noted that the extension of the line $T_{\beta\pi}(x)$ to the T = 0-axis up to $x = x_b$ (the line of second-order phase transition cannot end at a point) naturally leads to the concept of a quantum critical point ($x = x_b$, T = 0) for a higher doping level x_b compared to x_0 .

In the case of a short-range rather than a long-range OAF order, the phase transition inside the superconducting state does not occur, and yet the broad region of developed fluctuations at temperatures above T_c allows interpretation of the pseudogap state with conditional separation into strong and weak pseudogaps, reflecting one of the admissible versions of the phase diagram of cuprates [42].

The conception of large-momentum superconducting pairing in screened Coulomb repulsion [20], which naturally leads to a two-component order parameter reflecting the charge and current degrees of freedom of the relative pair motion, agrees well on the whole with experimental data for the phase diagram and the physical properties of cuprates.

The work was supported by the Russian Foundation for Basic Research (grants Nos 05-02-17077a and 06-02-17186a).

References

- Ginzburg V L, Landau L D Zh. Eksp. Teor. Fiz. 20 1064 (1950) [Translated into English: Collected Papers of L.D. Landau (Oxford: Pergamon Press, 1965) p. 546]
- 2. Bardeen J, Cooper L N, Schrieffer J R Phys. Rev. 108 1175 (1957)
- Abrikosov A A Zh. Eksp. Teor. Fiz. 32 1442 (1957) [Sov. Phys. JETP 5 1174 (1957)]
- Gor'kov L P Zh. Eksp. Teor. Fiz. 36 1918 (1959) [Sov. Phys. JETP 9 1364 (1959)]
- 5. Bednorz J G, Müller K A Z. Phys. B 64 189 (1986)
- Ginzburg V L, Kirzhnits D A (Eds) Problema Vysokotemperaturnoi Sverkhprovodimosti (The Problem of High-Temperature Superconductivity) (Moscow: Nauka, 1977) [Translated into English: High-Temperature Superconductivity (New York: Consultants Bureau, 1982)]
- Bulaevskii L N, Ginzburg V L, Sobyanin A A Zh. Eksp. Teor. Fiz. 94 355 (1988) [Sov. Phys. JETP 67 1499 (1988)]; Bulaevskii L N, Ginzburg V L, Sobyanin A A, Stratonnikov A A Usp. Fiz. Nauk 157 539 (1989) [Sov. Phys. Usp. 32 277 (1989)]
- Ginzburg V L Zh. Eksp. Teor. Fiz. 14 177 (1944); J. Phys. USSR 8 148 (1944); Pis'ma Zh. Eksp. Teor. Fiz. 49 50 (1989) [JETP Lett. 49 58 (1989)]; Usp. Fiz. Nauk 168 363 (1998) [Phys. Usp. 41 307 (1998)]
- 9. Ginzburg V L et al. Solid State Commun. 50 339 (1984)
- 10. Corson J et al. Nature 398 221 (1999)
- 11. Xu Z A et al. *Nature* **406** 486 (2000)
- 12. Wang Y et al. Science 299 86 (2003)
- 13. Wang Y et al. Phys. Rev. Lett. 95 247002 (2005); cond-mat/0503190
- 14. Anderson P W Science **235** 1196 (1987)
- 15. Dagotto E Rev. Mod. Phys. 66 763 (1994)
- 16. Kohn W, Luttinger J M Phys. Rev. Lett. 15 524 (1965)
- 17. Moskalenko V A Fiz. Met. Metalloved. 8 503 (1959)
- 18. Suhl H, Matthias B T, Walker L R Phys. Rev. Lett. 3 552 (1959)
- Damascelli A, Hussain Z, Shen Z-X *Rev. Mod. Phys.* **75** 473 (2003)
 Belyavsky V I, Kopaev Yu V *Usp. Fiz. Nauk* **176** 457 (2006) [*Phys.*
- *Usp.* **49** 441 (2006)] 21. Basov D N, Timusk T *Rev. Mod. Phys.* **77** 721 (2005)
- 22. Loram J W et al. *Physica C* **341–348** 831 (2000)
- 23. Belyavsky V I, Kapaev V V, Kopaev Yu V Pis'ma Zh. Eksp. Teor. Fiz. 81 650 (2005) [JETP Lett. 81 527 (2005)]
- 24. Sadovskiĭ M V Usp. Fiz. Nauk 171 539 (2001) [Phys. Usp. 44 515 (2001)]
- 25. Emery V J, Kivelson S A Nature **374** 434 (1995)
- 26. Chakravarthy S et al. Phys. Rev. B 63 094503 (2001)
- 27. Lee P A, Nagaosa N, Wen X-G Rev. Mod. Phys. 78 17 (2006)
- 28. Boebinger G S et al. Phys. Rev. Lett. 77 5417 (1996)
- Belyavskii V I et al. Zh. Eksp. Teor. Fiz. 126 672 (2004) [JETP 99 585 (2004)]
- 30. Ivanov D A, Lee P A, Wen X-G Phys. Rev. Lett. 84 3958 (2000)

- Belyavsky V I, Kopaev Yu V, Smirnov M Yu Zh. Eksp. Teor. Fiz. 128 525 (2005) [JETP 101 452 (2005)]
- 32. Belyavsky V I, Kopaev Yu V, Smirnov M Yu *Phys. Rev. B* 72 132501 (2005)
- Belyavsky V I et al. Zh. Eksp. Teor. Fiz. 124 1149 (2003) [JETP 97 1032 (2003)]
- 34. Belyavsky V I, Kopaev Yu V Phys. Rev. B 67 024513 (2003)
- 35. Belyavsky V I et al. Phys. Lett. A 342 267 (2005)
- 36. Franck J P, Lawrie D D J. Supercond. 8 591 (1995)
- 37. Berk N F, Schrieffer J R Phys. Rev. Lett. 17 433 (1966)
- Chubukov A V, Pines D, Schmalian J, in *The Physics of Super*conductors Vol. 1 Conventional and High-T_c Superconductors (Eds K H Bennemann, J B Ketterson) (Berlin: Springer-Verlag, 2003) p. 495
- 39. Zhao G Phys. Rev. B 64 024503 (2001)
- 40. Brandow B H Phys. Rev. B 65 054503 (2002)
- 41. Belyavsky V I, Kopaev Yu V Zh. Eksp. Teor. Fiz. **127** 45 (2005) [JETP **100** 39 (2005)]
- Norman M R, Pines D, Kallin C Adv. Phys. 54 715 (2005); condmat/0507031

PACS numbers: **41.60. – m**, **52.20. – j**, 52.27.Lw 10.1070/PU2007v050n05ABEH006215

Polarization effects in a medium: from Vavilov – Cherenkov radiation and transition radiation to dust-particle pairing, or the development of one of V L Ginzburg's ideas from 1940 to 2006

V N Tsytovich

1. Polarization around particles

In the future general particle theory, with each particle consisting of all the other particles, any particle, being an excitation of the system, will be surrounded by the polarization of these other particles. So far, only the notion of the polarization produced around particles traveling through a medium has been elaborated (Fig. 1a). When the states of the particles change, their polarization 'coats' also change. Figure 1 shows the interaction of particles with external forces, with emitted radiation or incident radiation, with either individual incident particles or a large number of incident particles (i.e., particle fluxes) — the oval S in Fig. 1b. The interparticle interaction depends strongly on perturbations of the polarization cloud during the interaction. The physics of such interactions was first considered by Ginzburg [1].

