Physics – Uspekhi **50** (9) 965 – 976 (2007)

CONFERENCES AND SYMPOSIA

PACS number: 01.10.Fv

Scientific session of the Physical Sciences Division of the Russian Academy of Sciences (25 April 2007)

A scientific session of the Physical Sciences Division of the Russian Academy of Sciences (RAS) was held on April 25, 2007 in the Conference Hall of the P N Lebedev Physical Institute, RAS. The following reports were presented at the session.

(1) **Novikov I D, Kardashev N S, Shatskii A A** (Astro-Cosmic Center, P N Lebedev Physical Institute, RAS, Moscow) "The multicomponent Universe and the astrophysics of wormholes";

(2) **Lukash V N, Mikheeva E V** (Astro-Cosmic Center, P N Lebedev Physical Institute, RAS, Moscow) "Dark matter: from initial conditions to structure formation in the Universe".

An bridged version of these reports is given below.

PACS numbers: **04.20.–q, 04.70.–s, 98.80.–k**
DOI: 10.1070/PU2007v050n09ABEH006381

The multicomponent Universe and the astrophysics of wormholes

I D Novikov, N S Kardashev, A A Shatskii

1. Introduction

Solutions to the pressing problems of astrophysics may prove to be paradoxical and strange from the viewpoint of current scientific dogmas.

Astrophysics (cosmology, in particular) has seen many unusual discoveries. Here is a short history of the wormhole problem in astrophysics.

The paper by Einstein and Rosen [1] published in 1935 may be considered the first serious work on wormholes. To describe a hypothetical object, they introduced a "mathematical representation of physical space by a space of two identical sheets connected by a *bridge*" [1], i.e., the researchers use the term 'bridge' to describe what we would today call a wormhole.

This was a brilliant idea, but the mathematical model proposed by the researchers was ill-posed.

The first modern work devoted to this problem was the paper by Wheeler [2] published in 1955. There for the first time a diagram of a wormhole was given. Two years later, Misner and Wheeler, in their well-known paper [3], introduced the term 'wormhole' to the physical community.

Our report is an attempt to prove that some astrophysical objects may be entrances to wormholes. These wormholes seem to be remnants from the inflation phase of the Universe's evolution. The chaotic inflation model is the base of modern cosmology and assumes that there is an infinite number of universes, in addition to ours, which appear in a scalar field in different regions and at different instants of time and form what is known as 'spacetime foam' [4 – 6]. Primary spacetime tunnels (wormholes) probably exist in the initial scalar field [7] and, possibly, have been retained since inflation [8, 9], connecting different regions of our and other universes (see Fig. 1), which opens the unique possibility of studying the multicomponent Universe and detecting new types of objects — entrances to tunnels.

However, analysis of wormhole models reveals that for wormholes to exist, matter with an unusual equation of state is needed [10 – 12]. The equation of state must be anisotropic, and $w_{\|} = p_{\|}/\varepsilon$ must be smaller than -1 (as with phantom matter). Here, $p_{\|}$ is the total pressure along the tunnel, and ε is the total energy density of all the components of matter in the tunnel. So far, the existence of such matter has only been an assumption [13]. In what follows, we use the term 'phantom energy' when dealing with an isotropic equation of state, $p/\varepsilon < -1$, and the term 'phantom matter' when we are dealing with an anisotropic equation of state. The units of measurement are selected such that $c = 1$ and $G = 1$.

In our report we consider models in which the main wormhole material with all the necessary properties is a strong magnetic field that penetrates everything, while phantom matter and phantom energy are needed only as a small addition, and models in which the main material is phantom energy with an equation of state close to the vacuum one ($p/\varepsilon = -1$) and the addition of magnetic field energy density. Some observed astronomical objects may turn out to be entrances to tunnels.

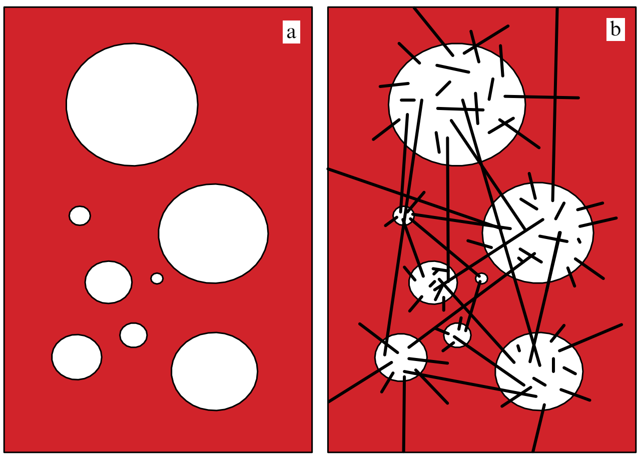

Figure 1. Models of a chaotic inflationary multicomponent Universe (a) without tunnels, and (b) with wormholes.

Uspekhi Fizicheskikh Nauk **177** (9) 1017 – 1028 (2007)
Translated by E Yankovsky; edited by A Radzig

We do not examine the stability of wormholes. Models of wormholes exist for which stability has been proved to exist (see, e.g., paper [14]).

Our hypothesis has been discussed in greater detail in Refs [15 – 17].

