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# Instabilities of a multicomponent plasma with accelerated particles and magnetic field generation in astrophysical objects

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## Contents



**Abstract.** **A system of MHD equations for the description of a magnetized nonequilibrium astrophysical plasma with neutral atoms and suprathermal (in particular, relativistic) particles is formulated. The instabilities of such a plasma, which arise from the presence of neutral and relativistic components, are considered. It is shown that the presence of nonthermal particles interacting with the thermal plasma component via regular and fluctuating electromagnetic fields is responsible for the emergence of specific mechanisms of MHD wave generation. The main generation mechanisms of static and turbulent magnetic fields near shock wave fronts in the Galaxy and interplanetary space are analyzed. We discuss the application of the generation effects of long-wave magnetic fluctuations to the problems of magnetic field origin and relativistic particle acceleration in astrophysical objects of various natures.**

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## 1. Introduction

An appreciable admixture of neutral atoms and molecules, along with a charged component, is abundant in many astrophysical objects. In addition to hot, rarefied, and fully ionized caverns (temperature $T \approx 10^6$ K, ion concentration $n \approx 2 \times 10^{-3}$ cm$^{-3}$) in the galactic disk there is a warm phase ($T \approx 10^4$ K, $n \approx 0.2$ particles cm$^{-3}$) with the degree of ionization on the order of 0.1, which occupies a few dozen percent of the volume (see Ruzmaikin et al. [1]). In cold neutral clouds, which are the main star formation sites and quite often contain young active stars which undergo rapid evolution, the density of matter is even 2 – 3 orders of magnitude higher, while the temperature and the degree of ionization are lower. The degree of ionization of the substance in the photospheres of the Sun and numerous stars is also quite low. It is equal to only about $10^{-3}$ in the solar photosphere. In all previously mentioned objects, and in many others, apart from the partially ionized background plasma there are also suprathermal particles, both relativistic (cosmic rays) and nonrelativistic, whose sources may reside both inside the object under consideration and far outside it. It is clear that similar conditions may also exist in objects of terrestrial origin (the ionosphere).

The aim of our work consists in analyzing the low-frequency magnetohydrodynamic (MHD) oscillations which may be excited, sustained, and enhanced in a partially ionized plasma with populations of suprathermal particles. The presence of a neutral component can substantially (by many orders of magnitude) strengthen the dissipation of electromagnetic energy due to ion collisions with neutral atoms. This effect was estimated and investigated earlier (see the monographs by Cowling [2] and Pikel'ner [3]). On the basis of the above studies one might draw the conclusion that the only effect of the admixture of neutrals is that the dissipation of MHD oscillations becomes stronger. However, the situation changes when an extrinsic current produced by an extraneous (relative to the plasma) source (in our case, the source of accelerated particles) is induced in the plasma.

The electrical conductivity $\sigma$ is quite high in a high-temperature fully ionized plasma, resulting in a strong screening of external current by the background particles. The screening is maximized in the dissipation-free limit $\sigma \to \infty$, and there persists only the Hall component of extrinsic current, which has a weak effect on the electromagnetic field and low-frequency oscillations in the plasma.

In the presence of an appreciable neutral component and a constant magnetic field, the transverse (relative to the magnetic field) magnetic viscosity of the medium strongly increases, with the result that the external current screening effect proportionally weakens. As a result, the external accelerated-particle current gains the capability of maintaining and amplifying oscillations in the plasma. Another factor which serves to significantly decrease the transversal plasma conductivity is the effective scattering of plasma particles by small-scale electromagnetic field fluctuations, in particular, by collisionless MHD turbulence. The occurrence of small-scale turbulence in the solar wind plasma was revealed by direct observations in the interplanetary medium. That a very broad spectrum of electromagnetic fluctuations is present in the interstellar plasma also follows from direct and indirect data. The wandering of magnetic lines of force plays a role similar to scattering. They are caused by the random harmonics of the magnetic field with scale lengths significantly exceeding the gyroradii of thermal particles.

This review is concerned with an analysis of the consequences emerging from the multicomponent nature of real cosmic plasmas. The materials generalized in the review show that the presence of nonthermal particles interacting with the thermal plasma component via regular and fluctuating electromagnetic fields is responsible for specific mechanisms of MHD wave generation. In particular, in the vicinity of shock wave fronts which accelerate energetic particles there arises the possibility of nonresonance excitation of long-wave MHD Alfvén type fluctuations with the increment proportional to the effective magnetic viscosity of the plasma and the fraction of nonthermal particles. We analyze the effective electrical conduction and magnetic viscosity in a multicomponent magnetized plasma with a neutral component and MHD fluctuations. The existence of a small, dynamically insignificant neutral component may efficiently suppress the transversal Coulomb conductivity in a magnetized plasma and realize the possibility of efficient nonresonance excitation of long-wave MHD fluctuations in a medium with nonthermal particles. For scales shorter than the Coulomb collision length, the conductivity and kinetic properties of the plasma are determined by scattering from field fluctuations. In this case, the macroscopic description of the system in the framework of collisionless hydrodynamics allows us to investigate the possibility of nonresonance MHD-mode excitation in the medium with a nonthermal component. We discuss the linkage between the generation effects of long-wave magnetic fluctuations and the problems of the origin of magnetic fields and relativistic particle acceleration in astrophysical objects of various natures.

Presently, the most popular mechanism of particle acceleration by shock fronts [4–7] and other statistical mechanisms are effective only when the scattering of accelerated particles is strong enough, which is required for a long confinement of the particles in the acceleration region [8, 9]. Since particle scattering by the MHD turbulence is resonant in nature, MHD oscillations with a wavelength on the order of the Larmor radius of accelerated particles are needed for effective scattering of high-energy particles.

Until recently, the primary emphasis in the literature was placed on resonance mechanisms, both linear [10–14] and nonlinear [15, 16], of turbulence generation by accelerated particles. Being attractive as a source of small-scale fluctuations, the resonance mechanisms are not necessarily effective for the production of long-wave modes responsible for the formation of particle spectra in the region of extremely high energies. As a rule, the accelerated-particle spectrum decreases steeply with energy in the range of maximum particle energies, even though the mechanism of acceleration by a strong shock wave may provide a gently sloping spectrum of particles in the domain of their effective fluctuation-induced scattering. In addition to the resonance production of the modes, this circumstance calls for an efficient mechanism of their conversion to long-wave fluctuations [14] or an efficient mechanism for the amplification of weak background long-wave fluctuations. An advantage of nonresonance mechanisms for the generation of long-wave magnetic field fluctuations is the capability of transferring a substantial amount of energy from nonthermal particles to the magnetic field, even for the spectra of particles accelerated by a shock wave with a compression ratio of $\lesssim 4$ without an appreciable modification of the shock prefront. Large-scale magnetic field fluctuations may be generated by the majority of particles accelerated by the shock wave, irrespective of the magnitude of their gyroradius. Therein lies an important feature of nonresonance mechanisms. This property is quite significant in the determination of the highest energies of the particles accelerated by an MHD shock wave. The problem of determining the maximum energies of particles accelerated by the shocks waves of supernova remnants, which was formulated more than 20 years ago by Lagage and Cesarsky [17] (see also the monograph by Berezinskii et al. [18] and the review by Blandford and Eichler [19]), is still a topical problem [20–22]. In our review, we shall consider at length the nonresonance mechanisms of turbulence generation in plasmas with suprathermal and neutral components. The mechanisms of nonresonance generation of large-scale fluctuations depend on the dispersion and dissipative properties of the multicomponent turbulent plasma with the possible presence of a neutral component. In Section 2 we give the derivation of the basic equations that describe the dynamics and kinetic properties of the multicomponent turbulent plasma. We discuss the generalized Ohm law for a multicomponent magnetized collisionless plasma, which is of significance in astrophysical applications. Considered in Section 12 are possible observational manifestations of the nonresonance mechanisms of MHD mode generation in supernova remnants and shock waves observed in the heliosphere, which allow detailed comparisons.

The possibilities for the formation of specific MHD turbulence spectra and the acceleration of relativistic particles in the weakly ionized plasma of molecular clouds were pointed out in Ref. [23]. The presence of a neutral component is also significant in the formation of the nonthermal emission spectra of supernova remnants interacting with molecular clouds [24].

The nonresonant mechanisms of fluctuation generation in plasmas with a nonequilibrium high-energy component may be caused by buoyancy effects of the multicomponent system with relativistic particles [25] and the occurrence of the current of nonthermal energetic particles [26–28], as well as by the instabilities of MHD plasma flows modified by cosmic ray fluxes in the neighborhood of shock waves [29]. Relativistic particles may play a significant part in disturbing the hydrostatic magnetized-plasma equilibrium with the appreciable pressure of cosmic rays in the large-scale gravitational field of the galaxy [25]. The cosmic-ray pressure effect gives rise to the effective critical value of the polytropic index below which a Rayleigh–Taylor type instability develops in the system. The critical polytropic index may depend on the character of cosmic ray propagation regime (diffusive or convective — see the discussion in Ref. [18]). In this review we do not consider the instabilities related to the gravitational field and restrict ourselves to electromagnetic interactions. Our main concern is with the analysis of multicomponent plasma systems with nonthermal particle fluxes and, in particular, with MHD shock waves. Emphasis is also placed on the derivation of equations which describe the macroscopic flows of multicomponent partially ionized plasma systems with nonthermal particles (Sections 2 and 5), because different dynamic equations are quite often employed in research dedicated to this problem. The generation effects of MHD perturbations by extrinsic currents in a plasma are considered with the inclusion of screening in Sections 3–5 and 7–11. Applications of the generation mechanisms of MHD perturbations to the problem of the enhancement of large-scale magnetic fields in the shells of supernova remnants and shock waves in the interplanetary medium (the only natural laboratory where it is possible to directly measure plasma, nonthermal particle, and field parameters) are discussed in Sections 6 and 12.

Including the kinetics of energetic nonthermal particles in the interstellar plasma may be beneficial to the understanding of the origin problem of magnetic fields on a galactic scale and in galactic clusters [30]. The formation problem of large-scale static magnetic fields with the participation of suprathermal particles is discussed in Section 6 of the present review. The observed structure of galactic magnetic field calls for the inclusion of a large number of modes in the dynamo model or for an alternative model in which the field will be randomly anisotropic, being related to shear and compressible large-scale flows [31]. In modern models of a nonlinear dynamo effect, a significant part can be played by large-scale flows, like a galactic fountain [32]. One would also expect appreciable effects related to energetic particles in the multicomponent plasma which forms large-scale flows (see Ref. [33]). Finally, in Section 6 we consider the reconstruction mechanisms of large-scale stationary magnetic fields in the vicinity of nonthermal particle sources in the arms of the Galaxy.

## 2. Ohm's law and inclusion of an extrinsic current

In the framework of magnetic hydrodynamics, the influence of the microscopic properties of conducting media on the macroscopic dynamics of the system is normally contained in Ohm's law in the generalized form, which defines the relation between the electromagnetic field and the electric current density in a conducting medium (see Ref. [34]). In this case, significant distinctions arise in the MHD description of systems with close Coulomb collisions and of collisionless plasma in which relaxation and transfer processes are determined by long-wave fluctuations of the electromagnetic field of collective excitations. In astrophysical systems with a wide range of spatial scales, which spans many decades, either of the regimes may be realized. In this section we consider Ohm's law in the generalized form both for a magnetized multicomponent collisional plasma with neutral atoms and for a system void of Coulomb collisions and with relaxation due to fluctuating long-wave microfields. We begin with the case of a plasma with Coulomb collisions.

### 2.1 Ohm's law in a partially ionized collisional plasma

We take advantage of the quasihydrodynamic approximation (see Ref. [35]) to establish the relation between the electromagnetic field and the electric current density in a moving, partially ionized three-component medium (e stands for electrons, i for ions, and a for neutral atoms). It is derived from the kinetic Boltzmann equation with a model Bhatnagar–Groos–Krook collision integral. For our purpose it would suffice to employ the continuity equation and the equations of motion of the medium components:

$$\frac{\partial n_\alpha}{\partial t} + \nabla n_\alpha \mathbf{u}_\alpha = 0\,, \qquad \alpha = \mathrm{e}, \mathrm{i}, \mathrm{a}\,, \tag{1}$$

$$n_\alpha m_\alpha \left[\frac{\partial \mathbf{u}_\alpha}{\partial t} + (\mathbf{u}_\alpha \nabla)\mathbf{u}_\alpha\right]_l = e_\alpha n_\alpha \left(\mathbf{E} + \frac{1}{c}\,\mathbf{u}_\alpha \times \mathbf{B}\right)_l$$
$$- \nabla_l P_\alpha - \nabla_k \Pi_{kl}^{(\alpha)} - n_\alpha m_\alpha \sum_\beta \frac{(\mathbf{u}_\alpha - \mathbf{u}_\beta)_l}{\tau_{\alpha\beta}}\,, \tag{2}$$

where $P_\alpha$ is the partial pressure of sort $\alpha$ particles, $\Pi_{kl}^{(\alpha)}$ is the viscous stress tensor, and $\tau_{\alpha\beta}$ are the average times of collisions between the particles of sorts $\alpha$ and $\beta$. The momentum conservation law leads to the interdependence of the collision times:

$$n_\alpha m_\alpha \tau_{\alpha\beta}^{-1} = n_\beta m_\beta \tau_{\beta\alpha}^{-1}\,. \tag{3}$$

In the subsequent discussion we go over to a single-fluid model and consider the conditions whereby all three components move as an aggregate continuum medium. We therefore introduce the bulk velocity of the medium

$$\mathbf{u} = \frac{n_\mathrm{i} m_\mathrm{i} \mathbf{u}_\mathrm{i} + n_\mathrm{a} m_\mathrm{a} \mathbf{u}_\mathrm{a} + n_\mathrm{e} m_\mathrm{e} \mathbf{u}_\mathrm{e}}{n_\mathrm{i} m_\mathrm{i} + n_\mathrm{a} m_\mathrm{a} + n_\mathrm{e} m_\mathrm{e}}\,, \qquad \mathbf{u}_\alpha = \mathbf{u} + \mathbf{v}_\alpha\,, \tag{4}$$

and assume that the deviations of the hydrodynamic velocities of all three components from the bulk velocity are small:

$$v_\alpha \ll u\,, \qquad n_\mathrm{i} m_\mathrm{i} \mathbf{v}_\mathrm{i} + n_\mathrm{a} m_\mathrm{a} \mathbf{v}_\mathrm{a} + n_\mathrm{e} m_\mathrm{e} \mathbf{v}_\mathrm{e} = 0\,. \tag{5}$$

Substituting expression (4) into Eqns (1) and (2) and neglecting the terms quadratic in small additions permits writing down three equations of motion for the components of the medium:

$$n_{\rm e}m_{\rm e}\left[\frac{\partial\mathbf{u}}{\partial t}+(\mathbf{u}\nabla)\mathbf{u}\right]+n_{\rm e}m_{\rm e}\left[\frac{\partial\mathbf{v}_{\rm e}}{\partial t}+(\mathbf{u}\nabla)\mathbf{v}_{\rm e}+(\mathbf{v}_{\rm e}\nabla)\mathbf{u}\right]$$
$$=-en_{\rm e}\left(\mathbf{E}+\frac{1}{c}\,\mathbf{u}\times\mathbf{B}\right)-\frac{en_{\rm e}}{c}\,\mathbf{v}_{\rm e}\times\mathbf{B}$$
$$-\nabla P_{\rm e}+\eta_{\rm e}\left(\Delta\mathbf{u}+\frac{1}{3}\,\nabla(\nabla\mathbf{u})\right)$$
$$-n_{\rm e}m_{\rm e}\,\frac{\mathbf{v}_{\rm e}-\mathbf{v}_{\rm i}}{\tau_{\rm ei}}-n_{\rm e}m_{\rm e}\,\frac{\mathbf{v}_{\rm e}-\mathbf{v}_{\rm a}}{\tau_{\rm ea}}\,, \tag{6}$$

$$n_{\rm i}m_{\rm i}\left[\frac{\partial\mathbf{u}}{\partial t}+(\mathbf{u}\nabla)\mathbf{u}\right]+n_{\rm i}m_{\rm i}\left[\frac{\partial\mathbf{v}_{\rm i}}{\partial t}+(\mathbf{u}\nabla)\mathbf{v}_{\rm i}+(\mathbf{v}_{\rm i}\nabla)\mathbf{u}\right]$$
$$=en_{\rm i}\left(\mathbf{E}+\frac{1}{c}\,\mathbf{u}\times\mathbf{B}\right)+\frac{en_{\rm i}}{c}\,\mathbf{v}_{\rm i}\times\mathbf{B}$$
$$-\nabla P_{\rm i}+\eta_{\rm i}\left(\Delta\mathbf{u}+\frac{1}{3}\,\nabla(\nabla\mathbf{u})\right)$$
$$-n_{\rm i}m_{\rm i}\,\frac{\mathbf{v}_{\rm i}-\mathbf{v}_{\rm e}}{\tau_{\rm ie}}-n_{\rm i}m_{\rm i}\,\frac{\mathbf{v}_{\rm i}-\mathbf{v}_{\rm a}}{\tau_{\rm ia}}\,, \tag{7}$$

$$n_{\rm a}m_{\rm a}\left[\frac{\partial\mathbf{u}}{\partial t}+(\mathbf{u}\nabla)\mathbf{u}\right]+n_{\rm a}m_{\rm a}\left[\frac{\partial\mathbf{v}_{\rm a}}{\partial t}+(\mathbf{u}\nabla)\mathbf{v}_{\rm a}+(\mathbf{v}_{\rm a}\nabla)\mathbf{u}\right]$$
$$=-\nabla P_{\rm a}+\eta_{\rm a}\left(\Delta\mathbf{u}+\frac{1}{3}\,\nabla(\nabla\mathbf{u})\right)$$
$$-n_{\rm a}m_{\rm a}\,\frac{\mathbf{v}_{\rm a}-\mathbf{v}_{\rm e}}{\tau_{\rm ae}}-n_{\rm a}m_{\rm a}\,\frac{\mathbf{v}_{\rm a}-\mathbf{v}_{\rm i}}{\tau_{\rm ai}}\,. \tag{8}$$

The terms of order $v_\alpha/u$ in small dissipative summands containing the dynamic viscosity $\eta_\alpha$ were neglected in the above equations.

We next take into account the quasineutrality of the medium and introduce the total mass density

$$\rho=n_{\rm i}(m_{\rm i}+m_{\rm e})+n_{\rm a}m_{\rm a}\approx n_{\rm i}m_{\rm i}+n_{\rm a}m_{\rm a} \tag{9}$$

and the electric current density

$$\mathbf{j}=en_{\rm i}(\mathbf{v}_{\rm i}-\mathbf{v}_{\rm e})\,, \tag{10}$$

as well as the total pressure $P=P_{\rm e}+P_{\rm i}+P_{\rm a}$ and the total viscosity $\eta=\eta_{\rm e}+\eta_{\rm i}+\eta_{\rm a}$. Term-by-term addition of Eqns (6)–(8) yields the equation of motion of the medium as a whole:

$$\rho\left[\frac{\partial\mathbf{u}}{\partial t}+(\mathbf{u}\nabla)\mathbf{u}\right]=\frac{1}{c}\,\mathbf{j}\times\mathbf{B}-\nabla P+\eta\left(\Delta\mathbf{u}+\frac{1}{3}\,\nabla(\nabla\mathbf{u})\right). \tag{11}$$

From Eqns (1) we obtain the continuity equation

$$\frac{\partial\rho}{\partial t}+\nabla\rho\mathbf{u}=0\,. \tag{12}$$

The friction forces between medium components have not entered into Eqn (11) by virtue of fulfilling conditions (3).

To obtain the generalized Ohm law, we take advantage of Eqns (6), (7), and (11). In Eqn (11), the last dissipative term on the right-hand side is taken to be small and is dropped. The quantity $\partial\mathbf{u}/\partial t+(\mathbf{u}\nabla)\mathbf{u}$ in Eqns (6) and (7) is expressed with the aid of Eqn (11) and the terms of the form $\partial\mathbf{v}_{\rm i,e}/\partial t+(\mathbf{u}\nabla)\mathbf{v}_{\rm i,e}$ are discarded. The last-named approximation implies that the frequencies $\omega$ of the oscillations under discussion are low in comparison with collision frequencies $\tau_{\alpha\beta}^{-1}$, and the wavelengths are long in comparison with the particle transport mean free paths $\Lambda_\alpha=v_{T\alpha}/\sum_\beta\tau_{\alpha\beta}^{-1}$:

$$\omega\ll\tau_{\rm ei}^{-1},\,\tau_{\rm ea}^{-1}\,,\qquad\lambda=\frac{2\pi}{k}\gg\Lambda_{\rm e},\,\Lambda_{\rm i}\,, \tag{13}$$

where $v_{T\alpha}$ are the thermal velocities. Lastly, we introduce the mass fraction of the neutral component

$$F=\frac{n_{\rm a}m_{\rm a}}{n_{\rm a}m_{\rm a}+n_{\rm i}m_{\rm i}}\approx\frac{n_{\rm a}}{n_{\rm a}+n_{\rm i}}\,, \tag{14}$$

on the assumption that there is only one sort of singly charged ions in the plasma, so that $m_{\rm a}\approx m_{\rm i}$. We also assume that the ratio between the concentrations of ions and neutral atoms persists in the collisional plasma in the presence of low-frequency oscillations, i.e., $n'_{\rm a}/n'_{\rm i}=n_{\rm a}/n_{\rm i}$ and $F=\text{const}$, where $n'_{\rm i,a}$ are the concentration perturbations.

Then we express the electron and neutral component velocities, $\mathbf{v}_{\rm e}$ and $\mathbf{v}_{\rm a}$, in terms of the current density and the ion velocity $\mathbf{v}_{\rm i}$:

$$\mathbf{v}_{\rm e}=\mathbf{v}_{\rm i}-\frac{\mathbf{j}}{en_{\rm i}}\,,\qquad\mathbf{v}_{\rm e}-\mathbf{v}_{\rm a}\approx\frac{\mathbf{v}_{\rm i}}{F}-\frac{\mathbf{j}}{en_{\rm i}}\,,\qquad\mathbf{v}_{\rm i}-\mathbf{v}_{\rm a}\approx\frac{\mathbf{v}_{\rm i}}{F}\,. \tag{15}$$

Terms of the order of $m_{\rm e}/m_{\rm i}$ were dropped. In what follows we also discard the small terms on the order of $(m_{\rm e}/m_{\rm i})^{1/2}\ll 1$. Around this order of magnitude has, in particular, the ratio

$$\frac{m_{\rm e}\tau_{\rm ia}}{m_{\rm i}\tau_{\rm ea}}=\frac{m_{\rm e}v_{T{\rm e}}\Lambda_{\rm ia}}{m_{\rm i}v_{T{\rm i}}\Lambda_{\rm ea}}=\left(\frac{m_{\rm e}}{m_{\rm i}}\right)^{1/2}\frac{\sigma_{\rm ea}}{\sigma_{\rm ia}}\approx\left(\frac{m_{\rm e}}{m_{\rm i}}\right)^{1/2},$$

where $\sigma_{\rm ia,ea}$ denote the cross sections for collisions between charged and neutral particles.

On term-by-term addition of Eqns (6) and (7), upon the above simplifications we derive from the resultant equation the relation between the ion velocity $\mathbf{v}_{\rm i}$ and the current:

$$\mathbf{v}_{\rm i}\approx\frac{F^2\tau_{\rm ia}}{n_{\rm i}m_{\rm i}c}\,\mathbf{j}\times\mathbf{B}+\frac{Fm_{\rm e}\tau_{\rm ia}}{en_{\rm i}m_{\rm i}\tau_{\rm ea}}\,\mathbf{j}\,. \tag{16}$$

By eliminating the velocity $\mathbf{v}_{\rm i}$ from Eqn (7) with the account for relation (16) we arrive at the generalized Ohm law — the relationship between the electromagnetic field, the current, and the hydrodynamic parameters of the medium:

$$\mathbf{E}+\frac{1}{c}\,\mathbf{u}\times\mathbf{B}+\frac{1}{en_{\rm i}}\,\nabla\big[(1-F)P-P_{\rm i}\big]=\frac{\mathbf{j}}{\sigma}+\frac{1}{n_{\rm i}ec}\,\mathbf{j}\times\mathbf{B}$$
$$+\frac{F^2\tau_{\rm ia}}{n_{\rm i}m_{\rm i}c^2}\,\mathbf{B}\times(\mathbf{j}\times\mathbf{B})\,. \tag{17}$$

The pressure gradients also enter into this relationship to produce an additional effective electric field. The quantity

$$\sigma=\frac{e^2n_{\rm i}\tau_{\rm e}}{m_{\rm e}}\,,\quad\text{where}\quad\tau_{\rm e}=\frac{\tau_{\rm ei}\tau_{\rm ea}}{\tau_{\rm ei}+\tau_{\rm ea}} \tag{18}$$

is the plasma conductivity with due regard for neutral atoms and in the absence of an external magnetic field. In a 'cold' plasma, wherein the pressure $P$ can be neglected, Ohm's law takes on the form

$$\mathbf{E} + \frac{1}{c}\,\mathbf{u}\times\mathbf{B} = \frac{\mathbf{j}}{\sigma} + \frac{1}{n_{\mathrm{i}}ec}\,\mathbf{j}\times\mathbf{B} + \frac{F^2\tau_{\mathrm{ia}}}{n_{\mathrm{i}}m_{\mathrm{i}}c^2}\,\mathbf{B}\times(\mathbf{j}\times\mathbf{B})\,. \quad (19)$$

If the electric current $\mathbf{j} = \mathbf{j}_{\parallel} + \mathbf{j}_{\perp}$ is separated into two components parallel and perpendicular to the magnetic field $\mathbf{B}$, Ohm's law can be presented as

$$\mathbf{E} + \frac{1}{c}\,\mathbf{u}\times\mathbf{B} = \frac{\mathbf{j}_{\parallel}}{\sigma} + \frac{B}{n_{\mathrm{i}}ec}\,\mathbf{j}\times\mathbf{e}_{\parallel} + \frac{\mathbf{j}_{\perp}}{\sigma_{\perp}^{\mathrm{eff}}}\,, \quad (20)$$

where $\mathbf{e}_{\parallel} = \mathbf{B}/B$, and

$$\frac{1}{\sigma_{\perp}^{\mathrm{eff}}} = \frac{1}{\sigma} + \frac{F^2B^2\tau_{\mathrm{ia}}}{n_{\mathrm{i}}m_{\mathrm{i}}c^2}\,. \quad (21)$$

Hence, it follows that the quantities $\sigma$ and $\sigma_{\perp}^{\mathrm{eff}}$ play the respective parts of the effective longitudinal and transversal conductivities of a partially ionized plasma. The relationships inverse to equation (20) are of the form

$$\mathbf{j}_{\parallel} = \sigma\mathbf{E}_{\parallel}\,,$$
$$\mathbf{j}_{\perp} = \sigma_{\perp}\left(\mathbf{E}_{\perp} + \frac{1}{c}\,\mathbf{u}\times\mathbf{B}\right) - \sigma_{\perp}\,\frac{B\sigma_{\perp}^{\mathrm{eff}}}{ecn_{\mathrm{i}}}\left(\mathbf{E}\times\mathbf{e}_{\parallel} - \frac{B}{c}\,\mathbf{u}_{\perp}\right), \quad (22)$$

where

$$\frac{1}{\sigma_{\perp}} = \frac{1}{\sigma_{\perp}^{\mathrm{eff}}}\left[1 + \left(\frac{B\sigma_{\perp}^{\mathrm{eff}}}{ecn_{\mathrm{i}}}\right)^2\right]. \quad (23)$$

In the absence of the neutral component, $F = 0$, $\sigma_{\perp}^{\mathrm{eff}} = \sigma = e^2n_{\mathrm{i}}\tau_{\mathrm{ei}}/m_{\mathrm{e}}$, $\sigma_{\perp}^{-1} = \sigma^{-1}(1 + \omega_{B\mathrm{e}}^2\tau_{\mathrm{ei}}^2)$, and we have the well-known relation for the transverse current

$$\mathbf{j}_{\perp} = \frac{\sigma}{1+\omega_{B\mathrm{e}}^2\tau_{\mathrm{ei}}^2}\left(\mathbf{E}_{\perp} + \frac{1}{c}\,\mathbf{u}\times\mathbf{B}\right) - \frac{\sigma\omega_{B\mathrm{e}}\tau_{\mathrm{ei}}}{1+\omega_{B\mathrm{e}}^2\tau_{\mathrm{ei}}^2}\left(\mathbf{E}\times\mathbf{e}_{\parallel} - \frac{B}{c}\,\mathbf{u}_{\perp}\right), \quad (24)$$

where $\omega_{B\mathrm{e}} = eB/m_{\mathrm{e}}c$ is the electron cyclotron frequency.

Relationships (17)–(23) show that the inclusion of neutral particles substantially complicates the current–electromagnetic field linkage and enhances its nonlinearity. We therefore write out the system of equations in a linearized form with the inclusion of a weak external current $\mathbf{j}^{\mathrm{ext}}$, which will allow us to investigate low-frequency low-amplitude oscillations. The quantities $\mathbf{b} = \mathbf{B} - \mathbf{B}_0$, $\mathbf{E}$, and $\mathbf{u}' = \mathbf{u} - \mathbf{u}_0$, as well as density and pressure perturbations $\rho'$ and $P'$, are assumed to be small. We assume the cold medium approximation: the thermal conduction is neglected, the velocity of sound $c_{\mathrm{s}}$ is below the Alfvén velocity $v_{\mathrm{A}} = B_0/\sqrt{4\pi\rho}$, and the kinematic viscosity $\nu \to 0$. In this case, one has $P' \approx c_{\mathrm{s}}^2\rho'$. The background quantities $\mathbf{B}_0$, $\mathbf{u}_0$, $\rho$, and $P$ are taken to be uniform. It is pertinent to note that we consider a collisional plasma and take into account collisions between all three components. The Alfvén velocity is therefore defined in terms of the total density of the medium, including the neutral component.