2. Ginzburg's paper of 1940

In Ginzburg's 1940 paper "Quantum theory of the supersonic radiation of an electron uniformly traveling through a medium", quantum energy and momentum conservation laws for radiation in a medium, $\varepsilon_{\mathbf{p}} = \varepsilon_{\mathbf{p}'} + \hbar\omega_{\mathbf{k}}$ and $\mathbf{p} = \mathbf{p}' + \hbar\mathbf{k}$, were first used; in the system of units where $\hbar = 1$, they become $\varepsilon_{\mathbf{p}} = \varepsilon_{\mathbf{p}'} + \omega_{\mathbf{k}}$ and $\mathbf{p} = \mathbf{p}' + \mathbf{k}$, which in the classical limit ($\mathbf{k} \ll \mathbf{p}, \omega_{\mathbf{k}} \ll \varepsilon_{\mathbf{p}}$) leads to the classical Tamm – Frank condition $\omega_{\mathbf{k}} = (\mathbf{kv}), \mathbf{v} = d\varepsilon_{\mathbf{p}}/d\mathbf{k}$ for Vavilov – Cherenkov radiation. Of significance here is (i) the introduction of the photon momentum in the medium and (ii) the clear statement that an exchange of energy and momentum occurs only between the particle and the radiation. Subsequent research led to a deeper understanding and generalization of these statements.

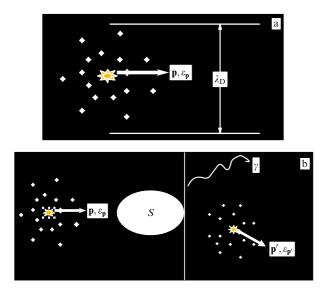


Figure 1. (a) A particle with a momentum **p** and energy ε_p freely moving through a medium is always surrounded by a polarization with some effective radius λ_D (the Debye radius in a plasma). (b) Scheme of the interaction of particles surrounded by polarization clouds: shown at the left is a particle prior to the interaction and at the right after the interaction, which results in the radiation of a mode of the medium (a wave propagating through the medium).

Concerning the first item, there is a remark in [1] that "...in a medium the photon momentum is $\hbar\omega n/c$ rather than $\hbar\omega/c$... the notion of photons with a momentum $\hbar\omega n/c$ is valid to the same degree as its related notion of the speed of light c/n, which is, strictly speaking, incorrect." Numerous subsequent investigations into the radiation in media with spatial dispersion confirmed this statement and enabled obtaining the general result that even in the classical description, the energy radiation power \dot{E} and the momentum radiation rate \dot{P} obey the relations $\dot{E} = \int \omega_{\mathbf{k}} w_{\mathbf{k}} \, d\mathbf{k}$ and $\dot{\mathbf{P}} = \int \mathbf{k} w_{\mathbf{k}} \, d\mathbf{k}$, where $w_{\mathbf{k}}$ is the radiation probability. This result applies to any modes of the medium (for instance, phonons in solids or plasmons in a plasma) and even to those hydrodynamic modes whose electromagnetic momentum is zero or is negligible and is related to particle displacements. The second item turned out to be most important from the standpoint of physical consequences: any modes of the medium can be radiated by any heavy particles, the polarization of high-frequency waves being produced only by light particles (for instance, electrons) and therefore being determined by the mass of light particles. According to Ref. [1], only the particle and radiation can exchange momentum and energy, although polarization can be produced by light particles and determined by their mass (for instance, the radiation of an ion is determined by the electron mass).

This result appears to be more important than the widespread opinion that the most significant fact is that uniformly moving particles can emit radiation. This result also applies to other processes like transition (inherently polarization-related) scattering, polarization Bremsstrahlung, and the interaction of particles via their polarization clouds. All these lines of research have been under steady development, beginning with Ginzburg's paper [1], and are being pursued at present, including their numerous astrophysical applications. The most important of these areas are briefly discussed in the present report.

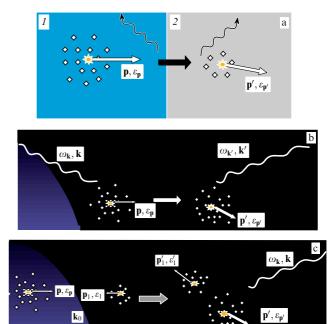


Figure 2. (a) During particle transit from medium 1 to medium 2, modes (waves) in both media propagate from the interface and surface waves travel along the boundary between the media. The energy and momentum conservation law should account for a change in the particle polarization cloud in transit from medium 1 to medium 2. (b) Transition scattering scheme: the incident wave gives rise to perturbations of the particle polarization cloud, which changes the scattering of heavy particles. (c) Scheme of particle polarization Bremsstrahlung, in which perturbations of all colliding-particle clouds during collisions play an essential role; the bound electrons of the polarization clouds of colliding atoms and ions also participate in the perturbations.

3. Patterns of transition radiation, transition scattering, and polarization Bremsstrahlung

Transition radiation, which was first considered by Ginzburg and I M Frank [2], is an example of a process related to polarization cloud variations in the transit of a particle from one medium to another, which leads to the radiation of modes in both media (Fig. 2a). The modes may be any modes of either media or surface modes. The energy and momentum conservation laws are satisfied only when the changes in the energy and momentum for the polarization 'fur coats' are taken into account, which is proven in Ref. [3]. Any mode or wave in the medium also carries a polarization wave with it; this polarization wave may be scattered due to the oscillations of the polarization cloud of the particle, which is transition radiation [4] (Fig. 2b). The transition radiation for ions in a plasma can be determined by the electron mass when the wavelength exceeds the polarization cloud size; therefore, the cross section for the scattering by ions can be greater than or of the order of the Thomson cross section for the scattering by electrons in the vacuum [5]. The transition scattering interferes with the ordinary scattering caused by the perturbation of motion of the scattering particle itself, and this interference suppresses the scattering by electrons.

