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Forward and backward waves: three definitions and their interrelation and applicability

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Contents

Abstract. **The three known property-specific definitions for forward and backward waves propagating through various media and waveguides are reviewed. Criteria by which these waves can be identified according to their definitions are introduced. It is shown that in some cases using these criteria simultaneously can yield inconsistent or even opposite results. Usability conditions and ranges of applicability of these criteria and the above definitions are specified by employing the example of electromagnetic waves and waveguides.**

1. Introduction

There are three different definitions of forward and backward time-harmonic waves propagating in various media and wave-channeling structures — waveguides. The first definition, which can be thought of as classical [1 – 4], amounts to the following. A wave is considered a forward (backward) wave if the directions of its phase and group velocities are the same (opposite). It is implicitly assumed that the velocities, being vector quantities, are collinear. This definition is generalized in a natural manner to the cases of noncollinear velocities: the scalar product of the phase and group velocities is positive for a forward wave, and negative for a backward wave [5].

Another definition differs from the one given above in that the direction of the group velocity is replaced with the direction of the wave energy (power) flux [6]. And, finally, the third definition differs from the classical one (and the second one) in that the direction of the group velocity (and of the energy flux) is replaced with the direction of exponential decay of the wave's field, caused by dissipation and absorption of the wave energy in the medium or waveguide [7, 8]. And although at first glance the three definitions do not differ very much from the standpoint of physics, they must be distinguished because they are related to different properties of the wave. One must also distinguish the conditions in which they can be applied.

The first definition is based on the dispersion properties of the wave propagation constant with respect to frequency. The group velocity characterizes the process of propagation of a group of harmonic waves in the description of pulse propagation [9]. This definition is usually utilized when the loss in wave energy in the medium or waveguide is not taken into account. In this case, the group velocity proves to be equal to the energy velocity of the wave, i.e., the rate at which the wave transfers its energy [9 – 17]. The second definition is based directly on the energy properties of waves and can also be used when energy loss is taken into account. The third definition is grounded on the analytical properties of the complex-valued wave propagation constant [7, 8]. It is applied when describing waves with allowance for losses or, as is usually said, in lossy media and waveguides.

On the whole, the above definitions of forward and backward waves augment each other, thus expanding the range of their applicability for the above types of wave. In some cases, however, simultaneous use of these definitions leads to contradictions in identifying the type of wave, which is especially true of backward waves (see below). Hence, as said earlier, one must distinguish between the usability conditions and ranges of applicability of each definition, which makes it possible to have unique and consistent concepts concerning forward and backward waves.

Note that in the present work we take into account both known reasons for the emergence of electromagnetic backward waves: the special dispersion properties of the parameters of the medium (the permittivity and the permeability)

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and the structure of waveguides that lead to anomalous frequency dispersion of the wave propagation constant, and the presence of a negative medium, i.e., a medium with negative values of the permittivity and the permeability [6] or the real parts of these parameters with allowance for wave energy loss [7, 8]. Here, we consider media in which the permittivity and the permeability (or their real parts) are negative simultaneously, since we examine waves that propagate through media or waveguides either with no damping or with weak damping associated only with wave energy loss.

2. First definition

Let the wave factor in the expression for the field of time-harmonic waves propagating in homogeneous media and of waves propagating along the axis of regular or periodically irregular waveguides, for example, along the z-axis (with x and y being the transverse coordinates), be of the form

$$\exp\left[\mathrm{i}(\omega t - \gamma z)\right], \tag{1}$$

where ω is the field's circular frequency ($\omega > 0$), and γ is the wave propagation constant which for waves propagating in lossless media and waveguides is a real quantity. The wave phase and group velocities are described (see Refs [1–5, 9–17]) by the following expressions

$$v_{\mathrm{ph}} = \left(\frac{\gamma}{\omega}\right)^{-1}, \qquad v_{\mathrm{gr}} = \left(\frac{\mathrm{d}\gamma}{\mathrm{d}\omega}\right)^{-1}. \tag{2}$$

The above first definition of forward and backward waves suggests the following identification criterion by which the type of wave is determined: if the sign of the coefficient

$$K = \frac{\mathrm{d}\gamma^2}{\mathrm{d}\omega^2} = \frac{\gamma\,\mathrm{d}\gamma}{\omega\,\mathrm{d}\omega} = \frac{1}{v_{\mathrm{ph}} v_{\mathrm{gr}}} \tag{3}$$

is positive, the wave is a forward one, and if it is negative, the wave is backward one. The criterion can be expressed by the following set of inequalities

$$K \gtrless 0, \tag{4}$$

where the upper inequality corresponds to a forward wave, and the lower inequality to a backward wave.