2. A model of a spherically symmetric magnetic wormhole

A static spherically symmetric wormhole is described by the same equations of state as a spherical relativistic star, and the major difference is, by definition, that such a wormhole has a spatial 'neck' creating a multiply connected topology and that there is no horizon.[1] According to Morris and Thorne [18], the metric can be written out as follows:

$$ds^2 = \exp\big(2\phi(r)\big)\,dt^2 - \frac{dr^2}{1 - b(r)/r} - r^2\,d\Omega^2\,, \tag{1}$$

where r is the radial coordinate, $\phi(r)$ is what is known as the redshift function, and $b(r)$ is the shape function. The wormhole neck corresponds to the minimum $r = r_0 = b(r_0)$ and $b'(r_0) \leqslant 1$. The fact that an object has a horizon is expressed by the condition $\phi \to -\infty$ or $\exp\phi \to 0$, so for a wormhole the function ϕ must be finite. For the spherically symmetric case, the diagonal terms of the energy – momentum tensor are given by [18]

$$\begin{aligned} &8\pi\varepsilon(r) = \frac{db}{dr}\frac{1}{r^2}\,, \qquad 8\pi p_\parallel(r) = -\frac{b}{r^3} + 2\left(1 - \frac{b}{r}\right)\frac{d\phi}{dr}\frac{1}{r}\,, \\ &8\pi p_\perp(r) = \left(1 - \frac{b}{r}\right)\left[\frac{d^2\phi}{dr^2} + \left(\frac{d\phi}{dr}\right)^2 + \frac{1}{r}\frac{d\phi}{dr}\right] \\ &\qquad - \frac{1}{2r^2}\left(r\frac{db}{dr} - b\right)\left(\frac{d\phi}{dr} + \frac{1}{r}\right), \end{aligned} \tag{2}$$

where $\varepsilon(r)$ is the energy density.

We introduce the wormhole mass $M(r)$ for an external observer as follows:

$$M(r) = M_0 + \int_{r_0}^{r} 4\pi\varepsilon r^2\,dr\,, \tag{3}$$

with $M_0 = r_0/2$.

To make numerical calculations more convenient, we introduce the variable $x = r_0/r$, with the result that the interval $r_0 \leqslant r < \infty$ is transformed into $0 < x \leqslant 1$, and instead of Eqns (2) we have

$$\begin{aligned} &8\pi\varepsilon r_0^2 = -\frac{b'x^4}{r_0}\,, \\ &8\pi p_\parallel r_0^2 = -\frac{bx^3}{r_0} - 2x^3\left(1 - \frac{bx}{r_0}\right)\phi'\,, \\ &8\pi p_\perp r_0^2 = \left(1 - \frac{bx}{r_0}\right)\left[x^4\phi'' + x^3\phi' + x^4(\phi')^2\right] \\ &\qquad + \frac{0.5x^3(xb' + b)(1 - x\phi')}{r_0}\,. \end{aligned} \tag{4}$$

Here, a prime indicates a derivative with respect to x. In Ref. [19], it was shown that in a spherically symmetric wormhole with $w_\parallel = \text{const}$ and $w_\perp = \text{const}$ the following inequalities determining the possible equations of state and their anisotropy must hold:

$$-2w_\perp < w_\parallel < -1\,. \tag{5}$$

The left inequality in Eqn (5) specifies the finiteness of the wormhole mass as $r \to \infty$, while the right inequality indicates the absence of a horizon.

It is highly interesting that for a magnetic (or electric) field the conditions specified by (5) are 'almost' met. If the direction of the field coincides with r, the energy – momentum tensor defines the equation of state:

$$w_\parallel = -1\,, \quad w_\perp = 1\,, \quad \varepsilon = \frac{H^2 + E^2}{8\pi}\,, \tag{6}$$

which to within a small negative addition to $w_\parallel$ satisfies conditions (5) for a wormhole.

In Ref. [20], a model of a wormhole consisting of phantom matter with an anisotropic equation of state, namely

$$1 + \delta = -\frac{p_\parallel}{\varepsilon} = \frac{p_\perp}{\varepsilon}\,, \tag{7}$$

was examined, and it was found that for a wormhole to exist it is sufficient that the parameter δ be as small as desired.

We introduce the following notation: $x_h = r_0/r_h > 1$, the ratio of the wormhole neck radius to the radius of the horizon of the respective Reissner – Nordström black hole [21] with a magnetic charge $Q = r_h$. The value of ε is determined by the relationships

$$\varepsilon = \varepsilon_0\,x^4\left(\frac{x_h - 1}{x_h - x}\right)^\delta, \qquad \varepsilon_0 = \frac{1}{8\pi r_0^2(1 + \delta)}\,. \tag{8}$$

The wormhole mass M_∞ for an observer located far from the neck falls within the interval

$$M_0 \leqslant M_\infty \leqslant 2M_0\,. \tag{9}$$

The left inequality in Eqn (9) follows from definition (3), and the right inequality follows from formulas (8).

In this regard, it is possible to conclude that an electromagnetic field may make up a substantial or even the major part of wormhole matter.

The parameters of the neck of a magnetic wormhole with different masses M_0 are listed in Table 1. From Eqn (8) we can derive expressions (with allowance for the constants c and G) for r_0, H_0, the mass density ρ_0, the frequency ν_G of radial vibrations with a minimum amplitude, the gyrofrequency ν_H in the neck in its rest frame, and the frequency ν_c (for an external observer) of revolution along the lower stable circular orbit:

$$\begin{aligned} &r_0 = \frac{G}{c^2}\,2M_0\,, \\ &H_0 = \frac{c^4}{G^{3/2}}\,(2M_0)^{-1}\,, \\ &\rho_0 = \frac{c^6}{8\pi G^3}\,(2M_0)^{-2}\,, \\ &\nu_G = \frac{c^3}{2\sqrt{2}\,\pi G}\,(2M_0)^{-1}\,, \\ &\nu_H = \frac{ec^3}{2\pi m_e G^{3/2}}\,(2M_0)^{-1}\,, \\ &\nu_c = \frac{\sqrt{3}\,c^3}{32\pi G}\,(2M_0)^{-1} = \sqrt{\frac{3}{128}}\,\nu_G\,. \end{aligned} \tag{10}$$

[1] Here and in what follows we mean an observer's horizon.

