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New approaches to electroweak symmetry breaking

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Contents

Abstract. **Although the spectacular experimental achievements of particle physics in the last decade have strengthened the Standard Model (SM) as an adequate description of nature, they have also revealed that the SM matter represents a mere 5% or so of the energy density of the Universe, which clearly points to some physics beyond the SM despite the desperate lack of direct experimental evidence. The sector responsible for the spontaneous breaking of the SM electroweak symmetry is likely to be the first to provide experimental hints at this new physics. The aim of this review is, after briefly introducing SM physics and the conventional Higgs mechanism, to give a survey of recent ideas on how the dynamics of electroweak symmetry breaking can be explained.**

1. Introduction

Particle physics has lived through a decade of great experimental success [1]: the discovery of the top quark, the observation of solar and atmospheric neutrino oscillations, the measurement of direct CP violation in the K system, the measurement of CP violation in the B system, evidence of an accelerated phase in the expansion of the universe, determination of the dark energy and dark matter composition of the universe, etc. These results have strengthened the Standard Model (SM) of particle physics as a successful description of nature. Yet, these results have also led to the conclusion that the SM matter only represents about 5% of the energy density of the Universe and therefore physics beyond the SM is called for, albeit direct evidence of such physics is desperately missing.

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With the exception of gravitational interactions, the SM describes all the fundamental interactions among elementary particles. These interactions correspond to gauge interactions mediated by gauge bosons: the photon, the $W^{\pm}$ and Z^0 bosons, and the gluons associated respectively with the electromagnetic, weak, and strong interactions. These gauge bosons ensure the invariance of the theory under the $U(1)_Y \times SU(2)_L \times SU(3)_C$ local symmetry. While the photon and gluons are massless, the $W^{\pm}$ and Z^0 bosons have a mass of the order of 10^{-25} kg, i.e., 100 GeV in particle physics units. These masses can be understood as a consequence of the fact that $U(1)_Y \times SU(2)_L$ is not an exact symmetry of nature. Broken symmetries are not particular to particle physics and are encountered in classical physics: while the laws of mechanics are invariant under rotations in the three dimensions of space, we clearly distinguish the vertical direction on Earth from the two horizontal ones: SO(3) is broken down to SO(2). Similarly, the vacuum of the SM is actually only invariant under the $U(1)_{em}$ subgroup describing the conservation of the electric charge, while applying other transformations of $U(1)_Y \times SU(2)_L$ does not leave this vacuum invariant. For instance, a scalar field could have a vacuum expectation value that identifies a preferred direction among the four generators of $U(1)_Y \times SU(2)_L$. The interactions of this scalar field with the other elementary particles, quarks, leptons, and gauge fields would then be responsible for their masses. Such a scalar particle is known as the Higgs particle.

The Standard Model can be divided into three sectors: the gauge sector, the flavor sector, and the electroweak symmetry breaking sector. While the first two have been well tested in accelerator experiments (such as LEP, SLD, BABAR, BELLE), the sector of electroweak symmetry breaking (EWSB) is currently the subject of intense scrutiny, not only because particle physicists hope to discover the Higgs boson at the Large Hadron Collider (LHC), soon to be operational at CERN, but also because this sector could well provide us with the first hints in a detector of a new physics beyond the Standard Model. Indeed, the usual Higgs mechanism jeopardizes our current understanding of the SM at the quantum level and requires the existence of additional structures (new particles, new symmetries, new dimensions) to stabilize the weak scale. Better than a long introduction, the following tautology reveals that an understanding of the dynamics of EWSB is still missing:

Why is the EW symmetry broken?

Because the Higgs potential is unstable at the origin.

Why is the Higgs potential unstable at the origin?

Because otherwise the EW symmetry would not be broken.

One should understand here that the Higgs mechanism is only a description of EWSB and not an explanation of it because, in particular, there is no dynamics to explain the instability of the Higgs potential at the origin.

The Higgs sector involves two experimentally unknown parameters, the Higgs boson mass ($M_{\rm H}$) and the cutoff scale (Λ) of the SM itself, i.e., the scale at which the new physics is to show up. Yet, these two parameters are subject to theoretical consistency constraints — the well-known unitarity, triviality, stability, and naturality bounds — as well as indirect experimental constraints through the electroweak precision data.

Integrating out any heavy particle generates new nonrenormalizable interactions between the light SM particles. If the SM is to make any sense as an effective theory at low energy, these new interactions have to leave the SM gauge symmetry unbroken. Yet, they can break some (accidental, approximate) SM global symmetries and, depending on which global symmetry is actually broken, these new interactions can manifest themselves at rather low energy (Table 1). The EWSB sector seems to be a good place to look for manifestation of the new physics in the energy range that will be explored at the LHC.

Table 1. Examples of nonrenormalizable interactions between SM particles obtained after integrating out some heavy degrees of freedom and limits on the corresponding scales that suppress them. These interactions can be classified according to the global symmetries they break. Rather low scales can affect the EWSB sector. (From Ref. [2].)

Broken symmetry	Operators	Scale Λ, TeV
B, L	$(QQQL)/\Lambda^2$	10^{13}
Flavor (1st, 2nd family), CP	$(\bar{d}s\bar{d}s)/\Lambda^2$	1000
Flavor (2nd, 3rd family)	$m_{\rm b}(\bar{s}\sigma_{\mu\nu}F^{\mu\nu}b)/\Lambda^2$	50
Custodial SU(2)	$(h^\dagger D_\mu h)^2/\Lambda^2$	5
None	$(h^\dagger h)^3/\Lambda^2$	—

This raises three important questions that we address in this review:

• Given the experimental results from LEP, SLD, and Tevatron, what are the constraints on a new physics in the EWSB sector?

• Is it possible to add a new physics around the TeV scale that stabilizes the EW scale?

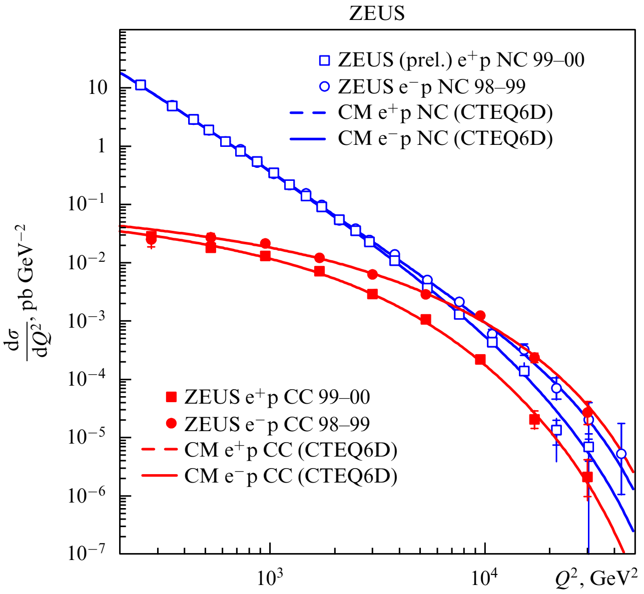


Figure 1. Unification of electromagnetic and weak interactions as seen by the ZEUS collaboration [7].

• What is the potential for discovering a new physics in the EWSB sector at the LHC?

This review is organized as follows:[1] Section 2 contains a short introduction to the Higgs mechanism and discusses the limits on the Higgs mass and the cutoff scale of the Standard Model; Section 3 explains how the new physics can address the problems encountered in the Higgs sector and gives the general low-energy structure of the corrections induced in the gauge sector by the new physics; Section 4 discusses models where the Higgs field appears as a component of a gauge field along an extra dimension; finally, Section 5 reports on Higgsless models, where the EWSB is no longer achieved through a Higgs mechanism but results from nontrivial boundary conditions for the gauge fields at the boundaries of a fifth dimension. Another class of EWSB models has become quite popular in recent years, namely the Little Higgs models. Since some very good reviews are already available on that subject [2, 6], only a short account of these models is given in this review in Section 3.2.

2. Electroweak symmetry breaking and Higgs physics

2.1 SM Higgs physics

2.1.1 Higgs mechanism. Experiments (Fig. 1) show that above approximately 100 GeV, electromagnetism and weak interaction are 'unified,' while at lower energy, the photon is quite different from the Z^0 and the $W^\pm$ bosons; in particular, they have quite different masses:

$$\begin{aligned} &M_\gamma < 6\times10^{-17}\ \text{eV}\,,\\ &M_{W^\pm} = 80.425\pm0.038\ \text{GeV}\,, \qquad (1)\\ &M_{Z^0} = 91.1876\pm0.0021\ \text{GeV}\,. \end{aligned}$$

[1] Some useful reviews/lecture notes on electroweak physics can be found in Ref. [3]. The reader interested in an introduction to technicolor models can have a look at [4]. Sections 4 and 5 involve physics in extra dimensions. The material of these two sections is self-contained; more advanced introductions to extra dimensions can be found in Refs [5].

The simplest way to explain the possible origin of these masses is to rely on the Higgs mechanism [8]: the initial Lagrangian is invariant under the full $SU(2)_L \times U(1)_Y$ group, but the vacuum only preserves a $U(1)_{em}$ subgroup. This *spontaneous* breaking can result from a scalar field taking a nonzero vacuum expectation value (*vev*). More precisely, we assume that the theory involves a scalar field H of hypercharge $1/2$ transforming as a doublet under $SU(2)_L$. Using a $SU(2)_L \times U(1)_Y$ transformation, the *vev* of H can always be brought to the form

Symmetry of the Lagrangian	**Symmetry of the vaccum**
$SU(2)_L \times U(1)_Y$	$U(1)_{em}$
Higgs doublet	Higgs *vev*
$H = \begin{pmatrix} H_+ \\ H_0 \end{pmatrix}$	$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$

The nonzero *vev* of the Higgs boson is a consequence of the form of the Higgs potential

$$V(H) = \lambda \left(H^\dagger H - \frac{v^2}{2} \right)^2 , \tag{2}$$

which has the shape of a Mexican hat and exhibits a local maximum at the origin.

The gauge boson masses are generated through the covariant derivative of the scalar field[2] ($W^\pm_\mu = (W^1_\mu \mp \mathrm{i} W^2_\mu)/\sqrt{2}$)

$$D_\mu H = \partial_\mu H - \frac{\mathrm{i}}{2} \begin{pmatrix} g W^3_\mu + g' B_\mu & \sqrt{2} g W^+_\mu \\ \sqrt{2} g W^-_\mu & -g W^3_\mu + g' B_\mu \end{pmatrix} H , \tag{3}$$

and $|D_\mu H|^2$ contains the quadratic mass terms

$$\frac{1}{4} g^2 v^2 \, W^+_\mu W^{-\mu} + \frac{1}{8} (W^3_\mu \, B_\mu) \begin{pmatrix} g^2 v^2 & -g g' v^2 \\ -g g' v^2 & g'^2 v^2 \end{pmatrix} \begin{pmatrix} W^{3\mu} \\ B^\mu \end{pmatrix} . \tag{4}$$

Thus, the gauge boson spectrum contains

• a pair of electrically charged gauge bosons, the $W^\pm$'s, having the mass

$$M_W^2 = \frac{1}{4} g^2 v^2 ; \tag{5}$$

• a pair of electrically neutral gauge bosons, the photon γ and the Z boson, which are the linear combinations of W^3 and B that diagonalize mass matrix (4):

$$Z_\mu = c_W W^3_\mu - s_W B_\mu , \qquad \gamma_\mu = s_W W^3_\mu + c_W B_\mu , \tag{6}$$

where the weak mixing angle is the ratio of the $SU(2)_L$ and $U(1)_Y$ gauge couplings:

$$c_W = \cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}} , \qquad s_W = \sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}} . \tag{7}$$

[2] Throughout this review, we use the mostly minus signature metric $(+, -, \ldots, -)$. The Greek indices $\mu, \nu, \ldots$, denote our usual 4D spacetime coordinates, while coordinates along the extra dimensions are denoted by lowercase Latin indices. Uppercase indices denote both 4D and extra dimensional coordinates.

As a consequence of the unbroken $U(1)_{em}$ symmetry, the photon remains massless and the mass of the Z boson is given by

$$M_Z^2 = \frac{1}{4} (g^2 + g'^2) v^2 . \tag{8}$$

2.1.2 Counting the degrees of freedom. In the breaking of $SU2_L \times U(1)_Y$ down to $U(1)_{em}$, three gauge directions are broken, which requires 'eating' three Goldstone bosons. These Goldstone bosons are provided by the Higgs doublet that involves four real scalar degrees of freedom, out of which three are the eaten Goldstone bosons, which become the longitudinal polarizations of the massive gauge bosons. Therefore, in total, there is one remaining physical real scalar degree of freedom: the famous 'Higgs boson.' It describes the fluctuations around a nontrivial vacuum.

This decomposition can be made explicit in the parameterization

$$H = \exp(\mathrm{i}\pi^a \sigma^a) \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix} . \tag{9}$$

In the unitary gauge, the π^a are nonphysical and correspond to the eaten Goldstone bosons, while h is the physical Higgs boson. The physical Higgs boson has some nontrivial interactions with the gauge bosons and with the quarks and leptons through the Yukawa couplings. It also has nontrivial self-interactions, which are obtained by expanding the scalar potential around its minimum:

$$V = \lambda \left(H^\dagger H - \frac{v^2}{2} \right)^2 = \lambda v^2 h^2 + \lambda v h^3 + \frac{1}{4} \lambda h^4 . \tag{10}$$

In particular, the Higgs mass is

$$M_H^2 = 2\lambda v^2 . \tag{11}$$

2.1.3 Custodial symmetry. From the spectrum of the gauge bosons given above, we easily obtain the value of the ρ parameter

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = \frac{(1/4) g^2 v^2}{(1/4)(g^2 + g'^2) v^2 g^2/(g^2 + g'^2)} = 1 . \tag{12}$$

There is an empirical theorem that says that whenever you find either 0 or 1 at the end of a computation, it means that either you made a mistake or there is a symmetry that can explain the result. In the present case, there is clearly no mistake, and hence there must be a symmetry behind this particular value of the ρ parameter. Indeed, an $SU(2)$ custodial symmetry is present in the Higgs sector of the theory [9]. The Higgs doublet contains four real degrees of freedom and the point is that the Higgs potential is actually invariant under any rotation of these four components, and hence under a global $SO(4)$ symmetry, which mathematically is nothing but $SU(2)_L \times SU(2)_R$, the $SU(2)_L$ being the gauge symmetry of the Standard Model. The origin of the $SU(2)_R$ symmetry can also be made more transparent as follows. A doublet of $SU(2)$ is a pseudo-real representation, which means that the complex conjugate of the doublet is equivalent to the doublet itself. In practice, if we consider the doublet to be a field H_i with a lower SU(2) index, then the complex conjugate automatically has an upper index. However, using the SU(2) epsilon tensor $\epsilon_{ij} = \mathrm{i}\sigma^2_{ij}$, this can be lowered again, and therefore H_i and $\mathrm{i}\epsilon_{ij}(H^*)^j$ transform in the same way. This means that in addition to $\mathrm{SU(2)_L}$ acting on the usual SU(2) index, there is another $\mathrm{SU(2)_R}$ symmetry that mixes H with $\mathrm{i}\sigma^2 H^*$. We can write a 2×2 matrix as

$$\mathcal{H} = \begin{pmatrix} \mathrm{i}\sigma^2 H^* & H \end{pmatrix} = \begin{pmatrix} H_0^* & H_+ \\ -H_+^* & H_0 \end{pmatrix}, \tag{13}$$

on which the $\mathrm{SU(2)_L}\times\mathrm{SU(2)_R}$ symmetry acts as

$$\mathcal{H} \to U_\mathrm{L}\,\mathcal{H}\,U_\mathrm{R}^\dagger\,. \tag{14}$$

The $\mathrm{SU(2)_L}\times\mathrm{SU(2)_R}$ invariance of the Higgs potential is made explicit by noting that

$$\mathcal{H}^\dagger\mathcal{H} = H^\dagger H\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{15}$$

such that the Higgs potential becomes

$$V = \frac{\lambda}{4}\left(\mathrm{Tr}\,\mathcal{H}^\dagger\mathcal{H} - v^2\right)^2. \tag{16}$$

The Higgs *vev*

$$\langle H\rangle = \begin{pmatrix} 0 \\ \dfrac{v}{\sqrt{2}} \end{pmatrix}, \quad \text{i.e.,} \quad \langle\mathcal{H}\rangle = \frac{v}{\sqrt{2}}\begin{pmatrix} 1 & \\ & 1 \end{pmatrix}, \tag{17}$$

obviously breaks $\mathrm{SU(2)_L}\times\mathrm{SU(2)_R}$ down to the diagonal subgroup $\mathrm{SU(2)_D}$ ($U_\mathrm{L} = U_\mathrm{R}$ in (14) leaves the vacuum invariant). Under this unbroken symmetry, the three gauge bosons $(W^1_\mu, W^2_\mu, W^3_\mu)$ transform as a triplet, which in particular imposes the same mass term for all W^i. The mass term for the W^3 gauge boson can be obtained from the mass matrix in the basis of (γ, Z):

$$\begin{aligned}(Z_\mu\,\gamma_\mu)&\begin{pmatrix} M_\mathrm{Z}^2 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} Z^\mu \\ \gamma^\mu \end{pmatrix} \\ &= (W^3_\mu\,B_\mu)\begin{pmatrix} c_\mathrm{W}^2 M_\mathrm{Z}^2 & -c_\mathrm{W}s_\mathrm{W}M_\mathrm{Z}^2 \\ -c_\mathrm{W}s_\mathrm{W}M_\mathrm{Z}^2 & s_\mathrm{W}^2 M_\mathrm{Z}^2 \end{pmatrix}\begin{pmatrix} W^{3\mu} \\ B^\mu \end{pmatrix}.\end{aligned} \tag{18}$$

The $\mathrm{SU(2)_D}$ symmetry finally imposes the relation

$$M_\mathrm{W}^2 = c_\mathrm{W}^2 M_\mathrm{Z}^2\,, \tag{19}$$

which is just $\rho = 1$. Actually, the custodial symmetry is explicitly broken by the $\mathrm{U(1)}_Y$ interaction (H and H^* have opposite charges) and by the Yukawa couplings to quarks and leptons. As a consequence, at the one-loop level, there are corrections to $\rho = 1$. Actually, the screening theorem [10] states that these corrections only appear through some logarithms and thus the deviation from $\rho = 1$ remains rather small.

More generally, if $\mathrm{SU(2)_L}\times\mathrm{U(1)}_Y$ is broken not only through a doublet but also through a collection of scalar fields in the $2s_i + 1$ representation of $\mathrm{SU(2)_L}$, carrying a hypercharge y_i and acquiring a *vev* v_i, we can similarly arrive at the ρ parameter now being given by

$$\rho = \frac{\sum_i\left(s_i(s_i+1) - y_i^2\right)v_i^2}{\sum_i 2y_i^2 v_i^2}\,. \tag{20}$$

In particular, any nondoublet *vev* must contribute to a deviation from $\rho = 1$, the only exceptions occurring when $s(s+1) = 3y^2$, e.g., $s = 3$ and $y = 2$.

2.2 Limits on the Higgs mass and the scale of the new physics

2.2.1 Unitarity limit. There is a fundamental motivation to generate the W and Z masses through a Higgs mechanism because these masses are actually inconsistent with the content of known particles, and extra degrees of freedom, like the Higgs boson, are needed to soften the UV behavior of massive gauge bosons [11, 12]. One polarization of a massive spin-1 particle indeed grows with the energy of the particle.

Three polarizations of a massive spin-1 particle

(ϵ_μ is the polarization vector, k_μ is the gauge boson momentum)

$$A_\mu = \epsilon_\mu \exp\left(\mathrm{i}k_\mu x^\mu\right)$$

$$\epsilon_\mu\epsilon^\mu = -1 \text{ and } k^\mu\epsilon_\mu = 0$$

$$k^\mu = (E, 0, 0, k), \text{ with } k_\mu k^\mu = E^2 - k^2 = M^2$$

two transverse polarizations	one longitudinal polarization
$\epsilon_1^\mu = (0,1,0,0)$ $\epsilon_2^\mu = (0,0,1,0)$	$\epsilon_\perp^\mu = \left(\dfrac{k}{M}, 0, 0, \dfrac{E}{M}\right) \approx \dfrac{k^\mu}{M} + \mathcal{O}\left(\dfrac{E}{M}\right)$

We note that in the R_ξ gauge, the time-like polarization, $\epsilon^\mu\epsilon_\mu = 1$, $k^\mu\epsilon_\mu = M$, is arbitrarily heavy and decouples. With the longitudinal polarization being proportional to the energy, when we look at the scattering of these longitudinal polarizations, we generically end up[3] with a tree-level amplitude that grows as E^4 (E being the energy in the center-of-mass frame),

$$\mathcal{A} = \mathcal{A}^{(4)}\left(\frac{E}{M}\right)^4 + \mathcal{A}^{(2)}\left(\frac{E}{M}\right)^2 + \ldots\,, \tag{21}$$

and that becomes bigger than unity at the energy scale of the order of the gauge boson mass (Fig. 2). Thus, in the absence of any additional fundamental degrees of freedom, the theory enters a strongly coupled regime (with a restoration of unitarity by higher loop effects and/or by the exchange of bound states) and we lose perturbative control of the theory (this is referred to as the *perturbative unitarity* violation). This is precisely where the Higgs boson is useful: it gives an additional contribution to the scattering amplitude that exactly cancels the previously energy-growing amplitude. Thus, we are left with the amplitude $\mathcal{A} = g^2 M_\mathrm{H}^2/(4M_\mathrm{W}^2)$ that remains finite at arbitrarily high energy. This finite amplitude still has to be small enough to maintain the perturbative unitarity: that is, the Higgs boson must be light enough, because if it is too heavy, the scattering amplitude is already too big when the Higgs boson becomes efficient in the cancellation of the growing amplitude. To give a quantitative estimate of the unitarity limit on the Higgs mass, we need to decompose the scattering amplitude into partial waves [13]:

$$\mathcal{A} = 16\pi\sum_{l=0}^{\infty}(2l+1)\,P_l(\cos\theta)\,a_l\,, \tag{22}$$

[3] Actually, if the self-interactions of the gauge bosons are generated by the gauge invariant kinetic term $F_{\mu\nu}F^{\mu\nu}$, the quartic and cubic vertices are related to each other such that the E^4 terms in the scattering amplitude cancel and we are left with a scattering amplitude that only grows as E^2. This residual E^2 amplitude can only be canceled by the exchange of a scalar degree of freedom associated with a Higgs mechanism [12].

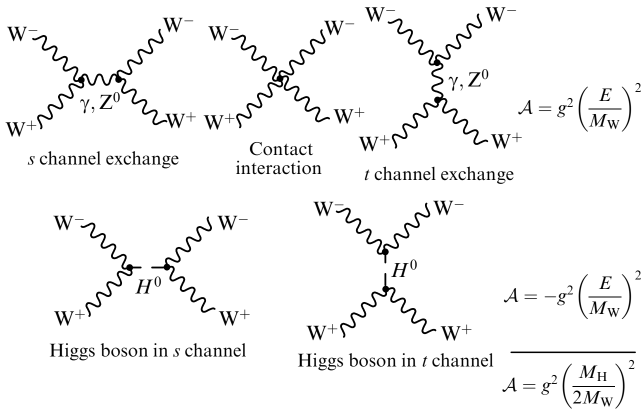


Figure 2. Scattering of the longitudinal components of the W boson in the Standard Model. The contact interaction and the exchange of other spin-1 particles make a contribution to the scattering amplitude that grows as E^2 (the expected E^4 term is canceled because of the relation between the quartic and cubic gauge boson self-interactions). The exchange of the physical Higgs boson also makes an E^2 contribution to the W scattering amplitude. In the Higgs mechanism, the masses of the gauge bosons are related to their couplings to the Higgs boson, which as a consequence ensures that the E^2 contributions exactly cancel. The residual amplitude remains finite at arbitrarily large energy. The Higgs boson unitarizes the W scattering, as long as its mass is small enough. The conventions for the Feynman diagrams are as follows: a wiggly line denotes a spin-1 field, a plain line denotes a spin-1/2 field, and a dashed line denotes a spin-0 field.

where P_l are the Legendre polynomials ($P_0(x) = 1$, $P_1(x) = x$, $P_2(x) = 3x^2/2 - 1/2, \ldots$). Using the orthonormality relation satisfied by the Legendre polynomials, we obtain the partial wave amplitudes

$$a_l = \frac{1}{32\pi}\int_{-1}^{1} \mathrm{d}(\cos\theta)\, P_l(\cos\theta)\mathcal{A}\,. \tag{23}$$

The differential cross section is related to the scattering amplitude $\mathcal{A}$ by

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{64\pi^2 s}\,|\mathcal{A}|^2\,, \tag{24}$$

which finally leads to

$$\sigma = \frac{16}{s}\sum_{l=0}^{\infty}(2l+1)|a_l|^2\,. \tag{25}$$

The optical theorem

$$\sigma = \frac{1}{s}\,\mathrm{Im}\left(\mathcal{A}(\theta = 0)\right) \tag{26}$$

simply amounts to

$$\mathrm{Im}\,a_l = |a_l|^2\,, \tag{27}$$

which can be rewritten as

$$(\mathrm{Im}\,a_l - 1/2)^2 + (\mathrm{Re}\,a_l)^2 = 1/4\,. \tag{28}$$

In the complex plane, a_l is thus located on the circle of radius $1/2$ centered at $(0, 1/2)$. Therefore,

$$|\mathrm{Re}\,a_l| \leqslant 1/2\,. \tag{29}$$

For the SM without a Higgs boson,

$$a_0 = \frac{g^2 E^2}{16\pi\, M_W^2}\,, \tag{30}$$

which shows that perturbative unitarity cannot be maintained without a Higgs boson above ~ 620 GeV (i.e., $\sqrt{s} \sim 1.2$ TeV). With a Higgs boson, we obtain

$$a_0 = \frac{g^2 M_H^2}{64\pi\, M_W^2}\,, \tag{31}$$

which leads to the upper limit for the Higgs mass:

$$M_H \leqslant 1.2\ \text{TeV}\,. \tag{32}$$

Actually, considering the channel

$$2W^+W^- + ZZ \to 2W^+W^- + ZZ$$

yields the more stringent limit [13],

$$M_H \leqslant 780\ \text{GeV}\,. \tag{33}$$

In any case, these limits give an order-of-magnitude estimate and should not be regarded as strict limits because nonperturbative effects do not turn on abruptly anyway. One should rather understand that the numerical values obtained here are the *raison d'être* of the LHC: we know that around 1 TeV, the dynamics of EWSB will be revealed.