We employ the equation of motion (11), generalized Ohm's law (17), and Maxwell equations

$$\nabla\mathbf{B} = 0\,, \qquad \nabla\times\mathbf{B} = \frac{4\pi}{c}\,(\mathbf{j} + \mathbf{j}^{\mathrm{ext}})\,, \qquad \nabla\times\mathbf{E} = -\frac{1}{c}\,\frac{\partial\mathbf{B}}{\partial t}\,. \quad (25)$$

Then, the linearized equations are written out in the form†

$$\frac{\partial\mathbf{b}}{\partial t} + (\mathbf{u}_0\nabla)\mathbf{b} - B_0\big[(\mathbf{e}_{\parallel}\nabla)\mathbf{u}' - \mathbf{e}_{\parallel}\nabla\mathbf{u}'\big]$$
$$= \nu_{\mathrm{m}}\Delta\mathbf{b} + (\nu_{\mathrm{eff}} - \nu_{\mathrm{m}})\big[\mathbf{e}_{\parallel}\Delta b_{\parallel} - (\mathbf{e}_{\parallel}\nabla)\nabla_{\perp} b_{\parallel} + (\mathbf{e}_{\parallel}\nabla)^2\mathbf{b}_{\perp}\big]$$
$$- \frac{cB_0}{4\pi en_{\mathrm{i}}}\,(\mathbf{e}_{\parallel}\nabla)(\nabla\times\mathbf{b}) +$$
$$+ \frac{4\pi\nu_{\mathrm{m}}}{c}\,\nabla\times\left[\mathbf{j}^{\mathrm{ext}} - (\mathbf{j}^{\mathrm{ext}}\mathbf{e}_{\parallel})\,\frac{\mathbf{b}}{B_0} - \mathbf{e}_{\parallel}\,\frac{\mathbf{b}_{\perp}\mathbf{j}^{\mathrm{ext}}}{B_0}\right] + \frac{B_0}{n_{\mathrm{i}}e}\,(\mathbf{e}_{\parallel}\nabla)\,\mathbf{j}^{\mathrm{ext}}$$
$$+ \frac{4\pi}{c}\,(\nu_{\mathrm{eff}} - \nu_{\mathrm{m}})\big[\mathbf{e}_{\parallel}\big(\mathbf{e}_{\parallel}[\nabla\times\mathbf{j}^{\mathrm{ext}}]\big) + (\mathbf{e}_{\parallel}\nabla)(\mathbf{e}_{\parallel}\times\mathbf{j}^{\mathrm{ext}})\big]\,, \quad (26)$$

$$\frac{\partial\mathbf{u}'}{\partial t} + (\mathbf{u}_0\nabla)\,\mathbf{u}' = -\frac{c_{\mathrm{s}}^2}{\rho}\,\nabla\rho' + \frac{B_0}{4\pi\rho}\,(\nabla\times\mathbf{b})\times\mathbf{e}_{\parallel}\,, \quad (27)$$

$$\frac{\partial\rho'}{\partial t} + \mathbf{u}_0\nabla\rho' + \rho\nabla\mathbf{u}' = 0\,. \quad (28)$$

Here, the unit vector $\mathbf{e}_{\parallel}$ is aligned with $\mathbf{B}_0$, and

$$\nu_{\mathrm{m}} = \frac{c^2}{4\pi\sigma}\,, \qquad \nu_{\mathrm{eff}} = \frac{c^2}{4\pi\sigma_{\perp}^{\mathrm{eff}}} \quad (29)$$

are the Coulomb collisional magnetic viscosity and the effective magnetic viscosity with the inclusion of the neutral component, respectively. The quantity $\sigma_{\perp}^{\mathrm{eff}}$ is given by formula (21) in which $B = B_0$.

To make more lucid the relatively complex structure of Eqn (26), we write it out for the special case of an incompressible medium in a simplified geometry, when the plasma current $\mathbf{j}$ and the extrinsic current $\mathbf{j}^{\mathrm{ext}}$ are directed transversely to the field $\mathbf{B}_0$:

$$\frac{\partial\mathbf{b}}{\partial t} + (\mathbf{u}_0\nabla)\,\mathbf{b} = (\mathbf{B}_0\nabla)\,\mathbf{u}' + \nu_{\mathrm{eff}}\Delta\mathbf{b}$$
$$- \frac{c}{4\pi en_{\mathrm{i}}}\,(\mathbf{B}_0\nabla)(\nabla\times\mathbf{b}) + \frac{4\pi\nu_{\mathrm{eff}}}{c}\left[\nabla\times\mathbf{j}^{\mathrm{ext}} - \frac{\mathbf{j}^{\mathrm{ext}}\mathbf{e}_{\parallel}}{B_0}\,\nabla\times\mathbf{b}\right]$$
$$+ \frac{1}{en_{\mathrm{i}}}\,(\mathbf{B}_0\nabla)\,\mathbf{j}^{\mathrm{ext}}\,. \quad (30)$$

This equation differs from that which describes the magnetic field in fully ionized plasma by the value of effective magnetic viscosity. Let us estimate the ratio

$$\frac{\nu_{\mathrm{eff}}}{\nu_{\mathrm{m}}} = 1 + \frac{F^2B_0^2\tau_{\mathrm{ia}}\sigma}{n_{\mathrm{i}}m_{\mathrm{i}}c^2} = 1 + F^2(\omega_{\mathrm{i}}\tau_{\mathrm{ia}})(\omega_{\mathrm{e}}\tau_{\mathrm{ei}})\,, \quad (31)$$

where $\omega_{\mathrm{i,e}} = eB_0/m_{\mathrm{i,e}}c$ are the cyclotron frequencies, for several typical astrophysical objects (see Table 1). A purely hydrogenous medium was considered in all cases, with the exception of the ionosphere. A value of $\sigma_{\mathrm{ia}} \approx 10^{-14}$ cm$^2$ was taken for the ion–atom collision cross section. The last column of Table 1 gives the quantity $K = F^2(\omega_{\mathrm{i}}\tau_{\mathrm{ia}})(\omega_{\mathrm{e}}\tau_{\mathrm{ei}})$ which characterizes the contribution of neutral atoms to the evolution of the magnetic field. The penultimate row of Table 1 lists the parameters of a photoionized intergalactic medium for a density which corresponds to the domain 200 times above the mean baryon density at $z = 0$.

As is clear from Table 1, in all the cases considered above, except for the solar photosphere, the contribution of the

† Two formulas (26) and (30) were corrected by the authors when proof reading the English-translated text. (*Editor's note.*)

**Table 1.**

| Object | $B_0$, G | $n_a$, cm$^{-3}$ | $n_i$, cm$^{-3}$ | $F$ | $T$, K | $\Lambda_{ia}$, cm | $\omega_i\tau_{ia}$ | $\omega_e\tau_{ei}$ | $K$ |
|---|---|---|---|---|---|---|---|---|---|
| Neutral clouds | $1.5\times10^{-5}$ | 20 | 0.05 | $\approx 1$ | 100 | $5\times10^{12}$ | $4\times10^{6}$ | $5\times10^{4}$ | $2\times10^{11}$ |
| Warm intercloud medium | $2\times10^{-6}$ | 0.2 | 0.03 | 0.85 | $10^4$ | $5\times10^{14}$ | $6\times10^{6}$ | $10^7$ | $5\times10^{13}$ |
| Solar photosphere | 1.0 | $10^{16}$ | $10^{13}$ | $\approx 1$ | $6\times10^{3}$ | $1\times10^{-2}$ | $8\times10^{-5}$ | $8\times10^{-3}$ | $6\times10^{-7}$ |
| Cosmological medium ($z<1$) | $10^{-9}$ | $5\times10^{-9}$ | $4\times10^{-5}$ | $10^{-4}$ | $10^4$ | $2\times10^{22}$ | $1\times10^{11}$ | $5\times10^{5}$ | $10^9$ |
| Terrestrial ionosphere | 1.0 | $2\times10^{8}$ | $10^6$ | $\approx 1$ | $10^3$ | $5\times10^{5}$ | $10^4$ | $5\times10^{3}$ | $5\times10^{7}$ |

neutral component is quite significant even when the fraction of neutral atoms is smaller than that indicated in the table. This circumstance is of significance for particle acceleration processes near the shock fronts, where the degree of ionization may be augmented due to the heating of the medium.

The contribution of the Hall terms to Eqn (30) is defined by the ratio

$$\frac{cB_0}{4\pi e n_i \nu_{\rm eff}} \approx \frac{1}{F^2\omega_i\tau_{ia}}\,. \qquad (32)$$

For $F^2\omega_i\tau_{ia} \gg 1$, the terms indicated play a small part. However, the Hall terms prevail over all the remaining ones in the parameter range defined by inequalities $K \gg 1$ and $F^2\omega_i\tau_{ia} \ll 1$.

An important property of equation (30) consists in the effective magnetic viscosity $\nu_{\rm eff}$ being a common factor for the dissipative term $\Delta\mathbf{b}$ and the term with extrinsic current $\nabla\times\mathbf{j}^{\rm ext}$. That is why the $\nu_{\rm eff}\big(\Delta\mathbf{b} + (4\pi/c)\nabla\times\mathbf{j}^{\rm ext}\big)$ tandem can either strengthen dissipation or convert it into oscillation build-up, depending on the properties of the extrinsic current.

## 2.2 Ohm's law in a collisionless magnetized plasma

In several cases of importance, the macroscopic description of a rarefied plasma with rare Coulomb collisions becomes possible due to the effects of particle scattering by the stochastic fluctuations of an electromagnetic field, fluctuations which accompany collective plasma oscillations (see Refs [34, 36–39]). Below in this section we shall calculate the effective conductivity of such a plasma for the simplest stochastic field models under the assumption that the plasma medium is statistically homogeneous.

We proceed from the kinetic equation for plasma electrons and ions taking into account the fluctuating electromagnetic fields and neglecting Coulomb collisions:

$$\frac{\partial f_{e,i}}{\partial t} + \mathbf{v}\,\frac{\partial f_{e,i}}{\partial \mathbf{r}} + e(\mathbf{E}+\mathbf{e})\,\frac{\partial f_{e,i}}{\partial \mathbf{p}} + \frac{e}{c}\big[\mathbf{v}\times(\mathbf{B}+\mathbf{b})\big]\,\frac{\partial f_{e,i}}{\partial \mathbf{p}} = 0\,. \qquad (33)$$

Here, $\mathbf{B}$ and $\mathbf{E}$ are the magnetic and electric large-scale regular fields, and $\mathbf{b}$ and $\mathbf{e}$ are the fluctuating fields with zero average values upon averaging over the ensemble of fluctuations. The Coulomb collision integral is taken to be zero. In the complete formulation of the problem, the fluctuating fields should be assumed to be self-consistent and dependent on the particle distribution functions $f_{e,i}$, and an investigation should be made of a strongly nonlinear evolution of the system, which is presently impossible. We restrict ourselves to the simplest model of a statistically homogeneous and stationary system, which serves to illustrate the influence of collective effects on the plasma conductivity. This will enable us to qualitatively analyze the influence of these effects on the generation of a magnetic field in collisionless plasmas with nonthermal particles — the main objective of our paper. Assuming the amplitudes of fluctuating fields to be small, we can perform averaging of Eqn (33) over the ensemble of fluctuations in the framework of a quasilinear theory. This leads to the equation for averaged distribution functions $F_{e,i} = \langle f_{e,i}\rangle$:

$$\frac{\partial F_{e,i}}{\partial t} + e\mathbf{E}\,\frac{\partial F_{e,i}}{\partial \mathbf{p}} + \frac{e}{c}(\mathbf{v}\times\mathbf{B})\,\frac{\partial F_{e,i}}{\partial \mathbf{p}} = \mathrm{St}\,[F_{e,i}]\,. \qquad (34)$$

The effective collision integral $\mathrm{St}\,[F_{e,i}]$ is of the form

$$\mathrm{St}\,[F_{e,i}] = \left(\frac{e}{c}\right)^2 \left[\frac{\partial^2}{\partial v_\alpha\,\partial v_\beta}(\Phi_{\alpha\beta}F_{e,i}) + e_{\sigma\mu\alpha}v_\mu\,\frac{\partial^2}{\partial v_\sigma\,\partial v_\beta}(\Psi_{\alpha\beta}F_{e,i})\right], \qquad (35)$$

where

$$\Phi_{\alpha\beta} = \int_0^\infty \mathrm{d}\tau\,\Bigg[\Big(\langle\mathbf{e}_\alpha\mathbf{e}_\gamma\rangle\mathbf{b}_{0\gamma} + \langle\mathbf{e}_\alpha\mathbf{b}_\gamma^\perp\rangle\,\mathbf{E}_\gamma^\perp B\Big)\mathbf{b}_{0\beta} + \langle\mathbf{e}_\alpha\mathbf{b}_\nu\rangle\mathbf{b}_{0\nu}\mathbf{b}_{0\gamma}e_{\gamma\sigma\mu}\left(\frac{v_\mu}{c} - e_{\mu\lambda\nu}\mathbf{b}_{0\nu}\,\frac{\mathbf{E}_\lambda^\perp}{B}\right)(\delta_{\sigma\beta} - \mathbf{b}_{0\sigma}\mathbf{b}_{0\beta})\Bigg]\,, \qquad (36)$$

$$\Psi_{\alpha\beta} = \int_0^\infty \mathrm{d}\tau\,\Bigg[\Big(\langle\mathbf{b}_\alpha\mathbf{e}_\gamma\rangle\mathbf{b}_{0\gamma} + \langle\mathbf{e}_\alpha\mathbf{b}_\gamma^\perp\rangle\,\frac{\mathbf{E}_\gamma^\perp}{B}\Big)\mathbf{b}_{0\beta} + \langle\mathbf{b}_\alpha\mathbf{b}_\nu\rangle\mathbf{b}_{0\nu}\mathbf{b}_{0\gamma}e_{\gamma\sigma\mu}\left(\frac{v_\mu}{c} - e_{\mu\lambda\nu}\mathbf{b}_{0\nu}\,\frac{\mathbf{E}_\lambda^\perp}{B}\right)(\delta_{\sigma\beta} - \mathbf{b}_{0\sigma}\mathbf{b}_{0\beta})\Bigg]\,. \qquad (37)$$

Here, we introduced a unit vector $\mathbf{b}_0 = \mathbf{B}/B$ of the average field. The pair correlation functions $\big\langle\mathbf{b}_\alpha(\mathbf{r},t)\,\mathbf{b}_\beta(\mathbf{r}',t')\big\rangle$, $\big\langle\mathbf{e}_\alpha(\mathbf{r},t)\,\mathbf{e}_\beta(\mathbf{r}',t')\big\rangle$, and $\big\langle\mathbf{b}_\alpha(\mathbf{r},t)\,\mathbf{e}_\beta(\mathbf{r}',t')\big\rangle$ of the fluctuating fields are assumed to be statistically homogeneous and stationary. They depend on the differences $\tau = t - t'$ and $\mathbf{r} - \mathbf{r}' = \mathbf{R}(\tau)$, where $\mathbf{R}(\tau)$ is the law of particle motion in the fields $\mathbf{E}$ and $\mathbf{B}$. The motion of magnetized particles in the weakly inhomogeneous and slowly varying fields $\mathbf{B}(\mathbf{r},t)$ and $\mathbf{E}(\mathbf{r},t)$ may be described in the drift approximation.

Let us now consider two simple models of fluctuating fields and calculate the plasma conductivities by expressing them in terms of the correlation characteristics of the random fields.

**2.2.1 Small-scale static inhomogeneities.** The plasma moves with a constant and uniform velocity $\mathbf{u} = \mathrm{const}$, $u \ll c$, relative to a reference system selected. In the co-moving system there are constant and homogeneous fields $\mathbf{B}'$, $\mathbf{E}' \perp \mathbf{B}'$, with $E' \ll B'$, as well as static magnetic fluctuations $\mathbf{b}'$ whose spatial scales do not exceed the gyroradii of thermal particles (small-scale random inhomogeneities). There are no electric fluctuations in the co-moving frame of reference: $\mathbf{e}' = 0$. In the selected ('laboratory') system of coordinates there are, correct to terms of first order in the parameter $u/c \ll 1$, the following fields:

$$\mathbf{B} \approx \mathbf{B}', \quad \mathbf{b} \approx \mathbf{b}', \quad \mathbf{E} \approx \mathbf{E}' - \frac{\mathbf{u}}{c} \times \mathbf{B}, \quad \mathbf{e} \approx -\frac{\mathbf{u}}{c} \times \mathbf{b}. \tag{38}$$

A full-length calculation of the collisional term for this case can be found in § 7 of monograph [8]. The equation for the averaged distribution function may be brought to the form (hereinafter we shall drop the subscripts e and i for simplicity)

$$\frac{\partial F}{\partial t} + \mathbf{v}\frac{\partial F}{\partial \mathbf{r}} + e\mathbf{E}'\frac{\partial F}{\partial \mathbf{p}} = \omega_B \mathbf{b}_0 \left[\mathbf{v} - \mathbf{u}, \frac{\partial}{\partial \mathbf{v}}\right] F + \frac{e^2 L_c \langle b^2 \rangle}{(m_{e,i} c)^2} \left[\mathbf{v} - \mathbf{u}, \frac{\partial}{\partial \mathbf{v}}\right]_\alpha \frac{1}{|\mathbf{v} - \mathbf{u}|} \left[\mathbf{v} - \mathbf{u}, \frac{\partial}{\partial \mathbf{v}}\right]_\alpha F, \tag{39}$$

where $L_c$ is the correlation length of a stochastic magnetic field. In view of the small scale of the fluctuations, the equation is insensitive to their spectral composition.

Our concern will be with the stationary and uniform particle distribution, when the distribution function depends only on the momentum $F(\mathbf{p})$. We pass on to a new independent variable

$$\mathbf{v}' = \mathbf{v} - \mathbf{u} = \frac{\mathbf{p}}{m} - \mathbf{u}, \tag{40}$$

which stands for the particle velocity in the co-moving system of coordinates. Equation (39) takes on a compact form

$$\frac{e}{m_{e,i}} \mathbf{E}' \frac{\partial F}{\partial \mathbf{v}'} = \omega_B \mathbf{b}_0 \widehat{\mathcal{O}} F + \frac{1}{2\tau_r(v')} \widehat{\mathcal{O}}^{\,2} F, \tag{41}$$

where

$$\widehat{\mathcal{O}} = \mathbf{v}' \times \frac{\partial}{\partial \mathbf{v}'} = \mathbf{p}' \times \frac{\partial}{\partial \mathbf{p}'} \tag{42}$$

is the velocity rotation operator, and

$$\tau_r^{-1}(v') = \frac{2e^2 L_c \langle b^2 \rangle}{(mc)^2 v'} \tag{43}$$

is the time of distribution function relaxation caused by particle scattering from small-scale inhomogeneities.

In the stationary regime, the distribution function in the co-moving frame of reference possesses a small anisotropy, which corresponds to the smallness of the electric field $\mathbf{E}' \neq 0$. The distribution function may therefore be represented in the form

$$F = \frac{1}{4\pi} \left[ N(v') + \frac{3\mathbf{v}'}{v'^2} \mathbf{J}(v') \right], \tag{44}$$

where $J \ll v'N$, while the quantities $J$ and $N$ depend only on the modulus $v' = |\mathbf{v} - \mathbf{u}|$.

Substitution of expression (44) into Eqn (41) permits finding the equation for the particle density flux $\mathbf{J}$ induced by the electric field:

$$\mathbf{J} + \omega_B \tau_r \mathbf{b}_0 \times \mathbf{J} = -\frac{e\tau_r v'}{3} \frac{\partial N}{\partial v'} \mathbf{E}'. \tag{45}$$

The particle flux is perpendicular to the large-scale magnetic field. The electric current density $\mathbf{j}$ produced by the particles of one sort can be calculated by the formulas

$$\mathbf{j} = e \int \mathbf{v}' F \, \mathrm{d}^3 p' = e \int \mathbf{J} p'^2 \, \mathrm{d}p'. \tag{46}$$

These formulas do not take into account the currents

$$\pm e\mathbf{u} \int N_{e,i} \, p'^2 \, \mathrm{d}p' = \pm e n_{e,i} \mathbf{u}$$

arising from the overall motion of the medium, which cancel out in a quasineutral plasma.

By using Eqn (45) and expression (46), we arrive at the Ohm law in the ordinary form:

$$\mathbf{j} = \sigma_\perp \mathbf{E}' - \sigma_H \mathbf{b}_0 \times \mathbf{E}', \quad \mathbf{E}' = \mathbf{E} + \frac{\mathbf{u}}{c} \times \mathbf{B}, \tag{47}$$

where the conductivities are given by

$$\begin{aligned} \sigma_\perp &= \int \frac{\sigma(p')}{1 + \omega_B^2 \tau_r^2} \, p'^2 \, \mathrm{d}p', \\ \sigma_H &= \int \frac{\sigma(p')\omega_B \tau_r}{1 + \omega_B^2 \tau_r^2} \, p'^2 \, \mathrm{d}p', \\ \sigma(p') &= -\frac{e^2 \tau_r p'}{3m} \frac{\partial N}{\partial p'} > 0. \end{aligned} \tag{48}$$

For a strong magnetization ($\omega_B \tau_r \gg 1$), we introduce the averaged velocity $\overline{v}'$ by the formula

$$\frac{1}{\overline{v}'} = -\frac{1}{3n} \int \frac{p'^3}{v'} \frac{\partial N}{\partial p'} \, \mathrm{d}p' \tag{49}$$

to obtain

$$\sigma_\perp = \frac{ne^2 L_c}{m\overline{v}'} \frac{\langle b^2 \rangle}{B^2} \tag{50}$$

— an analog of the Drude formula in which the role of a collision time is played by the quantity

$$\tau = \frac{L_c \langle b^2 \rangle}{\overline{v}' B^2}.$$

Formula (50) contains the particle mass in the denominator and shows that electrons play the dominant part in the electrical conduction under consideration. In the same limit $\sigma_H = ecn/B$, and the Hall current is produced due to the electric particle drift under the crossed fields $\mathbf{E}' \times \mathbf{B}$:

$$\mathbf{j}_H = \frac{ecn}{B^2} \mathbf{E}' \times \mathbf{B}.$$

The Hall currents of electrons and ions cancel out.

**2.2.2 Large-scale Alfvén waves with random phases.** In contrast to the previous case, let us consider nonstatic fluctuations, where there are magnetic field $\mathbf{b}'(\mathbf{r}', t)$ and electric field $\mathbf{e}' = \mp \mathbf{v}_A \times \mathbf{b}'/c$ in the co-moving system of coordinates, where $\mathbf{v}_A = \mathbf{B}/\sqrt{4\pi\rho}$ is the Alfvén velocity, and $\mathbf{r}' = \mathbf{r} - \mathbf{u}t$ is the radius vector. In the observer frame of reference we have

$$\mathbf{b} = \mathbf{b}', \quad \mathbf{e} = -\frac{1}{c}(\mathbf{u} \pm \mathbf{v}_A) \times \mathbf{b}. \tag{51}$$

Upon averaging over turbulent oscillations, the equation for the distribution function which depends only on the momentum can be written down in the form similar to Eqn (41), though with a more complex collision integral:

$$\frac{e}{m}\,\mathbf{E}'\,\frac{\partial F}{\partial \mathbf{v}'} - \omega_B \mathbf{b}_0 \widehat{\mathcal{O}} F = \frac{e^2}{(mc)^2}\left[\mathbf{v}' \mp \mathbf{v}_{\mathrm{A}}, \frac{\partial}{\partial \mathbf{v}'}\right]_\alpha$$
$$\times \int \big\langle b_\alpha(\mathbf{r}', t)\, b_\beta(\mathbf{r}'_0, t_0)\big\rangle\, G(\mathbf{r}', \mathbf{v}', t;\, \mathbf{r}'_0, \mathbf{v}'_0, t_0)$$
$$\times \left[\mathbf{v}'_0 \mp \mathbf{v}_{\mathrm{A}}, \frac{\partial}{\partial \mathbf{v}'_0}\right]_\beta F(\mathbf{r}'_0, \mathbf{v}'_0, t_0)\, \mathrm{d}^3 r'_0\, \mathrm{d}^3 v'_0\, \mathrm{d}t_0 \equiv \mathrm{St}\,[F]\,. \quad (52)$$

Here, $G(\mathbf{r}', \mathbf{v}', t;\, \mathbf{r}'_0, \mathbf{v}'_0, t_0)$ is the Green function which describes the particle motion in crossed uniform fields $\mathbf{E}'$ and $\mathbf{B}$ over a time $\tau = t - t_0$. The Green function is of the form

$$G(\mathbf{r}', \mathbf{v}', t;\, \mathbf{r}'_0, \mathbf{v}'_0, t_0) = \delta(\mathbf{v}' - \mathbf{V} - \mathbf{v}'_0)\,\delta(\mathbf{r}' - \mathbf{R} - \mathbf{r}'_0)\,\Theta(\tau)\,, \quad (53)$$

where $\mathbf{V}$ and $\mathbf{R}$ are the velocity and radius-vector changes during the time $\tau$:

$$\mathbf{V}(\tau, \mathbf{v}'_0) = \mathbf{v}'_{0\perp}(\cos\omega_B\tau - 1) - \mathbf{b}_0 \times \mathbf{v}'_0 \sin\omega_B\tau\,,$$
$$\mathbf{R}(\tau, \mathbf{v}'_0) = \frac{1}{\omega_B}\,\mathbf{v}'_{0\perp}\sin\omega_B\tau + \frac{1}{\omega_B}\,\mathbf{b}_0 \times \mathbf{v}'_0(\cos\omega_B\tau - 1) + \mathbf{v}'_{0\parallel}\tau\,. \quad (54)$$

In a statistically homogeneous medium, the correlation tensor of Alfvén modes depends on difference arguments:

$$\big\langle b_\alpha(\mathbf{r}', t)\, b_\beta(\mathbf{r}'_0, t_0)\big\rangle = K_{\alpha\beta}\big(\mathbf{R}(\tau, \mathbf{v}'_0), \tau\big)\,. \quad (55)$$

We shall assume that the correlation lengths $L_{\mathrm{c}}$ in the longitudinal and transverse directions are comparable and they are long in comparison with the gyroradii of thermal particles: $L_{\mathrm{c}} \gg r_{\mathrm{g}}$. In this case, one may neglect the transverse particle displacement in the co-moving system of coordinates and put $\mathbf{R} \approx \mathbf{b}_0 v'_{0\parallel}\tau$. The correlation time $\tau_{\mathrm{c}}$ will be determined by the highest of the following three velocities: the particle velocity $v'_T$ in the co-moving frame of reference, the Alfvén velocity $v_{\mathrm{A}}$, and the velocity $u$ of the medium. In cold plasmas ($v'_T \ll v_{\mathrm{A}}$ or $v'_T \ll u$), in particular, the correlation time $\tau_{\mathrm{c}} \approx L_{\mathrm{c}}/u$ or $\tau_{\mathrm{c}} \approx L_{\mathrm{c}}/v_{\mathrm{A}}$ is independent of the thermal velocity and does not necessitate averaging with the distribution function. One more simplification emerges when the distribution of Alfvén-mode wave vectors is axially symmetric about the direction of the field $\mathbf{B}$. In this case, the correlation tensor takes on the form

$$K_{\alpha\beta}\big(\mathbf{R}(\tau, \mathbf{v}'_0), \tau\big) = K(v'_{0\parallel}\tau, \tau)(\delta_{\alpha\beta} - b_{0\alpha}b_{0\beta})\,. \quad (56)$$

We next pass on to the equation for the electric current by taking advantage of its representation (46) in terms of the distribution function. We perform integration with respect to $\mathrm{d}^3 r'_0\, \mathrm{d}^3 v'$ and transform the collision integral in the following way:

$$e\int v'_\lambda\, \mathrm{St}\,[F]\,\mathrm{d}^3 v' = \frac{e^3}{(mc)^2}\, e_{\alpha\mu\lambda}\, e_{\beta\kappa\sigma}\int \mathrm{d}^3 v'_0\, F(\mathbf{v}'_0)(\mathbf{v}'_0 - \mathbf{v}_{\mathrm{A}})_\kappa$$
$$\times \frac{\partial}{\partial v'_{0\sigma}}\int_0^\infty \mathrm{d}\tau\, K_{\alpha\beta}(v'_{0\parallel}\tau, \tau)\big(\mathbf{v}'_0 + \mathbf{V}(\tau, \mathbf{v}'_0) \mp \mathbf{v}_{\mathrm{A}}\big)_\mu\,. \quad (57)$$

The velocity $\mathbf{V}$ contains terms proportional to $\cos\omega_B\tau$ and $\sin\omega_B\tau$. In the integration with respect to $\mathrm{d}\tau$, these terms will be of a $(\omega_B\tau_{\mathrm{c}})^{-1} \ll 1$-order infinitesimal and may be neglected. By using representation (56) for the correlation tensor, we bring expression (57) to a simple form

$$e\int v'_\lambda\, \mathrm{St}\,[F]\,\mathrm{d}^3 v' = -\frac{e^3}{(mc)^2}\int \mathrm{d}^3 v'\, F(\mathbf{v}')v'^{\perp}_\lambda\, \frac{\partial}{\partial v'_\parallel}\big(v'_\parallel \bar{K}(v'_\parallel)\big)\,, \quad (58)$$

where

$$\bar{K}(v'_\parallel) = \int_0^\infty K(v'_\parallel\tau, \tau)\,\mathrm{d}\tau \quad (59)$$

and the designation of the integration variable on the right-hand side of expression (58) is simplified.