The quantum scattering conservation laws $\varepsilon_{\mathbf{p}} + \omega_{\mathbf{k}} = \varepsilon_{\mathbf{p}'} + \omega_{\mathbf{k}'}$ and $\mathbf{p} + \mathbf{k} = \mathbf{p}' + \mathbf{k}'$, which are similar to those first used by Ginzburg for wave emission, lead to the law for the total scattering probability in the classical limit: $\omega_{\mathbf{k}} - \omega_{\mathbf{k}'} = (\mathbf{k} - \mathbf{k}')\mathbf{v}$. In particle collisions, the oscillation of the polarization cloud of each of the colliding particles

makes a contribution to the amplitude of polarization-Bremsstrahlung radiation (Fig. 2c) [6]. The quantum conservation laws

$$\begin{split} \varepsilon_{\mathbf{p}} &= \varepsilon_{\mathbf{p}_1} = \varepsilon_{\mathbf{p}'} + \varepsilon_{\mathbf{p}'_1} + \omega_{\mathbf{k}} , \\ \mathbf{p}' &= \mathbf{p} - \mathbf{k}_0 , \quad \mathbf{p}'_1 = \mathbf{p}_1 + \mathbf{k}_0 - \mathbf{k} , \end{split}$$

where \mathbf{k}_0 is the momentum transferred from one particle to another during collisions, lead to the same relation for the total emission probability in collisions in the classical limit as for the Bremsstrahlung with the perturbations of polarization clouds neglected: $\omega_{\mathbf{k}} = (\mathbf{k} - \mathbf{k}_0)\mathbf{v} + \mathbf{k}_0\mathbf{v}'$. The polarization clouds are like a 'transmission link,' but their perturbations may determine the energy and momentum exchange between the final states of radiation and particles [1].

4. Examples of transition scattering in plasmas

The measurement of the so-called Thomson scattering in plasma, which was carried out by British scientists invited to the USSR at the dawn of thermonuclear research, was used to prove the need for a sufficiently high temperature in tokamaks and lent impetus to the entire scope of research into controlled nuclear fusion. It is theorized that the scattering by individual particles is the Thomson scattering, i.e., occurs due to particle oscillations in the field of the incident wave, but in a plasma (or in another medium), this scattering corresponds to the scattering by density fluctuations and the resultant difference arises from the fact that electron density fluctuations may also be caused by ions.

The question arises as to the role of transition scattering in the total scattering. It is pertinent to note in this connection that, amazingly, some physicists are unaware of the foundations of the physical processes considered in Ref. [1] long ago. Physically, it is clear that for wavelengths longer than the polarization cloud size, the electrons of the cloud oscillate coherently in the wave field and the ions may scatter more intensely than the electrons, for which the polarization cloud oscillates in antiphase relative to the oscillations of the scattering electrons. The answer to this question, which was given in 1985 (published in the proceedings of the conference on transition radiation [7] held in Erevan), is as follows. When transition radiation is taken into account, the formulas used for the so-called 'Thomson scattering by fluctuations' can be rewritten as the sum of the scattering by electrons and ions. The scattering probability for electrons then contains the sum of the Thomson scattering and transition scattering amplitudes, while for ions the scattering is entirely determined by the transition scattering amplitude. This is also evident from the formula

$$\frac{Q}{Q_0} \propto r_0^2 \left[\left(1 - \frac{1 - \epsilon_{\rm e}}{\epsilon} \right)^2 f_{\rm e} + \left(\frac{1 - \epsilon_{\rm e}}{\epsilon} \right)^2 f_{\rm i} \right],$$

given in textbooks [8] on scattering in plasmas, where Q is the scattered radiation intensity, Q_0 is the incident radiation intensity, r_0 is the classical electron radius, ϵ_e and ϵ are the electron and total permittivities at the frequency and wave numbers of beats, and f_e and f_i are the electron and ion distribution functions. The difference between scattering by fluctuations and by separate particles is thereby eliminated: the total scattering is the sum of scatterings by separate particles. In the first approximation, this statement is valid for any medium. The transition scattering has resonances (zeroes of ϵ), which completely describe the experimentally

measured Raman scattering from plasma modes. This treatment is not merely a different interpretation of scattering, because electrons and ions experience different types of additional actions (collisions at least).

The lack of understanding of transition scattering by astrophysicists is exemplified by the response of the editors of *Astrophysical Journal* to a paper submitted to that journal concerning the generalization of the Sunyaev–Zel'dovich effect to the low-frequency domain, where the transition scattering by ions with a cross section of the order of the Thomson scattering by electrons becomes dominant. The editors of one of the leading journals in astrophysics considered it possible to reply that "the authors may be right, but neither the Editors nor the referees can understand how ions can have such a large scattering cross section." The paper was published in the journal *Physics of Plasmas* [9]. This is indicative of the glacial pace with which physical notions laid back in 1940 [1] make their way to astrophysics.

5. Generalization of Einstein's notions of induced processes to nonequilibrium plasma states

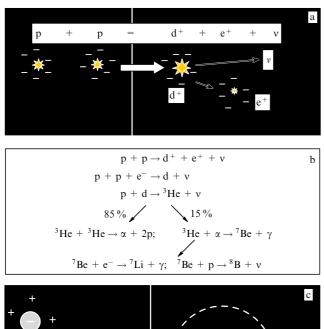
Last year was the centenary of three Einstein's 1905 discoveries, including the discovery of stimulated processes, which provided the basis for modern laser physics. Plasma is the only medium where the smallness of field energy in comparison with the particle energy allows constructing an entirely analytic theory of nonequilibrium stimulated processes, including nonequilibrium distributions of plasma modes (determined by nonequilibrium numbers $N_{\mathbf{k}}$ of quanta) and plasma particles whose distribution is defined by nonequilibrium distributions f_e and f_i , with the inclusion of all stimulated processes [10]. The central results is the proof that this construction is possible only if polarization effects are taken into account in all processes. The probability of polarization scattering by ions appearing in Section 4 enters the nonequilibrium equation for the ion distribution function f_i . This leaves no room for doubt about the validity of the interpretation of scattering as a process whose inherent part is transition scattering; precisely the plasma ions gain energy and momentum in the course of such a scattering. Although the last statement may be derived using the results in Ref. [10], it was not explicitly formulated until 2005 (see report [11]). The experimental data published to date well indicate that ions are responsible for the stimulated transition scattering of plasma modes.

6. Examples of polarization Bremsstrahlung

Because the wavelength of a Bremsstrahlung photon is longer than the dimension of an atom, the role of a polarization cloud may also be played by bound electrons: for complex atoms, this effect was termed the atomic Bremsstrahlung or the polarization Bremsstrahlung radiation. The latter term reflects the fact that interference occurs, i.e., the amplitudes of the Bremsstrahlung and polarization radiation are added to each other.

