contrast, the refractive index describes the phase variation of a field of a rather special type, a running monochromatic plane wave, and nothing more than that.

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