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PACS numbers: 74.20.De, 74.20.Mn, **74.72.–h**
DOI: 10.1070/PU2007v050n05ABEH006223

Ginzburg – Landau equations for high-temperature superconductors

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The phenomenological theory of superconductivity [1] formulated by V L Ginzburg and L D Landau in 1950 (long before the appearance of the Bardeen – Cooper – Schrieffer

(BCS) microscopic theory of superconductivity [2]) predetermined many prospective directions in condensed state physics. The complex order parameter introduced in paper [1] made it possible to describe the transition to the superconducting state as the establishment of phase coherence in an electronic system, while taking account of the gradient contribution to the free energy functional (in the spirit of Ornstein and Zernicke fluctuation theory) allowed consideration of the behavior of a superconducting system in inhomogeneous external fields, in particular, the Meissner effect. Such parameters of the Ginzburg – Landau theory as the coherence length and the penetration depth permitted seeing the difference in the behavior of different superconductors and making their simple classification (type I and type II superconductors [3]). The Ginzburg – Landau equations (derived in 1958 by L P Gor'kov [4] proceeding from the microscopic theory) are the principal instrument for interpretation of experimental data and underlie numerous technical applications.

The 1986 discovery of high-temperature superconductivity [5] and the consequent active experimental and theoretical studies of this unique phenomenon (following the way largely paved by the group of theoreticians headed by Ginzburg [6]) led to the necessity of explaining the properties of new superconductors that did not fit in the usual BCS scheme.

Ginzburg was one of the first to pay attention to the then unknown temperature range lying above the superconducting transition temperature T_c, in which strong fluctuation effects show themselves [7]. It is currently believed that the understanding of the nature of this region of the pseudogap state of high-temperature superconducting (HTSC) cuprates can provide insight into the microscopic mechanism of the superconductivity of these compounds.

Ginzburg's interest in the thermoelectric phenomena in superconductors [8] and in the giant diamagnetism of ordered states with orbital currents [9], which he has shown for over half a century, is now shared by many research workers in connection with the observed anomalous Nernst effect [10 – 12] and the nonlinear-in-field diamagnetism [13] in the region of the strong pseudogap of HTSC cuprates.

To explain the whole set of HTSC cuprate properties in both pseudogap and superconducting states, various theoretical schemes have been proposed, which are mostly based on the assumption that these properties are basically determined by strong electron correlations in copper – oxygen planes [14].

The Coulomb repulsion restricting the double occupation of the copper atom lattice sites in cuprate planes leads to the fact that the parent compound appears to be an antiferromagnetic (AF) insulator. With increasing concentration of carriers incorporated through doping, the long-range AF order is replaced by the short-range order, and the dielectric gap is preserved, thus offering the conditions for the occurrence of superconductivity with an unusual energy-gap symmetry [15]. Hence, strong Coulomb correlations lead not only to a rise of insulating state, but also to cuprate superconductivity.

The possibility of the occurrence of superconductivity in pairing repulsion, first noticed by Landau, was investigated by Kohn and Luttinger [16] for an isotropic degenerate electron gas, and by Moskalenko [17] and Suhl et al. [18] for metals with a two-band electronic spectrum. The estimates obtained in these works lead to rather low T_c values.

Here, we present the phenomenology of large-momentum superconducting pairing during Coulomb repulsion in the framework of the Ginzburg – Landau scheme and consider its application to the interpretation of the phase diagram of doped cuprate compounds.

For finite sections of the Fermi contour in the form of a rounded-corner square [19], which is typical of cuprates, the nesting condition

$$\varepsilon(\mathbf{Q}+\mathbf{p})+\varepsilon(\mathbf{p})=2\mu \tag{1}$$

holds true, where $\varepsilon(\mathbf{p})$ is the dispersion law, and μ is the chemical potential, which leads to dielectric instability of the system. The momentum $\mathbf{Q}$ determines the period of state with a long-range dielectric order. Furthermore, for finite sections of the Fermi contour the mirror nesting condition [20]

$$\varepsilon\left(\frac{\mathbf{K}}{2}+\mathbf{k}\right)=\varepsilon\left(\frac{\mathbf{K}}{2}-\mathbf{k}\right) \tag{2}$$

is fulfilled, which corresponds to the fact that a pair of likely charged particles with momenta $\mathbf{k}_{\pm}=\mathbf{K}/2\pm\mathbf{k}$ belonging to the Fermi contour has the total momentum $\mathbf{K}$ when the momentum $\mathbf{k}$ of the relative motion is determined in a certain part of the Brillouin zone (the kinematically restricted region). The mirror nesting produces instability with respect to singlet superconducting pairing with pair momentum $\mathbf{K}$.