This criterion gets much simpler when we are dealing with plane electromagnetic waves propagating in homogeneous isotropic media for which there exists a simple dispersion relation

$$\gamma^2 = \omega^2 \varepsilon \mu \tag{5}$$

connecting the wave propagation constant with the parameters of the medium: the permittivity ε, and the permeability μ. In this case, the coefficient

$$K = \frac{1}{2}\left(\frac{\mathrm{d}\omega\varepsilon}{\mathrm{d}\omega}\,\mu + \varepsilon\,\frac{\mathrm{d}\omega\mu}{\mathrm{d}\omega}\right) \tag{6}$$

is directly related to the frequency dispersion of the medium or the material dispersion.

The diagrams in Fig. 1 show the typical frequency-dependent functions $\omega\varepsilon(\omega)$ and $\omega\mu(\omega)$ for a lossless medium. The curves correspond to an electromagnetic model of a medium with dipole particles (molecules or the electromagnetic particles of an artificial medium) that have electrical and magnetic resonances at frequencies ω_ε and ω_μ, respectively. From Eqn (6) it follows that if the frequency intervals $(\omega_\varepsilon, \bar{\omega}_\varepsilon)$ and $(\omega_\mu, \bar{\omega}_\mu)$, where ε and μ possess negative values, overlap (this occurs, for example, in an artificial chiral medium [18, 19]), the right-hand side of Eqn (6) is negative at frequencies lying in the overlap region, which corresponds to a backward wave. Here, the wave is a propagating one, since according to relation (5) it has a real-valued propagation constant. Thus, for $v_{\mathrm{gr}} > 0$ we have

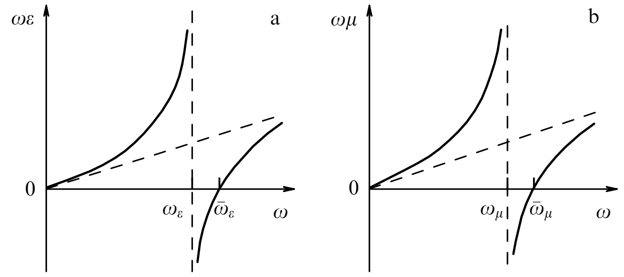

Figure 1. Dispersion curves of the permittivity (a) and the permeability (b) for a dipole electromagnetic model of a medium without accounting for wave energy loss in the medium.

$$v_{\mathrm{ph}} = \frac{\omega}{\gamma} < 0, \qquad \gamma = -\omega\left|(\varepsilon\mu)^{1/2}\right|.$$

For waves in closed and open waveguides that have a homogeneous or piecewise homogeneous (in the cross section) medium [8–17], the dispersion equation and its solution can be represented in the form

$$\begin{aligned} &F(\gamma^2, k_1^2, k_2^2, \ldots, \varepsilon_1, \varepsilon_2, \ldots, \mu_1, \mu_2, \ldots, a_1, a_2, \ldots) = 0, \\ &\gamma^2 = f(k_1^2, k_2^2, \ldots, \varepsilon_1, \varepsilon_2, \ldots, \mu_1, \mu_2, \ldots, a_1, a_2, \ldots), \end{aligned} \tag{7}$$

where $k_n^2 = \omega^2 \varepsilon_n \mu_n$, with ε_m and μ_l being the permittivity and the permeability of the homogeneous parts of the medium; a_χ are frequency-independent geometric parameters describing the dimensions of the waveguide and the dimensions of the homogeneous parts of the medium in the cross section, and n, m, l, and χ are natural numbers. For instance, for waves in a round metal waveguide with an axisymmetric double-layer medium [14–16], one finds

$$F\big((\gamma a)^2, (k_1 a)^2, (k_2 a)^2, \varepsilon_1, \varepsilon_2, \mu_1, \mu_2, a/b\big) = 0, \tag{8}$$

where a is the radius of the inner rod, and b is the waveguide's radius. For an open round homogeneous magneto-dielectric waveguide, in Eqn (8) we must put $\varepsilon_2 = \varepsilon^0$, $\mu_2 = \mu^0$, and $a/b = 0$, where ε^0 and μ^0 are the free-space parameters [9, 11].