Intensive theoretical investigations were performed and repeatedly borne out in experiments by a large team of the Leningrad Physicotechnical Institute and the teams of several Moscow institutes, including the Lebedev Physics Institute, the General Physics Institute, and the Kurchatov Institute. The main results are expounded in the collective monograph Ref. [6]. The following two examples serve to illustrate the possibility of manifestation of qualitatively new effects. (1) In electron collisions with partially ionized atoms (ions) in a plasma, when the screening of an atomic nucleus is partly produced by bound electrons and partly by plasma electrons, the bound and free electrons may act coherently in the polarization Bremsstrahlung (the radiation intensity is proportional to the squared sum of the numbers of the bound and free electrons) [6, Ch. 6]. This occurs, of course, at a high speed of the incident particle, when its energy is much higher than the binding energy. In this case, the electrons bound prior to the collision remain such after the collision.

(2) In a plasma containing dust particles, the particles can lead to polarization Bremsstrahlung due to polarization charge oscillations in collisions of heavy dust particles. Dust particles carry very large negative charges (up to $Z_d \approx$ $10^4 - 10^6$ in units of the electron charge), which are balanced by the cloud of the electrons and ions surrounding the dust particle. The Bremsstrahlung involves the standard smallness with respect to the coupling constant and the polarization radiation amplitude is of the same order of magnitude as the Bremsstrahlung amplitude, but the intensity, being proportional to the squared charge of each of colliding particles,



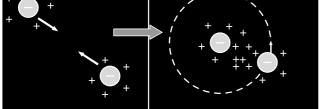


Figure 3. (a) Example of nuclear collisions at the beginning of the hydrogen cycle; prior to and after collisions, all charged particles of the nuclear reaction are surrounded by polarization clouds, which affect these reaction rates. (b) Scheme of the hydrogen cycle of nuclear reactions in the interior of the sun. Each of the nuclei is 'bare' (there are no bound electrons and the screening is effected by free negative charges of the plasma). (c) Formation of the bound states of two dust particles in their collisions, which is due to their attraction for an excessive density of positive ions of the polarization cloud between the interacting dust particles. The momentum and energy of a particle captured in an attractive potential well may decrease due to the emission of dust sound waves or friction against neutral gas atoms.

contains a very large factor Z_d^4 . In experiments, the polarization – Bremsstrahlung energy loss of dust particles due to the emission of low-frequency modes may be comparable to their energy loss due to deceleration in a neutral gas [12].

7. Particle collisions in a plasma

Nonradiative collisions that are also affected by the polarization clouds of particles are possible (Fig. 3a). It is generally accepted presently that particle interactions in collisions correspond to dynamically screened interactions, the polarization clouds in collisions of two selected (usually called probe) particles being formed by fluctuations of all the other plasma particles (see Ref. [5]). The two-particle collision cross sections contain the factor $1/|\epsilon_{\mathbf{k},\mathbf{k}\,\mathbf{v}}|^2$ and are therefore determined by the distributions of all the other plasma particles.

8. Effect of polarization on nuclear reaction rates

Normally, the polarization clouds affect nuclear tunneling, which is responsible for nuclear reactions (Fig. 3b). This is significant at high plasma densities of the order of the densities existing in the interior of stars, which was first pointed out by Salpeter [13] in 1954 and formed the basis for the modern scenario of stellar evolution. For the solar interior, corrections for the hydrogen cycle reactions (Fig. 3b) range from 5% to 20% [13]. The Debye screening was postulated in [13], although from the modern standpoint, such a screening must be derived in the fluctuation theory that takes nuclear reactions into account as well. The first such investigations [14] into the screening kinetics of nuclear reactions in a plasma exposed the main error in Ref. [13], which becomes evident when invoking the fluctuation theory that determines the final (and rather long) time of polarization screening formation.

Of significance in this problem is the understanding of the fundamental propositions that the description in quantum physics is probabilistic; specifically, the probabilistic nature of tunneling is an indication that although the tunneling time is short for a high barrier, its probability is low, which leads to low nuclear reaction rates. According to Ref. [14], the screening formation time due to fluctuations is much longer than the tunneling time. There also emerges a new effect: fluctuation correlations lead to an effect of the same order of magnitude as the increase in the tunneling probability due to the polarization lowering of the potential barrier. This effect was considered for the averaged Debye potential in Ref. [13]. As shown in Ref. [14], for a weak screening (roughly speaking, applicable to the solar interior), the amplitude of the correlation effect is precisely equal to the amplitude in Ref. [13]. But the sign of the correlation effect amplitude was calculated erroneously in Ref. [14], which would have been insignificant if both effects had been combined independently. However, the amplitudes, not the probabilities, are summed, and this led to destructive interference of the two effects in Ref. [14] (roughly speaking, if the Salpeter amplitude is taken to be 1, then $|1 - 1|^2 = 0$). With this error corrected in Ref. [15], the constructive interference of the two effects resulted in an unacceptable result: a four-fold increase in the corrections, $|1 + 1|^2 = 4$. A way out was also found in Ref. [15], where it was proven that the probabilities of the process do not change (with the fluctuation interpretation without the introduction of the unproven averaged screening) and the Salpeter effect is nonexistent. Then the correlations 'recover' the Salpeter result $(|1|^2 = 1)$. But this is valid only

for a weak screening, whereas the Salpeter effect is also used in the astrophysics of dense star evolution in conditions of strong screening. It has become necessary to replace it by strong correlation effects, whose theory is not yet fully elaborated, although there is a wide spectrum of laboratory experimental investigations [16].

9. Interaction and pairing of dust particles

In recent years, considerable experimental and theoretical study has been devoted to dust plasmas (see reviews [17, 18]). The central problem is the interaction of dust particles, which are macroscopic objects (with the number of atoms greater than $10^9 - 10^{11}$) with high negative charges (over $10^4 - 10^6$ electron charges) and with dimensions much shorter than the dimension of the polarization screening cloud. Each particle produces plasma flows and interacts with plasma flows, and polarization charges interact not with individual plasma particles but coherently with many of them or, to state it in different terms, with plasma flows. In the experiments conducted, the free path of the flows is indeed short and the flow field is the additional field whose interaction with the polarization field may change the interaction of dust particles (see the schematic in Fig. 3c). In this case, it turns out that repulsion becomes attraction at long distances, where the flows affect the particle interaction most efficiently (Fig. 4a), which signifies the possibility of particle pairing with the formation of bound states like dust molecules by like-charged particles and the possibility of forming larger dust particle complexes up to crystals. The experimental discovery of these crystals in 1996 [19-22], which were termed plasma crystals, posed the problem of explaining the physics of their formation. The change of the isolated dust particle interaction in plasmas due to plasma flows was first considered by

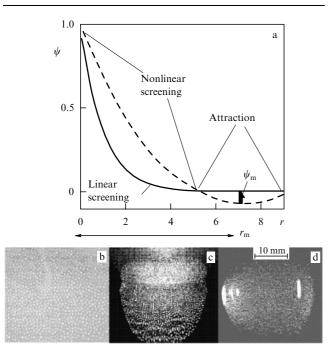


Figure 4. (a) Schematic representation of the screening factor ψ in the interaction of dust particles, $V = Z_d^2 e^2 \psi(r/\lambda_{Di})/r$; the interparticle distance *r* is given in terms of λ_{Di} . (b, c, d) Examples of the plasma crystals observed in Refs [19, 22, 33], respectively. Shown in Fig. 4b is the distribution in one of the crystal planes; in Figs 4c and 4d, the vertical axis is oriented along the force of gravity in the laboratory experiments in Refs [22, 33].