The nesting and mirror nesting of the Fermi contour make possible the development of instability in both the superconducting and a certain insulating channel of pairing upon Coulomb repulsion. In the insulating channel no logarithmic singularity is induced by the mirror nesting which (as distinct from the ordinary nesting) cannot therefore be the reason for a radical transformation of the phonon spectrum.

An approximate mirror nesting takes place in only finite sections of the Fermi contour, and hence the finite density of noncondensate particles is retained up to $T=0$, which is reflected in Drude type behavior of optical conductivity [21] and a quasilinear temperature dependence of heat capacity [22] of cuprates in the superconducting state.

The characteristic form of the superconducting region in the phase diagram of cuprates is determined by two competing factors: with increased doping, the momentum space area making an effective contribution to the order parameter increases, while the length of the Fermi contour sections with mirror nesting decreases. An approximate mirror nesting can lead to superconductivity with large (but generally incommensurate) pair momentum. A further evolution of the Fermi contour with doping [23] makes the channel of pairing with large momentum ineffective. The usual channel of Cooper pairing with zero pair momentum in the electron – phonon interaction (EPI) may also turn out to be ineffective because of the smallness of the Tolmachev logarithm which restricts the coupling constant from below.

Apart from the spin antiferromagnetic and superconducting states with a long-range order, the phase diagram of cuprates with hole doping shows a pseudogap state restricted from above by a certain temperature T^*. The fact that some phase transition corresponds to this temperature has no convincing experimental confirmation, which gives grounds for treating T^* as the temperature of crossover between the pseudogap states for $T_c<T<T^*$ and the normal Fermi liquid for $T>T^*$. The pseudogap behavior can be associated with the insulating short-range order [24] or with the developed fluctuations of the superconducting order parameter for $T>T_c$, which appears possible for a low superfluid density (a low phase stiffness), for which reason the loss of phase coherence occurs earlier than the pair-break of the Cooper pair [25]. In this case, incoherent pairs (a fluctuating superconducting order) can exist in a certain temperature range above T_c. The characteristic width of this interval has the order of T_c and proves to be much lower than T^* in underdoped compounds.

If, as is assumed in Ref. [26], the pseudogap manifests a hidden (hardly detectable) long-range dielectric antiferromagnetic order in the form of a density wave of orbital current with d-wave symmetry, then T^* has the meaning of phase transition temperature. The orbital antiferromagnetism possibly manifests itself as only the short-range order [27], in particular, as the insulating state of the Abrikosov vortex core (which considerably lowers its energy and has an experimental confirmation [28]).

The pseudogap region can conditionally be divided into the regions of a strong pseudogap for $T_c < T < T^*_{str}$, in which the developed fluctuations of the superconducting order parameter induce an increase in the diamagnetic response and a giant Nernst effect, and a weak pseudogap for $T^*_{str} < T < T^*$ with anomalies of some physical properties. The upper boundary T^*_{str} of the strong pseudogap is the temperature of crossover between the regions of weak and developed fluctuations of the superconducting order parameter.

In the scheme of large-momentum pairing, the screened Coulomb repulsion, as distinct from the pairing attraction, allows not only the bound state, but also the long-lived quasistationary states of incoherent pairs [29], which broaden substantially the region of developed fluctuations of the superconducting order parameter at temperatures above T_c and can be associated with the state of the strong pseudogap.

The hidden long-range order in the form of a current-density wave with d-wave symmetry can manifest itself in the relative phase of two components of the superconducting order parameter [31, 32]. The zeros of the superconducting (for extended s-wave symmetry) and orbital antiferromagnetic (corresponding, according to Ref. [26], to the flux-phase [27] possessing d-wave symmetry) order parameters do not coincide, which can be associated with the relative insensitivity of cuprate superconductivity to scattering by nonmagnetic impurities.

The necessary (and sufficient in the case of mirror nesting) condition of superconductivity under repulsion is the existence of at least one negative eigenvalue of the pairing interaction operator. The eigenfunction corresponding to the negative eigenvalue has the line of zeros crossing the Fermi contour in the domain of kinematic constraint. The superconducting energy gap appears to be a function with alternating signs of momentum of the relative motion of a pair inside this region, which vanishes at several points of the Fermi contour [20].