Let us consider the limiting cases of cold and hot plasmas. In the former case, the thermal velocity in the co-moving frame of reference is low in comparison with the Alfvén and medium transport velocities: $v' \ll v_{\mathrm{A}}, u$. Then, the time integral (59) can be simplified, so that

$$\bar{K}(v'_\parallel) \approx \int_0^\infty K(0, \tau)\,\mathrm{d}\tau = \langle b^2\rangle\tau_{\mathrm{c}}\,, \quad (60)$$

where the last equality may be treated as the definition of the correlation time, whose order of magnitude is $\tau_{\mathrm{c}} \approx L_{\mathrm{c}}/u$ (if $u \gtrsim v_{\mathrm{A}}$). In this case, according to representation (46), integral (58) is expressed in terms of the transverse electric current:

$$e\int \mathbf{v}'\, \mathrm{St}\,[F]\,\mathrm{d}^3 v' = -\frac{1}{\tau_{\mathrm{r}}}\,\mathbf{j}\,, \quad (61)$$

where

$$\frac{1}{\tau_{\mathrm{r}}} = \frac{e^2\tau_{\mathrm{c}}\langle b^2\rangle}{(mc)^2}$$

is the reciprocal of the relaxation time. We integrate in a similar way the terms appearing on the left-hand side of Eqn (52) to obtain the equation for the electric current:

$$\mathbf{j} + \omega_B\tau_{\mathrm{r}}\mathbf{b}_0 \times \mathbf{j} = \frac{e^2 n\tau_{\mathrm{r}}}{m}\,\mathbf{E}'\,. \quad (62)$$

From this equation we obtain Ohm's law in the form of expression (47), the conductivities for a strong magnetization ($\omega_B\tau_{\mathrm{r}} \gg 1$) being of the form

$$\sigma_\perp = \frac{e^2 n\tau_{\mathrm{c}}}{m}\,\frac{\langle b^2\rangle}{B^2}\,, \qquad \sigma_{\mathrm{H}} = \frac{ecn}{B} \quad (63)$$

and quite similar to those obtained in the context of the model of small-scale static inhomogeneities.

In a hot plasma, the thermal electron velocity is high in comparison with $v_{\mathrm{A}}$ and $u$. The time integral (59) is approximated by the expression

$$\bar{K}(v'_\parallel) \approx \int_0^\infty K(v'_\parallel\tau, 0)\,\mathrm{d}\tau = \langle b^2\rangle\,\frac{L_{\mathrm{c}}}{|v'_\parallel|}\,, \quad (64)$$

with

$$\frac{\partial}{\partial v'_\parallel}\big(v'_\parallel \bar{K}(v'_\parallel)\big) = 2L_{\mathrm{c}}\langle b^2\rangle\,\delta(v'_\parallel)\,.$$

By calculating expression (58) with the use of distribution function (44), we arrive at

$$e\int \mathbf{v}'\, \mathrm{St}\,[F]\,\mathrm{d}^3 v' = -\frac{3e^2 L_{\mathrm{c}}\langle b^2\rangle}{2(mc)^2}\int e\mathbf{J}_\perp(v_\perp)v_\perp\,\mathrm{d}v_\perp\,. \quad (65)$$

Since the electric current $\mathbf{j} = e \int \mathbf{J}(v) v^2 \, \mathrm{d}v$, the integral in expression (65) differs from the current by only some average thermal velocity $\overline{v}_\perp$; to calculate it requires, as in the case of expression (49), specifying the distribution function of the background particles. Eventually we obtain the transversal conductivity in the form

$$\sigma_\perp = \frac{3e^2 n L_\mathrm{c}}{2m\overline{v}_\perp} \frac{\langle b^2 \rangle}{B^2} \, . \tag{66}$$

Consideration of the limiting cases of cold and hot plasmas permits writing down the interpolation formula which is suitable for estimating the transversal conductivity of a collisionless plasma involving Alfvén turbulence:

$$\sigma_\perp \approx \frac{e^2 n L_\mathrm{c}}{m_\mathrm{e}\overline{v}} \frac{\langle b^2 \rangle}{B^2} \, , \tag{67}$$

where the highest of three velocities $v_{T\mathrm{e}}$, $v_\mathrm{A}$, and $u$ should be substituted for $\overline{v}$.

Throughout the foregoing text we only considered the case of a statistically homogeneous medium. When the fields **E** and **B**, as well as the particle distributions, are spatially inhomogeneous, in Ohm's law there appear additional terms arising from this large-scale inhomogeneity, which are not given here.

The expression for the longitudinal current and the longitudinal conductivity should be considered simultaneously with the equation for the longitudinal electric field (the polarization field). This more complicated problem will not be considered here (see Refs [39, 40] which discuss the more general case of the collisional integral). For the problems considered in this review it would suffice to know the transverse currents.

Let us give some more information about the plasma parameters of astrophysical objects. In a rarefied hot plasma, the mean free path between the Coulomb collisions is $\Lambda_\mathrm{c} \approx 1.4 \times 10^{12} \, T^2 / n\lambda_{10}$ (cm), when the temperature is measured in eV, the concentration $n$ in cm$^{-3}$, and $\lambda_{10}$ is the Coulomb logarithm divided by 10. The temperature range of interest for astrophysical plasma objects extends from about 1 eV to $\sim 10$ keV — the gas temperature in rich galaxy clusters. The characteristic plasma densities for the majority of objects with collisionless shock waves lie in the conventional range between $10^{-7}$ cm$^{-3}$ in the intergalactic medium and $\sim 10^{15}$ cm$^{-3}$ in accretion disks. The estimated magnetic fields range from $10^{-9}$ G for the intergalactic space to appreciably more than 1 G in stellar atmospheres.

The frequency $\tau_\mathrm{r}^{-1}$ of thermal ion scattering by magnetic fluctuations with scales much longer than the particle's gyroradius in the mean magnetic field can, as was done in the foregoing, be estimated by the formula

$$\tau_\mathrm{r}^{-1} \approx \frac{e^2}{m_\mathrm{i}^2 c^2} \int_0^\infty K(v'_\parallel \tau, 0) \, \mathrm{d}\tau \approx \frac{e^2 \langle b^2 \rangle \tau_\mathrm{c}}{m_\mathrm{i}^2 c^2} \, , \tag{68}$$

which is valid when $\tau_\mathrm{r}^{-1} \ll \omega_B$. Hence, we have the following limitation on the correlation time $\tau_\mathrm{c}$ and the r.m.s. fluctuation amplitude: $\langle b^2 \rangle / B_0^2 \ll (\omega_B \tau_\mathrm{c})^{-1}$. For single-scale fluctuations, the correlation time is defined by this single scale. For extended Kolmogorov type fluctuation spectra, which subside with decreasing scale:

$$K(k) = \langle b^2 \rangle \frac{\alpha - 1}{k^\alpha} \, , \quad \alpha \geqslant 1 \, , \quad \frac{2\pi}{L_\mathrm{c}} \leqslant k \leqslant \frac{2\pi}{l_\mathrm{min}} \, , \quad L_\mathrm{c} \gg l_\mathrm{min} \, , \tag{69}$$

the correlation time is defined by the fluctuations with the longest scale. In particular, for $\alpha \to 1$ (Bohm's case) we have

$$(\alpha - 1)^{-1} \to \ln \frac{L_\mathrm{c}}{l_\mathrm{min}} \, , \qquad \tau_\mathrm{c} = \frac{L_\mathrm{c}}{2\overline{v} \ln (L_\mathrm{c}/l_\mathrm{min})} \, . \tag{70}$$

## 3. Linear modes and their damping

We consider the natural oscillations of a partially ionized medium in the absence of dissipation ($\nu_\mathrm{m} = \nu_\mathrm{eff} = 0$) and extrinsic current ($\mathbf{j}^\mathrm{ext} = 0$). Assuming that $\mathbf{u}', \mathbf{b}, \rho' \propto \exp(\mathrm{i}\mathbf{kr} - \mathrm{i}\omega t)$, from Eqns (26)–(28) we obtain the system of algebraic equations for the components of the field **b**:

$$\left( \omega' - \frac{v_\mathrm{A}^2 k_\parallel^2}{\omega'} \right) \mathbf{b} = \frac{v_\mathrm{A}^2 \omega' (k^2 \mathbf{e}_\parallel - k_\parallel \mathbf{k}) b_\parallel}{\omega'^2 - c_\mathrm{s}^2 k^2} + \mathrm{i} \, \frac{B_0 c k_\parallel}{4\pi n_\mathrm{i} e} \, \mathbf{k} \times \mathbf{b} \, , \tag{71}$$

where $\omega' = \omega - \mathbf{k}\mathbf{u}_0$ is the frequency in the co-moving frame of reference. By projecting Eqn (71) onto the directions $\mathbf{e}_\parallel$ and $\mathbf{k} \times \mathbf{e}_\parallel$, we find the dispersion relation

$$(\omega'^2 - v_\mathrm{A}^2 k_\parallel^2) \left( \omega'^2 - v_\mathrm{A}^2 k_\parallel^2 - \frac{\omega'^2 v_\mathrm{A}^2 k_\perp^2}{\omega'^2 - c_\mathrm{s}^2 k^2} \right) = \omega'^2 v_\mathrm{A}^2 k_\parallel^2 \, \frac{(ck)^2}{(1 - F)\omega_{0\mathrm{i}}^2} \, . \tag{72}$$

Here, the right-hand side is written out in terms of the ion plasma frequency

$$\omega_{0\mathrm{i}}^2 = \frac{4\pi n_\mathrm{i} e^2}{m_\mathrm{i}} \, . \tag{73}$$

When the scale of field inhomogeneity is sufficiently large, $k \ll \omega_{0\mathrm{i}} \sqrt{1 - F}/c$, the right-hand side of the equality is small in comparison with the terms which enter onto the left-hand side and it may be placed equal to zero. This signifies neglecting the Hall current and yields the known dispersion laws for three MHD modes — the Alfvén mode and two magnetosonic ones:

$$\omega_\mathrm{A} = \pm v_\mathrm{A} |k_\parallel| \, , \qquad \omega_\mathrm{f,\,s} = \pm v_\mathrm{f,\,s} \, k \, ,$$
$$v_\mathrm{f,\,s}^2 = \frac{1}{2} (v_\mathrm{A}^2 + c_\mathrm{s}^2) \pm \frac{1}{2} \left[ (v_\mathrm{A}^2 + c_\mathrm{s}^2)^2 - 4 v_\mathrm{A}^2 c_\mathrm{s}^2 \cos^2 \vartheta \right]^{1/2} , \tag{74}$$

where $\vartheta$ is the angle between the vectors **k** and $\mathbf{B}_0$. In the dissipation-free limit, the phase velocities are no different from the velocities of the modes in fully ionized plasmas. However, the damping is substantially stronger.

To estimate the wave damping coefficients, we include in Eqn (71) the dissipative terms from Eqn (26) but omit the weak Hall current which is not responsible for dissipation:

$$(\omega'^2 - v_\mathrm{A}^2 k_\parallel^2) \, \mathbf{b} = \frac{v_\mathrm{A}^2 \omega'^2 (k^2 \mathbf{e}_\parallel - k_\parallel \mathbf{k}) b_\parallel}{\omega'^2 - c_\mathrm{s}^2 k^2} - \mathrm{i}\omega' \left\{ \nu_\mathrm{m} k^2 \mathbf{b} + (\nu_\mathrm{eff} - \nu_\mathrm{m}) \left[ (k^2 \mathbf{e}_\parallel - k_\parallel \mathbf{k}) b_\parallel + k_\parallel^2 \mathbf{b}_\perp \right] \right\} . \tag{75}$$

To the Alfvén mode there corresponds $b_\parallel = 0$, the vector **b** in this mode being aligned with $\mathbf{k} \times \mathbf{e}_\parallel$. Retaining in Eqn (75) the corresponding terms and putting $\omega' = \omega_\mathrm{A} - \mathrm{i}\gamma_\mathrm{A}$, where $\gamma_\mathrm{A} \ll |\omega_\mathrm{A}|$, we find the damping coefficient

$$\gamma_\mathrm{A} = \frac{1}{2} (\nu_\mathrm{eff} k_\parallel^2 + \nu_\mathrm{m} k_\perp^2) \, . \tag{76}$$

The damping is primarily accounted for by the high effective collisional viscosity. This takes place for $k_\parallel^2/k_\perp^2 \ll \nu_{\rm m}/\nu_{\rm eff}$. In the absence of the neutral component ($\nu_{\rm eff} = \nu_{\rm m}$) we obtain the known expression $\gamma_{\rm A} = \nu_{\rm m} k^2/2$.

In the case of magnetosonic waves, $b_\parallel \neq 0$ and the vector $\mathbf{b}$ lies in the $(\mathbf{k}, \mathbf{e}_\parallel)$ plane. The damping coefficient of the fast and slow modes is given by the expressions

$$\gamma_{\rm f,\,s} = \nu_{\rm eff} k^2 \, \frac{v_{\rm f,\,s}^2 - c_{\rm s}^2}{4v_{\rm f,\,s}^2 + 2c_{\rm s}^2 + 2v_{\rm A}^2} \, . \tag{77}$$

For the slow mode, which corresponds to the minus sign ahead of the square root in expression (74), the above formula is valid for $\gamma_{\rm s} > 0$ and $\gamma_{\rm s} \ll |\omega_{\rm s}|$.

Prior to investigating the effects of a magnetic-field and MHD-wave generation in multicomponent plasma systems, which is the main objective of our work, it would be instructive to consider the comparatively limited problem involving prescribed extraneous currents which act on the electron – ion magnetoactive plasma.

## 4. MHD wave excitation by an external current in a single-fluid plasma model

MHD waves in highly conducting liquids and a plasma may be generated both by direct mechanical excitation and by way of external electric currents. The generation problem has been discussed in different works [41 – 43] since the discovery of MHD waves by H Alfvén in 1942. Both excitation mechanisms play a significant part in cosmic plasmas, but in a collisionless plasma the mechanism of field excitation by electric currents is quite often the dominant one.

We shall briefly discuss the wave generation problem with the aim of determining the relative role of resonance and nonresonance mechanisms of wave excitation by external current $\mathbf{j}^{\rm ext}$. In the formulation of the problem considered by Akhiezer et al. [43], we have a plasma with ions of one sort and a given extrinsic current. Then, the excitation of linear MHD waves may be investigated by employing the equation of motion (11) of a single-fluid medium, in which the last terms on the left-hand side, responsible for viscous dissipation, may be dropped. The equation for the velocity $\mathbf{u}(\mathbf{r}, t)$ of the medium differs from Eqn (11), in particular, in that the force $(1/c)(\mathbf{j}^{\rm ext} \times \mathbf{B})$, which acts on the external current carrier and not on the plasma, is missing from it:

$$\rho \left[ \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\nabla)\mathbf{u} \right] = \frac{1}{c}\, \mathbf{j} \times \mathbf{B} - \nabla P \, . \tag{78}$$

In this case, the magnetic field is determined by the total current:

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\, (\mathbf{j} + \mathbf{j}^{\rm ext}) \, ,$$

which permits excluding from Eqn (78) the proper plasma current $\mathbf{j}$ and obtaining the equation relating the extrinsic current $\mathbf{j}^{\rm ext}$ to the velocity $\mathbf{u}$ and the field $\mathbf{B}$ in the plasma.

The induction equation, which is derived with the use of Ohm's law in its ordinary representation $\mathbf{j} = \sigma(\mathbf{E} + \mathbf{u} \times \mathbf{B}/c)$, takes on the form

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times [\mathbf{u} \times \mathbf{B}] + \nu_{\rm m} \Delta \mathbf{B} + \frac{4\pi \nu_{\rm m}}{c}\, \nabla \times \mathbf{j}^{\rm ext} \, . \tag{79}$$

In Eqn (79), we took into account the terms corresponding to the plasma finite conductivity effects, which are required in the analysis of the resonance effect of wave generation. However, unlike Ref. [43], apart from the magnetic diffusion effect which is described by the term $\nu_{\rm m}\Delta\mathbf{B}$, we also retained the last term which contains the extrinsic current and is proportional to the magnetic viscosity $\nu_{\rm m}$. This summand is different from conventional dissipative terms proportional to $\eta k^2$ and $\nu_{\rm m} k^2$ in that it is a source of the field in the induction equation.

Let us consider the external current in the form of a traveling wave, $\mathbf{j}^{\rm ext} = \mathbf{j}_0 \exp\left[ {\rm i}\omega(z/v_{\rm ph} - t) \right]$, which permits introducing in explicit form the phase velocity $v_{\rm ph}$ of the source. The system of coordinates is so selected that its $Oz$-axis is aligned with the direction of current wave propagation, and the unperturbed magnetic field $\mathbf{B}_0$ lies in the $xz$ plane. Upon the conventional linearization procedure, $\mathbf{B} = \mathbf{B}_0 + \mathbf{b}$, we obtain the equations for the vectors $\mathbf{b}$ and $\mathbf{u}$ of the Alfvén wave in a homogeneous stationary plasma with the magnetic field $\mathbf{B}_0$. The Alfvén mode can be marked out as the mode with the polarization along the $Oy$-axis:

$$\frac{\partial u_y}{\partial t} - \frac{B_{0z}}{4\pi\rho_0}\, \frac{\partial b_y}{\partial z} = \frac{j_x^{\rm ext} B_{0z} - j_z^{\rm ext} B_{0x}}{\rho_0 c} \, , \tag{80}$$

$$\frac{\partial b_y}{\partial t} - B_{0z}\, \frac{\partial u_y}{\partial z} = \nu_{\rm m}\, \frac{\partial^2 b_y}{\partial z^2} - \frac{4\pi \nu_{\rm m}}{c} \left( \frac{\partial j_z^{\rm ext}}{\partial x} - \frac{\partial j_x^{\rm ext}}{\partial z} \right) . \tag{81}$$

We define the external current in the form of a traveling shear wave with $j_z^{\rm ext} = 0$ and $j_x^{\rm ext} \neq 0$, which secures the fulfilment of the condition $\nabla \mathbf{j}^{\rm ext} = 0$. The solution which describes the forced oscillations in the medium is of the form

$$u_y = -\frac{B_{0z}}{{\rm i}\omega\rho_0 c}\, \frac{v_{\rm ph}^2}{v_{\rm ph}^2 - v_{\rm A}^2 + {\rm i}\omega\nu_{\rm m}}\, j_x^{\rm ext} \, , \tag{82}$$

$$b_y = \frac{4\pi v_{\rm ph}}{{\rm i}\omega c}\, \frac{v_{\rm A}^2 - {\rm i}\omega\nu_{\rm m}}{v_{\rm ph}^2 - v_{\rm A}^2 + {\rm i}\omega\nu_{\rm m}}\, j_x^{\rm ext} \, . \tag{83}$$

These quantities are related as

$$u_y = \mp \frac{b_y}{\sqrt{4\pi\rho}}\, \frac{v_{\rm A} v_{\rm ph}}{v_{\rm A}^2 - {\rm i}\omega\nu_{\rm m}} \, , \tag{84}$$

where $v_{\rm A} = |B_{0z}|/\sqrt{4\pi\rho}$.

The relationship obtained is different from the relationship inherent in the eigenmode in a dissipation-free medium, i.e., in the Alfvén wave, and goes over into it for $\nu_{\rm m} \to 0$ and $v_{\rm ph} \to v_{\rm A}$.

We also consider wave excitation by an external current aligned with the direction of wave propagation, i.e., $j_z^{\rm ext} \neq 0$ and $j_x^{\rm ext} = 0$ (here, $\nabla\, \mathbf{j}^{\rm ext} \neq 0$). In this case, the corresponding relationships take on the form

$$u_y = \frac{B_{0x}}{{\rm i}\omega\rho_0 c}\, \frac{v_{\rm ph}^2 - {\rm i}\omega\nu_{\rm m}}{v_{\rm ph}^2 - v_{\rm A}^2 + {\rm i}\omega\nu_{\rm m}}\, j_z^{\rm ext} \, , \tag{85}$$

$$b_y = -\frac{B_{0x} B_{0z}}{{\rm i}\omega\rho c v_{\rm ph}}\, \frac{v_{\rm ph}^2}{v_{\rm ph}^2 - v_{\rm A}^2 + {\rm i}\omega\nu_{\rm m}}\, j_z^{\rm ext} \, , \tag{86}$$

$$u_y = -b_y\, \frac{v_{\rm ph}^2 + {\rm i}\omega\nu_{\rm m}}{B_{0z} v_{\rm ph}} \, . \tag{87}$$

The resultant relationships show that forced oscillations in the medium are excited by external currents with an arbitrary phase velocity. The wave propagates with the phase velocity of the current. When the phase velocity of external current coincides with the Alfvén velocity of the medium there occurs a resonance. The wave amplitude at resonance is inversely proportional to the magnetic viscosity and is therefore directly proportional to the electrical conductivity of the medium. In a dissipation-free medium ($v_{\mathrm{m}} \to 0$), the wave may exist ($u_y$ and $b_y$ may be nonzero) even in the absence of external current ($j^{\text{ext}} \to 0$). However, only the eigenmodes with $v_{\mathrm{ph}} = v_{\mathrm{A}}$ are possible in this case, and the relation between the velocity and magnetic field amplitudes corresponds to the Alfvén wave:

$$u_y = \mp \frac{b_y}{\sqrt{4\pi\rho}} \,. \tag{88}$$

In the presence of external current, the field–velocity relation is different from formula (88) in the general case. The situation is similar to that which takes place in the emission of electromagnetic waves in a vacuum. In the domain occupied by current, the electromagnetic oscillations do not generally possess the properties of vacuum modes and transform into them only some distance away from this domain (in the wave zone).

We emphasize once again: the amplitude of the Alfvén wave under resonance excitation by an external current in a medium with one sort of ions is higher, the higher the conductivity of the medium. A qualitatively different situation takes place in multicomponent systems with nonthermal accelerated particles, which we now turn our attention to.

## 5. Allowance for the accelerated component in MHD equations

Let us assume that the medium under consideration contains, apart from the background nonrelativistic plasma and the neutral component, some population of accelerated charged particles, relativistic or nonrelativistic, whose velocities $v$ far exceed all the remaining velocities: $v \gg v_{\mathrm{A}}, c_{\mathrm{s}}, u$. Being significantly different from the background particles by energy, these particles should be considered the fourth medium component with specific properties. In view of the high energy of these particles, we neglect their collisions with the particles of the background medium but include their interaction with small-scale stochastic turbulent fields. This may be done by taking advantage of the kinetic equation, averaged over turbulent fields, for the averaged distribution function $f$ of the particles involved (see Ref. [8]):

$$\frac{\partial f}{\partial t} + \mathbf{v}\,\frac{\partial f}{\partial \mathbf{r}} + e\mathbf{E}\,\frac{\partial f}{\partial \mathbf{p}} - \frac{ec}{\mathcal{E}}\,\mathbf{B}\,\mathcal{O}f = I[f]\,. \tag{89}$$

Here, $\mathbf{B}$ and $\mathbf{E}$ are the large-scale regular magnetic and electric fields, $I[f]$ is the collision integral which takes into account the interaction of accelerated particles with MHD turbulence, and $\mathcal{E}$ is the total accelerated-particle energy. Specific expressions for this quantity within the lowest-order approximation in turbulence amplitude may be found in monograph [8], and the nonlinear corrections are given in review [44]. In the case under consideration, the electric field in the ionized medium owes its origin to the transport velocity and may be written as

$$\mathbf{E} = -\frac{1}{c}\,\mathbf{u} \times \mathbf{B}\,, \tag{90}$$

making it possible to combine the last two terms on the left-hand side of Eqn (89) into one, if need be:

$$\frac{\partial f}{\partial t} + \mathbf{v}\,\frac{\partial f}{\partial \mathbf{r}} = \frac{e}{c}\,\mathbf{B}\left[(\mathbf{v} - \mathbf{u}) \times \frac{\partial}{\partial \mathbf{p}}\right] f + I[f]\,. \tag{91}$$

Hence, it follows that the equation terms proportional to $u$ contain the small parameter $u/v$.

To incorporate the accelerated particles into the general system of equations obtained in the preceding sections, we impart the hydrodynamic form to Eqn (91). To do this we go over to the momentum balance equation for accelerated particles by multiplying Eqn (91) by the momentum of a single particle and integrating it with respect to $\mathrm{d}^3 p$. We denote the momentum density by

$$\boldsymbol{\mathcal{P}}(\mathbf{r}, t) = \int \mathbf{p} f\,\mathrm{d}^3 p\,, \tag{92}$$

and the momentum flux density by

$$\Pi_{\alpha\beta} = \int p_\alpha v_\beta f\,\mathrm{d}^3 p \tag{93}$$

to arrive at

$$\frac{\partial \mathcal{P}_\alpha}{\partial t} + \nabla_\beta \Pi_{\alpha\beta} = \frac{1}{c}\,(\mathbf{j}^{\text{ext}} \times \mathbf{B})_\alpha - eN\mathbf{E}_\alpha + \int p_\alpha I[f]\,\mathrm{d}^3 p\,, \tag{94}$$

where

$$\mathbf{j}^{\text{ext}} = e\int \mathbf{v} f\,\mathrm{d}^3 p \tag{95}$$

is the electric current induced by accelerated particles, and $N$ is their local concentration. The last term in Eqn (94) may be approximately written down, as follows from its dimensionality and physical considerations, in the form

$$\int p_\alpha I[f]\,\mathrm{d}^3 p \approx -\frac{\mathcal{P}_\alpha}{\tau_{\mathrm{s}}}\,. \tag{96}$$

Here, $\tau_{\mathrm{s}}$ is the time of accelerated-particle distribution function isotropization under the action of scattering turbulent fields.

Let us compare the resultant Eqn (94) with the momentum balance equation for the background particles, which is obtained by summing the right- and left-hand sides of Eqns (11) and (13):

$$\frac{\partial}{\partial t}(\rho u_\alpha) + \nabla_\beta(\rho u_\alpha u_\beta) = -\nabla_\alpha P + \frac{1}{c}(\mathbf{j} \times \mathbf{B})_\alpha\,. \tag{97}$$

We discarded from Eqn (11) the dissipative term, assuming the medium to be cold enough. We add Eqn (94) termwise to Eqn (97) to obtain

$$\frac{\partial}{\partial t}(\rho u_\alpha + \mathcal{P}_\alpha) + \nabla_\beta(\rho u_\alpha u_\beta + \Pi_{\alpha\beta}) = -\nabla_\alpha P + \frac{1}{c}\left[(\mathbf{j} + \mathbf{j}^{\text{ext}}) \times \mathbf{B})\right]_\alpha - \frac{\mathcal{P}_\alpha}{\tau_{\mathrm{s}}}\,. \tag{98}$$

The term $eN\mathbf{E}$ was cancelled by the corresponding contribution from the background particles, because the system under discussion is electroneutral as a whole. In view of the fact that the spatial scales under discussion are far greater than the

Debye length, local macroscopic electroneutrality $(N + n_i - n_e = 0)$ also occurs.

To somewhat simplify the resultant equation of motion for a four-component medium, we take into consideration the distinguishing features of an accelerated-particle distribution function in astrophysical objects. The first feature is the low accelerated-particle concentration in comparison with the background-particle concentration. In the galactic disk, $N/(n_i + n_a) \approx 10^{-9}$ on the average. In the acceleration of particles at the shock front, this ratio depends heavily on the form of the spectrum and the highest energy to which the acceleration occurs. We estimate it for typical conditions by proceeding from the energy conservation law which may be written out in the form of the relationship

$$\eta \, \frac{m_p u^2}{2} \, n_0 u = \overline{T} N_0 u' \,, \tag{99}$$

where $\eta$ is the fraction of the primary energy flux transferred to accelerated particles, $u$ and $u'$ are the velocities of the medium ahead of and behind the front, $n_0$ is the average proton concentration ahead of the front calculated with the inclusion of the neutral component, and $\overline{T}$ is the average kinetic energy of the accelerated particles. For a power law spectrum of accelerated protons, i.e.

$$\begin{aligned} & f_0(p,\theta) = \frac{1}{4\pi} \, N(p)(1 + A\cos\theta) \,, \\ & N(p) = (\alpha - 3) N_0 \, \frac{p_0^{\alpha-3}}{p^{\alpha}} \,, \qquad p_m \geqslant p \geqslant p_0 \,, \end{aligned} \tag{100}$$

we obtain

$$\begin{aligned} & \overline{T} \approx \frac{\alpha - 3}{5 - \alpha} \left( \frac{m_p c}{p_0} \right)^{5-\alpha} \frac{p_0^2}{2 m_p} \,, \quad 5 > \alpha > 4 \,, \quad p_0 \ll m_p c \,, \\ & \overline{T} \approx m_p c^2 \, \ln \left( \frac{p_m}{p_0} \right) , \quad \alpha = 4 \,, \\ & \overline{T} \approx \frac{\alpha - 3}{4 - \alpha} \left( \frac{p_0}{p_m} \right)^{\alpha-3} \mathcal{E}_m \,, \quad 3 < \alpha < 4 \,, \quad p_m \gg m_p c^2 \,. \end{aligned} \tag{101}$$

We shall not consider extreme situations with a strong front modification by accelerated particles $(\alpha < 4)$, restricting ourselves to a weak front distortion $(\alpha \approx 4)$ and a moderate fraction $(\eta \approx 0.1)$ of the energy flux transferred to the accelerated particles. Under these assumptions, one has

$$\frac{N_0}{n_0} \approx \frac{0.2}{\ln (p_m/p_0)} \left( \frac{u}{c} \right)^2 . \tag{102}$$

An important characteristic of the distribution function of the particles accelerated by diffusion mechanisms (for instance, in the first-order Fermi acceleration near the shock front) is the high degree of its isotropy. This is due to the fact that the particles must undergo strong scattering capable of confining them within the bounded acceleration region for a long time. That is why the dimensionless anisotropy parameter $A$ in expression (100) is small, as a rule. In the Galaxy, it has, on average, a value of $A \lesssim 10^{-3}$ for proton energies $\mathcal{E} \lesssim 10^4$ GeV and is on the order of $u/c \ll 1$ near the shock front.