2.2.2 Triviality limit. The other standard limits on the Higgs mass come from the study of radiative corrections to the Higgs potential. At the quantum level, the coefficients of the Higgs potential

$$V(h) = -\frac{1}{2}\,\mu^2 h^2 + \frac{1}{4}\,\lambda h^4 \tag{34}$$

change with the energy. At the one-loop level, the renormalization group equation for the Higgs quartic coupling is given by (Fig. 3)

$$16\pi^2\,\frac{\mathrm{d}\lambda}{\mathrm{d}\ln Q} = 24\lambda^2 - (3g'^2 + 9g^2 - 12y_t^2)\,\lambda + \frac{3}{8}\,g'^4 + \frac{3}{4}\,g'^2 g^2 + \frac{9}{8}\,g^4 - 6y_t^4 + \ldots\,(\text{smaller Yukawa})\,. \tag{35}$$

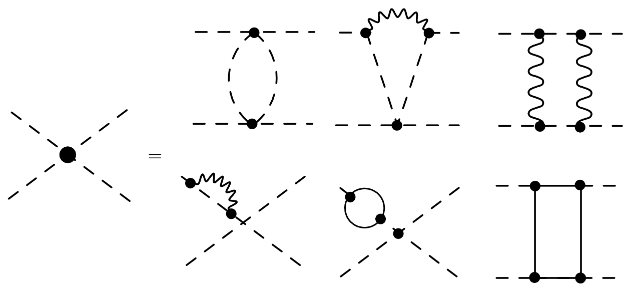


Figure 3. One-loop corrections to the Higgs quartic coupling.

In the limit of a large Higgs mass, the first term in the right-hand side dominates and makes the Higgs mass increasing with Q, finally leading to an instability that can be seen as follows. The solution of the λ-dominated renormalization group equation (RGE) is ($M_{\rm H}$ and v are the Higgs mass and the Higgs *vev* at the weak scale)

$$\lambda(Q) = \frac{M_{\rm H}^2}{2v^2 - (3/2\pi^2)\, M_{\rm H}^2 \ln (Q/v)}\,, \tag{36}$$

which exhibits a Landau pole at

$$Q = v \exp \frac{4\pi^2 v^2}{3M_{\rm H}^2}\,. \tag{37}$$

A new physics should appear before that point to prevent the instability from developing. We thus obtain an upper limit for the cutoff scale of the SM [14]:

$$\Lambda \leqslant v \exp \frac{4\pi^2 v^2}{3M_{\rm H}^2}\,. \tag{38}$$

For a fixed value of the SM cutoff, this relation gives an upper limit on the Higgs mass (see Fig. 5). In particular, we cannot take $\Lambda \to \infty$, since in this microscopic limit we necessarily have $\lambda = 0$ (trivial theory) and therefore no EWSB can occur.

2.2.3 Stability limit. In the previous section, we considered the large Higgs mass limit. We now consider the small Higgs mass limit. In that limit, the quartic coupling RGE is dominated by the top Yukawa coupling, which makes the Higgs mass decreasing with Q and another instability develops. To obtain the energy dependence of λ, we further need the RG evolution of the top Yukawa coupling. At one loop (Fig. 4), we have

$$16\pi^2 \frac{{\rm d}y_{\rm t}}{{\rm d}\ln Q} = \frac{9}{2}\, y_{\rm t}^3 + \ldots \text{(smaller Yukawa)}\,. \tag{39}$$

We now obtain

$$y^2(Q) = \frac{y_0^2}{1 - (9/16\pi^2)\, y_0^2 \ln (Q/Q_0)}\,, \tag{40}$$

$$\lambda(Q) = \lambda_0 - \frac{(3/8\pi^2)\, y_0^4 \ln (Q/Q_0)}{1 - (9/16\pi^2)\, y_0^2 \ln (Q/Q_0)}\,. \tag{41}$$

For large energy, the Higgs quartic coupling is driven to negative values and the Higgs potential is unbounded from below. Again, new physics is needed before the energy at which $\lambda = 0$ ($M_{\rm H}$ and $y_{\rm t}$ are the Higgs mass and the top Yukawa coupling at the weak scale):

$$\Lambda \leqslant v \exp \frac{4\pi^2 M_{\rm H}^2}{3y_{\rm t}^4 v^2}\,. \tag{42}$$

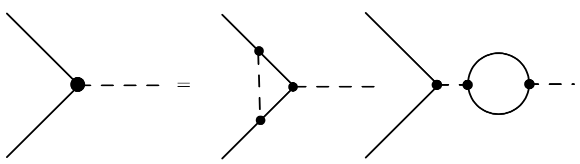


Figure 4. One-loop corrections to the top Yukawa coupling.

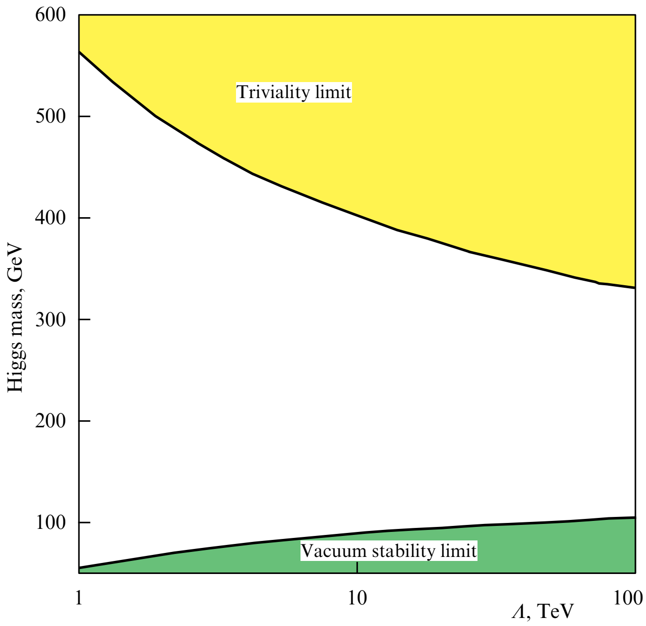


Figure 5. Triviality and stability limits on the physical Higgs mass as a function of the SM cutoff Λ. (From Ref. [16].)

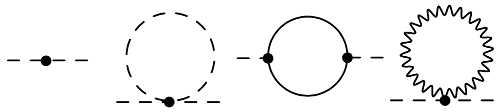


Figure 6. One-loop corrections to the Higgs mass. The three diagrams are quadratically divergent and make the Higgs mass highly UV sensitive.

For a fixed value of the SM cutoff, this relation gives a lower limit on the Higgs mass (Fig. 5). This is the vacuum stability limit [15].

The two instabilities of the Higgs quartic coupling that we just mentioned can be fixed, e.g., if we find a symmetry such that λ is related to the gauge coupling, e.g., $\lambda = g^2$, then λ would automatically inherit the good UV asymptotically free behavior of the gauge coupling. Such a relation holds in supersymmetric theories and also in 6D gauge – Higgs unification models, where the Higgs boson is identified as a component of the gauge field along some extra dimensions (see Section 4).

2.2.4 Quadratic divergence and the hierarchy problem. So far, we have looked only at the running Higgs quartic coupling, a dimensionless parameter. The radiative corrections are actually more severe for the (tachyonic) mass term of the Higgs potential because this term turns out to be highly dependent on UV physics, which leads to the so-called hierarchy problem [17]. The one-loop corrections to $M_{\rm H}$ are shown in Fig. 6, and we obtain

$$\delta M_{\rm H}^2 = \left(\frac{1}{4}\,(9g^2 + 3g'^2) - 6y_{\rm t}^2 + 6\lambda\right) \frac{\Lambda^2}{32\pi^2}\,. \tag{43}$$

As an example, for a 10 TeV cutoff, the gauge, top, and Higgs contributions to the Higgs mass square corrections are, respectively, of the order of $(600~{\rm GeV})^2$, $-(1.5~{\rm TeV})^2$, and $(800~{\rm GeV})^2$, all quite far from what the Higgs mass should be. The SM particles give unnaturally large corrections to the

Higgs mass: they destabilize the Higgs *vev* and tend to push it towards the UV cutoff of the SM. Some precise adjustment (fine-tuning) between the bare mass and the one-loop correction is needed to maintain the *vev* of the Higgs around the weak scale: if we take two large numbers, their sum/difference is of the same order unless these numbers are almost equal up to several significant digits (see Ref. [18] for a recent estimate of the amount of fine-tuning within the SM and various BSM models).

The hierarchy problem is a generic technical problem in any theory involving some light scalar fields.

In the study of any theory beyond the Standard Model, one needs to be able to quickly estimate the quadratically divergent corrections to the scalar potentials. One can explicitly calculate some Feynman diagrams or, more conveniently, rely on the computation of the Coleman–Weinberg potential [19]. At the one-loop level, this effective potential for a scalar field ϕ is given by

$$V(\phi) = \int \frac{\mathrm{d}^4 k_E}{2(2\pi)^4}\,\mathrm{STr}\,\ln\left(k_E^2 + M^2(\phi)\right), \tag{44}$$

where the supertrace, i.e., the trace with an extra minus sign for the fermionic degrees of freedom, is taken over all the particles that acquire a mass when ϕ is away from the origin. After integrating over $\mathrm{d}^4 k_E$, we obtain

$$V = -\frac{\Lambda^4}{128\pi^2}\,\mathrm{STr}\,1 + \frac{\Lambda^2}{64\pi^2}\,\mathrm{STr}\,M^2(\phi) + \frac{1}{64\pi^2}\,\mathrm{STr}\,M^4(\phi)\,\ln\frac{M^2(\phi)}{\Lambda^2}\,,$$

where we easily identify the quadratically divergent corrections to the scalar potential.

We explicitly consider the case of the Higgs boson potential in the SM. We only need to know the H-dependent masses of the different particles.

***H*-dependent masses**

particles	number of polarizations	off-shell mass
$W^\pm$	3×2	$M_W^2 = \frac{1}{4}g^2H^2$
Z^0	3	$M_Z^2 = \frac{1}{4}(g^2+g'^2)H^2$
top	4×3	$M_t^2 = \frac{1}{2}y_t^2H^2$
Higgs	1	$M_H^2 = \lambda(3H^2 - v^2)$
Goldstone	3	$M_G^2 = \lambda(H^2 - v^2)$

We note that these masses are computed for a generic value of H: in particular, away from the true vacuum $H = v$, the Goldstone bosons are not massless and contribute to the Coleman–Weinberg potential [20]. Summing over all the particles, we obtain the sought quadratically divergent correction

$$V_{\Lambda^2} = \frac{1}{2}\left(\frac{1}{4}(9g^2 + 3g'^2) - 6y_t^2 + 6\lambda\right)\frac{\Lambda^2}{32\pi^2}\,H^2\,, \tag{45}$$

in agreement with the diagram computation.

3. New physics and EWSB

3.1 Stabilization of the Higgs potential by symmetries

We have learnt in the previous sections that the description of EWSB with a Higgs boson suffers from several instabilities at the quantum level. Extra structures (particles and/or symmetries) are needed to stabilize the Higgs potential. To keep radiative corrections under control, a theorist can use two tools:

• The **spin trick** [21]: in general, a particle of spin s has $2s+1$ degrees of polarization with the only exception being a particle moving at the speed of light, in which case fewer polarizations may be physical. Conversely, if a symmetry leads to the decoupling of some polarization states, then the particle necessarily propagates at the speed of light and thus remains massless. For instance, gauge invariance ensures that the longitudinal polarization of a vector field is nonphysical and chiral symmetry keeps only one fermion chirality: both spin-1 and spin-1/2 particles are protected from dangerous radiative corrections. Unfortunately, this spin trick cannot be used for a spin-0 particle like the SM Higgs scalar boson.

• The **Goldstone theorem** [22]: when a global symmetry is spontaneously broken, the spectrum contains a *massless* spin-0 particle. However, here again, it seems difficult to invoke this trick to protect the SM Higgs boson from radiative corrections since a Nambu–Goldstone boson can only have some derivative couplings, unlike the Higgs field. Little Higgs models have been constructed to circumvent these difficulties; they provide realistic examples of the Higgs boson as a (pseudo) Nambu–Goldstone boson. A short account of these models is given below in Section 3.2.

In the late 1960s, the Coleman–Mandula and Haag–Lopuszanski–Sohnius theorems [23] taught us how to apply the spin trick to spin-0 particles: the four-dimensional Poincaré symmetry has to be enlarged. The first construction of this type consists in embedding the 4D Poincaré algebra into a superalgebra. Then the supersymmetry between fermions and bosons extends the spin trick to scalar particles. Actually, there exists an even simpler way to enlarge the Poincaré symmetry by invoking extra dimensions: the 5D Poincaré algebra obviously contains the 4D Poincaré algebra as a subalgebra. After compactification of the extra dimensions, from the 4D standpoint, the higher-dimensional gauge field decomposes into a 4D gauge field (the components along our 4D world) and 4D scalar fields (the components along the extra dimensions). The symmetry between vectors and scalars allows extending the spin trick to spin-0 particles.

Neither supersymmetry nor higher-dimensional Poincaré symmetry are exact symmetries of nature. Therefore, if they ever have a role to play, they have to be broken. In order not to lose any of their benefits, this breaking has to proceed without reintroducing any strong UV dependence into the renormalized scalar mass square: we need a *soft breaking*. This question has been well studied in supersymmetric theories, and we discuss the soft breaking of higher-dimensional gauge theories in Section 4. In Section 5, we briefly report on Higgsless models [24, 25], where the EWSB is no longer achieved through a Higgs mechanism but results from nontrivial boundary conditions for the gauge fields at the boundaries of a fifth dimension.

3.2 Little Higgs models

By analogy with the pions of QCD, the lightness of the Higgs boson could be explained if it were a Nambu – Goldstone boson (NGB) corresponding to the spontaneously broken global symmetry of the new strongly interacting sector. This is not a new idea *per se*, but a new ingredient, the notion of *collective breaking*, has been added in recent years to construct realistic models allowing large nonderivative couplings that at the same time are still free of quadratic divergences at one loop. The idea is that some interaction terms are introduced to break the global symmetries from which the Nambu – Goldstone boson originates, but two or more such interactions should be turned on simultaneously for the tentative NGB to acquire mass. The one-loop radiative corrections should then involve two symmetry breaking interactions to generate a mass term. The absence of quadratic divergences now follows from the fact that there are no quadratically divergent diagrams involving two symmetry breaking couplings: therefore, the corrections to the NGB Higgs mass are only logarithmic. This way, we can obtain a light composite Higgs boson compatible with a strong coupling scale around 10 TeV.

Diagrammatically, the cancellation of the quadratic divergences is due to a set of new TeV-scale particles: gauge bosons, vector-like quarks, and extra massive scalars, which are related to SM particles by the original global symmetry. Noteworthy and contrary to supersymmetry, the cancellation of the divergences is achieved by particles of the same spin. These new particles of around one TeV, with definite couplings to ordinary particles as dictated by the global symmetries of the theory, are perfect goals for the LHC.

Very good reviews are already available on Little Higgs models [2, 6], and the reader is referred to them for further details. We just mention that the compatibility of Little Higgs models with experimental data is significantly improved when the global symmetry of the models involves a custodial symmetry as well as a T-parity under which, by analogy with the R-parity in SUSY models, the SM particles are even and their partners are odd.

3.3 EW precision tests

We have seen that we need new particles to stabilize the weak scale. They have to be massive to evade direct searches. They still influence SM physics and can be ‘detected’ through precision measurements.

3.3.1 An example of EW corrections induced by a heavy particle. As an example, we take an extra heavy B$'$ gauge boson. The full Lagrangian is

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}\, W_3(p^2 - M_{\mathrm{W}}^2) W_3 - t_0 M_{\mathrm{W}}^2 W_3 B \\ &-\frac{1}{2}\, B(p^2 - t_0^2 M_{\mathrm{W}}^2) B + g J_3 W_3 + g' J_y B \\ &-\frac{1}{2}\, B'(p^2 - M^2) B' + g' J_y B' \,, \end{aligned} \tag{46}$$

where $t_0 = g'/g$ (in what follows, we also use $c_0 = g/\sqrt{g^2+g'^2}$ and $s_0 = g'/\sqrt{g^2+g'^2}$), J_y and J_3 are the usual fermion currents coupled to B_μ and $W_{3\mu}$, $J_y^\mu = \sum_i y_i \bar{f}_i \bar{\sigma}^\mu f_i$, and $J_3^\mu = \sum_i T_{3Li} \bar{f}_i \bar{\sigma}^\mu f_i$. We now integrate out the heavy particle, which means that we freeze its dynamics and replace B' by the solution of its equation of motion:

$$\frac{\partial \mathcal{L}}{\partial B'} = 0 \;\Leftrightarrow\; B' = \frac{g' J_y}{p^2 - M^2}\,. \tag{47}$$

Substituting this expression in the original Lagrangian and expanding for $M \gg p$, we obtain the effective Lagrangian

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}\, W_3(p^2 - M_{\mathrm{W}}^2) W_3 - t_0 M_{\mathrm{W}}^2 W_3 B \\ &-\frac{1}{2}\, B(p^2 - t_0^2 M_{\mathrm{W}}^2) B + g J_3 W_3 + g' J_y B - \frac{(g' J_y)^2}{2M^2}\,. \end{aligned} \tag{48}$$

Using the equation of motion for B, $g' J_y = t_0 M_{\mathrm{W}}^2 W_3 + (p^2 - t_0^2 M_{\mathrm{W}}^2) B$, we can actually write the four-fermion interaction as a modification of the propagator of the gauge bosons:

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}\, W_3 \left(p^2 - M_{\mathrm{W}}^2 \left(1 - \frac{t_0^2 M_{\mathrm{W}}^2}{M^2}\right)\right) W_3 \\ &- t_0 M_{\mathrm{W}}^2 \left(1 + \frac{p^2 - t_0^2 M_{\mathrm{W}}^2}{M^2}\right) W_3 B \\ &-\frac{1}{2}\, B \left(p^2 \left(1 - 2\,\frac{t_0^2 M_{\mathrm{W}}^2}{M^2}\right) - t_0^2 M_{\mathrm{W}}^2 \left(1 - \frac{t_0^2 M_{\mathrm{W}}^2}{M^2}\right) + \frac{p^4}{M^2}\right) B \\ &+ g J_3 W_3 + g' J_y B + \mathcal{O}(p^6) + \mathcal{O}\left(\frac{1}{M^4}\right). \end{aligned} \tag{49}$$

The mass matrix based on (W_3, B) is therefore given by

$$\left(1 - \frac{t_0^2 M_{\mathrm{W}}^2}{M^2}\right) \begin{pmatrix} M_{\mathrm{W}}^2 & -t_0 M_{\mathrm{W}}^2 \\ -t_0 M_{\mathrm{W}}^2 & t_0^2 M_{\mathrm{W}}^2 \end{pmatrix}. \tag{50}$$

We note that the determinant of this mass matrix vanishes, as it should to maintain the masslessness of the photon. Furthermore, we also note that the weak mixing angle is unaffected:

$$Z = c_0 W_3 - s_0 B\,, \qquad \gamma = s_0 W_3 + c_0 B\,. \tag{51}$$

It is essential here to ensure that the photon actually couples to the electric charge $T_{3L} + Y$. The photon remains massless, but the mass of the Z boson is modified by the existence of the heavy B' field:

$$M_Z^2 = \frac{1}{c_0^2}\, M_{\mathrm{W}}^2 \left(1 - \frac{t_0^2 M_{\mathrm{W}}^2}{M^2}\right), \qquad M_\gamma^2 = 0\,. \tag{52}$$

At low energy, we thus obtain a deviation from $\rho = 1$ because we now have

$$\rho \equiv \frac{M_{\mathrm{W}}^2}{M_Z^2 c_0^2} \approx 1 + \frac{t_0^2 M_{\mathrm{W}}^2}{M^2}\,. \tag{53}$$

The deviation from $\rho = 1$ is usually called the T parameter,

$$\rho \equiv 1 + \alpha_{\mathrm{em}} T\,. \tag{54}$$

In our example, we obtain

$$T = \frac{t_0^2 M_{\mathrm{W}}^2}{\alpha_{\mathrm{em}} M^2}\,. \tag{55}$$

The upper limit [1] on the T parameter, $T \leqslant 0.2$, gives a lower limit for the mass of the heavy B' field, $M \geqslant 1.1$ TeV. That is a rather generic result: the limit on the new physics needed to stabilize the weak scale is at least one order of magnitude above the weak scale. This has been called the *little hierarchy problem* or *LEP paradox* [26].

In deriving effective Lagrangian (49), we used the equation of motion for B, a light degree of freedom. We must see whether this is a legitimate approach. Another way to arrive at Eqn (49) is actually to first integrate out the combination of B and B' that does not couple to the fermions,

$$B_{\text{in}} = B + B', \qquad B_{\text{out}} = B - B'. \tag{56}$$

This time, integrating out B_{out} does not generate any four-fermion interactions and the Lagrangian directly becomes that in Eqn (49) with B replaced with B_{in} (the two Lagrangians actually start to differ around $1/M^6$).

This second approach, sometimes called the 'holographic' approach, might also seem doubtful since we are integrating out a state that is not a mass eigenstate. But we can easily convince ourselves that it is perfectly legitimate by considering this even simpler case:

$$\mathcal{L} = -\frac{1}{2}\, B\Pi_{BB}B + g'J_yB - \frac{1}{2}\, B'\Pi_{B'B'}B' + g'J_yB'\,.$$

We first integrate out the heavy mass eigenstate B' and, as before, obtain an effective Lagrangian with a four-fermion interaction:

$$\mathcal{L} = -\frac{1}{2}\, B\Pi_{BB}B + g'J_yB + \frac{(g'J_y)^2}{2\Pi_{B'B'}}\,.$$

Instead of using the equation of motion for B to eliminate the four-fermion interaction, we now perform a field redefinition of B,

$$B = \sqrt{\frac{\Pi_{B'B'}}{\Pi_{B'B'} + \Pi_{BB}}}\,\tilde{B} + \left(\frac{g'}{\Pi_{BB}} + \sqrt{\frac{g'(\Pi_{B'B'} + \Pi_{BB})}{\Pi_{B'B'}\Pi_{BB}^2}}\right) J_y\,, \tag{57}$$

to obtain the simple effective Lagrangian

$$\mathcal{L} = -\frac{1}{2}\,\tilde{B}(\Pi_{BB}^{-1} + \Pi_{B'B'}^{-1})^{-1}\tilde{B} + g'J_y\tilde{B}\,.$$

Remarkably enough, this is exactly the result we would have obtained more simply by directly integrating out the 'holographic' combination $B_{\text{out}} = B - B'$.