Table 1. Parameters of the neck of a magnetic wormhole with different masses.

$M_\infty = 2M_0$	r_0, cm	H_0, G	$\rho(r_0)$, g cm^{-3}	ν_G, Hz	ν_H, Hz	ν_c, Hz
6×10^{42} g $= 3\times10^9 M_\odot$ (quasar)	4.5×10^{14}	7.8×10^9	2.7×10^{-3}	7.6×10^{-6} (1.5 days)	2.2×10^{16}	1.16×10^{-6} (9.8 days)
10^{39} g $= 5\times10^5 M_\odot$ ($e^\pm$ pair production)	7.4×10^{10}	4.4×10^{13}	9.7×10^4	0.045 (22 s)	1.3×10^{20}	6.9×10^{-3} (2.4 min)
2×10^{33} g $= M_\odot$ (Sun)	1.5×10^5	2.3×10^{19}	2.4×10^{16}	2.3×10^4	6.6×10^{25}	3.5×10^3
6×10^{27} g $= M_\oplus$ (Earth)	0.45	7.8×10^{24}	2.7×10^{27}	7.6×10^9	2.2×10^{31}	1.16×10^9
5×10^{10} g (positronium)	3.5×10^{-18}	10^{42}	4.4×10^{61}	9.7×10^{26}	2.7×10^{48}	1.5×10^{26}
1.8×10^3 g ($\mu^\pm$ pair production)	$1.3\times10-25$	2.6×10^{49}	3×10^{76}	2.6×10^{34}	7.3×10^{55}	4×10^{33}
2×10^{-5} g (Planck mass)	1.5×10^{-33}	2.3×10^{57}	2.4×10^{92}	2.3×10^{42}	6.6×10^{63}	3.5×10^{41}

The wormhole parameters in Table 1 have been estimated for a quasar nucleus, objects with a critical field (and respective wormhole mass) needed for electron–positron pair production, objects whose mass is on order of the Sun's and Earth's masses, objects with a critical magnetic field (and respective wormhole mass) in which a positronium atom is stable, objects with a critical magnetic field (and mass) needed for monopole–antimonopole pair production, and objects with Planck mass.

Expression (1) for the metric was deduced with respect to Schwarzschild's coordinate system. The physical coordinate l measuring the distance along the tunnel encompasses both entrances ($-\infty < l < +\infty$) and is related to the radial coordinate r by the following formula

$$l(r) = \pm \int_{r_0}^{r} \frac{\mathrm{d}r}{\sqrt{1 - b_\pm(r)/r}} \,. \qquad (11)$$

For $r \gg r_0$, we have $l \to r$, and for a model described by formulas (8) with a small δ, the magnetic field strength equals

$$H \approx \frac{2M_0\sqrt{G}}{r^2} \,. \qquad (12)$$

If we assume that the electric field is weak, the restriction related to electron–positron pair production is lifted, but Table 1 lists only the limiting intensity of the field, $H_{\rm lim} = m_{\rm e}^2 c^3/(\mu\hbar) \approx 4.4\times10^{13}$ G, corresponding to this restriction. This field strength determines the specific conditions related to the fact that for $H > H_{\rm lim}$ the Landau excitation level exceeds the electron rest energy. In a field stronger than 10^{42} G, the positronium atom becomes stable and the medium gets filled by these atoms produced from vacuum [22, 23]. In magnetic fields stronger than the critical field $H_{\max}$, the vacuum breaks down and monopole pairs are produced [24–26]. If the mass m_μ of a stable colorless monopole of the 't Hooft–Polyakov type [27–29] is of order 10^{16} GeV $\sim 10^{-8}$ g, the magnetic charge $\mu = (3/2)\hbar c/e \sim 10^{-7}$, and $H_{\max} = m_\mu^2 c^3/(\mu\hbar) \approx 2.6\times10^{49}$ G, then the maximum mass of a magnetic wormhole with such a field in the neck is, according to formulas (10), $2M_0 \approx 1.8$ kg. The monopoles being produced will leave the wormhole, thus reducing its mass. In relation to other quantum processes, the stability of such small wormholes is not obvious, with the result that the minimum wormhole mass may prove to be much larger than 1.8 kg. The lower limit for the mass of composite wormholes is, obviously, even smaller. We also note that the absence of a horizon for wormholes means that they do not evaporate (the Hawking effect). Hence, primary wormholes with a small mass could have survived up to this day, in contrast to primordial black holes whose lower bound on mass is about 10^{15} g.

3. Entrances into tunnels and black holes with a magnetic field in the Galaxy and in galactic nuclei

The model considered is Section 2 presupposes that among galactic and extragalactic objects one can detect entrances into tunnels or black holes formed as a result of evolution from primordial wormholes. From the above discussion it also appears that these black holes may differ from primordial black holes in the magnitude and structure of the magnetic field.