If the functions in Eqns (7) and (8) depend analytically on their independent variables, the wave identification factor is given by

$$K = \frac{\mathrm{d}\gamma^2}{\mathrm{d}\omega^2} = \sum_n N_n^C N_n^M + \sum_m M_m^C M_m^M + \sum_l L_l^C L_l^M, \tag{9}$$

where

$$N_n^C = -\left(\frac{\partial F}{\partial \gamma^2}\right)^{-1} \frac{\partial F}{\partial k_n^2} = \frac{\partial f}{\partial k_n^2} = \frac{\partial \gamma^2}{\partial k_n^2}\,,$$

$$M_m^C = -\left(\frac{\partial F}{\partial \gamma^2}\right)^{-1} \frac{\partial F}{\partial \varepsilon_m} = \frac{\partial f}{\partial \varepsilon_m} = \frac{\partial \gamma^2}{\partial \varepsilon_m}\,, \tag{10}$$

$$L_l^C = -\left(\frac{\partial F}{\partial \gamma^2}\right)^{-1} \frac{\partial F}{\partial \mu_l} = \frac{\partial f}{\partial \mu_l} = \frac{\partial \gamma^2}{\partial \mu_l}\,,$$

$$N_n^M = \frac{\mathrm{d}k_n^2}{\mathrm{d}\omega^2} = \frac{1}{2}\left(\frac{\mathrm{d}\omega\varepsilon_n}{\mathrm{d}\omega}\,\mu_n + \varepsilon_n\,\frac{\mathrm{d}\omega\mu_n}{\mathrm{d}\omega}\right),$$

$$M_m^M = \frac{1}{2\omega}\,\frac{\mathrm{d}\varepsilon_m}{\mathrm{d}\omega}\,, \qquad L_l^M = \frac{1}{2\omega}\,\frac{\mathrm{d}\mu_l}{\mathrm{d}\omega}\,. \tag{11}$$

The derivatives in formulas (10) reflect the structural dispersion of the waveguide, while in formulas (11) they reflect the material dispersion of the medium. In particular, if the material dispersion is ignored, one obtains

$$K = \sum_n N_n^C \varepsilon_n\,\mu_n = \sum_n \frac{\partial \gamma^2}{\partial k_n^2}\,\varepsilon_n\,\mu_n\,. \tag{12}$$

It is interesting to note that for waves in closed metal waveguides filled with a homogeneous medium [9–11], one has

$$\gamma^2 = k^2 - g^2\,, \tag{13}$$

where g is frequency-independent, and the coefficient

$$K = \frac{\mathrm{d}\gamma^2}{\mathrm{d}k^2}\,\frac{\mathrm{d}k^2}{\mathrm{d}\omega^2} = \frac{\mathrm{d}k^2}{\mathrm{d}\omega^2} = \frac{1}{2}\left(\frac{\mathrm{d}\omega\varepsilon}{\mathrm{d}\omega}\,\mu + \varepsilon\,\frac{\mathrm{d}\omega\mu}{\mathrm{d}\omega}\right), \tag{14}$$

so that the types of waves are determined, as it is in the case of a plane wave, only by the sign of the material dispersion of the medium (cf. Ref. [8]).

3. Energy velocity

The energy velocity, i.e., the velocity with which the wave transfers energy [9–17], is defined as

$$v_{\mathrm{en}} = \frac{P}{W}\,, \tag{15}$$

where for plane electromagnetic waves one has

$$P = \frac{1}{2}\,\mathrm{Re}\,(\mathbf{E} \times \mathbf{H}^*)\,\mathbf{z}\,, \tag{16}$$

$$W = \frac{1}{4}\left(\frac{\mathrm{d}\omega\varepsilon}{\mathrm{d}\omega}\,|\mathbf{E}|^2 + \frac{\mathrm{d}\omega\mu}{\mathrm{d}\omega}\,|\mathbf{H}|^2\right), \tag{17}$$

and for waves in waveguides it follows that

$$P = \frac{1}{2}\int_S \mathrm{Re}\,(\mathbf{E} \times \mathbf{H}^*)\,\mathbf{z}\,\mathrm{d}S\,, \tag{18}$$

$$W = \frac{1}{4L}\int_V \left(\frac{\mathrm{d}\omega\varepsilon}{\mathrm{d}\omega}\,|\mathbf{E}|^2 + \frac{\mathrm{d}\omega\mu}{\mathrm{d}\omega}\,|\mathbf{H}|^2\right)\mathrm{d}V\,. \tag{19}$$