Equation (98) allows some simplifications. With the aid of distribution function (100), we shall estimate the momentum density for accelerated particles and find the ratio between it and the momentum density for the background medium:

$$\frac{\mathcal{P}}{n_0 m_p u} \approx \frac{N_0}{n_0} \, \frac{cA}{u} \, \ln \left( \frac{p_m}{p_0} \right) \approx 0.2 \, \frac{Au}{c} \ll 1 \,,$$

if use is made of the estimate (102). By virtue of the relative smallness of accelerated-particle momentum density, it may be neglected on the right- and left-hand sides of Eqn (98). However, the term $\nabla_\beta \Pi_{\alpha\beta} = \nabla_\alpha P_{cr}$ should not be dropped, because the pressure due to the accelerated particles may amount to a significant fraction of the dynamic pressure of the gas-dynamic flow. Eventually, we obtain the equation of motion for the four-component continuous medium in the form

$$\begin{aligned} \rho \left[ \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\nabla)\mathbf{u} \right] &= \frac{1}{c} (\mathbf{j} + \mathbf{j}^{ext}) \times \mathbf{B} - \nabla(P + P_{cr}) \\ &+ \eta \left( \Delta \mathbf{u} + \frac{1}{3} \, \nabla(\nabla \mathbf{u}) \right) . \end{aligned} \tag{103}$$

In comparison with Eqn (11), in Eqn (103) there appear the total pressure and the total current, which include the contributions of accelerated particles. This corresponds to the assumption that the accelerated particles interact with the background plasma via the regular field and the stochastic fields. That is why it turns out that the magnetic force acting on the current produced by these particles is applied to the plasma as a whole. In Eqn (103), we wrote the dissipative term as well, since it underwent no changes as a result of the above transformations. To Eqn (103) must be added Maxwell's equations (25). The background-particle current $\mathbf{j}$ is related to electromagnetic fields by Ohm's law (17) or (22), but the accelerated-particle current $\mathbf{j}^{ext}$ does not obey this law and should be separately calculated employing the distribution function.

## 6. Restructuring of the static magnetic field by relativistic particles near the shock front

The high efficiency of mechanical energy transfer to accelerated particles was noted even in the first papers concerned with charged particle acceleration by strong MHD shock fronts in turbulent media [13, 45–47]. The accelerated component may acquire an energy which amounts to a few dozen percent of the shock wave energy. This leads to the result that the accelerated particles exert an appreciable retraction on the velocity of the plasma stream and the turbulence near the shock front. Different aspects of these phenomena have been investigated by many authors [48–50]. In the papers mentioned above and in other papers it was shown that the accelerated particles deform the hydrodynamic flow velocity profile and may, under certain conditions, be responsible for a complete smearing of the thermal jump. Relativistic particles intensively generate hydromagnetic (primarily Alfvén) perturbations via resonance and nonresonance mechanisms (see Refs [10–12, 27, 28] and others).These issues will be systematically outlined in the sections of our review that follow.

In this section we consider, following Ref. [51], one more aspect of this problem, namely, the possibility that the electric current of accelerated relativistic particles generates a static magnetic field ahead of the shock front. The spatial scale of this field is on the order of the prefront thickness $l \approx \kappa_1/u_1$, where $\kappa_1$ is the relativistic-particle diffusion tensor component normal to the front, and $u_1$ is the normal component of hydrodynamic plasma velocity. For nonrelativistic shock waves, this scale is far greater than the gyroradius of relativistic particles. The field generated in the prefront is added to the primary magnetic field and may substantially modify it. The plasma flow carries the modified field behind the front, and its magnitude and geometry have an important bearing on the acceleration of particles and their emission by nonthermal mechanisms.

An upper bound on the turbulence energy $\mathcal{W}_{\max}$ and the magnetic induction $\mathcal{B}_{\max}$ which may be generated by the accelerated particles themselves can be obtained from the energy conservation law. The energy density of the gas flow ahead of the front amounts to $n_0 m_p u_1^2/2$ (we neglect the internal energy). In our case, the part of the intermediate agent is played by relativistic particles, which can account for a couple dozen percent of the flow energy [50, 52]. Taking this fraction as $\eta \approx 20\%$ and assuming that $u_1 \approx 3 \times 10^8$ cm s$^{-1}$ (the value typical for supernova outbursts and strong stellar winds, see Lozinskaya [53]), we arrive at the estimates of the quantities we are seeking:

$$\mathcal{W}_{\max} \approx \eta\, \frac{n_0 m_p u_1^2}{2}\,, \qquad \mathcal{B}_{\max} \approx \sqrt{4\pi\eta n_0 m_p u_1^2}\,. \tag{104}$$

These estimates for the three principal known phases of the interstellar medium are collected in Table 2. They are several orders of magnitude higher than the observed values $B_{\text{obs}}$ (see, for instance, Refs [1, 31, 54]) in the corresponding phases, which may be indicative of the significance of the effect under discussion. That is why it is desirable to find for this problem a self-consistent solution which is based on the conservation laws and takes into account the dependence of the problem parameters on the magnetic field being calculated. This has been possible to achieve for the generation of a static magnetic field in the simple model considered below.

### 6.1 Formulation of the problem

In the context of a stationary self-consistent model we shall consider the one-dimensional problem of magnetic field generation by the electric current induced by accelerated relativistic particles with all possible energies. Let a plane shock front ($z = 0$) be a source of accelerated protons whose average energy will be specified below. Atomic nuclei heavier than protons and electrons make up only a small fraction of accelerated particles and will be neglected. The background plasma and background neutral medium will also be assumed to be purely hydrogenic. The plasma moves along the $Oz$-axis and the normal component of its velocity experiences a jump $\Delta u = u_z(0) - u_2 > 0$ at the front and is constant behind the front. The velocity $u_z(z)$ ahead of the front ($z < 0$) may smoothly decrease from a value $u_1$ far away from the front to some value $u_z(0) < u_1$ if an appreciable fraction of the energy of the shock wave goes into the acceleration of particles and the generation of a magnetic field.

The particles are accelerated in the vicinity of the front, and therefore the source of energetic particles at the front is specified in the form

$$Q(z) = q_0 \delta(z)\,, \tag{105}$$

which corresponds to a uniform particle injection from the background plasma into the regime of acceleration by the shock front. The injection power $q_0$ may be expressed in terms of a dimensionless parameter $\chi < 1$, namely, the fraction of particle flux injected into the acceleration regime: $q_0 = n_0 u_1 \chi$, where $n_0$ is the total concentration of hydrogen ions and neutral atoms ahead of the front. The neutral atoms are ionized at the front. Accelerated protons are drawn from the thermal background, and therefore at the front it is required to define the thermal proton sink

$$Q_{\text{th}}(z) = -Q(z)\,, \tag{106}$$

to ensure the conservation of electric charge and the number of particles.

Let us assume that ahead of the front there is an initial uniform magnetic field $\mathbf{B}_1$ whose direction is defined by polar angles $\theta$ and $\alpha$, and turbulent pulsations containing small-scale random magnetic fields which scatter particles. Relativistic and thermal particles will execute random wandering in the turbulent medium. It is essential that the magnetic field direction not be coincided with the normal to the shock front, because in this case, for a uniform particle injection into the acceleration regime at the front, the current may flow only along the $Oz$-axis and would not induce a magnetic field on the strength of symmetry. The diffusion coefficients $\kappa$ for relativistic protons and similar coefficients $D^{\text{p}}$ and $D^{\text{e}}$ for background nonrelativistic protons and electrons have different values along and across the large-scale magnetic field. Furthermore, they are different in the domains ahead of and behind the front. Owing to diffusion anisotropy, the relativistic and background particles will induce an electric current which may have projections on all three axes. The aim of our work is to estimate the magnetic field produced by this current.

The equation relating the magnetic field to the extrinsic current and the velocity of the medium is written within the $v_{\text{eff}} \gg v_{\text{m}}$ approximation and neglecting the Hall term in Eqn (20). We also neglect the contribution of turbulence to the conductivity and magnetic viscosity, which was considered in Section 2.2. Under these approximations, the magnetic field equation, which may be obtained from Eqns (20) and (25), takes on the form

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times (v_{\text{eff}} \nabla \times \mathbf{B})_\perp + \nabla \times [\mathbf{u} \times \mathbf{B}] + \frac{4\pi}{c}\, \nabla \times v_{\text{eff}}\, \mathbf{j}_\perp^{\text{ext}}\,, \qquad \nabla \cdot \mathbf{B} = 0\,. \tag{107}$$

Here, the subscript $\perp$ denotes the vector perpendicular to the field $\mathbf{B}$, $\mathbf{u}$ is the regular velocity of the medium in the prefront, and the quantities $\mathbf{B}$, $\mathbf{u}$, and $\mathbf{j}^{\text{ext}}$ depend on one coordinate, $z$. The magnetic viscosity, which is proportional to $B^2(z)$, also depends on this coordinate.

**Table 2.**

| $n_0$, protons cm$^{-3}$ | $\mathcal{B}_{\max}$, G | $\mathcal{W}_{\max}$, eV cm$^{-3}$ | $B_{\text{obs}}$, G | Note |
|---|---|---|---|---|
| 20 | $2.8 \times 10^{-3}$ | $2 \times 10^4$ | $1.5 \times 10^{-5}$ | Neutral clouds |
| 0.2 | $2.8 \times 10^{-4}$ | 200 | $(2-6) \times 10^{-6}$ | 'Warm' intercloud medium |
| $2 \times 10^{-3}$ | $2.8 \times 10^{-5}$ | 2 | ? | Hot caverns |

### 6.2 Calculation of the total current

The total extrinsic current in the plasma is induced by relativistic protons, as well as by background protons and electrons: $\mathbf{j}^{\text{ext}} = \mathbf{j}^{\text{p}} + \mathbf{j}^{\text{e}}$. The proton- and electron-induced currents may be written out as ($z \neq 0$)

$$j_\alpha^{\text{p}} = -e\kappa_{\alpha\beta}\,\frac{\partial N}{\partial x_\beta} - eD_{\alpha\beta}^{\text{p}}\,\frac{\partial n_{\text{p}}}{\partial x_\beta} + eu_\alpha(N + n_{\text{p}}) + \sigma_{\alpha\beta}^{\text{p}} E_\beta\,, \tag{108}$$

$$j_\alpha^{\text{e}} = eD_{\alpha\beta}^{\text{e}}\,\frac{\partial n_{\text{e}}}{\partial x_\beta} - eu_\alpha n_{\text{e}} + \sigma_{\alpha\beta}^{\text{e}} E_\beta\,, \tag{109}$$

where $N$ is the number density of accelerated protons, $n_{\text{p}}$ and $n_{\text{e}}$ are the nonequilibrium additions to the concentrations of the background protons and electrons, and $\mathbf{E}$ is the electric field which emerges due to charge separation and satisfies the equation of electrostatics:

$$\nabla\mathbf{E} = 4\pi e(N + n_{\text{p}} - n_{\text{e}})\,. \tag{110}$$

Here, we consider the integral concentrations of arbitrary-energy particles. The quantities $\sigma^{\text{e}}$ and $\sigma^{\text{p}}$ are the conductivities of the background plasma, related to its diffusion coefficients as [55]

$$\sigma_{\alpha\beta}^{\text{e,p}} = \frac{D_{\alpha\beta}^{\text{e,p}}}{4\pi r_{\text{D}}^2}\,, \tag{111}$$

where $r_{\text{D}} = \sqrt{T/4\pi n_0 e^2}$ is the Debye radius, $T$ is the temperature, and $n_0$ is the equilibrium number density of the background protons or electrons.

In the stationary case, the total current $\mathbf{j}^{\text{ext}} = \mathbf{j}^{\text{p}} + \mathbf{j}^{\text{e}}$, as well as the currents of relativistic and nonrelativistic protons and electrons taken separately, satisfies the equations

$$\nabla\,\mathbf{j}^{\text{ext}} = \nabla\,\mathbf{j}^{\text{p}} = \nabla\,\mathbf{j}^{\text{e}} = 0\,. \tag{112}$$

As noted above, we shall consider the one-dimensional case and treat the number densities and the currents as functions of one spatial coordinate, $z$. The relativistic particles are scarcely affected by the self-consistent electric field, and in the equation for the number density $N$ it would therefore suffice to account for only the diffusion of the particles and their convective transfer by the motion of the medium. For a steady-state acceleration, the equation is of the form

$$\frac{\mathrm{d}}{\mathrm{d}z}\left\{-\kappa_{1,2}\,\frac{\mathrm{d}N_{1,2}}{\mathrm{d}z} + u_{1,2}N_{1,2}\right\} = q_0\delta(z)\,. \tag{113}$$

Here, the subscripts 1, 2 refer to the domains ahead of (1) and behind (2) the shock front, and $\kappa_{1,2}$ are the values of diffusion coefficient $\kappa_{zz}$ in the corresponding domains. Similar notation will be used for the coefficients $D^{\text{p}}$ and $D^{\text{e}}$ in the subsequent discussion. The diffusion coefficients $\kappa$ and $D$ themselves are considered to mean the quantities which are averaged over the energy spectra, respectively, of relativistic and thermal particles and which depend on $z$ in the general case.

On the right-hand side of Eqn (113) we replace the source by the boundary conditions [8]

$$N_1 = N_2\,, \qquad \kappa_1\,\frac{\mathrm{d}N_1}{\mathrm{d}z} - \kappa_2\,\frac{\mathrm{d}N_2}{\mathrm{d}z} - \Delta u N_{1,2} = q_0 \quad \text{for} \quad z = 0\,, \tag{114}$$

to find the solutions of equation (113):

$$\begin{aligned} &N_1(z) = N_0\exp\left[-\zeta(z)\right], \qquad N_0 = \frac{q_0}{u_2}\,, \\ &\zeta(z) = \int_z^0 \frac{u_z(z')\,\mathrm{d}z'}{\kappa_1(z')}\,, \qquad z \leqslant 0; \\ &N_2 = N_0\,, \qquad z \geqslant 0\,. \end{aligned} \tag{115}$$

The condition $\zeta(l) \approx 1$ defines the distance $l$ by which the accelerated particles are able to move upstream before they are carried behind the shock front.

The concentrations of nonrelativistic particles readjust to the distribution of relativistic protons and should be determined in the $z > 0$ domain using Eqns (112). These equations have the form

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}z}\left(-eD_1^{\text{p}}\,\frac{\mathrm{d}n_{\text{p}}}{\mathrm{d}z} + eu_z(z)n_{\text{p}} + \sigma_{zz}^{\text{p}} E_z\right) = 0\,, \\ &\frac{\mathrm{d}}{\mathrm{d}z}\left(eD_1^{\text{e}}\,\frac{\mathrm{d}n_{\text{e}}}{\mathrm{d}z} - eu_z(z)n_{\text{e}} + \sigma_{zz}^{\text{e}} E_z\right) = 0\,. \end{aligned} \tag{116}$$

We integrate this system and take advantage of formula (111) to obtain

$$E_z = 4\pi e r_{\text{D}}^2\left(\frac{\mathrm{d}n_{\text{p}}}{\mathrm{d}z} - \frac{u_z(z)}{D_1^{\text{p}}}\,n_{\text{p}}\right) = -4\pi e r_{\text{D}}^2\left(\frac{\mathrm{d}n_{\text{e}}}{\mathrm{d}z} - \frac{u_z(z)}{D_1^{\text{e}}}\,n_{\text{e}}\right). \tag{117}$$

This permits, in view of formulas (108) and (109), writing out the components of the total current:

$$\begin{aligned} &j_x^{\text{ext}} = -e\kappa_{xz}\,\frac{\mathrm{d}N}{\mathrm{d}z} + eu_z(z)\left\{\frac{D_{xz}^{\text{e}}}{D_1^{\text{e}}}\,n_{\text{e}} - \frac{D_{xz}^{\text{p}}}{D_1^{\text{p}}}\,n_{\text{p}}\right\}, \\ &j_y^{\text{ext}} = -e\kappa_{yz}\,\frac{\mathrm{d}N}{\mathrm{d}z} + eu_z(z)\left\{\frac{D_{yz}^{\text{e}}}{D_1^{\text{e}}}\,n_{\text{e}} - \frac{D_{yz}^{\text{p}}}{D_1^{\text{p}}}\,n_{\text{p}}\right\}, \\ &j_z^{\text{ext}} = 0\,, \end{aligned} \tag{118}$$

where $N(z)$ is given by formulas (115). Attempts to perform the subsequent calculation in the general form do not meet with success, and it calls for additional assumptions.

(1) The parameter

$$\left(\frac{ur_{\text{D}}}{\kappa_1}\right)^2 = \left(\frac{3ur_{\text{D}}}{v\Lambda_\parallel}\right)^2 \ll 1 \tag{119}$$

is small for the interstellar medium phases under consideration. For relativistic particles and nonrelativistic shock waves, the ratio $3u/v < 1$. The parameter $r_{\text{D}}/\Lambda_\parallel$ has a very small value not exceeding $10^{-13}$, if the average over the galactic disk, $\Lambda_\parallel \approx 10^{18}$ cm, is involved (see Ref. [18]). Even an increase in the magnetic field strength and the turbulent pulsations by several orders of magnitude near the front cannot compensate for this smallness.

From the smallness of the above parameter, on the strength of Eqns (110) and (117) there follows the plasma quasineutrality condition

$$N + n_{\text{p}} - n_{\text{e}} \approx 0\,, \tag{120}$$

which is fulfilled with an accuracy of order the magnitude of the aforementioned parameter. The inhomogeneity scale for the background particle distribution is the same as for accelerated protons, and therefore to an order of magnitude one has $n'_{\rm p,e} \approx (u/\kappa_1) n_{\rm p,e}$.

When the thermal-particle diffusion coefficients are low enough, namely

$$D_1^{\rm p,e} \ll \kappa_1 \,, \tag{121}$$

the terms with derivatives in Eqn (117) may be neglected, which gives $n_{\rm p}/n_{\rm e} \approx -D_1^{\rm p}/D_1^{\rm e}$. The quasineutrality condition permits finding

$$n_{\rm p} \approx -N(z)\,\frac{D_1^{\rm p}}{D_1^{\rm p}+D_1^{\rm e}}\,, \qquad n_{\rm e} \approx N(z)\,\frac{D_1^{\rm e}}{D_1^{\rm p}+D_1^{\rm e}} \tag{122}$$

and obtaining under this approximation the current density in the prefront domain:

$$\begin{aligned} j_x^{\rm ext} &= eu_z(z)\left\{\frac{D_{xz}^{\rm e}+D_{xz}^{\rm p}}{D_1^{\rm e}+D_1^{\rm p}} - \frac{\kappa_{xz}}{\kappa_1}\right\} N(z)\,, \\ j_y^{\rm ext} &= eu_z(z)\left\{\frac{D_{yz}^{\rm e}+D_{yz}^{\rm p}}{D_1^{\rm e}+D_1^{\rm p}} - \frac{\kappa_{yz}}{\kappa_1}\right\} N(z)\,. \end{aligned} \tag{123}$$

(2) When condition (121) is not fulfilled but the diffusion of thermal particles is ambipolar, as is the case in neutral clouds and the thermal phase [1], protons and electrons diffuse jointly with equal effective diffusion coefficients $D_{\mu\nu}^{\rm e} \approx D_{\mu\nu}^{\rm p}$ which by an order of magnitude are close to the lesser of them. In this case, with the help of the quasineutrality condition, from Eqns (118) we obtain

$$\begin{aligned} j_x^{\rm ext} &= eu_z(z)\left\{\frac{D_{xz}^{\rm p}}{D_1^{\rm p}} - \frac{\kappa_{xz}}{\kappa_1}\right\} N(z)\,, \\ j_y^{\rm ext} &= eu_z(z)\left\{\frac{D_{yz}^{\rm p}}{D_1^{\rm p}} - \frac{\kappa_{yz}}{\kappa_1}\right\} N(z)\,. \end{aligned} \tag{124}$$

In the case of ambipolar diffusion, expressions (123) and (124) coincide.

The resultant expressions show that a nonzero current density in the model under discussion may emerge only in the presence of cross diffusion coefficients $\kappa_{xz}$, $\kappa_{yz}$ and of similar coefficients for background particles. In this case, the curl of the current, which enters into Eqn (107) as a source, is nonzero. Let $\Lambda$ denote the transport mean free path of particles along the regular magnetic field $\mathbf{B}$; then, in the system of axes, one of which coincides with $\mathbf{B}$, we will have the longitudinal, transverse, and Hall diffusion coefficients (see, for instance, Ref. [8])

$$\begin{aligned} \kappa_\parallel &= \frac{1}{3}\, v\Lambda\,, \\ \kappa_\perp &= \kappa_\parallel\,\frac{r_{\rm g}^2}{r_{\rm g}^2+\Lambda^2}\,, \\ \kappa_{\rm H} &= \kappa_\parallel\,\frac{r_{\rm g}\Lambda}{r_{\rm g}^2+\Lambda^2}\,. \end{aligned} \tag{125}$$

Here, $v$ is the velocity of a particle, and $r_{\rm g} = cp/eB_0$ is its gyroradius. The thermal-particle diffusion coefficients are expressed in a similar way, but their Larmor radii are substantially shorter than for relativistic protons and their mean free paths are determined by entirely different physical factors. That is why it is highly unlikely that the terms in braces in expressions (123) and (124) cancel out.

The diffusion coefficients entering into formulas (123) and (124) are expressed through relationships (125) as follows:

$$\begin{aligned} \kappa_{xz} &= (\kappa_\parallel - \kappa_\perp)\sin\theta\cos\theta\cos\alpha - \kappa_{\rm H}\sin\theta\sin\alpha\,, \\ \kappa_{yz} &= (\kappa_\parallel - \kappa_\perp)\sin\theta\cos\theta\sin\alpha + \kappa_{\rm H}\sin\theta\cos\alpha\,, \\ \kappa_1 &= \kappa_\parallel\cos^2\theta + \kappa_\perp\sin^2\theta\,. \end{aligned} \tag{126}$$

The angles $\theta$ and $\alpha$ define the direction of the large-scale field in the coordinate system with the polar axis aligned with the normal to the plane of the shock front. The background-particle diffusion coefficients possess a similar structure. On permutation of the indices $x$, $z$ and $y$, $z$ on the left-hand sides of formulas (126), the sign in front of $\kappa_{\rm H}$ on the right-hand sides should be changed.

We note that the electric current in a system in which the electric charge is compensated for with a high degree of accuracy and its macroscopic displacements are lacking arises from the nonuniformity of the distribution of the Larmor circles of individual particles, both accelerated and background. The current under discussion is similar to the magnetization current $\mathbf{j} = c\,\mathrm{rot}\,\mathbf{M}$ in macroscopic electrodynamics, where $\mathbf{M}$ is the nonuniform magnetization of the medium.

The currents induced by relativistic protons and background particles enter into expressions (123) and (124) with different signs, which reflects the screening effect. Nevertheless, any appreciable local cancellation of these currents in a large spatial domain, in our view, necessitates special conditions. This would signify the readjustment of background-particle diffusion coefficients to the values of relativistic-proton diffusion coefficients. As noted in the foregoing, no physical mechanisms have been established for such a readjustment. At the same time, the electric charge balance condition (120) is fulfilled with a high degree of accuracy due to the action of Coulomb forces. For these reasons we do not assume, as some general principle, that the total current is equal to zero, $\mathbf{j}^{\rm ext} + \mathbf{j}^{\rm p} + \mathbf{j}^{\rm e} = 0$, as is done in several papers. This situation is only possible in some special cases (a uniform magnetic field and so on).

### 6.3 Self-consistent magnetic field calculation

We write out the conservation laws assuming that the plasma ahead of the shock front is cold and its pressure is negligible, $P_{\rm g} \approx 0$. The plasma velocity $u_1$ far away from the front has only one $z$-component, i.e., is normal to the front. However, not far from the front there appears a tangential component defined by the magnetic field. The corresponding system of equations should include the magnetic field and in this sense generalize the relations given in Ref. [50]. We have the following system which expresses the constancy of mass, momentum, and energy fluxes:

$$J = m_{\rm p} n(z) u_z(z) = m_{\rm p} n_0 u_1 = \mathrm{const}\,, \tag{127}$$

$$Ju_z(z) + P_{\rm c}(z) + \frac{B_{\rm t}^2(z)}{8\pi} = Ju_1 + \frac{B_{1\rm t}^2}{8\pi}\,, \tag{128}$$

$$J\mathbf{u}_{\rm t}(z) - \frac{1}{4\pi}\,B_{1z}\mathbf{B}_{\rm t}(z) = -\frac{1}{4\pi}\,B_{1z}\mathbf{B}_{1\rm t}\,, \tag{129}$$

$$\begin{aligned} &\frac{1}{2}\,Ju_z^2(z) + \frac{\gamma_{\rm c}}{\gamma_{\rm c}-1}\,u_z(z)P_{\rm c}(z) - \frac{\kappa_1(z)}{\gamma_{\rm c}-1}\,\frac{{\rm d}P_{\rm c}}{{\rm d}z} \\ &\qquad + \frac{1}{4\pi}\,u_z(z)B_{\rm t}^2(z) + q_m(z) = \frac{1}{2}\,Ju_1^2 + \frac{1}{4\pi}\,u_1 B_{1\rm t}^2\,. \end{aligned} \tag{130}$$

Here, $J = m_{\rm p} n_0 u_1$ is the mass flux density, $\gamma_{\rm c}$ is the index of the Poisson adiabat for the relativistic gas of accelerated particles, and $q_m(z)$ is the runaway-particle energy flux. To calculate it requires knowing the accelerated-particle energy spectrum. In formulas (127)–(130), $\mathbf{B}_{\rm t} = \big(B_x(z), B_y(z)\big)$ denotes the projection of the total magnetic field onto the $(x, y)$ plane parallel to the shock front. It includes both the initial field $\mathbf{B}_1$ and the additional field generated by the accelerated particles. Similarly, one finds that

$$B(z) = \sqrt{B_x^2(z) + B_y^2(z) + B_{1z}^2}$$

is the absolute value of the total magnetic field.

Maxwell's equations (107) in combination with Eqn (129) permit an easy derivation of the first integrals for the one-dimensional case under consideration:

$$\begin{aligned}
&\nu_{\rm eff}(z)\,\frac{{\rm d}B_x}{{\rm d}z} - \nu_{\rm eff}(z) B_y \left(\frac{{\rm d}B_x}{{\rm d}z}\,\frac{B_y}{B^2} - \frac{{\rm d}B_y}{{\rm d}z}\,\frac{B_x}{B^2}\right)\\
&- u_z(z) B_x + \frac{B_{1z}^2}{4\pi J}\,B_x\\
&= \frac{4\pi}{c}\,\nu_{\rm eff}(z)\, j_y^{\rm ext}(z) - \frac{4\pi}{cB^2}\,\nu_{\rm eff}(z)\big(j_x^{\rm ext} B_x + j_y^{\rm ext} B_y\big) B_y - u_1 B_{1x}\,, \qquad (131)\\
&\nu_{\rm eff}(z)\,\frac{{\rm d}B_y}{{\rm d}z} + \nu_{\rm eff}(z) B_x \left(\frac{{\rm d}B_x}{{\rm d}z}\,\frac{B_y}{B^2} - \frac{{\rm d}B_y}{{\rm d}z}\,\frac{B_x}{B^2}\right)\\
&- u_z(z) B_y + \frac{B_{1z}^2}{4\pi J}\,B_y\\
&= -\frac{4\pi}{c}\,\nu_{\rm eff}(z)\, j_x(z) + \frac{4\pi}{cB^2}\,\nu_{\rm eff}(z)\big(j_x^{\rm ext} B_x + j_y^{\rm ext} B_y\big) B_x - u_1 B_{1y}\,,
\end{aligned}$$

with $B_z(z) = B_{1z} = \text{const}$. In the self-consistent calculation, the dependence of all diffusion coefficients on the total magnetic field should be taken into account. In particular, the trigonometric functions in formulas (126) should be expressed as follows:

$$\cos\theta = \frac{B_{1z}}{B(z)}\,, \qquad \sin\theta\cos\alpha = \frac{B_x(z)}{B(z)}\,, \qquad \sin\theta\sin\alpha = \frac{B_y(z)}{B(z)}\,. \qquad (132)$$

According to the estimate made at the beginning of this section, the ratio $B_{1z}^2/4\pi J u_1 \lesssim 10^{-4}$ in the main phases of the interstellar medium, and therefore the terms $B_{1z}^2 B_{x,y}/4\pi J u_1$ are discarded in what follows.

The solution of Eqns (131) essentially depends on the ratio between the accelerated-particle and magnetic-field diffusion coefficients. The inhomogeneity in magnetic field distribution arises from accelerated particles, and therefore the derivative $|B'_{x,y}| \lesssim (u_1/\kappa_1)|B_{x,y}|$. From this estimate it follows that the ratio $\nu_{\rm eff}/\kappa_1$ determines the relative contribution of the terms that do and do not contain the derivatives of the magnetic field with respect to $z$ on the left-hand sides of Eqn (131). We shall consider two cases.

(1) $\nu_{\rm eff}/\kappa_1 \ll 1$. The terms with the derivative may be neglected, which gives the solution

$$\begin{aligned}
B_x(z) &= -\frac{4\pi\nu_{\rm eff}(z)}{c u_z(z)}\left\{ j_y^{\rm ext} - \frac{B_y}{B^2}\big(j_x^{\rm ext} B_x + j_y^{\rm ext} B_y\big)\right\} + \frac{u_1}{u_z(z)}\,B_{1x}\,,\\
B_y(z) &= \frac{4\pi\nu_{\rm eff}(z)}{c u_z(z)}\left\{ j_x^{\rm ext} - \frac{B_x}{B^2}\big(j_x^{\rm ext} B_x + j_y^{\rm ext} B_y\big)\right\} + \frac{u_1}{u_z(z)}\,B_{1y}\,.
\end{aligned} \qquad (133)$$

(2) $\nu_{\rm eff}/\kappa_1 \gg 1$. This case calls for a numerical solution, but for a semiqualitative analysis of the physical picture it would suffice to estimate the derivatives by the order of magnitude: ${\rm d}B_{x,y}/{\rm d}z \approx (u_1/\kappa_1) B_{x,y}$. We substitute this estimate into Eqn (131) and find an approximate solution:

$$\begin{aligned}
B_x(z) &= \frac{4\pi\kappa_1(z)}{c u_1}\left\{ j_y^{\rm ext} - \frac{B_y}{B^2}\big(j_x^{\rm ext} B_x + j_y^{\rm ext} B_y\big)\right\} + B_{1x}\,,\\
B_y(z) &= -\frac{4\pi\kappa_1(z)}{c u_1}\left\{ j_x^{\rm ext} - \frac{B_x}{B^2}\big(j_x^{\rm ext} B_x + j_y^{\rm ext} B_y\big)\right\} + B_{1y}\,.
\end{aligned} \qquad (134)$$

Even after the simplifications made above, equalities (133) and (134) are complex integral equations in $B_x(z)$ and $B_y(z)$, because the particle diffusion coefficients depend on these quantities. The coefficient $\kappa_1$ defines, via integral (115), the thickness of the layer in which the field is changed by relativistic particles. However, when our concern is only with the magnetic field at the front ($z = 0$), the integral (115) vanishes and relationships (133) and (134) transform to transcendental equations in $B_x(0)$ and $B_y(0)$.