3.3.2 General structure of the EW corrections. Under mild assumptions (universality, heaviness of the new physics, flavor universality, CP invariance), we can obtain the general form of the corrections induced by the new physics [27–29] (see [30] for recent reviews). The most general $\mathrm{U}(1)_{\text{em}}$-invariant quadratic Lagrangian for the SM gauge bosons

$$\mathcal{L} = -\frac{1}{2}\, W_3^\mu\, \Pi_{33}(p^2)\, W_{3\mu} - W_3^\mu\, \Pi_{3B}(p^2)\, B_\mu - \frac{1}{2}\, B^\mu\, \Pi_{BB}(p^2)\, B_\mu - W_+^\mu\, \Pi_{+-}(p^2)\, W_{-\mu} \tag{58}$$

involves four vacuum polarizations, which we expand in powers of momentum

$$\Pi_V(p^2) = \Pi_V(0) + p^2\Pi_V'(0) + \frac{1}{2}(p^2)^2\Pi_V''(0) + \mathcal{O}(p^6)\,. \tag{59}$$

Therefore, 12 coefficients should describe the most general low-energy effective Lagrangian. But three of them can actually be removed by normalizing the gauge bosons (which corresponds to the identification of the three SM parameters g, g', and v),

$$\Pi'_{+-}(0) = \Pi'_{BB}(0) = 1\,, \qquad \Pi_{+-}(0) = -M_{\mathrm{W}}^2 = -(80.425\ \text{GeV})^2. \tag{60}$$

The remaining 9 parameters are not yet fully independent because we need to impose the masslessness of the photon and its coupling to $Q = T_{3L} + Y$. These two consistency constraints are explicitly given by

$$g'^2\Pi_{33} + g^2\Pi_{00} + 2gg'\Pi_{30} = 0$$

and

$$g\Pi_{00} + g'\Pi_{30} = 0\,.$$

We are thus left with a total of 7 arbitrary coefficients [29]. They are given in Table 2 along with the dimension-six operators that generate them. For instance, in the example of a heavy B' field discussed in the previous section, we obtain

$$t_0^2\widehat{S} = \widehat{T} = t_0^2 Y = \frac{t_0^2 M_{\mathrm{W}}^2}{M^2}\,, \qquad \widehat{U} = V = X = W = 0\,. \tag{61}$$

In universal models, i.e., when the new physics couples to the SM fermions only through the combinations that appear in SM fermionic currents

$$J_y^\mu = \sum_i y_i \bar{f}_i\,\bar{\sigma}^\mu f_i\,, \qquad J_a^\mu = \sum_d \bar{f}_d\,\bar{\sigma}^\mu\sigma^a f_d\,, \tag{62}$$

all the corrections induced by heavy particles are encoded in the oblique parameters; for nonuniversal models, more effects might appear as vertex corrections (see Ref. [31] for the details). In universal models, four oblique parameters are dominant over the other ones, which can also be understood from the fact that $\hat{U}$, V, and X are not generated by dimension-six operators, and we generically expect them to be further suppressed compared with the other four coefficients: $\hat{U} \sim (M_{\mathrm{W}}^2/\Lambda^2)\hat{T}$, $V \sim (M_{\mathrm{W}}^4/\Lambda^4)\hat{T}$, and $X \sim (M_{\mathrm{W}}^2/\Lambda^2)\hat{S}$. In nonuniversal models, this is no longer true [31], and $\hat{U}$, V, and X can be of the same order as $\hat{S}$, $\hat{T}$, W, and Y.

Electroweak precise measurements can be analyzed using the parameterization just described and the results of the fits (assuming the existence of a light/heavy Higgs boson) are [29] (see also [33] for another recent analysis of EW precision measurements).

Fit	$10^3\widehat{S}$	$10^3\widehat{T}$	$10^3 Y$	$10^3 W$
115 GeV Higgs	0.0 ± 1.3	0.1 ± 0.9	0.1 ± 1.2	-0.4 ± 0.8
800 GeV Higgs	-0.9 ± 1.3	2.0 ± 1.0	0.0 ± 1.2	-0.2 ± 0.8

Table 2. Seven coefficients parameterize the most general low-energy Lagrangian beyond the SM.

Coefficients	Dimension-6 operators	$SU(2)_c$	$SU(2)_L$
$\widehat{S} = \frac{g}{g'}\,\Pi'_{3B}(0)$	$\frac{(H^\dagger\tau^a H)W^a_{\mu\nu}B_{\mu\nu}}{gg'}$	+	−
$\widehat{T} = \frac{1}{M_W^2}\left(\Pi_{33}(0) - \Pi_{+-}(0)\right)$	$\lvert H^\dagger D_\mu H\rvert^2$	−	−
$\widehat{U} = \Pi'_{+-}(0) - \Pi'_{33}(0)$	Dim. 8	−	−
$V = \frac{M_W^2}{2}\left(\Pi''_{33}(0) - \Pi''_{+-}(0)\right)$	Dim. 10	−	−
$X = \frac{M_W^2}{2}\,\Pi''_{3B}(0)$	Dim. 8	+	−
$Y = \frac{M_W^2}{2}\,\Pi''_{BB}(0)$	$\frac{(\partial_\rho B_{\mu\nu})^2}{2g'^2}$	+	+
$W = \frac{M_W^2}{2}\,\Pi''_{33}(0)$	$\frac{(D_\rho W^a_{\mu\nu})^2}{2g^2}$	+	+

Note. $SU(2)_L \times U(1)_Y$-invariant higher-dimensional operators can give rise to these corrections. We note that these corrections have definite symmetry properties under the gauge $SU(2)_L$ and the $SU(2)_c$ custodial symmetry. The more usual S, T, and U coefficients are obtained as $S = 4s_W^2\hat{S}/\alpha_{em} \approx 119\,\hat{S}$, $T = \hat{T}/\alpha_{em} \approx 129\,\hat{T}$, and $U = -4s_W^2\hat{U}/\alpha_{em} \approx -119\,\hat{U}$. From [29] ([29] uses a noncanonical normalization of the gauge bosons, hence the different factors of g and g' appearing in the definition of $\hat{S}, \hat{T}, \ldots, W$).

3.3.3 An example of EW corrections induced by a higher-dimensional operator. As a concrete example of the analysis above, we find the effects of the higher-dimensional operator [28]

$$\mathcal{L}_T = \frac{a}{\Lambda^2}\lvert H^\dagger D_\mu H\rvert^2\,, \tag{63}$$

where a is a dimensionless coefficient. After EWSB, $\langle H\rangle = (0, v/\sqrt{2})$, the operator $\mathcal{L}_T$ simply occurs as an additional mass term for the Z^0 gauge boson:

$$\mathcal{L}_T = \frac{av^4}{8\Lambda^2}(g'B_\mu - gW^3_\mu)^2\,. \tag{64}$$

Because the W mass and the weak mixing angle remain unchanged, we easily obtain the correction to the ρ parameter induced by $\mathcal{L}_T$,

$$\rho = \frac{M_W^2}{M_Z^2 c_W^2} = 1 - \frac{av^2}{\Lambda^2}\,, \tag{65}$$

which corresponds to a nonvanishing T parameter

$$T = \frac{\rho - 1}{\alpha_{em}} = -\frac{av^2}{\alpha_{em}\Lambda^2}\,. \tag{66}$$

This is also what we could derive using the formalism in Table 2, because

$$\Pi_{+-} = -\frac{1}{4}\,g^2v^2 \quad\text{and}\quad \Pi_{33} = -\frac{1}{4}\,g^2v^2 + \frac{ag^2v^4}{4\Lambda^2}\,. \tag{67}$$

It was noted in Ref. [32] that field redefinitions amount to the following two relations among various dimension-six operators:

$$-\frac{2gscv^2}{\alpha}\,\mathcal{O}_S - \frac{g'v^2}{\alpha}\,\mathcal{O}_T + g'\sum_f \mathrm{i}y_f(h^\dagger D^\mu h)\bar{f}\bar{\sigma}^\mu f + \text{h.c.} = 2\mathrm{i}B_{\mu\nu}D^\mu h^\dagger D^\nu h\,, \tag{68}$$

$$-\frac{4g'scv^2}{\alpha}\,\mathcal{O}_S + g\sum_d \mathrm{i}(h^\dagger\sigma^a D^\mu h)\,\bar{f}_d\,\bar{\sigma}^\mu\sigma^a f_d + \text{h.c.} = 4\mathrm{i}W^a_{\mu\nu}D^\mu h^\dagger\sigma^a D^\nu h\,, \tag{69}$$

where

$$\mathcal{O}_S = \frac{\alpha}{4scv^2}(h^\dagger\sigma^a h)W^a_{\mu\nu}B^{\mu\nu}\,,\qquad \mathcal{O}_T = -\frac{2\alpha}{v^2}\,\lvert h^\dagger D_\mu h\rvert^2\,. \tag{70}$$

The peculiarity of these two relations is that the terms appearing in the right-hand side only affect the triple gauge self-interactions of the bosons, which are poorly measured compared with the other electroweak quantities. This way, it is possible to disguise oblique corrections into yet unconstrained operators.

4. Gauge – Higgs unification models

The components of the gauge fields along some extra dimensions are seen from the 4D standpoint as some 4D scalar fields (we call them gauge scalars). It is only above the compactification scale, when the extra dimensions open up, that the higher-dimensional gauge structure reveals itself. We now describe models of gauge – Higgs unification, where the Higgs boson is identified as some gauge scalar.[4] This approach is actually quite old [35, 36], but it is only recently within the context of orbifolds [37] that it has been implemented in realistic models [38 – 45]. A series of questions immediately arise:

- Which gauge group contains the Higgs boson?
- How many extra dimensions do we need? How are they compactified?
- What are the radiative corrections?
- How can matter be incorporated? How are the Yukawa couplings generated?

[4] See [34] for a recent pedagogical introductions to this approach.

It is interesting to note that the deconstruction versions of these gauge – Higgs unification models led to the idea of Little Higgs models [46]. The symmetry protecting the Higgs mass is a discrete shift symmetry and the construction is much less constrained by the absence of the 5D Lorentz invariance.

4.1 Orbifold breaking. 5D SU(3) model

Both 4D vectors and 4D scalars originating from higher-dimensional gauge fields belong to the adjoint representation of the gauge group, while the SM Higgs boson is in the fundamental representation of the weak symmetry. To identify the Higgs boson as a component of a gauge field in extra dimensions, we thus need to enlarge the $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_Y$ gauge symmetry to a bigger group G. This bigger group can be broken in different ways: (i) by introducing a higher-dimensional Higgs field; (ii) by a Green – Schwarz mechanism; (iii) by compactification on a nontrivial background manifold; or (iv) by compactification on an orbifold. This last method is not only well motivated in the string context but also offers the advantage of easily accommodating the presence of 4D chiral matter. [5]

The simplest example of an orbifold is S^2/Z_2, i.e., a circle $(-\pi R \leqslant y \leqslant \pi R)$ with a parity identification $(y \sim -y)$ (Fig. 7). The identification of the points y and $-y$ means that the values of any field evaluated at these points have to be physically equivalent, i.e., equal up to a global symmetry transformation: $\phi(x,-y) = U\phi(x,y)$. For consistency, U has to be a Z_2 symmetry, $U^2 = 1$. We note that there are two special points on the circle, 0 and πR, which are identified with themselves: they are *fixed points* of the orbifold. The invariance of the kinetic term dictates the transformation of the various components of the gauge field:

$$A_\mu(x,-y) = UA_\mu(x,y)U^{-1}\,, \qquad A_5(x,-y) = -UA_5(x,y)U^{-1}\,. \tag{71}$$

In a Kaluza – Klein (KK) decomposition, the 4D mass is related to the derivative of the field along the extra dimension, and hence a massless mode should be independent of y. From orbifold boundary conditions (71), we have the 4D massless vectors corresponding to the generators of the gauge group that *commute* with the orbifold matrix U, while the 4D massless gauge scalars correspond to the generators that *anticommute* with U. We consider the example of an SU(3) gauge group broken by the orbifold projection $U = \mathrm{diag}\,(-1,-1,1)$ down to $\mathrm{SU}(2)\times\mathrm{U}(1)$: from the eight gauge components of A_5, only an SU(2) scalar doublet remains massless.

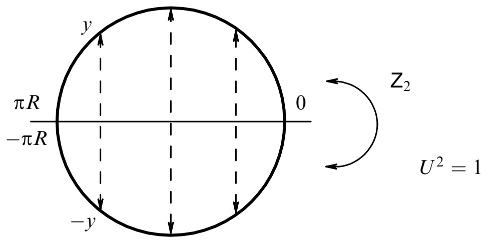


Figure 7. The simplest example of an orbifold: the points $-y$ and y on the circle are identified. The fields at the identified points have to be equal up to a global Z_2 symmetry of the theory; for instance, the gauge field components satisfy $A_\mu(x,-y) = UA_\mu(x,y)U^{-1}$ and $A_5(x,-y) = -UA_5(x,y)U^{-1}$, where U, a global symmetry of the theory, is the orbifold projection. The zero modes of A_μ correspond to the gauge directions that commute with the orbifold projection, $[A_\mu,\ U] = 0$, while the zero modes of A_5 anticommute with U, $\{A_5,\ U\} = 0$.

$$\mathbf{SU(3) \to SU(2)\times U(1)}$$

$$U = \mathrm{diag}\,(-1,-1,1)$$

$$[A_\mu, U] = 0 \quad A_\mu = \frac{1}{2}\begin{pmatrix} A_\mu^3 + \dfrac{A_\mu^8}{\sqrt{3}} & A_\mu^1 - \mathrm{i}A_\mu^2 & \\ A_\mu^1 + \mathrm{i}A_\mu^2 & -A_\mu^3 + \dfrac{A_\mu^8}{\sqrt{3}} & \\ & & -\dfrac{2A_\mu^8}{\sqrt{3}} \end{pmatrix} \quad \mathrm{SU}(2)\times\mathrm{U}(1)$$

$$\{A_5, U\} = 0 \quad A_5 = \frac{1}{2}\begin{pmatrix} & & A_5^4 - \mathrm{i}A_5^5 \\ & & A_5^6 - \mathrm{i}A_5^7 \\ A_5^4 + \mathrm{i}A_5^5 & A_5^6 + \mathrm{i}A_5^7 & \end{pmatrix} \quad \frac{\mathrm{SU}(3)}{\mathrm{SU}(2)\times\mathrm{U}(1)}$$

It is tempting to identify the massless SU(2) doublet contained in A_5 as the Higgs doublet: $H_0 = (A_5^6 - \mathrm{i}A_5^7)/2$ and $H_+ = (A_5^4 - \mathrm{i}A_5^5)/2$. For that, we need to know its U(1) charge. Under any transformation of SU(3), A_5 transforms as $\delta_T A_5 = g[T, A_5]$. In particular, under U(1) in $\mathrm{SU}(2)\times\mathrm{U}(1)$, with $T = T_8 = \mathrm{diag}\,(1,1,-2)/(2\sqrt{3}\,)$, we obtain

$$\delta_T \begin{pmatrix} & & H_+ \\ & & H_0 \\ H_+^* & H_0^* & \end{pmatrix} = g\,\frac{3}{2\sqrt{3}}\begin{pmatrix} & & H_+ \\ & & H_0 \\ -H_+^* & -H_0^* & \end{pmatrix}. \tag{72}$$

Therefore, the U(1) charge of the doublet is equal to $\sqrt{3}/2$. We need to change the normalization of U(1) for the charge of the doublet to be 1/2; this is achieved by setting $\mathrm{U}(1)_Y = T_8/\sqrt{3}$. The gauge coupling of $\mathrm{U}(1)_Y$ is thus $g' = \sqrt{3}g$. Because we embedded $\mathrm{SU}(2)_\mathrm{L}\times\mathrm{U}(1)_Y$ in a simple group, we have a prediction for the weak mixing angle,

$$\sin^2\theta_\mathrm{W} = \frac{g'^2}{g^2 + g'^2} = \frac{3g^2}{g^2 + 3g^2} = \frac{3}{4}\,. \tag{73}$$

This value is quite far from the experimental one $(\sin^2\theta_\mathrm{W} \approx 0.23)$, which certainly invalidates this simple SU(3) gauge – Higgs unification model. Furthermore, with this embedding of $\mathrm{SU}(2)_\mathrm{L}\times\mathrm{U}(1)_Y$ in SU(3), there is no way to obtain quarks and leptons from irreducible SU(3) representations.

At this point, we can envision at least two ways to proceed: (i) add another U(1) factor to SU(3); (ii) examine other embeddings of $\mathrm{SU}(2)_\mathrm{L}\times\mathrm{U}(1)_Y$ into simple groups. Although the former rules out one nice aspect of the gauge – Higgs unification models, the prediction of the weak mixing angle, recent developments seem to indicate that it is the right direction to follow, while, as we see in what follows, a radiative instability spoils the most promising models of the second class. Finally, a third way to go is to modify the geometry of the extra-dimensional space.

[5] See Section 5.5.1 for an explanation on how chiral matter is obtained in orbifold models.

Before proceeding with the construction of gauge – Higgs unification models, we note that the orbifold projection can be reinterpreted as simple boundary conditions on an interval.

G → H orbifold breaking

H subgroup

$A^H_\mu(-y) = A^H_\mu(y)$ $\partial_5 A^H_\mu|_{y=0,\pi R} = 0$

equivalent to

$A^H_5(-y) = -A^H_5(y)$ $A^H_5|_{y=0,\pi R} = 0$

G/H coset

$A^{G/H}_\mu(-y) = -A^{G/H}_\mu(y)$ $A^{G/H}_\mu|_{y=0,\pi R} = 0$

equivalent to

$A^{G/H}_5(-y) = A^{G/H}_5(y)$ $\partial_5 A^{G/H}_5|_{y=0,\pi R} = 0$

It is also possible to accommodate a Scherk – Schwarz twist, i.e., $\phi(y + 2\pi R) = T\phi(y)$. The twist manifests itself by different boundary conditions at the ends of the interval.

We also mention that orbifold breaking has been applied to break Grand Unified (GU) symmetries (see [47] for a review). In that case, the compactification is close to the GUT scale, while in gauge – Higgs models, it is of the order of the weak scale.

4.2 6D G_2 model

We restrict our analysis to an Abelian orbifold using inner automorphism (the orbifold matrix U is an element of the group itself). It is well known that the rank of the gauge group is then preserved. Because SU(2) × U(1) is of rank two, we need to look for a simple group of rank two. There are only four possibilities: SO(4), SO(5), SU(3), and the exceptional group G_2 (for an explicit matrix realization of G_2 that exhibits its SU(3) subgroup, see [40]). The first three cases either do not provide an SU(2) scalar doublet or lead to a prediction of the weak mixing angle too far from the experimental value.

The most interesting possibility relies on G_2 [35, 40], which can be broken down to SU(2) × U(1) by compactification on a two-dimensional orbifold, $T^2/\mathbb{Z}_4$ (Fig. 8), T^2 being a square torus and $\mathbb{Z}_4$ a rotation by 90° (there is no way to break G_2 to SU(2) × U(1) just using a $\mathbb{Z}_2$ projection: a projection of order at least four is required, which can only be implemented in the presence of at least two extra dimensions). There are again two *fixed points* on the torus left invariant by the rotation. The action of the orbifold on the fundamental representation of G_2 is defined by the matrix $U = \mathrm{diag}\,(\mathrm{i}, \mathrm{i}, -1, -\mathrm{i}, -\mathrm{i}, -1, 1)$. The low-energy gauge group is found to be SU(2) × U(1) and the gauge couplings are such that the weak mixing angle satisfies $\sin^2\theta_W = 1/4$. The massless spectrum also contains two SU(2) scalar doublets, h and H, carrying the respective hypercharges 1/2 and 3/2; h is thus a perfect candidate to be identified with the SM Higgs doublet.

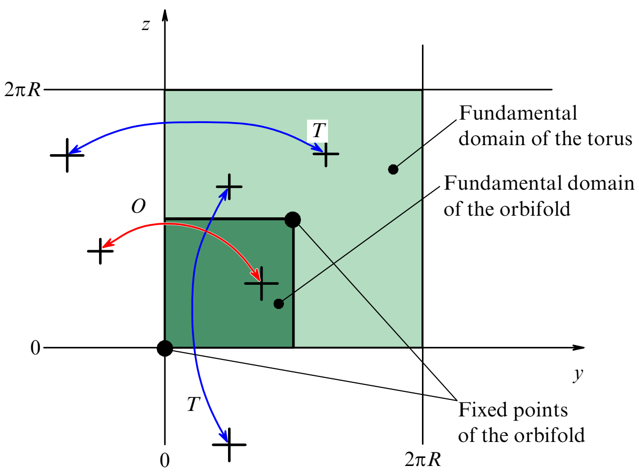


Figure 8. $T^2/\mathbb{Z}_4$ orbifold: besides the torus identification T, $y \sim y + 2\pi R$ and $z \sim z + 2\pi R$, the orbifold projection O identifies the points $(y, z) \sim (-z, y)$ up to a global $\mathbb{Z}_4$ symmetry U: $A_\mu(-z, y) = UA_\mu(y, z)U^\dagger$, $A_y(-z, y) = -UA_z(y, z)U^\dagger$, and $A_z(-z, y) = UA_y(y, z)U^\dagger$. With $U = \mathrm{diag}\,(\mathrm{i}, \mathrm{i}, -1, -\mathrm{i}, -\mathrm{i}, -1, 1)$, G_2 is broken down to SU(2) × U(1).

$G_2 \to \mathrm{SU}(2) \times \mathrm{U}(1)$

G_2 gauge group

adjoint = 14 fundamental = 7

SU(3) decomposition

$14 = 8 + 3 + \bar{3}$ $7 = 3 + \bar{3} + 1$

SU(2) × U(1) decomposition

T_8 normalization

$$14 = \left(3_0 + (2 + \bar{2})_{\sqrt{3}/2} + 1_0\right) + \left(2 + \bar{2}\right)_{1/2\sqrt{3}} + \left(1 + \bar{1}\right)_{-1/\sqrt{3}}$$

$$7 = \left(2 + \bar{2}\right)_{1/2\sqrt{3}} + \left(1 + \bar{1}\right)_{-1/\sqrt{3}} + 1_0$$

weak mixing angle

$$\mathrm{U}(1)_Y = \sqrt{3}\,T_8 \Leftrightarrow \sin^2\theta_W = \frac{1}{4}$$

Another benefit of having two extra dimensions is that the non-Abelian interaction piece contained in the gauge kinetic term automatically generates a quartic coupling. In our model after compactification, the potential for the canonically normalized scalars is given by

$$V = \frac{1}{6}\,g^2\left(|h|^4 + 3|H|^4 + 3(H^\dagger\sigma^a h)(h^\dagger\sigma^a H) - 6|h|^2|H|^2\right), \tag{74}$$

where σ^a, $a = 1, 2, 3$, are the Pauli matrices and g is the gauge coupling of the low-energy SU(2) gauge group in 4D. As in supersymmetry, we find that the Higgs quartic coupling is related to the square of the gauge coupling, although the details of the potential are different (in particular, there is a single doublet of hypercharge 1/2 in the present model).

4.3 Radiative corrections

There are two types of operators involving the 4D gauge scalar fields that can be generated radiatively [48]:

(i) some bulk operators,

(ii) some operators localized at *fixed points* of the orbifold.

The higher-dimensional gauge invariance acts on gauge bosons and gauge scalars as

$$\delta A^A_M = \partial_M \epsilon^A + g f^{ABC} A^B_M \epsilon^C, \tag{75}$$

where f^{ABC} are the structure constants. For consistency, the gauge transformation parameter ϵ^C satisfies the same boundary conditions as A_μ:

$$G \to H: \quad \partial_5 \epsilon^{H}\big|_{\text{fixed point}} = 0\,, \qquad \epsilon^{G/H}\big|_{\text{fixed point}} = 0\,. \tag{76}$$

Therefore, it is in fact gauge invariance that protects the first kind of operators: indeed, above the compactification scale, the gauge-scalar fields actually occur as some components of the higher-dimensional gauge field and the Slavnov – Taylor identities, e.g., forbid the appearance of any mass term. Below the compactification scale, however, we have to deal with ordinary scalars, which acquire some radiative but finite (because of a cutoff at the compactification scale) mass. All the bulk operators are thus generated by IR effects and are finite. Another way to see this is to note that the only gauge-invariant operator that can give rise to a Higgs potential must be nonlocal in the extra dimensions and expressed in terms of the Wilson line $\mathcal{P}\exp(\mathrm{i}\int \mathrm{d}x^i A_i)$. Being a nonlocal operator, the Higgs potential is finite to all orders in the perturbation theory, is UV insensitive, and is calculable once the degrees of freedom around the weak scale are known.