The presence of a radial magnetic field can be detected from the specific law of the field intensity variation ($H \propto r^{-2}$) and the same sign of the magnetic field on all sides. The rotation of a monopole generates a dipole electric field which can accelerate the relativistic particles. What is important is that a dipole electric field (in contrast to the quadrupole field for a disk) accelerates electrons in the direction of one pole, while protons and positrons are accelerated in the direction of the other pole. This allows an explanation within the framework of the model in question of the origin of unidirectional jets from some sources (e.g., the quasar 3C273) [30].

The presence of an accretion disk complicates the picture. A quadrupole electric field generates bidirectional jets of electrons or protons/positrons (depending on the sign of the quadrupole). As a result, we may be faced with different jet structures. The interaction effects of the electromagnetic fields of a wormhole/black hole and the accretion disk are probably very strong.

The difference between the entrance to a wormhole and a black hole can be detected by the absence of a horizon: a luminous source falling into a wormhole will be observed continuously but with variable redshift or even blue shift. Here, however, we must assume that the tunnel is transparent.

A blue shift may emerge if the mass of the opposite entrance (in relation to the observer) to the wormhole is larger than the mass of the closest entrance. If the tunnel is transparent and there are accretion disks at both entrances, the redshifts for the spectra of these disks will also be different: two different redshifts of a single source related to the wormhole could be detected.

The observed image of a wormhole may exhibit an inner structure with angular dimensions much smaller than those determined by the gravitational diameter.

In this respect, the observed data on the gravitational lensing of the quasar Q0957+561 with a redshift $z = 1.4141$ [31] is very interesting, since it follows that the sizes of this quasar are much smaller than the Schwarzschild diameter. A specific feature of a wormhole that demonstrates strong relativistic effects and at the same time the absence of a horizon is the possibility of periodic vibrations of a test mass in relation to the neck (see Table 1 and Appendix 1). Here, the redshift of this mass also varies periodically. If the structure of the wormhole is close to horizon formation, the values of these shifts, their period, and the flux oscillations may be very large. In view of this one should mention the observations of quasiperiodic variations in the flux of such objects as IDV-sources (IDV stands for Intra Day Variability), say from the Lacerta object 0716+714 [32].

When sources move along circular orbits about the entrance to a tunnel (see Table 1 and Appendix 2), a compact source will also have a variable flux and redshift. Finally, an external observer can detect radiation at gyrofrequencies and phenomena related to $e^{\pm}$- and $\mu^{\pm}$-pair production.

4. Conclusions and prospects

We believe that the main conclusion that can be drawn from the above discussion is the possibility of detecting among known galactic and extragalactic objects usually identified as black holes with stellar masses and masses on the order of galactic nuclei new types of primary cosmological objects — entrances into wormholes or specific black holes that have formed from wormholes. The conclusion is also important here that there is a strong magnetic field with a radial structure (the Hedgehog model [33, 34]), which ensures, with allowance for rotation, the generation of jets of relativistic particles (unidirectional or bidirectional outbursts). For such objects, the wormhole models assume the presence of specific effects associated with the absence of a horizon and ensuring that the sources of radiation are visible at any point of the tunnel, provided that the medium is transparent. What they also assume is that the variations of the spectrum, flux, and polarization of the luminous source follow a definite law. New effects of gravitational lensing on wormholes are expected to be detected (see Refs [20, 35]). Also, it is possible that quasiperiodic vibrations of luminous objects in relation to the wormhole neck will be discovered.

It is possible to study the structure of candidates, i.e., sources related to the entrances to wormholes (or black holes) with a radial magnetic field, with instruments whose angular resolution amounts to several milliseconds of arc and even better; such a study is scheduled for conducting with the space interferometers in the RadioAstron [36] and Millimetron [37] projects.

It seems possible that the radiation sources will be discovered that are associated with double entrances into tunnels; these entrances form systems with strong magnetic dipole radiation and ejection of relativistic $e^{\pm}$. The final stage of evolution of such systems would conclude with creating the black hole and generating the high-power electromagnetic pulses.

It is also important to emphasize that the presence of wormholes with strong magnetic fields makes it possible to assume that the elementary magnetic monopoles predicted in Refs [33, 34] may have been absorbed by these objects in the process of cosmological evolution.

Another area of research is related to the spectral and polarization monitoring of the radiation from these sources.

The discovery of tunnels opens the path to possible studies of the entire multicomponent Universe.

Appendices

Appendix 1. The observation of a body vibrating with respect to the neck of a wormhole

A specific phenomenon may be the vibrations of bodies near the neck of a wormhole (radial orbits). Signals from such sources, reaching an external observer, exhibit a characteristic periodicity in their spectrum. All other objects except wormholes (stars and black holes) irrevocably absorb the radiation falling on them. Periodic radial vibrations constitute a specific feature of wormholes.

To simplify matters, let us take a test body with zero angular momentum. To solve the equations of motion we use the Hamilton – Jacobi method in curved space [38].

For the body's velocity we then have

$$\frac{\partial r}{\partial t} = \pm \exp\phi \sqrt{\left(1 - \frac{b}{r}\right)\left[1 - \exp(2\phi)\left(\frac{m_0}{E_0}\right)^2\right]}\,, \tag{13}$$

where m_0 and E_0 are the rest mass and total energy of the body, respectively. Since, for a wormhole, $\exp\phi > 0$ in the entire space, the equation $\dot{r} = 0$ has three roots: two of them correspond to the second factor in the square brackets of the integrand in Eqn (13) being zero (stopping of the body on two sides of the neck), and the third corresponds to the first factor in the integrand being zero (stopping at the neck).