The kinematic constraint is sufficient for one negative eigenvalue to separate from the spectrum of the kernel of the screened Coulomb pairing interaction [33]. Such pairing interaction can approximately be described by a degenerate kernel with two even (with respect to the transformation $\mathbf{k} \to -\mathbf{k}$) eigenfunctions with eigenvalues of opposite signs. Thus, the superconducting ordering upon pairing Coulomb repulsion corresponds to a two-component complex order parameter (conventional superconductivity upon pairing attraction due to EPI is described by a one-component order parameter).

Pairing repulsion leads to the existence of three singular lines with common intersection points in each domain of kinematic constraint corresponding to one of the crystal equivalent pair momenta. One of these lines is part of the Fermi contour on which the pair kinetic energy

$$2\xi(\mathbf{k}) = \varepsilon\left(\frac{\mathbf{K}}{2} + \mathbf{k}\right) + \varepsilon\left(\frac{\mathbf{K}}{2} - \mathbf{k}\right) - 2\mu$$

vanishes because of the mirror nesting (when crossing this line the quasiparticle charge reverses sign). The second singular line is the line of zeros of the order parameter (the intersection points of this line with the Fermi contour correspond to a gapless spectrum of quasiparticles). The group velocity of the quasiparticle vanishes in the line of minima of the quasiparticle energy as a function of momentum [20]. The coherence factors exhibit a nontrivial dependence on the momentum with inhomogeneous distribution of particles in momentum space, which leads to asymmetry of tunnel conductivity, to a peak-dip-hump structure of tunnel and photoemission spectra, and also to a restriction of Andreev reflection in cuprates [20]. The transition to a superconducting state causes a shift (linear in the absolute value of the order parameter) in the chemical potential depending on the ratio of areas of the occupied and vacant parts of the domain of kinematic constraint [34].

In each domain of kinematic constraint one can determine the order parameter in the form of the product of wave functions of the relative motion and free motion of the center-of-mass of a pair with momentum $\mathbf{K}_j$ and radius vector $\mathbf{R}$. In the mean-field approximation, the wave function $\Psi_j(\mathbf{k})$ of the relative motion is proportional to the nontrivial solution of the self-consistent equation. With allowance for the degeneracy due to crystal symmetry, the order parameter is written down as

$$\Psi(\mathbf{R}, \mathbf{k}) = \sum_j \gamma_j \exp(\mathrm{i}\mathbf{K}_j\mathbf{R})\, \Psi_j(\mathbf{k}), \qquad (3)$$

where the domain of definition of momentum $\mathbf{k}$ of the relative motion is the union of all the domains of kinematic constraint, and the coefficients γ_j are determined by the interaction removing the degeneracy typical of pairing with large momentum.

Under dominating EPI-induced attraction, which itself can lead to conventional s-wave superconductivity, all the coefficients γ_j prove to be identical. The function $\Psi_j(\mathbf{k})$ has a line of zeros crossing the Fermi contour in the corresponding domain of kinematic constraint, so that the order parameter has zeros on the Fermi contour (distributed symmetrically about quadrants of the Brillouin zone) and remains invariant under rotation through the angle $\pi/2$ in momentum space. Such order parameter corresponds to extended s-wave symmetry.

The scheme of large-momentum pairing with allowance made for the contribution of the EPI mechanism of pairing [35] provides an explanation of the occurrence of the isotope effect in cuprates, including the negative isotope effect [36].

If the dominating pairing perturbation is the exchange by AF magnons [37, 38], the coefficients γ_j corresponding to neighboring Ξ_j regions have different signs. In this case, when turning through the angle $\pi/2$, the order parameter changes sign and four more zeros are added to the zeros due to pairing repulsion at the intersection points of the Fermi contour with the diagonals of the Brillouin zone. Then, the order parameter can be attributed to the extended d-wave symmetry. In different compounds (or in the bulk or the near-surface layer of one compound) both types of symmetry show themselves [39, 40].