As regards the brane-localized operators, the situation is more complicated. At the fixed point, the bulk gauge group is partially broken: there is only the $\mathrm{SU}(2)\times\mathrm{U}(1)$ subgroup left unbroken. The unbroken gauge group acts linearly on the gauge-scalar doublet and certainly does not forbid any quadratically divergent localized mass term to be generated:[6]

$$\delta_H A_\mu^H = \partial_\mu \epsilon^H + g f^{HHH} A_\mu^H \epsilon^H\,, \qquad \delta_H A_\mu^{G/H} = 0\,, \tag{77}$$

$$\delta_H A_5^H = 0\,, \qquad \delta_H A_5^{G/H} = g f^{G/HG/HH} A_5^{G/H} \epsilon^H\,. \tag{78}$$

Even though H is the only unbroken gauge symmetry at the fixed points, there is still some residual symmetry left over from the full G gauge symmetry in the bulk: indeed, the broken generators of the higher-dimensional gauge invariance act on the gauge scalars as a shift symmetry proportional to the derivative of the gauge parameters [48]:

$$\delta_{G/H} A_\mu^H = 0\,, \qquad \delta_{G/H} A_\mu^{G/H} = 0\,, \tag{79}$$

$$\delta_{G/H} A_5^H = 0\,, \qquad \delta_{G/H} A_5^{G/H} = \partial_5 \epsilon^{G/H}\,. \tag{80}$$

This Peccei – Quinn-like symmetry suffices to prevent the appearance of local mass counterterms. To construct an invariant, we need to use an object that transforms homogeneously under the gauge symmetry, like the gauge field strength tensor F_{MN}. In 5D orbifolds, there is no possible local counterterm involving the gauge field strength, because it is an antisymmetric object, while we have only one index at our disposal. In 6D orbifolds, however, the brane-localized operators [40, 49]

$$\mathrm{Tr}\,(U^k F_{56})\,, \qquad k = 1, 2, 3, \tag{81}$$

are perfectly allowed and are invariant under the local gauge transformations:

$$\mathrm{Tr}\,(UF_{56}) \to \mathrm{Tr}\,\big(Ug(0)F_{56}\,g^{-1}(0)\big) = \mathrm{Tr}\,\big(UF_{56}\,g^{-1}(0)g(0)\big) = \mathrm{Tr}\,(UF_{56})\,,$$

[6] Because $[H,H]\in H$ and $[H,G/H]\in G/H$, the only nonvanishing structure constants are f^{HHH} and $f^{G/HG/HH}$ and their cyclic permutations.

Guage contribution Ghost contribution

Figure 9. One-loop diagrams contributing to the tadpole contained in the operator $\mathrm{Tr}\,(UF_{56})$ localized at the fixed point. These diagrams are quadratically divergent and reintroduce a radiative instability in the Higgs potential.

where in the first equality we used the fact that U and g commute *at the fixed points* (to be specific, we considered that the fixed point is at the origin). These operators are potentially quite dangerous since they correspond to a tadpole for some massive KK gauge scalars along the unbroken U(1) directions and, through the non-Abelian part of F_{56}, to a mass for the massless gauge-scalars. And by power-counting, these operators are *quadratically divergent*.

In the case of a T^2/Z_2 orbifold, the parity invariance $(y,z)\to(-y,z)$ can be defined consistently with the orbifold projection

$$\begin{array}{ccc} & \mathsf{Z}_2 & \\ & \phi(x,-y,-z) = U\phi(x,y,z) & \\ P \swarrow & & \searrow P \\ \phi(x,y,-z) = U\phi(x,-y,z) & & U\phi(x,-y,z) \end{array}$$

and forbids the appearance of operators (81). On a T^2/Z_4 orbifold, the parity symmetry is no longer consistent with the orbifold projection:

$$\begin{array}{ccc} & \mathsf{Z}_4 & \\ & \phi(x,-z,y) = U\phi(x,y,z) & \\ P \swarrow & & \searrow P \\ \phi(x,z,y) = U^{-1}\phi(x,-y,z) & & U\phi(x,-y,z) \end{array}$$

and no discrete symmetry protects operators (81). A direct computation (Fig. 9) of the tadpole piece contained in (81) reveals that at the one-loop level, the contributions of the ghost fields and of the gauge fields add up to indeed generate the operator $\mathrm{Tr}\,(UF_{56})$ at the fixed points.[7] Thus, a quadratic divergence is reintroduced. It destabilizes the Higgs potential at one loop. A possible way to (partially) cure this instability is to engineer a setup such that the sum of quadratically divergent tadpoles at different fixed points vanishes (global cancellation) [42].

In conclusion, although gauge symmetry breaking on 5D orbifolds, as well as on some 6D orbifolds, is soft in the sense that the radiative corrections to the Higgs potential are finite, in the case of the potentially most interesting orbifold, T^2/Z_4, the gauge symmetry breaking is not soft.

[7] In the original computation [40] of the tadpole, we found that the two contributions cancel each other. But because no symmetry protects this tadpole operator, by virtue of the theorem mentioned in Section 2.1.3, our vanishing result was a sign of a mistake in our computation. More careful computations [42, 50] indeed yielded a nonvanishing quadratically divergent tadpole.

4.4 Introducing matter and Yukawa interactions

The introduction of quarks and leptons and the generation of their masses are not straightforward [40, 42]. If the matter were in the bulk, the Higgs *vev* would generate fermion masses that are controlled by the higher-dimensional gauge coupling and we could have group theory factors associated with different representations at our disposal in order to generate different masses, which certainly cannot account for the diversity of the matter spectrum. The other possibility is to resort to fermions localized at the orbifold fixed points. As regards the Yukawa interactions, one could try to directly linearly couple the SM fermions to the Higgs field at the fixed points. But introducing Yukawa couplings this way explicitly breaks the residual Peccei – Quinn shift symmetry G/H. One would like to seek operators that do not break this shift symmetry: this can be achieved by using operators that involve Wilson lines between the fixed points (the two fixed points may also coincide) $W = P\exp(\mathrm{i}\int A_i\,\mathrm{d}x^i)$. Actually, it is natural to expect the appearance of these operators once some massive bulk fermions that could mix with fields at the fixed points are integrated out in a Froggatt – Nielsen-like way. The masses of the light fermions can be obtained either by small mixing with the bulk fermions or by a large bulk mass, which exponentially suppresses the effective Yukawa, and it is possible to easily reproduce the hierarchy of the matter spectrum [42]. An interesting flavor structure in the first two generations has been revealed in the study of a particular SU(3) model in 5D [43].

4.5 Experimental signatures

Collider signals have not been studied in detail yet (possibly due to the lack of a fully realistic model). We can nevertheless make some predictions about a generic gauge – Higgs unification model:

- We should observe KK excitations of the W and Z bosons around 500 GeV – 1 TeV;
- We should observe spin-1 KK excitations of the G/H coset; in particular, there should be some gauge bosons with the EW quantum numbers of the Higgs doublet;
- We should observe extra scalar fields;
- We should observe some bulk fermions that mix with SM fermions to generate their masses.

4.6 Recent developments and open issues

In view of the quadratic divergence of the localized tadpole in the most promising 6D model, recent studies of gauge–Higgs unification models have focused mainly on 5D. The main issue of 5D models is to accommodate the heaviness of the top quark and the Higgs boson.

Regarding the top mass, with the Yukawa constants generated via gauge coupling, it is hard to engineer a setup with a top quark heavier than the W boson. A possible way out is to embed the top quark in a large representation such that the effective Yukawa constant is enhanced by a group factor. For instance, in the SU(3) model in [44], the prediction $M_{\mathrm{t}} = 2M_{\mathrm{W}}$ was obtained at the tree level. The main drawback of this possibility is that the large representation lowers the scale where the extra-dimensional theory becomes strongly coupled. Moreover, some rather large deviations in the coupling of the left bottom quark to Z, $\mathrm{Zb_l\bar{b}_l}$, are introduced at the tree level. Another possibility pursued in [45] is to explicitly give up the Lorentz invariance along the extra dimension. In this case, each fermion effectively feels an extra dimension of different length, alleviating the relation between the top quark and W-boson masses. The strong coupling scale is also lowered in that case and the Lorentz breaking reintroduces a UV sensitivity of the Higgs potential at higher loops (as in Little Higgs models). And again, corrections to $\mathrm{Zb_lb_l}$ and four-fermion operators generated by the KK gauge bosons impose a limit on the scale of the fifth dimension of the order of a few TeV.

Regarding the Higgs mass, it generically turns out to be too small, below the value currently excluded by LEP, because the quartic interaction is now generated at the one-loop level (contrarily to the $\mathrm{G_2}$ 6D model, where it was present at the tree level). Since the entire potential (mass and quartic) is loop-generated, the potential also generically prefers large values of the Higgs *vev* relative to the compactification scale, such that the scale of the new physics stays dangerously low. It was shown in [44] that the Higgs mass can be increased by the presence of several (twisted) bulk fermions. In the Lorentz-violating model in Ref. [45], the Higgs mass is set by the scale of the top quark.

Another avenue that has been explored in [51, 52] is to embed the idea of gauge – Higgs unification in a warped extra dimension. The nice thing is that the warping enhances both the Higgs boson and the top-quark mass. But the nontrivial background also induces corrections to EW precision observables. Via the AdS/CFT correspondence (see Section 5.4.1 for a short introduction to it), these models are now reinterpreted as weak couplings, dual to the old composite Higgs models of Georgi – Kaplan [53]. One highly valuable benefit of warped extra dimensions is the ability to move the scale of the new physics to very high energy, with the possibility of accommodating unification in particular.

One final comment concerns the dynamics of the EW phase transition in these gauge – Higgs unification models. As in Little Higgs theories, the structure of the radiatively generated Higgs potential is richer than just the ϕ^4 Mexican-hat potential with the presence of a series of nonrenormalizable interactions. It was shown that a moderately first-order EW phase transition can then be obtained even for reasonably large values of the Higgs mass [54] (see also [55]). This revives the possibility of EW baryogenesis to generate the asymmetry between matter and antimatter.

5. 5D Higgsless models

The idea behind Higgsless theories [24, 25] is that a momentum along an extra dimension is equivalent to a mass in 4D, and we can therefore generate a mass by giving a momentum to a particle in an extra dimension. As in quantum mechanics, a nonzero momentum along a compact direction can result from nontrivial boundary conditions (BCs). Therefore, the work consists in identifying the appropriate boundary conditions and the geometry of the extra dimension to reproduce the spectrum and the couplings of the SM. Short presentations of Higgsless models can be found in [56 – 58]. A more comprehensive review, with a significant overlap with the present lectures, is [59].

In the previous section, we discussed 5D models where part of the gauge group is broken by orbifold compactification, i.e., by some particular boundary conditions.[8] This raises the hope to achieve a breaking of the EW symmetry directly by BCs. Before facing the details of an explicit construction, we note the principal obstacles occurring in the construction of a realistic model [47].

Rank reduction. In usual (abelian) orbifold compactifications, the rank of the gauge group cannot be reduced unless the orbifold projection corresponds to an outer automorphism of the gauge symmetry. For a given algebra, the number of automorphisms is limited and, in particular, it is not possible to break $SU(2)\times U(1)$ down to $U(1)$. Therefore, more general BCs than those obtained from simple orbifold projection have to be considered.

Nonrational mass ratio. In usual KK compactifications, the spectrum is dictated by the geometry of the extra dimensions and the mass gap between two KK states is given by an integer times the inverse size of the extra dimension. It therefore seems nontrivial to obtain a mass ratio of the W and Z bosons that is related to the gauge couplings.

Unitarity restoration. We have shown in Section 2.2.1 that the Higgs boson is essential in restoring perturbative unitarity in the longitudinal massive gauge boson scattering. Thus, the question that we want to raise is whether such a breaking of the gauge symmetries via BCs in 5D theories yields a consistent theory; in other words, whether a momentum along a fifth dimension is UV-safer than a regular 4D gauge boson mass. To verify that such a breaking is indeed soft, we need to investigate the issue of the unitarity of scattering amplitudes in such 5D gauge theories compactified on an interval, with nontrivial BCs. We derive the general expression for the amplitude for elastic scattering of longitudinal gauge bosons and write the necessary conditions for the cancellation of the terms that grow with energy. We find that all the consistent BCs are unitary in the sense that all terms proportional to E^4 and E^2 vanish. In fact, any theory with only Dirichlet or Neumann BCs is unitary. Surprisingly, this also includes theories where the boundary conditions can be regarded as coming from a very large expectation value of a brane-localized Higgs field. In the limit as the expectation value diverges, there are no scalar degrees of freedom at low energy, hence the name of Higgsless theories.

There are several aspects of Higgsless theories that are not covered in this section, for instance, the construction of 4D models from deconstruction, the construction of 6D models, or some applications to GUT breaking. The reader is referred to the existing literature [60, 61]. Higgsless models only address the issue of EWSB. Ultimately, they should be embedded into a GUT, and the $L-R$ structure seems to indicate that an SO(10) or a Pati–Salam embedding is the right direction to take [62].

[8] It might be useful to stress the difference between gauge–Higgs unification models and Higgsless models: in gauge–Higgs models, a bigger gauge group G is broken to $SU(2)\times U(1)$ by an orbifold/boundary condition, while the actual EW breaking is achieved via a Higgs mechanism, the Higgs boson being identified as some massless component of the bigger gauge group along the extra dimensions; the orbifold projection has indeed been carefully chosen such that a massless mode of A_5 remained massless. In Higgsless models, on the contrary, boundary conditions are chosen such that no component of A_5 remained in the physical spectrum (the massless ones are thrown away by the boundary conditions, while the massive ones are eaten up to give the longitudinal polarizations of massive gauge bosons).

5.1 Gauge symmetry breaking by boundary conditions

5.1.1 Boundary conditions for a scalar field. We start with the bulk action for the scalar field

$$\mathcal{S}_{\text{bulk}} = \int \mathrm{d}^4x \int_0^{\pi R} \mathrm{d}y \left(\frac{1}{2}\, \partial^M \phi\, \partial_M \phi - V(\phi) \right), \tag{82}$$

where we assume that the interval runs between 0 and πR. For simplicity, we first assume that there is no term added at the boundary of the interval. We apply the variational principle to this theory:

$$\delta\mathcal{S} = \int \mathrm{d}^4x \int_0^{\pi R} \mathrm{d}y \left(\partial^M \phi\, \partial_M \delta\phi - \frac{\partial V}{\partial \phi}\, \delta\phi \right). \tag{83}$$

Separating the ordinary 4D coordinates from the fifth coordinate (and integrating by parts in the ordinary 4D coordinates, where we apply the usual requirements that the fields vanish for large distances), we obtain

$$\delta\mathcal{S} = \int \mathrm{d}^4x \int_0^{\pi R} \mathrm{d}y \left[-\partial_\mu \partial^\mu \phi \delta\phi - \frac{\partial V}{\partial \phi}\, \delta\phi - \partial_y \phi\, \partial_y \delta\phi \right]. \tag{84}$$

Since we have not yet decided what boundary conditions we want to impose, we have to keep the boundary terms when integrating by parts in the fifth coordinate y:

$$\delta\mathcal{S} = \int \mathrm{d}^4x \int_0^{\pi R} \mathrm{d}y \left[-\partial_M \partial^M \phi - \frac{\partial V}{\partial \phi} \right] \delta\phi - \int \mathrm{d}^4x \left[\partial_y \phi\, \delta\phi \right]_0^{\pi R}. \tag{85}$$

To ensure that the variational principle is satisfied, we need $\delta\mathcal{S}=0$, but because this consists of a bulk and a boundary piece, it is required:

- that the bulk equation of motion have the standard form $\partial_M \phi\, \partial^M \phi = -\partial V/\partial\phi$;
- that the boundary variation also vanish. This implies that we need to choose the BC such that

$$\partial_y \phi\, \delta\phi\big|_{\text{bd}} = 0\,. \tag{86}$$

We say that a boundary condition is *natural* if it is obtained by letting the boundary variation of the field $\delta\phi|_{\text{bd}}$ be arbitrary. In this case, the natural BC is $\partial_y\phi = 0$: a flat or Neumann BC. But at this stage, this is not the only possibility: we could also satisfy (86) by imposing $\delta\phi|_{\text{bd}} = 0$, which would follow from the Dirichlet BC $\phi|_{\text{bd}} = 0$. Thus, we have two possible BCs for a scalar field on an interval with no boundary terms:

- Neumann BC $\partial_y \phi|_{\text{bd}} = 0$,
- Dirichlet BC $\phi|_{\text{bd}} = 0$.

However, we want to allow only natural boundary conditions in the theory, because they are the ones that do not lead to explicit (hard) symmetry breaking once more complicated fields like gauge fields are allowed. Thus, in order to still allow the Dirichlet BC, we need to reinterpret it as the natural BC for a theory with additional terms in the Lagrangian added at the boundary. The simplest possibility is to add a *mass term* to modify the Lagrangian as

$$\mathcal{S} = \mathcal{S}_{\text{bulk}} - \int \mathrm{d}^4x\, \frac{1}{2}\, M_1^2 \phi^2\big|_{y=0} - \int \mathrm{d}^4x\, \frac{1}{2}\, M_2^2 \phi^2\big|_{y=\pi R}. \tag{87}$$

This makes an additional contribution to the boundary variation of the action, which is now given by

$$\delta\mathcal{S}_{\text{bd}} = -\int \mathrm{d}^4x\, \delta\phi(\partial_y\phi + M_2^2\phi)\big|_{y=\pi R} + \int \mathrm{d}^4x\, \delta\phi(\partial_y\phi - M_1^2\phi)\big|_{y=0}\,. \quad (88)$$

Thus, the natural BCs are given by

$$\partial_y\phi + M_2^2\phi = 0 \quad \text{at} \quad y = \pi R\,, \qquad \partial_y\phi - M_1^2\phi = 0 \quad \text{at} \quad y = 0\,. \quad (89)$$

Clearly, as $M_i \to \infty$, we recover the Dirichlet BCs in the limit. This is the way we always understand the Dirichlet BCs: we interpret them as the case with infinitely large boundary-induced mass terms for the fields.

We now consider what happens when we add a *kinetic term* at the boundary for ϕ. For simplicity, we set the mass parameters on the branes to zero and take the action in the form

$$\mathcal{S} = \mathcal{S}_{\text{bulk}} + \int \mathrm{d}^4x\, \frac{1}{2M}\, \partial_\mu\phi\, \partial^\mu\phi\big|_{y=0}\,. \quad (90)$$

We note that the boundary term had to be added with a definite sign, that is, we assume that the arbitrary mass parameter M is positive. This is in accordance with our expectations that kinetic terms have to have positive signs in order to avoid ghostlike states. For simplicity, we have only added a kinetic term on one of the branes, but of course we could easily repeat the following analysis for the second brane. The boundary variation at $y = 0$ is modified to

$$\delta\mathcal{S}_{y=0} = \int \mathrm{d}^4x\, \delta\phi\left(\partial_y\phi - \frac{1}{M}\,\Box_4\phi\right)\bigg|_{y=0}\,. \quad (91)$$

Thus the natural BC is given by

$$\partial_y\phi = \frac{1}{M}\,\Box_4\phi\,. \quad (92)$$

Using the bulk equation of motion (in the presence of no bulk potential) $\Box_5\phi = \Box_4\phi - \phi'' = 0$, we can also write this BC as $M\phi' = \phi''$. The final form of the BC is obtained by using the KK decomposition of the field ϕ, where the 4D modes ϕ_n are usually assumed to have the x dependence $\phi_n \exp(\mathrm{i}p_n x)$, where $p_n^2 = m_n^2$ is the nth KK mass eigenvalue. Using this form, we have the BC

$$\partial_y\phi = \frac{1}{M}\,\Box_4\phi = -\frac{p_n^2}{M}\,\phi = -\frac{m_n^2}{M}\,\phi\,. \quad (93)$$

In either form, this BC is quite peculiar: it depends on the actual mass eigenvalue in the final form, or involves second derivatives in the first form. This could be dangerous, because we know from the theory of differential equations that usually BCs that only involve first derivatives automatically lead to a Hermitian differential operator on an interval. The standard argument is that the second-derivative operator $\mathrm{d}^2/\mathrm{d}y^2$ is Hermitian if the scalar product

$$\langle f, g\rangle = \int_0^{\pi R} \mathrm{d}y\, f^*(y) g(y) \quad (94)$$

satisfies the relation

$$\left\langle f, \frac{\mathrm{d}^2}{\mathrm{d}y^2}\, g\right\rangle = \left\langle \frac{\mathrm{d}^2}{\mathrm{d}y^2}\, f, g\right\rangle, \quad (95)$$

which is indeed satisfied if the functions f and g have boundary conditions of the form

$$f'\big|_{0,\pi R} = \alpha f\big|_{0,\pi R}\,. \quad (96)$$

The usual properties of the real-valuedness of eigenvalues and completeness of eigenfunctions follow from the hermiticity of the scalar product. For boundary conditions of type (93), the scalar product has to be supplemented with a boundary term to remain Hermitian:

$$\langle f, g\rangle = \int_0^{\pi R} \mathrm{d}y\, f(y) g(y) + \frac{1}{M}\, fg\big|_0\,. \quad (97)$$

In particular, the completeness property of the eigenfunctions is now expressed as

$$\sum_n g_n(x) g_n(y) = \delta(x - y) - \frac{1}{M}\,\delta(x) \sum_n g_n(0) g_n(y)\,. \quad (98)$$

5.1.2 Boundary conditions for a gauge field. The same exercise can be repeated [47] for a spin-1 particle.[9] We just have to be somewhat more careful and fix a gauge to deal with gauge degrees of freedom in an appropriate way. A gauge field A_M in 5D contains a 4D gauge field A_μ and a 4D scalar A_5. The 4D vector contains a whole KK tower of massive gauge bosons; however, as we see below, the KK tower of the A_5 is 'eaten' by the massive gauge fields and (except for a possible zero mode) is nonphysical. That this is what happens can be guessed from the fact that the Lagrangian contains a mixing term between the gauge fields and the scalar, reminiscent of the usual 4D Higgs mechanism. The Lagrangian is given by the standard expression

$$\mathcal{S} = \int \mathrm{d}^5x \left(-\frac{1}{4}\, F^a_{MN} F^{MN\,a}\right) = \int \mathrm{d}^5x \left(-\frac{1}{4}\, F^a_{\mu\nu} F^{\mu\nu\,a} - \frac{1}{2}\, F^a_{\mu 5} F^{\mu 5\,a}\right), \quad (99)$$

where the field strength has the usual form $F^a_{MN} = \partial_M A^a_N - \partial_N A^a_M + g_5 f^{abc} A^b_M A^c_N$ and g_5 is the 5D gauge coupling, which has mass dimension $-1/2$. The theory is nonrenormalizable, and therefore has to be considered a low-energy effective theory valid below a cutoff scale, which we calculate in what follows.

To determine the gauge-fixing term, we consider the mixing term between the 4D scalar and the 4D gauge fields:

$$\int \mathrm{d}^5x \left(-\frac{1}{2} F^a_{\mu 5} F^{\mu 5 a}\right) = \int \mathrm{d}^5x\, \partial_5 A^a_\mu\, \partial^\mu A^{5a} + \ldots\,. \quad (100)$$

Integrated by parts, the mixing term becomes

$$[\partial_\mu A^{\mu a} A^a_5]_0^{\pi R} - \int_0^{\pi R} \mathrm{d}y\, \partial^\mu A^a_\mu\, \partial_5 A^a_5\,. \quad (101)$$

[9] The reader interested in boundary conditions for a spin-2 field can have a look at [63]. In Section 5.5.1, we explain how to obtain boundary conditions for spin 1/2.

The *bulk* mixing can be canceled by adding a gauge-fixing term of the form

$$\mathcal{S}_{GF}^{\text{bulk}} = \int \mathrm{d}^5 x \left(-\frac{1}{2\xi} (\partial_\mu A^{\mu a} - \xi \partial_5 A_5^a)^2 \right). \tag{102}$$

This term is chosen such that the A_5-independent piece agrees with the usual Lorentz gauge-fixing term, and such that the cross term exactly cancels the mixing term from (101). Thus, in the R_ξ gauge, which is what we have defined, the propagator for the 4D gauge fields is the usual one.

Varying the full action, we then obtain the bulk equations of motion and the possible BCs. After integrating by parts, we find that at the quadratic level, $\delta\mathcal{S}_{\text{bulk}} + \delta\mathcal{S}_{GF}^{\text{bulk}}$ is given by

$$\int \mathrm{d}^5 x \left[\left(\partial_\mu (\partial^\mu A^{\nu a} - \partial^\nu A^{\mu a}) - \partial_5^2 A^{\nu a} + \frac{1}{\xi} \partial^\nu \partial_\sigma A^{\sigma a} \right) \delta A_\nu^a - (\partial_\sigma \partial^\sigma A_5^a - \xi \partial_5^2 A_5^a) \delta A_5^a \right]. \tag{103}$$

The bulk equations of motion just state that the coefficients of δA_ν^a and δA_5^a in the above equation vanish. We can see that the A_5^a field has the term $\xi \partial_5^2 A_5^a$ in its equation of motion. This implies that if the wave function is not flat (e.g., the KK mode is not massless), then the field is not physical (since in the unitary gauge $\xi \to \infty$, this field has an infinite effective 4D mass and decouples). This shows that, as mentioned above, the scalar KK tower of A_5^a is completely unphysical, owing to the 5D Higgs mechanism, except perhaps for the zero mode for A_5^a. Whether there is a zero mode depends on the BC for the A_5 field. In Higgsless models, there is no A_5 zero mode.