Let us discuss the stopping at the neck more thoroughly. This is a unique phenomenon related solely to the curvature of space. First, we verify that this stopping takes a finite time. For this we examine the integral

$$\Delta t = \int_{r_0}^{r_1} \frac{\mathrm{d}r}{\dot{r}}\,. \tag{14}$$

The integral is not divergent if the r-derivative of $1 - b/r$ is nonzero in the neck. That this is the case follows from the expression for the wormhole metric. From the fact that integral (14) is finite follows the finiteness of the time it takes the body to stop in the neck (and, for similar reasons, the entire cycle of vibrations is also finite).

Expression (13) does not give the physical velocity of the body, since the coordinate r is not the physical radial coordinate — it is l that represents such a coordinate in Eqn (11). According to this, we find the physical velocity $\dot{l}$ of the body along the radius:

$$\dot{l} = \pm \exp\phi \sqrt{1 - \exp(2\phi)\left(\frac{m_0}{E_0}\right)^2}\,. \tag{15}$$

This velocity no longer vanishes at the wormhole neck.

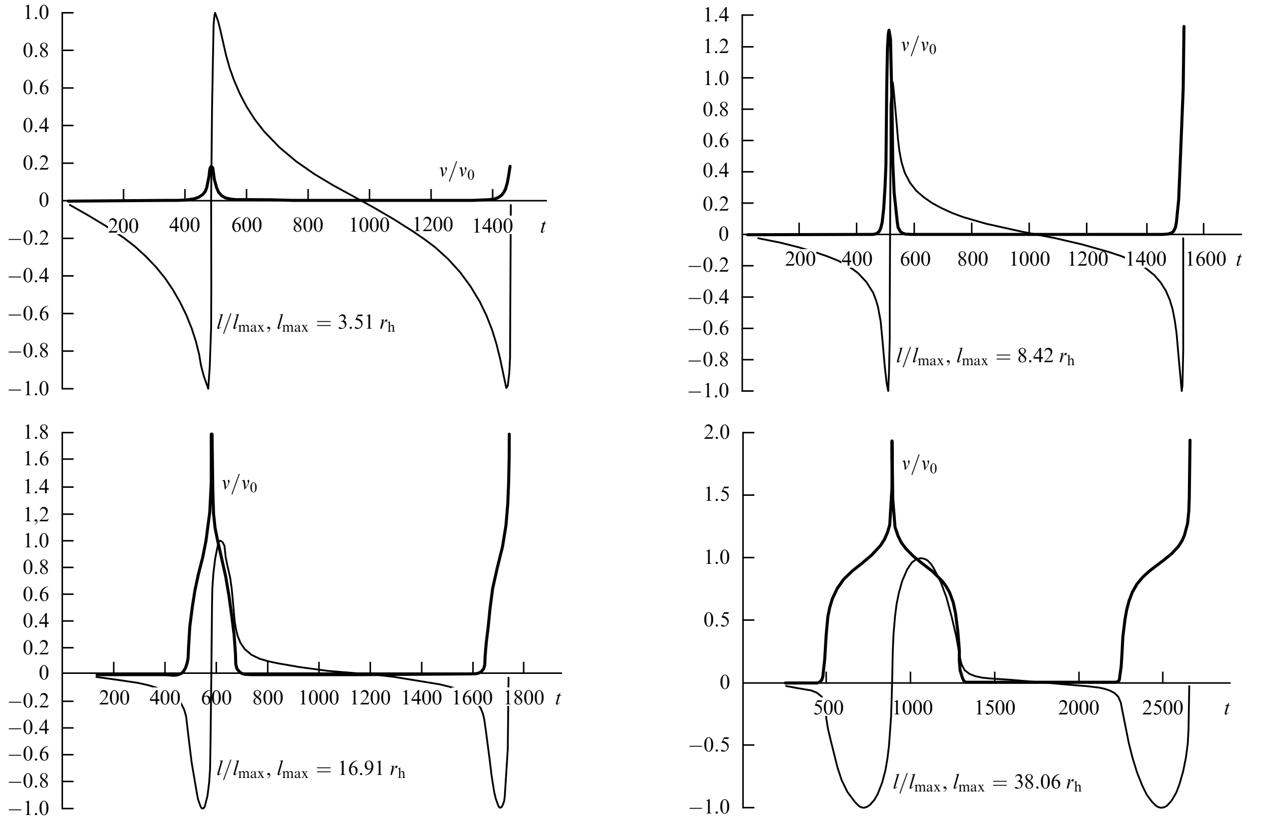

Figure 2. Dependences of the frequency shift ν/ν_0 (heavy curves) and of the shift in the physical radial coordinate l/l_{max} (light curves) on time t (in units of r_h/c). The diagrams are constructed for $\delta = 0.001$.

The redshift of the signal emitted by the body is determined by two factors:

(i) the Doppler shift from the motion of the source: the factor $\sqrt{1-v^2}/(1 \pm v)$, where $v = \dot{r}\sqrt{|g_{rr}/g_{tt}|}$ is the physical velocity of the body in its proper time (here the plus sign corresponds to the body's motion away from the observer, and the minus sign toward the observer, and g_{rr} and g_{tt} are the components of the metric tensor);

(ii) the gravitational redshift: the factor $\exp\phi$.

Hence, the frequency of the signal for a distant observer will be determined by the expression

$$\nu = \nu_0 \, \frac{\exp(2\phi)(m_0/E_0)}{1 \pm \sqrt{1 - \exp(2\phi)(m_0/E_0)^2}} \,, \tag{16}$$

where ν_0 is the frequency of the signal measured on the moving body. This shows that at the wormhole neck, in contrast to the black hole horizon, the frequency of the signal does not go to zero for a distant observer.