Pitaevskii [23] in 1960, but the effect of the flows on the interaction of isolated particles is substantially different from that for a collection of dust particles.

Indicated in Fig. 4a is the domain of collective interaction for two probe particles, which emerges in the presence of many-particle flows and is basically similar to collective interaction in ordinary plasmas (see Section 7). The collision integrals that describe the dust particle interactions contain not only the mean-square fluctuations of polarization fields but also the mean products of the polarization fields and the flow fields, which are related to each other. A significant feature of the interaction is the nonlinearity of the polarization cloud; this effect first considered in Ref. [24] for artificial earth satellites (their dimensions are also shorter than the screening radius), and is commonly called the Gurevich screening in the literature.

10. Plasma dust crystals

and explanation of phase transition parameters

It was believed that the high charges of plasma dust particles could lead to strong correlations and could be responsible for the transition of dust to a crystal state even at low dust density [25]. It was assumed that the coupling constant (nonideality constant) $\Gamma = Z_d^2 e^2 / T_d r_m$ (where $r_m = (4\pi n_d/3)^{-1/3}$ is the average distance between the dust particles and *T* is the dust temperature) in this case should amount to at least 4–10, as for ordinary phase transitions to the solid state. The ease of crystal production from dust in plasmas by the mere injection of dust particles into an ordinary high-frequency discharge in a low-temperature plasma [19] (it was even sufficient to inject printer toner [26]) and especially the transition parameters themselves [27, 28] turned out to be quite unexpected (examples of dust crystals obtained in different experiments are given in Figs 4b–4d).

The first surprising thing is that the observed values of the parameter Γ are extremely large (from $\approx 3 \times 10^3 - 10^4$ up to $\approx 10^5$). Second, the value of r_m is relatively large and exceeds the linear screening radius by a factor of 8–10. If the dust particle field is assumed to be completely screened at these distances, it is unclear why the particles do not come closer to each other. Lastly, the dust temperature T_d on crystal melting turned out to be rather low, of the order of 0.1-1 eV. Although this temperature is much higher than room temperature (≈ 0.02 eV) and the crystals are rather 'firm,' it is much lower than the maximum energy that corresponds to the approach of particles for a distance of the order of their radius, which is estimated as $3Z_dT_e \approx 50$ keV for the values $Z_d \approx 3 \times 10^3$ and $T_e \approx 2$ eV, typical for the experiments conducted.

This brings up the question: Is it mere coincidence that the large observed magnitude of Γ agrees with the ratio between the maximum interaction energy and the melting temperature? The crystal formation can hardly be called a manifestation of strong coupling, because the interaction might be many times stronger. The interaction is most likely the unscreened Coulomb interaction, which is confirmed by abundant experimental evidence for the attraction of like-charged dust particles [29, 30]. Initially, attempts were made to fit the observed Γ values to the value $\Gamma = 170$ predicted numerically for a one-component plasma model by invoking the Debye screening. However, the value of the screening length should then be restricted by certain bounds: the screening radius may differ from the interparticle distance no greater than several-fold, which gives an unacceptably

long screening radius. Furthermore, it became evident that such screening, unlike the linear Debye screening, is nonlinear, because the ratio between the potential energy and the temperature of screening ions ranges from ≈ 300 to 3-10 in a broad domain around the dust particle. The best explanation was obtained with the inclusion of the interaction of flux fields and polarization fields for nonlinear screening [31, 32] responsible for dust particle attraction, which is schematically shown in Fig. 4a. This model not only predicts the correct value of the interparticle distance but also easily explains other observations: the large value of $\Gamma = 1/\psi_m$ [31, 33] and the low ratio between the melting temperature and the maximum interaction energy, which turns out to be equal to $\psi_{\rm m}$, this ratio coinciding with $1/\Gamma$. The screening nonlinearity is significant in this case because it determines the polarization charge distribution in the nonlinear domain near the dust particle and defines the interparticle distance at which an energy minimum of the attractive well occurs, close to the observed interparticle distance $r_{\rm m}$.

The attraction of dust particles exists irrespective of whether the screening is linear or nonlinear, the nonlinearity normally being strong in laboratory experiments, where $\tau = T_i/T_e \approx 10^{-2}$, and most often weak under astrophysical conditions. In all cases, the particle fields are modified by the flows such that they become long-range and extend to distances much longer than the Debye screening radius. There has been significant progress in solving this problem, but it is still unclear how to introduce, even if approximately, free energy in a manifestly non-Hamiltonian system in order to gain the possibility of using standard models of phase transitions. It is pertinent to note that this research shows promise for studying the effects of pairing (of electrons, in particular) in other nonequilibrium systems.

11. Effective gravitational instability in a dust plasma

The long-range attraction of dust particles and the fact that a certain part of their field is not completely screened and extends to long distances may lead to a new, gravitational-type instability in dust plasmas [34, 35]. Only dust particles are subject to this instability, while the ordinary universal gravitational instability acts on any mass. Of course, attraction may also be transferred to other particles via the interaction with the dust particles. In laboratory conditions, this instability may lead to the formation of different structures that may experience a phase transition to the crystal state on further cooling of the dust particles. In astrophysical conditions, it may be associated with the observed structuring of dust clouds with dimensions much shorter than the Jeans length for the ordinary gravitational instability.