To eliminate the *boundary* mixing term in (101), we also need to add a boundary gauge-fixing term with an a priori unrelated boundary gauge-fixing coefficient ξ_{bd},

$$\mathcal{S}_{GF}^{\text{bd}} = -\frac{1}{2\xi_{\text{bd}}} \int \mathrm{d}^4 x \, (\partial_\mu A^{\mu a} \pm \xi_{\text{bd}} A_5^a)^2 |_{0,\pi R}, \tag{104}$$

where the minus sign is for $y = 0$ and the plus is for $y = \pi R$. The boundary variations are then given by

$$\left(\pm \partial_5 A^{\mu a} + \frac{1}{\xi_{\text{bd}}} \partial^\mu \partial_\nu A^{\nu a} \right) \delta A_\mu^a |_{0,\pi R} - (\pm \xi \partial_5 A_5^a + \xi_{\text{bd}} A_5^a) \delta A_5^a |_{0,\pi R}.$$

The natural boundary conditions in an arbitrary gauge ξ, ξ_{bd} are given by

$$\partial_5 A^{\mu a} \pm \frac{1}{\xi_{\text{bd}}} \partial_\nu \partial^\mu A^{\nu a} = 0, \qquad \xi \partial_5 A_5^a \pm \xi_{\text{bd}} A_5^a = 0. \tag{105}$$

This simplifies considerably if we go to the unitary gauge at the boundary given by $\xi_{\text{bd}} \to \infty$. In this case, we are left with the simple set of boundary conditions

$$\partial_5 A^{\mu a} = 0, \qquad A_5^a = 0. \tag{106}$$

These are the boundary conditions that are usually imposed for gauge fields in the absence of any boundary terms. We note that we again could have chosen some nonnatural boundary conditions, where instead of requiring the boundary variation to be arbitrary we would require the boundary variation itself (and thus some of the fields on the boundary) to vanish. It turns out that these boundary conditions would lead to a hard (explicit) breaking of gauge invariance, and we do not therefore consider them in what follows. We see below how these simple BCs are modified if scalar fields are added on the branes.

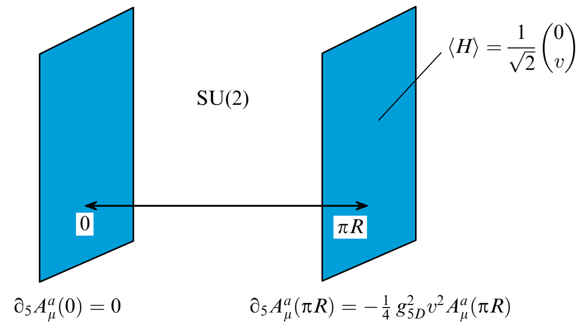


Figure 10. Example of a Higgs mechanism localized on a boundary. For a finite Higgs *vev*, we obtain a mixed BC which, in the infinite-*vev* limit, simply becomes a Dirichlet BC: all the gauge bosons that couple to the Higgs boson have a wave function that vanishes at the point where the Higgs boson is localized. In that limit, there is no scalar degree of freedom in the low-energy effective action and the gauge symmetry is entirely broken by the BCs; the mass of the lightest KK state is simply inversely proportional to the size of the extra dimension.

5.1.3 Higgs mechanism localized on a boundary: scalar decoupling limit. We now consider the case where scalar fields that develop vacuum expectation values are added at the boundary [24, 47, 64, 65]. Instead of repeating a full and general analysis (which can be found in [65]), we present a concrete example. We consider the SU(2) gauge group with Newmann BCs for the A_μ components at both ends of the interval (Fig. 10). We then assume that at $y = \pi R$, SU(2) is fully broken by the *vev* of a Higgs doublet. As in the scalar case, the boundary mass generated by the Higgs *vev* induces a mixed BC of the form

$$\partial_5 A_\mu^a(\pi R) = -\frac{1}{4} g_{5D}^2 v^2 A_\mu^a(\pi R). \tag{107}$$

The canonically normalized ($\int_0^{\pi R} f_k^2(y) = 1$) KK modes are given by

$$A_\mu^a(x, y) = \sum_{k=1}^{\infty} f_k(y) A_\mu^{(k)}(x) \tag{108}$$

with

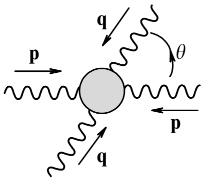
$$f_k(y) = \frac{\sqrt{2}}{\sqrt{\pi R \big(1 + 16 M_k^2/(g_{5D}^4 v^4)\big) + 4/(g_{5D}^2 v^2)}} \frac{\cos(M_k y)}{\sin(M_k \pi R)}. \tag{109}$$

The BC at the origin, $y = 0$, is trivially satisfied, and the condition at $y = \pi R$ determines the mass spectrum through the equation

$$M_k \tan(M_k \pi R) = \frac{1}{4} g_{5D}^2 v^2. \tag{110}$$

In the large *vev* limit, we obtain the wave functions at the $y = \pi R$ boundary decreasing as $1/v^2$

$$f_k(\pi R) \sim 2\sqrt{\frac{2}{\pi R}} \frac{2k+1}{g_{5D}^2 R v^2}, \tag{111}$$



$$\epsilon_\mu = \left(\frac{|\mathbf{p}|}{M_n}, \frac{E}{M_n}\frac{\mathbf{p}}{p}\right)$$

$$p_\mu^{\text{in}} = \left(E, 0, 0, \pm\sqrt{E^2 - M_n^2}\right)$$

$$q_\mu^{\text{out}} = \left(E, \pm\sqrt{E^2 - M_n^2}\sin\theta, 0, \pm\sqrt{E^2 - M_n^2}\cos\theta\right)$$

Figure 11. Elastic scattering of longitudinal modes of KK gauge bosons, $n + n \to n + n$, with the gauge index structure $a + b \to c + d$. The E-dependence can be estimated from $\epsilon \sim E, p_\mu \sim E$ and a propagator $\sim E^{-2}$.

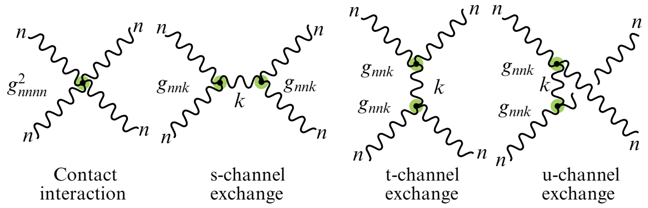


Figure 12. The four diagrams contributing at the tree level to the elastic scattering amplitude of the nth KK mode.

while

$$M_k \sim \frac{2k+1}{2R}\left(1 - \frac{4}{g_{5D}^2 \pi R v^2}\right). \tag{112}$$

This limit exactly corresponds to a Dirichlet BC: in the large-*vev* limit, the wave functions of the gauge bosons that couple to the Higgs vanish. It can also be verified that A_5 actually obeys a Neumann BC in that limit. In our example, however, because of the other Dirichlet BC at $y = 0$, there is still no physical massless mode for A_5, while the would-be massive ones are 'eaten' to give the longitudinal polarizations of the massive A_μ field. What allows us to decouple the Higgs degree of freedom from the low-energy action is that, contrary to 4D, the masses of the gauge bosons are not proportional to the Higgs *vev*.

5.2 Unitarity restoration by KK modes. Sum rules of Higgsless theories

Our aim is to build a Higgsless model of electroweak symmetry breaking using BC breaking in extra dimensions. But there is a problem in theories with massive gauge bosons without a Higgs scalar: the scattering amplitude of longitudinal gauge bosons increases with the energy and violates unitarity at a low scale [11–13] (see Section 2.2.1). We must first understand what happens to this unitarity limit[10] in a theory with extra dimensions [24, 67–69]. For simplicity, we focus on the elastic scattering of the longitudinal modes of the nth KK mode (Fig. 11). The E-dependence can be estimated from $\epsilon \sim E$, $p_\mu \sim E$ and a propagator $\sim E^{-2}$. This way, we find that the amplitude can increase as E^4; for $E \gg M_{\text{W}}$, we can then expand the amplitude in decreasing powers of E as

$$\mathcal{A} = \mathcal{A}^{(4)}\,\frac{E^4}{M_n^4} + \mathcal{A}^{(2)}\,\frac{E^2}{M_n^2} + \mathcal{A}^{(0)} + \mathcal{O}\left(\frac{M_n^2}{E^2}\right). \tag{113}$$

In the SM (and any theory where the gauge kinetic terms form the gauge-invariant combination $F_{\mu\nu}^2$), the $\mathcal{A}^{(4)}$ term automatically vanishes, while $\mathcal{A}^{(2)}$ is only canceled after taking the Higgs exchange diagrams into account.

In the case of a theory with an extra dimension with BC breaking of the gauge symmetry, there are no Higgs exchange diagrams; however, we must sum up the exchanges of all KK modes, as in Fig. 12. As a result, we find the following expression for the terms in the amplitudes that grow with the energy:

$$\mathcal{A}^{(4)} = \mathrm{i}\left(g_{nnnn}^2 - \sum_k g_{nnk}^2\right) a^{(4)}(\theta), \tag{114}$$

[10] There are also some unitarity issues associated with the masses of the fermions (see [66]).

with

$$a^{(4)}(\theta) = f^{abe} f^{cde}(3 + 6\cos\theta - \cos^2\theta) + 2(3 - \cos^2\theta)\, f^{ace} f^{bde}. \tag{115}$$

For the term $\mathcal{A}^{(4)}$ to vanish, it suffices to ensure the following sum rule between the couplings of the various KK modes [24]:

$$E^4 \text{ sum rule}: \quad g_{nnnn}^2 = \sum_k g_{nnk}^2. \tag{116}$$

Assuming $\mathcal{A}^{(4)} = 0$, we obtain

$$\mathcal{A}^{(2)} = \frac{\mathrm{i}}{M_n^2}\left(4 g_{nnnn} M_n^2 - 3\sum_k g_{nnk}^2 M_k^2\right) a^{(2)}(\theta) \tag{117}$$

with

$$a^{(2)}(\theta) = f^{ace} f^{bde} - \sin^2\frac{\theta}{2}\; f^{abe} f^{cde}, \tag{118}$$

where g_{nnnn}^2 is the quartic self-coupling of the nth massive gauge field and g_{nnk} is the cubic coupling between the KK modes. In theories with extra dimensions, these are of course related to the extra-dimensional wave functions $f_n(y)$ of the various modes as

$$g_{mnk} = g_5 \int \mathrm{d}y\, f_m(y) f_n(y) f_k(y), \tag{119}$$

$$g_{mnkl}^2 = g_5^2 \int \mathrm{d}y\, f_m(y) f_n(y) f_k(y) f_l(y). \tag{120}$$

The most important point about the amplitudes in (114)–(117) is that they only depend on an overall kinematic factor multiplied by an overall expression of the couplings (the dynamics factors from the kinematics). Assuming that relation (116) holds, we can find a sum rule that ensures the vanishing of the $\mathcal{A}^{(2)}$ term [24]:

$$E^2 \text{ sum rule}: \quad g_{nnnn} M_n^2 = \frac{3}{4}\sum_k g_{nnk}^2 M_k^2. \tag{121}$$

Amazingly, higher-dimensional gauge invariance ensures that both these sum rules are satisfied as long as the breaking of the gauge symmetry is spontaneous. For example, it is easy to show the first sum rule via the completeness of the wavefunctions $f_n(y)$:

$$\int_0^{\pi R} \mathrm{d}y\, f_n^4(y) = \sum_k \int_0^{\pi R} \mathrm{d}y \int_0^{\pi R} \mathrm{d}z\, f_n^2(y) f_n^2(z) f_k(y) f_k(z), \tag{122}$$

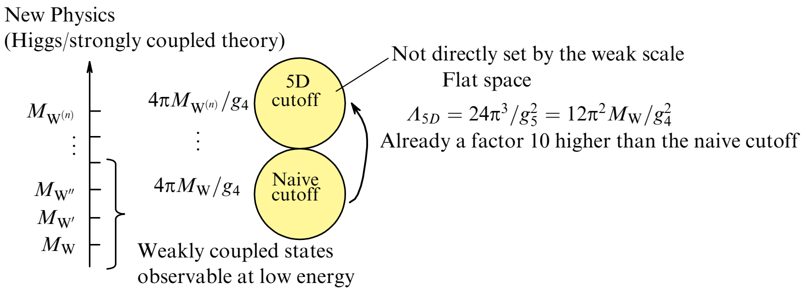


Figure 13. The scattering amplitude of the longitudinal components of the lightest massive KK gauge boson would naively become nonperturbative at the energy scale $4\pi M_W/g_4$. However, before reaching that scale, the exchange of the KK excitations starts canceling the scattering amplitude. The story repeats itself until the heaviest KK mode below the 5D cutoff is reached, for which no heavier excitations can intervene to smooth its scattering amplitude. Thus, the perturbative unitarity breakdown has been delayed and pushed to a scale that is not directly related to the mass of the lightest massive gauge bosons. A detailed analysis [68] of inelastic channels confirms the loss of perturbative unitarity at an energy scale related to the 5D cutoff.

and using the completeness relation

$$\sum_k f_k(y) f_k(z) = \delta(y - z) \tag{123}$$

we can see that the two sides indeed agree. It can be shown similarly [24] that the second sum rule is also satisfied if the boundary conditions are natural and all terms in the Lagrangian (including boundary terms) are gauge invariant. We insist on the particular case of a Higgs mechanism localized at the boundary: for a finite Higgs *vev*, the cancellation of the E^2 term requires the exchange of the brane Higgs scalar degree of freedom; however, in the infinite-*vev* limit, the contribution of the Higgs exchange to the scattering amplitude actually cancels and we are left with simple Dirichlet BCs, for which the scattering amplitude is unitarized by the sole exchange of spin-1 KK excitations.

At this point, it should be noted that the two sum rules cannot be satisfied with a finite number of KK modes. This is in full agreement with the old theorem by Cornwall et al. [12], who established that the only way to restore perturbative unitarity in the scattering of massive spin-1 particles is through the exchange of a scalar Higgs boson. Our 5D theory is nonrenormalizable anyway, and hence it is valid up to a finite cutoff. Our result actually shows that through the exchange of the KK gauge bosons, the perturbative unitarity breakdown is postponed from the energy scale of the order of the mass of the lightest KK state to the true 5D cutoff of the order of the mass of the heaviest KK state (Fig. 13).

We see from the above analysis that in any gauge-invariant extra-dimensional theory, the terms in the amplitude that grow with the energy cancel. However, this does not automatically mean that the theory itself is unitary. The reason is that there are two additional worries: even if $\mathcal{A}^{(4)}$ and $\mathcal{A}^{(2)}$ vanish, $\mathcal{A}^{(0)}$ could be too large and spoil the unitarity. This is what happens in the SM if the Higgs mass is too large. In the extra-dimensional case, this would mean that the extra KK modes would make the scattering amplitude flatten out to a constant value. But if the KK modes themselves are too heavy, then this flattening out occurs too late, when the amplitude already violates the unitarity. The other issue is that in a theory with extra dimensions, there are infinitely many KK modes and hence, as the scattering energy grows, we should worry not only about the elastic channel but also about the ever growing number of possible inelastic final states. A full analysis taking both effects into account was performed in [68], where it was shown that after taking the opening up of the inelastic channels into account, the scattering amplitude increases linearly with energy and always violates unitarity at some energy scale. This is a consequence of the intrinsic non-renormalizability of the higher-dimensional gauge theory.

It was found in [68] that the unitarity violation scale due to the linear increase in the scattering amplitude is equal (up to a small numerical factor of the order 2–4) to the cutoff scale of the 5D theory obtained from the naive dimensional analysis (NDA). This cutoff scale can be estimated as follows. The one-loop amplitude in 5D is proportional to the 5D loop factor

$$\frac{g_5^2}{24\pi^3}\,. \tag{124}$$

The dimensionless quantity obtained from this loop factor is

$$\frac{g_5^2 E}{24\pi^3}\,, \tag{125}$$

where E is the scattering energy. The cutoff scale can be obtained by calculating the energy scale at which this loop factor becomes about 1 (that is, the scale at which the loop and tree-level contributions become comparable). From this, we obtain

$$\Lambda_{\mathrm{NDA}} = \frac{24\pi^3}{g_5^2}\,. \tag{126}$$

We can express this scale by using the matching of the higher-dimensional and the lower-dimensional gauge couplings. In the simplest theories, this is usually given by

$$g_5^2 = \pi R g_4^2\,, \tag{127}$$

where πR is the length of the interval and g_4 is the effective 4D gauge coupling. Hence, the final expression for the cutoff scale can be given as

$$\Lambda_{\mathrm{NDA}} = \frac{24\pi^2}{g_4^2 R}\,. \tag{128}$$

We see in what follows that in the Higgsless models, $1/R$ is replaced by M_W^2/M_{KK}, where M_W is the physical W mass and M_{KK} is the mass of the first KK mode beyond W. Thus, the cutoff scale is indeed lower if the mass of the KK mode used for unitarization is higher. However, this Λ_{NDA} could be significantly higher than the cutoff scale in the SM without a Higgs boson, which is around 1.2 TeV. We return to a more detailed discussion of $\Lambda_{\rm NDA}$ in Higgsless models at the end of this section.

5.3 Toy models

Having seen that KK gauge bosons can be used to delay the unitarity violation scale basically up to the cutoff scale of the higher-dimensional gauge theory, we start seeking a model that actually has these properties and resembles the SM. It should have a massless photon, a massive charged gauge boson to be identified with the W boson, and a somewhat heavier neutral gauge boson to be identified with the Z boson. Most importantly, we need to have the correct SM mass ratio (at the tree level)

$$\frac{M_{\rm W}^2}{M_{\rm Z}^2} = \cos^2\theta_{\rm W} = \frac{g^2}{g^2+g'^2}\,, \tag{129}$$

where g is the $\mathrm{SU}(2)_{\rm L}$ gauge coupling and g' is the $\mathrm{U}(1)_Y$ gauge coupling of the SM. We use BCs to achieve this. As stressed in the introduction, this seems to be very difficult at first sight, since we need to somehow obtain a theory where the masses of the KK modes are related to the gauge couplings. Usually, the KK masses are simply integer or half-integer multiples of $1/R$.

5.3.1 En route to a Higgsless model. Considering a very naive toy model with the SU(2) gauge group in the bulk (Fig. 14) as an example, we can have the following BCs for the various gauge directions:

$$\partial_y A_\mu^3 = 0 \quad \text{at} \;\; y = 0, \pi R\,, \tag{130}$$

$$\partial_y A_\mu^{1,2} = 0 \;\; \text{at} \;\; y = 0\,, \qquad A_\mu^{1,2} = 0 \;\; \text{at} \;\; y = \pi R\,. \tag{131}$$

Solving the bulk equations of motion and enforcing the BCs, we obtain the KK decomposition

$$A_\mu^1(x,y) = \sum_{k=0}^{\infty} \frac{1}{\sqrt{\pi R}} \sin\frac{(2k+1)y}{2R}\Big(W_\mu^{+\,(k)}(x) + W_\mu^{-\,(k)}(x)\Big)\,, \tag{132}$$

$$A_\mu^2(x,y) = \sum_{k=0}^{\infty} \frac{1}{\sqrt{\pi R}} \sin\frac{(2k+1)y}{2R}\Big(W_\mu^{+\,(k)}(x) - W_\mu^{-\,(k)}(x)\Big)\,, \tag{133}$$

$$A_\mu^3(x,y) = \sum_{k=0}^{\infty} \sqrt{\frac{2}{2^{\delta_{k0}}\pi R}} \cos\frac{ky}{R}\,\gamma_\mu^{(k)}(x)\,. \tag{134}$$

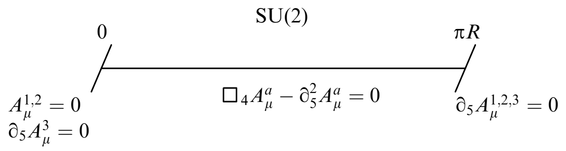


Figure 14. Example of the breaking of SU(2) to U(1) achieved by boundary conditions. The spectrum consists of a massless gauge boson and all its KK excitations, a pair of electrically charged massive gauge bosons, and all of their KK excitations. With a lot of imagination one could see something starting to resemble the SM with a massless photon, a pair of massive $\mathrm{W}^{\pm}$, and the first KK excitation of the photon that can be seen as a Z. This model is still quite far from reality (incorrect W/Z mass ratio, resonances that are too light), but it illustrates the basic idea that the masses of the W and Z bosons are generated by the boundary conditions and not through the usual Higgs mechanism.

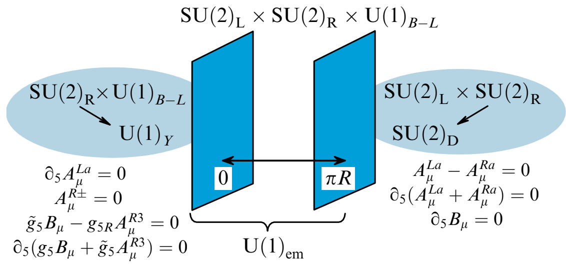


Figure 15. The symmetry breaking structure of the flat space Higgsless toy model [24].

This spectrum somewhat resembles that of the SM in the sense that there is a massless gauge boson that can be identified with the photon γ, a pair of charged massive gauge bosons that can be identified with the $\mathrm{W}^{\pm}$ bosons, and a massive neutral gauge boson that can be identified with the Z boson. However, we can see that the mass ratio of W and Z is

$$\frac{M_{\rm Z}}{M_{\rm W}} = 2\,, \tag{135}$$

and another problem is that the first KK modes of W and Z are such that

$$\frac{M_{\rm Z'}}{M_{\rm Z}} = 2\,, \qquad \frac{M_{\rm W'}}{M_{\rm W}} = 3\,. \tag{136}$$

Hence, besides the totally incorrect W/Z mass ratio, there are additional KK states at masses of the order of 250 GeV, which is phenomenologically unacceptable. We see below that both these problems can be resolved by going to a warped Higgsless model with custodial SU(2).

5.3.2 Flat Higgsless model. It is clear from the above discussion that in order to find a Higgsless model with the correct W/Z mass ratio, we need to find an extra-dimensional model incorporating the custodial SU(2) symmetry [70]. Once such a construction is found, the gauge boson mass ratio is automatically the right one. Therefore, we need to somehow involve $\mathrm{SU(2)_R}$ in the construction. The simplest possibility is to put the entire $\mathrm{SU(2)_L \times SU(2)_R \times U(1)}_{B-L}$ gauge group in the bulk of an extra dimension [24]. To mimic the symmetry-breaking pattern in the SM most closely, we assume that the symmetry breaking on one of the branes is given by $\mathrm{SU(2)_L \times SU(2)_R \to SU(2)_D}$, with $\mathrm{U(1)}_{B-L}$ unbroken. On the other boundary, we need to reduce the bulk gauge symmetry to that of the SM, and thus have the symmetry breaking pattern $\mathrm{SU(2)_R \times U(1)}_{B-L} \to \mathrm{U(1)}_Y$, which is illustrated in Fig. 15.

We let A_M^{Ra}, A_M^{La}, and B_M denote the respective gauge bosons of $\mathrm{SU(2)_R}$, $\mathrm{SU(2)_L}$, and $\mathrm{U(1)}_{B-L}$; g_{5L} and g_{5R} are the gauge couplings of the two SU(2), and $\tilde{g}_5$ is the gauge coupling of $\mathrm{U(1)}_{B-L}$. To obtain the desired BCs as discussed above, we need to follow the procedure outlined in Section 5.1. We assume that there is a boundary Higgs boson on the left brane in the representation $(1,2)_{1/2}$ under $\mathrm{SU(2)_L \times SU(2)_R \times U(1)}_{B-L}$, which breaks $\mathrm{SU(2)_R \times U(1)}_{B-L}$ into $\mathrm{U(1)}_Y$. We could also use the more conventional triplet representation under $\mathrm{SU(2)_R}$, which allows obtaining neutrino masses later on. On the right brane, we assume that there is a bi-doublet Higgs field in the representation $(2,2)_0$, which breaks the electroweak symmetry as in the SM: $SU(2)_L \times SU(2)_R \to SU(2)_D$. We then take all the Higgs *vev*'s to infinity in order to decouple the boundary scalars from the theory, and impose the natural boundary conditions as described above. We thus obtain

at $y = 0$:

$$\partial_z(g_{5R}B_\mu + \tilde{g}_5 A_\mu^{R3}) = 0\,, \quad \partial_z A_\mu^{La} = 0\,, \quad A_\mu^{R1,2} = 0\,,$$
$$\tilde{g}_5 B_\mu - g_{5R}A_\mu^{R3} = 0\,; \tag{137}$$

at $y = \pi R$:

$$\partial_z(g_{5R}A_\mu^{La} + g_{5L}A_\mu^{Ra}) = 0\,, \quad \partial_z B_\mu = 0\,,$$
$$g_{5L}A_\mu^{La} - g_{5R}A_\mu^{Ra} = 0\,. \tag{138}$$

As usual, the BCs for the A_5 and B_5 components are the opposite of the 4D gauge field BCs, i.e., all Dirichlet conditions are replaced by Neumann ones and vice versa.