The expansion of the order parameter in terms of the complete orthonormal system of two eigenfunctions $\varphi_s(\mathbf{k})$ of the degenerate kernel $U(\mathbf{k}-\mathbf{k}')$ of the pairing-interaction operator allows defining the order parameter by two of its complex components depending, in the case of a spatially inhomogeneous system, on the radius vector of the center of mass:

$$\Psi(\mathbf{R},\mathbf{k}) = \sum_{s=1}^{2} \Psi_s(\mathbf{R})\,\varphi_s(\mathbf{k})\,. \tag{4}$$

The whole dependence on the momentum of relative motion is transferred to the eigenfunctions defined without regard to the self-consistent equation.

The two-dimensional (calculated for one cuprate plane) free-energy density in the Ginzburg–Landau functional can be given as follows:

$$f = f_0 + f_{\rm g} + f_{\rm m}\,, \tag{5}$$

where f_0 are contributions of the second and fourth order in $\Psi_s(\mathbf{R})$, $f_{\rm g}$ is the gradient term, and $f_{\rm m}$ is the magnetic field energy density.

Expansion of the free-energy density in powers of the order parameter can generally be represented in the form

$$f_0 = \sum_{ss'} A_{ss'}\Psi_s^*\Psi_{s'} + \frac{1}{2}\sum_{ss'tt'} B_{ss'tt'}\Psi_s^*\Psi_{s'}^*\Psi_t\Psi_{t'}\,. \tag{6}$$

Here, the matrices $A_{ss'}$ and $B_{ss'tt'}$ are functions of temperature and doping.

Retaining in the gradient term only the contribution of the second-order in $\nabla\Psi_s$, which is sufficient for a slowly varying $\Psi_s(\mathbf{R})$, we can write the gradient term as

$$f_{\rm g} = \frac{\hbar^2}{4m}\sum_{ss'}[\widehat{D}\Psi_s]^\dagger M_{ss'}[\widehat{D}\Psi_{s'}]\,, \tag{7}$$

where the elements of the matrix $M_{ss'}$ also depend on the temperature and doping, and the covariant differentiation operator has the form

$$\widehat{D} = -{\rm i}\nabla - \frac{2e}{\hbar c}\,\mathbf{A}\,. \tag{8}$$

Here, $\mathbf{A} = \mathbf{A}(\mathbf{R})$ is the vector potential determining the induction of the magnetic field $\mathbf{B} = {\rm rot}\,\mathbf{A}$. Field $\mathbf{A}$ characterizes not only the external magnetic field, but also the internal magnetic field associated with the possible occurrence of spontaneous orbital currents.

The change of the two-dimensional density of the medium free energy in a magnetic field is written out as

$$f_{\rm m} = \frac{z_0}{8\pi}\,({\rm rot}\,\mathbf{A})^2\,, \tag{9}$$

where z_0 is the distance between the neighboring planes.

The matrices determining the expansion of the free energy in power series of the order parameter were calculated in Ref. [41] in the weak coupling approximation.

The components of the order parameter have a common phase factor $\Psi_s = \psi_s \exp({\rm i}\Phi)$. The phase Φ referring to the motion of the center of mass of pairs is associated with establishment of phase coherence in the system of pairs upon transition to the superconducting state. The complex coefficients ψ_s are characterized by the absolute values related to each other by the normalization condition $|\psi_1|^2 + |\psi_2|^2 = n_{\rm sf}/2$ and by the relative phase β: $\psi_2 = \psi_1\exp({\rm i}\beta)$. Thus, for a given superfluid density $n_{\rm sf}$, the relative orbital motion of the pair is characterized by two independent parameters: by one of the modulus (ψ_1 or ψ_2), and by the relative phase β.

The occurrence of a nonzero modulus of the order parameter is associated with violation of gauge symmetry upon transition to the superconducting state, i.e., with the charge degree of freedom of a pair. It is natural to assume that the phase β, which shows up in the gradient term, is associated with the orbital current degree of freedom of the relative motion of the pair.

The state of a spatially homogeneous system is determined by the minimum condition of free-energy density (5). For a temperature of $T > T_{\rm sc}$, where $T_{\rm sc}$ is the superconducting phase transition temperature, the elements of the matrix $A_{ss'}$ are greater than zero, and the minimum of function (5) corresponds to the obvious trivial solution $\psi_1 = \psi_2 = 0$ with an indefinite relative phase β. For $T < T_{\rm sc}$, a nontrivial solution occurs for which the equilibrium values of ψ_1, ψ_2, and β are determined by the values of the matrices $A_{ss'}$ and $B_{ss'tt'}$.