In order to find, for an external observer, the time dependence of the redshift ν/ν_0 of the body, we must add to t the time Δt it takes light to travel from point r to point r_{max} of the body. The corresponding diagrams are shown in Fig. 2.

For extremely small amplitudes $r_1 - r_0$, the vibrations of the test body become harmonic. This is achieved if the following inequalities hold:

$$r - r_0 \leqslant r_1 - r_0 \ll r_0 - r_h \,, \quad 1 - x \leqslant 1 - x_1 \ll x_h - 1 \,, \tag{17}$$

where $x_1 = r_0/r_1$. In this case, near the stopping points of the body the components of its velocity $\dot{r}$ (13) can be represented by series expansions in powers of $1 - x$ (or in powers of $x - x_1$). Leaving only the leading terms, we find that

$$\exp\phi \approx x_h - 1 \,, \qquad 1 - \frac{b}{r} \approx (x_h - 1)(1 - x) \,,$$
$$1 - \exp(2\phi)\left(\frac{m_0}{E_0}\right)^2 \approx \frac{2(x - x_1)}{x_h - 1} \,. \tag{18}$$

This leads us to the equation of motion for a harmonic oscillator with stopping points at $x = 1$ and $x = x_1$:

$$(\dot{r})^2 \approx 2(x_h - 1)^2(1 - x)(x - x_1) \,,$$
$$r_h \ddot{r} = \dot{r}\,\frac{\partial \dot{r}}{\partial x} = \frac{1}{2}\,\frac{\partial(\dot{r})^2}{\partial x} \,, \tag{19}$$
$$(1 - x)^{\cdot\cdot} = -\omega_0^2 (1 - x) \,,$$

where $\omega_0 = \sqrt{2}\,(x_h - 1)/r_h$. To an external observer, the period of such oscillations appears as

$$T_1 = \frac{\sqrt{2}\,\pi r_h}{c(x_h - 1)} \,. \tag{20}$$

Here, the oscillations of the physical coordinate l are also harmonic, which follows from Eqns (11), (15), and (18). The body oscillates from $-l_1$ to $+l_1$, and the condition of smallness for l_1, corresponding to Eqn (17), is given by the inequality $l_1 \ll r_h$ (thus, in terms of these coordinates the amplitude is not necessarily extremely small).

The oscillations of the coordinate l have twice as big a period ($T_2 = 2T_1$), since the physical velocity $\dot{l}$ has two stopping points (rather than three, as $\dot{r}$ has). This can easily be verified by calculating the period directly from Eqn (14).

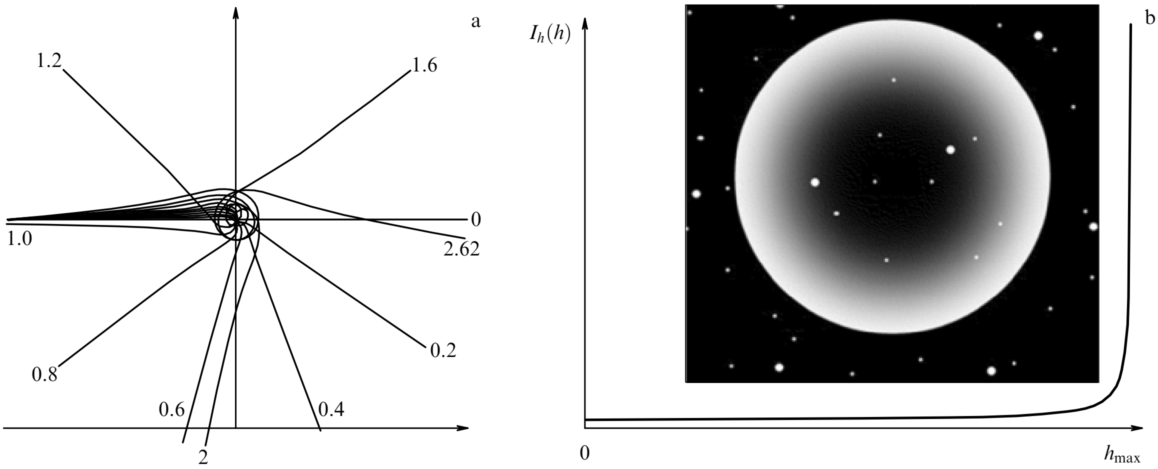

Figure 3. (a) Deflection of photons traveling through the neck of a wormhole (the number alongside each curve specifies the value of the impact parameter). (b) Dependence of the intensity I_h of light traveling through the neck of a wormhole on the impact parameter h.

Appendix 2. Circular orbits of a test particle about a wormhole

Let us find the limiting circular orbits of a test particle about a wormhole. For this we first solve this problem in the metric of the limiting Reissner – Nordström spacetime [with the Hamilton – Jacobi method (see Ref. [38])].

For the specific angular momentum of the particle we introduce the notation $\lambda = L/(m_0 r_{\rm h} c)$.

In accordance with Refs [39, 40], the specific effective potential energy $U(x)$ of the test particle and its derivative take, respectively, the form

$$U(x) = (1-x)\sqrt{1+\lambda^2 x^2}\,, \qquad U'(x) = -\frac{2\lambda^2 x^2 - \lambda^2 x + 1}{\sqrt{1+\lambda^2 x^2}}\,. \tag{21}$$

The roots for the unstable and stable circular orbits can be found from the condition $U'(x) = 0$:

$$x_{\pm} = 0.25\left(1 \pm \sqrt{1-\frac{8}{\lambda^2}}\right). \tag{22}$$

Hence follows that $\lambda^2 \geqslant 8$.