The next step in determining the mass spectrum is to find the right KK decomposition of this model. First, none of the A_5 and B_5 components have a flat BC on both ends. This means that there is no zero mode in these fields and, as we have seen, all the massive scalars are unphysical since they are just gauge artifacts (supplying the longitudinal components of the massive KK towers). The main point to observe about the KK decomposition of the gauge fields is that the BCs mix up the states in the various components. This implies that a single 4D mode lives in several different 5D fields. Because there is no mixing in the bulk and we are currently discussing a flat 5D background, the wave functions are of the form $f_k(y) \propto a\cos M_k y + b\sin M_k y$. If we make the simplifying assumption that $g_{5L} = g_{5R} = g_5$, then the KK decomposition is somewhat simpler than the most general one, and is given by

$$B_\mu(x,y) = g_5\, a_0\gamma_\mu(x) + \tilde{g}_5 \sum_{k=1}^{\infty} b_k \cos\left(M_k^Z(y-\pi R)\right) Z_\mu^{(k)}(x)\,, \tag{139}$$

$$A_\mu^{L3}(x,y) = \tilde{g}_5\, a_0\gamma_\mu(x) - g_5 \sum_{k=1}^{\infty} b_k \frac{\cos(M_k^Z y)}{2\cos(M_k^Z \pi R)} Z_\mu^{(k)}(x)\,, \tag{140}$$

$$A_\mu^{R3}(x,y) = \tilde{g}_5 a_0\gamma_\mu(x) - g_5 \sum_{k=1}^{\infty} b_k \frac{\cos\left(M_k^Z(y-2\pi R)\right)}{2\cos(M_k^Z\pi R)} Z_\mu^{(k)}(x)\,, \tag{141}$$

$$A_\mu^{L\pm}(x,y) = \sum_{k=1}^{\infty} c_k \cos(M_k^W y)\, W_\mu^{(k)\pm}(x)\,, \tag{142}$$

$$A_\mu^{R\pm}(x,y) = \sum_{k=1}^{\infty} c_k \sin(M_k^W y)\, W_\mu^{(k)\pm}(x) \tag{143}$$

(we let $A_\mu^{L,R\pm}$ denote the linear combinations $(A^{L,R1} \mp iA^{L,R2})/\sqrt{2}$).

The BCs further impose a mass spectrum that is made up of a massless photon, the gauge boson associated with the unbroken $U(1)_Q$ symmetry, and some towers of massive charged and neutral gauge bosons, $W^{(k)}$ and $Z^{(k)}$, respectively. The $W^\pm$ masses are solutions of the quantization equation

$$\cos(2M_W\pi R) = 0\,, \tag{144}$$

whence

$$M_k^W = \frac{2k-1}{4R}\,, \qquad k = 1,2,\ldots\,. \tag{145}$$

The quantization equation giving the masses of the neutral gauge bosons is somewhat more complicated due to the mixing of the various U(1) factors of the bulk gauge group:

$$\tan^2(M_Z\pi R) = 1 + \frac{\tilde{g}_5^2}{g_5^2}\,. \tag{146}$$

The KK Z-boson masses are thus given by

$$M_k^Z = \left(M_0 + \frac{k-1}{R}\right)_{k=1,2,\ldots}\,,$$
$$M_k^{Z'} = \left(-M_0 + \frac{k}{R}\right)_{k=1,2,\ldots}\,, \tag{147}$$

where

$$M_0 = \frac{1}{\pi R}\arctan\sqrt{1+\frac{2\tilde{g}_5^2}{g_5^2}}\,.$$

We note that $1/(4R) < M_0 < 1/(2R)$ and hence the Z′'s are heavier than the Z's ($M_k^{Z'} > M_k^Z$). We also see that the lightest Z is heavier than the lightest W ($M_1^Z > M_1^W$), in agreement with the SM spectrum. But the mass ratio W/Z is given by

$$\frac{M_W^2}{M_Z^2} = \frac{\pi^2}{16}\arctan^{-2}\sqrt{1+\frac{g_{4D}'^2}{g_{4D}^2}} \sim 0.85\,, \tag{148}$$

and hence the ρ parameter is

$$\rho = \frac{M_W^2}{M_Z^2\cos^2\theta_W} \sim 1.10\,. \tag{149}$$

To arrive at these expressions, we have assumed that the SM quarks and leptons are localized on the $SU(2)_L \times U(1)_Y$ boundary, leading to the following relations between the 4D and 5D gauge couplings:

$$g_4 = \frac{g_5}{\sqrt{\pi R}}\,, \qquad g_4' = \frac{\sqrt{2}\,\tilde{g}_5}{\sqrt{\pi R}}\,. \tag{150}$$

The W/Z mass ratio is close to its SM value, but the ten percent deviation is still huge compared to the experimental precision. The reason for this deviation is that while the bulk and the right $SU(2)_D \times U(1)_{B-L}$ brane are symmetric under the custodial SU(2) group, the left $SU(2)_L \times U(1)_Y$ brane is not, and the KK wave functions have a significant component around the left brane, which gives rise to the large deviation from $\rho = 1$. Therefore, we must find a way of ensuring that the KK modes of the gauge fields do not 'feel' the left brane very much, but are repelled from there, and only the lightest (almost zero) modes γ, Z, and $W^\pm$ have a large overlap with the left brane.

In summary, the flat Higgsless model suffers from two serious drawbacks: (i) KK excitations of the W and Z bosons are too light, (ii) the deviation of the ρ parameter from its custodial value is too great. We now see that embedding the model in a warped space actually cures these two problems simultaneously.

5.4 Warped Higgsless model with custodial symmetry

5.4.1 Some aspects of the AdS/CFT correspondence. To ensure an unbroken custodial symmetry, we need to devise a set-up such that the KK modes are localized away from the point where the custodial symmetry is broken. We can consider adding large kinetic terms localized on the $SU(2)_L \times U(1)_Y$ boundary, which indeed repeal the wave functions of massive KK gauge bosons [71]. Actually, there is an even simpler way to localize the wave functions: to warp the space. Indeed, in an anti-de Sitter space, the KK wave functions are Bessel functions of order one that are generically exponentially peaked at one end of the interval. From the standpoint of the AdS/CFT correspondence [72, 73], this localization property allows inferring the global and local symmetries left out by particular BCs.

As illustrated in Fig. 16, we consider a gauge symmetry G in the bulk of AdS_5. The BCs on the UV brane break G to a subgroup H that is further broken to H_0 by the BCs on the IR brane. The corresponding 4D CFT has the H gauge invariance and the G/H global symmetry. The interpretation of the IR BCs is that H is actually spontaneously broken down to H_0. If matter is added in the 5D theory, it can be interpreted as composite states of the CFT if it is localized close to the IR brane or as elementary fields coupled to the CFT if it is localized on the UV brane. For instance, a 5D Higgs localized on the IR brane can be seen as the dual version of composite Higgs models [51, 70]. When the Higgs vacuum expectation value is sent to infinity, the 4D theory is more a technicolor-like model. In that sense, the 5D warped Higgsless model can be seen as a weakly coupled dual version of the theory of 'technicolor' [25].

5.4.2 Towards a realistic Higgsless model. From the correspondence discussed above, we can now find the sought theory relatively easily [25]. We want a theory that has the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ global symmetry, with the $SU(2)_L \times U(1)_Y$ subgroup weakly gauged and broken by BCs on the IR brane. To have the full global symmetry, we need to take $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ in the bulk of AdS_5. To ensure the absence of unwanted gauge fields at low energies, we need to break $SU(2)_R \times U(1)_{B-L}$ to $U(1)_Y$

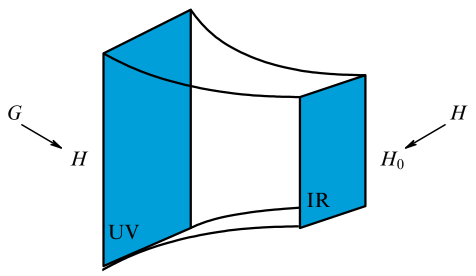


Figure 16. Correspondence between a 5D gauge theory in an AdS space and a 4D CFT. The UV brane is interpreted as a UV cutoff of the CFT, while the IR brane mimics the spontaneous breaking of the conformal symmetry by the CFT. The subgroup H unbroken on the UV brane corresponds to the gauge symmetry of the CFT, while the coset G/H is the global symmetry of the CFT. The strong dynamics of the CFT spontaneously breaks H to H_0 at the IR scale. 5D fields localized close to the IR boundary correspond to composites states of the CFT, while UV matter is given by elementary fields coupled to the CFT.

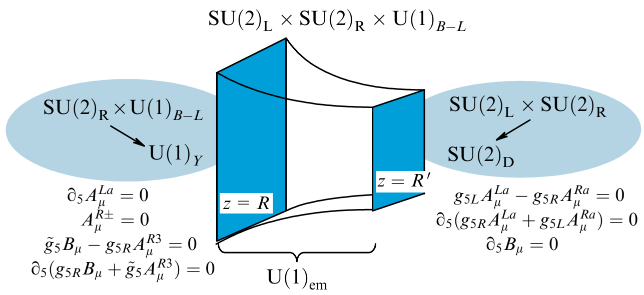


Figure 17. The symmetry-breaking structure of the warped Higgsless model [25]. We consider a 5D gauge theory in the fixed gravitational AdS background. The UV brane is located at $z = R$ and the IR brane is located at $z = R'$. R is the AdS curvature scale. In conformal coordinates, the AdS metric is given by $ds^2 = (R/z)^2 (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2)$.

on the UV brane, which we do through BCs as in the flat case. Finally, the boundary conditions on the TeV brane reduce $SU(2)_L \times SU(2)_R$ to $SU(2)_D$. This setup is illustrated in Fig. 17. We note that it is practically identical to the flat-space toy model considered above, except that the theory is in the AdS space.

The only difference between the flat and the warped Higgsless models is the shape of the wave functions. Indeed, the solution of the bulk equations of motion in the AdS space involves some Bessel functions of order 1:

$$\psi_k^{(A)}(z) = z\left(a_k^{(A)} J_1(q_k z) + b_k^{(A)} Y_1(q_k z)\right), \qquad (151)$$

where A labels the corresponding gauge boson.

Due to the mixing of the various gauge groups, the KK decomposition is slightly complicated, but can be obtained by simply enforcing the BCs with wave functions of form (151):

$$B_\mu(x, z) = g_5\, a_0 \gamma_\mu(x) + \sum_{k=1}^{\infty} \psi_k^{(B)}(z)\, Z_\mu^{(k)}(x), \qquad (152)$$

$$A_\mu^{L3}(x, z) = \tilde{g}_5\, a_0 \gamma_\mu(x) + \sum_{k=1}^{\infty} \psi_k^{(L3)}(z)\, Z_\mu^{(k)}(x), \qquad (153)$$

$$A_\mu^{R3}(x, z) = \tilde{g}_5\, a_0 \gamma_\mu(x) + \sum_{k=1}^{\infty} \psi_k^{(R3)}(z)\, Z_\mu^{(k)}(x), \qquad (154)$$

$$A_\mu^{L\pm}(x, z) = \sum_{k=1}^{\infty} \psi_k^{(L\pm)}(z)\, W_\mu^{(k)\pm}(x), \qquad (155)$$

$$A_\mu^{R\pm}(x, z) = \sum_{k=1}^{\infty} \psi_k^{(R\pm)}(z)\, W_\mu^{(k)\pm}(x). \qquad (156)$$

Here, $\gamma(x)$ is the 4D photon, which has a flat wave function due to the unbroken $U(1)_Q$ symmetry, and $W_\mu^{(k)\pm}(x)$ and $Z_\mu^{(k)}(x)$ are the KK towers of the massive W and Z gauge bosons, the lowest of which are supposed to correspond to the observed $W^\pm$ and Z^0 bosons. Enforcing the BCs with these wave functions leads to the quantization equations, from which the spectrum is obtained. For the W's, we have

$$(R_0 - \tilde{R}_0)(R_1 - \tilde{R}_1) + (\tilde{R}_1 - R_0)(\tilde{R}_0 - R_1) = 0, \qquad (157)$$

where the ratios $R_{0,1}$ and $\tilde{R}_{0,1}$ are given by

$$R_i \equiv \frac{Y_i(MR)}{J_i(MR)}, \qquad \tilde{R}_i \equiv \frac{Y_i(MR')}{J_i(MR')}. \qquad (158)$$

In the leading order in $1/R$ and for $\ln(R'/R) \gg 1$, the lightest solution of this equation for the mass of the $W^\pm$ bosons is

$$M_W^2 \approx \frac{1}{R'^2 \ln(R'/R)} . \quad (159)$$

We note that this result is independent of the 5D gauge coupling and depends only on the scales R and R'. Taking $R = 10^{-19}$ GeV^{-1} yields $R' = 2 \times 10^{-3}$ GeV^{-1}. The equation determining the masses of the KK tower for the Z boson (the states that are mostly A^{L3} or A^{R3}) is given by

$$2\tilde{g}_5^2 (R_0 - \tilde{R}_1)(\tilde{R}_0 - R_1) = g_5^2 \left[(R_0 - \tilde{R}_0)(R_1 - \tilde{R}_1) + (\tilde{R}_1 - R_0)(\tilde{R}_0 - R_1) \right] . \quad (160)$$

The lowest mass of the Z tower is approximately given by

$$M_Z^2 = \frac{g_5^2 + 2\tilde{g}_5^2}{g_5^2 + \tilde{g}_5^2} \frac{1}{R'^2 \ln(R'/R)} . \quad (161)$$

Finally, there is a third tower of states, corresponding to the excited modes of the photon (the particles that are mostly B-type), whose masses are given by

$$R_0 = \tilde{R}_0 . \quad (162)$$

This does not have a light mode [the zero mode corresponding to the massless photon has been explicitly separated in (152)–(154)].

If the SM fermions are localized on the Planck brane, then the leading-order expressions for the effective 4D couplings are

$$\frac{1}{g^2} = \frac{R \ln(R'/R)}{g_5^2} , \qquad \frac{1}{g'^2} = R \ln \frac{R'}{R} \left(\frac{1}{g_5^2} + \frac{1}{\tilde{g}_5^2} \right) , \quad (163)$$

and hence the 4D Weinberg angle is given by

$$\sin\theta_W = \frac{\tilde{g}_5}{\sqrt{g_5^2 + 2\tilde{g}_5^2}} = \frac{g'}{\sqrt{g^2 + g'^2}} \quad (164)$$

(see Sections 5.5 and 5.6 for more details). We can see that in the leading order, the SM expression for the W/Z mass ratio is reproduced in this theory as expected. In fact, the full structure of the SM coupling is reproduced in the leading order in $1/\ln(R'/R)$, which implies that at the leading-log level, there is no S-parameter either. An S-parameter in this language would have manifested itself in an overall shift in the coupling of the Z boson compared to its SM value calculated from the W and γ couplings, which are absent at this order of approximation. The corrections to the SM relations occur in the next order of the log expansion. Since $\ln(R'/R) \sim \mathcal{O}(10)$, this correction could still be too large to match the precision electroweak data. We discuss the issue of electroweak precision observables in Section 5.6

The KK masses of the W bosons (and also the Z boson because of the custodial SU(2) symmetry) are approximately given by

$$M_{W^{(n)}} = \frac{\pi(n + 1/2)}{2R'} , \qquad n = 1, 2, \ldots . \quad (165)$$

It follows that the ratio between the physical W mass and the first KK mode is given by

$$\frac{M_W}{M_{W'}} = \frac{4}{3\pi} \frac{1}{\sqrt{\ln(R'/R)}} . \quad (166)$$

We can see that warping achieves two desirable properties: it enforces the custodial SU(2) and thus automatically generates the correct W/Z mass ratio, and it also pushes up the masses of the KK resonances of W and Z. This implies that we can obtain a theory where the W$'$ and Z$'$ bosons are not so light that they would already be excluded by the LEP or the Tevatron experiments. In the flat model, the mass spectrum involved only one scale and the mass gap between the W and Z bosons and their next KK excitations was too small. In the warped models, the presence of two scales (the size of the extra dimension and the curvature scale of the space) allows increasing the mass gap. The $\ln(R'/R)$ suppression of the W mass compared to the W$'$ mass can actually be easily understood from the naive dimensional analysis and the AdS/CFT correspondence described in Section 5.4.1. Indeed, the W mass originates from the IR spontaneous breaking of gauge symmetry weakly coupled to the CFT, and hence

$$M_W^2 = \frac{g_4^2}{R_{IR}^2} . \quad (167)$$

Furthermore, the 4D gauge coupling is obtained from the 5D one (from NDA, $g_5^2 = R_{UV}$) and the normalization condition of the (flat) wave function of the massless gauge boson:

$$\frac{1}{g_4^2} = \frac{\int_{R_{UV}}^{R_{IR}} dz\, R_{UV}/z}{g_5^2} . \quad (168)$$

Hence,

$$M_W^2 = \frac{1}{R_{IR}^2 \ln(R_{IR}/R_{UV})} . \quad (169)$$

Finally, we can return to the issue of perturbative unitarity in these models. In the flat-space case, we have seen that the unitarity violation scale is basically given by the NDA cutoff scale (126). But in a warped extra dimension, all scales are dependent on the position along the extra dimension, and hence the lowest cutoff scale is at the IR brane, given by

$$\Lambda_{NDA} \sim \frac{24\pi^3}{g_5^2} \frac{R}{R'} . \quad (170)$$

Using our expressions for the 4D couplings and the W and W$'$ masses above, we can see that [68, 74]

$$\Lambda_{NDA} \sim \frac{12\pi^4 M_W^2}{g^2 M_{W'}} . \quad (171)$$

From the above formula, it is clear that the heavier the resonance, the lower the scale where the perturbative unitarity is violated. This also gives a rough estimate, valid up to a numerical coefficient, of the actual scale of non-perturbative physics. An explicit calculation of the scattering amplitude, including inelastic channels, shows that this is indeed the case and the numerical factor is found to be roughly 1/2 [68].

Since the ratio of the W boson to the first KK mode mass squared is of the order of

$$\frac{M_{\rm W}^2}{M_{\rm W'}^2} = \mathcal{O}\left(\frac{1}{\ln(R'/R)}\right), \tag{172}$$

increasing the value of R (corresponding to lowering the 5D UV scale) significantly increases the NDA cutoff. With R chosen to be the inverse Planck scale, the first KK resonance occurs around 1.2 TeV, but for larger values of R, this scale can be safely reduced to below 1 TeV (see Fig. 21).

5.5 Fermion masses

In the SM, quarks and leptons acquire a mass, after EWSB, through their Yukawa couplings to the Higgs boson. In the absence of a Higgs boson, we cannot write any Yukawa coupling and one should expect the fermions to remain massless. However, as for the gauge fields, appropriate boundary conditions force the fermions to acquire a momentum along the extra dimension; this is how the fermions become massive from the 4D standpoint. We now review this construction [25, 71, 75, 76] (see also [77] for earlier works on fermions in an RS warped background). A general discussion on spin-1/2 boundary conditions can be found in [75].

The SM fermions cannot be completely localized on the UV boundary: because the unbroken gauge group on that boundary coincides with the SM $\mathrm{SU}(2)_{\rm L} \times \mathrm{U}(1)_Y$ symmetry, the theory on that brane should be chiral and there is no way for the chiral zero-mode fermions to acquire a mass. The SM fermions cannot stay on the IR brane either, because the unbroken $\mathrm{SU}(2)_{\rm D}$ gauge symmetry imposes an isospin-invariant spectrum and the up-type and down-type quarks become degenerate, as the electron and the electron neutrino are. The only possibility is thus to embed the SM fermions into 5D fields living in the bulk and feeling the gauge symmetry breakings on both boundaries. Since the irreducible spin-1/2 representations of the 5D Lorentz group correspond to a 4-component Dirac spinor, extra fermionic degrees of freedom are needed to complete the SM chiral spinors to 5D Dirac spinors, and we therefore return to a vector-like spectrum. However, as is well known, orbifold-like projections can eliminate half of the spectrum at the lowest KK level to actually provide a 4D effective chiral theory.

5.5.1 Chiral fermions from the orbifold projection/boundary conditions. A 5D Dirac spinor decomposes under the 4D Lorentz subgroup into two two-component spinors (technically speaking, the simplest 5D irreducible spin-1/2 representation breaks up as $(0,1/2)+(1/2,0)$ under the 4D Lorentz subgroup),

$$\Psi = \begin{pmatrix} \chi_\alpha \\ \bar\psi^{\dot\alpha} \end{pmatrix}, \tag{173}$$

in the 5D chiral representation of the Dirac matrices:

$$\Gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar\sigma^\mu & 0 \end{pmatrix}_{\mu=0,1,2,3}, \quad \Gamma^5 = \mathrm{i}\begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix}, \tag{174}$$

where $\sigma^i = -\bar\sigma^i$ are the usual Pauli spin matrices,[11] while $\sigma^0 = \bar\sigma^0 = -\mathbf{1}$.

[11] Explicitly,

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Under the Z_2 orbifold projection $y \sim -y$ discussed in Section 4.1, in order to leave the 5D Dirac equation invariant, Ψ has to satisfy

$$\Psi(-y) = -\mathrm{i}\Gamma^5 \Psi(y), \tag{175}$$

i.e.,

$$\chi(-y) = \chi(y), \quad \psi(-y) = -\psi(y). \tag{176}$$

Therefore, only χ has a KK zero mode:

$$\chi(x,y) = \sum_{n=0}^{\infty} \cos\frac{ny}{R}\,\chi^{(n)}(x), \qquad \psi(x,y) = \sum_{n=1}^{\infty} \sin\frac{ny}{R}\,\psi^{(n)}(x). \tag{177}$$

We now briefly explain how we can recover this result using boundary conditions. With a mass in the bulk, the 5D Lagrangian for Ψ is given by

$$\mathcal{S} = \int \mathrm{d}^5 x \left(\frac{\mathrm{i}}{2}(\bar\Psi\Gamma^M\partial_M\Psi - \partial_M\bar\Psi\Gamma^M\Psi) - m\bar\Psi\Psi\right), \tag{178}$$

which in 4D components is written as[12]

$$\mathcal{S} = \int \mathrm{d}^5 x \Big(-\mathrm{i}\bar\chi\bar\sigma^\mu\partial_\mu\chi - \mathrm{i}\psi\sigma^\mu\partial_\mu\bar\psi + (\psi\overleftrightarrow{\partial_5}\chi - \bar\chi\overleftrightarrow{\partial_5}\bar\psi) + m(\psi\chi + \bar\chi\bar\psi)\Big), \tag{179}$$

where $\overleftrightarrow{\partial_5} = (1/2)\big(\overrightarrow{\partial_5} - \overleftarrow{\partial_5}\big)$. The variation of this 5D Lagrangian leads to the bulk equations of motion

$$-\mathrm{i}\bar\sigma^\mu\partial_\mu\chi - \partial_5\bar\psi + m\bar\psi = 0, \quad -\mathrm{i}\sigma^\mu\partial_\mu\bar\psi + \partial_5\chi + m\chi = 0. \tag{180}$$

In addition, requiring that the variation of the Lagrangian at the boundary also vanish gives

$$-\delta\psi\chi + \psi\delta\chi + \delta\bar\chi\bar\psi - \bar\chi\delta\bar\psi = 0. \tag{181}$$

Naively, one might think that because there are two independent spinors χ and ψ, two independent boundary conditions for each spinor would be required. But because the bulk equations of motion are only first order, there is only one integration constant. Therefore, for the Dirac pair $(\chi, \bar\psi)$, there is only one boundary condition $f(\chi,\psi) = 0$ at each boundary, where f is some function of the spinors and their conjugates. The form of f together with the bulk equations of motion in (180) then determines all the arbitrary coefficients in the complete solution of the spinor equation of motion on the interval. For instance, we can require that the spinor ψ vanish on both boundaries. Then the bulk equations of motion yield

$$(\partial_5 + m)\chi\big|_{0,\pi R} = 0. \tag{182}$$

[12] Usually, the terms with left-acting derivatives are integrated by parts, such that all derivatives act to the right. But because we are working here in a compact space with boundaries, the integration by parts produces boundary terms that cannot be neglected. We note that both (178) and (179) are Hermitian.

Table 3. Embedding of the SM fermions into 5D Dirac spinors.

Particle	Bulk $L \times R \times (B-L)$	UV brane $L \times Y$	IR brane $D \times (B-L)$	Q_{em}
$\begin{pmatrix}\chi_u \\ \chi_d\end{pmatrix}_L$	$(\square, 1, 1/6)$	$(\square, 1/6)$	$(\square, 1/6)$	$2/3$, $-1/3$
$\begin{pmatrix}\psi_u \\ \psi_d\end{pmatrix}_L$	$(\bar{\square}, 1, -1/6)$	$(\bar{\square}, -1/6)$	$(\bar{\square}, -1/6)$	$-2/3$, $1/3$
$\begin{pmatrix}\chi_u \\ \chi_d\end{pmatrix}_R$	$(1, \square, 1/6)$	$(1, 2/3)$, $(1, -1/3)$	$(\square, 1/6)$	$2/3$, $-1/3$
$\begin{pmatrix}\psi_u \\ \psi_d\end{pmatrix}_R$	$(1, \bar{\square}, -1/6)$	$(1, -2/3)$, $(1, 1/3)$	$(\bar{\square}, -1/6)$	$-2/3$, $1/3$

$Q_{em} = Y + T_{3L} \qquad Y = (B-L) + T_{3R}$

Note. We have indicated the quantum numbers of the different components under the bulk $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetry, the subgroup $SU(2)_L \times U(1)_Y$ that remains unbroken on the UV boundary, the subgroup $SU(2)_D \times U(1)_{B-L}$ unbroken on the IR brane, and, finally, the electric charge. The shaded spinors are the fields with the right quantum numbers to be identified as massless SM fermions, while the other spinors correspond to partners needed to complete 5D Dirac spinors. The latter become massive by the orbifold projection/boundary conditions. Through the Dirac mass added at the IR boundary, there is a mixing between the would-be zero modes and some partners, and the fermion that is to be eventually identified as the SM u_L is a mixture of χ_{u_L} and a small amount of χ_{u_R}. Since this last field has incorrect SM quantum numbers, we would end up with deviations in the couplings of the fermions to the gauge bosons. These deviations are particularly sizable for the third generation due to the heaviness of the top quark.