To calculate them we preset the fraction $\eta$ of the dynamic pressure of the plasma flow, which is transferred to accelerated particles and the magnetic field they generate:

$$\eta = \frac{P_{\rm c}(0) + B^2(0)/8\pi - B_1^2/8\pi}{J u_1}\,. \qquad (135)$$

From Eqn (128) we find

$$\frac{u_z(0)}{u_1} = 1 - \eta\,. \qquad (136)$$

In the subsequent estimates the value of $\eta$ will be set at a level not exceeding 20%, although it may be several times greater under intense acceleration of particles.

The flux density $q_0 = N_0 u_2$ of particles injected into the acceleration regime can also be expressed in terms of the relative value $\eta$ of the dynamic pressure. We take into account the relation between the pressure $P_{\rm c}$ of relativistic particles and their average kinetic energy $\overline{T}$:

$$P_{\rm c} = \frac{1}{3}\int v p N(p)\, p^2\, {\rm d}p = \frac{\overline{vp}}{3}\,N_0 = (\gamma_{\rm c} - 1)\overline{T} N_0\,. \qquad (137)$$

With the help of relationship (135) we find

$$q_0 = \frac{n_0 u_2}{\gamma_{\rm c} - 1}\,\frac{m_{\rm p} u_1^2}{\overline{T}}\left(\eta - \frac{B_{\rm t}^2(0) - B_{1{\rm t}}^2}{8\pi J u_1}\right), \qquad (138)$$

where $u_2$ is the plasma velocity behind the shock front.

We begin estimating the field at the front in the case that $\nu_{\rm eff} \gg \kappa_1$. We consider the regime of intense particle acceleration at the front, capable of furnishing a small value of the diffusion coefficient. In this regime, the transport mean free path assumes the least possible value of $\Lambda \approx r_{\rm g}$, i.e., comes to be on the order of the gyroradius (the Bohm limit). The reasoning in favor of the realization of this situation is given in Section 11 of our review.

In this case, relationships (125) give

$$\kappa_\parallel = \frac{\overline{vp}}{3}\,\frac{c}{eB} = 2\kappa_\perp = 2\kappa_{\rm H}\,. \qquad (139)$$

Here, the over-bar denotes averaging over the spectrum of accelerated particles. It may be performed with the aid of relationship (137). As regards the tensor $D^{\rm p}_{\mu\nu}$ we make only one assumption: we put $D^{\rm p}_{\rm H} \approx 0$ because of the smallness of the Larmor radii of thermal particles and because these coefficients for protons and electrons are opposite in sign. In this case, it is not necessary that $D^{\rm p}_{\parallel} \gg D^{\rm p}_{\perp}$, since the random component of the magnetic field may partly isotropize these coefficients. Under the assumptions that $B_{1z} \neq 0$, $B_{1y} \neq 0$, and $B_{1x} = 0$ we also introduce the dimensionless quantities

$$X = \frac{B_x(0)}{B(0)} , \qquad Y = \frac{B_y(0)}{B(0)} ,$$
$$B^2(0) = \frac{B_{1z}^2}{1 - X^2 - Y^2} , \qquad d = \frac{D^{\rm p}_{\perp}}{D^{\rm p}_{\parallel} - D^{\rm p}_{\perp}} \geqslant 0 , \tag{140}$$

and the parameter with the dimensionality of the square of the magnetic field induction, viz.

$$b^2 = 2\pi \gamma_{\rm c} J u_1 \eta + \frac{\gamma_{\rm c} B_1^2}{4} . \tag{141}$$

This parameter characterizes the free energy density of the flow of matter with an initial magnetic field. The index of the Poisson adiabat for the relativistic gas is $\gamma_{\rm c} \approx 4/3$. The magnetic field $b$ is of the same order of magnitude as the previously calculated field $\mathcal{B}_{\max}$ [see formula (104) and Table 2]. For a strong shock wave, it is higher by at least two orders of magnitude than the field observed in the corresponding phases.

The possible values of the field at the shock front should be determined from the system of equations (134), into which one should substitute the current components (124), put $z = 0$, and express all quantities in terms of $X$, $Y$, $d$, and $b$ with the help of formulas (140) and (141). Eventually, an awkward system of equations containing fractions and radicals results. We give here simple linear equations written down in the approximation $|X| \ll 1$, $|Y| \ll 1$:

$$X = -\frac{b^2}{B_{1z}^2} \left[ X + \frac{B_{1z}(d-1)}{|B_{1z}|(d+1)} \, Y \right] ,$$
$$Y = \frac{b^2}{B_{1z}^2} \left[ -Y + \frac{B_{1z}(d-1)}{|B_{1z}|(d+1)} \, X \right] + \frac{B_{1y}}{|B_{1z}|} . \tag{142}$$

This system has the unique solution

$$B_x(0) = -B_{1y} \, \frac{B_{1z}|B_{1z}|\delta_*}{b^2(1+\delta_*^2)} , \qquad B_y(0) = B_{1y} \, \frac{B_{1z}^2}{b^2(1+\delta_*^2)} , \tag{143}$$

where $\delta_* = (d-1)/(d+1)$. Here, $b^2 \gg B_{1z}^2$ and the parameter $\delta_*$ does not exceed unity: $0 \leqslant \delta_*^2 \leqslant 1$. That is why the parallel components of the initial field at the front turn out to be suppressed by the secondary field generated by the electric current of accelerated particles. The suppression factor (on the order of $10^4$ under intense acceleration) is quite significant. Even when the fraction of energy transferred to accelerated particles lowers to $\eta = 10^{-4}$, there persists a tenfold suppression of the field components directed along the front. Other shock transition regimes, if any, are to be determined from the unsimplified system of equations. But its numerical solution shows that all real roots of this system are small in comparison with unity. All possible solutions are therefore exhausted by formulas (143) — that is, near the front there actually persists only the magnetic field component normal to the front for uniform particle injection.

This state of a strongly nonequilibrium plasma system is an example of self-organization of a plasma with a magnetic field. The transition of the system to this state has a simple physical meaning. For a normal orientation of the magnetic field relative to the shock front, the subsequent field generation terminates by virtue of the system symmetry, and such a configuration is stable in this sense. The system tends to change to precisely this stable state. If there is a tangential field component far away from the front, it decreases in magnitude as the front is approached. The degree of suppression is determined by the energy stored by the accelerated particles inducing the secondary field.

The resultant solution does not allow the passage to the limit $B_{1z} \to 0$, because in this case the conditions $|X| \ll 1$ and $|Y| \ll 1$ are incompatible with the equation $X^2 + Y^2 = 1$, which follows from expressions (140). That is why the case of a front-parallel initial magnetic field should be treated separately.

At $B_{1z} = B_{1x} = 0$, the value of $B_x(0) = 0$ transforms the first equation (134) to an identity. From the second equation, upon substituting into it the corresponding quantities, in particular $B_y(0) = \pm B(0)$, we obtain two quadratic equations

$$(1 \mp 4) B^2 + 4 B_{1y} B - \frac{1}{\gamma_{\rm c}} \, b^2 = 0 . \tag{144}$$

One of them has a solution satisfying the physical requirement $B(0) > 0$:

$$B(0) = \sqrt{\frac{4}{25} \, B_{1y}^2 + \frac{4b^2}{5\gamma_{\rm c}}} - \frac{2}{5} \, B_{1y} \approx \frac{2b}{\sqrt{5\gamma_{\rm c}}} \gg B_{1y} . \tag{145}$$

The remaining roots are either negative or complex.

The field generated by accelerated particles in this case, i.e., in the absence of a primary field component normal to the front, may exceed by approximately 100 times typical fields in the cold and warm phases of the interstellar medium. However, the question of whether this state is stable is still open. Conceivably, the small fluctuations in the normal component, as well as the density fluctuations in the prefront medium, might transfer the system to the above-considered state with $B_{1z} \neq 0$, in which the tangential field component at the front is strongly suppressed.

We next investigate the case with $\kappa_1 \gg \nu_{\rm eff}$. The generated field is, according to expression (133), proportional to the effective magnetic viscosity $\nu_{\rm eff}$. We represent it, according to formulas (21) and (29), in the form $\nu_{\rm eff}(z) = \nu_0 B^2(z)/B_1^2$, where $B_1$ is the average field in the corresponding phase, and $\nu_0$ is the magnetic viscosity corresponding to the average field. The main difference from the previous case consists in the different dependences of $\nu_{\rm eff}$ and $\kappa_1$ on the magnetic field. We perform the calculation to find for the case of $B_{1z} \neq 0$:

$$B_x(0) = B_{1y} \, \frac{2 B_1^2 B_{1z} \delta_*}{B_{1z}^2 H (1-\eta)(1+\delta_*^2)} ,$$
$$B_y(0) = -B_{1y} \, \frac{2 B_1^2}{|B_{1z}| H (1-\eta)(1+\delta_*^2)} , \tag{146}$$

where

$$H = \frac{1}{h} \left( 8\pi J u_1 \eta + B_1^2 \right) , \qquad h = \frac{2(\gamma_{\rm c} - 1) c \overline{T}}{e \nu_0} . \tag{147}$$

The average accelerated-particle energy, which enters into formula (147), depends on the shape of the energy spectrum. For a sufficiently strong acceleration, the exponent of the momentum spectrum in the phase space is close to four. In this case, the average energy is given by [50]

$$\overline{T} = \frac{cp_0}{3(\gamma_c - 1)} \ln\left(\frac{2p_m}{m_p c}\right),$$

where $p_0 \approx m_p u_1$ is the injection momentum, and $p_m$ is the greatest momentum reached in the course of the acceleration (in the subsequent discussion we assume that $p_m = 100 m_p c$). This yields $\overline{T} \approx 0.8 \times 10^{-4}$ erg. The ratio $H/B_1 \approx 10^2 - 10^5 \gg 1$ for typical parameter values in the warm phase and in neutral clouds. Again, the tangential field component at the front is therefore strongly suppressed, with only the normal component persisting.

But result (146) is inapplicable for $B_{1z} \to 0$, and the orientation of the initial field along the front should be considered separately. We set $B_{1z} = B_{1x} = 0$, $|B_{1y}| = B_1 \neq 0$, and introduce new variables $X = B_x(0)/B_1$, $Y = B_y(0)/B_1$. We seek the solution with $X = 0$. For the determination of $Y$ we obtain the equation

$$Y^4 - \frac{hH}{B_1^2}\, Y^2 \pm \frac{h}{B_1}\, Y \mp \frac{h}{B_1(1-\eta)} = 0\,. \tag{148}$$

Here, the upper signs correspond to the condition $B_y(0) > 0$ and the lower signs to the opposite condition, $B_y(0) < 0$. The coefficients of Eqn (148) significantly differ in magnitude: $hH/B_1^2 > 10^4$, $h/B_1 < 10$ in the warm phase and neutral clouds for $\eta \approx 20\%$. This equation therefore has large and small roots which may be approximately found by dropping in turn the small terms in the equation. For $Y \ll 1$, we discard the term $Y^4$ in Eqn (148) and from the quadratic equation we obtain

$$B_y(0) = \mp B_{1y} \sqrt{\frac{|B_{1y}|}{H(1-\eta)}} \ll B_1\,. \tag{149}$$

For $Y \gg 1$, we retain the first two terms in Eqn (148) to find

$$B_y(0) = \pm\sqrt{hH}\,, \qquad \left|B_y(0)\right| \gg B_1\,. \tag{150}$$

Therefore, in this case there are two shock front states with low and high magnetic fields. The possibility of several stationary solutions is a natural result for an open and strongly nonequilibrium system. The realizability of the corresponding regimes in nature should be elucidated by investigating their immunity to small perturbations. This is a separate problem which is not considered here (for an example of the solution to a similar stability problem of a shock front with relativistic particles, see Ref. [56]).

The magnetic field generated in the prefront of the shock wave is carried behind the front by the flux of matter. Its subsequent evolution depends on the physical conditions behind the front and, in particular, on the structure of the velocity field. The field in spherical supernova remnants was calculated in Ref. [57] for an arbitrary structure in the domain ahead of the shock front.

The theoretically predicted result concerning the suppression of the tangential magnetic field signifies, as applied to the quasispherical supernova remnants, that the field should be radial in structure near the remnant boundary. This conclusion is confirmed by the observed data on the polarization of synchrotron radio emission. The radial magnetic field in young supernova remnants (Tycho, Kepler, 1006, Cassiopeia A) was noted by Reynolds and Gilmore [58]. (For a more detailed discussion, see Ref. [57].)

It should be emphasized that the assumption about the uniform injection of particles into the acceleration regime is highly important to the problem under consideration. When the injection is nonuniform, which may be caused, for instance, by the density nonuniformity of the medium ahead of the front, in the prefrontal region there are bound to emerge currents parallel to the front, which may be responsible for the generation of an additional parallel field.

A significant limitation on the investigation conducted is also the assumption that the plane front is unbounded in dimensions, from which follows one-dimensionality of the problem. In reality, owing to the boundedness of the shock fronts in real objects, each such front is a source of large-scale electric current which spreads over the entire Galaxy and is able to generate a magnetic field away from the front, and not only in the prefrontal region (see Ref. [59]). The solution of these more realistic problems in a nonlinear self-consistent formulation with nonuniform (and nonstationary) injection and with shock fronts bounded in dimensions is supposedly possible only by numerical methods.

## 7. Accelerated-particle current driven by a weak MHD wave

We now address ourselves to the investigation of nonstationary turbulent motions and calculate the electric current of accelerated particles, which emerges under the action of a weak MHD wave, in order to elucidate the possibility of exciting oscillations. The distribution function unperturbed by the field of the MHD wave is taken in the form of expression (100). It is assumed to be stationary and is characterized by the anisotropy parameter $A \ll 1$ and a power-law shape of the particle momentum spectrum ($\theta$ is the angle between the uniform field $\mathbf{B}_0$ and the particle momentum).

The accelerated-particle distribution function $f(\mathbf{r}, \mathbf{p}, \theta, \phi, t)$ perturbed by an external field satisfies the kinetic equation

$$\frac{\partial f}{\partial t} + \mathbf{v}\,\frac{\partial f}{\partial \mathbf{r}} + e\mathbf{E}\,\frac{\partial f}{\partial \mathbf{p}} - \frac{ec}{\mathcal{E}}\,(\mathbf{B}_0 + \mathbf{b})\,\mathcal{O} f = I[f]\,, \tag{151}$$

where $\mathcal{O}$ is the momentum rotation operator defined according to expression (42), and $\mathbf{E}$, $\mathbf{b}$ is the external electromagnetic field of the MHD wave.

Let us linearize kinetic equation (151) by assuming the external field to be weak and separating out from the distribution function $f = f_0 + \delta f$ the small part $\delta f$ caused by this field:

$$\frac{\partial \delta f}{\partial t} + \mathbf{v}\,\frac{\partial \delta f}{\partial \mathbf{r}} - \frac{ec}{\mathcal{E}}\,\mathbf{B}_0 \mathcal{O} \delta f = -e\mathbf{E}\,\frac{\partial f_0}{\partial \mathbf{p}} + \frac{ec}{\mathcal{E}}\,\mathbf{b}\mathcal{O} f_0 + I[\delta f]\,. \tag{152}$$

The last term in Eqn (152) takes into account the relaxation of the distribution function $\delta f$ resulting from the interaction of accelerated particles with the background particles and stochastic fields. We assume this effect to be weak and write down the collision integral in the relaxation frequency approximation: $I[\delta f] \to -\nu \delta f$, where $\nu \to +0$. This term will subsequently allow us to correctly integrate singular expressions. We consider a plane MHD wave in which the field vectors $\mathbf{E}, \mathbf{b} \propto \exp(\mathrm{i}\mathbf{kr} - \mathrm{i}\omega t)$ are related by Maxwell's equations

$$\mathbf{k} \times \mathbf{E} = \frac{\omega}{c}\,\mathbf{b}\,, \qquad \mathbf{E} = -\frac{\omega}{ck}\,\mathbf{e}_\parallel \times \mathbf{b}\,. \tag{153}$$

Here, $\mathbf{k}$ is a real vector, and the frequency $\omega$ may assume a complex value.

From Eqn (152) it follows that the coordinate and time dependence of the distribution function is the same as for the electromagnetic field. The equation may therefore be written in the form

$$\frac{\partial}{\partial\phi}\,\delta f - \frac{1}{\Omega}\left[\nu - \mathrm{i}(\omega - k_\parallel v_\parallel - k_\perp v_\perp \cos\phi)\right]\delta f = Q(\phi)\,, \tag{154}$$

where $\phi$ is the azimuth angle of relativistic particle momentum counted about the regular magnetic field, so that

$$\mathbf{B}_0 \boldsymbol{\mathcal{O}} \delta f = B_0\,\frac{\partial}{\partial\phi}\,\delta f\,, \qquad \Omega = \frac{ceB_0}{\mathcal{E}}\,,$$

and the right-hand side of Eqn (154) contains known quantities:

$$Q(\phi) = \left(\frac{e\mathbf{E}}{\Omega} - \frac{ec}{\mathcal{E}\Omega}\,\mathbf{b} \times \mathbf{p}\right)\frac{\partial f_0}{\partial \mathbf{p}}\,. \tag{155}$$

For definiteness, henceforward we shall assume all accelerated particles to be strongly relativistic protons and make use of the formulas $v \approx c$, $\mathcal{E} \approx cp$, and $p_0 \approx m_\mathrm{p} c$.

The solution to Eqn (152) may be written out in the form

$$\delta f = \int_{\pm\infty}^{\phi} Q(\phi')\exp\left[h(\phi) - h(\phi')\right]\mathrm{d}\phi'\,, \tag{156}$$

where

$$h(\phi) = \frac{1}{\Omega}\left\{\left[\nu - \mathrm{i}(\omega - k_\parallel v_\parallel)\right]\phi + \mathrm{i}k_\perp v_\perp \sin\phi\right\}, \tag{157}$$

and the signs $\pm$ of the lower limit of integration are selected so that the integral converges at infinity. Calculating the quantity $Q(\phi)$ with the use of distribution function (100) and relationships (153) yields

$$\begin{aligned} Q(\phi) &= \frac{eN_0 p_0^{\alpha-3}}{4\pi\Omega p^{\alpha+1}} \\ &\times \left\{A - \frac{\omega k_\parallel}{ck^2}(\alpha - 3)\left[\alpha + (\alpha+1)A\cos\theta\right]\right\}\sin\theta(\mathbf{b}\mathbf{e}_\phi) \\ &- \frac{(\alpha-3)eN_0 p_0^{\alpha-3}}{4\pi\Omega p^{\alpha+1}}\,\frac{\omega k_\perp}{ck^2}\left[\alpha + (\alpha+1)A\cos\theta\right]\sin\theta\sin\phi(\mathbf{b}\mathbf{e}_\parallel)\,, \end{aligned} \tag{158}$$

where $\mathbf{e}_\phi$ is the unit vector in the $\mathbf{e}_\parallel \times \mathbf{p}$ direction. Subsequent integration with respect to the azimuth angle $\phi'$ in expression (156) may be performed with the help of the known expansion of the exponential in the Bessel functions:

$$\begin{aligned} \delta f &= -\frac{eN_0 p_0^{\alpha-3}}{4\pi\Omega p^{\alpha+1}}\left\{A - \frac{\omega k_\parallel}{ck^2}(\alpha-3)\left[\alpha + (\alpha+1)A\cos\theta\right]\right\}\sin\theta \\ &\times \sum_{n=-\infty}^{\infty}\left(\frac{\mathrm{i}J_n' k_\parallel}{(a+\mathrm{i}n)k_\perp}\,b_\parallel + \frac{nJ_n}{\beta(a+\mathrm{i}n)k_\perp}\,\mathbf{b}[\mathbf{e}_\parallel \times \mathbf{k}]\right) \\ &\times \exp(-\mathrm{i}n\phi + \mathrm{i}\beta\sin\phi) \\ &+ \frac{(\alpha-3)eN_0 p_0^{\alpha-3}}{4\pi\Omega p^{\alpha+1}}\,\frac{\omega k_\perp}{ck^2}\left[\alpha + (\alpha+1)A\cos\theta\right]\sin\theta\,\mathbf{b}_\parallel \\ &\times \sum_{n=-\infty}^{\infty}\frac{\mathrm{i}J_n'}{a+\mathrm{i}n}\exp(-\mathrm{i}n\phi + \mathrm{i}\beta\sin\phi)\,. \end{aligned} \tag{159}$$

Here, we introduced the notation

$$a = \frac{1}{\Omega}\left[\nu - \mathrm{i}(\omega - k_\parallel v_\parallel)\right], \qquad \beta = \frac{k_\perp v_\perp}{\Omega}\,, \tag{160}$$

with the Bessel function $J_n(\beta)$ and its derivative everywhere having $\beta$ as their argument.

The electric current of accelerated particles is calculated by the well-known formula

$$\mathbf{j}^{\mathrm{ext}} = \int e\mathbf{v}\delta f(p,\theta,\phi)\,p^2\,\mathrm{d}p\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi\,. \tag{161}$$

Further in this section we shall consider the case where the extrinsic current is produced by strongly relativistic protons and the background medium is purely hydrogenic, i.e., $m_\mathrm{i} = m_\mathrm{p}$, $p_0 \approx m_\mathrm{p}c$, $v \approx c$, $\mathcal{E} \approx cp$, and $\omega_\mathrm{i} = eB_0/m_\mathrm{p}c$. The integration with respect to $\mathrm{d}\phi$ reduces the calculation of the current to taking the double integrals

$$\begin{aligned} \mathbf{j}_\perp^{\mathrm{ext}} &= \frac{1}{2}\,e^2 cN_0 p_0^{\alpha-3}\int_{p_0}^{p_\mathrm{m}}\frac{\mathrm{d}p}{\Omega p^{\alpha-1}} \\ &\times \int_0^\pi \sin^3\theta\,\mathrm{d}\theta\left[\left(A - \frac{\omega k_\parallel}{ck^2}(\alpha-3)\left[\alpha + (\alpha+1)A\cos\theta\right]\right)\right. \\ &\times \sum_{n=-\infty}^{\infty}\left(\frac{\mathrm{i}J_n J_n'}{\beta(a+\mathrm{i}n)}\,\mathbf{b}_\perp + \frac{n^2 J_n^2}{\beta^2(a+\mathrm{i}n)}\,\mathbf{e}_\parallel \times \mathbf{b}\right. \\ &\left.- \frac{J_{n+1}J_{n-1}k_\parallel b_\parallel}{(a+\mathrm{i}n)k_\perp^2}\,\mathbf{e}_\parallel \times \mathbf{k}\right) + (\alpha-3)\left[\alpha + (\alpha+1)A\cos\theta\right]\frac{\omega}{ck}\,b_\parallel \\ &\left.\times \sum_{n=-\infty}^{\infty}\frac{\mathrm{i}J_n'}{(a+\mathrm{i}n)k}\left(\frac{nJ_n\mathbf{k}_\perp}{\beta} - \mathrm{i}J_n'\mathbf{e}_\parallel \times \mathbf{k}\right)\right], \end{aligned} \tag{162}$$

$$\begin{aligned} j_\parallel^{\mathrm{ext}} &= \frac{1}{2}\,e^2 cN_0 p_0^{\alpha-3}\int_{p_0}^{p_\mathrm{m}}\frac{\mathrm{d}p}{\Omega p^{\alpha-1}} \\ &\times \int_0^\pi \sin^2\theta\cos\theta\,\mathrm{d}\theta\left[\left(A - \frac{\omega k_\parallel}{ck^2}(\alpha-3)\left[\alpha + (\alpha+1)A\cos\theta\right]\right)\right. \\ &\times \sum_{n=-\infty}^{\infty}\left(\frac{\mathrm{i}J_n' J_n k_\parallel}{(a+\mathrm{i}n)k_\perp}\,b_\parallel + \frac{nJ_n^2}{\beta(a+\mathrm{i}n)k_\perp}\,\mathbf{b}[\mathbf{e}_\parallel \times \mathbf{k}]\right) \\ &\left.+ (\alpha-3)\left[\alpha + (\alpha+1)A\cos\theta\right]b_\parallel\,\frac{\omega k_\perp}{ck^2}\sum_{n=-\infty}^{\infty}\frac{\mathrm{i}J_n' J_n}{a+\mathrm{i}n}\right]. \end{aligned} \tag{163}$$

Although the resultant expressions are rather cumbersome, their general structure is rather simple:

$$\begin{aligned} \mathbf{j}_\perp^{\mathrm{ext}} &= -\sigma_{\mathrm{cr}}\mathbf{e}_\parallel \times \mathbf{b} + g\mathbf{b} + \chi b_\parallel\,\frac{\mathbf{e}_\parallel \times \mathbf{k}}{k_\perp}\,, \\ j_\parallel^{\mathrm{ext}} &= \epsilon b_\parallel + \eta\,\frac{\mathbf{b}(\mathbf{e}_\parallel \times \mathbf{k})}{k_\perp}\,, \end{aligned} \tag{164}$$

where $g$, $\sigma_{\mathrm{cr}}$, $\chi$, $\epsilon$, and $\eta$ are the complex kinetic coefficients whose representation in the form of double integrals is easily established by comparing expressions (162)–(164). Generally, their calculation should by performed by numerical methods. However, on simplifying the geometry it is possible to obtain a relatively simple result in the analytical form.

Let the wave vector direction make a small angle with the direction of an external magnetic field $\mathbf{B}_0$, i.e., $k_\perp \ll |k_\parallel|$. We shall consider sufficiently small $k$, for which $\beta \approx (k_\perp/k)kr_g \ll 1$, where $r_g = cp/eB_0$ is the gyroradius of a relativistic particle. In this case, the wavelength of the MHD oscillation may be on the order of or greater than the relativistic particle gyroradius:

$$kr_g \lesssim 1 . \tag{165}$$

In the lowest approximation in the parameter $\beta$, the extrinsic current is strongly simplified: $j_\parallel^{\text{ext}} \approx 0$, and

$$\begin{aligned}\mathbf{j}_\perp^{\text{ext}} = \frac{1}{4}\, e^2 c N_0 p_0^{\alpha-3} \int_{p_0}^{p_m} \frac{\mathrm{d}p}{p^{\alpha-1}} \\ \times \int_0^\pi \sin^3\theta \,\mathrm{d}\theta \left(A - \frac{\omega k_\parallel}{ck^2}(\alpha-3)\left[\alpha + (\alpha+1)A\cos\theta\right]\right) \\ \times \frac{\mathbf{b}_\perp + a\mathbf{e}_\parallel \times \mathbf{b}}{\Omega(a^2+1)} .\end{aligned} \tag{166}$$

To calculate the remaining integrals, we can conveniently transform the denominator of the integrand:

$$\begin{aligned}\frac{1}{\Omega(1+a^2)} &= \frac{1}{2\Omega}\left[\frac{1}{1+\mathrm{i}a} + \frac{1}{1-\mathrm{i}a}\right] \\ &= \frac{1}{2}\left[\frac{1}{\Omega + \omega - kcx + \mathrm{i}\nu} + \frac{1}{\Omega - \omega + kcx - \mathrm{i}\nu}\right],\end{aligned} \tag{167}$$

where $x = \cos\theta$. At this stage, we may direct the low relaxation frequency $\nu$ to zero and employ Sokhotskii's formulas. Furthermore, we take into account the inequality $\omega \ll \Omega$, which may be written in the form $kr_g \ll c/v_A$ and which is fulfilled by virtue of inequality (165), since $v_A \ll c$ in our case. This permits representing the denominators of interest in the form

$$\begin{aligned}\frac{1}{\Omega(1+a^2)} &= \frac{1}{2ck}\left[\frac{\mathcal{P}}{x+\xi} - \frac{\mathcal{P}}{x-\xi} + \mathrm{i}\pi\left[\delta(x+\xi) - \delta(x-\xi)\right]\right], \\ \frac{a}{\Omega(1+a^2)} &= -\frac{\mathrm{i}}{2ck}\left[\frac{\mathcal{P}}{x+\xi} + \frac{\mathcal{P}}{x-\xi} + \mathrm{i}\pi\left[\delta(x+\xi) + \delta(x-\xi)\right]\right].\end{aligned} \tag{168}$$

Here, we introduced the designation $\xi = \Omega/ck$ and made use of the condition $\omega \ll \Omega$ and the principal-value symbol $\mathcal{P}$.

We apply representations (168) to bring the relativistic-proton current to the form

$$\begin{aligned}\mathbf{j}^{\text{cr}} = \mathbf{b}\,\frac{\omega_0^2}{32\pi\omega_i}\left(\frac{ck}{\omega_i}\right)^{\alpha-3}\left[A - \alpha(\alpha-3)\,\frac{\omega k_\parallel}{ck^2}\right] \\ \times\left\{\int_0^{\xi_0}\left[4\xi + 2(1-\xi^2)\,\ln\left|\frac{\xi+1}{\xi-1}\right|\right]\xi^{\alpha-3}\,\mathrm{d}\xi - \mathrm{i}\,\frac{4\pi}{\alpha(\alpha-2)}\right\} \\ + \mathrm{i}\mathbf{e}_\parallel \times \mathbf{b}\,\frac{(\alpha+1)(\alpha-3)\omega_0^2\omega}{32\pi\omega_i^2}\left(\frac{ck}{\omega_i}\right)^{\alpha-4} \\ \times A\left\{\int_0^{\xi_0}\left[\frac{8}{3} - 4\xi^2 - 2\xi(1-\xi^2)\,\ln\left|\frac{\xi+1}{\xi-1}\right|\right]\xi^{\alpha-3}\,\mathrm{d}\xi \right. \\ \left. + \mathrm{i}\,\frac{4\pi}{\alpha^2-1}\right\},\end{aligned} \tag{169}$$

where we introduced the notation

$$\omega_0^2 = \frac{4\pi e^2 N_0}{m_p}\,, \qquad \omega_i = \frac{eB_0}{m_p c}\,, \qquad \xi_0 = \frac{\omega_i}{ck}\,. \tag{170}$$

The frequency $\omega_0$ differs from the ion plasma frequency $\omega_{0i}$ in that the latter contains the background-proton concentration $n_i$, while $\omega_0$ contains the concentration of relativistic protons.