Here, for a plane wave, P is the power flux through a unit area that is perpendicular to the direction of the flux, and W is the wave energy density, while for a wave propagating in a waveguide, P is the power flux through the waveguide's cross-sectional area S (for an open waveguide, through the entire cross section), W is the wave energy per unit length of a waveguide, L is the length of a section of the waveguide with volume V, and for a periodic waveguide L is the length of a period. As usual, $\mathbf{E}$ and $\mathbf{H}$ are the complex-valued amplitudes of the electric and magnetic field strengths in the waves, and $\mathbf{z}$ is the unit vector along the z-axis. The medium may be inhomogeneous over the waveguide's cross section, but it must be homogeneous or periodically inhomogeneous along the waveguide's axis.

From formulas (15)–(19) it follows that they are valid for both positive ($\varepsilon, \mu > 0$) and negative ($\varepsilon, \mu < 0$) media. When the derivatives $\mathrm{d}\omega\varepsilon/\mathrm{d}\omega$ and $\mathrm{d}\omega\mu/\mathrm{d}\omega$ are positive (see Fig. 1), the wave energy W is positive for both positive values of ε and μ and negative values of these parameters, which agrees with the physical ideas concerning the energy of an electromagnetic field. Notice that a negative electromagnetic medium was first predicted theoretically from the model of an artificial chiral isotropic medium with a frequency dispersion of the parameters [18, 19], so that in Ref. [17] I introduced formulas for the energy W that are more general than Eqns (17) and (19) and are also valid for a chiral medium whose properties include the chiral parameter ρ, in addition to ε and μ.

As is known (see Refs [9–17]), when ε and μ are real, i.e., in the case of waves propagating through lossless media and waveguides, one finds

$$v_{\mathrm{gr}} = v_{\mathrm{en}}\,. \tag{20}$$

As the field frequency approaches the resonance values ω_ε and ω_μ (see Fig. 1), the energy of the wave's field tends to infinity (at finite values of $|\mathbf{E}|$ and $|\mathbf{H}|$), while the group and energy velocities tend to zero. Clearly, all this is an idealization of the physical process.

In this connection, let us examine the example of a plane wave, to which leads the attempt to generalize the concepts of group and energy velocities to include lossy media. When losses in the medium are taken into account, the parameters ε and μ of the medium and, hence, the wave propagation constant γ are complex-valued quantities

$$\varepsilon = \varepsilon' + \mathrm{i}\varepsilon''\,, \qquad \mu = \mu' + \mathrm{i}\mu''\,, \qquad \gamma = \gamma' + \mathrm{i}\gamma''\,, \tag{21}$$

with ε'', μ'', $\gamma'' < 0$. When the losses are small, the following inequalities are valid:

$$|\varepsilon''| \ll |\varepsilon'|\,, \qquad |\mu''| \ll |\mu'|\,, \qquad |\gamma''| \ll |\gamma'|\,. \tag{22}$$

In this case, the real parts of the derivatives of $\omega\varepsilon(\omega)$ and $\omega\mu(\omega)$ near the resonance frequencies ω_ε and ω_μ take the negative values (Fig. 2). Within this frequency range with

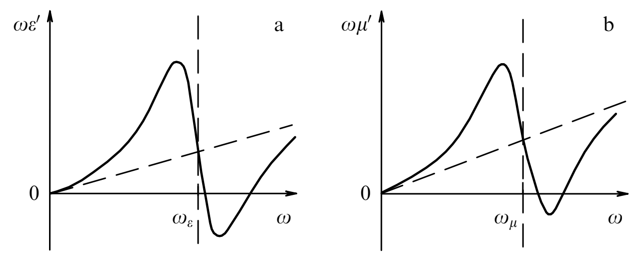

Figure 2. Dispersion curves of the permittivity (a) and the permeability (b) for a dipole electromagnetic model of a medium accounting for wave energy loss.

anomalous dispersion of the medium parameters, the real part of W also proves to be negative, which indicates that it is impossible to interpret it as the energy of the wave's field. Hence, formula (17) for the energy W and the energy velocity (15) have no physical meaning in this frequency range (cf. Refs [9–11]).