For simplification, we can put $\psi_1 = \psi_2 \equiv \psi$. In the case of a spatially homogeneous system without an external magnetic field, the summands $f_{\rm g}$ and $f_{\rm m}$ are absent in expansion (5). The free-energy density can then be rewritten in the form

$$f_0 = a_1\psi^2 + \frac{1}{2}(B + 2C\cos\beta + D\cos^2\beta)\psi^4\,, \tag{10}$$

where $a_1 = A_{11} + A_{22}$, $B = B_{1111} + 2B_{1122} + B_{2222}$, $C = 2(B_{1112} + B_{1222})$, and $D = 4B_{1122}$. Notice that the simplest approximation corresponding to a symmetric occupation of the domain of kinematic constraint gives $B \neq 0$ and $C = D = 0$. Therefore, for the analysis of possible states in the phase diagram it is necessary to remove this restriction.

The study of function (10) for an extremum at $T < T_{\rm sc}$ reveals that the minimum is reached for $\beta = \pi$ and $\psi \neq 0$, when the condition $C \geqslant D$ holds true or for $\beta < \pi$ and $\psi \neq 0$ if $C \leqslant D$. In the latter case, the relative phase is determined by the relationship $\cos\beta = -C/D$. To distinguish between the two thermodynamically equilibrium SC phases, we shall introduce the order parameter $\alpha = \pi - \beta$. Thus, for $C \geqslant D$ we have $\alpha = 0$, while for $C < D$ we have $\alpha \neq 0$.

The deviation of the relative phase β from π permits an obvious interpretation. The change in the phase of the electron annihilation operator at the site of the crystal lattice $\mathbf{n}$ can be due to the vector potential $\mathbf{A}(\mathbf{n})$ of the magnetic field occurring with the appearance of the orbital antiferromagnetic (OAF) ordering [26]. In the superconducting state, the OAF ordering can appear as AF-correlated circulations of orbital currents [30] surviving also for $T > T_{\rm sc}$.

The occurrence of orbital currents in the superconducting state leads to the necessity of allowing for in the Ginzburg–Landau functional the contribution due to the energy of their magnetic field. This contribution is formally taken into account in the free-energy density by the term $f_{\rm m}$ if we understand $\mathbf{B}$ as magnetic induction of the field of orbital currents. A simple addition to f_0 of a summand of the form $f_{\rm m}(\alpha) = \varkappa\alpha^2$ with positive $\varkappa$ excludes the minimum of the free-energy density for $\alpha \neq 0$. This naturally necessitates a consideration of competition between two pairing channels:

the large-momentum superconducting pairing, and the insulating OAF pairing with the order parameter α.

Since spontaneous orbital currents can also occur in the absence of superconducting order, the free-energy density (in the absence of superconductivity) near an OAF transition may be represented as an expansion in even powers of α:

$$f_{\rm d} = a_2\alpha^2 + \frac{1}{2}\,b_2\alpha^4\,, \qquad (11)$$

where b_2 is a positive doping function, and the coefficient a_2 near the line of an insulating phase transition can be represented as $a_2 = \tau_2 a'$, where $a' > 0$ and $\tau_2 = \big(T - T_{\rm d}(x)\big)/T_{\rm d}(x)$, with $T_{\rm d}(x)$ being the temperature of transition into the OAF state.

The relation between the two types of ordering is determined by the gradient term $f_{\rm g}$ in which the contribution of spontaneous currents to the spatially homogeneous system should be retained. This leads to the appearance in the free energy of the summand $b_{12}\psi^2\alpha^2$, where b_{12} is a doping-dependent phenomenological parameter determined by the matrix $M_{ss'}$.

Thus, the free-energy density describing the competition between the superconducting and OAF-ordered states up to and including fourth-order terms assumes the form

$$f = a_1\psi^2 + a_2\alpha^2 + \frac{1}{2}\,b_1\psi^4 + b_{12}\psi^2\alpha^2 + \frac{1}{2}\,b_2\alpha^4\,, \qquad (12)$$

where the coefficient b_1, as can be seen from expression (10), is determined by the nonzero elements of the matrix $B_{ss'tt'}$. The expansion (12) makes sense only in a small neighborhood of both phase transitions, where the lines $T_{\rm sc}(x)$ and $T_{\rm d}(x)$ either intersect or run near each other.