The dispersion equation for the effective dust instability coincides in form with the well-known equation for the ordinary gravitational instability $\omega^2 = k^2 v_{s,eff}^2 - G_{eff} m_d n_d$, which is written, for example, for dust particles with a specific mass m_d , size a, and density n_d . The effective speed of sound corresponds to the dust sound speed [35],

$$v_{\rm s, eff}^2 = \frac{Z_{\rm d} P T_{\rm i}}{m_{\rm d} s_{\rm eff}} , \quad s_{\rm eff} = \frac{1+P}{1+z} ,$$

where $P = n_d Z_d/n_i$ is the parameter characterizing the relative charge fraction on dust particles (normally, of the order of unity) and $z = Z_d e^2/aT_e$ is the dimensionless charge of the dust particles (equal to about 2–4). The effective

gravitational constant G_{eff} depends on the coupling constant $k_{\text{eff}}\lambda_{\text{D}}$ of flows to electromagnetic fields ($1/k_{\text{eff}}$ is of the order of the most effective length of the interaction of electrostatic fields and flows) [32]:

$$G_{\rm eff} = \frac{Z_{\rm d}^2 e^2 (k_{\rm eff} \lambda_{\rm D})^2}{m_{\rm d}^2 s_{\rm eff}} \left(k_{\rm eff} \lambda_{\rm D}\right)^2 = \frac{\alpha_{\rm d} z a^2 T_{\rm e}}{T_{\rm i} \lambda_{\rm Di}^2}$$

where α_d is a numerical constant, which is estimated as $\alpha_d \approx 0.16$ and depends on the coefficients determining the charging rate of dust particles and the force of their entrainment by ion flows. The effective Jeans length $L_{\text{eff}} \approx 1/k_{\text{eff}}$, independent of the mass of dust particles and only slightly dependent on their size, may be estimated using the number of ions inside the sphere of the ion Debye radius $N_i = n_i 4\pi \lambda_{\text{Di}}^3/3$ as

$$L_{\rm eff} \approx \lambda_{\rm Di} \; \frac{N_{\rm i}}{Z_{\rm d}} \; \sqrt{\frac{T_{\rm e}(1+z)}{T_{\rm i} \, \alpha_{\rm d} P \left(1-P\right)}}.$$

For typical parameters of laboratory experiments, $a \approx 10 \,\mu\text{m}$, $z \approx 3$, $T_e \approx 3 \text{ eV}$, $m_d \approx 2 \times 10^{-9}$ g, and $P \approx 0.5$, we obtain $G_{\text{eff}} \approx 72.6$ dyn cm² g⁻², i.e., G_{eff} is approximately nine orders of magnitude greater than the ordinary gravitational constant $G = 6.67 \times 10^{-8}$ dyn cm² g⁻². The effective Jeans length and the effective dust particle attraction correspond to those appearing in the explanation for dust crystallization in laboratory experiments. In astrophysical conditions, the effective Jeans length is estimated differently for dust clouds of various types but is normally in the $10^{14} - 10^{17}$ cm range and, as a rule, turns out to be much shorter than the gravitational Jeans length.

Polarization effects also have numerous astrophysical applications.

12. Transition scattering in the interior of the sun and solar neutrinos

Thermonuclear reactions in the interior of the Sun [in its central part, up to approximately $(1/3)R_{\odot}$] heat the interior to $T \approx 1.5$ keV, and the energy transfer to the solar surface is radiatively effected due to scattering by electrons and ions. The scattering by ions is transition scattering and practically replaces the scattering by electrons in the frequency range $\omega_{\rm pe} < \omega \ll \omega_{\rm pe} c / v_{T_{\rm e}}$, where $\omega_{\rm pe}$ is the electron plasma frequency, c is the speed of light, and v_{T_c} is the average thermal electron velocity. The solar opacity is determined by scattering processes, and this coefficient is used in solar models to determine the interior temperature from the observed luminosity and thereby to determine the neutrino flux. Depending on the temperature, most critical is the yield of highest-energy neutrinos in the hydrogen cycle (the boron neutrinos from the decay of ⁷Be, producing ⁸B, in particular), which were measured in David's first experiments with a deficit of 2-3.

A very strong temperature dependence of the boron neutrino yield corresponds to the fact that a temperature decrease by 1-2 K in the solar interior results in a reduction in the number of energetic neutrinos by about a factor of 2. The 1-2 K temperature decrease does not contradict solar seismology data, although the accuracy of solar vibration mode measurements decreases sharply for the modes that extend to the central solar region. That is why the role of ions in the scattering and transfer of radiation in the solar interior did not attract attention until 1987 in connection with the problem of solar neutrino deficit [36]. In Ref. [36], the problem was considered in the framework of scattering by electron fluctuations under the assumption that ions affect these fluctuations. As already discussed in detail, this essentially amounts to describing the transition scattering by ions in the first approximation; in this approximation, the results in Ref. [36] are correct and take the transition scattering by ions into account. The criterion that the scattering by ions is dominant is not quite well satisfied in the solar interior, and therefore about 30-60% of the radiation energy is scattered by ions and accordingly 40-70% by electrons, depending on the radiation frequency. This is because the frequency range for the propagation of electromagnetic waves in the solar interior is rather narrow: $\omega_{\rm pe} \approx 9.78 \times 10^{17} < \omega < T/\hbar \approx 7.62 \times 10^{18} \, {\rm s}^{-1}$, while $c/v_{T_e} \approx 10.1$, and therefore the condition $\omega \ll \omega_{\rm pe}(c/v_{T_e})$ for strong dominance of the scattering by ions is not satisfied, strictly speaking. In the calculations in Ref. [36], the effects of transition scattering by ions in the models of radiation transfer in the solar interior are included only in the first approximation. Further improvements in opacity (after the required revision of the interpretation) called for a clear differentiation between the effects affecting the radiation transfer by ions and those affecting the radiation transfer by electrons. An investigation into additional corrections to the opacity coefficient (there are nine of them in all), which was generalized in Ref. [37], shows that the total of all corrections may amount to 7-12%, which yields an interior temperature decrease by 1-2 K, which is required by high-energy solar neutrinos, for practically invariable proton neutrino fluxes.

13. Polarization corrections to thermonuclear reactions and solar neutrinos

A polarization effect in the solar interior that affects the neutrino yield is the correlation of fluctuations producing the polarization clouds, which, as discussed above, coincide with the Salpeter factor [13] only in the first order. A more detailed analysis of the correlation effects in polarizations and their role in all reactions of the hydrogen cycle (Fig. 3b) was performed in Ref. [15]. It revealed that the corrections to almost all of these reactions are 1.25 - 1.37 times greater than the Salpeter corrections. The latter increase from 5% early in the hydrogen cycle to 20% at the end of the cycle. According to Ref. [15], only correlation effects can be responsible for corrections to the thermonuclear reaction rates, and therefore the corrections increase from 6.5% to 25%. This does not have a marked effect on the predictions for the neutrino radiation from the first reactions of the hydrogen cycle and is in reasonable agreement with observations.