Next, if in allowing for small medium loss we think of the phase velocity as specified by the formula

$$v_{\mathrm{ph}} = \operatorname{Re}\left(\frac{\gamma}{\omega}\right)^{-1} = \frac{\omega\gamma'}{|\gamma|^2} \cong \left(\frac{\gamma'}{\omega}\right)^{-1}, \tag{23}$$

where the correction term with a quadratically small quantity on the order of $(\gamma''/\gamma')^2$ is discarded, while the group velocity is assumed to be given by the formula

$$v_{\mathrm{gr}} = \operatorname{Re}\left(\frac{\mathrm{d}\gamma}{\mathrm{d}\omega}\right)^{-1} = \frac{\mathrm{d}\gamma'}{\mathrm{d}\omega}\left|\frac{\mathrm{d}\gamma}{\mathrm{d}\omega}\right|^{-2} \cong \left(\frac{\mathrm{d}\gamma'}{\mathrm{d}\omega}\right)^{-1}, \tag{24}$$

where the quadratically small correction term is also dropped, for the criterion (4) we obtain

$$K \cong \frac{\mathrm{d}\gamma'^2}{\mathrm{d}\omega^2} \cong \operatorname{Re}\frac{\mathrm{d}\gamma^2}{\mathrm{d}\omega^2}\,. \tag{25}$$

Here, for the plane wave (5), one has

$$\operatorname{Re}\frac{\mathrm{d}\gamma^2}{\mathrm{d}\omega^2} \cong \frac{1}{2}\left(\frac{\mathrm{d}\omega\varepsilon'}{\mathrm{d}\omega}\,\mu' + \varepsilon'\,\frac{\mathrm{d}\omega\mu'}{\mathrm{d}\omega}\right). \tag{26}$$

If we now apply criterion (4) to Eqn (26), in the frequency range with anomalous dispersion (see Fig. 2) we find that for $\varepsilon', \mu' > 0$ the wave proves to be of the backward type, while for $\varepsilon', \mu' < 0$ it is a forward wave, i.e., everything is reversed in relation to the frequency range with normal dispersion. Since, as shown earlier, the energy velocity (15) loses all meaning in the anomalous dispersion range but relationship (20) still holds, we may conclude that the physical consequences of using the concept of group velocity in the form (24) in the anomalous dispersion range are meaningless. Notice that this conclusion is related not simply to the quantitative aspect of the correction problem but with the fundamental qualitative aspect of the effect of allowing for losses on the description of the wave properties.

4. Second definition

When wave energy losses are taken into account, expressions (16) and (18) for the power flux P retain their physical meaning, in contrast to expressions (17) and (19) for the wave energy W, both in the normal dispersion range and in the anomalous dispersion range. Allowance for small loss only changes somewhat the quantitative value of P. Hence, for lossy media and waveguides, i.e., with allowance for losses in them, it is natural to use not the first definition of wave types but the second type, replacing the direction (sign) of the group velocity with the direction (sign) of the power flux. Here, for waves propagating in lossless media and waveguides, the directions (signs) of the group velocity and power flux coincide as they do, incidentally, in the normal dispersion range when losses are taken into account.

If the direction of the power flux has been fixed along, say, the z-axis, then $P > 0$ for both forward and backward waves [7, 8]. In this case, the type of wave is determined by the sign of

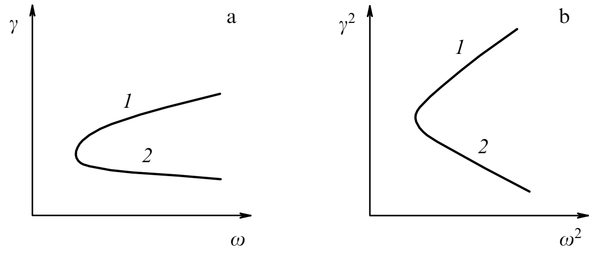

Figure 3. Dispersion curves of the propagation constants for forward and backward waves channelled along a metal waveguide with an inner dielectric rod and a planar waveguide with a negative guiding layer.