Doping causes the suppression of orbital antiferromagnetism and it is therefore natural to assume $T_{\rm d}(x)$ and $T_{\rm sc}(x)$ to be decreasing functions of doping. Suppose that for small x the insulating order with the transition temperature $T_{\rm d}(x)$ dominates over superconductivity with the transition temperature $T_{\rm sc}(x)$ and is quickly suppressed by doping. This implies the possibility for the lines $T_{\rm d}(x)$ and $T_{\rm sc}(x)$ to intersect at a certain point (a tetracritical point c) corresponding to the doping x_0.

Minimization of function (12) gives rise to four different phases in the phase diagram.

(1) For $T > \max\big(T_{\rm d}(x), T_{\rm sc}(x)\big)$, the minimum is reached at $\alpha = 0$ and $\psi = 0$, which corresponds to the normal (N) phase. The section $T_{\rm d}(x)$ for $x < x_0$ is the line of phase transition from the N phase to the insulating OAF phase (α phase corresponding to a weak pseudogap), while the line $T_{\rm sc}(x)$ for $x > x_0$ corresponds to the phase transition from the N phase to the superconducting π phase.

(2) The insulating α phase penetrates the temperature range below $T_{\rm sc}(x)$ (the region of a strong pseudogap). The position of the $T_{\rm C}(x)$ line with $x < x_0$ corresponding to a phase transition from the α phase to the superconducting β phase is determined by the condition $b_2a_1 - b_{12}a_2 = 0$. In the α phase, $\psi = 0$ and $\alpha^2 = -a_2/b_2$.

(3) The sector β corresponds to the superconducting β phase in which

$$\psi^2 = -\frac{b_2a_1 - b_{12}a_2}{b_1b_2 - b_{12}^2}\,, \qquad \alpha^2 = -\frac{b_1a_2 - b_{12}a_1}{b_1b_2 - b_{12}^2}\,, \qquad (13)$$

and superconductivity coexists with spontaneous orbital antiferromagnetism. The temperature $T_{\rm C}$ of superconducting phase transition from α to β phase is below $T_{\rm sc}$. Similarly, the temperature $T_{\beta\pi}$ of the phase transition between two superconducting states (β and π phases) is less than $T_{\rm d}$.

(4) In the superconducting π phase, the order parameter has the form $\alpha = 0$, $\psi = -a_1/b_1$. Part of the π phase between $T_{\rm d}(x)$ and $T_{\beta\pi}(x)$ for $x > x_0$ penetrates the temperature range below $T_{\rm d}(x)$.

Apart from the four thermodynamically distinct phases, the diagram shows two regions that can be interpreted as regions of developed fluctuations of the superconducting order parameter (the region between the $T_{\rm sc}(x)$ and $T_{\rm C}(x)$ lines for $x < x_0$) and the OAF order parameter (the region between the $T_{\rm d}(x)$ and $T_{\beta\pi}(x)$ lines for $x > x_0$). In the first of these regions it is the order parameter ψ that fluctuates: incoherent superconducting pairs exist in the form of quasistationary states at temperatures exceeding $T_{\rm C}$ [29]. The fluctuation state of superconducting pairs corresponds to the saddle point (on the ψ-axis) of the free-energy density as a function of ψ and α, close in energy to the minimum on the α-axis. The temperature $T_{\rm sc}$ up to which developed fluctuations of SC pairs exist is not the phase transition temperature and corresponds to the crossover between the two states of the insulating α phase: weak and strong pseudogaps. It should be noted that quasistationary states can also occur at temperatures above $T_{\rm sc}$ [29], thus extending the region relevant to the strong pseudogap.

In the region of developed fluctuations of the insulating order parameter α [between the lines $T_{\rm d}(x)$ and $T_{\beta\pi}(x)$ inside the superconducting state], the free-energy density passes a minimum on the ψ-axis and the saddle point on the α-axis. The free-energy values in the minimum and at the saddle point are close to each other within this region and the line $T_{\rm d}(x)$ has the meaning of crossover which limits conditionally the π-phase region with developed fluctuations of the insulating OAF order parameter α. These fluctuations appear as quasistationary states of orbital circular currents and correspond to the current circulations in the superconducting state, which were investigated in Ref. [30]. Such fluctuations occurring in the mean-field scheme are due to the competition between two ordered states. The second-order phase transition between two superconducting states at $T_{\beta\pi}(x)$ separates the region of conventional superconductivity (π phase), which is in fact described by the one-component order parameter (ψ), from the coexistence region of the insulating state and the SC state (β phase), whose description essentially requires no less than a two-component order parameter. Above the doping level corresponding to the $\beta \to \pi$ transition, a broad region of phase diagram exists which also shows up developed fluctuations. Since such a transition proceeds between two superconducting states, the phase interruption is due not to the motion of the center of mass but to the relative pair motion, i.e., to fluctuations of the relative phase β in the form of quasistationary states of circular orbital currents. Phase interruption of the superconducting order parameter (this phase is due to the motion of the center of mass of the pair) leading to the destruction of superconductivity results from the occurrence of Abrikosov vortices, which is the cause of the anomalous strengthening of the Nernst effect.