Therefore, the last stable orbit corresponds to

$$x_- = \frac{1}{4}\,, \qquad r_- = 4r_{\rm h}\,, \qquad \lambda^2 = 8\,, \tag{23}$$

while the last unstable circular orbit corresponds to

$$x_+ = \frac{1}{2}\,, \qquad r_+ = 2r_{\rm h}\,, \qquad \lambda^2 \to \infty\,. \tag{24}$$

The same method can be used to derive an expression for $\dot{\varphi} = \text{const}$ on a circular orbit, and an expression for the revolution period

$$\tau = \int_0^{2\pi} \frac{\mathrm{d}\varphi}{\dot{\varphi}} = \frac{2\pi r^2 E_0/L}{c^2(1-r_{\rm h}/r)^2}\,, \tag{25}$$

where $E_0 = U(r)$ for the specific circular orbit.

From this we can find the periods (measured by a clock of a distant observer) of the last stable and unstable circular orbits:

$$\tau(4r_{\rm h}) = \frac{32\pi r_{\rm h}}{\sqrt{3}c}\,, \qquad \tau(2r_{\rm h}) = \frac{8\pi r_{\rm h}}{c}\,. \tag{26}$$

When $r \geqslant 2r_{\rm h}$, for a wormhole we can ignore the corrections related to δ. Hence, results (23) – (26) would also hold for wormholes.

Appendix 3. The characteristic distribution of the intensity of light passing through a wormhole

The angles of the deflection of photons traveling through the neck of a wormhole with the parameters $w_\perp = -w_\parallel = 2$ have been calculated in Ref. [19] (Fig. 3a). Using this result, one can construct the angular dependence of the intensity of light traveling through the wormhole's neck (see Ref. [20]).

We examine the case of a uniform (on the average) distribution of light sources from the side of a wormhole opposite an observer (e.g., the cosmological background). Then, in the spherically symmetric version, the wormhole neck on the average is illuminated uniformly from all sides.

We analyze the light that travels through the wormhole neck in the equatorial plane (i.e., in the plane $\theta = \pi/2$). On the observer's side, the intensity of the light passing through the neck will depend on the impact parameter h (measured in units of the neck radius r_0).

Let φ be the angle of deflection of a photon after it has passed through the neck (as a result of gravitational lensing), and $I_{\rm tot}$ be the total intensity of light from different directions of angle φ, with $I_\varphi \equiv \mathrm{d}I_{\rm tot}/\mathrm{d}\varphi$ being the intensity density per unit angle. We have $I_\varphi = \text{const}$ (in view of the fact that the intensity does not depend on the direction of light propagation beyond the neck).

Then, one obtains

$$\frac{\mathrm{d}I_{\rm tot}}{\mathrm{d}h} \equiv I_h(h) = \frac{\mathrm{d}I_{\rm tot}}{\mathrm{d}\varphi}\frac{\mathrm{d}\varphi}{\mathrm{d}h} = \text{const}\,\frac{\mathrm{d}\varphi}{\mathrm{d}h}\,.$$

The function $\varphi(h)$ was found in Ref. [19]:

$$\varphi(h) = 2\int_1^\infty \frac{h}{x^2\sqrt{(1-y/x)\big(\exp(-2\phi) - h^2/x^2\big)}}\,\mathrm{d}x\,. \tag{27}$$

Introducing the notation $1/x \equiv q$, we get

$$I_h(h) = C \int_0^{q_0} \frac{\exp(-2\phi)\,\mathrm{d}q}{\sqrt{1-qy}\left[\exp(-2\phi) - q^2h^2\right]^{3/2}} \,. \qquad (28)$$

This intensity distribution in h is minimal at the zero impact parameter and maximal at maximum impact parameters corresponding to the width of the wormhole neck, with this result being independent of the wavelength of the light traveling through the wormhole.

A characteristic diagram for $I_h(h)$ is shown in Fig. 3b. It corresponds to a wormhole with metric (1). Thus, the observer will see a ring of light with sharp outer edges and smeared inner edges.

With sufficiently high resolution of the observational devices, this fact can make it possible to distinguish between wormholes and black holes in, say, active galactic nuclei.