the phase velocity or, which is the same, by the sign of the real part of the wave propagation constant γ' in Eqn (23), since $\omega > 0$. This suggests that, contrary to the first definition but in agreement with the second definition, in a lossy medium in the range of anomalous dispersion of $\omega\varepsilon'$ and $\omega\mu'$ (see Fig. 2), as well as in the normal dispersion range, plane waves are of the forward type when $\varepsilon', \mu' > 0$, and of the backward type when $\varepsilon', \mu' < 0$. As shown in Ref. [7], in the case of small losses, we arrive at

$$\frac{\gamma'}{\omega} \cong \begin{cases} \left|(\varepsilon'\mu')^{1/2}\right| & \text{if}\ \ \varepsilon', \mu' > 0\,, \\ -\left|(\varepsilon'\mu')^{1/2}\right| & \text{if}\ \ \varepsilon', \mu' < 0\,. \end{cases} \tag{27}$$

Only material dispersion is possible in the case of plane waves. On the other hand, when the properties of waves in a waveguide are studied, usually only the structural dispersion (12) is taken into account, while the medium parameters ε and μ are assumed to be frequency-independent [8–16]. In this case, Eqn (19) for W assumes the form

$$W = \frac{1}{4L}\int_V \left(\varepsilon|\mathbf{E}|^2 + \mu|\mathbf{H}|^2\right)\mathrm{d}V\,. \tag{28}$$

When ε and μ are real and positive, the energy W is positive, too, and has physical meaning, as well as equality (20) holds true, with the result that both the second and the first definitions of waves can be utilized. To establish the type of wave, it is advisable to use (and this is commonly done; see Refs [8–16]) the first definition, since in this case it is enough to know the dispersion relation alone. However, to establish the direction of the power flux and the sign of P, we must also know (or calculate) the field functions.

For instance, for waves propagating in a metal waveguide, either circular or rectangular, with an inserted dielectric rod ($\varepsilon, \mu > 0$), the propagation constants of some waves have dispersion curves similar to those depicted in Fig. 3 [14–16]. According to criterion (4), from Fig. 3 it follows that the first branch of the curves corresponds to a forward wave, and the second branch to a backward wave.

However, if the permittivity ε and the permeability μ of an inserted rod for a closed waveguide or of a rod guiding the waves for an open waveguide are negative, the situation is quite different. Now Eqn (28) does not describe the wave energy and the energy velocity (15), along with equality (20), lose all physical meaning. In this case, as shown in Ref. [8] for the example of an open planar waveguide with a negative ($\varepsilon, \mu < 0$) guiding layer, the second definition yields the opposite result: the first branch of the curves in Fig. 3 corresponds to a backward wave, and the second branch to a forward wave. This also follows from the third definition of the type of waves [8] when loss is taken into account.

5. Third definition

According to what we said earlier, the application of the third definition of forward and backward waves is possible when we allow for wave energy losses [7, 8] and is based on the use of the analytical properties of the complex-valued wave propagation constant

$$\gamma = \gamma' + \mathrm{i}\gamma'' \tag{29}$$

as a function of the parameters of the medium and the waveguide. When γ is complex-valued, the wave factor (1) in the wave's field function assumes the form

$$\exp\left[\mathrm{i}(\omega t - \gamma' z)\right] \exp \gamma'' z\,, \tag{30}$$

where with $\gamma'' < 0$ the wave field decreases in the direction of the z-axis. From the physics of the problem it follows that this is possible if the energy (power) flux of the wave points in the same direction. This suggests that if the sign of the real part of the wave propagation constant differs from the sign of the imaginary part, the wave is of the forward type; if these parts have the same sign, the wave is a backward one. Since the dispersion relation usually contains γ^2, we can introduce a criterion similar to inequalities (4) for identifying the wave type:

$$\operatorname{Im}\gamma^2 = 2\gamma'\gamma'' \lessgtr 0\,, \tag{31}$$

where the upper inequality corresponds to a forward wave, and the lower inequality to a backward wave. Notice that if the wave factor is represented in the form

$$\exp\left[\mathrm{i}(\gamma z - \omega t)\right] = \exp\left[\mathrm{i}(\gamma' z - \omega t)\right] \exp\left(-\gamma'' z\right) \tag{32}$$

instead of expression (1), the inequality signs in formula (31) must be changed to their opposites.