As a result, for the current induced by relativistic particles we obtained the expression

$$\mathbf{j}^{\text{cr}} = -(\sigma'_{\text{cr}} + \mathrm{i}\sigma''_{\text{cr}})\mathbf{e}_\parallel \times \mathbf{b} + (g' + \mathrm{i}g'')\,\mathbf{b}\,, \tag{171}$$

which is linear in the magnetic field, with the kinetic coefficients

$$\begin{aligned}\sigma'_{\text{cr}} &= \frac{(\alpha-3)\omega_0^2}{8(\alpha-1)\omega_i}\left(\frac{ck}{\omega_i}\right)^{\alpha-3}\frac{\omega A}{ck}\,, \\ \sigma''_{\text{cr}} &= \frac{(\alpha+1)(\alpha-3)\omega_0^2\omega}{60\pi(\alpha-4)\omega_i^2}\,A\,, \\ g' &= \frac{\omega_0^2}{12(\alpha-3)\pi\omega_i}\left(A - \frac{\alpha(\alpha-3)\omega k_\parallel}{ck^2}\right), \\ g'' &= -\frac{\omega_0^2}{8\alpha(\alpha-2)\omega_i}\left(\frac{ck}{\omega_i}\right)^{\alpha-3}\left(A - \frac{\alpha(\alpha-3)\omega k_\parallel}{ck^2}\right).\end{aligned} \tag{172}$$

In the passage to the limit $\alpha \to 4$, it is required to make the change $(\alpha-4)^{-1} \to \ln\xi_0 + 8/15$.

The kinetic coefficients $\sigma_{\text{cr}}$ and $g$ are pseudo-scalars, because the current is written down in terms of the magnetic induction pseudo-vector. On the right-hand sides of expressions (169) and (172), the projections of the polar vectors onto the direction of the regular magnetic field $\mathbf{B}_0$ are pseudo-scalars, i.e., the quantities $A$ and $k_\parallel$. The accelerated-particle current is also nonzero for an isotropic unperturbed distribution function, to which there corresponds $A = 0$. The dispersion properties of a multicomponent plasma are anisotropic and allow perturbation-induced currents. In this case, the preferential direction is produced by external fields, primarily by the quasiuniform magnetic field $\mathbf{B}_0$.

Let us estimate the relative magnitude of the kinetic coefficients obtained. The exponent $\alpha$ has a value of 4.7 in galactic cosmic rays for proton energies $\mathcal{E} \lesssim 3 \times 10^6$ GeV, and a value of 4 in the acceleration at a strong shock front. The ratio $g''/g' \approx (ck/\omega_i)^{\alpha-3}$ becomes small for $k \ll \omega_i/c \approx 1/r_g(p_0)$, where $r_g(p_0) \approx 10^{12}$ cm for an average field $B_0 \approx 3 \times 10^{-6}$ G in the galactic disk. The quantities $\sigma'_{\text{cr}}$ and $\sigma''_{\text{cr}}$ are smaller than $g''$ by a factor on the order of $\omega/ck \approx v_A/c \ll 1$. Therefore, the main part in the case involved is played by the coefficient $g'$, and $g''$ turns out to rank next in significance. The signs of each of the four coefficients for a real value of the frequency $\omega$ may be positive or negative, depending on the signs and absolute values of the factors $\omega$, $k_\parallel$, and $A$.

## 8. Linear growth rate of MHD oscillations

The current (171) was obtained in the geometry wherein the MHD wave propagated at a small angle to the direction of the field $\mathbf{B}_0$. That is why use can be made of Eqn (30). In the initially immobile medium $u_0 = 0$, and the velocity of matter in the wave is determined from Eqn (27):

$$\mathbf{u}' = -\frac{B_0 k_\parallel}{4\pi\rho\omega}\,\mathbf{b}\,. \tag{173}$$

We substitute the current (171) and the velocity (173) into Eqn (30) and project both parts of the equation onto the axes perpendicular to $\mathbf{B}_0$. Taking into consideration that $\mathbf{k} \times \mathbf{b} \approx k_\parallel \mathbf{e}_\parallel \times \mathbf{b}$, $b_\parallel \approx 0$ in the geometry involved, we obtain the system of equations in the form

$$C_1 b_x + C_2 b_y = 0\,, \qquad -C_2 b_x + C_1 b_y = 0 \tag{174}$$

for the field components. Here, the notation was used:

$$\begin{aligned} C_1 &= \omega^2 - v_{\mathrm{A}}^2 k^2 + \frac{B_0 k_\parallel \omega}{e n_{\mathrm{i}}}\, g' + \frac{4\pi v_{\mathrm{eff}} k_\parallel \omega}{c}\, \sigma'_{\mathrm{cr}} \\ &\quad - \mathrm{i}\left[ v_{\mathrm{eff}} k^2 \omega - \frac{B_0 k_\parallel \omega}{e n_{\mathrm{i}}}\, g'' - \frac{4\pi v_{\mathrm{eff}} k_\parallel \omega}{c}\, \sigma''_{\mathrm{cr}} \right], \\ C_2 &= \frac{B_0 k_\parallel \omega}{e n_{\mathrm{i}}}\, \sigma'_{\mathrm{cr}} - \frac{4\pi v_{\mathrm{eff}} k_\parallel \omega}{c}\, g' \\ &\quad + \mathrm{i}\left[ \frac{B_0 k_\parallel \omega}{e n_{\mathrm{i}}}\, \sigma''_{\mathrm{cr}} + \frac{B_0 c k_\parallel^2 \omega}{4\pi e n_{\mathrm{i}}} - \frac{4\pi v_{\mathrm{eff}} k_\parallel \omega}{c}\, g'' \right]. \end{aligned} \tag{175}$$

The condition that the determinant of the system is equal to zero may be written in the form of two equalities $C_1 \mp \mathrm{i} C_2 = 0$, which give the dispersion relations between the frequency and the wave vector for MHD eigenmodes:

$$\begin{aligned} &\omega^2 - v_{\mathrm{A}}^2 k^2 \pm \frac{B_0 c k_\parallel \omega}{4\pi e n_{\mathrm{i}}} + \frac{B_0 k_\parallel \omega}{e n_{\mathrm{i}}} (g' \pm \sigma''_{\mathrm{cr}}) \\ &\quad + \frac{4\pi v_{\mathrm{eff}} k_\parallel \omega}{c} (\sigma'_{\mathrm{cr}} \mp g'') \\ &\quad - \mathrm{i}\left[ v_{\mathrm{eff}} k^2 \omega \left( 1 - \frac{4\pi k_\parallel}{c k^2} (\sigma''_{\mathrm{cr}} \mp g') \right) - \frac{B_0 k_\parallel \omega}{e n_{\mathrm{i}}} (g'' \mp \sigma'_{\mathrm{cr}}) \right] = 0\,. \end{aligned} \tag{176}$$

Generally, these dispersion equations give several oscillation branches. They are easy to analyze only when the terms due to the Hall current and the current of accelerated particles can be taken as small. Right away we emphasize that the latter condition is not always fulfilled. However, if the imaginary part of the frequency, $\gamma = \mathrm{Im}\,\omega$, as well as the real correction $\omega^{(1)}$ to the frequency $\omega^{(0)} = \pm v_{\mathrm{A}} k$ are small, they can be easily determined from relation (176):

$$\begin{aligned} \gamma &= -\frac{1}{2}\, v_{\mathrm{eff}} k^2 \left( 1 - \frac{4\pi k_\parallel}{c k^2} (\sigma''_{\mathrm{cr}} \pm g') \right) + \frac{B_0 k_\parallel \omega^{(0)}}{e n_{\mathrm{i}}} (g'' \pm \sigma'_{\mathrm{cr}})\,, \\ \omega^{(1)} &= \mp \frac{B_0 c k_\parallel}{8\pi e n_{\mathrm{i}}} - \frac{2\pi v_{\mathrm{eff}} k_\parallel}{c} (\sigma'_{\mathrm{cr}} \mp g'') - \frac{B_0 k_\parallel}{e n_{\mathrm{i}}} (g' \pm \sigma''_{\mathrm{cr}})\,. \end{aligned} \tag{177}$$

In each of equalities (177) it is possible to discard the last (Hall) terms which possess smallness of order $(\omega_{\mathrm{i}} \tau_{\mathrm{ia}})^{-1} \ll 1$ (an estimate of $\omega_{\mathrm{i}} \tau_{\mathrm{ia}}$ is given in Table 1) in comparison with another single-type terms:

$$\gamma = -\frac{1}{2}\, v_{\mathrm{eff}} k^2 \left( 1 - \frac{4\pi k_\parallel}{c k^2} (\sigma''_{\mathrm{cr}} \pm g') \right). \tag{178}$$

When the relative magnitudes of the kinetic coefficients $g$ and $\sigma_{\mathrm{cr}}$, noted at the end of the previous section, are also taken into account, we arrive at a conclusion that the term containing $g'$ makes the main contribution to the dispersion relation. The terms containing $g''$, $\sigma''_{\mathrm{cr}}$, and $\sigma'_{\mathrm{cr}}$ are small. It is significant that the term with $g'$ may vary in sign. That branch of the oscillations to which the negative value of the factor $1 - 4\pi k_\parallel g'/ck^2 \approx 1 - 4\pi |g'|/ck$ corresponds is unstable, and its amplitude will build up for wave vector values satisfying the inequality

$$k < k_{\mathrm{cr}} = \frac{4\pi |g'|}{c}\,. \tag{179}$$

The imaginary part $\gamma = \mathrm{Im}\,\omega$ of the frequency in this case is positive and for $\gamma \ll |\omega|$ assumes the value

$$\gamma = \frac{1}{2} \left( \frac{4\pi |g'|}{ck} - 1 \right) v_{\mathrm{eff}} k^2 \approx \frac{2\pi |g'| v_{\mathrm{eff}} k}{c}\,. \tag{180}$$

The last-given approximate value of the increment takes place for $k \ll k_{\mathrm{cr}}$.

As follows from formulas (172), the conclusion about the existence of growing modes holds also for an isotropic accelerated-particle distribution function, i.e., at $A = 0$. To verify this conclusion and formula (178), let us calculate the increment by other means, on the basis of the energy balance of the magnetic wave field without invoking dispersion relation (176). Let there be a wave propagating in the direction $\mathbf{B}_0$ or in the opposite direction:

$$\mathbf{b} = \mathbf{b}_0 \exp\left(\mathrm{i}\mathbf{k}\mathbf{r} - \mathrm{i}\omega t\right), \qquad \mathbf{k} = k_\parallel \mathbf{e}_\parallel\,, \qquad k_\parallel \lessgtr 0\,, \tag{181}$$

In this case, the real $\omega' = \mathrm{Re}\,\omega$ and imaginary $\gamma = \mathrm{Im}\,\omega$ parts of the complex frequency may be arbitrary in sign. At this stage of our calculation, the magnitude of $\gamma$ is assumed to be unknown but small: $|\gamma| \ll \omega$. Let us consider the time variation of magnetic energy averaged over the oscillation period $T = 2\pi c/\omega' \ll \gamma^{-1}$. From the explicit form of the magnetic field (181) we find

$$\frac{\partial}{\partial t} \frac{|b|^2}{16\pi} = \frac{1}{8\pi}\, \mathrm{Re}\, \mathbf{b}^* \frac{\partial \mathbf{b}}{\partial t} = 2\gamma\, \frac{|b|^2}{16\pi}\,. \tag{182}$$

On the other hand, by using Maxwell's equations (25) we bring Eqn (182) to the form

$$\frac{\partial}{\partial t} \frac{|b|^2}{16\pi} = -\frac{c}{8\pi}\, \mathrm{Re}\left( \mathbf{E} [\nabla \times \mathbf{b}^*] \right). \tag{183}$$

Here, we took into consideration that $\nabla [\mathbf{E} \times \mathbf{b}^*] = 0$ on the strength of the uniformity of the system. We next express the electric field in terms of $\nabla \times \mathbf{b}$, and the extrinsic current (171) with the help of Ohm's law (20) and the second equation (25):

$$\mathbf{E} = \frac{c}{4\pi \sigma_\perp^{\mathrm{eff}}}\, \nabla \times \mathbf{b} - \frac{1}{\sigma_\perp^{\mathrm{eff}}}\, \mathbf{j}^{\mathrm{ext}} - \frac{B_0}{c}\, \mathbf{u}' \times \mathbf{e}_\parallel\,. \tag{184}$$

The Hall terms were discarded here owing to their smallness. We eliminate the velocity $\mathbf{u}'$ with the aid of expression (173). Upon substitution of the electric field into Eqn (183) we have

$$\frac{\partial}{\partial t} \frac{|b|^2}{16\pi} = -2\gamma\, \frac{|b|^2}{16\pi} - \frac{c^2 k^2}{2\pi \sigma_\perp^{\mathrm{eff}}} \frac{|b|^2}{16\pi} - \frac{c k_\parallel}{8\pi \sigma_\perp^{\mathrm{eff}}}\, \mathrm{Re}\, \mathrm{i} (\mathbf{e}_\parallel \times \mathbf{b}^*)\, \mathbf{j}^{\mathrm{ext}}\,. \tag{185}$$

The last term on the right-hand side is transformed with allowance made for the circular polarization of the MHD wave. This gives $\mathrm{Re}\,\mathrm{i}(\mathbf{e}_\parallel \times \mathbf{b}^*)\mathbf{b} = \mp |b|^2$, with the signs corresponding to different senses of the rotation of the vector $\mathbf{b}$ around $\mathbf{B}_0$. Eventually, we obtain

$$\mathrm{Re}\, \mathrm{i} (\mathbf{e}_\parallel \times \mathbf{b}^*)\, \mathbf{j}^{\mathrm{ext}} = |b|^2 (\sigma'_{\mathrm{cr}} \mp g')\,.$$

We equate the right-hand sides of Eqns (182) and (185) to find the increment (decrement) which precisely coincides with expression (178).

The quantity $\nu_{\rm eff}$ may be far greater than the magnetic viscosity of a fully ionized plasma due to the neutral component and a strong magnetization (see Table 1), and therefore the situation is not ruled out whereby $\gamma > {\rm Re}\,\omega$ and even $\gamma \gg {\rm Re}\,\omega$. This corresponds to a rapid aperiodic build-up of turbulent pulsations which will differ radically in properties from Alfvén and magnetosonic low-amplitude waves. The linear theory developed above permits determining only the threshold for the emergence of this strong instability.

It is well to bear in mind, however, that the great magnitude of $\nu_{\rm eff}$ is due to ion collisions with neutral atoms. This process is quantitatively characterized by the collision mean free path $\Lambda_{\rm ia}$. The collisional mechanism works effectively for oscillations with a wavelength $\lambda > \Lambda_{\rm ia}$, i.e., provided

$$k < k_{\rm s} = \frac{2\pi}{\Lambda_{\rm ia}} \,. \tag{186}$$

When $k_{\rm s} < k_{\rm cr}$, it is precisely condition (186) rather than condition (179) that limits the action of the nonresonance mechanism from the side of high $k$ values. For $k > k_{\rm s}$, however, when the plasma may be treated as being collisionless, the resonance oscillation build-up mechanism considered below in Section 10 comes into effect.

Let us estimate here the increment of MHD oscillation growth in a warm partially ionized phase of the interstellar medium of the galactic disk, which occupies a substantial part (a few dozen percent) of the disk volume. Using the data given in Table 1, we estimate the effective magnetic viscosity $\nu_{\rm eff} \approx 10^{21}$ cm$^2$ s$^{-1}$. The relativistic particle number density in the interstellar space is assumed to be $N_0 \approx 10^{-10}$ cm$^{-3}$ and the exponent of the momentum spectrum $\alpha \approx 4.7$ (see the reference book [60]). We also estimate $v_{\rm A} \approx 2\times10^6$ cm s$^{-1}$ and $\omega_{\rm i} \approx 0.02$ rad s$^{-1}$. The anisotropy $A$ of galactic cosmic rays is known from observations and amounts to about $10^{-3}$ for proton energies close to 1 TeV [18]. Of the same order of magnitude is the second term $\alpha(\alpha-3)\omega k_\parallel/ck^2 \approx \alpha(\alpha-3)v_{\rm A}/c \approx 5\times10^{-4}$ in parentheses in the expression for $g'$ [formula (172)]. Using the above data we find $g' \approx 2.6\times10^{-8}$ s$^{-1}$ and the shortest wavelength $\lambda_{\rm cr} = 2\pi/k_{\rm cr} \approx 2\times10^{17}$ cm $\approx 0.1$ pc from which the amplitude growth of magnetic inhomogeneities can set in. According to observations, the longest scale of stochastic inhomogeneities in the disk is on the order of 100 pc [18]. The typical time of inhomogeneity build-up for the minimal wavelength is around $\gamma^{-1} \approx 1.5\times10^6$ years, and for a wavelength on the order of 100 pc this time runs to $10^9$ years, which supposedly does not exceed the disk lifetime. A natural result of the build-up of large-scale magnetic inhomogeneities due to the anisotropic cosmic ray distribution is an approximate equidistribution of the energy densities in these two subsystems (magnetic turbulence and relativistic particles), which is observed in the galactic disk.

We estimate the increment-to-frequency ratio in the case involved to obtain $\gamma/\omega \approx 3\times10^{-3}$. For the realization of the situation $\gamma/\omega > 1$ discussed above, it would suffice, all other conditions being the same, to increase the concentration of accelerated particles by three orders of magnitude, which is quite possible in cosmic ray sources.

## 9. Excitation of nonresonance oscillations ahead of the shock wave front

From the previous section it follows that the MHD-oscillation excitation mechanism under discussion operates even in the galactic disk where the density of relativistic particles is quite low on average. Clearly this turbulence enhancement mechanism is applicable near shock fronts which are sources of accelerated particles and where their number is much greater than, on average, in the Galaxy (see Ref. [28]). The neutral component required for the operation of the mechanism is present in the supernova remnants interacting with neutral clouds (for instance, IC 433), which may be confirmed by the emission spectra of these remnants [24]. Some fraction of the neutral atoms of hydrogen, helium, and metals reaches the shock front also in the case of a supernova remnant embedded in a rarefied medium. They are observed in optical and UV spectra of the shock front as the superposition of a broad line and a narrow line (in particular, for the $\rm H_\alpha$ line) in the SN 1006, Kepler, Tycho, RCW 86, Cygnus Loop, etc. remnants and represent an efficient method of estimating the shock front velocity [61, 62].

We calculate the accelerated-particle current in the plane prefront of a nonrelativistic shock wave propagating through a partially ionized turbulent medium. In the exact definition of this problem, it is essentially nonlinear and calls for a self-consistent simultaneous calculation of the absolute number and spectrum of accelerated particles in combination with the spectrum and intensity of MHD turbulence which determines the diffusion coefficient and thereby the efficiency and rate of particle acceleration. At present, attempts to solve the self-consistent nonlinear problem fail, and therefore we *a priori* assume the occurrence of sufficiently strong turbulence to subsequently verify (at the level of estimates) from the calculated increment the consistency of the assumptions made. We employ the frame of reference in which the shock front is immobile. The accelerated particles in the turbulent medium possess a weakly anisotropic distribution function which may be written down as (see Ref. [8])

$$f_0(z,\mathbf{p}) = \frac{1}{4\pi}\left[N(p,z) + \frac{3}{pv}\,\mathbf{pJ}(p,z)\right], \qquad J \ll vN\,, \tag{187}$$

where

$$J_\alpha = -\kappa_{\alpha\beta}\,\frac{\partial N}{\partial x_\beta} - \frac{p}{3}\,\frac{\partial N}{\partial p}\,u_{0\alpha} \tag{188}$$

is the differential flux density of accelerated particles, $\kappa_{\alpha\beta}$ is their diffusion tensor, and $\mathbf{u}_0$ is the velocity of the medium. In order to simplify the geometry of the system, we shall assume that the velocity $\mathbf{u}_0$ of the medium and the regular magnetic field $\mathbf{B}_0$ are directed normally to the plane front.

The isotropic part $N(p,z)$ of the distribution function in the region ahead of the shock front is easily calculated in the stationary case in the probe-particle approximation:

$$N(p,z) = (\alpha-3)N_0\,\frac{p_0^{\alpha-3}}{p^\alpha}\,\exp\left[\int_0^z \frac{u_0\,{\rm d}z'}{\kappa_\parallel(p,z')}\right], \tag{189}$$
$$z \leqslant 0\,, \quad p_0 \leqslant p \leqslant p_{\rm m}\,.$$

Here, $N_0$ is the concentration of relativistic particles with all energies, $\alpha = 3u_0/\Delta u$ is the exponent of the momentum spectrum, and $\Delta u > 0$ is the velocity jump at the front.

Solution (189) corresponds to the case where only a small fraction of the shock energy is expended on particle acceleration and the velocity of the medium in the prefront region may be treated as being approximately constant: $u_0 \approx$ const. For a moderately strong front, the exponent $\alpha > 4$, and in the case of a strong shock wave $\alpha \leqslant 4$ (see, for instance, Ref. [50]). The specific value of $\alpha$ depends not only on the Mach number of the wave, but also on the rate of particle injection into the acceleration regime. For $\alpha < 4$, the bulk of energy belongs to the highest-energy particles with $\mathcal{E} \lesssim \mathcal{E}_{\rm m} = cp_{\rm m}$, and a few dozen percent of the total stream energy is expended on the particle acceleration. We shall restrict ourselves to the values $\alpha \geqslant 4$ and assume a moderate acceleration rate whereat the total kinetic energy of accelerated particles does not exceed 10% of the total energy of the system but the highest particle momentum $p_{\rm m}$ is far greater than the injection momentum: $p_{\rm m} \gg p_0 \approx m_{\rm p}c$. For these values of the exponent of the momentum spectrum, the total energy of accelerated particles at the front ($z = 0$) is a logarithmic function of $p_{\rm m}$:

$$w_{\rm cr} \approx \int_{p_0}^{p_{\rm m}} cpN(p,0)\,p^2\,{\rm d}p = N_0 m_{\rm p} c^2 \ln \frac{p_{\rm m}}{p_0} \,. \tag{190}$$

At present, attempts to reliably calculate the turbulence spectrum and the accelerated-particle diffusion coefficient defined by this spectrum do not meet with success, and we are therefore led to take it from model considerations. There is good reason to consider the most commonly employed models.

(1) Strong turbulence, whereat the diffusion coefficient approaches the Bohm limit, i.e., the transport mean free path $\Lambda(p)$ of a particle is on the order of its Larmor radius:

$$\Lambda(p) \approx r_{\rm g}(p) = \frac{cp}{eB}\,, \qquad \kappa_\parallel = \frac{c\Lambda}{3}\,, \qquad p_0 \leqslant p \leqslant p_{\rm m}\,, \tag{191}$$

where $p_0$ and $p_{\rm m}$ bound the range of momenta under consideration, the case $p_{\rm m} \gg p_0$ being of peculiar interest. The turbulent and regular fields are taken to be on the same order of magnitude: $B \approx B_0$. This condition, generally speaking, is not at variance with the assumption that the energy fraction transferred to accelerated particles is small: in a strong shock wave under typical astrophysical conditions, the mechanical energy density is several orders of magnitude higher than the energy density of the primary magnetic field (see the estimates at the beginning of Section 6).

In Section 11, on the basis of a semiphenomenological scheme for the description of statistically homogeneous incompressible MHD turbulence, we shall give a substantiation of the possible realization of the Bohm diffusion coefficient in the conditions under consideration.

(2) The transport mean free path and the longitudinal diffusion coefficient are constant in the energy range involved:

$$\Lambda_\parallel = {\rm const}\,, \qquad \kappa_\parallel = \frac{c\Lambda_\parallel}{3} = {\rm const}\,, \qquad p_0 \leqslant p \leqslant p_{\rm m}\,. \tag{192}$$

This situation is realized for magnetic turbulence with the spectrum of the form $\langle b^2 \rangle_k \sim k^{-2}$. In this case, the transport mean free path would be naturally identified with the Larmor radius of the highest-energy particles: $\Lambda_\parallel \approx r_{\rm g}(p_{\rm m})$. The field in the largest-scale turbulence harmonics is comparable with the regular field in this case, but the smaller-scale harmonics are weak and ensure a strongly anisotropic diffusion with a constant longitudinal mean free path $\Lambda_\parallel \approx r_{\rm g}(p_{\rm m}) = {\rm const}$ for particles with $p \ll p_{\rm m}$.

Naturally, both assumptions are inherently model in character, because attempts to self-consistently and simultaneously treat the particle acceleration and the turbulence generation do not meet with success.

The most significant distinction between distribution function (187) and distribution function (100) considered above is the former's inhomogeneity: in the prefront region, accelerated particles occupy a layer of finite thickness on the order of $l(p) \approx \kappa(p)/u_0$, which depends on their energy. As follows from the results obtained in the previous section, the instability increment is proportional to the number density $N(z)$ of accelerated particles with all energies, and therefore of interest is the distribution of this quantity in the prefront for different diffusion coefficients. The case of a constant diffusion coefficient (192) is the most simple: the particle distribution over the prefront decreases exponentially, viz.

$$N(z) = N_0 \exp\left(\frac{z}{l_{\rm m}}\right), \qquad l_{\rm m} = \frac{c}{3u_0}\,\Lambda = {\rm const}\,. \tag{193}$$

The prefront thickness is given by the value of $l_{\rm m}$, which is independent of the shape of the accelerated-particle spectrum and is defined by the highest-energy particles.

The situation is somewhat more complicated in the case of the Bohm diffusion model. By integrating, in view of the dependence $\kappa_\parallel(p) = cr_{\rm g}(p)/3 \propto p$, expression (189) with respect to $p^2\,{\rm d}p$, we find

$$N(z) = (\alpha - 3) N_0 \zeta_0^{3-\alpha} \left[ \Gamma(\alpha - 3, \zeta_{\rm m}) - \Gamma(\alpha - 3, \zeta_0) \right], \tag{194}$$

where $\Gamma(\alpha - 3, \zeta)$ is the incomplete gamma-function (see the *Tables of Integrals, Sums, Series and Products* (Moscow: GIFML Publ., 1963) by Gradshtein and Ryzhik), and

$$\zeta_0(z) = \frac{3eu}{c^2 p_0} \int_z^0 B(z')\,{\rm d}z' \geqslant 0 \qquad (z \leqslant 0) \tag{195}$$

is the dimensionless distance. The quantity $\zeta_{\rm m}$ differs from expression (195) by the substitution of $p_{\rm m}$ for $p_0$. In the limiting cases, we have

$$\Gamma(\alpha - 3, \zeta) \approx \begin{cases} \Gamma(\alpha - 3) - \dfrac{\zeta^{\alpha-3}}{\alpha - 3}\,, & \zeta \ll 1\,, \\ \zeta^{\alpha-4} \exp(-\zeta)\,, & \zeta \gg 1\,, \end{cases} \tag{196}$$

where $\Gamma(\alpha - 3)$ is the ordinary gamma-function. From these asymptotics we obtain $N(z) \approx N_0$ for short distances ($\zeta \ll 1$, $\zeta_{\rm m} \ll 1$), and

$$N(z) \approx (\alpha - 3) N_0 \zeta^{3-\alpha} \zeta_{\rm m}^{\alpha-4} \exp(-\zeta_{\rm m}) = (\alpha - 3) N_0 \left(\frac{p_0}{p_{\rm m}}\right)^{\alpha-3} \frac{\exp(-\zeta_{\rm m})}{\zeta_{\rm m}}$$

for long distances ($\zeta_0 \gg \zeta_{\rm m} \gg 1$). In the intermediate range $\zeta_0 \gg 1$, $\zeta_{\rm m} \ll 1$ of principal interest, we find

$$N(z) \approx \Gamma(\alpha - 2) N_0 \zeta_0^{\alpha-3} = \frac{\Gamma(\alpha - 2) N_0 (c^2 p_0)^{\alpha-3}}{\left(3u_0 e \int_z^0 B(z')\,{\rm d}z'\right)^{\alpha-3}}\,. \tag{197}$$

For a moderate acceleration at a strong front ($\alpha \approx 4$) and a uniform magnetic field, we have a slow particle concentration decrease $N(z) \propto |z|^{-1}$ in the domain under consideration. However, when the magnetic field decreases with distance to the shock front, which is natural in the generation of magnetic fluctuations by accelerated particles, the decrease in particle concentration will slow down still further. To summarize these estimates, it is valid to say that the accelerated-particle number density is approximately constant in a layer of thickness $l_0 \approx cr_g(p_0)/3u_0$ and decreases by a slower than $1/|z|$ law in the layer between $l_0$ and $l_m \approx cr_g(p_m)/3u_0 \gg l_0$. The exponential decay takes place at the distances $|z| \gg l_m$. Therefore, the prefront thickness is determined in this case, too, primarily by the highest-energy particles.