References

1. Einstein A, Rosen N *Phys. Rev.* **48** 73 (1935)
2. Wheeler J A *Phys. Rev.* **97** 511 (1955)
3. Misner C W, Wheeler J A *Ann. Phys.* (New York) **2** 525 (1957)
4. Wheeler J A *Ann. Phys.* (New York) **2** 604 (1957)
5. Vilenkin A *Phys. Rev. D* **27** 2848 (1983)
6. Linde A D *Phys. Lett. B* **175** 395 (1986)
7. Visser M *Lorentzian Wormholes: from Einstein to Hawking* (Woodbury, NY: AIP, 1995)
8. Lobo F S N *Phys. Rev. D* **71** 084011 (2005)
9. Shinkai H, Hayward S A *Phys. Rev. D* **66** 044005 (2002)
10. Rahaman F et al. *Phys. Lett. B* **633** 161 (2006); gr-qc/0512075
11. Kuhfittig P K F *Phys. Rev. D* **73** 084014 (2006); gr-qc/0512027; Lobo F S N *Phys. Rev. D* **71** 124022 (2005); gr-qc/0506001
12. Visser M, Kar S, Dadhich N *Phys. Rev. Lett.* **90** 201102 (2003); gr-qc/0301003
13. Jassal H K, Bagla J S, Padmanabhan T *Phys. Rev. D* **72** 103503 (2005)
14. Armendáriz-Picón C *Phys. Rev. D* **65** 104010 (2002); gr-qc/0201027
15. Kardashev N S, Novikov I D, Shatskii A A *Astron. Zh.* **83** 675 (2006) [*Astron. Rep.* **50** 601 (2006)]
16. Kardashev N S, Novikov I D, Shatskiy A A *Int. J. Mod. Phys. D* **16** 909 (2007)
17. Kardashev N S, Novikov I D, Shatskiy A A, astro-ph/0610441
18. Morris M S, Thorne K S *Am. J. Phys.* **56** 395 (1988)
19. Shatskii A A *Astron. Zh.* **81** 579 (2004) [*Astron. Rep.* **48** 525 (2004)]
20. Shatskii A A *Astron. Zh.* **84** (2) 99 (2007) [*Astron. Rep.* **51** 81 (2007)]
21. Misner C W, Thorne K S, Wheeler J A *Gravitation* (San Francisco: W.H. Freeman, 1973) [Translated into Russian: Vol. 3 (Moscow: Ainshtain, 1997)]
22. Shabad A E, Usov V V, hep-th/0512236
23. Shabad A E, Usov V V, in *Particle Physics at the Year of 250th Anniversary of Moscow University: Proc. of the 12th Lomonosov Conf. on Elementary Particle Physics, Moscow, Russia, 25–31 August 2005* (Ed. A I Studenikin) (Singapore: World Scientific, 2006); astro-ph/0601542
24. Bander M, Rubinstein H R *Phys. Lett. B* **280** 121 (1992)
25. Duncan R C, astro-ph/0002442
26. Peng Q, Chou C *Astrophys. J.* **551** L23 (2001)
27. 't Hooft G *Nucl. Phys. B* **79** 276 (1974)
28. Polyakov A M *Pis'ma Zh. Eksp. Teor. Fiz.* **20** 430 (1974) [*JETP Lett.* **20** 194 (1974)]
29. Kibble T W B *J. Phys. A: Math. Gen.* **9** 1387 (1976)
30. Stawarz L *Astrophys. J.* **613** 119 (2004)
31. Schild R E, Leiter D J, Robertson S L, astro-ph/0505518
32. Ostorero L et al., astro-ph/0602237
33. Kardashev N S "Posleslovie k russkomu izdaniyu" ("Afterward to the Russian edition"), in: Burbidge G, Burbidge M *Kvazary* (Quasi-Stellar Objects) (Moscow: Mir, 1969)
34. Kovalev Yu A, Kovalev Yu Yu, Nizhelsky N A *Publ. Astron. Soc. Jpn.* **52** 1027 (2000)
35. Cherepashchuk A M *Vestn. Mosk. Univ. Fiz. Astron.* **60** (2) 62 (2005) [*Moscow Univ. Phys. Bull.* **60** (2) 74 (2005)]
36. RadioAstron Mission, http://www.asc.rssi.ru/radioastron/description/intro_eng.htm
37. Millimetron Project, http://www.asc.rssi.ru/millimetron/eng/millim_eng.htm
38. Landau L D, Lifshitz E M *Teoriya Polya* (The Classical Theory of Fields) (Moscow: Nauka, 1995) [Translated into English (Boston: Butterworth–Heinemann, 1987)]
39. Frolov V P, Novikov I D *Black Hole Physics: Basic Concepts and New Developments* (Dordrecht: Kluwer, 1998)
40. Carr B J "Primordial black holes: do they exist and are they useful?", astro-ph/0511743

PACS numbers: **95.35 + d**, **98.80. – k**
DOI: 10.1070/PU2007v050n09ABEH006382

Dark matter: from initial conditions to structure formation in the Universe

V N Lukash, E V Mikheeva

1. "Bring me that, don't know what"

We are at the verge of a discovery capable of changing the essence of our world view. We are talking about the nature of dark matter.

Recently, astronomy has taken important steps in observational justification for dark matter, and today the presence of dark matter in the Universe can be considered a firmly established fact. What makes the situation so special is that astronomers *observe* structures made of matter *unknown* to physicists. This has posed the problem of identifying the physical nature of such matter.

Modern elementary particle physics knows of no particles that have the properties of dark matter. This requires an extension of the Standard Model. But how and in what direction should we move, and what and where should we look for? The heading of this section, a quotation from a Russian fairy tale, very fittingly reflects the current situation.

Physicists are looking for the unknown particles, having only a general idea about the properties of the observed matter. But what are these properties?

The only thing we know for certain is that dark matter interacts with a luminous matter (baryons) in the gravitational way and constitutes a cold medium with a cosmological density several times higher than the baryon density. In view of such simple properties, dark matter directly affects the development of the gravitational potential of the Universe. The contrast of its density grew with time, leading to the formation of gravitationally bound systems of dark-matter halos.

It should be emphasized that this process of development of gravitational instability could be triggered in the Friedmann Universe only in the presence of primordial density perturbations, whose very existence is in no way related to dark matter but instead is caused by Big Bang physics. Hence, we face another important question of how these seed perturbations, from which the structure of dark matter developed, emerged.

We will turn our attention to the problem of generation of primordial cosmological perturbations somewhat later. Let us now return to dark matter.

Baryons are trapped by the gravitational wells of dark matter concentrations. Hence, although the particles of dark