To apply the third definition (the same goes for the first definition), it is enough to know the dispersion equation alone or its solution for the wave propagation constants. But the analysis and, even more so, the solution of a complex dispersion equation, i.e., an equation for the complex-valued wave propagation constant with complex-valued parameters of the medium, are much more complicated (see Refs [7, 8]) than for a real-valued dispersion equation. However, when the loss is small (the imaginary parts of the quantities in formulas (21) and (22) are small), one can use the analytical properties of γ and perturbation-theory methods to obtain the imaginary parts of γ and γ^2 on the basis of a real dispersion equation. Of course, this is possible if perturbation techniques work, i.e., if allowing for losses does not lead to fundamentally new results, such as, as noted earlier, for plane waves in the frequency range of a medium with anomalous dispersion of the permittivity and permeability (see Fig. 2). In contrast to this case, when only the structural dispersion for waves in a waveguide is taken into account, i.e., when the dispersion of the wave propagation constant depends solely on the geometric structure of the waveguide and on the shape of medium inhomogeneity in the waveguide's cross section, perturbation techniques can be reasonably employed [8, 9, 17, 20, 21].

Let us illustrate this conclusion with the example of a dispersion equation and its solution in the form of Eqn (7). For small imaginary parts of

$$\varepsilon = \varepsilon' + \mathrm{i}\varepsilon''\,, \qquad \mu = \mu' + \mathrm{i}\mu''\,, \tag{33}$$

where $\varepsilon'', \mu'' < 0$, $|\varepsilon''| \ll |\varepsilon'|$, and $|\mu''| \ll |\mu'|$, we obtain

$$\operatorname{Im}\gamma^2 = \sum_n N_n^C \operatorname{Im} k_n^2 + \sum_m M_m^C \varepsilon_m'' + \sum_l L_l^C \mu_l''\,, \tag{34}$$

with

$$\operatorname{Im} k_n^2 = \omega^2 (\varepsilon_n' \mu_n'' + \mu_n' \varepsilon_n'')\,. \tag{35}$$

Here, the structural coefficients N_n^C, M_m^C, and L_l^C are calculated in the same way as in expressions (10), with the use of the real values of γ^2, k_n^2, ε, and μ.

Using criterion (31) and in view of expressions (33)–(35) it is much easier to identify the types of waves than it does by studying in detail the dispersion equation and its solutions, as was carried out in Refs [7, 8] for plane waves propagating in a negative ($\varepsilon', \mu' < 0$) medium and for waves in a planar waveguide with a guiding layer of a negative medium.

Above, we examined the interconnections between the first and second definitions of forward and backward waves via Eqns (15)–(20), and between the second and third definitions via Eqn (30). For the first and third definitions, such relationships are given by expressions (9)–(11) and (34), which reflect in a similar way the structural properties of waveguides, while the material properties of media are reflected differently: in the first definition this is done through the material dispersion of the medium, and in the second through the material losses (wave energy losses in the medium).

To what we have said we should add that if neither the material dispersion of the medium nor wave energy loss are taken into account, then according to formula (12), in the absence of any difference in the structural coefficients N_n^C entering Eqns (10) and (12) for waveguides filled with a negative ($\varepsilon_n, \mu_n < 0$) medium, criterion (4) obtained on the basis of the first definition of the type of wave does not make it possible to identify backward waves, i.e., to distinguish them from forward waves. Waves in homogeneous media (5) and waves in closed waveguides (13) provide examples of this fact. In a more general case, such a situation occurs when the functions describing a dispersion equation and its solution (7) do not contain ε_m and μ_l arguments separately from k_n^2 or when these functions are symmetric with respect to the substitutions

$$\varepsilon_m \to -\varepsilon_m\,, \qquad \mu_l \to -\mu_l\,. \tag{36}$$

6. Conclusion

The above treatment of the problems associated with the three known definitions of forward and backward waves has confirmed the intuitive assumption that the second definition is the more general one and combines the other two. The first definition is equivalent to the second one when wave energy losses in media and waveguides are not accounted for. If, in addition, the frequency dispersion of the permittivity and the permeability of the medium are ignored, then the first definition is equivalent to the second only if both parameters of the medium and waveguide are positive ($\varepsilon, \mu > 0$). The third definition is equivalent to the second one when wave energy losses in media are taken into account.

The criteria for identifying the types of waves, derived from the first and third definitions, can be effectively used in specific studies of waves, since they substantially simplify the calculations associated with the identification of forward and backward waves propagating in various media and wave-channeling structures.

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