We now turn to the calculation of accelerated-particle current in the prefront region. We take advantage of the above-noted approximate prefront uniformity over a thickness $l_m$ and calculate the current near the front, in the domain $|z| \ll l_m$. This calculation may be performed in much the same way as the solution of the similar problem in Section 7, with the wavelengths of the MHD oscillations under discussion being bounded from above by the prefront thickness: $k \gtrsim 2\pi/\lambda_m$, $\lambda_m \approx l_m$. The addition $\delta f$ to the accelerated-particle distribution function should contain the same exponential nonuniform factor as the unperturbed functions (187) and (189): $\delta f \propto \exp\left[\mathrm{i}(\mathbf{kr} - \omega t) + u_0 z/\kappa_\parallel\right]$. That is why, the infinitesimal $v$ in Eqn (154) is replaced by the finite quantity

$$v = \frac{v_\parallel u_0}{\kappa_\parallel(p)} \,. \tag{198}$$

According to expressions (187)–(189), the unperturbed distribution function takes the form

$$f_0(z, p, \theta) = \frac{(\alpha - 3) N_0 p_0^{\alpha-3}}{4\pi p^\alpha} \left[1 + (\alpha - 3)\,\frac{u_0}{v}\cos\theta\right] \exp\frac{u_0 z}{\kappa_\parallel(p)} \,. \tag{199}$$

Comparing this expression with formula (100) shows that the anisotropy of the distribution function in the prefront region of the shock wave is given by

$$A = (\alpha - 3)\,\frac{u_0}{v} \,. \tag{200}$$

We next present the results relating separately to the cases of relativistic ($v \approx c$, $p_0 \approx m_p c$, $\mathcal{E} \approx cp$) and nonrelativistic ($v \ll c$, $p = m_p v$, $\mathcal{E} \approx m_p c^2$, $p_0 \approx m_p u_0$, $p_m \ll m_p c$) particles. In what follows, the upper value $\alpha + 1$ in a two-component column corresponds to relativistic particles, and the lower value $\alpha + 2$ to nonrelativistic ones. For the quantity $Q(\phi)$ we have

$$\begin{aligned} Q(\phi) = {} & \frac{(\alpha - 3) e N_0 p_0^{\alpha-3}}{4\pi p^\alpha \Omega} \\ & \times \left\{ E_\parallel \left[(\alpha - 3)\,\frac{u_0}{v} - \alpha\cos\theta - (\alpha - 3)\begin{pmatrix}\alpha + 1\\ \alpha + 2\end{pmatrix}\frac{u_0}{v}\cos^2\theta\right] \right. \\ & - \mathbf{Ee}_\perp \left[\alpha + (\alpha - 3)\begin{pmatrix}\alpha + 1\\ \alpha + 2\end{pmatrix}\frac{u_0}{v}\cos\theta\right]\sin\theta \\ & \left. + (\alpha - 3)\,\frac{cp}{\mathcal{E}}\,\frac{u_0}{v}\,\mathbf{be}_\phi \sin\theta \right\}, \qquad |z| \ll l_m \,. \end{aligned} \tag{201}$$

The last inequality signifies that we are considering the domain near the front, where the exponential in expression (199) may be replaced by unity. Expressions (199)–(201) embrace the cases of the Bohm and constant diffusion coefficients.

In the subsequent discussion we assume, as in Section 4, that $k_\perp \ll |k_\parallel|$. Expression (167) will now contain a finite quantity $v$ defined by formula (198). By comparing its absolute value with other items in the denominators, we ascertain that for relativistic particles the imaginary part in expression (167) is small in comparison with the real part: $v_\parallel u_0/\kappa_\parallel \Omega \approx 3u_0/c \ll 1$ for the Bohm diffusion coefficient, and $v_\parallel u_0/\kappa_\parallel \Omega \approx 3u_0 p/cp_m \ll 1$ for a constant transport mean free path. For nonrelativistic particles, the requisite smallness will take place provided $\Lambda(p) \gg 3r_{g0}$, where $r_{g0} = cm_p u_0/eB$ is the gyroradius of a proton with the velocity $u_0$. In all these cases, Sokhotskii's formulas (168) may be used as approximate relationships (bearing in mind that the small quantity $v$ may now have either sign).

Upon performing the corresponding calculations we arrive at the previous expression (171) for the current, where the kinetic coefficients have the following values:

$$\begin{aligned} \sigma'_{cr} &= \frac{(\alpha - 3)^2 \omega_0^2 \omega}{8(\alpha - 1)\omega_i^2}\left(\frac{ck}{\omega_i}\right)^{\alpha-4}\frac{u_0}{c} \,, \\ \sigma''_{cr} &= \frac{(\alpha + 1)(\alpha - 3)^2 \omega_0^2 \omega u_0}{60\pi(\alpha - 4)\omega_i^2 c} \,, \\ g' &= \frac{\omega_0^2}{12\pi\omega_i}\left(\frac{u_0}{c} - \frac{\alpha\omega k_\parallel}{ck^2}\right), \\ g'' &= -\frac{(\alpha - 3)\omega_0^2}{8\alpha(\alpha - 2)\omega_i}\left(\frac{ck}{\omega_i}\right)^{\alpha-3}\left(\frac{u_0}{c} - \frac{\alpha\omega k_\parallel}{ck^2}\right) \end{aligned} \tag{202}$$

in the relativistic case, and

$$\begin{aligned} \sigma'_{cr} &= \frac{(\alpha - 3)\omega_0^2 \omega}{8\alpha\omega_i^2}\left(\frac{u_0 k}{\omega_i}\right)^{\alpha-3}\frac{u_0}{c} \,, \\ \sigma''_{cr} &= \frac{(\alpha + 2)(\alpha - 3)\omega_0^2 \omega u_0}{60\pi\omega_i^2 c} \,, \\ g' &= \frac{(\alpha + 2)(\alpha - 3)\omega_0^2}{12\pi\omega_i}\left(\frac{u_0}{c} - \frac{\alpha\omega k_\parallel}{ck^2}\right), \\ g'' &= -\frac{(\alpha - 3)\omega_0^2}{8\alpha(\alpha - 2)\omega_i}\left(\frac{u_0 k}{\omega_i}\right)^{\alpha-3}\left(\frac{u_0}{c} - \frac{\alpha\omega k_\parallel}{ck^2}\right) \end{aligned} \tag{203}$$

in the nonrelativistic case. When passing to $\alpha = 4$, it is required to make the change $(\alpha - 4)^{-1} \to \ln\xi_0 + 8/15$.

The local oscillation increment near the shock front should now be calculated from the dispersion relation

$$\omega'^2 - v_A^2 k_\parallel^2 + \mathrm{i}\nu_{\mathrm{eff}} k^2 \omega' \left(1 \pm \frac{4\pi k_\parallel g'}{ck^2}\right) = 0 \tag{204}$$

similar to Eqn (176), in which we discarded all terms that are small under ordinary conditions. Because of the general transfer of the medium in the prefront region with the velocity $u_0$, in the dispersion relation (204) there enters a frequency $\omega' = \omega - u_0 k_\parallel$ containing the Doppler shift. The quantity $g'$ in expressions (202) and (203) was defined in terms of the initial frequency $\omega$. For $v_A \ll u_0$, which is often the case, expressions (179) and (180) for the critical wave vector and the oscillation increment remain valid. The constraint (186) is also in force.

## 10. Resonance generation of oscillations by relativistic particles

We compare the new nonresonance turbulence enhancement mechanism under consideration with the previously known mechanism of MHD wave generation by accelerated particles, whose theory was elaborated in Refs [10–14]. To this end, let us calculate the increment of the resonance low-frequency ($\omega \ll \omega_{\rm i}$) wave build-up by relativistic particles in a fully ionized homogeneous collisionless plasma by employing the Maxwell equations (25) but not using, of course, Ohm's law. The model of a collisionless medium also applies to the plasma with a neutral component considered in the previous section, provided the oscillation wavelengths $\lambda = 2\pi/k$ are shorter than the particle transport mean free paths $\Lambda_{\rm ia}$ and $\Lambda_{\rm ei}$.

We shall express the electric current $\mathbf{j}$ of the background particles in a collisionless plasma in terms of its permittivity (see, for instance, Akhiezer et al. [43]) and the external harmonic electric field:

$$\mathbf{j}_\perp = -\frac{\mathrm{i}\omega}{4\pi}\,(\varepsilon_\perp - 1)\,\mathbf{E}_\perp + \frac{\omega}{4\pi}\,q\mathbf{E}\times\mathbf{e}_\parallel\,,\qquad j_\parallel = -\frac{\mathrm{i}\omega}{4\pi}\,(\varepsilon_\parallel - 1)E_\parallel\,. \tag{205}$$

Only the transverse current is required for our purposes; in the frequency range $\omega \ll \omega_{\rm i}$ and in a cold plasma, it is defined by the quantities

$$\varepsilon_\perp - 1 \approx \left(\frac{c}{v_{\rm A}}\right)^2, \qquad q \approx \left(\frac{c}{v_{\rm A}}\right)^2\frac{\omega}{\omega_{\rm i}}\,, \tag{206}$$

where $q\mathbf{e}_\parallel = \mathbf{q}$ is the gyration vector. The relativistic-particle current (171) should also be expressed in terms of the electric field:

$$\mathbf{j}_\perp^{\rm cr} = \frac{ck_\parallel}{\omega}\left[(\sigma'_{\rm cr} + \mathrm{i}\sigma''_{\rm cr})\,\mathbf{E}_\perp - (g' + \mathrm{i}g'')\,\mathbf{E}\times\mathbf{e}_\parallel\right]. \tag{207}$$

It should be recalled that the relativistic-particle current (207) is produced by waves propagating at small angles to the vector $\mathbf{B}_0 = B_0\mathbf{e}_\parallel$ in the forward or backward direction. Here, we are not concerned with longitudinal oscillations. We only consider large-scale MHD type modes, in which the quasi-neutrality of the medium is fulfilled with a high accuracy. We can therefore make use of the condition $\nabla\mathbf{E} = \mathrm{i}\mathbf{k}\mathbf{E} = 0$. System of equations (25) takes the following form on substituting expressions (205) and (207), as well as eliminating the magnetic field of the wave:

$$k^2\mathbf{E}_\perp = \frac{\omega^2}{c^2}(\varepsilon_\perp - 1)\,\mathbf{E}_\perp + \frac{\mathrm{i}\omega^2}{c^2}\,q\mathbf{E}\times\mathbf{e}_\parallel + \mathrm{i}\,\frac{4\pi k_\parallel}{c}\left[(\sigma'_{\rm cr} + \mathrm{i}\sigma''_{\rm cr})\,\mathbf{E}_\perp - (g' + \mathrm{i}g'')\,\mathbf{E}\times\mathbf{e}_\parallel\right]. \tag{208}$$

We write it down as

$$A_1\mathbf{E}_\perp + \mathrm{i}A_2\mathbf{E}\times\mathbf{e}_\parallel = 0\,, \tag{209}$$

where

$$A_1 = \omega^2 - v_{\rm A}^2k^2 - \frac{4\pi k_\parallel v_{\rm A}^2}{c}\,(\sigma''_{\rm cr} - \mathrm{i}\sigma'_{\rm cr})\,,\qquad A_2 = \frac{\omega^2 v_{\rm A}^2}{c^2}\,q - \frac{4\pi k_\parallel v_{\rm A}^2}{c}\,(g' + \mathrm{i}g'')\,. \tag{210}$$

We vectorially multiply Eqn (209) by $\mathbf{e}_\parallel$ to obtain an equation of the form

$$-\mathrm{i}A_2\mathbf{E}_\perp + A_1\mathbf{E}\times\mathbf{e}_\parallel = 0\,. \tag{211}$$

The compatibility condition for Eqns (209) and (211) may be written in the form of two equalities

$$A_1 \pm A_2 = 0\,, \tag{212}$$

which show in combination with equation (209) that the eigenmodes are circularly polarized. They lead to dispersion relations

$$\omega^2 - v_{\rm A}^2k^2 \pm \frac{\omega^3}{\omega_{\rm i}} - \frac{4\pi k_\parallel v_{\rm A}^2}{c}\,(\sigma''_{\rm cr} \pm g') + \mathrm{i}\,\frac{4\pi k_\parallel v_{\rm A}^2}{c}\,(\sigma'_{\rm cr} \mp g'') = 0\,. \tag{213}$$

In the absence of relativistic particles, the dispersion relations take on the form

$$\omega^2 - v_{\rm A}^2k^2 \pm \frac{\omega^3}{\omega_{\rm i}} = 0\,, \tag{214}$$

where the last term on the left-hand side (the item due to the Hall current) is small in the $\omega \ll \omega_{\rm i}$ frequency range. For an arbitrary direction of wave propagation, Eqn (214) assumes the form

$$\left(\frac{\omega^2}{v_{\rm A}^2} - k_\parallel^2\right)\left(\frac{\omega^2}{v_{\rm A}^2} - k^2\right) - \frac{\omega^4}{v_{\rm A}^4}\,\frac{\omega^2}{\omega_{\rm i}^2} = 0\,. \tag{215}$$

Neglecting the small Hall term, from the dispersion relation we obtain the frequencies of two modes,

$$\omega_{\rm A} = \pm|k_\parallel|v_{\rm A}\,, \qquad \omega_{\rm ms} = \pm kv_{\rm A}\,, \tag{216}$$

the Alfvén and fast magnetosonic modes, with no damping in a cold plasma. For a longitudinal propagation, to which Eqn (214) corresponds, the mode frequencies degenerate and become equal.

The inclusion of accelerated-particle current gives rise to imaginary terms in dispersion relation (213), which may have different signs. This signifies that growing and damped oscillation branches emerge. Their growth (damping) rates $\gamma$ and corrections $\omega^{(1)}$ to the real parts of the frequencies are easily found when they are small:

$$\gamma = \frac{2\pi k_\parallel v_{\rm A}^2}{c\,\omega^{(0)}}\,(\pm g'' - \sigma'_{\rm cr})\,,\qquad \omega^{(1)} = \frac{2\pi k_\parallel v_{\rm A}^2}{c\,\omega^{(0)}}\,(\sigma''_{\rm cr} \pm g') \mp \frac{\omega^{(0)\,2}}{2\omega_{\rm i}}\,, \tag{217}$$

where $\omega^{(0)} = \pm kv_{\rm A}$. Positive $\gamma$ values correspond to oscillation build-up. Since the kinetic coefficients $g''$ and $\sigma'_{\rm cr}$, as well as the frequency $\omega^{(0)}$, may have different signs, it is always possible to select the signs in Eqn (217) in such a way that $\gamma > 0$ and oscillation build-up occurs. This is also possible in the case of an isotropic accelerated-particle distribution function ($A = 0$), because there is medium anisotropy produced by the magnetic field $\mathbf{B}_0$.

Expression (217) for the increment may be obtained from the magnetic energy balance in the MHD wave following the scheme [see expressions (181)–(185)] identical to that employed for the nonresonance increment. All one needs to do is to express the magnetic energy in terms of the electric field:

$$\frac{|b|^2}{16\pi} = \frac{c^2 k^2 |E|^2}{16\pi|\omega|^2} \approx \frac{c^2|E|^2}{16\pi v_{\mathrm{A}}^2} \,.$$

We simplify the expression for the growth rate (217) by taking advantage of the estimate of the relative magnitude of kinetic coefficients at the end of Section 7:

$$\gamma \approx \pm \frac{2\pi k_{\parallel} v_{\mathrm{A}}^2}{c\,\omega^{(0)}}\, g'' = \pm \frac{\pi\omega_0^2}{2\alpha(\alpha-2)\omega_{\mathrm{i}}} \left(\frac{ck}{\omega_{\mathrm{i}}}\right)^{\alpha-3} \left(A \pm \alpha(\alpha-3)\,\frac{v_{\mathrm{A}}}{c}\right) \frac{v_{\mathrm{A}}}{c} \,. \quad (218)$$

Here, the signs in the last parentheses and the signs in front of the whole expression were independently selected, because the sign of the frequency $\omega^{(0)} = \pm v_{\mathrm{A}}|k_{\parallel}|$ is independent of the sign of the projection $k_{\parallel}$ or the accelerated-particle flow rate sign (the anisotropy $A$). That is why there is no limitation on the magnitude or sign of the anisotropy parameter $A$ impeding the oscillation build-up. For an isotropic accelerated-particle distribution function in the system, the background medium anisotropy remains, which is produced by the field $\mathbf{B}_0$ and the Alfvén velocity. It is precisely this anisotropy that defines the oscillation excitation increment (218) at $A = 0$. The factor $(ck/\omega_{\mathrm{i}})^{\alpha-3}$, which depends on the exponent of the accelerated-particle spectrum, may be written down in different forms:

$$\left(\frac{ck}{\omega_{\mathrm{i}}}\right)^{\alpha-3} = (r_{\mathrm{g}0}k)^{\alpha-3} = \frac{N(p \geqslant p_{\mathrm{r}})}{N_0} \,, \quad (219)$$

where $r_{\mathrm{g}0} = cp_0/eB_0 = m_{\mathrm{p}}c^2/eB_0$ is the Larmor radius of lowest-energy particles, $p_{\mathrm{r}} = eB_0/ck$ is the resonance particle momentum defined by the condition $r_{\mathrm{g}}(p_{\mathrm{r}}) = k^{-1}$, and $N(p \geqslant p_{\mathrm{r}})$ is the number density of particles with momenta exceeding its resonance value.

We compare the increments in the nonresonance (177) and resonance (217) cases. The resonance increment is expressed in terms of the kinetic coefficients $g''$, $\sigma'_{\mathrm{cr}}$ of accelerated-particle current, which arose from the terms containing delta functions in formulas (168) for the transformation of singular denominators. These terms express the resonance conditions

$$\Omega \pm \omega \mp k_{\parallel} v_{\parallel} = 0 \,. \quad (220)$$

The nonresonance increment contains the kinetic coefficients $g'$, $\sigma''_{\mathrm{cr}}$, which arose from the nonresonance principal values in formulas (168). The second important distinction consists in the nonresonance increment containing, as a factor, the high effective magnetic viscosity. When the threshold condition (179) is fulfilled, $k < k_{\mathrm{cr}}$, this viscosity leads not to damping but to enhancement of oscillation excitation, impeding the screening of accelerated-particle current by the background particles.

We compare the effectiveness of resonance and nonresonance turbulence build-up near the shock front by employing the theory elaborated. Let there occur acceleration to an energy $\mathcal{E}_{\mathrm{m}} \approx 3 \times 10^6$ GeV (the energy of the bend in the proton spectrum) in the warm interstellar medium. We assume a moderate acceleration rate ($\eta = 0.1$ is the energy fraction transferred to accelerated particles) and a weak modification of the shock front by accelerated particles ($\alpha = 4$). In this case, according to estimate (102), the fraction

$$\frac{N_0}{n_0} \approx \frac{0.2}{\ln(p_{\mathrm{m}}/p_0)} \left(\frac{u}{c}\right)^2 \approx 1.4 \times 10^{-2} \left(\frac{u}{c}\right)^2$$

of the total proton concentration $n_0$ (i.e., the concentration of ionized and neutral atoms) ahead of the front transforms to the relativistic component. Taking advantage of this estimate and putting $A = (\alpha - 3)u/c = u/c$, most often with $u \gg v_{\mathrm{A}}$, we write out the ratio between the resonance increment and the oscillation frequency as

$$\left(\frac{\gamma}{\omega}\right)_{\mathrm{res}} \approx 5 \times 10^{-3}\, \frac{\omega_{0\mathrm{t}}^2}{\omega_{\mathrm{i}}^2} \left(\frac{u}{c}\right)^3 , \quad (221)$$

where $\omega_{0t}^2 = 4\pi n_0 e^2/m_{\mathrm{p}}$ is defined by the total number density $n_0$. From this estimate it follows that the increment-to-frequency ratio is inversely proportional to the squared magnetic field $B_0^2$, and the increment itself decreases with the magnetic field as $B_0^{-1}$. We calculate the requisite frequencies employing the data collected in Table 1 and find $\omega_{0\mathrm{t}}^2 \approx 6 \times 10^4$ (rad s$^{-1}$)$^2$, $\omega_{\mathrm{i}} \approx 2 \times 10^{-2}$ rad s$^{-1}$ to obtain

$$\left(\frac{\gamma}{\omega}\right)_{\mathrm{res}} \approx 7.5 \times 10^5 \left(\frac{u}{c}\right)^3 .$$

From formulas (172) and (180) we find the corresponding ratio in the nonresonance case:

$$\left(\frac{\gamma}{\omega}\right)_{\mathrm{nr}} \approx \frac{\omega_0^2}{6\omega_{\mathrm{i}}^2}\, \frac{v_{\mathrm{eff}}\omega_{\mathrm{i}}}{c v_{\mathrm{A}}}\, \frac{u}{c} \,. \quad (222)$$

Since $v_{\mathrm{eff}} \propto B_0^2$, the ratio (222) is independent of $B_0$ in this case. But it is sensitive to the fraction of neutral atoms, for it is proportional to the factor $F/(1-F)$. We make use of formula (21) and the data given in Table 1 and, in particular, put $F = 0.85$ to arrive at the estimate

$$\left(\frac{\gamma}{\omega}\right)_{\mathrm{nr}} \approx 7.5 \times 10^8 \left(\frac{u}{c}\right)^3 .$$

Neither ratio contains the wavelength (the wavenumber) and the nonresonance increment exceeds the resonance one by three orders of magnitude.

However, it is well to bear in mind that the growth rates derived above are applicable under different conditions, to which there correspond different oscillation wavelengths. The resonance increment was calculated for collisionless plasma and is therefore applicable to oscillations with wavelengths shorter than the least of the two transport mean free paths, $\Lambda_{\mathrm{ia}} \approx \Lambda_{\mathrm{ea}}$ and $\Lambda_{\mathrm{ei}}$. The nonresonance increment is applied when ion collisions with neutral atoms take place, in which case the oscillation wavelengths should exceed $\Lambda_{\mathrm{ia}}$. In the warm phase of the interstellar medium we have $\Lambda_{\mathrm{ia}} \approx 5 \times 10^{14}$ cm and $\Lambda_{\mathrm{ei}} \approx 5 \times 10^{16}$ cm.

In evaluating the supernova outburst, we take the mechanical explosion energy $E$ to be $10^{51}$ erg and the dumped mass to be $\Delta M = (0.1-1.0)M_{\odot}$ [53] to find the velocity of the free shell expansion at the initial stage of remnant dilation:

$$u_{\max} = \sqrt{\frac{2E}{\Delta M}} \approx (1-3) \times 10^9 \ \mathrm{cm\ s^{-1}} \,.$$

At this stage, $u/c \approx 0.03-0.1$, and both formulas, the resonance and nonresonance ones, lead to the ratio $\gamma/\omega \gg 1$, i.e., a very rapid field generation occurs. In this case, the analytical formulas (180) and (218), which were obtained by employing the condition $\gamma \ll \omega$, are inapplicable. At the Sedov stage, when the rake-in mass of the medium comes to exceed the shell mass, the front velocity decreases and its typical magnitudes are on the order of $u \approx (2-3) \times 10^8$ cm s$^{-1}$. In this case, $u/c \approx 10^{-2}$, and the resonance increment comes to be lower than the frequency, while the nonresonance increment is high in comparison with the frequency as before, provided the fraction of neutral atoms is sufficiently large and the factor $F/(1-F)$ in formula (222) is not too small. Neutral atoms are observed in the optical and UV spectra of a shock front as the superposition of a broad line and a narrow line (in particular, for the $H_\alpha$ line) in the remnants of SN 1006, Kepler, Tycho, RCW 86, Cygnus Loop, etc. and provide an effective means for estimating the velocity of a shock wave [61, 62].

## 11. Formation model of MHD fluctuation spectrum

The linear fluctuation growth increments for the magnetic field of the Alfvén type investigated above do not allow a conclusion about the enhanced field magnitudes. The magnitude of the steady-state magnetic field and its spectral properties are amenable to a consistent study only in the framework of nonlinear models. Accurate theories of the nonlinear time evolution normally invite three-dimensional nonstationary simulations of systems with widely varied scales and are still unrealizable even with the most powerful computers. That is why recourse is often made to strongly simplified estimates. The simplest coarse estimates of the magnitude of the magnetic field on the basis of the linear increment can be made on the assumption that a certain free-energy fraction of the plasma stream is transferred to MHD fluctuations. In this case, the instability saturation mechanism is not defined concretely and estimates are made of some spectrum-integrated energy density of the magnetic field. Estimates of this kind yield the upper limit for the magnitude of the magnetic field, provided the efficiency of energy transfer to the fluctuations exceeds several percent.

To estimate the nonlinear instability saturation level, advantage can also be taken of semiphenomenological schemes for the description of statistically homogeneous incompressible turbulence, which have been used for more than 50 years in the theory of developed Kolmogorov type turbulence produced by interscale spectral energy transfer (Kolmogorov [63], Monin and Yaglom [64]). These methods were employed, in particular, for the construction of model turbulence spectra of the interstellar medium [65, 66]. Modelled in this scheme is the spectral energy density $W_A(k, \mathbf{r}, t)$ of Alfvén type fluctuations. We shall relate it to a unit mass, as is customary in the theory of hydrodynamic turbulence. The foundation of the method is a model balance equation for the spectral energy density of fluctuations, which takes into account their generation, the spectral energy transfer over scales in the inertial range, and the possibility of mode damping:

$$\frac{\partial}{\partial t} W_A + (\mathbf{u}\nabla) W_A + \frac{\partial}{\partial k} \Pi_A(k) = 2\gamma(k) W_A \,. \tag{223}$$

To investigate the nonlinear instability saturation regimes, we take the spectral mode-energy transfer rate $\Pi_A(k)$ in the form commonly employed for the description of strong turbulence (see Ref. [71]):

$$\Pi_A(k) = C_K^{-3/2} k^{5/2} W_A^{3/2} \,. \tag{224}$$

The Kolmogorov constant $C_K$ represents the principal dimensionless model parameter in this theory. Investigations of the $C_K$ magnitude are of importance for applications, in the theory of turbulent transfer, in particular. The simulations of three-dimensional incompressible MHD turbulence performed in Ref. [67] yield $C_K = 3.6$. However, different results are also known: in particular, a value of $C_K \approx 1.7$ proposed in the review [68]. The growth and damping rates of the turbulent modes in Eqn (223) are included in $\gamma(k)$.

Equation (223) with the spectral mode-energy flux (224), supplemented with the corresponding initial and boundary conditions, allows a numerical solution, as well as an analytical solution by the method of characteristics. We shall consider the fluctuation spectrum formation in the vicinity of an MHD shock wave with accelerated particles. As shown in Section 9, the initial fluctuations in the wavenumber range $k_1 < k < k_{cr}$ build up with the increment (180) in the prefront of a shock wave (of size $L_1 = 2\pi/k_1$). In the rest frame of the shock wave, the plasma inflows a front with a velocity $u_1$ exceeding the sound and Alfvén velocities. In the case of a wave with a large Alfvén Mach number, the fluctuation build-up time is therefore limited and is approximately equal to $\tau_a = L_1/u_1$.

In the stationary regime, the turbulence level is determined by the balance between the mode amplitude growth due to the instability of a multifluid system (see Sections 8 and 9), the nonlinear mode-energy cascade towards shorter scales, and the convective transport of the enhanced modes beyond the boundary of the unstable region. Let the wave spectrum $W_\infty(k)$ unperturbed by the shock wave with accelerated particles be defined in the oncoming flow. The stationary solution of the nonlinear equation (223) in the prefrontal region $0 \leqslant z \leqslant L_1$ may be represented in the form of a one-parametric family of characteristics (with the parameter $s \geqslant 0$):

$$k = \left(s C_K^{-3/2} + k_0^{-2/3}\right)^{-3/2} , \tag{225}$$

$$W(s, k_0) = k^{-5/3} \left[ \frac{2}{3} \int_0^s \gamma\big(k(s)\big)\, \mathrm{d}s + k_0^{5/6} W_\infty^{1/2}(k_0) \right]^2 , \tag{226}$$

$$z = \frac{2u_1}{3} \int_0^s k^{5/6}(s)\, W^{1/2}\big(k(s)\big)\, \mathrm{d}s \,. \tag{227}$$

By performing integration successively in expressions (225)–(227) and solving the relations $z = z(s, k_0)$ and $k = k(s, k_0)$ for $s$ and $k_0$, it is possible to calculate with the aid of expression (226) the desired spectral energy density $W(z, k)$ for a given asymptotic spectrum of fluctuations in the oncoming flow $W_\infty(k)$.

For low-amplitude initial fluctuation distributions, a regime of linear growth is realized, which is limited by the finite time of fluctuation growth during the time $\tau_a$ of convective mode transfer in the prefront of the shock wave and which proceeds without significant nonlinear cascade effects. When the initial fluctuation level is not too low (which is often the case in shock waves in interstellar and interplanetary media), the stationary regime is possible, with fluctuation level saturation due to the nonlinear cascade. In this regime, the fluctuation spectrum formed in the instability region will depend only slightly on the initial spectrum shape, because only the initial amplitude is of importance.

Below we shall consider a simplified model convenient for a qualitative analysis of fluctuation spectrum parameters in the prefront of a strong shock wave in the Galaxy. Let the oncoming flow of matter be characterized by a statistically homogeneous background turbulence of the Kolmogorov type, which is sustained at a stationary level by different sources in the galactic disk (most likely, primarily by supernovae explosions). We accept this hypothesis as a simple working model which does not contradict the available observed data, being fully aware that the galactic medium is much more complex than an incompressible liquid. The sources of turbulence deposit their energy into perturbations with the fundamental (highest) scale, which is supposedly close to $L_0 \approx 100$ pc for the galactic disk.

In the presence of accelerated particles in the prefront, the turbulence will be strengthened by the nonresonance and resonance mechanisms in a layer of thickness $L_1$ and in some wavenumber range in accordance with the results obtained in Sections 8–10. With an increase in wavenumber, the increments change sign and transform into decrements, and the dissipation of turbulent energy will result in a spectrum cutoff.

Let the prefront thickness $L_1$ be small in comparison with the fundamental scale $L_0$, and let the corresponding wavenumbers satisfy the condition $k_1 \gg k_0$. On the interval $k_1 \gtrsim k \geqslant k_0$, in the stationary case Eqn (223) becomes simpler, the coordinate derivative vanishes due to the homogeneity of the background turbulence, and the equation with a source takes the form

$$\frac{\partial}{\partial k}\,\Pi_{\mathrm{A}}(k) = \epsilon\delta(k-k_0)\,, \quad \Pi_{\mathrm{A}}(k) = 0 \quad \text{for} \quad k < k_0\,. \tag{228}$$

Its solution corresponds to a constant flux (224) along the spectrum, $\Pi_{\mathrm{A}}(k) = \epsilon = \text{const}$, and leads to the Kolmogorov dependence of the energy density on the wavenumber:

$$W^0_{\mathrm{A}}(k) = C_{\mathrm{K}}\epsilon^{2/3}k^{-5/3}\,. \tag{229}$$

For $k \geqslant k_1$, the nonresonance mechanism of turbulence generation by accelerated particles is engaged in the prefront, and the system under consideration becomes inhomogeneous. To estimate the inhomogeneity, we average the stationary equation (223) with a source over the prefront thickness. As a result of averaging, the term with the coordinate derivative takes the form

$$-\frac{u_1}{L_1}\int_0^{L_1}\frac{\partial W_{\mathrm{A}}}{\partial z}\,\mathrm{d}z = -\frac{u_1}{L_1}\big[W_{\mathrm{A}}(L_1,k) - W_{\mathrm{A}}(0,k)\big]\,. \tag{230}$$

We identify the term $W_{\mathrm{A}}(0,k)$ with the averaged energy density in the prefront, and the quantity $W_{\mathrm{A}}(L_1,k)$ with the background Kolmogorov turbulence outside of the layer. Eventually, we arrive at the equation

$$\frac{\partial}{\partial k}\,\Pi_{\mathrm{A}}(k) = 2\gamma_{\mathrm{eff}}(k)W_{\mathrm{A}} + \epsilon\delta(k-k_0) + \frac{u_1}{L_1}\,W^0_{\mathrm{A}}(k)\,,$$
$$\Pi_{\mathrm{A}}(k) = 0 \quad \text{for} \quad k < k_0\,. \tag{231}$$

Here, $\gamma_{\mathrm{eff}}(k) = \gamma(k) - u_1/2L_1$ takes into account the removal of the MHD modes being generated from the prefrontal region, following which they cease to grow.