Our analysis is, strictly speaking, valid in only a small neighborhood of the tetracritical point c, and so the lines extended beyond this neighborhood have a rather conditional meaning reflecting the general tendencies of their behavior in the neighborhood of point c. In this connection, it should be noted that the extension of the line $T_{\beta\pi}(x)$ to the $T = 0$-axis up to $x = x_{\rm b}$ (the line of second-order phase transition cannot end at a point) naturally leads to the concept of a quantum critical point ($x = x_{\rm b}$, $T = 0$) for a higher doping level $x_{\rm b}$ compared to x_0.

In the case of a short-range rather than a long-range OAF order, the phase transition inside the superconducting state does not occur, and yet the broad region of developed fluctuations at temperatures above $T_{\rm c}$ allows interpretation of the pseudogap state with conditional separation into strong and weak pseudogaps, reflecting one of the admissible versions of the phase diagram of cuprates [42].

The conception of large-momentum superconducting pairing in screened Coulomb repulsion [20], which naturally leads to a two-component order parameter reflecting the charge and current degrees of freedom of the relative pair motion, agrees well on the whole with experimental data for the phase diagram and the physical properties of cuprates.

The work was supported by the Russian Foundation for Basic Research (grants Nos 05-02-17077a and 06-02-17186a).

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PACS numbers: **41.60.–m**, **52.20.–j**, 52.27.Lw
10.1070/PU2007v050n05ABEH006215

Polarization effects in a medium: from Vavilov – Cherenkov radiation and transition radiation to dust-particle pairing, or the development of one of V L Ginzburg's ideas from 1940 to 2006

V N Tsytovich

1. Polarization around particles

In the future general particle theory, with each particle consisting of all the other particles, any particle, being an excitation of the system, will be surrounded by the polarization of these other particles. So far, only the notion of the polarization produced around particles traveling through a medium has been elaborated (Fig. 1a). When the states of the particles change, their polarization 'coats' also change. Figure 1 shows the interaction of particles with external forces, with emitted radiation or incident radiation, with either individual incident particles or a large number of incident particles (i.e., particle fluxes) — the oval S in Fig. 1b. The interparticle interaction depends strongly on perturbations of the polarization cloud during the interaction. The physics of such interactions was first considered by Ginzburg [1].

2. Ginzburg's paper of 1940

In Ginzburg's 1940 paper "Quantum theory of the supersonic radiation of an electron uniformly traveling through a medium", quantum energy and momentum conservation laws for radiation in a medium, $\varepsilon_{\mathbf{p}} = \varepsilon_{\mathbf{p}'} + \hbar\omega_{\mathbf{k}}$ and $\mathbf{p} = \mathbf{p}' + \hbar\mathbf{k}$, were first used; in the system of units where $\hbar = 1$, they become $\varepsilon_{\mathbf{p}} = \varepsilon_{\mathbf{p}'} + \omega_{\mathbf{k}}$ and $\mathbf{p} = \mathbf{p}' + \mathbf{k}$, which in the classical limit ($\mathbf{k} \ll \mathbf{p}$, $\omega_{\mathbf{k}} \ll \varepsilon_{\mathbf{p}}$) leads to the classical Tamm – Frank condition $\omega_{\mathbf{k}} = (\mathbf{kv})$, $\mathbf{v} = {\rm d}\varepsilon_{\mathbf{p}}/{\rm d}\mathbf{k}$ for Vavilov – Cherenkov radiation. Of significance here is (i) the introduction of the photon momentum in the medium and (ii) the clear statement that an exchange of energy and momentum occurs only between the particle and the radiation. Subsequent research led to a deeper understanding and generalization of these statements.