But the correlation corrections for reactions with ⁷Be at the end of the hydrogen cycle are of the opposite sign (suppression rather than enhancement of the reactions) and are three times greater in absolute value than the Salpeter corrections. This is yet another effect that may account for the observed energetic neutrino deficit in David's experiments, irrespective of whether neutrino oscillations exist.

We note that there persist theoretical problems associated with low-energy neutrinos early in the hydrogen cycle. The correlation corrections coincide with the Salpeter ones only in the first order under the assumption of weak screening. However, earlier in the construction of solar models, it was noted that the screening is not very weak and the parameter characterizing its smallness is not much smaller than unity (is equal to about 1/7), and formulas interpolating between weak and strong Salpeter screening were used in constructing solar models. According to Ref. [15], the Salpeter screening is replaced by correlation effects whose theory may be adequately elaborated only for weak correlations. There is no well-established theoretical result for strong correlations that might be used for interpolating the weak-screening result.

14. Stellar evolution

The problem related to correlation effects and to strong screening is aggravated for stars in which the hydrogen cycle was completed and whose combustion is associated with the carbon cycle. The nucleus of carbon ¹²C has Z = 6, and the screening parameter is close to unity or much greater than unity. Weak screening may not be used, and there is still no good theory of strong screening related to strong correlations. The stellar evolution theory therefore invites a certain revision.

15. Sunyaev-Zel'dovich effect and transition scattering

The Sunyaev–Zel'dovich effect corresponds to photon reddening due to the induced scattering in radiation transit through a plasma. At present, only the Thomson scattering by electrons is taken into account and the effect is used for detecting the electron density. At frequencies $\omega < \omega_{pe}c/v_{T_e}$, transition scattering by ions is significant, while for $\omega \ll \omega_{pe}c/v_{T_e}$ transition scattering by ions prevails and the Sunyaev–Zel'dovich effect changes [9]. The wavelength threshold can be written in the form $\lambda > 47$ [m] $\sqrt{T[eV]/n[cm^{-3}]}$ and is manifested in the meter wavelength range for low temperatures and high densities of the plasma.

16. Transition scattering from dust particles forming noctilucent clouds

Noctilucent clouds are observed in the lower ionosphere at the altitude about 90 km in northern latitudes in summer. Radar detection by backscattering has revealed an abnormally high intensity of the scattered signal in comparison with the ordinary signal intensity due to scattering by electrons (approximately two orders of magnitude higher [38]). The simplest explanation is that the scattering is related to the transition scattering involving the nonlinear electron cloud of dust particles, which is consistent with the observation of a very small Doppler shift of the signal frequency indicating a very low scatterer speed. The scattering is proportional to $Z_d^2 n_d = Z_d P n_i$, and for the ordinary values $n_i \sim n_e$, $P \sim 1$, and $Z_d \sim 100$ yields an increase in the scattered signal in qualitative agreement with observations [38].

17. New dust structures. Dust stars

The attraction of dust particles via polarization 'fur coats' may have several astrophysical consequences presently amenable to measurements. The observation of diversified dust structures in laboratory conditions led to the assumption that structuring processes are an inherent property of dust plasmas and may be attributed to the effective gravitational instability of dust systems. There are no grounds to believe that such processes cannot develop under astrophysical conditions in dust clouds. The main implication of the ordinary gravitational instability is the structuring of matter in space, and it is easily seen that the effective gravitational instability in dust plasmas should lead to the structuring of dust clouds. The structural property of such clouds is indeed observed, but it has not been analyzed to what extent it is attributable to the effective gravitational instability of dust plasmas. A prerequisite for this analysis is the progress in describing systems with a size spread of dust particles, but investigations in this area are still in their infancy.

However, one may set up the problem of the final stage of the process, as well as raise the following question: If the ordinary gravitational instability can lead to star formation, can the effective gravitational dust instability lead to the formation of 'dust stars' as isolated objects surrounded by dust-free domains? So far, the existence of stable equilibrium in spherical dust structures has been proven to be possible for all its components — dust, plasma particles, and dust particle charge [38]. The generalization to systems with a size distribution of dust particles and their consequential charge distribution has not been made so far. However, some qualitative implications of investigations performed up to now allow the following preliminary conclusions:

(i) all dust structures should have sharp boundaries;

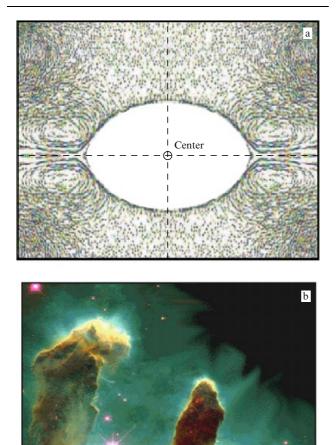


Figure 5. (a) Dust structures observed on the International Space Station. The central dust void, which is produced by plasma flows generated by ionization processes, has very sharp boundaries. On the outside, the dust plasma is surrounded by boundary voids, and convective dust cells are observed at the void periphery. Large-size dust structures break up into dust blobs (dust structures) and dust voids. (b) Example of dust structures with sharp boundaries in a dust nebula observed with the Hubble

telescope.

(ii) dust stars should 'feed on' external plasma streams (i.e., should absorb plasma unlike ordinary stars, being 'antistars' in this respect);

(iii) convective flows caused by the nonpotentiality of the electrostatic forces acting on the dust particles due to the spatial inhomogeneity of their charge should develop in the boundary regions.

Both effects — the sharp boundaries of the dust structures and the formation of dust convection in them — are borne out by experiments onboard the International Space Station (Fig. 5a) [40], while the sharp boundaries of space dust clouds are clearly illustrated by one of the pictures made by the Hubble telescope (Fig. 5b). It is believed that many dust structures (in particular, of the 'dust star' type) might be discovered near the closest stars with the use of the instrumentation of the recently launched Spitzer infrared telescope.

Today, it is quite difficult to answer the question of the possible evolution of 'dust stars' and their possible contribution to hidden mass.

Planar dust structures like planetary rings may exhibit structuring when the gravity of the central planet is weaker than the effects of mutual dust particle attraction. This does not apply to those rings in which large particles (stones) are the main components, whose motion is controlled primarily by the gravity of the central planet. A typical polarization effect like the excitation of the Mach cones of dust sound by a big stone [41] flying under one of Saturn's rings has now been planned for experiments using the Cassini space instrument, which has been successfully orbiting in the Cassini division for more than a year. The attraction of dust particles may show up in the Vavilov–Cherenkov radiation of the dust sound only when the spectrum is measured to sufficiently long wavelengths, of the order of those for which the effective gravitational instability discussed above may be significant.

The aim of this report was to show that the simple and yet extremely keen observations made by Ginzburg in 1940 have far-reaching implications and open up new vistas for laboratory and astrophysical investigations, including investigations into the pairing mechanisms of like-charged particles in nonequilibrium systems.