Equation (231) does not allow the separation of variables, but it may be easily solved by numerical methods and analyzed in the limiting cases. When the turbulence is not strengthened in the prefront, $\gamma(k) = 0$, the equation takes on the form of Eqn (228) and has the Kolmogorov spectrum (229) for its solution. In the second limiting case, the mode strengthening in the prefront may be quite significant, so that the energy density $W_{\mathrm{A}}$ will far exceed the background level: $W_{\mathrm{A}} \gg W^0_{\mathrm{A}}$. This situation is highly probable in the case of strong shock waves produced by supernova outbursts. According to the estimates made at the beginning of Section 6, the energy density of accelerated particles which strengthen the turbulence in the prefront is several orders of magnitude higher than the magnetic field and background turbulence energy densities. Under these conditions, it is possible to omit the inhomogeneous term containing $W^0_{\mathrm{A}}(k)$ (however, only on the wavenumber interval $k \geqslant k_1$) in equality (230) and Eqn (231):

$$\frac{\partial}{\partial k}\,\Pi_{\mathrm{A}}(k) = 2\gamma_{\mathrm{eff}}(k)W_{\mathrm{A}} + \epsilon\delta(k-k_0)\,. \tag{232}$$

By solving this equation with the flux (224) along the spectrum, we obtain the solution usability condition

$$\epsilon^{1/3} + \frac{2C_{\mathrm{K}}}{3}\int_{k_0}^{k}\gamma_{\mathrm{eff}}(k)\,k^{-5/3}\,\mathrm{d}k \geqslant 0\,, \tag{233}$$

and the solution itself

$$W_{\mathrm{A}}(k) = \frac{C_{\mathrm{K}}}{k^{5/3}}\left[\epsilon^{1/3} + \frac{2C_{\mathrm{K}}}{3}\int_{k_0}^{k}\gamma_{\mathrm{eff}}(k)\,k^{-5/3}\,\mathrm{d}k\right]^2, \tag{234}$$

where one must put $\gamma_{\mathrm{eff}}(k) = 0$ for $k < k_1$. On this spectral interval there is only the Kolmogorov turbulence.

The second term in brackets in expression (234) describes the turbulence generated by accelerated particles in the prefront. For a strong generation, it is far greater than the first term for $k > k_1$. The quantity $\epsilon$, viz. the flux of background turbulence energy along the spectrum, may be expressed in terms of the observable parameters of turbulence in the Galaxy. By integrating expression (229) over the entire spectrum from $k_0$ to $\infty$ and equating it to the observable turbulence energy density $\widetilde{B}_0^2/4\pi$, where $\widetilde{B}_0$ is the magnetic field induction of the largest-scale fluctuations, we obtain

$$\epsilon = \frac{\widetilde{B}_0^3 k_0}{(6\pi C_{\mathrm{K}}\rho)^{3/2}}\,, \tag{235}$$

where $\rho$ is the density of the medium. In the warm phase of the galactic disk, putting $k_0 \approx 2\times10^{-20}$ cm$^{-1}$, we find $\epsilon \approx 10^{-2}$ erg g$^{-1}$ s$^{-1}$.

When there are neutral atoms in the flux incident on the front, the increment $\gamma_{\mathrm{nr}}(k)$ is determined for $k > k_1$ by the nonresonance effect of oscillation build-up and is defined by formula (180). The interval of values in which this effect shows itself is bounded by the smallest of the quantities $k_{\mathrm{cr}}$ and $k_{\mathrm{s}}$ [see expressions (179) and (186)]. In the warm phase of the interstellar medium, $k_{\mathrm{s}} \ll k_{\mathrm{cr}}$, and for $k > k_{\mathrm{s}}$ the parameter $\nu_{\mathrm{eff}}$ therefore rapidly decreases to values $\nu_{\mathrm{m}} \ll \nu_{\mathrm{eff}}$ and the nonresonance build-up becomes ineffective. For $k > k_s$, the main part is played by the resonance mechanism and the corresponding increment $\gamma_{res}(k)$ is given by formula (217) depending on the spectrum of accelerated particles (with exponent $\alpha$). When $\alpha = 4$, which corresponds to a moderate acceleration at a strong shock front, $\gamma_{res}(k)$, as well as $\gamma_{nr}(k)$, is proportional to the wave vector.

The small-scale turbulence dissipation mechanism in this model is related to viscous loss due to the finite electrical conductivity. As shown in Sections 1–3, the effective conductivity is determined both by the possible presence of a neutral component and by turbulent fluctuations of the different scales. The resonance mode-energy absorption by particles may also be a significant factor, whose consistent inclusion calls for the solution of a self-consistent nonlinear problem of the retraction of turbulent fluctuations on the injection and acceleration of nonthermal particles. The nonlinear simulation of the resonance effects of mode generation with the inclusion of their retraction on particle acceleration by a strong shock wave was performed by the Monte Carlo method in Ref. [69]. The nonresonance mode generation mechanism, which was discussed at length in the foregoing, should also be included in the consistent model.

In the general case, the effective MHD-mode growth rate in a multifluid system (with allowance made for viscous damping), obtained in Sections 8 and 9, is, broadly speaking, a second-degree polynomial in the wavenumber $k$. When the MHD-mode generation effect prevails, the effective increment $\gamma_{eff}(k) \propto k^a$ in the $k \lesssim k_{cr}$ region (with $k_{cr} \gg k_1$). According to expression (234), the spectral energy density exhibits asymptotic behavior $W_A(k) \propto k^{2a-3}$. At $a = 1$, we have an intermediate asymptotics $W_A(k) \propto k^{-1}$. On the scale interval $k_1 < k < k_{cr}$, the fluctuation spectrum slopes more gently than $W_A(k) \propto k^{-1}$. The extent of the spectrum depends on the magnitude of $k_{cr}/k_1$ by approximately a linear law. The maximum of spectral mode amplitude is $W_m \propto k_{cr}^2$. As an illustration, Figure 1 shows the calculated spectral mode-energy densities (normalized on $W_{m30} = W_m$ at $k_{cr}/k_1 = 30$) for $k_{cr}/k_1 = 10$ and $k_{cr}/k_1 = 30$. In this calculation it was assumed that $\nu_{eff} k_1^2 \tau_a = 1$. This calculation shows the possibility of a strong increase in the energy density of instability-generated fluctuations on specific intervals of the wavenumber $k$ (the solid and dashed lines in Fig. 1) in comparison with the initial Kolmogorov fluctuation spectrum (the dotted line in Fig. 1).

In the framework of the quasilinear theory of resonance energetic-particle scattering by MHD waves, the energy dependence of the particle diffusion coefficient is defined by the mode spectrum $W_A(k)$ (see, for instance, monograph [8]). To the power function $W_A(k) \propto k^{-1}$ there corresponds, in the energy range of particles resonant with the corresponding modes, the Bohm diffusion law (191) discussed above. Therefore, the model predicts the Bohm diffusion coefficient with a linear dependence on the particle momentum even for moderate amplitudes of magnetic field fluctuations characteristic of the applicability of the quasilinear theory. The use of the Bohm diffusion coefficient is commonly considered to be phenomenologically reasonable for strong turbulence (see, for instance, reviews [48, 70], as well as Ref. [20]).

It should be emphasized once again that the model considered in this section can yield satisfactory quantitative estimates only when the energy of the turbulence generated is low in comparison with the energy of accelerated particles. When the densities of these two energies become approximately equal, using the linear increment in Eqn (231) proves to be incorrect. The level of turbulence saturation in this strongly nonlinear case may be coarsely estimated in the same way that the possible magnitude of the secondary magnetic field was estimated at the beginning of Section 6.

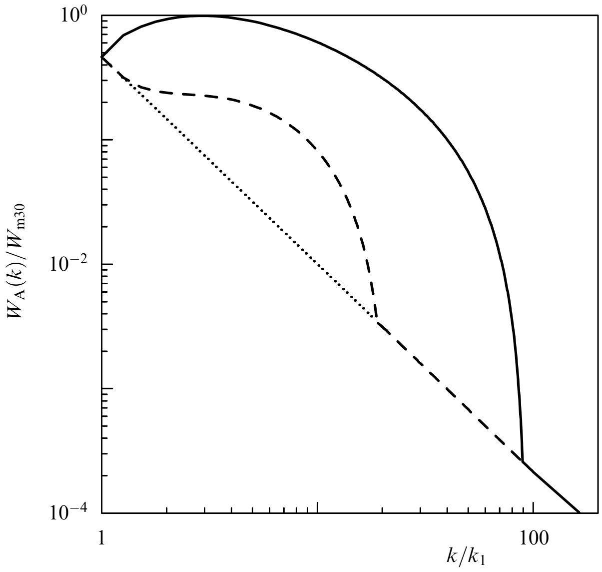


**Figure 1.** Model distribution of spectral energy density $W_A(k)$ in the shock prefront normalized on $W_m(k_{cr}/k_1 = 30) = W_{m30}$. The solid line denotes the distribution for $k_{cr}/k_1 = 30$, the dashed line the distribution for $k_{cr}/k_1 = 10$, and the dotted line the background Kolmogorov spectrum (see Section 11).

In this model, use was made of a simple local dependence of the spectral mode-energy transfer rate $\Pi_A(k)$ on the spectral energy density $W_A$. Nonlocal functional dependences are discussed at length in the book by Monin and Yaglom [71]; as a rule, their employment does not change the power in the asymptotic form of spectral energy density for short waves ($k \gg k_1$). Short-wave MHD turbulence appears to be essentially anisotropic and equation (231) invites modifications to take into account the anisotropy of the mode cascade for a high average magnetic field $B_0 \gg \delta B$ in the small-scale domain (see, in particular, the monograph [68] and references cited therein). According to Ref. [68], the degree of transverse turbulence anisotropy increases $\propto (k_\perp L_0)^{1/3}$ (where $L_0$ is an energy-containing scale). In this case, the small-scale MHD turbulence is described by a local two-dimensional model with a cascade over transverse wavenumbers. Large-scale strong MHD turbulence with $\delta B > B_0$ is supposedly close to quasi-isotropic turbulence with intermittent nonlinear structures.

The spectral energy density (234) permits estimating the attainable energy density of long-wave fluctuations under the assumption that the linear instability saturates due to the nonlinear effect of energy transfer over the Alfvén type mode spectrum and that no depletion of the energy source occurs. In real applications, the assumption that the system is incompressible is apparently not always realized. The longitudinal long-wave fluctuations of the magnetosonic type experience a substantially stronger Landau damping and in the turbulent medium they decay due to the Fermi acceleration of nonthermal particles (see Ref. [44]).

## 12. Enhancement of magnetic fields behind the fronts of astrophysical shock waves

Different-scale magnetic fields play a significant part in mass, energy-momentum, and angular momentum transfer in astrophysical objects of various natures in the accretion of matter on compact massive objects (e.g., Refs [72, 73]), in interstellar medium dynamics [1, 74, 75], and in galactic clusters [30]. We shall discuss the applications of the instability of a multicomponent plasma with accelerated particles to the problem of the origin of strong magnetic fields in the shells of supernova remnants which are traditionally considered as cosmic ray sources (see, for example, Refs [18, 20, 76]).

### 12.1 Magnetic fields in the shells of supernova remnants

Supernova remnants have long been known as powerful sources of nonthermal radio emission. In shell remnants, like Cassiopeia A (Cas A), the synchrotron emission of relativistic electrons is considered to be the principal generation mechanism of rf radiant flux. The possibility of constructing spatially resolved X-ray spectra of supernova remnants emerged with the commencement of operation, during the last decade, of orbital X-ray telescopes with about an arcsecond angular resolution (see, for instance, review [77]). As a result, in the nonthermal continuum of several remnants (SN 1006, Cas A, RCW 86, etc.) the X-ray radiation components with power-law spectra in which there were no spectral lines have been discovered. These components are commonly interpreted as synchrotron X-ray radiation [78 – 80].

In many cases, shock waves propagate through a partially ionized medium. For supernova remnants interacting with molecular clouds (for instance, IC 443), the existence of a neutral component ahead of the shock front may give rise to special features in a high-energy particle acceleration regime (see, for instance, Ref. [81]) and has a strong influence on the emission spectra of these remnants [24]. Some fractions of neutral hydrogen, helium, and metal atoms also reach the shock front of a supernova remnant that resides in a rarefied medium [62].

Let us consider the implications of the above mechanism of Alfvén type wave generation for the shock wave of the supernova remnant SN 1006 (G327.6 + 14.6). This is one of the young remnants known from ancient historical chronicles (see Ref. [53]), which appears to be classed with Ia type supernovae. The distance to the remnant is estimated at 2.1 kpc, and the dimension at about 18 pc [82]. A characteristic feature of the X-ray radiation from the SN 1006 supernova remnant is bright thin segments located in the northeastern (NE) and southwestern (SW) parts of the almost spherical shell with diameter close to 30′ of arc (Fig. 2). Detailed investigations of the bright NE domain were recently carried out with the Chandra X-ray Observatory (see Refs [83, 84]). The X-ray spectrum of the thin bright NE segment was dominated by a nonthermal continuum which is usually interpreted as the synchrotron emission of electrons with energies on the order of 10 – 100 TeV in the vicinity of a shock wave [79]. The high spatial resolution (on the order of 1″) of the CCD ACIS detector of the Chandra X-ray Observatory permitted K Long and co-workers to discover an abrupt jump in radiation intensity (see Ref. [83]). The authors revealed that the intensity of radiation with photon energies above 1.2 keV immediately ahead of the front did not

**Figure 2.** X-ray image of the SN 1006 supernova remnant, obtained with the Chandra satellite (Credit: NASA/CXC/Rutgers/J.Hughes et al.) in the 0.5 – 3 keV range (see Ref. [83]). The image clearly shows the nonthermal-continuum filaments interpreted as the synchrotron X-ray radiation of ultrarelativistic electrons accelerated by shock waves with enhanced magnetic fields in the prefront (see Section 12.1).

exceed 1.5% of the highest brightness in the domain immediately behind the shock front [83]. The width of the bright NE segment of X-ray synchrotron radiation was about 10″ (1″ $\approx 3.3 \times 10^{16}$ cm at an estimated distance of 2.1 kpc to SN 1006). The problem of the existence of a weak radio galactic halo in OCH and estimates of the diffusion coefficients for relativistic electrons were earlier discussed by Achterberg et al. [85]; however, the upper limit for the brightness of the synchrotron halo, established by Long et al. [83], is the most severe (see Ref. [86]).

The optical and UV spectra of SN 1006 [87, 88] show the presence of neutral atoms in the vicinity of the shock front. The remnant map in the $H_\alpha$ line, obtained by Winkler et al. [82], shows a more uniform distribution of the optical radiation over the limb of SN 1006. There are optical filaments in the NE part, which correlate with bright X-ray filaments, but there are also even brighter optical filaments in the SW part of the remnant, where X-ray filaments are not pronounced. The observations are consistent with the estimates of the neutral component fraction $F \sim 0.1$ in the prefront of a shock wave propagating with a velocity $v_{\rm sh} \sim 2300$ km s$^{-1}$. An estimate of the gas density ahead of the front of the NE sector of the shock wave in SN 1006 gives $n_{\rm i} \sim 0.1$ cm$^{-3}$. Using the hydrogen charge exchange rate constant for a temperature of about $10^4$ K, we obtain the charge-exchange mean free path of atomic hydrogen equal to the minimal instability wavelength $\lambda_0 = 2\pi k_0^{-1} \sim 2 \times 10^{16}$ cm (since $k_0 \gg k_{\rm cr}$), as well as an estimate of the magnetization factor $\omega_{B{\rm i}}\tau_{\rm i} \gtrsim 10^7 \times B(Fn_{-1})^{-1}$. Here, $B$ is measured in μG, and the plasma number density $n_{-1}$ in units of 0.1 cm$^{-3}$. Therefore, we employ relationship (30) to obtain the characteristic mode build-up time $\sim 6 \times 10^2 \times (N_0/n_{\rm i})^{-1}$ (s), which permits enhancing magnetic fields with scale lengths on the order of $\lambda_0$ during the lifetime of SN 1006 if the rate of proton injection into the regime of acceleration by the shock wave allows the values $N_0/n_i \gtrsim 10^{-7}$. By using relationship (99), one can ascertain that the energy density of accelerated particles in this case is equal to a fraction of one percent of the kinetic energy density of the oncoming plasma flow. If we restrict ourselves to the injection rates that allow cosmic-ray energy densities $w_{cr}$ on the order of several percent of the kinetic energy density ($\sim m_p n_i v_{sh}^2$) of the flow, we arrive at the possibility that magnetic field fluctuations with amplitudes $\delta B \sim 30$ μG can be generated ahead of the shock front. The compression $R \approx 4$ of the transverse field component at the jump in a strong shock wave (here, we consider a single-fluid wave without a lengthy prefront, because we are investigating the case of a low proton injection rate) will permit obtaining magnetic fields of about 100 μG in the region behind the shock front. Magnetic fields on the order of 100 μG behind the shock front of SN 1006 permit attributing the narrow X-ray continuum brightness distribution observed in the NE segment of the shock wave [83, 86, 89] to the rapid synchrotron cooling effect of relativistic electrons behind the shock front. In the case when magnetic fields are generated in a partially ionized medium, no appreciable pressure ($\sim m_p n_i v_{sh}^2$) of the nucleon component of cosmic rays is required in the prefront region, which was assumed in the model built in Ref. [89]. The upper limit for the ratio between the synchrotron luminosity in the prefront region and the highest luminosity in the transverse segment of the shock wave is evaluated as $R^{-\Gamma}$, where $\Gamma$ is the photon exponent of the synchrotron radiation spectrum above $\sim 1.5$ keV. In our case of compression in a strong single-fluid shock wave with $R \approx 4$ and a synchrotron exponent $\Gamma \sim 3$, we obtain the specific luminosity ratio of about 1.5%, which is consistent with the limit established by Long et al. [83]. In the comparison analysis of the synchrotron radiation (radio and X-ray) and optical ($H_\alpha$) maps, it should be borne in mind that the magnetic fluctuation growth rate is, apart from the neutral particle fraction $F$, also proportional to the local density of nonthermal particles, which is largely determined by the local injection rate. The mechanisms of particle injection into the regimes of acceleration by shock waves so far do not allow making quantitative predictions, but it is hypothesized that ion injection is stronger in the quasiparallel part of the shock wave (see Refs [48, 70, 90]).

The presence of radio wave filaments in the neighborhood of the galactic center may be an interesting implication of the physical mechanism of magnetic field generation by a shock wave in a plasma medium with a neutral component. The observations of Yusef-Zadeh et al. [91] are indicative of the possible relation of some of the filaments to supernova remnants. Neutral particles can substantially simplify the problem of magnetic field generation in the reverse shock wave propagating through the expanding supernova ejection in the nonlinear model of particle acceleration by the reverse shock [92].

### 12.2 Magnetic fluctuations in interplanetary shock waves

A natural (and quite frequently the only) laboratory for direct observations of collisionless MHD shock waves is the Sun with the processes proceeding in the near heliosphere. Collisionless waves and nonthermal particles in the interplanetary medium have been observed since the late 1960s, and a wealth of observational material has been accumulated (see, for instance, Refs [93–95]). The results of observations may be summarized as follows. Alfvén perturbations quite often prevail in the interplanetary plasma (see Refs [93, 94]); in regions of collisions of solar wind flows with different velocities, however, an important part is played by large-scale compressible perturbations. Outside of the regions of collision of fast flows and masses ejected by the solar corona, Leamon et al. [96] give a fluctuation power spectrum of the form $W(\nu)$ on the interval from $\nu^{(-1.46\pm0.01)}$ to $\nu^{(-1.93\pm0.02)}$ in the frequency range (in the rest frame of the detector aboard the *Wind* spacecraft) $0.01 < \nu < 0.4$ Hz. For $\nu > 0.4$ Hz, the power spectra were obtained from $\nu^{(-2.00\pm0.02)}$ to $\nu^{(-4.43\pm0.01)}$. The data by Leamon et al. [96] were derived from the analysis of 33 one-hour-long observations with the MFI magnetic field detector aboard the NASA's *Wind* spacecraft. These data were interpreted as relating to a turn-over from the inertial interval to the dissipative domain in the frequency range near $\nu \approx 0.4$ Hz. The mean spectrum in the inertial interval is $\nu^{-1.66}$, which is in perfect agreement with the Kolmogorov law.

The problem of spectral energy transfer and MHD turbulence dissipation is essential, in particular, to the understanding of the heating mechanisms of solar wind plasma. The heating mechanism is required for interpreting the observed decrease $\propto r^{-0.8}$ (or $r^{-1.0}$) in proton temperature with the heliocentric distance, which is much slower than the conventional law $T \propto r^{-2(\gamma_a-1)}$ for expanding wind with an adiabatic index $\gamma_a$ (see, for example, Refs [96, 97]). The heating of the solar wind plasma by the processes occurring in the dissipative domain of the turbulent cascade is a promising model. MHD turbulence modeling in the inertial interval and the dissipative domain is carried out with the inclusion of the two-dimensionality effects of the small-scale MHD mode cascade in the plane perpendicular to the average magnetic field [97–99].

The turbulence and accelerated-particle spectra in the immediate vicinity of the shock wave in the interplanetary medium on November 12, 1978 were comprehensively studied by Kennel et al. [100]. The authors revealed that the spectra of magnetic fluctuations in the vicinity of the shock wave in the $\nu < 0.1$ Hz frequency range, obtained from three 2.5-min intervals, are gently sloping ($\propto \nu^{-d}$ with an exponent $d \lesssim 1$). Furthermore, they discovered a strengthening of turbulence in the high-frequency ($\nu > 0.1$ Hz) range outside of the domain resonant with the gyrofrequencies of accelerated ions, where the magnetic-field power spectrum decreases rapidly with frequency. Ion fluxes accelerated by the shock to energies on the order of 150 keV were detected in the neighborhood of about $(2-3) \times 10^{10}$ cm, the prefront region dimensions depending on the particle energy. The ion spectrum is satisfactorily described by a power-law velocity distribution with an exponent $\approx 4.2$. Kennel et al. [100] estimated the accelerated-particle energy density in the 3–200-keV energy range at $1.6 \times 10^{-9}$ erg cm$^{-3}$. The measured number density of thermal particles was $n_i \approx 4$ cm$^{-3}$. The shock front velocity was estimated at 640 km s$^{-1}$, and the Alfvén Mach number of the shock at $M_A \sim 3.5$.

Let us consider the possibility that the mechanism described in Section 11 generates magnetic field fluctuations. It is possible to estimate the relative nonthermal-particle concentration $N_0/n_i \sim 10^{-3}$ and then find the magnitude of the critical wavenumber $k_{cr}$ (179) along with the mode build-up increment from formula (180). The estimate $k_{cr} \sim \omega_{pi}/c \times N_0/n_i \times M_A \sim 2 \times 10^{-10}$ cm$^{-1}$ yields a scale several times shorter than the domain of accelerated-particle distribution in the prefront. The frequencies of the growing MHD modes in the detector rest frame lie in the $(1-5)\times10^{-3}$ Hz range. For effective magnetic viscosities $\nu_{\mathrm{eff}}\sim10^{15}\ \mathrm{cm}^2\ \mathrm{s}^{-1}$, the MHD mode build-up increment (180) and the fluctuation spectrum amplitude (234) permit one to quantitatively describe the observed power spectra of large-scale magnetic field fluctuations. The calculation of effective magnetic viscosity in the framework of the fluctuation conductivity models for collisionless plasmas, considered in Section 2 [see formula (67)], allows explaining the above values of $\nu_{\mathrm{eff}}$ if it is assumed that the amplitudes of (small-scale) field fluctuations, which are responsible for the scattering of thermal electrons, are $\delta B/B\sim10^{-3}$. The fluctuation spectrum (234) predicted by the model is gently sloping in the low-frequency domain (with an exponent $\leqslant 1$) in accordance with observations [100, 101]. At the same time, Kennel et al. [100] pointed out that the spectrum of the MHD fluctuations resulted from resonance wave generation by accelerated particles would have an exponent 7/4, which is at variance with observations.

The nonresonance mechanisms of large-scale magnetic-field fluctuation generation by the current of accelerated particles in the neighborhood of MHD shocks, considered in Sections 7–9, permit forming magnetic fields with amplitudes which are many times higher than the unperturbed (asymptotic) magnitudes of magnetic field induction ahead of the shock front. An important feature of these mechanisms is that the generation of the magnetic field is not associated with a strong modification of the shock prefront by the pressure of accelerated particles, as is assumed in the models of Refs [15, 89] and in an earlier paper [102]. Therefore, strong large-scale magnetic fields may be generated in a shock prefront even for a particle acceleration efficiency on the order of 10% and particle spectra with exponents $\alpha\gtrsim4$ (and the nonthermal component pressure determined by particles with moderate energies). In resonance mechanisms, efficient generation of large-scale magnetic fields implies gently sloping particle spectra with nonthermal-particle pressure determined by high-energy particles. The nonresonance generation of small-scale fluctuations was recently considered by Bell [27], while anisotropic MHD cascades and the part played by two-wave scattering processes in turbulence formation are discussed in Refs [103, 104]. It is not inconceivable that the nearly Kolmogorov fluctuation spectrum observed in the interplanetary medium constitutes a complex superposition of an anisotropic transverse cascade and a gently sloping spectral distribution of Alfvén waves propagating along the average magnetic field [105].

The generation of magnetic fluctuations, of course, takes place immediately in the formation of the front of a collisionless shock wave in a plasma, which was considered by Sagdeev [36] as the main mechanism of collisionless relaxation in a shock wave. The Weibel instability of transverse mode growth in a plasma with an anisotropic velocity distribution [106, 107] supposedly plays a significant part in collisionless relaxation in the shock fronts. Numerical particle-in-cell (PIC) simulations of the structure of the collisionless shocks demonstrate the growth of small-scale filamentous magnetic-field structures on the scale of hundreds of inertial ion lengths $c/\omega_{\mathrm{pi}}$ (see, for instance, Refs [108, 109]). However, the level of magnetic field amplitudes far away from the front and the degree of particle thermalization, which are related to the decay of magnetic structures, invite further analysis [110]. Even the highest-power modern computers do not enable employing PIC codes for the simulation of astrophysical shock structures on the scales where accelerated particles play a significant role. Modeling that relies on the employment of kinetic equations considered in the foregoing therefore remains the main means of investigation of multicomponent plasmas with energetic particles.

## 13. Conclusions

Our review contains a derivation and detailed discussion of the basic equations describing the macroscopic dynamics of the cosmic plasma with the inclusion of nonthermal particles, fluctuating electromagnetic fields, and neutral atoms. We analyzed the implications of the multicomponent character of the cosmic plasma that pertain to the generation or enhancement of magnetic fields with different spatial scales. Primary emphasis was placed on the role of suprathermal (including relativistic) particles, as well as the role of the neutral component.

In this review it was demonstrated that both small additions of neutral particles and small-scale stochastic magnetic fields can change the effective electrical conductivity and magnetic viscosity of astrophysical plasmas by many orders of magnitude. This has the consequence that extrinsic currents substantially grow in importance; these are produced by suprathermal and, particularly, relativistic particles in the neighborhood of active astrophysical objects — the sources of accelerated particles. An extrinsic current induced by relativistic and background particles in a magnetized plasma generates a secondary large-scale magnetic field. As a result, the initial large-scale field may be enhanced or lowered by one or two orders of magnitude under typical conditions. Detailed observed data on magnetic fields in quite different astrophysical objects ranging from the cosmological medium to the heliomagnetosphere are given in Vallée's reviews [111].

The accelerated component of the cosmic plasma also exerts a strong influence on the MHD turbulence. In the neighborhood of shock fronts, which accelerate energetic particles, there emerges the possibility of the nonresonance excitation of large-scale Alfvén type MHD fluctuations with an increment proportional to the effective magnetic plasma viscosity and the fraction of suprathermal particles. The excitation of these oscillations substantially increases the efficiency of particle acceleration by shock fronts. The turbulence strengthening mechanism considered above does not involve an appreciable modification of the velocity profile and the transfer of the major part of gas-dynamic energy to the accelerated particles — therein lies its advantage over the processes of turbulence generation near the shock fronts discussed earlier.

We have come up with a formation model of MHD fluctuation spectrum and shown that the mechanism investigated leads, in a broad scale range, to a spectral energy density inversely proportional to the fluctuation wave vector. This spectrum shape may give rise to the Bohm dependence of the fast-particle diffusion coefficient on the momentum, whereat the transport mean free path gets on the order of the particle gyroradius.

In our review, turbulence was described in the conventional way involving the use of correlation tensors and spectral energy densities. Recent years have seen the development of a different approach in which the turbulent state is treated as the result of self-organization of a strongly nonequilibrium and nonlinear system. Readers may familiarize themselves with this circle of ideas and methods from the review [112].

The multicomponent plasma instabilities investigated are invoked to explain the observed data on the nonthermal radiation from supernova remnants and on fast particles, as well as on the MHD turbulence near shock fronts in the interplanetary space.

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