Solving the equations of motion with these boundary conditions results in a zero mode for χ, but not for ψ. That is, the low-energy theory is chiral.

As with gauge and scalar fields, there is a tower of massive Dirac fields in the 4D effective theory. The spectrum is obtained by solving the bulk equations, which dictates the general form of the wave functions, and by enforcing the boundary conditions. We perform this KK decomposition explicitly. The 5D spinors χ and ψ can be written as a sum of products of 4D KK Dirac fermions with 5D wave functions:

$$\chi = \sum_n g_n(y)\, \chi_n(x)\,, \qquad \bar{\psi} = \sum_n f_n(y)\, \bar{\psi}_n(x)\,. \tag{183}$$

The KK fermions satisfy the 4D Dirac equation

$$-\mathrm{i}\bar{\sigma}^\mu \partial_\mu \chi^{(n)} + m_n \bar{\psi}^{(n)} = 0\,, \qquad -\mathrm{i}\sigma^\mu \partial_\mu \bar{\psi}^{(n)} + m_n \chi^{(n)} = 0\,. \tag{184}$$

Substituting this decomposition in the 5D bulk equations of motion gives the equations

$$g_n' + m g_n - m_n f_n = 0\,, \qquad f_n' - m f_n + m_n g_n = 0\,. \tag{185}$$

The standard approach to solving this system of equations is to combine the two first-order coupled equations into two second-order uncoupled wave equations:

$$g_n'' + (m_n^2 - m^2) g_n = 0\,, \qquad f_n'' + (m_n^2 - m^2) f_n = 0\,. \tag{186}$$

The solution is simply a sum of sines and cosines, with the coefficients determined by reimposing the first-order equations and imposing the boundary conditions. For instance, if we require that $\psi = 0$ at both $y = 0$ and $y = \pi R$, we obtain

$$m_n = \sqrt{m^2 + \frac{n^2}{R^2}}\,, \qquad n = 1, 2, \ldots, \tag{187}$$

$$f_n(y) = a_n \sin\frac{ny}{R}\,, \tag{188}$$

$$g_n(y) = \frac{a_n}{m_n}\left(\frac{n}{R}\cos\frac{ny}{R} - m \sin\frac{ny}{R}\right), \tag{189}$$

and the remaining coefficient a_n is fixed by the normalization condition[13]

$$\int_0^{\pi R} \mathrm{d}y\, f_n^2(y) = 1\,. \tag{190}$$

Besides the massive spectrum, the boundary conditions also allow a zero mode for χ:

$$g_0(y) = \left[\frac{2m}{1 - \exp(-2m\pi R)}\right]^{1/2} \exp(-my)\,. \tag{191}$$

We note that the 5D mass does not contribute to the mass of the lightest fermion (which remains massless because of chirality), but dictates the shape of its wave function.

In conclusion, an orbifold projection or, equivalently, appropriate boundary conditions allow obtaining a 4D chiral spectrum from a 5D theory. This way, we can embed the SM quarks and leptons into 5D Dirac spinors following Table 3.

5.5.2 Fermions in the AdS background. In principle, when one is dealing with fermions in a nontrivial background, one needs to work with the 'square-root' of the metric, also known as vielbeins, and to introduce the spin connection to covariantize derivatives. Fortunately, in an AdS background,[14] the spin connection drops out from the spin-1/2

[13] The normalization of a 4D Dirac fermion actually imposes *two* equations:

$$\int_0^{\pi R} \mathrm{d}y\, f_n^2(y) = 1 \quad \text{and} \quad \int_0^{\pi R} \mathrm{d}y\, g_n^2(y) = 1\,.$$

However, thanks to the quantization equation, the second equation is redundant. This can be easily checked in the simple example presented here. In more complicated cases, the redundancy of the two normalization conditions is a good consistency check that the right KK decomposition has been obtained.

[14] We recall that in conformal coordinates, the AdS metric is given by

$$\mathrm{d}s^2 = \left(\frac{R}{z}\right)^2 \left(\eta_{\mu\nu}\, \mathrm{d}x^\mu\, \mathrm{d}x^\nu - \mathrm{d}z^2\right)$$

(R is the AdS curvature scale).

action, which simply becomes

$$\mathcal{S} = \int d^5x \, \frac{R^4}{z^4} \left(-i\bar{\chi}\bar{\sigma}^\mu \partial_\mu \chi - i\psi\sigma^\mu \partial_\mu \bar{\psi} + \left(\psi \overleftrightarrow{\partial_5} \chi - \bar{\chi} \overleftrightarrow{\partial_5} \bar{\psi}\right) + \frac{c}{z} (\psi\chi + \bar{\chi}\bar{\psi}) \right),$$

where the coefficient $c = mR$ is the bulk Dirac mass in units of the AdS curvature [and, as before, $\overleftrightarrow{\partial_5} = (\overrightarrow{\partial_5} - \overleftarrow{\partial_5})/2$]. The bulk equations of motion are

$$-i\bar{\sigma}^\mu \partial_\mu \chi - \partial_5 \bar{\psi} + \frac{c+2}{z} \bar{\psi} = 0,$$

$$-i\sigma^\mu \partial_\mu \bar{\psi} + \partial_5 \chi + \frac{c-2}{z} \chi = 0.$$

The KK decomposition is performed as in the flat case (183) with the wave functions now satisfying the coupled first-order differential equations

$$f_n' + m_n g_n - \frac{c+2}{z} f_n = 0, \qquad g_n' - m_n f_n + \frac{c-2}{z} g_n = 0, \tag{192}$$

which can be combined into uncoupled second-order differential equations

$$f_n'' - \frac{4}{z} f_n' + \left(m_n^2 - \frac{c^2 - c - 6}{z^2} \right) f_n = 0, \tag{193}$$

$$g_n'' - \frac{4}{z} g_n' + \left(m_n^2 - \frac{c^2 + c - 6}{z^2} \right) g_n = 0. \tag{194}$$

The solutions are now linear combinations of Bessel functions, as opposed to sin and cos functions:

$$g_n(z) = z^{5/2} \left(A_n J_{c+1/2}(m_n z) + B_n Y_{c+1/2}(m_n z) \right), \tag{195}$$

$$f_n(z) = z^{5/2} \left(C_n J_{c-1/2}(m_n z) + D_n Y_{c-1/2}(m_n z) \right). \tag{196}$$

First-order bulk equations of motion (192) further impose the conditions

$$A_n = C_n, \qquad B_n = D_n. \tag{197}$$

The remaining undetermined coefficients are fixed by the boundary conditions and the wave function normalization.

Finally, when boundary conditions permit, there can also be a zero mode. For instance, if $\psi|_{R,R'} = 0$, the zero mode is given by

$$g_0(y) = A_0 \left(\frac{z}{R} \right)^{2-c}, \qquad f = 0. \tag{198}$$

The coefficient A_0 is determined by the normalization condition

$$\int_R^{R'} dz \left(\frac{R}{z} \right)^5 \frac{z}{R} A_0^2 \left(\frac{z}{R} \right)^{4-2c} = A_0^2 \int_R^{R'} \left(\frac{z}{R} \right)^{-2c} dz = 1. \tag{199}$$

To understand from these equations where the fermions are localized, we study the behavior of this integral as we vary the limits of integration. If we send R' to infinity, we see that the integral remains convergent if $c > 1/2$, and the fermion is then localized on the UV brane. If we send R to zero, the integral is convergent if $c < 1/2$, and the fermion is localized on the IR brane. The value of the Dirac mass determines whether the fermion is localized towards the UV or IR branes. We note that the opposite choice of boundary conditions, which yields a zero mode ($\chi|_{R,R'} = 0$), results in a zero mode solution for ψ with localization at the UV brane when $c < -1/2$ and at the IR brane when $c > -1/2$. The interesting feature in the warped case is that the localization transition occurs not when the bulk mass passes through zero but at the points where $|c| = 1/2$. This is due to the curvature effects of the extra dimension. The CFT interpretation of the c parameter is an anomalous dimension that controls the amount of compositeness of the fermion [73].

5.5.3 Higgsless fermion masses. We have already explained how to embed the SM fermions into 5D Dirac spinors. To obtain the sought zero modes, the following boundary conditions have to be imposed:

$$\begin{pmatrix} \chi_{u_L} \\ \bar{\psi}_{u_L} \\ \chi_{d_L} \\ \bar{\psi}_{d_L} \end{pmatrix} \begin{matrix} + & + \\ - & - \\ + & + \\ - & - \end{matrix} \qquad \begin{pmatrix} \chi_{u_R} \\ \bar{\psi}_{u_R} \\ \chi_{d_R} \\ \bar{\psi}_{d_R} \end{pmatrix} \begin{matrix} - & - \\ + & + \\ - & - \\ + & + \end{matrix}, \tag{200}$$

with the + and − referring to Neumann and Dirichlet boundary conditions and the first/second sign denoting the BC on the UV/IR brane, respectively. These boundary conditions give massless chiral modes that match the SM fermion content. But u_L, d_L, u_R, and d_R are all massless at this stage, and we need to lift the zero modes to achieve the SM mass spectrum (Fig. 18). Although simply giving certain boundary conditions for the fermions allows lifting these zero modes, in the following discussion we talk about boundary operators and the boundary conditions that these operators induce. There are some subtleties in dealing with boundary operators for fermions. These arise from the fact that the fields themselves are not always continuous in the presence of a boundary operator because the equations of motion for fermions are first order. The most straightforward approach is to enforce the boundary conditions that give the zero modes as shown in Eqn (200) on the real boundary at $z = R, R'$ while the boundary operators are added on a fictitious brane at a distance ϵ away from it. The new boundary condition is then obtained by taking ϵ to be small. This physical picture is quite helpful in understanding what the different boundary conditions can do. The details can be found in [75].

The IR brane being vector-like, we can now form an $SU(2)_D$ mass term that mixes the L and R SM helicities.

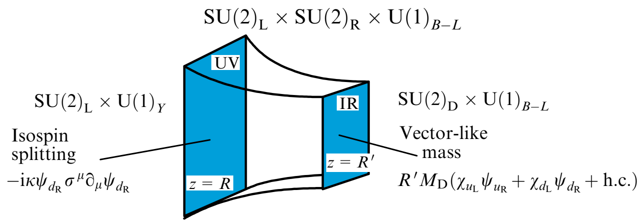


Figure 18. Brane-localized operators needed to increase the masses of the SM fermions.

However, this Dirac mass term has to be the same for the up and down quarks (the mass term is isospin invariant). Fortunately, the $SU(2)_R$ invariance is broken on the UV brane and we can introduce operators there that distinguish between u_R and d_R. Technically, the effect of the brane-localized operators is to modify the BCs. Explicitly, the IR Dirac mass affect the BCs as follows:

$$\begin{array}{lcl} \chi_L+ & & \\ \psi_L - \psi_L|_{IR}=0 & \text{discontinuities} & \psi_L|_{IR} = -M_D R' \psi_R|_{IR} \\ & M_D \quad \text{in} & \\ \chi_R - \chi_R|_{IR} = 0 & \Longrightarrow & \chi_R|_{IR} = M_D R' \chi_L|_{IR} \\ & \chi_L \& \psi_R & \\ \psi_R+ & & \end{array} .$$

In the same way, the UV brane operator modifies the BCs as follows:

$$\begin{array}{lcl} & \text{discontinuities} & \\ \chi_{u_R}- & & \\ \chi_{u_R}|_{UV}=0 & \kappa \quad \text{in} & \chi_{u_R}|_{UV} = \kappa m \psi_{u_R}|_{UV} \, . \\ \psi_{u_R}+ & \Longrightarrow & \\ & \psi_{u_R} & \end{array}$$

It is now easy to enforce these modified boundary conditions using the general form of wave functions (195) and (196) that satisfy the bulk equations of motion. For fermions localized towards the UV brane ($c_L > 1/2$ and $c_R < -1/2$), we obtain the approximate expression

$$m \approx \frac{\sqrt{2c_L - 1}}{\sqrt{\kappa^2 - 1/(2c_R + 1)}} M_D \left(\frac{R_{UV}}{R_{IR}} \right)^{c_L - c_R - 1} . \tag{201}$$

The spectrum of the light generations of quarks can be easily reproduced along these lines. But the top-quark mass poses a difficulty. Indeed, increasing M_D does not arbitrarily increase the fermion mass because it becomes saturated: the situation is similar to what happens to a large Higgs *vev* localized on the boundary: the gauge boson masses remain finite even when the *vev* is sent to infinity. The maximum value of the fermion mass can be inferred by noting that in the infinite-M_D limit, there is a *chirality flip* in the BCs, which become

$$\begin{array}{llcll} \chi_L + & & & & \chi_L - \\ \psi_L - & \psi_L|_{IR} = -M_D R' \psi_R|_{IR} & M_D \to \infty & \psi_R|_{IR} = 0 & \psi_L + \\ \chi_R - & \chi_R|_{IR} = M_D R' \chi_L|_{IR} & \Longrightarrow & \chi_L|_{IR} = 0 & \chi_R + \\ \psi_R + & & & & \psi_R - \end{array}$$

and the corresponding mass is

$$m^2 = \frac{2}{R'^2 \ln (R'/R)} = 2M_W^2 \, , \tag{202}$$

where in the last equality we used the expression for the W mass in terms of R and R' and assumed $g_{5R} = g_{5L}$.

If we want to go above this saturated mass, we need to localize the fermions towards the IR brane. But even in this case, a sizable Dirac mass term on the TeV brane is needed to obtain a heavy enough top quark. The consequence of this mass term is the boundary condition for the bottom quarks:

$$\chi_{bR} = M_D R' \chi_{bL} \, . \tag{203}$$

This implies that if $M_D R' \sim 1$, then the left-handed bottom quark also has a sizable component in an $SU(2)_R$ multiplet, which, however, has a coupling to Z that is different from the SM value. Thus, there is a large deviation in the $Zb_L\bar{b}_L$ coupling. We note that the same deviation does not appear in the $Zb_R\bar{b}_R$ coupling, since the extra kinetic term introduced on the Planck brane to split the top and bottom quarks implies that the right-handed b quark mostly consists of the induced fermion on the Planck brane, which has the correct coupling to the Z boson.

The only way of circumventing this problem is to increase the value of $1/R'$, and thus decrease the necessary mixing on the TeV brane needed to obtain a heavy top quark. One way of increasing the value of $1/R'$ is by increasing the ratio g_{5R}/g_{5L} (at the price of also making the gauge KK modes heavier and hence the theory more strongly coupled). Another possibility for increasing the value of $1/R'$ is to separate the physics responsible for the electroweak symmetry breaking from that responsible for the generation of the top-quark mass. In technicolor models, this is usually achieved by introducing a new strong interaction called topcolor. In the extra-dimensional setup, this would correspond to adding two separate AdS_5 bulks, which meet at the Planck brane [65]. One bulk would then be mostly responsible for electroweak symmetry breaking and the other for generating the top-quark mass. The details of such models have been worked out in [65]. The main consequences of such models would be the necessary appearance of an isotriplet pseudo-Goldstone boson called the top-pion; depending on the detailed implementation of the model, there could also be a scalar particle (called the top-Higgs). This top-Higgs would, however, not play a major role in the unitarization of the gauge boson scattering amplitudes, but rather serve as the source for the top-quark mass only.

It was suggested recently that using a different embedding of the bottom and top left quarks into the $(2,2)_{2/3}$ representation of $SU(2)_L \times SU(2)_R \times U(1)_X$ can help to keep the $Zb_L\bar{b}_L$ coupling under control [82].

5.6 Electroweak precision tests

To compare Higgsless models to precision electroweak measurements, we need to compute the Peskin–Takeuchi parameters S, T, and U [71, 74, 78–81]. We use such parameters to fit the Z-pole observables at LEP1. In Ref. [29], Barbieri et al. proposed an enlarged set of parameters to also take differential cross section measurements at LEP2 into account. However, the only new information contained in the new parameters is the limit on four-fermion operators generated by the exchange of KK bosons, which we take into account to bound the lighter resonances at LEP2 and Tevatron. Effectively, our S, T, and U are linear combinations of the parameters in [29].

In Ref. [80], we computed the oblique corrections in the standard way, in terms of mass eigenstates, in the limit where the light fermions are localized on the Planck brane. The only relevant technical point in the calculation is the matching of the 4D gauge couplings. Indeed, if we write the couplings of the fermions, only two quantities are independent of the overall Z and W normalizations and are completely fixed by the boundary condition. These are the electric charge and the ratio between the hypercharge and T_3, that is, the couplings to the Z boson. Matching such quantities with the SM predictions, it is possible to disguise all the corrections as oblique parameters.

In the basic model with $g_{5L} = g_{5R} = g_5$ and vanishing localized kinetic terms, the leading contribution to S in the $1/\ln(R'/R) \approx 0.3$ expansion is

$$S \approx \frac{6\pi}{g^2 \ln(R'/R)} \approx 1.15\,, \tag{204}$$

while $T \approx U \approx 0$. This value of S is clearly too large to be compared with the experimental result.[15]

But the theory has more parameters: for instance, kinetic operators can be localized on the boundaries for the locally unbroken gauge symmetries:

$$\mathcal{L} = -\left[\frac{r}{4}\, {W^L_{\mu\nu}}^2 + \frac{r'}{4}\, {B^Y_{\mu\nu}}^2\right]\delta(z-R) - \frac{R'}{R}\left[\frac{\tau'}{4}\, {B_{\mu\nu}}^2 + \frac{\tau}{4}\, {W^D_{\mu\nu}}^2\right]\delta(z-R')\,.$$

We first study the effect of *asymmetric bulk gauge couplings* and *Planck brane kinetic terms*. The leading contribution to S is

$$S \approx \frac{6\pi}{g^2 \ln(R'/R)}\, \frac{2}{1 + g_{5R}^2/g_{5L}^2}\, \frac{1}{1 + r/\big(R \ln(R'/R)\big)}\,, \tag{205}$$

where, again, $T \approx U \approx 0$. Now, in the case of a large g_{5R}/g_{5L} ratio [or a large $\mathrm{SU(2)_L}$ kinetic term], S is suppressed. However, the W mass squared is also parametrically multiplied by the same factor. This means that the smaller the S parameter, the larger the scale of KK resonances, $1/R'$. Hence, in order to have small corrections, we possibly enter a strong coupling regime, where the above calculation becomes meaningless.

Another set of parameters is the *TeV kinetic terms*. Their contribution is more complicated, and we therefore show some results at the leading order for $\tau, \tau' \ll R \ln(R'/R)$. The $\mathrm{SU(2)_D}$ kinetic term appears in the linear order, and effectively multiplies Eqn (205) by the factor $1 + \tau/R$. On the other hand, the $\mathrm{U(1)}_{B-L}$ kinetic term contributes in the quadratic order. If only τ' is turned on, then

$$S \approx \frac{6\pi}{g^2 \ln(R'/R)} - \frac{8\pi}{g^2}\left(1 - \left(\frac{g'}{g}\right)^2\right)\frac{\tau'^2}{\big(R \ln(R'/R)\big)^2}\,, \tag{206}$$

$$T \approx -\frac{2\pi}{g^2}\left(1 - \left(\frac{g'}{g}\right)^4\right)\frac{\tau'^2}{\big(R \ln(R'/R)\big)^2}\,, \tag{207}$$

and $U \approx 0$. Hence, S vanishes at $\tau' \approx 0.15\, R \ln(R'/R)$. However, another effect is to make one Z$'$ lighter, namely the one that couples with the hypercharge.

We also numerically scanned the parameter space to seek a region where the model is not ruled out. For different values of g_{5R}/g_{5L},[16] we scanned the $\tau - \tau'$ space (Fig. 19). With $|S|$ and $|T|$ both required to be smaller than 0.3, there is an allowed region only for the large ratio $g_{5R}/g_{5L} > 2.5$, where the theory is most likely strongly coupled. These results are in agreement with similar studies in [79] and [29].

As originally proposed, the model does not seem to be upheld by the experiments, if we want strong coupling to occur above 3 TeV. However, this is not the end of the story since there is a solution [74, 84] to the S problem with additional beneficial side-effects. It has been known for a long time in Randall – Sundrum models with a Higgs boson that the effective S parameter is large and negative [85] if the fermions are localized on the TeV brane, as originally proposed. When the fermions are localized on the Planck brane, the contribution to S is positive and, hence, for some intermediate localization, the S parameter vanishes, as was first pointed out for RS models by Agashe et al. [70]. The reason for this is fairly simple. Because the W and Z wave functions are approximately flat and the gauge KK mode wave functions are orthogonal to them, the overlap of a gauge KK mode with two fermions approximately vanishes when the fermion wave functions are also approximately flat. The coupling of the gauge KK modes to the fermions induces a shift in the S parameter, and therefore, for approximately flat fermion wave functions, the S parameter must be small. We note that reducing the coupling to gauge KK modes not only reduces the S parameter but also weakens the experimental constraints on the existence of light KK modes. This case of delocalized bulk fermions is not covered by the no-go theorem in [29], since it was assumed there that the fermions are localized on the Planck brane.

To quantify these statements, it suffices to consider a toy model where all three families of fermions are massless and have a universal delocalized profile in the bulk. Before giving some numerical results, it is useful to understand the analytic behavior of S in interesting limits. For fermions almost localized on the Planck brane, it is possible to expand the result for the S parameter in powers of $(R/R')^{2c_\mathrm{L}-1} \ll 1$. The leading terms, also expanded in powers of $1/\ln$, are

$$S = \frac{6\pi}{g^2 \ln(R'/R)}\left(1 - \frac{4}{3}\,\frac{2c_\mathrm{L} - 1}{3 - 2c_\mathrm{L}}\left(\frac{R}{R'}\right)^{2c_\mathrm{L}-1} \ln\frac{R'}{R}\right), \tag{208}$$

and $U \approx T \approx 0$. The above formula is actually valid for $1/2 < c_\mathrm{L} < 3/2$. For $c_\mathrm{L} > 3/2$, the corrections are of the order of $(R'/R)^2$ and are numerically negligible. As we can see, as soon as the fermion wave function starts leaking into the bulk, S decreases.

Another interesting limit is when the profile is almost flat, $c_\mathrm{L} \approx 1/2$. In this case, the leading contributions to S are

$$S = \frac{2\pi}{g^2 \ln(R'/R)}\left(1 + (2c_\mathrm{L} - 1)\,\ln\frac{R'}{R} + \mathcal{O}\big((2c_\mathrm{L} - 1)^2\big)\right). \tag{209}$$

In the flat limit $c_\mathrm{L} = 1/2$, S is already suppressed by a factor of 3 with respect to the Planck-brane localization case. Moreover, the leading terms cancel at

$$c_\mathrm{L} = \frac{1}{2} - \frac{1}{2\ln(R'/R)} \approx 0.487\,. \tag{210}$$

For $c_\mathrm{L} < 1/2$, S becomes large and negative and, in the limit of TeV-brane localized fermions ($c_\mathrm{L} \ll 1/2$),

$$S = -\frac{16\pi}{g^2}\,\frac{1 - 2c_\mathrm{L}}{5 - 2c_\mathrm{L}}\,, \tag{211}$$

[15] Actually, this number should not be compared with the usual SM fit, but we should disentangle the contribution of the Higgs boson. Namely, it suffices to make the fit assuming a large Higgs mass, equal to the cut-off of the theory [29]. We also neglect loop corrections from the gauge KK modes.

[16] Using the Planck kinetic terms instead would only result in slightly different Z$'$ couplings, and hence different exclusion plots.