References

- Ginzburg V L Zh. Eksp. Teor. Fiz. 10 589 (1940); J. Phys. USSR 2 441 (1940)
- Ginzburg V L, Frank I M Zh. Eksp. Teor. Fiz. 16 15 (1946); J. Phys. USSR 9 353 (1945) (brief version)
- Ginzburg V L, Tsytovich V N Perekhodnoe Izluchenie i Perekhodnoe Rasseyanie (Transition Radiation and Transition Scattering) (Moscow: Nauka, 1984) [Translated into English (Bristol: A. Hilger, 1990)]
- Ginzburg V L, Tsytovich V N Zh. Eksp. Teor. Fiz. 65 1818 (1973) [Sov. Phys. JETP 38 909 (1974)]
- 5. Tsytovich V N Lectures on Non-Linear Plasma Kinetics (Berlin: Springer, 1995)
- Tsytovich V N, Oiringel' I M (Eds) Polyarizatsionnoe Tormoznoe Izluchenie Chastits i Atomov (Polarization Bremsstrahlung of Particles and Atoms) (Moscow: Nauka, 1987) [Translated into English (New York: Plenum Press, 1992)]
- Ginzburg V L, Tsytovich V N, in *Materialy Mezhdunarodnoi Konf.* po Perekhodnomu Izlucheniyu (Proc. of the Intern. Conf. on Transient Radiation) (Erevan: EGU, 1985) p. 511
- Sheffield J Plasma Scattering of Electromagnetic Radiation (New York: Academic Press, 1975) [Translated into Russian (Moscow: Atomizdat, 1978) p. 119]

- Bingham R, Tsytovich V N, de Angelis U, Forlani A, Mendonça J T Phys. Plasmas 10 3297 (2003)
- Tsytovich V N *Teoriya Turbulentnoi Plazmy* (Moscow: Atomizdat, 1971) [Translated into English: *Theory of Turbulent Plasma* (New York: Consultants Bureau, 1977)]
- Tsytovich V, in EPS 13 Beyond Einstein Physics for the 21st Century. 13 Gen. Conf. Eur. Phys. Soc., Bern, Switzerland 11-15 July 2005; Europhys. Conf. Abst. 29D 97 (2005)
- 12. Goree J, Morfill G, Tsytovich V N Fiz. Plazmy 24 534 (1998) [Plasma Phys. Rep. 24 490 (1998)]
- 13. Salpeter E E Austral. J. Phys. 7 373 (1954)
- Tsytovich V N, Bornatici M Fiz. Plazmy 26 894 (2000) [Plasma Phys. Rep. 26 840 (2000)]
- Tsytovich V N Zh. Eksp. Teor. Fiz. 121 1080 (2002) [JETP 94 927 (2002)]
- Fortov V, in Invited Papers from the 30th European Physical Society Conf. on Controlled Fusion and Plasma Physics (St. Petersburg, Russia, 7–11 July 2003); Plasma Phys. Controll. Fusion 45 (12A) A1 (2003)
- Tsytovich V N, Morfill G E, Thomas H Fiz. Plazmy 28 675 (2002) [Plasma Phys. Rep. 28 623 (2002)]; Morfill G E, Tsytovich V N, Thomas H Fiz. Plazmy 29 3 (2003) [Plasma Phys. Rep. 29 1 (2003)]; Thomas H, Morfill G E, Tsytovich V N Fiz. Plazmy 29 963 (2003) [Plasma Phys. Rep. 29 895 (2003)]; Tsytovich V N, Morfill G E, Thomas H Fiz. Plazmy 30 877 (2004) [Plasma Phys. Rep. 30 816 (2004)]
- Fortov V E et al. Usp. Fiz. Nauk 174 495 (2004) [Phys. Usp. 47 447 (2004)]; Phys. Rep. 421 1 (2005)
- 19. Thomas H et al. Phys. Rev. Lett. 73 652 (1994)
- 20. Chu J A, Lin I *Physica A* **205** 183 (1994)
- 21. Hayashi Y, Tachibana K Jpn. J. Appl. Phys. 33 L804 (1994)
- 22. Fortov V E et al. Phys. Lett. A 219 89 (1996)
- Pitaevskii L P Zh. Eksp. Teor. Fiz. 39 1450 (1960) [Sov. Phys. JETP 12 1008 (1961)]
- Al'pert Ya L, Gurevich A V, Pitaevskii L P *Iskusstvennye Sputniki v* Razrezhennoi Plazme (Artificial Satellites in Tenuous Plasmas) (Moscow: Nauka, 1964) [Translated into English: Space Physics with Artificial Satellites (New York: Consultants Bureau, 1965)]
- 25. Ikezi H Phys. Fluids 29 1764 (1986)
- 26. Tomme E B, Annaratone B M, Allen J E *Plasma Sources Sci. Technol.* **9** 87 (2000)
- 27. Tsytovich V N Pis'ma Zh. Eksp. Teor. Fiz. 81 563 (2005) [JETP Lett. 81 448 (2005)]
- 28. Tsytovich V Contrib. Plasma Phys. 45 533 (2005)
- 29. Law D et al., in XXIII Intern. Conf. on Phenomena in Ionized Gases; ICPIG, Contrib. Papers Vol. 1 (1997) p. 192
- 30. Annaratone B M et al. Phys. Rev. E 63 036406 (2001)
- 31. Tsytovich V N Zh. Eksp. Teor. Fiz. 131 312 (2007) [JETP 104 279 (2007)]
- Tsytovich V, Morfill G, in Invited Papers from the 31st European Physical Society Conf. on Plasma Physics, London, UK (28 June– 2 July 2004); Plasma Phys. Controll. Fusion. 46 (12B) B527 (2004)
- 33. Konopka U et al. Phys. Rev. E 61 1890 (2000)
- 34. Tsytovich V N, Watanabe K Contrib. Plasma Phys. 43 51 (2003)
- 35. Tsytovich V N Phys. Scripta 74 C81 (2006)
- 36. Boercker D B Astrophys. J. Lett. 316 L95 (1987)
- 37. Tsytovich V N et al. Usp. Fiz. Nauk 166 113 (1996) [Phys. Usp. 39 103 (1996)]
- 38. Havnes O, Aslaksen T, Brattli A Phys. Scripta T89 133 (2001)
- 39. Tsytovich V N Comments Phys. Pt. E Comments Plasma Phys. Controlled Fusion 1 1 (2000)
- 40. Tsytovich V N Fiz. Plazmy **31** 157 (2005) [Plasma Phys. Rep. **31** 133 (2005)]
- 41. Havnes O et al. Planet. Space Sci. 49 223 (2001)