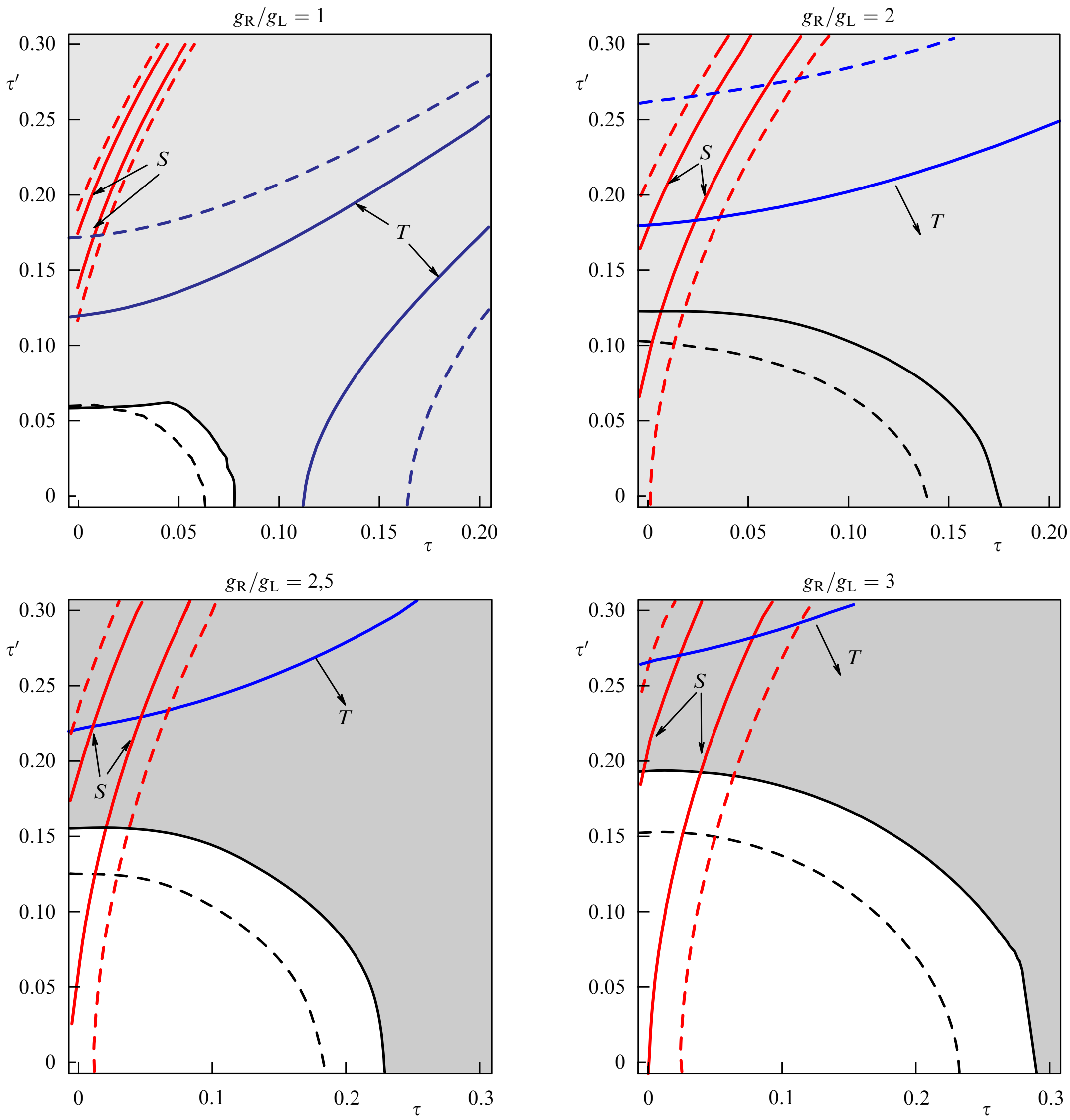


Figure 19. Combined plots of the experimental constraints on Higgsless models for different values of the g_{5R}/g_{5L} ratio, in the parameter space $\tau - \tau'$ [normalized by $R \ln(R'/R)$]. The solid contours for S (red) and T (blue) are at 0.25; the dashed contours are at 0.5. The black solid (dashed) line corresponds to a deviation in the differential cross section of 3% (2%) at LEP2. The shaded region is excluded by a deviation larger that 3% at LEP and/or direct search at Run1 of the Tevatron.

while in the limit as $c_L \to -\infty$,

$$T \to \frac{2\pi}{g^2 \ln(R'/R)} (1 + \tan^2 \theta_W) \approx 0.5 \,, \tag{212}$$

$$U \to -\frac{8\pi}{g^2 \ln(R'/R)} \frac{\tan^2 \theta_W}{2 + \tan^2 \theta_W} \frac{1}{c_L} \approx 0 \,. \tag{213}$$

In Fig. 20, we show the numerical results for the oblique parameters as functions of c_L. We can see that after vanishing at $c_L \approx 1/2$, S becomes negative and large, while T and U remain smaller. With R chosen to be the inverse Planck scale, the first KK resonance occurs around 1.2 TeV, but this scale can be safely reduced to below 1 TeV for larger values of R. Such resonances are weakly coupled to almost flat fermions and can easily avoid the strong limits from direct searches at LEP or Tevatron. If we imagine that the AdS space is a dual description of an approximate conformal field theory (CFT), then $1/R$ is the scale where the CFT is no longer approximately conformal and perhaps becomes asymptotically free. Thus, it is quite reasonable to think that the scale $1/R$ would be much smaller than the Planck scale.

In Fig. 21, we plot the value of the NDA scale in (126) and the mass of the first resonance in the $(c_L - R)$ plane. Increasing R also affects the oblique corrections. However, while it is always possible to reduce S by delocalizing the fermions, T increases and puts a limit on how far R can be increased. We also see from Fig. 21 that in the region where $|S| < 0.25$, the coupling of the first resonance with the light fermions is generically suppressed to less than 10% of the SM value. This means that the LEP limit of 2 TeV for SM-like Z$'$ resonances is also decreased by a factor of 10 at least (the correction to the differential cross section is roughly proportional to $g^2/M_{Z'}^2$). In the end, values of R as large as 10^{-7} GeV^{-1} are allowed, where the resonance masses are around 600 GeV. Therefore, following the analysis in [68], even if we take a factor of roughly 1/4 in the NDA scale into account, we see that the appearance of the strong coupling regime can be delayed up to 10 TeV.

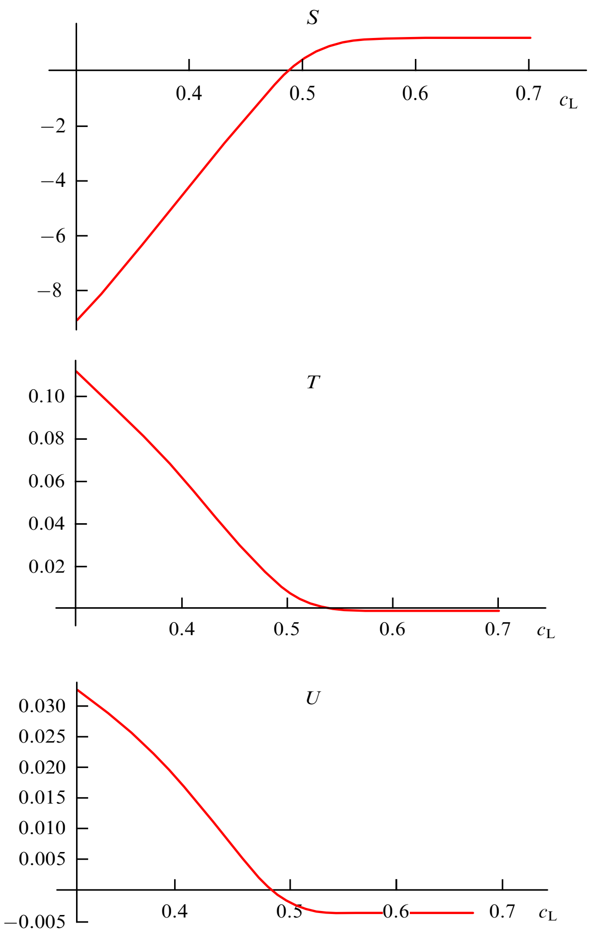


Figure 20. Plots of the oblique parameters as functions of the bulk mass of the reference fermion. The values on the right correspond to localization on the Planck brane. S vanishes at $c = 0.487$.

While the oblique corrections can be kept under control, at the price of some conspiracy in the localization of the SM quarks and leptons along the extra dimension, it is fair to say that, to date, the major challenge facing Higgsless models is actually the incorporation of the third family of quarks. A large top-quark mass can be obtained but at the price of distorting the coupling of the bottom quarks to the Z boson. One way to go could be to introduce a multi-throat setup [65], which allows different scales. It seems also that using a different embedding of the bottom and top left quarks into a $(2,2)_{2/3}$ of $SU(2)_L \times SU(2)_R \times U(1)_X$ can help to keep the $Zb_L\bar{b}_L$ coupling under control [82].

5.7 Collider signatures

The nonobservation of a physical scalar Higgs would be the first indication of a Higgsless scenario. Yet, *the absence of proof is not the proof of absence*, and some other models exist in which the Higgs boson is unobservable at the LHC, and we need to find other distinctive features of Higgsless models. This section closely follows the original works [83, 86] and reviews [57, 58].

The main predictions of Higgsless scenarios are

• the absence of a Higgs boson;

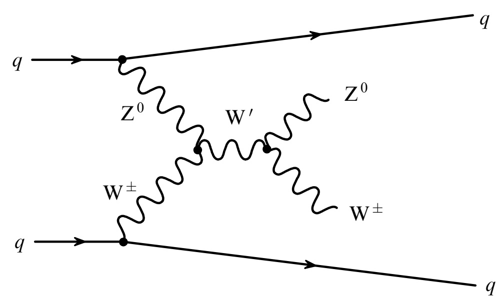
• the presence of spin-1 KK resonances with the W, Z quantum numbers;

• some slight deviations in the universality of the light fermion couplings to the SM gauge bosons; and

• some deviations in gauge boson self-interactions compared to the SM.

5D Higgsless models are nonrenormalizable and become strongly coupled at some cutoff scale Λ. But Higgsless models have been devised to push Λ high enough and to avoid any trouble with the EW precision test, which also means that the strong coupling sector will not be observable at the LHC. Still, beyond the SM spectrum, additional weakly coupled states are required to increase Λ, and they should be observable.

Many realizations of Higgsless models have been proposed, differing in the way the SM fermions are introduced or even in the number of extra dimensions. All these models have different particular signatures. However, the fundamental mechanism by which Λ is increased is a feature common to all these models: new massive spin-1 particles, with the same quantum numbers as the SM gauge bosons, appear at the TeV scale and their couplings to W, Z, and γ obey unitarity sum rules like (116)–(121), enforcing the cancellation of the energy-growing contributions to the scattering amplitudes of the longitudinal W and Z bosons. Vector boson fusion processes thus provide a model-independent test of the Higgsless scenario.



Generically, the sum rules are saturated by the inclusion of the first/a few resonance(s). Moreover, as required by the smallness of the oblique corrections, the couplings of the KK gauge bosons to the light SM fermions are also small. Therefore, model-independently, we can predict the existence of narrow and light resonances in the scattering of W and Z bosons and at least one of these resonances has to appear below approximately 1 TeV, because otherwise it would be inefficient to restore unitarity. For instance, the authors of [86] focused on the $W_L^\pm Z_L \to W_L^\pm Z_L$ elastic scattering (charged resonances like W′ are present and the final state is easily disentangled from the background).

In Fig. 22, we show the WZ elastic cross section and the number of events per 100 GeV bin in the $2j + 3l + \nu$ channel at the LHC with the integrated luminosity 300 fb^{-1} and appropriate cuts. It is found that with 10 fb^{-1} of data, corresponding to one year of running at low luminosity, the LHC will probe a Higgsless W′ up to 550 GeV, while covering the whole preferred range up to 1 TeV will require 60 fb^{-1}.

While a Higgsless W′ could hardly escape detection at the LHC, we will have to wait for a linear collider (the ILC) to precisely measure its couplings and thus to experimentally check the saturation of the unitarity sum rules.

Another way to seek the W′ and Z′ KK resonances is through Drell–Yan processes [83]. However, these analyses are more model-dependent since they rely on the couplings of the way the SM fermions are embedded in the model.

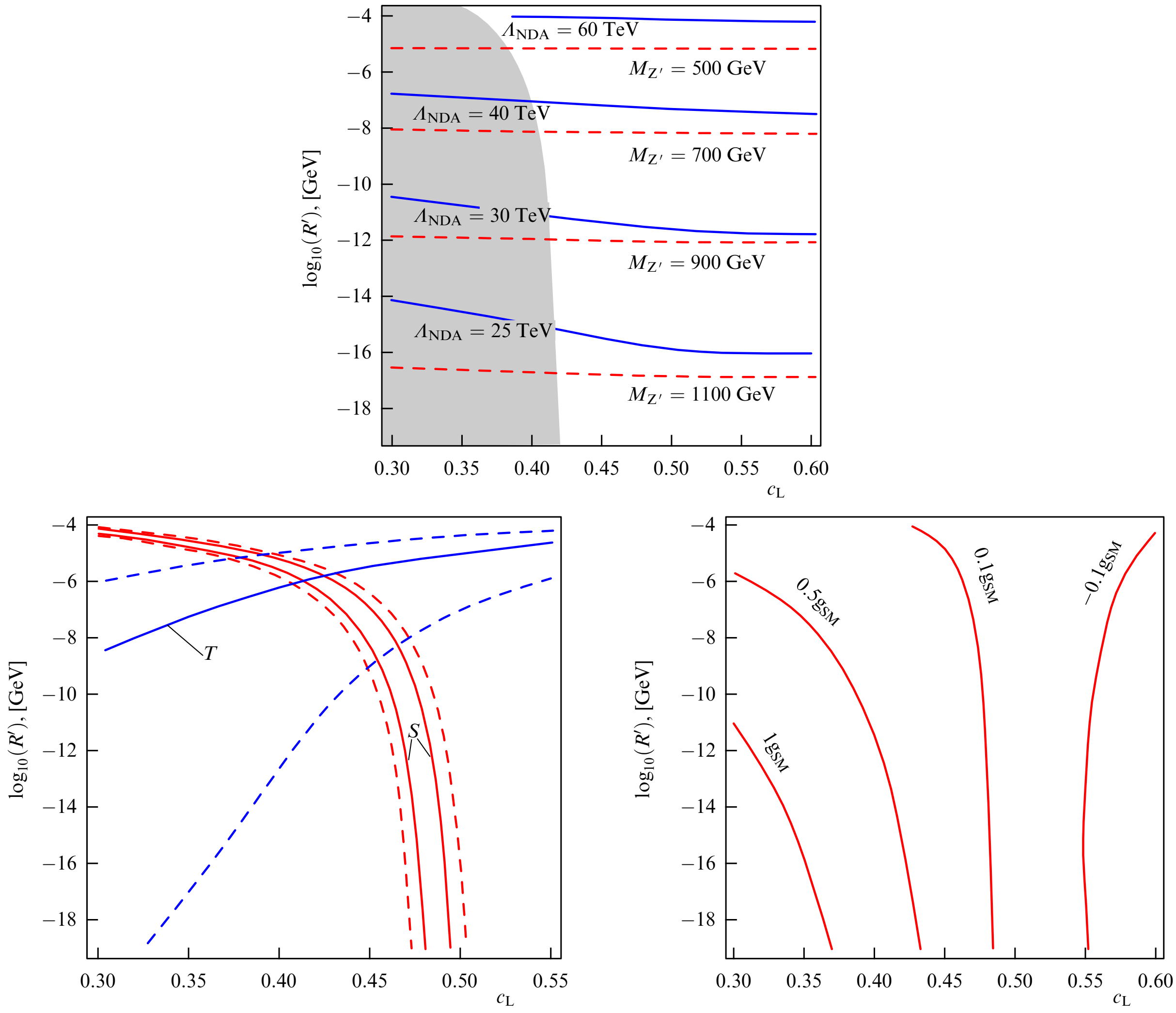


Figure 21. Top: contour plots of Λ_{NDA} (solid blue lines) and $M_{Z^{(1)}}$ (dashed red lines) in the parameter space $c_L - R$. The shaded region is excluded by direct searches of light Z′ at the LEP. Bottom left: contours of S (red), for $|S| = 0.25$ (solid) and 0.5 (dashed) and T (blue), for $|T| = 0.1$ (dotted), 0.3 (solid), and 0.5 (dashed), as functions of c_L and R are shown. Bottom right: contours for the generic suppression of fermion couplings to the first resonance with respect to the SM value can be seen. The region for c_L allowed by S is between 0.43 ± 0.5, where the couplings are suppressed by at least a factor of 10.

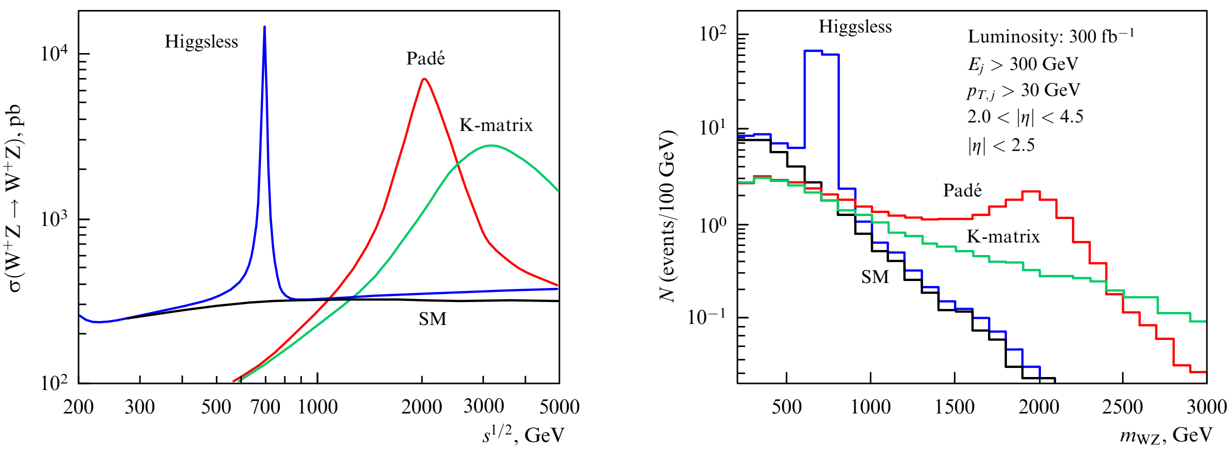


Figure 22. Left: $W_L^{\pm}Z_L \to W_L^{\pm}Z_L$ elastic cross sections in the SM (dotted), the Higgsless model with a 700 GeV W′ (blue) and two unitarization models: Padé (red) and K-matrix (green). Right: the number of events per 100 GeV bin in the $2j + 3l + \nu$ chanel at the LHC with the integrated luminosity 300 fb^{-1} and cuts as indicated in the figure (the different curves are the same as on the left). The couplings and the partial width of W′ can be predicted in a model-independent way from the unitarity sum rules: $g_{WZW'} < g_{WWZ} M_Z^2/(\sqrt{3} M_{W'} M_W)$, $\Gamma(W' \to WZ) \sim \alpha M_{W'}^3/(144 s_W^2 M_W^2)$. (From [86].)

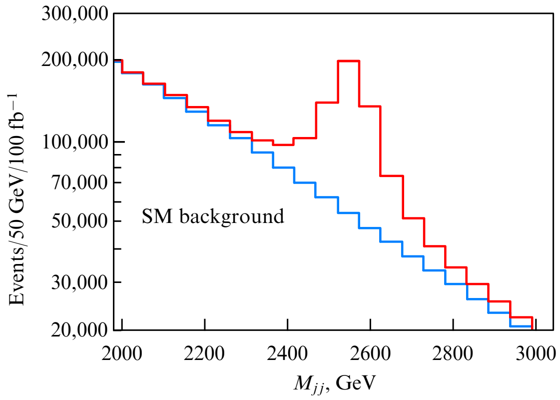


Figure 23. Dijet invariant mass spectrum at the LHC showing a prominent resonance due to the first gluon KK state. The black histogram corresponds to the SM background. (From [79].)

We also mention that KK gluons could easily show up as resonances in the dijet spectrum [83] (see Fig. 23). Again, the analysis depends on the localization of the fermions in the bulk.

Finally, another interesting prediction of Higgsless models is the presence of anomalous 4- and 3-boson couplings. Indeed, in the SM, the sum rules canceling the terms that increase with the fourth power of the energy are already satisfied by gauge invariance. To accommodate the contribution of the new states, the couplings between SM gauge bosons have to be corrected. Assuming that the sum rules are satisfied by the first resonance only, it is easy to evaluate such deviations:

$$\delta = \frac{\delta g_{\mathrm{WWZ}}}{g_{\mathrm{WWZ}}} \sim -\frac{1}{3}\,\frac{M_Z^2}{M_{Z'}^2}\,. \tag{214}$$

Here, δ is an overall shift in the coupling, and the deviation has been evaluated from the W elastic scattering sum rules. In Fig. 24, we plot the deviations in the WWWW and WWZ gauge couplings in the Higgsless model: the red lines encircle the preferred region by EWPTs. A deviation of the order of 1% to 3% is expected in the trilinear gauge couplings. This deviation is close to the present experimental limit from the LEP and might be probed at the LHC. The ILC will surely be able to measure such deviations. Here, we stress again that such deviations are a solid prediction of the Higgsless mechanism and are independent of the details of the specific Higgsless model.

6. Recent developments and conclusion

The legacy of LEP/SLC is an *impressive triumph of human endeavor*,[17] with the validation of the quantum nature of the Standard Model (SM) to its highest accuracy. Still, and despite all expectations, it leaves us with the most pressing question: How do elementary particles acquire mass? How is electroweak symmetry broken? The SM Higgs mechanism is only a description of EWSB and not an explanation of it since, in particular, there are no dynamics to explain the instability of the Higgs potential at the origin. The hierarchy problem tells us that it is less and less natural that no new particles emerge as we explore higher and higher energies. At the same time, however, electroweak precision measurements severely constrain the existence of such new particles. These constraints are nowadays so severe that the minimal supersymmetric standard model, which has for a long time been considered the paradigm of BSM physics, does not appear more natural than 1 in 100, in the absence of any anthropic selection. On the eve of setting up the LHC, this pang of conscience could have been quite discouraging. However, on the contrary, it has stimulated the creativity of BSM physicists and in the last few years numerous new ideas have emerged on both the phenomenological and the theoretical sides. Ingenious setups, sometimes with the rescue of some symmetries, succeed in hiding interesting new physics that would manifest itself at low energy by modifications of only quantities that have not been very well measured (gauge boson self-couplings, couplings of the top and bottom quarks to the W and Z bosons, etc.).

To conclude, we mention that the present overview of recent EWSB models inevitably suffers from a bias and that it

[17] R Rattazzi, talk at the International Europhysics Conference on High Energy Physics, July 21 – 27, 2005, Lisbon, Portugal.

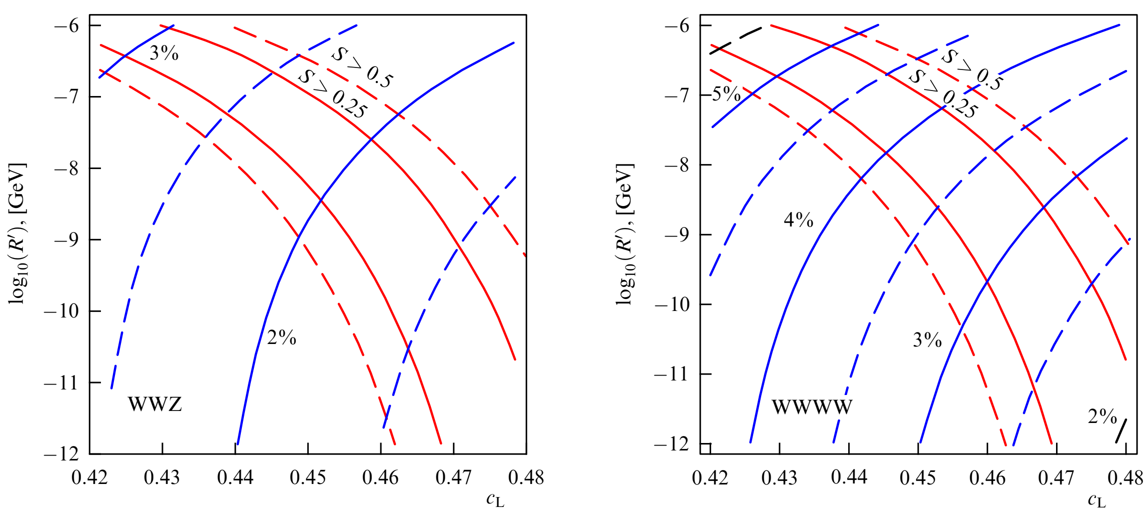


Figure 24. Deviations in the WWZ and WWWW gauge boson couplings in the Higgsless model as functions of c, the fermion localization parameter, and R, the energy scale on the UV brane. The red and blue lines are the regions preferred by S and T. (From [57].)

certainly cannot claim to be exhaustive. We have chosen a few models, mostly those on which we have worked. To repair any injustice, if it is ever possible, we must at least mention some other approaches that have appeared recently and that would definitively deserve other reviews on their own:

- fat Higgs models [87];
- gauge extensions of the minimal supersymmetric standard model [88];
- bosonic seesaw [89];
- supersymmetric Little Higgs models [90];
- twin Higgs models [91];
- 6D RG induced EWSB [92].

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